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Marcello D'Amato* **and Christian Di Pietro****

Abstract

We consider a model of occupational choice with credit market imperfections and local non convexities in education investment. The implications of individual heterogeneity for the evolution of wealth distribution and policies are studied. Convergence of the wealth distribution is obtained whenever the (exogenous) distribution of education costs entails the presence inefficient types, regardless of how "large" the support of the random ability parameter is. Conversely, poverty traps can emerge only if investment is efficient for every single agent in the economy. We explore conditions under which wealth accumulation eliminates the effects of financial market imperfection. Interestingly we show that, a necessary feature of steady states with occupational mobility is that wealth constraints, whenever they bind investment choices in the long run, they must bind for households in both occupations. Persistence of wealth constraints motivates our exploration of policies. Compared to the case of homogeneous ability, we show that heterogeneity requires more persistent policies to achieve similar results in terms of enhanced investment opportunities and income per capita. It is also shown that the scope for policies is larger under heterogeneity: policies can be effective in environments where they would fail in a world of homogeneous abilities.

JEL classification: D31, D91, I21, J24, O15.

Keywords: Heterogeneous Ability, Borrowing Constraints, Intergenerational Mobility, Wealth Inequality, Occupational Choice, Educational Investment.

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1 Introduction

In a world of credit market imperfections individual ability and wealth background represent the key elements to understand the aggregate dynamics of educational investments and the features of wealth distribution. Our aim here is to study the consequences of individual heterogeneity for the dynamics and the design of policies in a standard model of educational investment and occupational choice where individuals differ with respect to both education costs (ability) and lineage wealth.

To have an idea of the effects of heterogeneity on the evolution of wealth in models of occupational mobility with wealth constraints, consider the case of a distribution of abilities within a lineage compared to the case of an economy with a representative "average" type. For a given wealth distribution, the presence of kids smarter than average makes upward occupational mobility more likely in a household with little wealth. On the other hand the presence of kids with ability lower than average makes downward occupational mobility more likely in the relatively well to do family, whenever a larger return can be obtained from another asset different from human capital. Clearly, then, individual specific investment costs, in the context of occupational choice, represent a source of occupational mobility and, as such, they can provide a check to forces leading to path dependence of the wealth distribution and the emergence of poverty traps as in the form obtained, for example, in Galor and Zeira, (1993). Thus, by enhancing occupational mobility, heterogeneity can, even in models of "fixed thresholds" (in the terminology of Matsuyama, 2010) that are multidimensional, produce similar wealth dynamics as in models of mobility traps (Piketty, 1997 and Aghion and Bolton, 1997) and in models of short term persistence of wealth distribution with a convex investment technology (Becker and Tomes, 1979, and Loury 1981).

In such a context we address three specific issues: i. the implications of heterogeneity for the evolution of wealth distribution; ii. if and under which conditions the effects of credit constraints are not overcome in the long run by the accumulation of wealth at the lineage level; iii. what, if any, is the role for policy. We study a simple standard economic environment: a small open economy is considered where the size of indivisibility, financial market imperfections and returns to factors are set exogenously in the competitive equilibrium (i.e. we study a non interactive model of wealth distribution). In this context, the dynamics are shown to be quite different depending on whether or not the distribution of ability entails an inefficient type for whom the financial investment available in the economy is preferable as an alternative to educational investment.

Poverty traps are shown to disappear whenever the distribution of education costs entails the presence of ability types for which investment is inefficient; the result holding regardless of how "large" the support of the distribution of abilities is. Ergodicity with respect to the initial distribution occurs either in the form of stagnation-decline or in the form of a unique steady state with occupational (and wealth) mobility.

Conversely, a trap *can* still emerge, only if investment is efficient for every single agent in the economy, as when rate of returns on educational investment are large compared to the alternative financial investment, i.e. when income inequality is large.

In either cases- inefficient types being present or not- a scenario of decline emerges when both households in the unskilled occupations whose kids cannot invest (even in the most efficient type) *and* wealth bequest in middle class in the skilled occupation are below the level that would allow efficient investment to be financed.

Whether and under which conditions the evolution of the wealth distribution leads to the progressive elimination of the effects of financial market imperfections is a different issue from path dependence and deserves some attention on its own right.[1] Indeed, convergence of the wealth distribution does not imply, in general, that first best efficient investment can be financed in the long run in any lineage (e.g. Aghion and Bolton, 1997 in a model with ex-post heterogeneity).[2] The point is that, in a world of credit market imperfections, no matter what the specific microeconomics of the source of imperfection is, the commitment value of wealth generates a threshold below which investment cannot be financed (Matsuyama, 2010, Piketty 2000).

A result of our analysis is that the persistence of credit market imperfections in the limit depends on the forces that shape the level of wealth at the bottom *and* intermediate segments of the wealth distribution. Interestingly, in a steady state with occupational mobility we show that whenever wealth constrains choices in households in the unskilled occupation they necessarily bind investment in (some) households in the skilled occupation too.

An empirical counterpart of the result above is that *whenever the observed frequency of investment in skilled households correlates, at any point in time, with wealth background, then first best efficient cannot be obtained in any long run equilibrium with mobility.*

[1]In our context the accumulation of wealth through bequest is the instrument that households have to trigger upward mobility in the lineage. Although, due to the specification of the bequest motive, occupational mobility of generations in the lineage is an unintended consequence of the bequest choice.

[2]Financial market imperfections are doomed to persist in the long run by construction in models where non insurable large shocks (not considered in this paper) hit the returns to investment. See for example Loury (1981), Banerjee and Newman (1991), Aghion and Bolton (1997), Piketty (1997). Depending on the precise features of the shock, preferences and bequest motives, the role of wealth accumulation in such models can be that of an insurance device. Wealth accumulation at the lineage level can, in general, serve both as a source of finance in the presence of credit market imperfections and as an insurance device, whenever moral hazard aspects in production prevent complete insurance of idiosyncratic shocks. Our focus is on the former.

Finally, we explore the role of public policy in this context.[3] We share the common view that wealth constraints on investment induced by credit market imperfections are better tackled, where possible, by public intervention designed to improve directly the credit market failure where it arises, rather than leaning on the use of fiscal instrument and redistributive policies. However, in the case of educational investment, the latter approach is less likely to be successful, since the fraction of future income that can be pledged is low exacerbating incentive constraints in the process of repayment; direct tax-transfers instruments can be considered then.

The keypoint of heterogeneity in abilities for policy design- as for markets- is the information requirement and the incentive constraints under which they may have to be designed. We consider "simple policies" with minimum incentive requirements, i.e. involving taxes that do not prevent investment by any efficient type in the set of unconstrained households. Compared to the case of homogeneous abilities the set of wealth distributions that constitute a feasible tax base for policy is narrower. Whenever policies are feasible the presence of heterogeneity calls for policies that are more persistent than in the case of homogeneous investment costs. The result arises essentially from more restrictive feasibility constraints associated with the impossibility for the policy maker to observe agents' ability type and targeting the transfer scheme accordingly. More importantly, we also show that policy can be effective in environments where they would not be under the assumption of homogeneous costs. In particular, we characterize a set of conditions on the distribution of abilities such that a permanent policy is effective even when economic fundamentals [4] would dictate a declining path due to the absence of credit market for educational investment.

The rest of the paper is organized as follows: in section 2 we review the relevant literature, in section 3 we lay out the model, in section 4 we discuss the results about the dynamics of wealth distributions and occupational mobility in the case of heterogeneous investment costs, in section 5 we discuss the implications of heterogeneity for policy design. Section 6 discusses some extensions, section 7 concludes. All the proofs are reported in the Appendix.

2 Related Literature

The role of individual heterogeneity is studied in most of the literature focusing on the role of ex-post uninsurable shocks. Notable exception are Loury (1981) and Mookherjee and Napel (2007) who focus on ex-ante heterogeneity. The role of ex-post heterogeneity is briefly discussed, for example, in Galor

[3]It is well known that local indivisibilities and credit markets failure form an economic environment where temporary (both structural and policy) shocks can have permanent effects on investment, per capita income and equality of opportunity.

[4]Or, equivalently, temporary shocks to wealth, see note 15 below.

and Zeira (1993) on which we heavily draw for the modeling of the economic environment. There it is argued that provided that the variability of the individual (ex-post) non insurable shock (to wages) is not too large poverty traps are still obtained. Our analysis spells out all the emerging equilibrium configurations, and investigates some policy implications in greater detail for the case of ex-ante heterogeneity.

The work by Mookherjee and Napel (2007) analyzes the role of heterogeneity in shaping intergenerational mobility in the context of a model where pecuniary externalities to educational investment arise due to the presence of a decreasing returns to scale technology in both the modern and the traditional sector. Their analysis is conducted on the assumption that investment costs always ensure downward occupational mobility in skilled households. This latter, coupled with the assumption of positive occupational mobility at the bottom of the income distribution, entails the shrinking of the set of steady state from a continuum to a finite number, in a model with no financial bequest. They also briefly discuss policy implications arguing for the role of persistent policies, due to local uniqueness of the equilibria, ranked in terms of per capita income. The main difference with their analysis is in the presence of a financial bequest which, in our economy, defines a unique threshold for the unconstrained agents (who can, in principle, belong to both occupational groups).

Our analysis starts from a model which is quite different from those generating "low mobility traps" (as in Piketty, 1997, see, Piketty 2000, for a survey). The impact of initial distribution of wealth in these models works via the endogenous accumulation of aggregate capital and the determination of the interest rate, which is instead fixed in our model. The policy implications of heterogeneity, however, remind some of the conclusions drawn in these models (in particular Aghion and Bolton, 1997), on the possible relevance of persistent policies.

Models leading to poverty traps (Azariadis and Stachursky, 2006, Ravallion, 2006) have been an influential framework for policy design (see Barret et al., 2008 and references therein) since they are deemed to establish the relevance and to clarify scope and power of *temporary* policies to achieve permanent objectives. [5]

Indeed, subject to a proper scale dictated by fiscal budget constraints and provided that individual state (usually wealth) is observed by the policy maker, policy interventions are easy to enforce, featuring a very nice property: once broken the jaws of the trap, public policy and related institutions are no longer relevant and can be dismantled.[6]

[5]The role of large scale temporary policies has been important for the policy debate in developing countries where any form of initial accumulation of asset is prevented by fundamental conditions for a large fraction of the population (see for example, Sachs, 2005).

[6]The standard example of the effectiveness of policies arises in the case of homogeneous

This view about the role of transitory policies in environments leading to poverty traps has been challenged in models leading to convergence where long run intervention is required by the emergence of the so called mobility traps (see the survey in Piketty, 2000), similar conclusions are suggested by Mookherjee and Napel (2007) in a model with heterogeneity and endogenous factor prices. In models of mobility traps, ergodicity along with persistent inefficiencies calls for persistent redistributive policies, (as in Aghion and Bolton, 1997, Piketty 1997 or Loury, 1981) on the account of non insurable bad shocks restarting the history of lineages at any time at wealth levels low enough to constrain efficient investment. As already mentioned our model does not feature such shocks. The argument for persistent policies and institutions here is, therefore, different.

3 The Model

Our model economy is very similar to that considered in Galorand Zeira (1993) except for two features: there is no financial market whatsoever for educational investment and, more importantly, there is heterogeneity in the investment cost. The first feature is just a simplification and it does not affect the main results (see Piketty 2000, pp.459 and ss.), the second represents the focus of our investigation. Specifically, we consider a small open economy where a single good can be produced with two technologies, using skilled labor and physical capital in one case and unskilled labor in the other case. The returns to skilled labor are denoted by w, the returns to unskilled labor is denoted by v, the returns to capital is denoted by r, all assumed to be constant. The economy is populated by agents whose measure is normalized to 1 in every period t.

Each agent derive utility from their own consumption and bequest to their offspring according to:

$$U_{i,t} = \alpha \log c_{i,t} + (1-\alpha) \log b_{i,t} \quad \text{with} \quad \alpha \in (0,1) \tag{1}$$

where $c_{i,t}$ denotes consumption and $b_{i,t}$ denotes the bequest, $1-\alpha \in (0,1)$ is a measure of altruism within lineage.

Given the inherited bequest, the agent has to decide whether to invest in human capital or not and how much to bequeath to her own offspring given wage income resulting from occupation; the budget constraint of i in period t is given by

$$c_{i,t} + b_{i,t} = (1+r)(b_{i,t-1} - e \cdot x_i) + y_e \tag{2}$$

agents: if average wealth in the economy (or the scale of outside intervention through external resources) is large enough "one shot redistribution policies can have permanent effects" for achieving efficient levels of investment. Policies in this context also represent an efficient response to transitory adverse shocks that could otherwise drive the economy to decline. In the latter case "triage" policies and the like may involve reverse redistribution.

where x_i denotes the investment cost, $e \in \{0, 1\}$ denotes education investment. If $e = 0$ agents do not invest in human capital and $y_0 = v$, if $e = 1$ the agent bear the cost of investment x and she works as skilled yielding a wage $y_1 = w$.

The three main assumptions are therefore stated as:

Assumption 1. (Financial Market Imperfection) $b_{i,t-1} \geq e \cdot x_i$.

We model financial market imperfection simply as non existence of credit market (Loury, 1981, Mookherjee and Ray, 2003) for human capital investment. Wealth constrains investment choice; notice that the variation in x induces variation in the wealth threshold.

Assumption 2. (Heterogeneity in Investment Costs) x, a measure of ability, is a random variable distributed according to $G(x)$ on the support $\Delta x = [\underline{x}, \overline{x}]$, with $\underline{x} \geq 0$ and average $x_e = \int_{\underline{x}}^{\overline{x}} x dG(x)$.

This assumption captures the presence of indivisibilities in education investment (local non convexity) but adds heterogeneity as in Mookherjee and Napel (2007).[7]

Assumption 3. (Stability) $\rho := (1 + r)(1 - \alpha) < 1$.

Due to indivisibility, agents choose investment by solving:

$$\underset{e}{Max} \{U_{i,t}(e = 0), U_{i,t}(e = 1)\} \qquad (3)$$
$$s.to \; b_{i,t-1} \geq e \cdot x_i$$

It is easy to see that the unconstrained solution to (3) entails a cut off level for x;

$$U_{i,t}(e = 0) = U_{ti,}(e = 1) \Rightarrow \tilde{x_i} := \frac{w - v}{1 + r} \qquad (4)$$

whereas in the presence of binding constraint $\tilde{x_i} = b_{i,t-1}$. Summarizing:

Remark 1. *Educational investment occurs ($e = 1$) if and only if*

$$x_i \leq \min\{\tilde{x_i}, b_{i,t-1}\}. \qquad (5)$$

Intuitively, for given distributions of x and b, the demand for investment in human capital increases when the rate of return on education increases (the skill premium $w - v$ is larger) and when the return on financial wealth decrease (r is lower).

The system of family decisions defines a "non interactive dynamics" of the evolution of wealth distribution and hence the competitive equilibrium is simply obtained by aggregating individual choices. The object of analysis

[7]Notice that heterogeneity in investment costs is exogenous and does not depend on the inherited wealth (and therefore is independent on her ancestors' ability). Of course ancestor's ability affect their offspring's investment through bequeathed wealth.

is the evolution of a random variable $\mu_t(b)$ describing the distribution of bequests at each period t induced by $\mu_{t-1}(b)$, the past distribution at $t-1$, given exogenous prices in factor markets and other parameters of the model. The set of all exogenous parameters is denoted as $\Gamma = \{\alpha, r, w, v, G(x)\}$. To simplify notation index i is dropped henceforth.

Before studying the model with heterogeneity it is convenient to review some results on the model in the absence of heterogeneity as a benchmark.

3.1 The benchmark case of homogeneous costs of investment

As it is well known, in the case of homogeneous agents a few configurations emerge for the long run equilibrium depending on the forces that allow households to eliminate financial market imperfections through asset accumulation (returns on financial wealth, altruism, labor income and investment costs). Our benchmark is constructed so that the homogeneous cost of investment is equal to the average cost x_e. Notice first that for $x_e > \widetilde{x}$ the investment technology is *inefficient*[8] and $e = 0$ in every period, this case is trivial and disregarded. For $x_e \leq \widetilde{x}$, i.e. investment is *efficient*, the dynamics of wealth are governed by the map $\phi : b_{t-1} \to b_t$ defined as follows:

$$\phi(b;\, x_e) = \begin{cases} \phi_u(b;\, x_e) := (1-\alpha)v + \rho b & x_e > b \\ \phi_s(b;\, x_e) := (1-\alpha)w + \rho(b - x_e) & x_e \leq b \end{cases} \tag{6}$$

where the time subscript is eliminated for the subsequent exposition, to simplify the notation. Notice that the mapping $\phi(b;\, x_e)$ is stepwise linear increasing, with slope ρ and it exhibits a discontinuity at $b = x_e$, the wealth threshold. Three possible equilibrium configurations emerge summarized in the following:

Lemma 1. *Fix x_e so that $x_e < \widetilde{x}$, define FP as the set of fixed points of ϕ, denote $b_s = (1-\alpha)w$, $\underline{b} = \frac{(1-\alpha)v}{1-\rho}$. Then FP is characterized as follows:*

1) (Decline) If $x_e > b_s$, then $FP = \{\underline{b}\}$;

2) (Self sustaining growth) If $x_e < \underline{b}$, then $FP = \left\{\frac{(1-\alpha)w - \rho x_e}{1-\rho}\right\}$

3) (Poverty trap) If $x_e \in [\underline{b},\, b_s]$, then $FP = \left\{\underline{b},\, \frac{(1-\alpha)w-\rho x_e}{1-\rho}\right\}$.

Case 1) and 2) are ergodic, in case 3) the measure of skilled agent converging to either of the two steady state wealth levels depends on the initial distribution of wealth. This result is well known and the proof is omitted (e. g. Galor and Zeira, 1993). Notice that in case 1) credit market imperfections will drive the economy to stagnation. In case 2) *credit market imperfections become irrelevant for all agents in finite time*, in case 3) credit market imperfections only trap households starting in the unskilled occupations.

[8] The cost of investment is larger than the discounted present value of the return on investment.

4 The Evolution of Wealth Distribution in the Presence of Heterogeneity in Education Costs

With heterogeneity, equilibrium bequest and investment policies- for any given individual state (b, x) - are defined by the map Φ solving (3). The dynamics of wealth accumulation are defined by:

$$\Phi(b;\, x) = \begin{cases} \Phi_u := (1-\alpha)v + \rho b & x > \min\{\tilde{x}, b\}, \quad e = 0 \\ \Phi_s := (1-\alpha)w + \rho(b - x) & x \leq \min\{\tilde{x}, b\}, \quad e = 1 \end{cases} \tag{7}$$

Observe that Φ is a correspondence $b \to \Phi(b)$, where $\Phi(b)$ is the set of equilibrium bequest achievable by agents who received b, for different values of x. It is easy to see that the evolution of wealth, driven by Φ, follows a (linear) Markov process[9].

By studying $\Phi(b;\, x)$ a first characterization of the limit support of wealth distribution, denoted by S_∞, is obtained.

Denoted by S_0 the support of the initial distribution of wealth, for all $n > 0$ we put $S_n := \cup_{b \in S_{n-1}} \Phi(b)$. We indicate with B the interval $[\underline{b}, \overline{b}]$, where $\underline{b} = \frac{(1-\alpha)v}{1-\rho}$ and $\overline{b} = \frac{(1-\alpha)w - \rho \underline{x}}{1-\rho}$.

Lemma 2. *Suppose that S_0 is bounded, then the sequence $\{S_n\}$ converges to a unique limit set $S_\infty \subset B$.*

The result establishes that there exists a compact set such that the support of the limit distribution takes values in it. This set (S_∞) has not necessarily full measure on the borel sets with respect to B. Which subsets of B will feature positive measure will depend on the evolution of wealth distribution as shaped by the parameters of the model which we now study.

To this aim it is immediate to see that, again, three possibilities can emerge, depending on whether $\tilde{x} < \Delta x$, $\tilde{x} \in \Delta x$, $\tilde{x} > \Delta x$. The two interesting cases are:[10]

1. For $\tilde{x} \in \Delta x$ the rate of return of educational investment belongs to set of admissible investment costs. We denote this as a case of *low income inequality.*[11]

2. For $\tilde{x} > \Delta x$ the rate of return of educational investment is greater than all investment costs, the modern sector is efficient for any possible x. Then every agent faces an efficient investment technology, occupational mobility flows are governed by the dynamics of wealth constraints alone. We denote this as a case of *high income inequality.*

[9] See the appendix for a formal argument. The result is easily obtained by suitably adapting arguments in Loury (1981).

[10] If $\tilde{x} < \Delta x$ educational investment is inefficient, the economy trivially converges to $\underline{b}$. The case is not considered.

[11] Of course $\tilde{x}$ being the present value of education investment, depends on r as well.

Case 1 and 2 above are studied separately in the following two subsections.

4.1 The evolution of wealth distribution in the case of low income inequality ($\tilde{x} \in \Delta x$)

Individual variation of ability, regardless of inherited wealth, is such that in every lineage at any point in time there exist both efficient and inefficient types. Where the efficient type is defined as the cost level such that the return to educational investment is equal to the return on financial investment. Returns to educational investment depend on the skill premium, hence we denote this as a case of low income inequality. Occupational mobility flows, in this case, are regulated by both the (endogenous) process for the evolution of wealth (the tightness of credit market imperfection) and the exogenous process generating abilities. The evolution of wealth distribution in this case is characterized in the following

Proposition 1. *(Low income inequality). For $\tilde{x} \in \Delta x$ the dynamic of the evolution of wealth distributions is ergodic (with respect to μ_0). Moreover only two wealth distributions emerge, depending on Γ:*

i) (Decline) if $\underline{b} \leq \underline{x}$, then the dynamics of the wealth distributions converge to $\underline{b}$ with full measure;

ii) (Steady State with Wealth and Occupational Mobility) if $\underline{x} < \underline{b}$, then the system converges to $[\underline{b}, \overline{b}]$ with a unique measure μ^*.

In words, the proposition states that convergence is obtained in the case of low income inequality. Notice that decline can be part of the equilibrium configuration. The intuition is clear: since there always exist inefficient types, independently of the level of parental wealth, there will always exist downward *occupational* mobility flows in any range of the wealth distribution. Consequently, wealth accumulation cannot shelter households investment permanently, any lineage can experience histories after which their wealth converges to a neighborhood of $\underline{b}$. If $\underline{b} \leq \underline{x}$ the only possible limit distribution settles on $\underline{b}$ with full measure.

To achieve a stable limit wealth distribution downward mobility flows must be mirrored by equivalent upward mobility flows.

Upward mobility is possible if households with histories in the unskilled occupation accumulate enough wealth. Provided that $\underline{x} < \underline{b}$ then upward occupational mobility flows balance the downward occupational mobility and the economy can sustain investment in the long run and convergence of the wealth distribution;

It is worth noticing that, *the result holds regardless of how large the support* *measuring variation in abilities* Δx *is.* Even a small amount of heterogeneity forces the economy to converge.

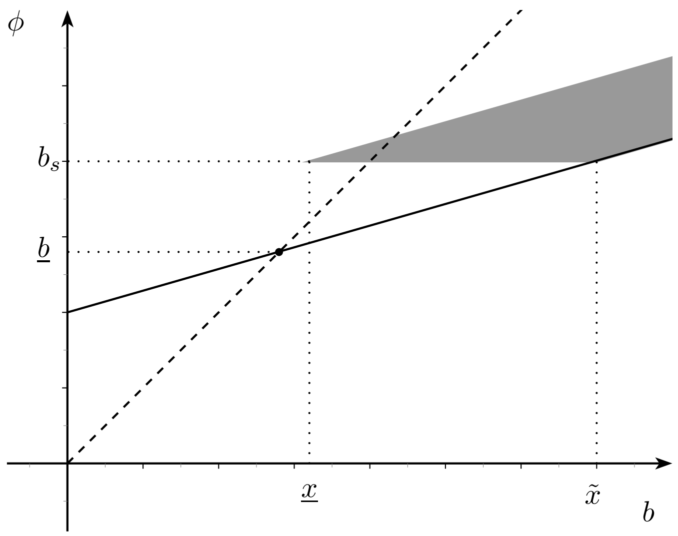

Figure 1. Image set of b under Φ in the case of low income inequality (Decline)

The grey area represents the set of bequeathed wealth by agents employed in the skilled sector. The black line represents the set of bequeathed wealth by agents in the unskilled sector. One important topological property is that the two sets are connected for $b \geq \tilde{x}$.[12] The economic counterpart of this property is that any agent will or will not face binding wealth constraints, regardless of her immediate ancestor's occupation.

In this respect the following result characterizes an interesting link among wealth segments and occupations:

Lemma 3. $b_s \geq \tilde{x}$ *if and only if* $\underline{b} \geq \tilde{x}$.

A necessary and sufficient condition for the accumulation of wealth in lineages to overcome credit market imperfections in the limit is $\frac{w-v}{w} < \rho$.

In words, the result states that- in the limit - the offspring of an unskilled household is wealth constrained ($b \leq \tilde{x}$) at $\underline{b}$ if and only if the offspring of the less wealthy households in the skilled occupation (an agent receiving wealth in a neighborhood of b_s) is wealth constrained too. This implies that,

[12]Notice that, by definition of $\tilde{x}$ the equilibrium bequest policy at $x = \tilde{x}$ must be the same regardless of the sector of occupation. This is a necessary feature for this case of low income inequality. The property induced on the structure of the image set of the mapping Φ allows, in principle, the extensions of the results in this section to more general individual transition functions generated by a more general structure of the fundamental economy (utility functional forms, bequest motive, endogenous wages). The existence of a path connecting the black line with the grey area will imply the ergodicity of the system even in a more general model.

contrary to the case of homogeneous costs (where, once a lineage's wealth is above a threshold than the lineage is not wealth constrained forever on), financial market imperfections do not necessarily disappear over the long run if lineages in the traditional sector are constrained.

An empirical implication is that whenever it is observed that educational investment varies across wealth groups within household with parents in the skilled occupation, then financial market imperfections must be binding for *both the lower and the middle class, in the limit distribution.* Therefore, if one lineage among the skilled, at a given point in time faces the consequences of financial market imperfection then it must be true that every lineage will face the consequences of financial market imperfections[13].

4.2 The evolution of wealth distribution in the case of high income inequality ($\tilde{x} > \Delta x$)

If educational investment is efficient for all agents financial market imperfections are all that matters in driving education investment choice and they are the only force that shape wealth mobility, the role of variation in ability is less important (but not null, of course). Therefore, initial wealth is crucial and we can expect that non ergodicity emerges in the form of poverty traps. The evolution of wealth distribution is characterized in the following

Proposition 2. *For high income inequality the following results hold:*

i) (Decline) if $\underline{b} \leq \underline{x}$ *and* $b_s < \overline{x}$ *then the system is ergodic with respect to* μ_0 *and the dynamics of the wealth distributions converge to* $\underline{b}$ *with full measure;*

ii) (Steady State with Occupational and Wealth Mobility) if $\underline{x} < \underline{b}$ *then the system is ergodic with respect to* μ_0 *and it converges to* $[\underline{b}, \overline{b}]$ *with a unique measure* μ^**. In particular if* $\overline{x} \leq b_s$ *the system converges to* $[b_s, \overline{b}]$*, in this case every agent will be employed in the skilled sector over the long run;*

iii) (Poverty Trap) if $\underline{b} \leq \underline{x} \leq \overline{x} < b_s$ *then the system is not ergodic and the dynamics of wealth distribution converges to* $\{\underline{b}\} \cup \left[\frac{(1-\alpha)w-\rho\overline{x}}{1-\rho}, \overline{b}\right]$ *with a measure that depends on* μ_0*.*

The mapping for the evolution of wealth is reported in figure 2.

[13]This property does not hinge on the assumption of constant return to factor and can be generalized. To get the intuition notice that with decreasing returns to scale in both sectors the skill premium is monotonically decreasing in the dynamics of the economy whenever it is not in decline. Suppose that, at a given point in time, for w and v such that $\widetilde{x} \in \Delta x$, Lemma 3 must hold. Then in any subsequent period with $w' < w$ and $v' > v$ skilled agents are still constrained.

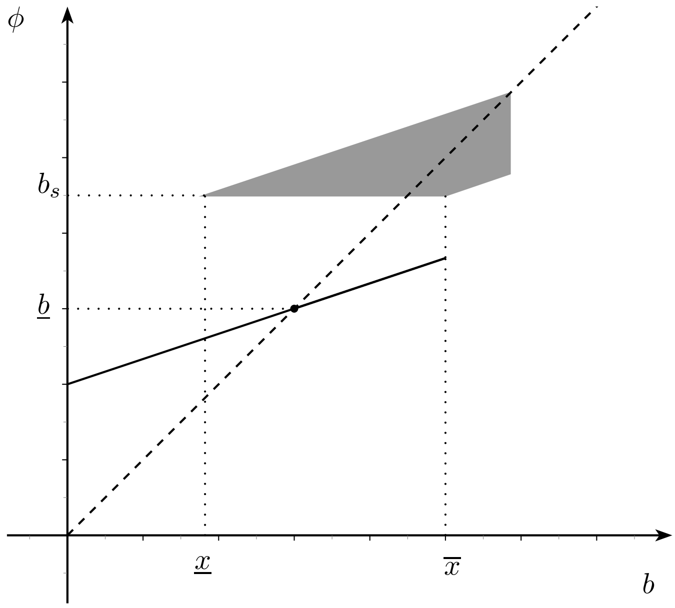

Figure 2. Image set of b under Φ in the case of high income inequality. (Steady State with Occupational and Wealth Mobility, Case ii)

Both case i) and case ii) in Proposition 2 also emerged in the regime of low income inequality. The reason for their emergence is, however, completely different in the two regimes of income inequality. With low income inequality, ergodicity is driven by the fact that downward occupational mobility is operative, regardless of wealth. With high income inequality, instead, dynamics are shaped by credit market imperfections, the only mechanism establishing whether occupational mobility channels are operative or not.

In case i) $\underline{b} \leq \underline{x}$ implies, as usual, that once a lineage gravitates in a neighborhood of $\underline{b}$ the occupational destiny is doomed and there is no way to escape that position in the wealth distribution; whereas $b_s < \overline{x}$ implies that the measure of credit constrained agents holding at b_s is positive, decline can be the only outcome in the long run.

Opposite considerations hold in case ii): if financial markets do not prevent investment in households with histories only in the unskilled occupations ($\underline{x} < \underline{b}$) but they affect households in the skilled occupations ($b_s < \overline{x}$) then a steady state with mobility is obtained, where financial market imperfections bind individual choices forever on. In other words: *a steady state with occupational mobility in the high inequality regime prevails only if financial market imperfections are not eliminated by wealth accumulation.* This is analogous, but not identical to what has been characterized in Lemma 3

The emergence of poverty traps (case iii)), instead, follows similar logic as in the case of homogeneous cost.

Taken together Propositions 1 and 2 have clear predictions linking the observation of occupational mobility to the evolution of wealth distribution and aggregate economic growth: 1) if at any point in time the economy features no upward *occupational* mobility flows and a positive (arbitrarily small) amount of downward *occupational* mobility flows, then the wealth accumulation process of the economy must be set on a declining path; 2) if different investment frequencies are observed in households in skilled occupations then financial market imperfection do not disappear over the long run. These two aspects motivate our exploration of policy in the next section.

5 Public Policies

In this section we derive some implications of heterogeneity in abilities for policy design in the simple model studied above. We argue that heterogeneity introduces additional constraints calling for larger policy persistence than in the homogeneous cost case, but it also makes redistributive policies somewhat more powerful.[14] Following the literature, a policy will be regarded as "effective" to the extent that it is able to support a larger measure of investment (and output) than in its absence. We concentrate on redistribution of wealth through a tax transfer scheme. This is equivalent to the case where non linear wealth tax revenues are used as a source of finance for public investment to reduce education costs direct and indirect (transportation infrastructure or location of schools in remote villages). We concentrate on simple incentive constrained policies, i.e. tax can never expropriate the resources for investment to the least (efficient) educational investment project, this clearly adds additional constraints compared to the case of homogenous (average) costs. In the set of initial wealth distributions where tax base satisfies this criterium, we will show, firstly, that, compared to homogeneous costs with the same average as $G(x)$, in the presence of heterogeneity, the required horizon for tax transfer policies to be effective must be, in a well defined sense, longer. In cases where ergodic steady states with mobility arise it is straightforward to argue that, whenever wealth constraints are operative in the limit distribution, persistent policies are required to achieve higher investment levels and equal opportunity due to ergodicity. However the issue is more subtle and argument for more persistent policies induced by heterogeneity can also be constructed for the cases where decline or poverty traps obtain. This is important since, when policies are more persistent, consider-

[14]Our discussion will not deal with aggregate shocks. However, it is clear that whether ergodicity is present or not hinges on the possibility that transitory shocks have permanent effects. The same argument provided to establish effectiveness of policy interventions can be used to characterize the effectiveness of policies to counteract the permanent effects of temporary shocks. An instance of such policies can be easily constructed in the context of the present model: consider the case (as it happens for a poverty trap scenario) of temporary exogenous large shocks driving wealth down to the level where investment is no longer feasible, our results imply that there exist conditions under which temporary redistributive policies avoid this form of decline.

ations about the bequest motive and associated distortions (e.g. crowding out effects on private savings) due to redistribution are more relevant for the preservation of the tax base, in the light of the expected nature of the intervention. [15]

Secondly we will show that, with heterogeneity, there is a larger scope for policies: in a well defined sense, for given average cost of investment, variation in ability is a source of heterogeneity in wealth that can be exploited for the design of effective redistributive policies in cases where policy is completely ineffective under homogeneity[16].

Before providing the details a few remarks are in order to clarify the limits of our analysis. Firstly remember that bequest motive is given by "joy of giving" motive[17] disregarding the crowding out effect of redistribution on savings. Secondly, we will assume that wealth is observable, whereas ability is not; i.e. we construct redistributive policies conditioned on the wealth state of the individual, but not on her ability. Relatedly, our focus is on feasibility constraints (i.e. policies are required to be budget balanced period by period), no optimality criterion is cast on the class of tax transfer policies we consider. In other words incentive constraints will be dealt with in the simplest possible way: we will always make sure that investment by high cost agents is, if efficient, never prevented by taxation.[18]

5.1 Some implications for policy in the case of homogeneous costs

Consider first, as a benchmark, the role of policy in the case of homogeneous costs. Let us assume that the initial wealth is large (just) enough to obtain feasibility of a one shot intervention, i.e. the economy is rich in wealth *just enough* to allow a one shot transfers large enough that all lineages overcome the wealth threshold making investment viable (more formally see Definition

[15] It is clear that the requirement of proper scale in policies aimed at dismantling the poverty trap is related to the same point. There exist of course other ways to eliminate poverty traps in the presence of non convexities, even in the case of homogeneous costs, requiring lower scale and longer horizon (decentralized mechanisms as Roscas, or different forms of public intervention designed in the form of "triage") with similar consequence on the relevance of distortionary effects of such policies.

[16] Even if "ability" would be partially endogenous so that wealth family can reduce x by suitable investment, if profitable, as in Becker and Tomes (1979) the same point would hold as long as wealth can be transmitted at a lower cost than ability. In general the argument stands whenever ability, although partially transmittable, includes some i.i.d. component. In other words the presence of i.i.d. components in a model with endogenous talent only relocates the problem of financial market imperfections on layer up in the education process.

[17] See e.g. Aghion and Bolton (1997) for a motivation of policy analysis under similar assumptions; see also Cremer and Pestieau (2003) and Matsuyama (2010) for surveys.

[18] This will guarantee that incentive compatibility constraints are satisfied in a weak sense (i.e. in any direct mechanism on x, under the proposed tax scheme, the agent is indifferent between truthtelling and misreporting her type at $\overline{x}$).

4 in appendix B). Let us define $n^*(\widetilde{\mu}_0)$ the minimum number of periods required for the transfer scheme to allow investment by efficient types (see Definition 6 in Appendix B). It is immediate to see that the following holds:

Remark 2. *Fix $\widetilde{\mu}_0$ as in Definition 4 and $n^*(\widetilde{\mu}_0)$ as in Definition 6, then there exist a transfer system T satisfying Definition 5 such that the following holds true:*

i) if $x_e < b_s$ (poverty trap and self sustaining growth) then $n^*(\widetilde{\mu}_0) = 1$;

ii) if $x_e > b_s$ (decline) then there exist no T which is feasible after a finite $n^d(\widetilde{\mu}_0)$ and such that $1 - \mu_{n^d+1}(b) > 0$ for $b > x_e$.

In words: i) states the well known result that if average initial wealth is larger than average cost (along with all initially distribution that stochastically dominate $\widetilde{\mu}_0(b)$) then a one shot redistributive policy removes inefficiency; ii) states that there is no feasible redistributive policy T that can prevent the decline. Again, the result is simple and well known and the proof is omitted.

The role of policy will be different in the presence of heterogeneity as we show next.

5.2 Some implications for policy in the presence heterogeneity

We consider a comparison performed under the same structural parameters. Initial wealth distribution $\widetilde{\mu}_0(b)$ satisfies Definition 4, notice that an additional restriction $1 - \mu_0(\overline{x}) > 0$ holds, i.e. there exists a positive measure of agents with wealth above $\overline{x}$ to provide the initial tax base[19]. Under such assumption there exists T such that preserves (efficient) investment by taxed agents if efficient (see Definition in Appendix B). The following result holds:

Proposition 3. *Fix $\widetilde{\mu}_0$ as in Definition 4', T as in Definition 5' and $n^*(\widetilde{\mu}_0)$ as in Definition 6'. If $\overline{x} \leq b_s$ and $\underline{x} > \underline{b}$ (poverty trap), then for any $G(x)$ satisfying Assumption 2, it must be $n^*(\mu_0) > 1$.*

In words, the intuitive reason for this result is that when x is not observable, in order to preserve incentives to invest to wealthy people the planner cannot tax wealth below $\overline{x}$. On the same ground, in order to allow investment in lineages with low wealth the minimum amount of wealth to be transferred is, again, $\overline{x}$. I.e. *heterogeneity makes feasibility constraints more binding for the policy maker.* Due to the lack of information by the policy maker makes redistributive policies more persistent than in the absence of

[19] Clearly if x is observable, i.e. with no additional informational constraint induced by heterogeneity, policy effectiveness can be achieved under the same conditions as in the case of homogeneous costs.

heterogeneity in order to achieve similar results (all efficient agents manage to invest).

A less immediate result holds for the case in which the economy is on a declining path (in both regimes of high and low inequality), it holds:

Proposition 4. *Fix $\widetilde{\mu}_0$ as in Definition 4', T as in Definition 5' and $n^*(\widetilde{\mu}_0)$ as in Definition 6'. If $b_s < \overline{x}$ and $\underline{x} > \underline{b}$ (decline), then for any $G(x)$ satisfying Assumption 3, it holds:*

i) (High inequality) if

$$b_s \geq (1-\rho)\overline{x} + \rho x_e \tag{8}$$

then T permanently in place is such that $e = 1$ for a strictly positive measure of agents in each period and therefore $\mu_t(\underline{b}) < 1$

ii) (Low inequality) if

$$b_s \geq (1-\rho \cdot G(\tilde{x}))\,\tilde{x} + \rho \cdot G(\tilde{x}) \cdot E[G \mid x \leq \tilde{x}] \tag{9}$$

then T permanently in place such that $e = 1$ for a strictly positive measure of agents in each period and therefore $\mu_t(\underline{b}) < 1$.

In words, the proposition above states that in a scenario of decline (regardless the level of income inequality) policy is effective as long as the minimum wealth inherited by the offspring of a household in the skilled sector is large enough. This contrasts sharply with the case of homogeneity[20] and it allows the economy to sustain investment for a subset of agents provided some conditions on Γ hold. The intuition is the following: suppose that, given the initial distribution of wealth, it is feasible for the policy maker to tax households in the top segments of wealth distribution (there exist a positive measure of $b > \overline{x}$) and finance investment in lower segments. If the condition $(1-\rho)\overline{x} + \rho x_e \leq b_s$ holds[21], then this policy can be replicated for an arbitrarily large number of periods to avoid decline. The sufficient condition under which the policy can be replicated requires that the investment cost is low enough for a sufficiently large measure of agents, or the minimum level of wealth accumulated by agents in the skilled sector is large enough

[20]In the case homogeneous costs and here too, "reverse redistribution" can also sustain investment in a scenario of decline, by transferring resources from households in the unskilled occupations to households in skilled occupations, whose wealth, in the dynamics, would fall below x in finite time, in the absence of transfers. On the account of indivisibility it may be efficient to extract resources from the bottom of wealth distribution to finance investment at the top, when the top is not rich enough to finance investment, as, for example, from farmers to warriors and clerks in Middle Ages. We do not deal with these aspects here.

[21]Trivial algebra shows that the condition $(1-\rho)\overline{x} + \rho x_e \leq b_s$ is consistent with $b_s < \overline{x}$ since $0 < \rho < 1$.

(so that the scale of intervention satisfies the fiscal budget constraint). Under such condition T supports the same measure of investment next period and so on. It is of some relevance to notice that similar results would hold in terms of the effectiveness of policy reply to transitory shocks to wealth. Finally, it holds:

Corollary 1. *In a scenario of decline, if conditions (8) and (9) in Proposition 4 hold with strictly inequality, then a permanent policy makes it feasible for all lineages to invest in the long run, whenever it is efficient.*

Summarizing, we have shown that heterogeneity has two important implications for the design of feasible redistributive policies in the presence of indivisibilities. Since they require more information to target tax and subsidies to wealth segments of the population than in the homogeneous case, redistributive policies are subject to additional constraints that require *larger policy persistence* in order to be effective. At the same time, policies can be *more effective* in the presence of heterogeneity.

We conclude therefore that, in the presence of heterogeneity- even if x is not observable- decline can be prevented by a system of transfer policies. The same transfer system would fail for the same aggregate economy in the absence of heterogeneity.

6 Extensions

Most of our results in section 4 can be extended to less restrictive hypothesis on the main structure of the model.

Considering a different bequest motive would not change the results on the dynamic equilibrium of the model (see Piketty, 2000) provided that the main qualitative features of the bequest function Φ defining the evolution of wealth within lineages would be preserved [22].

We assumed that individual characteristics only matter in the skilled occupation on the presumption that the unskilled occupation represents the traditional sector where productive capabilities can be transferred within families and the standard technology does not require special individual talents and financial investment to be learned and applied to production. Individual characteristics can be a feature of individual performance in both sectors, in such a model the result on the ergodicity in Proposition 1 would

[22]More precisely, the main results of section 4 would be maintained to the extent the fundamentals are such that the topological properties of the inverse image set of the Φ function are preserved. The presence of a discontinuity in the bequest function is induced by the presence of fixed cost and the absence of financial market. The location of this discontinuity with respect to the boundary of the support of the fixed cost would dictate the properties of the model in terms of ergodicity. See Bernheim and Ray (1987) for the monotonicity almost everywhere of the bequest strategy in the case of dynastic preferences.

not be changed (if any, ergodicity would obtain *a fortiori*). Results in Proposition 2 would still hold with suitable qualifications. [23].

In the course of the paper we considered an i.i.d. process for the transmission of abilities within lineages, a positive correlation of genetic endowment (to include, for example, the effect of nurturing, as in Becker and Tomes, 1979) would increase persistence of wealth distribution without affecting the main results.

Finally, the analysis of the model with endogenous returns to factor could be extended as in Galor and Zeira ([9]) and Mookherjee and Napel ([13]). Endogenous wages would change the nature of the model making it an interactive model of wealth dynamics where the equilibrium dynamics is described by a non linear Markov process as in Banerjee and Newman ([3]) and this is not pursued here. [24]

As for the implications of heterogeneity on policy design, the argument for extension is more delicate, as already argued in section 5. Indeed the more prolonged time horizon required in order for policies to be effective can induce a deadweight losses due to expected taxation under alternative assumptions on the bequest motive. Such modifications would generate a model where the trade off between tax rates and tax base is relevant for policy design. Notice however, that the presence of distortionary effects by restricting the tax base would make the feasibility constraint even tighter, increasing the required time horizon, which is what we argue in the present setting. The results on the larger scope allowed by heterogeneity would also be influenced restricting the sufficient conditions in Propositions 3 and 4.

7 Conclusions

We studied the implications of individual heterogeneity for the evolution of wealth distribution in a standard model with financial market imperfections and local non convexities in education investment technology. We considered heterogeneity in the cost of education investment.

Ergodicity obtains in the case of low income inequality, i.e. whenever the

[23]Here the discussion in Galor and Zeira (1993) p.43 applies along similar lines. In particular poverty traps obtain to the extent that market luck in the unskilled sector would not be large enough to overcome financial market imperfections in the households there employed

[24]It is easy to see that a continuum of steady state (limit distributions) can arise whenever the initial condition on factor allocation to occupation is such that the induced factor prices are consistent with the conditions for the poverty trap characterized in section 4 even with endogenous factor returns. For example in the case where the initial allocation is concentrated in the traditional sector and $w - v$ is large, the economy exhibits non ergodicity, as in the case of high income inequality analyzed above. If the initial distribution allows some upward occupational mobility wealthy agents in the traditional sector, on the other hand, ergodicity is more likely to emerge along the lines and under the conditions studied in Mookherjee and Napel ([13] 2007).

distribution of education costs entails the presence of ability types for which the returns on educational investment is lower than the returns on financial investment, regardless of how "large" the support of the distribution of abilities is. Conversely, a poverty trap can emerge only if income inequality is large, i.e. educational investment is preferred by every agent in the economy.

Does ergodicity of the wealth distribution imply that lineages overcome financial market imperfections in the long run in our setting? The answer is a qualified no. If income inequality is low, the implication is that financial market imperfection can disappear. However, it is established that whenever households in the unskilled occupations are wealth constrained so must be agents in the skilled occupation with low wealth (a middle class), in every long run equilibrium with occupational mobility. If income inequality is large, the effects of financial market imperfection cannot disappear in the long run. In the case of poverty traps (no occupational mobility) the effects of financial market imperfection will not disappear by definition, for the subset of trapped lineages, as in the case of homogeneous costs. In the case of ergodic distribution wealth constraints will be binding in the limit too whenever the steady state entails occupational mobility. Once again, every lineage will meet wealth constraint in any history *and* in the steady state.

The persistence of financial market imperfection in cases when ergodicity is induced by heterogeneity and a more general interest about the role of the latter in policy design motivated our exploration of policy issues.

In particular we show that, in a well defined sense, models with heterogeneity call for more persistent policies. We also show that redistributive policies can be more effective in environments with heterogeneous agents, specially in economies featuring poverty traps or stagnation. In the latter case permanently taxing wealth above a (arbitrary) threshold can make investment self sustaining for a positive measure of agents in a world doomed to stagnation, provided that the distribution of abilities features enough agents with low cost of investment.

The results suggest that indeed temporary policies are less effective in altering educational investment, per capita income and inequality in the presence of heterogeneity. If the failure of financial markets in the process of education investment and its persistence in the development path is an important feature of the economy and if investment's financing has to rely on a system of public transfer then, heterogeneity in ability calls for the establishment of long run institutions for their design and implementation.

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Appendix A.

In the following we introduce some notation and a list of definitions that have been used in the paper and will be used in the remaining of the Appendix. Moreover we report a few preliminary results on the Markov process for the evolution of wealth distribution.

Definition 1. *A transition probability on Z is a function $Q : Z \times \mathcal{Z} \to [0,1]$ such that 1. $Q(b,.)$ is a probability measure and 2. $Q(.,S)$ is a $\mathcal{Z}-$measurable function on $\mathbb{R}_+$. Where $\mathcal{Z}$ is the collection of Borel sets on Z.*

Let b be a real number and S a Borel set, Φ defines a transition probability in our model. In formula, for any $b \in Z$ and $S \in \mathcal{Z}$, $Q(b;\, S)$ is defined as follows

$$Q(b,\, S) = \int_{\Phi^{-1}(S;\, b)} dG(x) \tag{10}$$

where $\Phi^{-1}(S;\, b) := \{x \in [\underline{x}, \overline{x}] \mid \Phi(b;\, x) \in S\}$. It is immediate to prove that Q satisfies Definition 1.

The above definition allows us to define the object of our analysis, the evolution of wealth distribution, in the standard way (see Stokey and Lucas, 1989) as stated in the next two definitions.

Definition 2. *An equilibrium distribution of bequest at time t is a probability measure satisfying $\mu_t = T^*\mu_{t-1}$; where $T^*\mu_{t-1} = \int_Z Q(b,S)(d\mu_{t-1})$, i.e. T^* is the self adjoint operator on Q.*

Definition 3. *A steady state (invariant) distribution of bequest is a measure μ satisfying $\mu^* = T^*\mu^*$.*

In the remaining of this appendix proofs of Lemma 2, Proposition 1, Lemma 3 and Proposition 2, are provided as listed.

Proof of Lemma 2, Limit Support.
If the initial set $S_0 = [b_{\min}, b_{\max}]$, then for all $b \in S_0$ and $x \in \Delta x$ it holds

$$\Phi(b_{\min};\, \overline{x}) \leq \Phi(b;\, x) \leq \Phi(b_{\max};\, \underline{x})$$

it means that $S_n = [\Phi^n(b_{\min};\, \overline{x}), \Phi^n(b_{\max};\, \underline{x})]$ by trivial algebra and using the definition of Φ we get

$$\Phi^n(b_{\min};\, \overline{x}) = \rho^n b_{\min} + \left(\sum_0^{n-1} \rho^n\right) ((1-\alpha)v)$$

$$\Phi^n(b_{\max};\, \underline{x}) = \rho^n b_{\max} + \left(\sum_0^{n-1} \rho^n\right) ((1-\alpha)w - \rho\underline{x})$$

Since $\rho < 1$, take $n \to \infty$, then $\Phi^n(b_{\max}; \underline{x}) \to \overline{b}$ and $\Phi^n(b_{\min}; \overline{x}) \to \underline{b}$. $\square$

Proof of Proposition 1.
Part i) If $\underline{b} < \underline{x}$ for any initial wealth, b, there exists N_b such that after $n > N_b$ generations with low ability offspring the lineage will end up with wealth below $\underline{x}$.
If $\underline{b} = \underline{x}$ the unskilled households converge to $\underline{x}$ in this way they reduce the probability of investment. In the long rum the probability goes to zero for all the households.

Part ii) Following Stokey and Lucas (1989) if Condition M holds then T^* is a contraction, and if T^* is a contraction then the Markov process is ergodic.
Condition M: there exists $\epsilon > 0$ and an integer $N \geq 1$ such that for any $S \in \mathcal{B}$ either $Q^N(b, S) \geq \epsilon$, all $b \in B$, or $Q^N(b, S^c) \geq \epsilon$, all $b \in B$, $Q^N(b, S)$ is the probability that a lineage starting from b, after N generations arrives to a wealth level in $S \cap \Phi^N(b)$.

We proceed in two steps:

Step 1: If $\underline{x} < \underline{b}$, then $\lim_{N\to\infty} \Phi^N(b) = S_\infty$, for all $b \in B$.
Recall $\Phi(b) = \{\Phi(b;\, x) \mid x \in \Delta x\}$ then $\Phi^N(b) = \cup_{b'^{N-1}(b)} \Phi(b')$, for $N > 1$.
Since $\underline{x} < \underline{b}$ there is upward mobility, the probability to reach $\overline{b}$ is different from zero. On the other hand, there always exist a sequence of arbitrary lenght such that a lineage experiencing $x > \widetilde{x}$ has positive measure, due to $\widetilde{x} \in \Delta x$ inducing downward occupational mobility which, in turn, makes wealth state $\underline{b}$ reachable starting from any initial wealth in B, so $\lim_{N\to\infty} \Phi^N(b) = S_\infty$.

Step 2: Suppose that Condition M is not satisfied, then there must exist a borel set $S \cap S_\infty \neq \emptyset$, such that for any N we have:

$$b_N \in B : \quad \Phi^N(b_N) \cap S = \emptyset$$

Using Step 1, for all b_N we have that $\lim_{n\to\infty} \Phi^n(b_N) = S_\infty$, then the sequence $\{\Phi^N(b_N)\}_N$ converges to S_∞. We arrive to a contradiction

$$\emptyset = \lim_{N\to\infty} (S \cap \Phi^N(b_N)) = S \cap \lim_{N\to\infty} \Phi^N(b_N) = S \cap S_\infty \neq \emptyset.$$

$\square$

Proof of Lemma 3.
Recall that, $\underline{b} = \frac{(1-\alpha)v}{1-\rho}$ and $b_s = (1-\alpha)w$, then

$$\frac{(1-\alpha)v}{1-\rho} > (1-\alpha)w \Leftrightarrow (1-\alpha)w > \frac{w-v}{1+r}.$$

$\square$

Proof of Proposition 2.
Part i) The proof is the same as for point i) of Proposition 1.
Part ii) For $\overline{x} > b_s$ the proof is the same as for point ii) in Proposition 1. If $\overline{x} < b_s$ the offspring in skilled households will always invest, $\forall x \in \Delta x$, i.e. there is no downward occupational mobility. Moreover, since $\underline{x} < \underline{b}$ every lineage will be able to invest in the long run term.
Part iii) If $\underline{b} \leq \underline{x}$ and $\overline{x} < b_s$ then the limit distribution depends on the initial distribution μ_0. This is easy to show by considering two extreme cases. Consider μ_0 that has full measure on $[\underline{b}, \underline{x}]$, in this case no lineage can switch to skilled occupations and every lineage converges to $\underline{b}$.

On the other hand, consider μ_0 concentrated on $[b_s, \overline{b}]$, any agent can cover investment cost, no downward occupational mobility flows. The equilibrium distribution has support in $[b_s, \overline{b}]$. $\square$

Appendix B: Proofs of results in Section 5

Here we introduce some definitions used in propositions and lemmas in section 5. These definitions regards initial wealth, the class of tax transfer schemes within which the results are derived and the minimum number of periods to achieve the policy target. In the rest of the Appendix we report the proofs of Section 5.

As for the case of Homogeneous costs the following definitions are considered:

Definition 4. *Let $\widetilde{\mu}_0$ be a distribution of wealth such that $\int_0^\infty b\, d\widetilde{\mu}_0 = x_e$.*

Definition 5. *Let $T = \{\sigma, \tau\}$ be a tax transfer scheme such that $b - \tau \geq x_e$ and $\sigma - b \leq x_e$ (i.e. T preserves incentives to investment by taxed agents).*

Next define the number of periods the policy has to be in place in order to achieve investment whenever efficient

Definition 6. *Let $n^*(\widetilde{\mu}_0)$ the minimum number of periods such that the transfer system T is in place and $\mu_{n^*+1}(x_e) = 0$.*

As for the case of Heterogeneous costs the following definitions are considered:

Definition 4' *Let $\widetilde{\mu}_0$ be distribution of wealth such that: $\int_0^\infty b\, d\widetilde{\mu}_0 = x_e$ and $\int_{\overline{x}}^\infty d\widetilde{\mu}_0 > 0$.*

Definition 5' *Let $T = \{\sigma, \tau\}$ be a tax transfer scheme such that $b - \tau \geq \min\{\overline{x}, \widetilde{x}\}$, and $\sigma - b \leq \min\{\overline{x}, \widetilde{x}\}$.*

Definition 6' *Let $n^*(\widetilde{\mu}_0)$ the minimum number of periods such that the transfer system T is in place and $\mu_{n^*+1}(\overline{x}) = 0$.*

Endowed with these definitions we prove Propositions 3 and 4, Corollary 1 in sequence.

Proof of Proposition 3 (poverty trap).
Fix $\widetilde{\mu}_0$ according to Definition 4'. Hence it is possible for the policy maker to set $T = \{\sigma, \tau\}$ such that:

for all $b \geq \overline{x}$ then $\tau(b) = b - \overline{x}$; moreover if $b < \overline{x}$ then $\sigma(b) = \overline{x} - b$, for some b.

Since $\int_0^\infty b\, d\widetilde{\mu}_0 = x_e$, then there exists $y < \overline{x}$ such that it is feasible for the policy maker $\sigma(b) > 0$ for a measure of lineage given by $\int_0^y d\widetilde{\mu}_0 > 0$. Since $y < \overline{x}$, then it is immediate to conclude that $n^*(\mu_0) > 1$.

Notice that $\Phi(\overline{x};\, x) \geq b_s > \overline{x}$, implies that there are no downward mobility after the first intervention. The transfer scheme can, therefore, be replicated to make the measure of investing lineages increasing over time reaching 1 in finite time. After that the scheme can be dismantled. □

Proof of Proposition 4 (decline).
Remember that decline can occur both in the case of low and high income inequality. We prove the result for both cases separately. For both configurations we show that, in the set of policies satisfying Definition 5', there exist redistributive schemes such that permanent intervention prevents the economy to decline, i.e. the measure of non investing agent induced by the policy is $\mu_n(\underline{b}) < 1$. Suppose x is not observable, whereas individual wealth is observable.

Consider $\widetilde{\mu}_0$ satisfying Definition 4'. Without loss of generality define M_1 the measure of agents such that after redistribution their wealth is equal to $\min\{\overline{x},\, \tilde{x}\}$.

High income inequality ($\tilde{x} > \Delta x$). Consider the following redistributive scheme $T = \{\sigma, \tau\}$ such that:

for all $b \geq \overline{x}$ then $\tau(b) = b - \overline{x}$; moreover if $b < \overline{x}$ then $\sigma(b) = \overline{x} - b$ for some b

Define:

$$y_{\overline{x}} := \frac{b_s - (1-\rho)\overline{x}}{\rho}$$

Since for all $b \geq \overline{x}$ then $\tau(b) = b - \overline{x}$ and $1 - \mu_0(\overline{x}) > 0$ there exists, after tax a mass of agents bequething $\Phi(\overline{x};\, x)$.

Remember that under parameter configurations such that the economy is in decline it holds: $b_s \in \Delta x$, therefore simple algebra shows (use the definition of Φ) that there exists $y_{\overline{x}} \in \Delta x$ such that, for the mass of agents M_1 it holds:

$$\Phi(\overline{x};\, x) = \begin{cases} > \overline{x} & x < y_{\overline{x}} \\ < \overline{x} & x > y_{\overline{x}} \end{cases} \tag{11}$$

Which, in turn, implies that $G(x)$ is such that there exists a positive measure of lineages- in the mass of M_1, who bequeth a larger amount of wealth than the wealth (net of taxation) they received. In words: if the scheme is initially feasible the policy maker can replicate it for a subset of M_1. Next we show the sufficient condition such that M_1 is sustainable forever on. To this aim define $E(M_1)$ the tax base that can be raised by agents in M_1 and $U(M_1)$ the total amount of subsidies required to sustain investment in M_1. In particular: the measure of unconstrained agents in M_1 is given $M_1 \cdot G(y_{\overline{x}})$, for any $b \geq \overline{x}$ the tax that can be raised is $E[\Phi(\overline{x};\, x) - \overline{x} \mid x \leq y_{\overline{x}}]$. Therefore, for any given M_1 (depending on $\widetilde{\mu}_0$) total tax revenues next period will be given by

$$E(M_1) := \underbrace{M_1 \cdot G(y_{\overline{x}})}_{Pr(b \geq \overline{x})} \cdot \underbrace{E[\Phi(\overline{x};\, x) - \overline{x} \mid x \leq y_{\overline{x}}]}_{\text{Average Tax Revenue}}$$

where $G(y_{\overline{x}})$ measures the probability that an agent features investment cost below $y_{\overline{x}}$, $E[\Phi(\overline{x};\, x) - \overline{x} \mid x \leq y_{\overline{x}}]$ is the tax base for the next generation.

The subset of M_1 requiring a subsidy to investment is given by a measure $M_1(1 - G(y_{\overline{x}}))$, total susidies are given by:

$$U(M_1) :=) \underbrace{M_1 \cdot (1 - G(y_{\overline{x}})}_{Pr(b<\overline{x})} \cdot \underbrace{E[\overline{x} - \Phi(\overline{x};\, x) \mid x > y_{\overline{x}}]}_{\text{Average subsidy}}$$

where $E[\overline{x} - \Phi(\overline{x};\, x) \mid x > y_{\overline{x}}]$ is the conditional average subsidy.

It is immediate to see that the transfer scheme is self sustainable for a measure M_1 if the following feasibility constraint holds:

$$G(y_{\overline{x}}) \cdot E[\Phi(\overline{x};\, x) - \overline{x} \mid x \leq y_{\overline{x}}] \geq (1 - G(y_{\overline{x}})) \cdot E[\overline{x} - \Phi(\overline{x};\, x) \mid x > y_{\overline{x}}] \quad (12)$$

Use the definition of average tax

$$E[\Phi(\overline{x};\, x) - \overline{x} \mid x \leq y_{\overline{x}}] = \frac{\int_{\underline{x}}^{y_{\overline{x}}} [\Phi(\overline{x};\, x) - \overline{x}] dG}{G(y_{\overline{x}})} \quad (13)$$

and the definition of average subsidy:

$$E[\overline{x} - \Phi(\overline{x};\, x) \mid x > y_{\overline{x}}] = \frac{\int_{y_{\overline{x}}}^{\overline{x}} [\overline{x} - \Phi(\overline{x};\, x)] dG}{1 - G(y_{\overline{x}})} \quad (14)$$

Replace the last two equations into (12), simple algebra shows that feasibility holds if

$$b_s \geq (1 - \rho)\overline{x} + \rho x_e$$

Notice that this condition (and 12) does not depend neither upon M_1, nor upon $\widetilde{\mu}_0$. In other words the latter is a sufficient condition for any mass of investment M_1 (depending on $\widetilde{\mu}_0$) to be self sustainable.

Low Income Inequality ($\tilde{x} \in \Delta x$). The proof for this case reproduces the key steps taking into account that we assume to tax wealth above the highest efficient cost which now is $\tilde{x}$. If $\tilde{x} \in \Delta x$ notice that agents with $x > \tilde{x}$ will not invest, therefore the policy maker can devise a tax transfer scheme such that wealth is taxed above $\tilde{x}$ and subsidy is below $\tilde{x}$ and still preserve investment opportunities in wealthy lineages (those with $b > \tilde{x}$). Replicating the same argument as in the previous proof we conclude that a permanent tax transfer scheme satisfying feasibility constraints in each period exists if

$$G(y_{\tilde{x}}) \cdot E[\Phi(\tilde{x};\, x) - \tilde{x} \mid x \leq y_{\tilde{x}}] \geq (1 - G(y_{\tilde{x}})) \cdot E[\tilde{x} - \Phi(\tilde{x};\, x) \mid x > y_{\tilde{x}}] \quad (15)$$

Once again the condition does not depend upon M_1. Using the definition of average tax revenue and average subsidy get

$$E[\Phi(\tilde{x};\, x) - \tilde{x} \mid x \leq y_{\tilde{x}}] = \frac{\int_{\underline{x}}^{y_{\tilde{x}}} [\Phi(\tilde{x};\, x) - \tilde{x}] dG}{G(y_{\tilde{x}})} \tag{16}$$

and

$$E[\tilde{x} - \Phi(\tilde{x};\, x) \mid x > y_{\tilde{x}}] = \frac{\left[\int_{y_{\tilde{x}}}^{\tilde{x}} [\tilde{x} - \Phi(\tilde{x};\, x)] dG + \int_{\tilde{x}}^{\overline{x}} [\tilde{x} - b_s] dG\right]}{1 - G(y_{\tilde{x}})} \tag{17}$$

Then replacing (16) e (17) into (15) we get the sufficient conditionfor a permanent tax transfer scheme to be feasible avoiding decline for a subset of agents with positive measure:

$$b_s \geq (1 - \rho \cdot G(\tilde{x}))\, \tilde{x} + \rho \cdot G(\tilde{x}) \cdot E[G \mid x \leq \tilde{x}]$$

The proof can easily be generalised to an arbitrary level of wealth x' beyond which taxation $\tau = b - x' > 0$ applies.

□

Proof of Corollary 1.
If conditions (8) and (9) hold with strictly inequality, the policy maker can use the surplus from tax revenues to subsidize a strictly positive fraction $\epsilon(1 - M_1)$ agents in the second round.

The new mass of unconstrained agents is $M_1 + \epsilon(1 - M_1)$, so $(1-\epsilon)(1-M_1)$ is the measure of agents who do not receive the subsidy. It is easy to prove that after n generation the measure of agents who do not receive the subsidy is $(1 - \epsilon)^n(1 - M_1)$. Since $\epsilon < 1$, when $n \to \infty$ then $(1 - \epsilon)^n(1 - M_1) \to 0$.

□