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Managerial Compensations and Information Sharing under Moral Hazard: Is Transparency Good?

Salvatore Piccolo and Emanuele Tarantino

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University of Naples Federico II   University of Salerno   Bocconi University, Milan

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS – UNIVERSITY OF NAPLES
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unisa.it

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Abstract
We study the effects of information sharing on optimal contracting in a vertical hierarchies model with moral hazard and effort externalities. The paper has three main objectives. First, we determine and compare the equilibrium contracts with and without communication. We identify how each principal relates her agent's wage to the opponent's performance when they share information about agents' performances. It turns out that the type of effort externalities across organizations is the main determinant of the responsiveness of each agent's reward to the opponent's performance. Second, in order to throw novel light on the emergence of information sharing agreements, we characterize the equilibria of a non- cooperative game where principals first decide whether to share information and then offer contracts to their exclusive agents. We explore the implications of introducing certification costs and show that three types of equilibria may emerge depending on the nature and (relative) strength of effort externalities: principals bilaterally share information if agents' effort choices exhibit strong complementarity; only the principal with stronger monitoring power discloses information in equilibrium for intermediate levels of effort's complementarity; principals do not share information if efforts are substitutes and for low values of effort's complementarity. Moreover, differently from the common agency framework studied in Maier and Ottaviani (2009), in our model a prisoner's dilemma may occur when efforts are substitutes and certification costs are negligible: if a higher effort by one agent reduces the opponent's marginal productivity of effort the equilibrium involves no communication although principals would jointly be better off by sharing information. Finally, the model also offers novel testable predictions on the impact of competition on the basic trade-off between risk and incentives, the effects of organizations' asymmetries on information disclosure policies as well as on the link between corporate control and the power of incentives.

Keywords: Competing Hierarchies, Information Sharing, Moral Hazard

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* Università Cattolica del Sacro Cuore and CSEF. E-mail: piccolo@gmail.com.

** Università di Bologna, Dipartimento di Economia and TILEC. E-mail emanuele.tarantino@unibo.it.

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1 Introduction

The basic trade-off between risk and incentives, which shapes the way managerial compensations are optimally designed under moral-hazard, has been extensively studied in economics, finance and management. Stemming from Holmström (1979), this literature has been developed in various directions. For instance, by combining the stock price formation process with optimal contracting (e.g., Holmström and Tirole, 1993; Bolton et al., 2006), by studying the effects of product market competition on managerial slack (e.g., Hart, 1983; Schmidt, 1997; Raith, 2003; Piccolo et. al, 2008) or by emphasizing the relationship between authority and monetary incentives within complex organizations (see Bolton and Dewatripont, 2005, for a recent survey). Yet, so far, only very few papers have explored the link existing between the optimal design of managerial compensations and organizations' incentives to share information — e.g., Calzolari and Pavan (2006), Maier and Ottaviani (2009) and Piccolo (2011).

Over the last years, the emergence of trade associations and strategic alliances, together with the diffusion of information-intensive channels, has promoted the dissemination and exchange of information among firms in many industries — see, e.g., Jappelli and Pagano (2001), Briley et al. (1994) and Anderson et al. (2001). Disclosing private information and communicating interactively with cross-boundary partners has become a strategic management activity. In several cases, this involves channel partners who invest in bundles of sophisticated information technology[1] not only to disseminate information within a given organization, but also across its borders. Indeed, while traditional disclosure outlets through which firms convey information about their market performance, earnings and financial structure involve annual reports, special purpose reports and press release, in recent years corporate websites have become one of the main channels through which firms communicate — see, e.g., Aerts et. al (2004).

Information sharing agreements are usually seen as institutions that enhance efficiency thanks to learning and imitation, but it is also reasonable to believe that the information disclosed through these systems might allow firm owners to better discipline their management and, at the same time, to influence the strategic forces shaping the competitive arena. What is the effect of communication on the basic trade-off between risk and incentives in a model with competing vertical organizations? What are the costs and benefits of transparency when managerial activity generates cross-firm externalities? How does competition impact on the power of incentives under these agreements? Why do some firms disclose their market performance, while others prefer not to communicate such information?

We address these issues in a stylized vertical hierarchies set-up where two independent principals (owners or shareholders) deal with a pair of exclusive agents (managers) under the threat of moral hazard.[2] Managers' actions are unverifiable and contracts can depend only on imperfect measures of their efforts, performances are independently distributed and contracts are secret. The paper has three main objectives. First, we determine and compare equilibrium contracts with and without communication. When principals

[1] Such as telecommunication and satellite linkages, bar coding and electronic scanning systems, database management systems, etc.

[2] Exclusive deals are realistic in many other employment relationships because of labor natural indivisibility, human capital accumulation, specialization costs etc — see, e.g., Caillaud et al., (1995).

share information about agents' performances, we identify how each principal relates the own agent's wage to the opponent's performance. It turns out that the type of effort externalities across organizations is the main determinant of the responsiveness of each agent's reward to the opponent's performance. In particular, when managers' efforts are substitutes, it is less convenient for a principal to induce high effort relative to the single-hierarchy (monopoly) benchmark: in this case the marginal productivity of each agent's effort decreases with the opponent's effort. By contrast, when efforts have a complementary nature, there are positive externalities to be exploited across the two organizations. Hence, the equilibrium bonus exceeds the monopoly level.

Second, in order to throw novel light on the emergence of information sharing agreements and voluntary disclosure policies, we characterize the equilibria of a non-cooperative game where principals first decide whether to share information and then offer contracts to their exclusive agents. We explore the implications of introducing fixed certification costs and show that, depending on the nature and the (relative) strength of effort externalities, three types of equilibria may emerge: principals bilaterally share information if agents' effort choices exhibit strong complementarity; only the principal with stronger monitoring power discloses information in equilibrium for intermediate levels of effort's complementarity; principals do not share information if efforts are substitutes and for low values of effort's complementarity.

Interestingly, the existence of an equilibrium where only firms with stronger monitoring power disclose information is consistent with the recent empirical evidence. Many firms disclose voluntarily information beyond levels mandated by financial and accounting regulations. Still, other firms seem to disclose as little information as they can. For instance, using a sample of 152 Fortune 500 retailers' websites, Bodkin and Perry (2004) found that the more profitable companies were likely to use web-based communication systems. Hence, our model partly contributes to explain this evidence.

Moreover, the equilibrium characterization derived in the paper also shows that, differently from the common agency framework studied in Maier and Ottaviani (2009), with exclusive deals a prisoner's dilemma may occur when efforts are substitutes and certification costs are negligible: principals always benefit from sharing information, but in equilibrium they do not communicate if a higher effort by one agent reduces the opponent's marginal productivity of effort — i.e., when efforts are substitutes. Moreover, principals' gain from communication is decreasing with respect to own monitoring power and increasing with the opponent's one.

Third, the model offers a number of testable predictions on the impact of competition on the basic trade-off between risk and incentives, the effects of organization asymmetries on information disclosure policies as well as on the link between corporate control and the power of incentives. More precisely, we show that the type of interaction between managerial efforts contributes to explain the complex relationship between risk and incentives. In line with the evidence collected by the empirical literature investigating the validity of the basic agency model under moral hazard — see Prendergast (2002) for a survey — also in our model the prediction is ambiguous, and depends (among other things) on the interplay between managers' risk-aversion and the nature of firms' strategic interaction. In addition, we also derive predictions on the link between the strength of corporate control in one firm, which is measured in our set-up by the volatility of its manager's performance, and the bonus offered to the opponent manager. It turns out that if efforts are substitutes, an increase in one principal contractual power vis-à-vis her agent makes the opponent principal more willing to offer high-powered incentives, while the converse obtains with complementary efforts.

Finally, in an extension, we also characterize the equilibrium contracts and information sharing decisions under the hypothesis of correlation among performances. It turns out that, when the effort interaction term is negligible, the nature of correlation — i.e., negative or positive —- is a key determinant of the responsiveness of each agent's reward to the opponent's performance. Essentially, principals seek to design contracts in such a way to exploit correlation to reduce agents' risk-premium. Therefore, when performances are positively (resp. negatively) correlated, risk-diversification requires a principal to punish (resp. reward) her agent when the rival agent performs well. Moreover, we show that when certification costs are positive there is a unique inefficient equilibrium where principals do not share information. If certification costs are negligible, multiple communication equilibria may emerge, and the one with bilateral information sharing is the most efficient one. Besides providing additional insights about the way contract design and information sharing decisions interplay with vertical hierarchies, this also suggests that the insights obtained in the baseline model with independent performances carry over to a more general context where effort externalities and correlation among performances are both at play.

Summing up, the paper offers a theory of information sharing among vertical organizations under moral hazard. The model delivers a number of predictions not only on the internal organization of firms but also on the process of interaction and communication among independent organizations. These results provide ready to use guidelines for interpreting and designing future empirical investigations. Although we will develop the formal arguments in an abstract framework and focus on its implications for executive compensations, the scope of our conclusions has a wider scope and applies to any model involving horizontal externalities among agents dealing with exclusive principals, be it procurement contracting, manufacturers/retailers deals, patent licensing, insurance and credit relationships.

The rest of the paper is organized as follows. Section 2 relates our paper to the existing literature. Section 3 sets-up the model. Section 4 provides the equilibrium characterization. Section 5 presents the equilibrium analysis of the information sharing game. Section 6 extends the model to the case of correlated performances. Section 7 concludes. Proofs are in the Appendix.

2 Related literature

Stemming from Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), and Jappelli and Pagano (1993), numerous contributions have studied the costs and benefits of transparency in oligopolistic[3] and credit[4] markets. While in the banking literature information sharing about borrowers usually promotes

[3] See, e.g., Gal-Or (1985), Shapiro (1986), Raith (1996), Khun and Vives (1995), among others.

[4] See, e.g., Padilla and Pagano (1997) and (2000), Manove et al. (2001), Carlin and Rob (2009), Bennardo et al. (2011) among many others.

efficiency by allowing lenders to better screen investment projects (adverse selection) or to avoid the danger of opportunistic behavior by borrowers (moral hazard), in oligopolistic contexts communication may help firms to overcome coordination problems and, hence, facilitate implicit collusion. The beneficial role of experts, which acquire and disclose information to trading partners, has been studied in the intermediation literature — see, e.g., Lizzeri (1999) and Gromb and Martimort (2007). The related work on consumers' privacy considered, instead, environments where sellers can use information on individual purchasing history to engage in product customization and price discrimination — see, e.g., Acquisti and Varian (2005), Dodds (2003) and Taylor (2004). More recently, the role of communication has been studied in the 'networks' literature, where people in the same network exchange private information — see, e.g., Calvó et al. (2009) and Hagenbach and Koessler (2010).

These papers offer numerous insights on the emergence of information sharing agreements, but they all focus on the traditional case where *communicators* are *black-boxes* and are thus silent on the interplay between information exchange and agency conflicts *within* and *across* organizations. The literature linking internal organization issues with the complex question of communication between firms has started recently and is still in its infancy. Following the adverse selection tradition, Calzolari and Pavan (2006) consider a sequential common agency game where principals contracting with the same agent learn through costly contracting and then share the elicited information with rivals. In these games, players acquire private information via the contracting interaction with common parties, and create new private information by taking decisions that affect both rivals and contractual counterparts. Following this approach, Piccolo (2011) studies a vertical hierarchies model where competing principals learn their (exclusive) agents' costs through direct revelation mechanisms and then eventually share this information. In the moral hazard paradigm, Maier and Ottaviani (2009) study the costs and benefits of transparency in a common agency game where principals dealing with a common agent commit to share information about his performance in order to relax incentive constraints.[5]

Our paper complements this literature by studying the novel case where deals are exclusive, exactly as in Piccolo (2011), but the agency problem is one of moral hazard, as in Maier and Ottaviani (2009). The main differences between our model and those analyzed in Calzolari and Pavan (2006) and Piccolo (2011) are the following: first, we focus on moral hazard rather than adverse selection, second and perhaps more importantly, in our model agents are risk averse, whereas they both focus on the case of risk neutrality. As for the moral hazard literature, the major difference with Maier and Ottaviani (2009) lies in the contractual structure and the role of certification costs: they analyze an intrinsic common agency model, we focus on exclusive deals and show that, in these games, principals' equilibrium communication behavior might be asymmetric depending on their different degree of monitoring power. When certification costs are negligible, we also show that, in contrast to what happens in Maier and Ottaviani (2009) common agency game, inefficient equilibrium outcomes involving no communication may emerge.

[5] Bennardo et. al (2011) also study information sharing in a moral hazard context. However, in their multi-banking set-up principals share information about contractual rules and not about performance. In this respect our model has a different flavour.

3 The model

Players. There are two principals (owners or shareholders), P_1 and P_2, and two exclusive agents (managers), A_1 and A_2. Shareholders own all productive assets, but have no expertise to manage them. Hence, firms must be run by self-interested managers. Principal P_i's gross profit (y_i) is jointly determined by the agents' efforts (a_i, $i = 1, 2$) and by an additive random component (ε_i), namely:

$$y_i = a_i + \delta a_i a_j + \varepsilon_i, \quad \forall i, j = 1, 2, \quad i \neq j, \tag{1}$$

where $\varepsilon_i \sim N(0, \sigma_i^2)$, with $\sigma_i^2 \geq 0$ for each $i = 1, 2$. As in Maier and Ottaviani (2009), we assume that the random variables ε_1 and ε_2 are independent so as to isolate the pure effects of competition from those due to correlation.[6] Effort is unobservable and unverifiable. Hence, P_i cannot write a contract contingent on A_i's effort (a_i), but only on his performance (y_i). The variance component σ_i^2 reflects the strength of P_i's corporate control: a lower (resp. higher) σ_i^2 means that y_i is more (resp. less) informative about A_i's effort a_i. Or, alternatively, a low σ_i^2 implies a strong monitoring power of P_i vis-à-vis A_i.

The parameter $\delta \in \mathbb{R}$ measures the type of (strategic) interaction between the agents' efforts: $\delta < 0$ (resp. > 0) means that efforts create negative (resp. positive) externalities across the two organizations. Both cases have clear applications in practice. Negative externalities capture those instances in which firms compete on some dimensions. In the product market example, A_i's effort allows firm i to produce a more appealing product, which in turn makes A_j's effort less productive. By contrast, positive externalities capture those situations in which managers' conduct generates complementarities across firms — e.g., basic R&D investments, informative advertising campaigns.

Communication. We analyze a two stage game (hereafter $\mathcal{G}$) where principals first decide simultaneously and independently whether to share information. Once made, these first-stage decisions become observable to all players. Then, principals offer contracts, agents choose effort, performances realize and payments are made.

There are three communication regimes (subgames following the first-stage disclosure decisions) to be analyzed. The regime *without communication*: there is no exchange of information and each principal rewards own agent only on the basis of his own performance. The regime with *bilateral information sharing*: both principals commit to pool the information about their agents' performances — see, e.g., Maier and Ottaviani (2009). In this case, each agent A_i can be rewarded not only on the basis of his own performance (y_i), but also according to his opponent's performance (y_j). The regime with *unilateral information sharing* where only one principal commits to share information.

Finally, we assume that disclosing private information is costly: each principal who commits to share information must invest an amount $F \geq 0$. One can think of F, hereafter certification costs, as including all those legal and technological expenses necessary to provide timely and reliable information.

[6] Bolton and Dewatripoint (2005, Ch. 4) explain why correlation may facilitate the emergence of an information sharing agreement in these games. In an extension we show how the introduction of correlation affects our analysis.

Contracts. We restrict attention to the class of linear contracts — see, e.g., Holmström and Milgrom (1987).[7] Principals make take-it-or-leave-it offers to their exclusive agents. Contracts depend on the outcome of the first-stage information sharing game.

- If P_j commits not to share information, P_i's contract offer to A_i entails a wage which is a linear function of the realized profit y_i — i.e.,

$$w_i(y_i) = \alpha_i + \beta_i y_i, \quad \forall i = 1, 2. \tag{2}$$

Where α_i is the fixed wage and β_i is the (linear) incentive component (bonus).

- If P_j commits to share information, P_i's contract offer to A_i entails a wage which is a linear function of both y_i and y_j — i.e.,

$$w_i(y_i, y_j) = \alpha_i + \beta_i y_i + \gamma_i y_j, \quad \forall i, j = 1, 2, \quad i \neq j. \tag{3}$$

The parameter γ_i measures how A_i's incentive scheme reacts to A_j's performance; clearly, for $\gamma_i = 0$ the schemes in (2) and (3) are equivalent.
Contracts are secret. Hence, they have no strategic value.[8]

Timing. The timing of the game is as follows:

- ($T = 0$) Principals independently and simultaneously decide whether to share information.
- ($T = 1$) The information sharing decisions become common knowledge to all players and accordingly principals (simultaneously) offer contracts.
- ($T = 2$) Agents choose efforts, profits realize and principals share information if they committed to do so. Finally, payments are made.

Each player's outside option is normalized to zero with no loss of insights. For obvious reasons we rule out the possibility of side transfers across players belonging to different organizations.

Equilibrium concept. The equilibrium concept is Perfect Bayesian equilibrium (PBE) with the added *passive beliefs* refinement — i.e., when an agent is offered a contract different from the one he expects in equilibrium, he does not revise his beliefs about the contract offered to the other agent — see Katz (1991) among others.

[7] We will make assumptions on the agents' certainty utility function such that this restriction is with no loss of generality when contracts are secret.

[8] The commitment value of observable contracts have been extensively analyzed in the traditional IO literature — see, e.g., Fershtman and Judd (1987) among many others. We will abstract from this issue by assuming that contracts are secret.

Assumptions. Principals are risk neutral: P_i maximizes expected (gross) profits net of the linear wage — i.e.,

$$\mathbb{E}[y_i - w_i\,(.)\,], \quad \forall i = 1, 2.$$

Agents are risk averse with CARA preferences and additively separable effort cost. Hence, agent A_i's certainty utility is:

$$u_i(w_i, a_i) = 1 - e^{-r(w_i - \psi(a_i))}, \quad \forall i, j = 1, 2, \quad i \neq j,$$

where the function $\psi\,(a_i)$ measures the effort cost (disutility) in monetary terms. The parameter $r > 0$ indicates the absolute risk-aversion index which, for simplicity, is common to both agents. These hypotheses are usually justified with the argument that shareholders can better diversify risks relative to their employees.

We shall assume that the effort cost is quadratic $\psi\,(a_i) = a_i^2/2$. Finally, to keep the analysis tractable, it will be convenient to derive our results for δ small, so that expected profits will be computed through Taylor expansions. This will allow us to identify the first-order effects of information sharing and neglect the second-order ones.

The single-hierarchy (monopoly) benchmark. Before turning to the equilibrium analysis, recall that in the standard case of no competition ($\delta = 0$), the agent's effort choice satisfies the first-order condition $a_i = \beta_i$, and the optimal contract entails:

$$\beta_i^* = a_i^* = \frac{1}{1 + r\sigma_i^2}, \quad \forall i = 1, 2. \tag{4}$$

Hence, both a higher volatility (σ_i^2) and a higher risk aversion index (r) induce P_i to offer a low-powered incentive scheme. This is because more uncertainty makes the realized profit y_i a worse indicator of A_i's effort and greater risk-aversion commands a larger risk-premium for the agent — see, e.g., Bolton and Dewatripont (2005, Ch. 4) and Laffont and Martimort (2002, Ch. 4). In the next section we will study how the introduction of the effort interaction term δ shapes the equilibrium contracts with and without information sharing.

4 Equilibrium contracts

In this section we characterize the equilibrium contracts in every possible subgame following the first-stage information sharing decisions. Namely, the regime without information sharing where principals do not communicate, the bilateral information sharing regime where both principals commit to share information and the unilateral information sharing regime where only one principal shares information.

4.1 No information sharing

Consider first the regime where principals do not share information. Note that, given the CARA hypothesis, it is convenient to carry out the analysis in terms of the certainty equivalent agents obtain upon choosing a given level of effort. Using the wage function in (2) together with the performance structure in (1), we have that:

$$w_i(y_i) = \alpha_i + \beta_i a_i + \delta\beta_i a_i a_j + \beta_i \varepsilon_i, \quad \forall i,j=1,2, \quad i \neq j.$$

Hence,

$$CE_i^n = \alpha_i + \beta_i a_i(1+\delta a_j) - \tfrac{a_i^2}{2} - \tfrac{r}{2}\sigma_i^2\beta_i^2. \tag{5}$$

The last term in the equation above is the risk premium required by A_i for the uncertainty borne.

Each principal chooses the compensation scheme so as to maximize (net) expected profits subject to the participation and incentive compatibility constraints. Formally, principal P_i solves:

$$\mathcal{P}_i^n \qquad \begin{cases} \max\limits_{(\alpha_i,\beta_i)} \mathbb{E}_{y_i}[y_i - w_i(y_i)] \\ \text{s.t.} \\ CE_i^n \geq 0 \qquad (PC_i), \\ a_i \in \arg\max\limits_{a_i \geq 0} CE_i^n \qquad (IC_i). \end{cases}$$

For any pair of contracts $w_1(.) = \alpha_1 + \beta_1 y_1$ and $w_2(.) = \alpha_2 + \beta_2 y_2$, incentive compatibility implies the following second-stage reaction functions:[9]

$$a_i^n(a_j^n) = \beta_i + \delta\beta_i a_j^n, \quad \forall i,j=1,2, \quad i \neq j, \tag{6}$$

where a_j^n denotes A_j's equilibrium effort level under no communication. Hence, in the standard case where $\beta_i > 0$, efforts are strategic substitutes (resp. complements) if $\delta < 0$ (resp. > 0). Note that, given A_j's equilibrium effort a_j^n, principal P_i can induce agent A_i to choose any desired effort simply by picking the right bonus β_i. If $\delta < 0$ making the agent's incentive scheme more responsive to his performance — i.e., increasing β_i — has two opposing effects on the equilibrium effort. Clearly, a higher β_i increases the intercept of the reaction function $a_i^n(.)$, which tends to increase the equilibrium effort everything else being kept equal (exactly as in the single-hierarchy benchmark). However, a higher β_i also makes the reaction function $a_i^n(.)$ steeper, which reduces the equilibrium effort when $\delta < 0$. Hence, if efforts are substitutes, competition limits the use of high-powered incentives under no information sharing.

The next lemma characterizes the second-stage efforts when both agents are offered the equilibrium contracts.

Lemma 1. *Let β_i^n $(i=1,2)$ be the equilibrium bonus under no information sharing. Agents' second-stage efforts are:*

$$a_i^n = \frac{\beta_i^n(1+\delta\beta_j^n)}{1-\delta^2\beta_i^n\beta_j^n}, \quad \forall i,j=1,2, \quad i \neq j.$$

[9] Given concavity of CE_i with respect to a_i, we can use the First-Order approach — see, e.g., Rogerson (1985) and Jewitt (1988).

If δ is small:

$$\frac{\partial a_i^n}{\partial \beta_i^n} > 0, \qquad sign\frac{\partial a_i^n}{\partial \beta_j^n} \approx sign\delta\beta_i^n.$$

A higher bonus β_i fosters agent A_i's equilibrium effort a_i^n exactly as in the single-hierarchy benchmark. Moreover, the impact of β_j on a_i^n depends on the sign of the slope of A_i's reaction function, that is on $\delta\beta_i^n$. In the standard case where agent A_i's reward is positively correlated with his performance — i.e., if $\beta_i > 0$ — a higher β_j induces A_i to work more (resp. less) if the effort externality is positive (resp. negative): a standard strategic effect similar in spirit to the one shaping firms' equilibrium behavior in oligopoly models.

Using the binding participation constraint, the fixed component α_i of A_i's wage is:

$$\alpha_i^n(a_j^n) = \tfrac{a_i^n(a_j^n)^2}{2} + \tfrac{r}{2}\sigma_i^2\beta_i^2 - \beta_i a_i^n(a_j^n)(1+\delta a_j^n), \quad \forall i,j = 1,2, \quad i \neq j.$$

Substituting $\alpha_i^n(a_j^n)$ into P_i's objective function, program $\mathcal{P}_i^n$ becomes:

$$\max_{\beta_i}\left\{\beta_i(1+\delta a_j^n) + \delta\beta_i(1+\delta a_j^n)a_j^n - \tfrac{r}{2}\sigma_i^2\beta_i^2 - \tfrac{\beta_i^2(1+\delta a_j^n)^2}{2}\right\}.$$

Optimizing with respect to β_i and rearranging terms, the first-order necessary and sufficient condition for a maximum entails:

$$(1+\delta a_j^n)^2 - r\sigma_i^2\beta_i - \beta_i(1+\delta a_j^n)^2 = 0, \quad \forall i,j = 1,2, \quad i \neq j. \tag{7}$$

Increasing the bonus β_i has three effects on P_i's expected profit. First, a high-powered incentive scheme — i.e., a larger β_i — elicits agent A_i's effort, which (*ceteris paribus*) triggers a better performance in expected terms: a *performance-enhancing* effect measured by the first term in (7). Clearly, this effect strengthens (resp. weakens) when efforts are complements (resp. substitutes) and A_j's effort increases (resp. diminishes). Second, a higher β_i also makes A_i more exposed to risk, hence P_i will need to pay a higher risk premium: a *risk-premium* effect measured by the second term in (7). Finally, a higher effort also means a higher disutility for A_i, which requires a higher reservation wage: an *effort-cost* effect captured by the last term in (7).

The next proposition characterizes the equilibrium bonus under no information sharing.

Proposition 1. *Assume that δ is small. When principals do not share information, the equilibrium bonus is:*

$$\beta_i^n \approx \beta_i^* + \underbrace{\frac{2\delta r\sigma_i^2}{(1+r\sigma_j^2)(1+r\sigma_i^2)^2}}_{Competitive\ effect}, \quad \forall i,j = 1,2, \quad i \neq j, \tag{8}$$

with $\beta_i^n \geq \beta_i^*$ *(resp.* $<$*) if* $\delta \geq 0$ *(resp.* $<$*). Moreover:*

$$sign\frac{\partial \beta_i^n}{\partial \sigma_j^2} \approx -sign\delta, \qquad sign\frac{\partial}{\partial \sigma_i^2}\left(\beta_i^n - \beta_i^*\right) \approx sign\delta\left(1 - r\sigma_i^2\right), \quad \forall i,j = 1,2, \quad i \neq j,$$

$$sign\frac{\partial}{\partial r}\left(\beta_i^n - \beta_i^*\right) \approx sign\delta(1 - r\sigma_i^2(1 + 2r\sigma_j^2)), \quad \forall i,j = 1,2, \quad i \neq j.$$

Equation (8) is a modified version of (4) obtained in the monopoly benchmark. In the competing hierarchies model at hand, the equilibrium bonus β_i^n accounts for a novel competitive effect. When efforts are substitutes, it is relatively less convenient for a principal to induce high effort. Hence, the bonus will be lower than in the single-hierarchy benchmark — i.e., $\beta_i^n < \beta_i^*$. By contrast, when efforts have a complementary nature, there are complementarities to be exploited across the two hierarchies. Hence, the equilibrium bonus exceeds the monopoly level — i.e., $\beta_i^n > \beta_i^*$.

Our model delivers a novel prediction on the link between the degree of corporate control in one firm, as measured by the volatility of its manager's performance, and the bonus offered to the opponent manager — i.e., on the effect of σ_j^2 on β_i^n. If efforts are substitutes ($\delta < 0$), an increase in P_j's contractual power vis-à-vis A_j makes P_i more willing to offer high-powered incentives. This is because when the agency conflict in firm j is more pronounced — i.e., σ_j^2 increases and it is more costly for P_j to induce effort provision by A_j — there is a competitive gain that P_i can grab by making A_i exert a higher effort. Conversely, when efforts are complements and σ_j^2 increases, P_i is less willing to offer high-powered incentives. This is because a higher σ_j^2 implies a lower a_j^n and hence weaker complementarities to be exploited.

Proposition 1 also suggests that the type of interaction between managerial efforts contributes to explain the complex relationship between risk and incentives. In line with the evidence collected by the empirical literature investigating the validity of the basic agency model under moral hazard — see Prendergast (2002) — also in our model the prediction is ambiguous. In particular, when managers' actions are strategic substitutes ($\delta < 0$), the impact of risk on incentives is weaker than what the standard monopolistic set-up would predict if managers are not too risk averse ($r < 1/\sigma_i^2$) and the opposite obtains otherwise. Intuitively, when efforts are substitutes and managers are not too (resp. very) risk averse, the cost of increasing agent A_i's exposure to risk via a larger bonus β_i^n is small (resp. large) relative to the competitive advantage that a higher effort would secure.

Differently, when efforts have a complementary nature ($\delta > 0$) the impact of risk on incentives is weaker than what the standard monopolistic set-up would predict if managers are very risk averse ($r > 1/\sigma_i^2$) and the opposite obtains otherwise (when $r < 1/\sigma_i^2$). Intuitively, if managers are very risk averse the cost of increasing A_i's risk exposure is large; hence, an increase in σ_i^2 induces P_i to reduce β_i^n because the negative impact of a lower bonus on A_i's effort is compensated by the effect of complementarity. When managers are not too risk averse, instead, the cost of increasing A_i's risk exposure is not so large; hence, an increase in σ_i^2 induces P_i to offer a larger bonus to countervail the implied reduction of A_j's effort.[10]

[10] Of course, there are other models that explain the empirical puzzle pointed out by Prendergast (2002). For instance, Szalay (2010) builds up an hybrid single-hierarchy model with moral hazard and adverse selection that also offers mixed predictions

Finally, the same type of arguments also explain why the equilibrium bonus with vertical hierarchies might be more or less responsive to the risk-aversion coefficient depending on the sign of δ and the magnitude of r relative to the monopoly benchmark.

4.2 Bilateral information sharing

We now consider the regime where both principals share information. Again, under the CARA specification the analysis can be conducted in terms of the certainty equivalent. Recall that in this regime the wage structure is such that:

$$w_i(y_i, y_j) = \alpha_i + \beta_i a_i + \delta\left(\beta_i + \gamma_i\right) a_i a_j + \beta_i \varepsilon_i + \gamma_i \varepsilon_j, \quad \forall i, j = 1, 2, \quad i \neq j.$$

Hence:

$$CE_i^s = \alpha_i + \beta_i a_i + \delta\left(\beta_i + \gamma_i\right) a_i a_j - \frac{a_i^2}{2} - \frac{r}{2}(\sigma_i^2 \beta_i^2 + \sigma_j^2 \gamma_i^2), \quad \forall i, j = 1, 2, \quad i \neq j. \tag{9}$$

There are two main differences to be emphasized between the above expression and the certainty equivalent obtained in the no information sharing regime. First, when principals share information about their agents' performance, agent A_i (partly) internalizes the effect of his effort choice on P_j's profit; and P_i can keep this effect under her control through the choice of the additional instrument γ_i. Second, A_i's risk premium depends not only on the variance of y_i but also on that of y_j. Hence, *ceteris paribus*, communication tends to increase risk, whereby making agents' effort more costly for both principals.

Principal P_i's optimization problem is:

$$\mathcal{P}_i^s \qquad \begin{cases} \max\limits_{(\alpha_i, \beta_i, \gamma_i)} \mathbb{E}_{(y_i, y_j)}[y_i - w_i\left(y_i, y_j\right)] \\ \text{s.t.} \\ CE_i^s \geq 0 \qquad (PC_i), \\ a_i \in \arg\max\limits_{a_i \geq 0} CE_i^s \qquad (IC_i). \end{cases}$$

Denote by a_j^s the equilibrium effort chosen by A_j in the information sharing regime. Using the first-order approach again, the incentive constraints yield the following second-stage reaction functions:

$$a_i^s(a_j^s) = \beta_i + \delta(\beta_i + \gamma_i) a_j^s, \quad \forall i, j = 1, 2, \quad i \neq j. \tag{10}$$

Note that the introduction of information sharing modifies the agents' effort choice in a non-trivial manner. Here, the slope of A_i's reaction function depends also on γ_i. Hence, in contrast to the no information sharing regime, principals can use two instruments to control effort. Essentially, by changing

on the relationship between risk and incentives. Moreover, Raith (2003) shows that the effect of competition on incentives might be sensitive to the specific measure of competition — i.e., fixed cost of entry or the degree of product differentiation in a Hotelling model. Our model, however, is the first to explicitly consider both the case of substitutability and that of complementarity.

β_i principal P_i pins down the intercept of A_i's reaction function and for any given β_i the new variable γ_i pins down its slope.

The next lemma characterizes the second-stage efforts under information sharing when both agents are offered the equilibrium contracts.

Lemma 2. *Let $w_i^s(.) = \alpha_i^s + \beta_i^s y_i + \gamma_i^s y_j$ ($i, j = 1, 2$ and $i \neq j$) be the equilibrium contracts under bilateral information sharing. Agents' second-stage efforts are:*

$$a_i^s = \frac{\beta_i^s(1+\delta\beta_j^s) + \delta\beta_j^s\gamma_i^s}{1-\delta^2\left(\beta_i^s+\gamma_i^s\right)\left(\beta_j^s+\gamma_j^s\right)}, \quad \forall i, j = 1, 2, \quad i \neq j.$$

If δ is small:

$$\begin{aligned} \frac{\partial a_i^s}{\partial \beta_i^s} &> 0, \qquad sign\frac{\partial a_i^s}{\partial \beta_j^s} \approx sign\delta(\beta_i^s+\gamma_i^s), \\ sign\frac{\partial a_i^s}{\partial \gamma_i^s} &\approx sign\delta\beta_j^s, \qquad sign\frac{\partial a_i^s}{\partial \gamma_j^s} \approx sign(\beta_i^s+\gamma_i^s). \end{aligned}$$

We conjecture, and verify ex-post, that $\beta_i^s > 0$ for each $i = 1, 2$.[11] In this case, a higher bonus β_i spurs agent A_i's equilibrium effort a_i^s, while the impact of β_j on a_i^s depends on the sign of the slope of A_i's reaction function, which under communication is given by the product $\delta(\beta_i^s + \gamma_i^s)$. What is novel here is the comparative statics with respect to γ_i and γ_j. Clearly, making agent A_i's reward more responsive to A_j's performance increases (resp. reduces) A_i's effort in the case of strategic complements (resp. strategic substitutes). The impact of γ_j on A_i's equilibrium effort, instead, depends on the sign of $\beta_i^s + \gamma_i^s$, which measures the overall weight that P_i assigns to the effort interaction term $\delta a_i a_j$.

We can now turn to solve the first-stage contract offer game. Again, the participation constraint is binding. Hence,

$$\alpha_i^s(a_j^s) = \tfrac{a_i^s(a_j^s)^2}{2} + \tfrac{r}{2}(\sigma_i^2\beta_i^2 + \sigma_j^2\gamma_i^2) - \beta_i a_i^s(a_j^s)(1+\delta a_j^s) - \gamma_i a_j^s(1+\delta a_i^s(a_j^s)), \quad \forall i, j = 1, 2, \quad i \neq j.$$

Substituting $\alpha_i^s(a_j^s)$ into P_i's objective, program $\mathcal{P}_i^s$ can be easily rewritten as:

$$\max_{(\beta_i,\gamma_i)} \left\{ (\beta_i + \delta a_j^s(\beta_i+\gamma_i))(1+\delta a_j^s) - \tfrac{r}{2}(\sigma_i^2\beta_i^2 + \sigma_j^2\gamma_i^2) - \tfrac{(\beta_i+\delta a_j^s(\beta_i+\gamma_i))^2}{2} \right\}.$$

Optimizing with respect to β_i and γ_i, the first-order necessary and sufficient conditions for an optimum under bilateral information sharing are:

$$(1+\delta a_j^s)^2 - r\sigma_i^2\beta_i - (\beta_i + \delta a_j^s(\beta_i+\gamma_i))(1+\delta a_j^s) = 0, \quad \forall i, j = 1, 2, \quad i \neq j, \tag{11}$$

$$\delta a_j^s(1+\delta a_j^s) - r\sigma_j^2\gamma_i - (\beta_i + \delta a_j^s(\beta_i+\gamma_i))\delta a_j^s = 0, \quad \forall i, j = 1, 2, \quad i \neq j, \tag{12}$$

[11] In Proposition 2 below it is shown that this conjecture is correct for δ small.

respectively. Equations (11) and (12) reflect the impact of information sharing on the equilibrium contracts. As in the no information sharing regime, a higher bonus β_i induces A_i to exert more effort, but it also calls for a higher risk premium.

The first-order condition with respect to γ_i derived in equation (12) represents a key feature of the regime with information sharing. It reflects the (marginal) costs and benefits of linking one agent's compensation to his opponent's performance. First, as explained in the analysis of equation (10), an increase in γ_i induces A_i to exert more costly effort and also impacts on the equilibrium effort put in by A_j. Hence, if efforts are complements (resp. substitutes) an increase in γ_i generates a positive (resp. negative) effect on a_j which, in turn, makes P_i better-off (resp. worse-off). Second, conditioning the wage $w_i(.)$ on y_j makes A_i more exposed to risk, so that P_i will have to pay a higher premium for this extra uncertainty.

The next proposition characterizes the key features of the equilibrium contracts.

Proposition 2. *Assume that δ is small. When principals share information, the equilibrium contracts entail $\beta_i^s \approx \beta_i^n$ $(i = 1, 2)$ and:*

$$\gamma_i^s \approx \underbrace{\frac{\delta\sigma_i^2}{\sigma_j^2(1 + r\sigma_i^2)(1 + r\sigma_j^2)}}_{\textit{Informational externality}}, \quad \forall i, j = 1, 2, \quad i \neq j. \tag{13}$$

Moreover,

$$\beta_i^s > 0, \qquad sign\gamma_i^s \approx sign\delta, \quad \forall i = 1, 2,$$

$$\frac{\partial|\gamma_i^s|}{\partial\sigma_j^2} < 0, \qquad \frac{\partial|\gamma_i^s|}{\partial\sigma_i^2} > 0, \qquad \frac{\partial|\gamma_i^s|}{\partial r} < 0, \quad \forall i, j = 1, 2 \quad i \neq j.$$

The main difference with the no information sharing regime is embedded in equation (13), which characterizes the relationship between A_i's wage and A_j's performance. One key effect shapes the sign of γ_i^s. This effect hinges upon the *informational externality* created by the effort interaction term. If efforts create positive (resp. negative) externalities, principal P_i would like to reward (resp. punish) agent A_i when A_j performs well. This is because when $\delta > 0$ (resp. $<$) a higher realization of y_j signals, *ceteris paribus*, a higher (resp. lower) a_i.

Note also that the larger is σ_j^2, the lower is γ_i^s in absolute value. This is because when σ_j^2 increases there is less to be learned about a_i from the realization of y_j. By contrast, the larger is σ_i^2, the higher if γ_i^s in absolute value. This is because shareholders with weaker monitoring power need to rely more heavily on external information to control their management. Finally, an increase of the risk-aversion coefficient r reduces the absolute value of γ_i^s because there is a large premium to give up when dealing with agents that are more reluctant to take extra risk.

4.3 Unilateral information sharing

In what follows, we consider the asymmetric case in which one principal (say P_1) shares information, whereas P_2 does not share information. Using the same techniques developed above, it is easy to show that A_1's effort choice follows the rule in (6) and that, by the same token, A_2's effort choice follows (10). Hence, in the asymmetric regime at hand, principal P_2 has a competitive advantage in the sense that she can use the additional information provided by P_1 to control A_2's effort, whereas P_1 can only condition A_1's wage on the performance of y_1. The next lemma follows:

Lemma 3. *Suppose that only P_1 discloses information. For any pair of equilibrium contracts $w_1^u(y_1) = \alpha_1^u + \beta_1^u y_1$ and $w_2^u(y_2, y_1) = \alpha_2^u + \beta_2^u y_2 + \gamma_2^u y_1$, agents' second-period efforts are:*

$$a_1^u = \frac{\beta_1^u(1+\delta\beta_2^u)}{1-\delta^2\beta_1^u(\gamma_2^u+\beta_2^u)}, \qquad a_2^u = \frac{\beta_2^u(1+\delta\beta_1^u)+\delta\beta_1^u\gamma_2^u}{1-\delta^2\beta_1(\gamma_2^u+\beta_2^u)}.$$

If δ is small:

$$\frac{\partial a_1^u}{\partial \beta_1^u} > 0, \quad sign\frac{\partial a_1^u}{\partial \beta_2^u} \approx sign\delta\beta_1^u, \quad \frac{\partial a_1^u}{\partial \gamma_2^u} > 0,$$

$$\frac{\partial a_2^u}{\partial \beta_2^u} > 0, \quad \frac{\partial a_2^u}{\partial \gamma_2^u} \approx sign\delta\beta_1^u, \quad sign\frac{\partial a_2^u}{\partial \beta_1^u} \approx sign\delta(\beta_2^u+\gamma_2^u).$$

The novel feature of this regime is the positive effect of γ_2^u on a_1^u — i.e., making agent A_2's reward more responsive to A_1's performance spurs A_1's equilibrium effort. This is because a higher γ_2^u increases the slope[12] of A_2's reaction function, which (*ceteris paribus*) tends to foster A_1's equilibrium effort regardless of the type of effort externality — i.e., irrespective of the sign of δ. The rest of the comparative statics results have the same interpretation as those presented in Lemma 1 and Lemma 2 above.

Coming back to the principals' optimization programs, it is easy to verify that P_1 solves $\mathcal{P}_1^n$ whereas P_2 solves $\mathcal{P}_2^s$. The solution of these programs leads to the following proposition.

Proposition 3. *Assume that δ is small. With unilateral information sharing: $\beta_1^u = \beta_1^s$, $\beta_2^u = \beta_2^s$ and $\gamma_2^u = \gamma_2^s$.*

Hence, the optimal contracts with unilateral information sharing are the same as those analyzed above. Note, however, that efforts are going to have an asymmetric structure because the principal that receives the information about the competing agent uses this extra evidence to limit the moral hazard problem with the own agent, whereas the principal who cannot rely on this extra information is bound to draw inference about own agent's effort only on the basis of the observed performance measure.

[12] Of course, when $\delta < 0$ this increase has to be considered in absolute value.

5 The value of communication

Building upon the characterization derived in the previous section we can now move to describe the equilibrium of game $\mathcal{G}$. Since in our setting communication decisions are taken unilaterally, principals' incentives to share information hinge only upon the effect that a disclosure decision has on the effort of the rival agent. This is because each principal chooses the optimal communication behavior for a given information sharing decision of the opponent, and hence the latter best-reply is shaped only by the impact of the disclosure decision on the behavior of the opponent's agent.

Principals' expected profits when they choose sharing (s) or not sharing (n) are given by:

		P_2: s	P_2: n
P_1	s	π_1^s, π_2^s	$\pi_1^{s,n}$, $\pi_2^{n,s}$
	n	$\pi_1^{n,s}$, $\pi_2^{s,n}$	π_1^n, π_2^n

In the rest of the analysis we assume, with no loss of insights, that $\sigma_2 \geq \sigma_1$. For throughout we have assumed that δ is small, we will also make the assumption that F is not too large to avoid the uninteresting case where information sharing is unviable under all circumstances.

The following proposition summarizes the results:

Proposition 4. *Assume that δ and F are small. Then, for $\sigma_2 \geq \sigma_1$ there exist two thresholds $\overline{\delta} \geq \underline{\delta} > 0$ with:*

$$\overline{\delta} \equiv \sqrt[3]{\frac{\sigma_2^2 F(1+r\sigma_1^2)\left(1+r\sigma_2^2\right)^4}{(1+2r\sigma_2^2)\sigma_1^2}}, \qquad \underline{\delta} \equiv \sqrt[3]{\frac{\sigma_1^2 F(1+r\sigma_2^2)\left(1+r\sigma_1^2\right)^4}{(1+2r\sigma_1^2)\sigma_2^2}},$$

such that game $\mathcal{G}$ features the following equilibrium properties:

- *If $\delta \geq \overline{\delta}$ there exists a unique PBE where both principals share information.*
- *If $\delta \leq \overline{\delta}$ there exists a unique PBE where principals do not share information.*
- *If $\delta \in (\underline{\delta}, \overline{\delta})$ and $F > 0$ there exists a unique PBE where P_1 shares information and P_2 does not.*
- *$\overline{\delta} \to \underline{\delta}$ for $\sigma_1 \to \sigma_2$. In this case there is only the equilibrium where both principals share information and the equilibrium without communication.*

The results in Proposition 4 are based on the trade-off between the following two forces. First, sharing information affects the equilibrium efforts: there is a strategic channel through which each principal can impact on the opponent agent's behavior by means of the disclosure decision. When efforts have a complementary nature ($\delta > 0$) principal P_i would like to commit to disclose A_i's performance because, by doing so, P_j will be able to elicit more effort from A_j. And this will in turn benefit also P_i via the positive externality. By contrast, when efforts are substitutes ($\delta < 0$) there is no incentive to share information because the implied higher effort by A_j would reduce P_i's profit. Second, disclosing information in a credible manner requires an investment that costs F, and this cost might outweight the positive (strategic) effect of information sharing.

Hence, on the balance, the equilibirum outcome of game $\mathcal{G}$ is shaped by the relative strength of these two effects. Clearly, if agents' effort choices exhibit strong complementarity both principals find it profitable to communicate in equilibrium — i.e., the former strategic effect dominates the latter for both organizations. For intermediate levels of complementarity only the principal with stronger monitoring power discloses information in equilibrium. On the one hand, a lower bound on δ is needed in order to make P_1, whose effort extraction ability is stronger than P_2's, willing to share information and exploit the beneficial role of complementarity. On the other hand, when δ is not too large P_2 has no incentive to share information because the benefit in terms of complementarity that this choice would imply is relatively smaller than the certification cost F. Finally, for low values of complementarity the strategic effect is so weak to make information sharing not profitable for both principals, and this is even more so with negative externalities ($\delta < 0$).

Figure 1 below illustrates the equilibrium outcome in a more compact fashion.

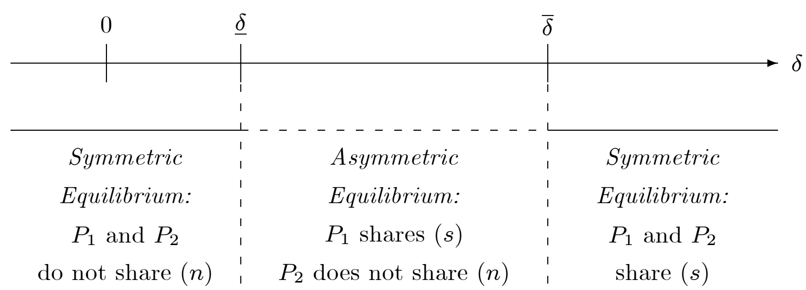

Figure 1: Communication Game—Illustration of the Equilibrium Analysis (case $F > 0$)

In order to relate and compare more closely our analysis with the predictions of the common agency model studied in Maier and Ottaviani (2009), it is interesting to derive the equilibrium characterization in the limiting case where certification costs are nil (exactly as in their set-up). In this case, only the strategic channel highlighted in the discussion of the results in Proposition 4 is at play and, in equilibrium, either principals do not communicate or they share information bilaterally. Hence, the asymmetric equilibrium with unilateral information disclosure discussed in Proposition 4 above disappears. The next proposition summarizes the result:

Proposition 5. *Assume that $F = 0$ and that δ is small. Game $\mathcal{G}$ features the following equilibrium properties:*

- *If $\delta > 0$ there is a unique PBE where both principals share information.*

- *If $\delta < 0$ there is a unique PBE where principals do not communicate.*

The economic intuition of this result rests upon the arguments used to explain Proposition 4. If efforts have a complementary nature principals communicate at equilibrium because sharing information allows them to exploit the positive externalities generated by the agents' effort choices. By contrast, when efforts are substitutes, there is no communication at equilibirum because each principal gains when the rival is unable to induce her own agent to exert a high effort.

Finally, to complete the comparison between our analysis and Maier and Ottaviani (2009), in the following corollary we study the efficiency properties of the equilibrium outcome obtained for $F = 0$.

Corollary 1. *Assume that $F = 0$ and that δ is small. For $\delta > 0$ the equilibrium of game $\mathcal{G}$ is efficient. For $\delta < 0$ the equilibrium is inefficient, with*

$$\pi_i^s - \pi_i^n \approx \frac{r\delta^2\sigma_i^4}{2\sigma_j^2(1+r\sigma_j^2)^2\left(1+r\sigma_i^2\right)^2} > 0, \qquad \forall i = 1,2 \quad j \neq i.$$

Moreover, $\pi_i^s - \pi_i^n$ is increasing in σ_i^2 and decreasing in σ_j^2.

This result brings out a potential conflict between private and collective interests to share information. Its intuition is straightforward. When certification costs are negligible, an information sharing agreement enables principals to limit agents' misbehavior and, at the same time, to internalize the externality produced by the effort choices. This produces an incentive to *coordinate* that makes communication worthwhile. Interestingly, this finding differs from the insights of the (intrinsic) common agency problem analyzed by Maier and Ottaviani (2009), where the efficient outcome from the principals' perspective is always implemented in equilibrium. In this sense exclusive deals appear to be less efficient than having a common agent.

Note also that the gain from sharing information increases with own volatility and decreases with the volatility of the partner with whom a principal shares information. This is because the worse is one principal's monitoring power vis-à-vis own agent, the larger is the benefit of having additional informative evidence that can be exploited to relax internal incentive constraints. By the same token, the gain from sharing information reduces when dealing with partners with inefficient monitoring technologies.

6 The role of correlation

In this section we briefly analyze the case where the random variables ε_1 and ε_2 are allowed to be correlated, and $\text{Cov}(\varepsilon_1, \varepsilon_2) = \rho\sigma_1\sigma_2$, with $\rho \in [-1, 1]$ being the correlation index. To isolate the effect of correlation on the equilibrium contracts we assume that $\delta = 0$, so as to neglect the role of competition. The rest of the set-up is as described in Section 3. As before, we assume that ρ is small enough, so as to compute profits by means of Taylor expansions.

First, note that the equilibrium with no communication boils down to the monopoly benchmark — i.e., $\beta_i = \beta_i^*$ for $i = 1, 2$. The case of bilateral information sharing is instead summarized in the next proposition:

Proposition 6. *Assume that ρ is small. When principals share information the equilibrium contracts entail $\beta_i^s = \beta_i^*$ $(i = 1, 2)$ and:*

$$\gamma_i^s \approx \underbrace{-\frac{\rho\sigma_i}{\sigma_j(1 + r\sigma_i^2)}}_{\textit{Diversification effect}}, \quad \forall i, j = 1, 2, \quad i \neq j. \tag{14}$$

Moreover, $sign\gamma_i^s \approx sign\rho$.

When performances are correlated and effort externalities are negligible, the main force shaping the equilibrium contract is a *diversification effect*. The interpretation can be borrowed from the standard CAPM analysis — see, e.g., Copeland and Weston (1988). Since performances are correlated, principals seek to design contracts in such a way to exploit correlation and reduce agents' risk-premium. Therefore, when performances are positively (resp. negatively) correlated, risk-diversification requires principal P_i to punish (resp. reward) agent A_i when A_j performs well.[13]

The next proposition characterizes the equilibrium outcome of game $\mathcal{G}$ when $\delta = 0$ and $\rho \neq 0$.

Proposition 7. *Assume that ρ is small. Game $\mathcal{G}$ features the following equilibrium properties:*

- *If $F > 0$ there is a unique PBE where principals do not share information, this equilibrium is inefficient in the sense that principals would jointly gain from sharing information.*
- *If $F = 0$ every profile of information sharing decisions is an equilibrium. However, the one where both principals share information is the Pareto dominant one, with*

$$\pi_i^s - \pi_i^n \approx \frac{r\rho^2\sigma_i^2}{2\left(1 + r\sigma_i^2\right)^2} > 0, \qquad \forall i = 1, 2.$$

The intuition of this result is straightforward. In absence of effort externalities the strategic channel by which each principal can affect the opponent agent's behavior by means of her disclosure decision is shut

[13] It can be verified that with both effort interaction and correlation the equilibrium contracts under bilateral and unilateral information sharing entail a bonus that is equal to that obtained in the no information sharing regime — i.e., $\beta_i = \beta_i^n$ — while the cross-performance weight is:

$$\gamma_i^s \approx \frac{\delta\sigma_i^2}{\sigma_j^2\left(1 + r\sigma_i^2\right)\left(1 + r\sigma_j^2\right)} - \frac{\rho\sigma_i}{\sigma_j\left(1 + r\sigma_i^2\right)}, \quad \forall i, j = 1, 2, \quad i \neq j. \tag{15}$$

Hence, the sign of γ_i^s depends on the relative strength of the *competitive* and *diversification* effects described above.

down. The only spillover generated by information sharing is due to the diversification effect discussed in Proposition 6. However, by its nature, the latter operates in the same way regardless of whether information is shared bilaterally or unilaterally.

The implication is that if certification costs are not negligible ($F > 0$) the game has a unique equilibrium with no communication. Instead, if they are negligible ($F = 0$) principals are indifferent between communicating agent's performances and staying silent. In the latter case, the Pareto dominant equilibrium is the one with bilateral information sharing: communication would allow them to better diversify risk and hence reduce agents' premium for the additional uncertainty. This result also suggests that, when δ and ρ are both different from zero but small, the insights obtained in the baseline model with independent performances carry over.

7 Concluding Remarks

The article has built a theory of information sharing among vertical organizations under moral hazard. The analysis delivers a number of novel predictions not only on the internal organization of firms but also on the process of interaction and communication among different organizations. The results provide ready to use guidelines for interpreting and designing future empirical investigations about: (i) the determinants of firms' incentives to share information with and without certification costs; (ii) the link between the power of incentives and competition; (iii) the impact of monitoring and contractual power on firms' internal organization.

8 Appendix

Proof of Lemma 1. In the following, let β_i^n denote the equilibrium bonus under no information sharing as resulting from the first-stage maximization problem of principal P_i. Incentive compatibility implies:

$$a_i(a_j^n) = \beta_i^n(1 + \delta a_j^n).$$

Plugging a_j^n into (6) and solving for a_i^n and a_j^n, we obtain that:

$$a_i^n = \frac{\beta_i^n(1 + \delta\beta_j^n)}{1 - \delta^2\beta_i^n\beta_j^n}.$$

The expression above characterizes the optimal value of effort put in by agent A_i in the regime without information sharing. To complete the proof, we perform the comparative statics of a_i^n with respect to β_i^n and β_j^n. The partial derivative of a_i^n with respect to β_i^n is equal to

$$\frac{\partial a_i^n}{\partial \beta_i^n} = \frac{1 + \beta_j^n\delta}{(1 - \beta_i^n\beta_j^n\delta^2)^2},$$

whose sign is unambiguously positive, because the denominator is always positive and

$$1 + \beta_j^n\delta \approx 1$$

for δ small. Finally, the derivative of a_i^n with respect to β_j^n is given by

$$\frac{\partial a_i^n}{\partial \beta_j^n} = \frac{\beta_i^n\delta(1 + \beta_j^n\delta)}{(1 - \beta_i^n\beta_j^n\delta^2)^2}.$$

In this case, the sign of the denominator is clearly positive and the sign of the numerator depends on the product between β_i^n and δ.■

Proof of Proposition 1. In the regime without information sharing, the first-order necessary and sufficient condition that determines the optimal value of β_i is equal to

$$(1 + \delta a_j^n)^2 - r\sigma_i^2\beta_i - \beta_i(1 + \delta a_j^n)^2 = 0.$$

We first solve for $\delta = 0$ and immediately obtain:

$$\beta_i^* = \frac{1}{1 + r\sigma_i^2}.$$

Then, taking the total derivative with respect to δ evaluated at $\delta = 0$ we have:

$$-(1 + r\sigma_i^2)\lim_{\delta\to 0}\frac{\partial\beta_i^n(\delta)}{\partial\delta} + 2(1 - \beta_i^*)\lim_{\delta\to 0} a_j^n = 0.$$

As $\lim_{\delta\to 0} a_j^n = \beta_j^*$, it follows that

$$-(1+r\sigma_i^2)\lim_{\delta\to 0}\frac{\partial\beta_i^n(\delta)}{\partial\delta}+2(\beta_j^*-\beta_i^*\beta_j^*)=0 \iff$$
$$\lim_{\delta\to 0}\frac{\partial\beta_i^n(\delta)}{\partial\delta}=\frac{2r\sigma_i^2}{(1+r\sigma_i^2)^2(1+r\sigma_j^2)}.$$

Therefore, the first-order approximation of $\beta_i^n(\delta)$ for δ small is equal to:

$$\beta_i^n \approx \beta_i^* + \delta\lim_{\delta\to 0}\frac{\partial\beta_i^n(\delta)}{\partial\delta}=\beta_i^*+\frac{2\delta r\sigma_i^2}{(1+r\sigma_i^2)^2(1+r\sigma_j^2)}.$$

It is easy to verify that the sign of $\beta_i^n-\beta_i^*$ depends on δ: if $\delta \geq 0$ (resp. $<$), the equilibrium bonus under no information sharing and competing hierarchies rises above (resp. below) the equilibrium bonus obtained in the monopoly benchmark.

We now turn to the comparative statics of β_i^n with respect to the volatility of agent A_j's performance (σ_j^2).

$$\frac{\partial\beta_i^n}{\partial\sigma_j^2}=-\frac{2\delta r^2\sigma_i^2}{(1+r\sigma_i^2)^2(1+r\sigma_j^2)^2},$$

whose sign depends on the sign $-\delta$. Instead, the derivative of the difference $\beta_i^n-\beta_i^*$ with respect to the volatility of agent A_i's performance is equal to

$$\frac{\partial}{\partial\sigma_i^2}(\beta_i^n-\beta_i^*)=\frac{2\delta r(1-r\sigma_i^2)}{(1+r\sigma_i^2)^3(1+r\sigma_j^2)},$$

whose sign depends on the sign $\delta(1-r\sigma_i^2)$. Finally, the derivative of $\beta_i^n-\beta_i^*$ with respect to the risk-aversion index r is:

$$\frac{\partial}{\partial r}(\beta_i^n-\beta_i^*)=\frac{2\delta\sigma_i^2(1-r\sigma_i^2(1+2r\sigma_j^2))}{(1+r\sigma_i^2)^3(1+r\sigma_j^2)^2}.$$

In this case, the sign of the derivative is determined by the sign of $\delta(1-r\sigma_i^2(1+2r\sigma_j^2))$. ■

Proof of Lemma 2. Let β_i^s and γ_i^s denote the equilibrium contractual bonuses under information sharing as resulting from the first-stage maximization problem of principal P_i.

Taking the first-order condition of (IC_i) in $\mathcal{P}_i^s$ with respect to a_i leads to

$$a_i^s(a_j^s)=\beta_i^s+\delta(\beta_i^s+\gamma_i^s)a_j^s.$$

Plugging a_j^s and solving for a_i^s and a_j^s gives

$$a_i^s=\frac{\beta_i^s(1+\delta\beta_j^s)+\beta_j^s\gamma_i^s\delta}{1-\delta^2(\beta_i^s+\gamma_i^s)(\beta_j^s+\gamma_j^s)}.$$

This expression characterizes the optimal value of effort put in by agent A_i in the regime with information sharing. The partial derivative of a_i^s with respect to β_i^s is equal to

$$\frac{\partial a_i^s}{\partial \beta_i^s} = \frac{1+\delta(\beta_j^s(1-\delta\gamma_i^s)-\delta\gamma_i^s\gamma_j^s)}{(1-\delta^2(\beta_i^s+\gamma_i^s)(\beta_j^s+\gamma_j^s))^2},$$

whose sign is positive for δ small.

The derivative of a_i^s with respect to β_j^s entails

$$\frac{\partial a_i^s}{\partial \beta_j^s} = \frac{\delta(\beta_i^s+\gamma_i^s)(1+\delta(\beta_i^s-\delta\beta_i^s\gamma_j^s-\delta\gamma_i^s\gamma_j^s))}{(1-\delta^2(\beta_i^s+\gamma_i^s)(\beta_j^s+\gamma_j^s))^2}$$

and the sign of this expression depends on the sign of the product between δ and $(\beta_i^s+\gamma_i^s)$. The derivative of a_i^s with respect to γ_i^s is equal to

$$\frac{\partial a_i^s}{\partial \gamma_i^s} = \frac{\delta(\beta_j^s(1+\delta\beta_i^s)+\delta\beta_i^s\gamma_j^s)}{(1-\delta^2(\beta_i^s+\gamma_i^s)(\beta_j^s+\gamma_j^s))^2}.$$

In this case, the sign of the derivative is univocally determined by the sign of $\delta\beta_j^s$. Finally, the derivative of a_i^s with respect to γ_j^s is given by the following expression:

$$\frac{\partial a_i^s}{\partial \gamma_j^s} = \frac{\delta^2(\beta_i^s+\gamma_i^s)(\beta_i^s+\delta\beta_i^s\beta_j^s+\delta\beta_j^s\gamma_i^s)}{(1-\delta^2(\beta_i^s+\gamma_i^s)(\beta_j^s+\gamma_j^s))^2},$$

whose sign is determined by the sign of $(\beta_i^s+\gamma_i^s)$. ■

Proof of Proposition 2. In the regime with bilateral information sharing, the first-order necessary and sufficient conditions with respect to β_i and γ_i are equal to, respectively:

$$(1+\delta a_j^s)^2 - r\sigma_i^2\beta_i - (\beta_i+\delta a_j^s(\beta_i+\gamma_i))(1+\delta a_j^s) = 0, \quad \forall i,j=1,2, \quad i\neq j \tag{A1}$$

and

$$\delta a_j^s(1+\delta a_j^s) - r\sigma_j^2\gamma_i - (\beta_i+\delta a_j^s(\beta_i+\gamma_i))\delta a_j^s = 0, \quad \forall i,j=1,2, \quad i\neq j. \tag{A2}$$

To take the first-order approximation of the optimal β_i and $\gamma_i$around $\delta=0$, we start by deriving the values of β_i and γ_i at $\delta=0$ using (A1) and (A2). This leads to

$$\beta_i^* = \frac{1}{1+r\sigma_i^2}, \qquad \gamma_i^* = 0.$$

Taking the total derivative of (A1) with respect to δ evaluated at $\delta=0$, we obtain the expression in what follows:

$$-(1+r\sigma_i^2)\lim_{\delta\to 0}\frac{\partial\beta_i^s(\delta)}{\partial\delta} + (2-2\beta_i^*-\gamma_i^*)\lim_{\delta\to 0} a_j^s = 0.$$

For $\lim_{\delta\to 0} a_j^s = \beta_j^*$, the expression above can be rewritten as

$$-(1+r\sigma_i^2)\lim_{\delta\to 0}\frac{\partial \beta_i^s(\delta)}{\partial\delta} + 2(\beta_j^* - \beta_i^*\beta_j^*) - \gamma_i^*\beta_j^* = 0 \iff$$

$$\lim_{\delta\to 0}\frac{\partial \beta_i^s(\delta)}{\partial\delta} = \frac{2r\sigma_i^2}{(1+r\sigma_i^2)^2(1+r\sigma_j^2)}.$$

Therefore, the first-order approximation of $\beta_i^s(\delta)$ around δ equal to 0 is given by:

$$\beta_i^s \approx \beta_i^* + \frac{2\delta r\sigma_i^2}{(1+r\sigma_i^2)^2(1+r\sigma_j^2)} = \beta_i^n,$$

with β_i^s positive for δ small. Note that the comparative statics of β_i^s is equivalent to the one given in Proposition 1 for β_i^n.

Next, consider γ_i^s. The total derivative of (A2) with respect to δ (evaluated at $\delta = 0$) is given by:

$$-r\sigma_j^2\lim_{\delta\to 0}\frac{\partial \gamma_i^s(\delta)}{\partial\delta} + (1-\beta_i^*)\lim_{\delta\to 0} a_j^s = 0.$$

As $\lim_{\delta\to 0} a_j^s = \beta_j^*$ it follows that

$$-r\sigma_j^2\lim_{\delta\to 0}\frac{\partial \gamma_i^s(\delta)}{\partial\delta} + (1-\beta_i^*)\beta_j^* = 0 \iff$$

$$\lim_{\delta\to 0}\frac{\partial \gamma_i^s(\delta)}{\partial\delta} = \frac{\sigma_i^2}{\sigma_j^2(1+r\sigma_i^2)(1+r\sigma_j^2)}.$$

Therefore, the first-order approximation of γ_i^s around $\delta = 0$ is equal to

$$\gamma_i^s \approx \gamma_i^* + \delta\lim_{\delta\to 0}\frac{\partial \gamma_i^s(\delta)}{\partial\delta} = \frac{\delta\sigma_i^2}{\sigma_j^2(1+r\sigma_i^2)(1+r\sigma_j^2)},$$

whose sign depends on the sign of δ.

In the following, we study the comparative statics of $|\gamma_i^s|$. It is straightforward to show that

$$\frac{\partial|\gamma_i^s|}{\partial\sigma_j^s} = -\frac{|\delta|\sigma_i^2(1+2r\sigma_j^2)}{\sigma_j^4(1+r\sigma_i^2)(1+r\sigma_j^2)^2} < 0, \quad \frac{\partial|\gamma_i^s|}{\partial\sigma_i^s} = \frac{|\delta|}{\sigma_j^2(1+r\sigma_i^2)^2(1+r\sigma_j^2)} > 0$$

and

$$\frac{\partial|\gamma_i^s|}{\partial r} = -\frac{|\delta|\sigma_i^2(\sigma_i^2+\sigma_j^2+2r\sigma_i^2\sigma_j^2)}{\sigma_j^2(1+r\sigma_i^2)^2(1+r\sigma_j^2)^2} < 0,$$

which concludes the proof. ■

Proof of Lemma 3. Let β_1^u, β_2^u and γ_2^u denote the equilibrium contractual bonuses offered under unilateral information sharing as resulting from the first-stage maximization problem of principals P_1 and P_2. As in the proof of Lemma 2, we conjecture that β_1^u and β_2^u are positive.

Given that P_1 discloses, we know that

$$a_1(a_2^u) = \beta_1^u + \delta\beta_1^u a_2^u.$$

At the same time, as P_i does not disclose then

$$a_2(a_1^u) = \beta_2^u + \delta(\beta_2^u + \gamma_2^u)a_1^u.$$

Solving for a_1^u and a_2^u gives

$$a_1^u = \frac{\beta_1^u(1+\delta\beta_2^u)}{1-\delta^2\beta_1^u(\beta_2^u+\gamma_2^u)}, \qquad a_2^u = \frac{\beta_2^u(1+\delta\beta_1^u)+\beta_1^u\gamma_2^u\delta}{1-\delta^2\beta_1^u(\beta_2^u+\gamma_2^u)}. \tag{A3}$$

The expressions in (A3) characterize the optimal values of effort put in by agents 1 and 2 in the regime with unilateral information sharing.

In what follows, we characterize the comparative statics of a_1^u and a_2^u. To begin with, the partial derivative of a_1^u with respect to β_1^u is equal to

$$\frac{\partial a_1^u}{\partial \beta_1^u} = \frac{1+\delta\beta_2^u}{(1-\delta^2\beta_1^u(\beta_2^u+\gamma_2^u))^2},$$

whose sign is positive for δ small.

Then, the derivative of a_1^u with respect to β_2^u is given by

$$\frac{\partial a_1^s}{\partial \beta_2^u} = \frac{\delta\beta_1^u(1+\delta\beta_1^u(1-\delta\gamma_2^u))}{(1-\delta^2\beta_1^u(\beta_2^u+\gamma_2^u))^2},$$

thus its sign depends on the sign of $\delta\beta_1^u$.

The derivative of a_1^u with respect to γ_2^u is equal to

$$\frac{\partial a_1^u}{\partial \gamma_2^u} = \frac{\delta^2(\beta_1^u)^2(1+\delta\beta_2^u)}{(1-\delta^2\beta_1^u(\beta_2^u+\gamma_2^u))^2}.$$

In this case, the sign of the derivative is clearly positive. We now turn to the comparative statics on a_2^u.

The derivative of a_2^u with respect to β_2^u is given by the following expression:

$$\frac{\partial a_2^u}{\partial \beta_2^u} = \frac{1+\delta\beta_1^u(1-\delta\gamma_2^u)}{(1-\delta^2\beta_1^u(\beta_2^u+\gamma_2^u))^2},$$

which is positive for δ small.

The derivative of a_2^u with respect to γ_2^u is equal to:

$$\frac{\partial a_2^u}{\partial \gamma_2^u} = \frac{\delta\beta_1^u(1+\delta\beta_2^u)}{(1-\delta^2\beta_1^u(\beta_2^u+\gamma_2^u))^2},$$

whose sign depends on $\delta\beta_1^u$.

Finally, the derivative of a_2^u with respect to β_1^u is given by:

$$\frac{\partial a_2^u}{\partial \beta_1^u} = \frac{\delta(1+\delta\beta_2^u)(\beta_2^u+\gamma_2^u)}{(1-\delta^2\beta_1^u(\beta_2^u+\gamma_2^u))^2}$$

and, for δ small, its sign is determined by the sign of $\delta(\beta_2^u+\gamma_2^u)$.■

Proof of Proposition 3. We first determine the contractual specifications decided by the principal that does not share information, P_j. The first-order necessary and sufficient conditions to determine β_j and γ_j are given in the following:

$$(1+\delta a_j^u)^2 - r\sigma_i^2\beta_i - (\beta_i + \delta a_j^u(\beta_i+\gamma_i))(1+\delta a_j^u) = 0, \quad \forall i,j = 1,2, \quad i \neq j \tag{A4}$$

and

$$\delta a_j^u(1+\delta a_j^u) - r\sigma_j^2\gamma_i - (\beta_i + \delta a_j^u(\beta_i+\gamma_i))\delta a_j^u = 0, \quad \forall i,j = 1,2, \quad i \neq j. \tag{A5}$$

Hence, at $\delta = 0$ (A4) and (A5) lead to:

$$\beta_j^* = \frac{1}{1+r\sigma_j^2}, \qquad \gamma_j^* = 0, \quad \forall j = 1,2.$$

At the same time, taking the total derivative of (A4) and (A5) with respect to δ evaluated at $\delta = 0$, we obtain:

$$\lim_{\delta\to 0}\frac{\partial \beta_j^u(\delta)}{\partial \delta} = \frac{2r\sigma_j^2}{(1+r\sigma_j^2)^2(1+r\sigma_i^2)}, \qquad \lim_{\delta\to 0}\frac{\partial \gamma_j^u(\delta)}{\partial \delta} = \frac{\sigma_j^2}{\sigma_i^2(1+r\sigma_j^2)(1+r\sigma_i^2)}.$$

Therefore, the first-order approximations of $\beta_j^u(\delta)$ and $\gamma_j^u(\delta)$ for δ small are equal to, respectively:

$$\beta_j^u \approx \beta_j^* + \frac{2\delta r\sigma_j^2}{(1+r\sigma_j^2)^2(1+r\sigma_i^2)} = \beta_j^s, \qquad \gamma_j^u \approx \frac{\delta\sigma_j^2}{\sigma_i^2(1+r\sigma_j^2)(1+r\sigma_i^2)} = \gamma_j^s.$$

We now turn to the determination of the contractual reward offered by the principal that shares information, P_i. In this case, the first-order necessary and sufficient condition for the determination of β_i is equal to

$$(1+\delta a_j^u)^2 - r\sigma_i^2\beta_i - \beta_i(1+\delta a_j^u)^2 = 0.$$

Then, taking the total derivative with respect to δ (evaluated at $\delta = 0$) and using β_i^* entails:

$$\lim_{\delta\to 0}\frac{\partial \beta_i^u(\delta)}{\partial \delta} = \frac{2r\sigma_i^2}{(1+r\sigma_i^2)^2(1+r\sigma_j^2)}.$$

Finally, the first-order approximation of $\beta_i^u(\delta)$ for δ small is equal to:

$$\beta_i^u \approx \beta_i^* + \frac{2\delta r\sigma_i^2}{(1+r\sigma_i^2)^2(1+r\sigma_j^2)} = \beta_i^s,$$

which completes the proof of Proposition 3. ■

Proof of Proposition 4. To begin with, note that the third order Taylor approximations around $\delta = 0$ of the differences $\pi_i^s - \pi_i^{n,s}$ and $\pi_i^{s,n} - \pi_i^n$ coincide and are equal to:

$$\pi_i^s - \pi_i^{n,s} = \pi_i^{s,n} - \pi_i^n \approx \frac{\delta^3(1+2r\sigma_i^2)\sigma_j^2}{\sigma_i^2(1+r\sigma_j^2)(1+r\sigma_i^2)^4} - F.$$

The expression above takes into account that, when she decides to share information, P_i needs to sink certification costs F. Thus, defining

$$\frac{(1+2r\sigma_i^2)\sigma_j^2}{\sigma_i^2(1+r\sigma_j^2)(1+r\sigma_i^2)^4} \equiv \kappa_i,$$

we can establish that to share (s) is a dominant strategy for principal P_i if

$$\delta^3\kappa_i - F \geq 0 \iff \delta \geq \sqrt[3]{\frac{F}{\kappa_i}},$$

with $\sqrt[3]{F/\kappa_i} > 0$ for all $F > 0$. Moreover, since

$$sign[\kappa_i - \kappa_j] = sign[\sigma_j - \sigma_i]$$

and $\sigma_2 \geq \sigma_1$, it turns out that

$$\overline{\delta} \equiv \sqrt[3]{\frac{F}{\kappa_2}} \geq \underline{\delta} \equiv \sqrt[3]{\frac{F}{\kappa_1}}.$$

This all implies that if $\delta \geq \overline{\delta}$ both principals 1 and 2 choose to share (s) at the equilibrium. Instead if $\delta \leq \underline{\delta}$ both principals prefer not to share (n) at the equilibrium. Finally, if $\delta \in (\underline{\delta}, \overline{\delta})$ an asymmetric equilibrium arises, in which principal 1 does not share (n) whereas principal 2 shares (s).

Notice that as $\underline{\delta} \to \overline{\delta} \equiv \hat{\delta}$, then the equilibria are two and feature both principals sharing information if $\delta \geq \hat{\delta}$ and both principals not sharing information otherwise. As a special case, we have that if $F = 0$ then $\hat{\delta} = 0$. ■

Proof of Proposition 5. To derive the equilibria in Proposition 5 we followed the steps in the proof of Proposition 4 using as restriction that $F = 0$.■

Proof of Corollary 1. Assume $F = 0$. It is easy to verify that the second-order Taylor approximation around $\delta = 0$ of the difference between the profits of P_i when both principals share (π_i^s) and the profits when both principals do not share (π_i^n) entails:

$$\pi_i^s - \pi_i^n \approx \frac{r\delta^2\sigma_i^4}{2\sigma_j^2(1+r\sigma_i^2)^2(1+r\sigma_j^2)^2}.$$

Clearly, this difference decreases with σ_j^2. Moreover,

$$\frac{\partial}{\partial\sigma_i^2}(\pi_i^s - \pi_i^n) = \frac{r\delta^2\sigma_i^2}{\sigma_j^2(1+r\sigma_i^2)^3(1+r\sigma_j^2)^2} > 0,$$

which concludes the proof. ■

Proof of Proposition 6. Assume $\delta = 0$ and $\rho \neq 0$. As in the standard case without effort externalities ($\delta = 0$) and a monopolistic hierarchy, A_i's effort choice satisfies the first-order condition if $a_i^s = \beta_i$, with $i = 1, 2$.

We can now solve the first-stage contract offer game in the case featuring both principals sharing information. The participation constraint is binding, thus

$$\alpha_i^s(a_j^s) = \tfrac{(a_i^s)^2}{2} + \tfrac{r}{2}(\sigma_i^2\beta_i^2 + \sigma_j^2\gamma_i^2 + 2\beta_i\gamma_i\sigma_i\sigma_j\rho) - \beta_i a_i^s - \gamma_i a_j^s, \quad \forall i,j = 1,2, \quad i \neq j.$$

Substituting $\alpha_i^s(a_j^s)$ into P_i's objective, the maximization program can be rewritten as:

$$\max_{(\beta_i,\gamma_i)} \left\{\beta_i - \tfrac{r}{2}(\sigma_i^2\beta_i^2 + \sigma_j^2\gamma_i^2 + 2\beta_i\gamma_i\sigma_i\sigma_j\rho) - \tfrac{\beta_i^2}{2}\right\}.$$

Optimizing with respect to β_i and γ_i, the first-order necessary and sufficient conditions for an optimum under bilateral information sharing are, respectively:

$$1 - r\sigma_i^2\beta_i - r\gamma_i\sigma_i\sigma_j\rho - \beta_i = 0, \quad \forall i,j = 1,2, \quad i \neq j, \tag{A6}$$

$$-r\sigma_j^2\gamma_i - r\beta_i\sigma_i\sigma_j\rho = 0, \quad \forall i,j = 1,2, \quad i \neq j. \tag{A7}$$

To compute the first-order approximation of the optimal β_i and γ_i, we start by deriving the values of β_i and γ_i at $\rho = 0$ using (A6) and (A7). This leads to

$$\beta_i^* = \frac{1}{1+r\sigma_i^2}, \qquad \gamma_i^* = 0, \quad \forall i = 1,2.$$

Taking the total derivative of (A6) and (A7) with respect to ρ evaluated at $\rho = 0$ we obtain, respectively:

$$-(1+r\sigma_i^2)\lim_{\rho\to 0}\frac{\partial\beta_i^s(\rho)}{\partial\rho} - r\gamma_i^*\sigma_i\sigma_j = 0 \quad \Longleftrightarrow \quad \lim_{\rho\to 0}\frac{\partial\beta_i^s(\rho)}{\partial\rho} = -\frac{r\gamma_i^*\sigma_i\sigma_j}{1+r\sigma_i^2}$$

and

$$-r\sigma_j^2\lim_{\rho\to 0}\frac{\partial\gamma_i^s(\rho)}{\partial\rho} - r\beta_i^*\sigma_i\sigma_j = 0 \quad \Longleftrightarrow \quad \lim_{\rho\to 0}\frac{\partial\gamma_i^s(\rho)}{\partial\rho} = -\frac{\beta_i^*\sigma_i}{\sigma_j}.$$

Using β_i^* and γ_i^*, it turns out that the first-order approximation of $\beta_i^s(\rho)$ around ρ equal to zero is equal to β_i^*, while the first-order approximation of $\gamma_i^s(\rho)$ around $\rho = 0$ is given by

$$\gamma_i^s \approx -\frac{\rho\sigma_i}{\sigma_j(1+r\sigma_i^2)},$$

whose sign depends on the sign of ρ. ■

Proof of Proposition 7. First, note that in the regime with no communication the second order Taylor approximation of P_i's profits around $\rho = 0$ entails

$$\pi_i^n \approx \frac{1}{2(1+r\sigma_i^2)},$$

which is equivalent to the profits of P_i in the monopoly benchmark.

Moreover, using the results of Proposition 6, we find that the second order Taylor expansion around $\rho = 0$ of P_i's profits when both principals communicate are given by:

$$\pi_i^s \approx \frac{1}{2(1+r\sigma_i^2)} + \frac{r\sigma_i^2\rho^2}{2(1+r\sigma_i^2)^2} - F.$$

In the asymmetric case in which principal P_i communicates and principal P_j does not communicate, the absence of externalities in effort provision, i.e. $\delta = 0$, implies that the second order approximation of P_i's expected profits is equal to:

$$\pi_i^{s,n} \approx \frac{1}{2(1+r\sigma_i^2)} - F.$$

Instead, if principal P_i does not share information while principal P_j shares information, the second order approximation of P_i's expected profits entails

$$\pi_i^{n,s} \approx \frac{1}{2(1+r\sigma_i^2)} + \frac{r\sigma_i^2\rho^2}{2(1+r\sigma_i^2)^2}.$$

It follows that if $F > 0$ the unique equilibrium of the communication game features both principals not sharing information (n). Instead, if $F = 0$ each principal is indifferent between sharing and not sharing information, however the equilibrium in which both P_1 and P_2 share information is Pareto dominant because

$$\pi_i^s - \pi_i^n \approx \frac{\sigma_i^2\rho^2}{2(1+r\sigma_i^2)^2} > 0.$$

■

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