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# Are Asymmetrically Informed Agents Envious?

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# *Are Asymmetrically Informed Agents Envious?*

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**Abstract**
In most economies, a fair allocation does not exist. Thus, it seems that we are condemned to live in an unfair world, since we are not happy with what we have and we look at the others with envious eyes. In this paper we want to give an hope for a more equitable society.

**Keywords**: Asymmetric information; fair allocation; constrained market equilibrium; Maximin and Bayesian expected utility function.

**JEL classification**: D63, D82.

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# 1 Introduction

The problem of fairness has been widely investigated and applied in many contexts, such as exchange economies [11], public good economies [2], as well as marriage markets [9]. The most appealing concept of fairness is due to Foley (1967) and Varian (1974), and it comes from only personal tastes. Indeed a fair allocation is an efficient distribution of resources among agents such that *"each individual prefers to keep his bundle rather than to receive the bundle of any other"*. Everybody wants to live in a fair world in which he is happy with what he has and can coexist with the others without any kind of envy. This is possible in a pure exchange economy simply because any competitive allocation resulting from an equal sharing of the total initial endowment is fair. Unfortunately, this is an ideal world. Indeed, when production is allowed, envy freeness may be incompatible with efficiency and therefore the set of fair allocations may be empty (see [10] and [11]). The same problem arises in condition of uncertainty or when agents are asymmetrically informed (see [4] and [7]). Our goal is to provide a condition which guarantees the existence of a fair distribution of resources among asymmetrically informed agents. We show that if there is a state of nature $\bar{\omega}$ that everybody can distinguish, then there is no tension anymore between interim efficiency and absence of envy. Hence, surprisingly, the presence of a common singleton in the private information partition of each individual is enough to ensure equity in the economy. But, this is not the end of the story. Indeed, one could observe that fairness is just a property that an equilibrium may or may not satisfy. Therefore, the mere existence of a fair allocation may be negligible if no equilibrium satisfies such a property. We exhibit an example of differential information economy in which, even if the set of fair allocations is non empty, the constrained market equilibrium, which is what really agents consume, is not fair. This example reinforces the idea that in differential information economies fairness is an utopia, and since everyone could agree that all economic activities or all contracts among individuals are made under uncertainty or incomplete information, it seems that we are condemned to live in an unfair society. In this paper we address the following questions: are really asymmetrically informed agents envious? Is there anything we can do to change such an unfair situation? In what do we make mistake? We show that the solution consists in changing our attitude towards uncertainty. If we abandon the Bayesian subjective expected utility (SEU) formulation and replace it by the maximin expected utility (MEU), not only a fair distribution of resources among asymmetrically informed agents always exists, but also the related notion of competitive equilibrium is fair. In other words, we propose a new notion of equilibrium which solves the conflict between efficiency and envy freeness when the equity of allocations is evaluated in an interim stage. The idea is to expect to receive the minimum in each possible situation.

## 2 Differential information economy

A differential information economy is a $t$-uple

$$\{I; \mathbb{R}^{\ell}_{+}; (\Omega, \mathcal{F}, \pi); (e_i, \mathcal{F}_i, u_i)_{i\in I}\},$$

where $I$ is the finite set of agents; $\mathbb{R}^{\ell}_{+}$ is the commodity space; $\Omega$ is the finite set of possible states of nature, $\mathcal{F}$ is the power set of $\Omega$, i.e., the set of all the possible events and $\pi$ is the common prior describing the probability of each state[1]. Each individual $i \in I$ is characterized by an initial endowment $e_i : \Omega \to \mathbb{R}^{\ell}_{+}$, a partition $\mathcal{F}_i$ of $\Omega$ which represents his private information and an utility function $u_i : \Omega \times \mathbb{R}^{\ell}_{+} \to \mathbb{R}$ representing his preferences. We assume that for each $i \in I$ and $\omega \in \Omega$, $u_i(\omega, \cdot)$ is continuous, strongly monotone and concave. For each state $\omega$ and each agent $i$, we denote by $\mathcal{F}_i(\omega)$ the element of $\mathcal{F}_i$ containing $\omega$. As usual, we can interpret the above economy as a two time period model ($t = 1, 2$), today $t = 1$ agent $i$ only knows that the realized state belongs to the event $\mathcal{F}_i(\omega)$, where $\omega$ is the true state at $t = 2$. With this information agents trade. At the ex post stage ($t = 2$), the state will be commonly known and consumption takes place. This requirement makes irrelevant incentive and measurability constraints.
An **allocation**[2] $x$ is a function $x : I \times \Omega \to \mathbb{R}^{\ell}_{+}$, and it is said to be **feasible** if $\sum_{i\in I} x_i(\omega) = \sum_{i\in I} e_i(\omega)$ for all $\omega \in \Omega$. For any allocation $x_i : \Omega \to \mathbb{R}^{\ell}_{+}$, agent $i$'s **(Bayesian) interim expected utility** function with respect to $\mathcal{F}_i$ at $x_i$ in state $\omega$ is given by

$$v_i(x_i|\mathcal{F}_i)(\omega) = \sum_{\omega'\in\mathcal{F}_i(\omega)} u_i(\omega', x_i(\omega'))\frac{\pi(\omega')}{\pi\big(\mathcal{F}_i(\omega)\big)}. \tag{1}$$

The **maximin utility** of each agent $i$ with respect to the partition $\mathcal{F}_i$ in state $\omega$ is given by

$$\underline{u}_i(\omega, x_i) = \min_{\omega'\in\mathcal{F}_i(\omega)} u_i(\omega', x_i(\omega')). \tag{2}$$

As the reader can see, (2) corresponds to an interim notion and does not require any expected utility representation.

### 2.1 The "Bayesian" case

We now recall the definitions of efficiency and fairness in a differential information economy, in which agents' preferences are represented by the Bayesian interim expected utility (1).

A feasible allocation $y$ interim Pareto dominates an allocation $x$ if every agent, given his private information, prefers $y$ over $x$ in each state, i.e., $v_i(y_i|\mathcal{F}_i)(\omega) >$

[1] Without loss of generality, we assume that $\pi(\omega) > 0$ for each state $\omega \in \Omega$.

[2] For each agent $i$, the function $x_i : \Omega \to \mathbb{R}^{\ell}_{+}$ is said to be the allocation of agent $i$, while the vector $x_i(\omega) \in \mathbb{R}^{\ell}_{+}$ is said to be the bundle of $i$ in state $\omega$.

$v_i(x_i|\mathcal{F}_i)(\omega)$ for each $i \in I$ and each $\omega \in \Omega$. A feasible allocation $x$ is said to be **interim efficient** if it is not interim Pareto dominated by any other allocation. Let $x$ be a feasible allocation and let $\omega$ be a state of nature. Then, an agent $i$ interim envies an agent $j$ at $\omega$ if $v_i(x_j|\mathcal{F}_i)(\omega) > v_i(x_i|\mathcal{F}_i)(\omega)$. The allocation $x$ is said to be **interim envy-free** if there is zero probability of an agent interim envying another, i.e., there does not exist a state $\omega$ at which some agent $i$ interim envies some agent $j$. A feasible allocation $x$ is said to be **interim fair** if it is both interim efficient and interim envy-free.

It is easy to show that the allocation which equally shares the aggregate endowment in each state is interim envy-free. Clearly, also the set of interim efficient allocation is non empty, but it is known that a fair allocation may not exist (see Example 1 in [4]). Although, De Clippel (2008) shows that interim efficiency is compatible with a weaker[3] notion of envy freeness: *there exists an interim Pareto optimal allocation such that it is impossible to find two agents $i$ and $j$ for which it is common knowledge[4] that $i$ interim envies $j$ (see Proposition 1 in [4]).* This result is obtained by using the notion of constrained market equilibrium introduced by Wilson (1978) which we recall below.

A pair $(p, x)$, where $p : \Omega \to \mathbb{R}^\ell_+$ is a non-zero function and $x$ is a feasible allocation, is said to be a **constrained market equilibrium** if for each $i \in I$ and $\omega \in \Omega$, $x_i$ maximizes $v_i(\cdot|\mathcal{F}_i)(\omega)$ subject to $\sum_{\omega' \in \mathcal{F}_i(\omega)} p(\omega') \cdot x_i(\omega') \leq \sum_{\omega' \in \mathcal{F}_i(\omega)} p(\omega') \cdot e_i(\omega')$.

A constrained market equilibrium exists and it is interim efficient, but it may not be interim envy free. For this reason, De Clippel evaluates envy freeness only in common knowledge events. However, this is not enough, since it might be the case that even if there is no envy in a commonly known event, agent $i$ could envy $j$ in a different event which is not "commonly known", and therefore there may still be envy.

## 2.2 A sufficient condition

Our goal is to provide a condition which guarantees the existence of a fair distribution of resources among asymmetrically informed agents. We show below that if there is a state of nature $\bar{\omega}$ that everybody can distinguish, i.e.,

$$(*) \qquad \{\bar{\omega}\} \in \mathcal{F}_i \quad \text{for all } i \in I,$$

then there is no tension anymore between interim efficiency and absence of envy in each event and not only in the "common knowledge"ones. We prove it by using

[3] A different approach is taken in [1] in a more general context.

[4] An event $E$ is said to be common knowledge if it can be written as a union of elements of $\mathcal{F}_i$ for each $i \in I$; i.e., $E \in \bigwedge_{i \in I} \mathcal{F}_i$.

the notion of competitive equilibrium[5]. Notice that in our context this notion is just technical, since in differential information economies agents do not consume competitive equilibrium allocations, but constrained market equilibrium allocations.

**Theorem 2.1** *Assume that condition* $(*)$ *holds, then there exists a fair allocation.*

**Proof:** First, notice that any competitive equilibrium $(p, x)$ resulting from an equal sharing of the initial endowments $\Big($i.e., $e_i(\omega) = \frac{1}{|I|}\sum_{i\in I} e_i(\omega)$ for each $i \in I$ and each $\omega \in \Omega\Big)$, is interim envy free. Indeed, assume on the contrary, that there exists a state $\omega$ and two agent $i$ and $j$ such that $v_i(x_j|\mathcal{F}_i)(\omega) > v_i(x_i|\mathcal{F}_i)(\omega)$. From (1) it follows that that there exists a state $\omega' \in \mathcal{F}_i(\omega)$ such that $u_i(\omega', x_j(\omega')) > u_i(\omega', x_i(\omega'))$, and hence $p(\omega') \cdot x_j(\omega') > p(\omega') \cdot e_i(\omega') = p(\omega') \cdot e_j(\omega')$. This is a contradiction, since $p(\omega) \cdot x_j(\omega) \leq p(\omega) \cdot e_j(\omega)$ in each state $\omega \in \Omega$. Moreover, notice that any equal income competitive equilibrium is interim efficient. Assume, by the way of contradiction, that there exists a feasible allocation $y$ such that $v_i(y_i|\mathcal{F}_i)(\omega) > v_i(x_i|\mathcal{F}_i)(\omega)$ for each $i \in I$ and each $\omega \in \Omega$. In particular, it holds in the state $\bar{\omega}$ of condition $(*)$, in which the above inequality reduces to be $u_i(\bar{\omega}, y_i(\bar{\omega})) > u_i(\bar{\omega}, x_i(\bar{\omega}))$ for all $i \in I$. Therefore, $p(\bar{\omega}) \cdot y_i(\bar{\omega}) > p(\bar{\omega}) \cdot e_i(\bar{\omega})$ for all $i \in I$, and hence $p(\bar{\omega}) \cdot \sum_{i\in I} y_i(\bar{\omega}) > p(\bar{\omega}) \cdot \sum_{i\in I} e_i(\bar{\omega})$, which contradicts the feasibility of the allocation $y$. Thus, any equal income competitive equilibrium allocation is fair, and from non emptiness of the set of competitive equilibrium we can deduce the existence of a fair allocation.
□

Therefore, surprisingly, the presence of a common singleton in the private information partition of each individual (see $(*)$) is enough to ensure equity in the economy. Although, this is not the end of the story. Indeed, one could observe that fairness is just a property that an equilibrium may or may not satisfy. Therefore, the mere existence of a fair allocation may be negligible if no equilibrium satisfies such a property. We now exhibit an example of differential information economy in which, even if the set of fair allocations is non empty, the constrained market equilibrium, which is what really agents consume, is not fair.

**Example 2.2** Consider an economy with three agents, three equiprobable states of nature $\{\omega_1, \omega_2, \omega_3\}$ and one good (money). The total endowment is equally shared among agents and it is 1200 dollars when the state is $\omega_1$; it is 1800 dollars when the

[5] A pair $(p, x)$, where $p : \Omega \to I\!R^{\ell}_{+}$ is a non-zero function and $x$ is a feasible allocation, is said to be a **competitive equilibrium** if for each $i \in I$ and $\omega \in \Omega$, $x_i(\omega)$ maximizes $u_i(\omega, \cdot)$ subject to $p(\omega) \cdot x_i(\omega) \leq p(\omega) \cdot e_i(\omega)$. It is well known that under standard assumptions, a competitive equilibrium exists.

state is $\omega_2$, and some positive quantity $c > 0$ if the state is $\omega_3$. Agent 3 knows the future state. Agents 1 and 2 do not. Agent 1 is risk neutral while agent 2 is risk averse. Formally, $\pi(\omega_1) = \pi(\omega_2) = \pi(\omega_3) = \frac{1}{3}$, and

$$\begin{aligned} u_1(\cdot, x_1) &= x_1; & \mathcal{F}_1 &= \{\{\omega_1, \omega_2\}; \{\omega_3\}\}; \\ u_2(\cdot, x_2) &= \sqrt{x_2}; & \mathcal{F}_2 &= \{\{\omega_1, \omega_2\}; \{\omega_3\}\}; \\ u_3(\cdot, x_3) &= x_3; & \mathcal{F}_3 &= \{\{\omega_1\}; \{\omega_2\}; \{\omega_3\}\}. \end{aligned}$$

The unique constrained market equilibrium allocation is $(300, 500, 400)$ in state $\omega_1$; $(700, 500, 600)$ in state $\omega_2$ and $(c/3, c/3, c/3)$ is state $\omega_3$. Even if this allocation is interim efficient, it is not interim envy free, since the third guy envies the bundle of the first agent in state $\omega_2$, i.e., $u_3(\omega_2, x_1(\omega_2)) = 700 > 600 = u_3(\omega_2, x_3(\omega_2))$. Notice that condition $(*)$ is satisfied, and according to Theorem 0.1 the set of fair allocations is not empty. Indeed, the unique fair allocation shares equally the aggregate endowment, i.e., $(400, 400, 400)$ in $\omega_1$; $(600, 600, 600)$ in $\omega_2$ and $(c/3, c/3, c/3)$ in $\omega_3$.

This means that although there exists an interim fair allocation, the unique constrained market equilibrium, which is what really agents consume, is not fair.

This example reinforces the idea that in differential information economies fairness is an utopia, and since everyone could agree that all economic activities or all contracts among individuals are made under uncertainty or incomplete information, it seems that we are condemned to live in an unfair society.

## 2.3 The "Maximin" case

In this section we address the following questions: are really asymmetrically informed agents envious? Is there anything we can do to change such an unfair situation? In what do we make mistake?

We show that the solution consists in changing our attitude towards uncertainty. If we abandon the Bayesian subjective expected utility (SEU) formulation and replace it by the maximin expected utility (MEU), not only a fair distribution of resources among asymmetrically informed agents always exists, but also the related notion of competitive equilibrium is fair. In other words, we consider a differential information economy in which agents' preferences are represented by the maximin utility (see 2) and we propose a new notion of equilibrium which solves the conflict between efficiency and envy freeness when the equity of allocations is evaluated in an interim stage. Below we introduce the notion of maximin fairness.

A feasible allocation $y$ maximin Pareto dominates an allocation $x$ if every agent prefers (with MEU formulation) $y$ over $x$ in each state, i.e., $\underline{u}_i(\omega, y_i) > \underline{u}_i(\omega, x_i)$ for each $i \in I$ and each $\omega \in \Omega$. A feasible allocation $x$ is said to be **maximin efficient** if it is not maximin Pareto dominated by any other allocation. Let $x$ be a feasible allocation and let $\omega$ be a state of nature. Then, agent $i$ maximin envies $j$ at $\omega$, if $\underline{u}_i(\omega, x_j) > \underline{u}_i(\omega, x_i)$. The allocation $x$ is said to be **maximin envy-free** if there does not exist a state $\omega$ at which some agent $i$ maximin envies some agent $j$. A feasible allocation $x$ is **maximin fair** if it is both maximin efficient and maximin envy-free.

In order to prove the existence of a maximin fair allocation, the following notion of equilibrium is needed (see [3]).

A pair $(p, x)$, where $p : \Omega \to I\!R^{\ell}_{+}$ is a non-zero function and $x$ is a feasible allocation is said to be a **maximin competitive equilibrium (MCE)** if for each agent $i \in I$ and state $\omega \in \Omega$, $x_i$ maximizes $\underline{u}_i(\omega, \cdot)$ subject to $p(\omega') \cdot y_i(\omega') \leq p(\omega') \cdot e_i(\omega')$ for all $\omega' \in \mathcal{F}_i(\omega)$. A feasible allocation $x$ is said to be a **maximin competitive equilibrium allocation** if there exists a price vector $p : \Omega \to I\!R^{\ell}_{+}$ such that $(p, x)$ is a maximin competitive equilibrium. It has been proved in [3] that a maximin competitive equilibrium exists. We now show that any maximin competitive equilibrium allocation is fair and hence we deduce the existence of a maximin fair allocation.

**Proposition 2.3** *If $(*)$ holds, any maximin competitive equilibrium allocation is maximin Pareto optimal.*

**Proof:** Let $(p, x)$ be a maximin competitive equilibrium and assume, on the contrary, that there exists a feasible allocation $y$ such that $\underline{u}_i(\omega, y_i) > \underline{u}_i(\omega, x_i)$ for all $i \in I$ and $\omega \in \Omega$. In particular, the above inequality holds in the state $\bar{\omega}$ of condition $(*)$. This implies that $p(\bar{\omega}) \cdot y_i(\bar{\omega}) > p(\bar{\omega}) \cdot e_i(\bar{\omega})$ for all $i \in I$, and hence $p(\bar{\omega}) \cdot \sum_{i \in i} y_i(\bar{\omega}) > p(\bar{\omega}) \cdot \sum_{i \in I} e_i(\bar{\omega})$, which clearly contradicts the feasibility of $y$. $\square$

We show below that any maximin competitive equilibrium with equal income is also maximin envy free, and hence maximin fair.

**Proposition 2.4** *If $(*)$ holds, any maximin competitive equilibrium allocation with equal income is maximin fair. The converse may not be true.*

**Proof:** Let $(p, x)$ be a maximin competitive equilibrium with equal income and assume on the contrary that $x$ is not maximin fair. Since any maximin competitive equilibrium allocation is maximin efficient (see Proposition 0.3), $x$ is not maximin envy free. Therefore, there exist two agents $i$ and $j$ and a state $\omega$ such that $\underline{u}_i(\omega, x_j) > \underline{u}_i(\omega, x_i)$. This implies that $x_j \notin \{y_i : p(\omega') \cdot y_i(\omega') \leq p(\omega') \cdot e_i(\omega')$ for all $\omega' \in \mathcal{F}_i(\omega)\}$, that is, there exists $\omega' \in \mathcal{F}_i(\omega)$ such that $p(\omega') \cdot x_j(\omega') > p(\omega') \cdot e_i(\omega') = p(\omega') \cdot e_j(\omega')$, which is a contradiction.
Hence, the set of equal income maximin competitive equilibrium allocations is contained in the set of maximin fair allocations. Actually, this inclusion is strict. Indeed, consider the economy described in Example 0.2, and notice that the unique maximin competitive equilibrium allocation, which assigns $(400, 400, 400)$ in state $a$; $(600, 600, 600)$ in state $b$ and $(c/3, c/3, c/3)$ in state $c$, is maximin fair. On the other hand the allocation which assigns $(400, 400, 400)$ in state $a$, $(400, 400, 1000)$ in state $b$ and $(c/3, c/3, c/3)$ in state $c$ is maximin fair, but it is clearly not a maximin competitive equilibrium allocation. $\square$

Since, under standard assumptions, a maximin competitive equilibrium exists (see [3]), the above Proposition also guarantees that the set of maximin fair allocations is non empty.

# 3 Conclusions and open questions

We have provided a condition which guarantees the existence of a (Bayesian) fair allocation and shown that this is not enough to have equity in the society, since the constrained market equilibrium, which is what really agents consume, may not be fair. On the other hand, we have observed that in a differential information economy, in which agents preferences are given by maximin utility, not only a maximin fair allocation exists, but more important the maximin competitive equilibrium is fair. However, equal income maximin competitive equilibrium are not the only efficient and equitable allocations. This suggests us that in order to get a characterization of equilibria in terms of fairness, coalitional notions of fairness are needed. We have already proved such an equivalence in [5] and [8] with two different approaches. In the above papers, we consider only Bayesian expected utility. I guess that the same holds true with maximin formulation and a continuum of agents.

Another interesting point is the following: we have assumed that ex post the realized state of nature is commonly known. This makes irrelevant incentive and measurability constraints. What does it happen if ex post each agent can only verify that the realized state of nature belongs to a certain event of his private partition? Typically, we require allocations to be $\mathcal{F}_i$-measurable. Does this measurability assumption create any problems? Since agents are asymmetrically informed, can agent $i$ compare his allocation $x_i$ with $j$'s allocation $x_j$, which may not be $\mathcal{F}_i$-measurable? Therefore, what is the correct definition of fair allocation in this context? We address all these questions in [1].

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