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# Monetary Policy and Stock-Price Dynamics in a DSGE Framework

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#### Abstract

This paper analyzes the role of stock prices in driving Monetary Policy for price stability in a Non-Ricardian DSGE model. It shows that the dynamics of the interest rate consistent with price stability requires a response to stock-price changes that depends on the shock driving them: a supply shock (e.g. productivity) does not require an additional, dedicated response relative to the standard Representative-Agent framework, while a demand shock does. Moreover, we show that implementing the exible-price allocation by means of an interest-rate rule that reacts to deviations of the stock-price level from the exible-price equilibrium incurs risks of endogenous instability that are the higher the less profitable on average equity shares. On the other hand, reacting to the stock-price growth rate is risk-free from the perspective of equilibrium determinacy, and can be beneficial from an overall real stability perspective.

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## 1 Introduction

Over the past three decades, with inflation successfully kept under control after the tumultuous 1970's, one of the major issues that Central Bankers had to learn to cope with was financial stability. The events of the last decade (the burst of the dotcom bubble in 2001 and the recent global financial meltdown), generated a revived interest, in the economic literature, in the links between monetary policy and stock-price dynamics, and gave new scope for a debate about the desirability that Central Banks be directly concerned with financial stability.<sup>1</sup> Among the others, one interesting issue being debated is the understanding of what should be the appropriate response of monetary policy makers in the face of real effects of large swings in stock prices, and whether an explicit concern about stock-price dynamics might improve macroeconomic performance.<sup>2</sup>

The issue was analyzed in a variety of setups, both theoretical and empirical, and the consequent debate is still very controversial, under many respects. The main stream of contributions analyzes the issue within a Dynamic New Keynesian (DNK) model with financial frictions,<sup>3</sup> where shocks to stock prices propagate to real activity by affecting the financial conditions of firms and thereby triggering a financial accelerator mechanism. Monetary policy in this context is analyzed by assessing the macroeconomic implications, for a calibrated economy, of augmenting a standard Taylor-type interest rule with an explicit response to deviations of the stock-price level from a given target.

The implications of this body of literature are not at all univocal and the debate does not seem settled yet. On one side, Bernanke and Gertler (1999) and (2001) conclude that since the macroeconomic relevance of stock-price dynamics relies on its links with inflation, a flexible inflation targeting approach is sufficient to achieve both price and financial stability, and that reacting to stock prices induces a perverse outcome in terms of output dynamics. Analogously, Gilchrist and Leahy (2002) find that both standard DNK models and economies featuring financial frictions, best replicate the dynamic properties of the benchmark RBC framework when no dedicated response is granted to stock-price dynamics. On the other hand, Cecchetti et Al. (2000), (2002), (2003), though emphasizing the difference between *targeting* stock-price stability and *reacting* to stockprice misalignments, strongly recommend that a Central Bank that recognizes a bubble in the dynamics of the stock market react to it; the conclusion is motivated on the grounds of simulations of the same model as in Bernanke and Gertler (1999), showing that the perverse outcome reported by the latter can be ruled out by simply adding a reaction to the output gap in the Taylor rule, and that adding a reaction to stock prices reduces overall volatility in the economy. To assess the links between stock prices, inflation and monetary policy, Carlstrom and Fuerst (2001) derive analytically the welfare-maximizing monetary policy in a flexible-price general equilibrium model with financial frictions. They show that, notwithstanding the absence of nominal rigidities – and hence the costs of inflation – a welfare-improving role for reacting to stock prices emerges, insofar

<sup>&</sup>lt;sup>1</sup>Mishkin and White (2002) highlight the difference between financial instability and stock market crashes, maintaining that the real concern of monetary policy makers should be the former, rather than the latter. The strong point they make is related to firms' balance sheets conditions and seems weaker when it comes to the possible real effects through households' wealth. Truth is, anyhow, that the stock market fragility is more often than not a highly sensitive indicator of financial instability, especially in periods of financial sophistication like the ones we live in.

<sup>&</sup>lt;sup>2</sup>See Nisticò (2005a) for an extensive survey.

 $<sup>^{3}</sup>$ The analytical framework exploited is the one developed in Bernanke, Gertler and Gilchrist (1999), augmented to allow for stochastic bubbles.

as it can counteract the inefficient response of the economy to shocks to the equity market, which propagate through the binding collateral constraints. With a more recent contribution, Carlstrom and Fuerst (2007) re-enter the debate and analyze the issue of equilibrium determinacy in a standard, representative-agent, DNK model in which the Central Bank responds also to stock prices. In their framework, however, the latter are redundant for the equilibrium allocation, unless monetary policy explicitly responds to them. Accordingly, a monetary policy rule including a response to some measure of stock-market dynamics is never optimal (whatever the concept of "optimality" considered). Indeed, in such setup, the authors show that reacting to stock prices raises the risks of inducing real indeterminacy in the system. This result had already been pointed out by Bullard and Schaling (2002) in an even simpler setup, in which stock prices are driven (to first order) only by the short-term interest rate.<sup>4</sup>

To our knowledge, therefore, all contributions analyzing the topic for micro-founded New-Keynesian setups, focus on the supply-side effects of stock-price dynamics, when considering any real effect at all.

A second stream of literature, to which this paper is also related, focuses on the analysis of a highly stylized and parsimonious Dynamic Stochastic New Keynesian model, describing the economy with two simple equations: an IS curve for the demand-side, and a New Keynesian Phillips Curve for the supply side. The major advantage of such model, and the reason of its widespread popularity for policy analysis, consists in its extreme tractability, which allows for analytical derivation of both endogenous dynamics and optimal monetary policy.<sup>5</sup> This model, which we will refer to as the *Standard Dynamic New Keynesian model* (SDNK), however, does not explicitly consider the dynamics of stock prices and their interplay with the business cycle and the conduct of monetary policy.

This paper aims at filling this gap, and presents a framework (which nests the *SDNK model* as a special case) which achieves both preservation of high tractability and explicit consideration of stock prices as a non-redundant variable for the business cycle. More specifically, we analyze the links between monetary policy, price stability, and stock-price dynamics within a tractable DSGE New-Keynesian model in which agents are non-ricardian and stock prices thereby affect real activity through wealth effects on consumption. In this way we establish an active role for stock prices in affecting the business cycle and a theoretical motive for the Central Bank to react to their dynamics. We analyze the implications of this extension for price stability and the role of the interplay between monetary policy and stock-price dynamics.<sup>6</sup>

We depart from the *SDNK model* mainly along two dimensions. First, while in the SDNK model profits from the monopolistic sector are uniformly distributed among households, here we assume there exists a market for shares on those profits: the stock market. The households then can choose to allocate their savings by either buying state-contingent assets or a portfolio of private stocks. This assumption implies endogenous stock-price dynamics.

<sup>&</sup>lt;sup>4</sup>Other contributions using different approaches (and drawing different conclusions) from each other can be found in Chadha, Sarno and Valente (2004), Cogley (1999), Filardo (2000), Faia and Monacelli (2005), Goodhart and Hofmann (2002), Gruen, Plumb and Stone (2005), Ludvigson and Steindel (1999), Miller, Weller and Zhang (2001), Mishkin (2001), and Schwartz (2002).

<sup>&</sup>lt;sup>5</sup>For a thorough analysis using this baseline model, a detailed discussion and complete references, see Wood-ford (2003), Ch. 4, and Galì (2003).

<sup>&</sup>lt;sup>6</sup>In the same spirit, Curdia and Woodford (2009, 2010, 2011) extend the SDNK model by introducing heterogeneity in households' patience and financial intermediaries to study analytically the role of unconventional monetary policy.

Second, we model the demand-side of the economy along the lines traced by Yaari (1965) and Blanchard (1985): every period, a constant fraction of agents in financial markets is randomly replaced by newcomers holding zero-wealth. While the SDNK model features a representative agent, here we introduce heterogeneity in households, related to the accumulated stock of financial wealth. The demand-side hence takes the form of a stochastic "perpetual youth" model, for which a closed-form solution for aggregate consumption within a closed economy is derived by Chadha and Nolan (2003) and Piergallini (2006).<sup>7</sup> The interplay between "newcomers" entering the markets with no wealth and "old traders" with accumulated wealth drives a wedge between the stochastic discount factor pricing all securities and the average marginal rate of intertemporal substitution in consumption, which in the case of infinitely-lived consumers coincide. In equilibrium, this wedge affects the growth rate of aggregate consumption, and makes stock prices a non-redundant asset even with complete markets. Recently, Castelnuovo and Nisticò (2010) have estimated an empirical version of the model presented here, using Bayesian Structural techniques, and have found this channel to be rather relevant, at least for the US: the model with wealth effect outperforms, from a bayesian perspective, all alternative empirical specifications, and the estimated replacement rate ranges between 10 and 20 percent, implying a response of output and inflation to a 1% financial shock of about .1 and .08 percent, respectively.

Within this framework we pinpoint the role of stock prices in determining the monetary policy consistent with price stability. We do so by comparing the implied *Wicksellian* Monetary Policy<sup>8</sup> with the one stemming from the benchmark Representative-Agent (RA) case, in which stock prices are redundant for the equilibrium allocation. Additionally, we also discuss the implementation of *Wicksellian* policy through simple and operational rules, studying the cyclical and dynamic implications of adding an explicit reaction to stock prices.

The main results are threefold.

First. The *Wicksellian* Monetary Policy implicit response to a given swing in stock prices depends on the structural shocks driving it. Demand shocks require an additional, dedicated response relative to the benchmark RA case, while supply shocks do not.

Second. The Taylor Principle is no longer a sufficient condition for equilibrium determinacy: it is shown, in fact, that for a given response to inflation, reacting too aggressively to deviations of the *stock-price level* from a chosen target may produce endogenous instability. We also show that the risks of inducing equilibrium indeterminacy by reacting to stock prices are the lower the higher the market power of listed firms (i.e. the more profitable on average their equity shares). The qualitative results of Carlstrom and Fuerst (2007) and Bullard and Schaling (2002), therefore, are confirmed even in a setup in which stock prices feed back into the demand-side of the economy (unlike in Carlstrom and Fuerst, 2007 and Bullard and Schaling, 2002).

Third. While responding to deviations of the stock-price *level* from a given target might induce indeterminacy, responding to deviations of the stock-price *growth rate* does not entail the same indeterminacy risks and can potentially imply substantial stability gains for inflation and interest rates.

The remaining of the paper is structured as follows. Section 2 presents a DSGE model of the business cycle in which the micro-founded stock-price dynamics have real effects on consumption.

<sup>&</sup>lt;sup>7</sup>Cardia (1991) derives an analogous solution within a small open economy framework.

<sup>&</sup>lt;sup>8</sup>In analogy with Woodford (2003), in the following we label as *Wicksellian* the monetary policy replicating the flexible-price allocation, by inducing the "natural" interest rate dynamics, consistent with flexible prices.

Section 3 analyzes the *Wicksellian* Monetary Policy, and discusses the issue of implementation and equilibrium determinacy. Section 4 analyzes the macroeconomic implications of adopting different instrument rules under alternative policy regimes. Section 5 finally summarizes and concludes.

## 2 A Structural Model with Stock-Wealth Effects.

#### 2.1 The Demand-Side.

The demand-side of the economy is a discrete-time stochastic version of the perpetual youth model introduced by Blanchard (1985) and Yaari (1965).<sup>9</sup> The economy consists of an indefinite number of cohorts, facing a constant probability  $\gamma$  of being replaced before the next period begins. To abstract from population growth the cohort size is set to  $\gamma$ .

Each household is assumed to have Cobb-Douglas preferences over consumption and leisure;<sup>10</sup> moreover, we assume that such preferences are subject to aggregate, exogenous stochastic shocks shifting the marginal utility of consumption,  $\mathcal{V}_t \equiv \exp(\nu_t)$ . These are *intertemporal* disturbances, which affect the equilibrium stochastic discount factor and, therefore, the dynamics of stock prices.

Consumers demand consumption goods and two types of financial assets: state-contingent bonds and equity shares issued by the monopolistic firms, to which they also supply labor. Equilibrium of this side of the economy, along a state equation for consumption, also yields a pricing equation for the equity shares.

Consumers entering financial markets in period j, therefore, seek to maximize their expected lifetime utility, discounted to account for impatience (as reflected by the intertemporal discount factor  $\beta$ ) and uncertain stay in financial markets (as reflected by the probability of "survival" across two subsequent periods:  $1 - \gamma$ ). To that aim, they choose a pattern for real consumption  $(C_{j,t})$ , hours worked  $(N_{j,t})$  and financial-asset holdings. The financial assets holdings at the end of period t consist of a set of contingent claims whose one-period ahead stochastic nominal payoff in period t + 1 is  $B_{j,t+1}^*$  and the relevant discount factor is  $\mathcal{F}_{t,t+1}$ , and a set of equity shares issued by each intermediate good-producing firm,  $Z_{j,t+1}(i)$ , whose real price at period t is  $Q_t(i)$ .

The optimization problem faced at time 0 by the j-periods-old representative consumer is therefore to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t (1-\gamma)^t \mathcal{V}_t \Big[ \delta \log C_{j,t} + (1-\delta) \log(1-N_{j,t}) \Big]$$

subject to a sequence of budget constraints of the form:

$$P_t C_{j,t} + E_t \{ \mathcal{F}_{t,t+1} B_{j,t+1}^* \} + P_t \int_0^1 Q_t(i) Z_{j,t+1}(i) \, \mathrm{d}i \le W_t N_{j,t} - P_t T_t + \Omega_{j,t}^*, \tag{1}$$

<sup>&</sup>lt;sup>9</sup>For other stochastic discrete-time versions of the perpetual youth model see Airaudo, Nisticò and Zanna (2007), Annicchiarico, Marini and Piergallini (2008), Cardia (1991), Castelnuovo and Nisticò (2010), Chadha and Nolan (2001) and (2003), Di Giorgio and Nisticò (2007, 2010), Nisticò (2011), and Piergallini (2006). For nonstochastic discrete-time versions see, among the others, Cushing (1999), Frenkel and Razin (1986), Smets and Wouters (2002).

<sup>&</sup>lt;sup>10</sup>The assumption of Cobb-Douglas preferences is key to retrieve time-invariant parameters characterizing the equilibrium conditions. See Smets and Wouters (2002) for a non-stochastic framework with CRRA utility. Ascari and Ranking (2007) have recently argued that models with perpetual youth and endogenous labor supply should feature GHH preferences, in order to avoid issues related to the negative labor supply of oldest generations. We acknowledge this contribution but choose to stick with standard preferences in order to preserve comparability with existing DNK literature.

where  $\gamma \in [0, 1]$ ,  $T_t$  denotes lump-sum taxes – which we assume are uniformly distributed across cohorts – and  $\Omega_{j,t}^*$  denotes the nominal financial wealth carried over from the previous period, defined as:

$$\Omega_{j,t}^* \equiv \frac{1}{1-\gamma} \left[ B_{j,t}^* + P_t \int_0^1 \left( Q_t(i) + D_t(i) \right) Z_{j,t}(i) \, \mathrm{d}i \right].$$
(2)

The first-order conditions for an optimum consist of the budget constraint (1) holding with equality, the intra-temporal optimality condition with respect to consumption and leisure

$$C_{j,t} = \frac{\delta}{1-\delta} \frac{W_t}{P_t} (1-N_{j,t}), \qquad (3)$$

and the inter-temporal conditions with respect to the two financial assets:

$$\mathcal{F}_{t,t+1} = \beta \frac{U_c(C_{j,t+1})\mathcal{V}_{t+1}}{U_c(C_{j,t})\mathcal{V}_t} = \beta \frac{P_t C_{j,t}}{P_{t+1} C_{j,t+1}} \exp(\nu_{t+1} - \nu_t)$$
(4)

$$P_t Q_t(i) = E_t \Big\{ \mathcal{F}_{t,t+1} P_{t+1} \Big[ Q_{t+1}(i) + D_{t+1}(i) \Big] \Big\}.$$
(5)

The nominal gross return  $(1 + r_t)$  on a safe one-period bond paying off one unit of currency in period t + 1 with probability 1 (whose price is therefore  $E_t \{\mathcal{F}_{t,t+1}\}$ ) is defined by the following non-arbitrage condition:

$$(1+r_t)E_t\{\mathcal{F}_{t,t+1}\} = 1.$$
 (6)

Equation (5), finally equates the nominal price of an equity share to its nominal expected payoff one period ahead, discounted by the stochastic factor  $\mathcal{F}_{t,t+1}$ , and defines the stock-price dynamics.

Using equations (5), (2) and (3), the equilibrium budget constraint (1) can be given the form of the following stochastic difference equation in the financial wealth  $\Omega_{i,t}^*$ :

$$\frac{1}{\delta} P_t C_{j,t} + E_t \Big\{ \mathcal{F}_{t,t+1} (1-\gamma) \Omega_{j,t+1}^* \Big\} = W_t - P_t T_t + \Omega_{j,t}^*.$$
(7)

Let's now define the nominal human wealth for cohort j at time t  $(h_{j,t}^*)$  as the expected present discounted value of the household's endowment of hours, net of taxes:

$$h_{j,t}^* \equiv E_t \Biggl\{ \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k} (1-\gamma)^k (W_{t+k} - P_{t+k} T_{t+k}) \Biggr\}.$$
 (8)

Notice that the assumption that taxes are common across cohorts implies that human wealth is also independent of age  $(h_{i,t}^* = h_t^*)$ .

As described in Piergallini (2006), using the equilibrium stochastic discount factor (4), a usual No-Ponzi-Game condition and the definition of human wealth (8), allows the stochastic difference equation in the financial wealth (7) to be solved forward, and to retrieve the equation that describes nominal individual consumption as a linear function of total real financial and human wealth:

$$C_{j,t} = \frac{\delta}{\Sigma_t} (\Omega_{j,t} + h_t), \tag{9}$$

in which let  $X \equiv X^*/P$ , for  $X = \Omega$ , h, and  $\Sigma_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^k (1-\gamma)^k \exp(\nu_{t+k} - \nu_t) \right\}$  is the reciprocal of the time-varying propensity to consume out of financial and human wealth, which is common

across cohorts (being a function of the aggregate preference shocks).<sup>11</sup> A current positive innovation in the preference shock, therefore, by reducing the present value of future stochastic payoffs, has the effect of increasing the current propensity to consume out of wealth.

It is useful, at this point, to characterize the agents interacting in the financial markets in period t as belonging to either one of two subsets: the "old traders" are those agents that are in the markets since at least one period (i.e.  $j \ll t$ ), while the "newcomers" are those that entered the markets in the current period (i.e. j = t). Since the latter enter the markets with no financial wealth ( $\Omega_{t,t} = 0$ ), in t they can consume only out of their human wealth:

$$C_{t,t} = \frac{\delta}{\Sigma_t} h_t. \tag{10}$$

On average, therefore, newcomers consume less than old traders because they hold a smaller amount of total wealth. This partition is important because all the action in this framework is going to originate from the difference between these two subsets of agents, and their interaction in financial markets.

#### 2.1.1 Aggregation across Cohorts

The aggregate *per-capita* levels across cohorts for each generation-specific variable  $X_j$  are computed as the weighted average  $X_t \equiv \sum_{j=-\infty}^t \gamma(1-\gamma)^{t-j} X_{j,t}$ , for all X = C, N, B, T, Z(i).

Hence, the solution of the consumers' problem implies the following aggregate relations:

$$C_t = \frac{\delta}{1-\delta} \frac{W_t}{P_t} (1-N_t), \tag{11}$$

$$C_t + B_t + \int_0^1 Q_t(i) Z_t(i) \, \mathrm{d}i = \frac{W_t}{P_t} N_t - T_t + \Omega_t$$
(12)

$$C_t = \frac{\delta}{\Sigma_t} (\Omega_t + h_t). \tag{13}$$

As its generation-specific counterpart, the aggregate budget constraint (12) can be given the form of a stochastic difference equation in aggregate wealth:

$$\frac{1}{\delta}C_t + E_t \{ \mathcal{F}_{t,t+1} \Pi_{t+1} \Omega_{t+1} \} = \frac{W_t}{P_t} N_t - T_t + \Omega_t,$$
(14)

in which  $\Pi \equiv 1 + \pi$  denotes the gross inflation rate and aggregate wealth is defined as

$$\Omega_t \equiv \left[ B_t + \int_0^1 \left( Q_t(i) + D_t(i) \right) Z_t(i) \,\mathrm{d}i \right]. \tag{15}$$

Finally, the above equation (14) and equation (13) form a system whose solution is the equation describing the dynamic path of aggregate consumption, in which the first term represents the financial wealth effects, which fade out as the replacement rate ( $\gamma$ ) goes to zero:<sup>12</sup>

$$(\Sigma_t - 1)C_t = \gamma \delta E_t \{ \mathcal{F}_{t,t+1} \Pi_{t+1} \Omega_{t+1} \} + (1 - \gamma) E_t \{ \mathcal{F}_{t,t+1} \Sigma_{t+1} \Pi_{t+1} C_{t+1} \}.$$
(16)

<sup>&</sup>lt;sup>11</sup>For details on the derivation refer to the Appendix and the working paper version of the paper (Nisticò, 2005b). For details on the derivation in a framework with constant propensity to consume out of wealth see Piergallini (2006).

<sup>&</sup>lt;sup>12</sup>For details refer to the Appendix.

## 2.2 The Supply-Side and Inflation Dynamics.

The supply-side of the economy consists of two sectors of infinitely-lived agents: a retail sector operating in perfect competition to produce the final consumption good and a wholesale sector hiring labor from the households to produce a continuum of differentiated intermediate goods.

In the retail sector the final consumption good  $Y_t$  is produced out of the intermediate goods through a CRS technology,  $Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$ , where  $\epsilon > 1$  reflects the degree of competition in the market for inputs  $Y_t(i)$ . Equilibrium in this sector yields the input demand function:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t,\tag{17}$$

and the aggregate price index:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} \,\mathrm{d}i \right]^{\frac{1}{1-\epsilon}}.$$
(18)

In this setting, any assumption about the property of this kind of firms is totally equivalent to one another, since the profits are driven to zero by perfect competition.

The firms in the wholesale sector produce a continuum of differentiated perishable goods out of hours worked, according to the following production function

$$Y_t(i) = A_t N_t(i), \tag{19}$$

in which  $A_t \equiv A \exp\{a_t\}$  reflects a labor-augmenting shock on productivity, which follow some stochastic process. Aggregating across firms and using equation (17) yields

$$A_t N_t = Y_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} \mathrm{d}i = Y_t \Xi_t, \tag{20}$$

where  $N_t \equiv \int_0^1 N_t(i) \, di$  is defined as the aggregate level of hours worked and  $\Xi_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di$  is an index of price dispersion over the continuum of intermediate goods-producing firms. Notice that, since  $\xi_t \equiv \log \Xi_t$  is of second-order, the linear aggregate production function is simply  $y_t = a_t + n_t$ .

In choosing the optimal level of labor to demand, each firm enters a competitive labor market and seeks to minimize total real costs subject to the technological constraint (19). The equilibrium real marginal costs, therefore, are constant across firms and given by:

$$MC_t = (1 - \tau^*) \frac{W_t}{A_t P_t},$$
 (21)

where  $\tau^*$  is the rate at which the government subsidizes employment.

The price setting mechanism follows Calvo's (1983) staggering assumption. When able to set its price optimally, each firm seeks to maximize the expected stream of future dividends (hence the real value of its outstanding shares), taking into consideration that the chosen price will have to be charged up until period t + k with probability  $\theta^k$ .

The dynamic problem faced by an optimizing firm at time t can therefore be stated as:

$$\max_{P_t^o(i)} E_t \left\{ \sum_{k=0}^{\infty} \theta^k \mathcal{F}_{t,t+k} Y_{t+k}(i) \left( P_t^o(i) - P_{t+k} M C_{t+k} \right) \right\},\$$

subject to the constraint coming from the demand for intermediate goods of the retail sector (17).

The first-order condition for the solution of the above problem implies that all firms revising their price at time t will choose a common optimal price level,  $P_t^o$ , set according to the following rule:

$$P_t^o = (1+\mu)E_t \sum_{k=0}^{\infty} \omega_{t,t+k} P_{t+k} M C_{t+k},$$
(22)

where  $(1 + \mu) \equiv \frac{\epsilon}{\epsilon - 1}$  is the steady state gross markup and the weights  $\omega_{t,t+k}$  depend on how the firms discount future cash flows expected in t for period t + k. In the limiting case of full price flexibility (arising for  $\theta = 0$ ) the price setting rule implies that all firms set their price as a constant markup over nominal marginal costs:  $P_t^n(i) = (1 + \mu)P_tMC_t^n = P_t.^{13}$  As a consequence, under full price flexibility real marginal costs are constant at their steady state level:

$$MC_t^n = \frac{1}{1+\mu}.$$
(23)

## 2.3 The Government and the Equilibrium

Following Galì (2003), we assume a public sector which consumes a fraction  $\varpi_t$  of total output  $(G_t = \varpi_t Y_t)$  and subsidizes employment at a constant rate  $\tau^*$ , appropriately chosen to correct the monopolistic distortion.<sup>14</sup> Public expenditures and subsidies are financed entirely through lump-sum taxation to the households; hence the government runs a balanced budget:

$$P_t T_t = P_t G_t + \tau^* W_t N_t. \tag{24}$$

In equilibrium, therefore, the net supply of state-contingent bonds is nil  $(B_t = 0)$ , while the equilibrium aggregate stock of outstanding equity for each intermediate good-producing firm must equal the corresponding total amount of issued shares, normalized to 1  $(Z_t(i) = 1 \text{ for all } i \in [0, 1])$ .

Finally, let's define total real dividend payments and the aggregate real stock-price index as the simple integration over the continuum of firms:<sup>15</sup>

$$D_t \equiv \int_0^1 D_t(i) \,\mathrm{d}i \qquad \qquad Q_t \equiv \int_0^1 Q_t(i) \,\mathrm{d}i. \tag{25}$$

Note that in equilibrium, given the pricing equation (5), the present discounted real value of future financial wealth,  $E_t \{ \mathcal{F}_{t,t+1} \Pi_{t+1} \Omega_{t+1} \}$ , is equal to the current level of the real stock-price index:

$$E_t \{ \mathcal{F}_{t,t+1} \Pi_{t+1} \Omega_{t+1} \} = \int_0^1 E_t \{ \mathcal{F}_{t,t+1} \Pi_{t+1} (Q_{t+1}(i) + D_{t+1}(i)) di \} = \int_0^1 Q_t(i) di = Q_t.$$
(26)

 $<sup>^{13}</sup>$ Henceforth we label the value that each variable takes in the flexible-price equilibrium as natural (or potential), and denote it by a superscript <sup>*n*</sup>.

<sup>&</sup>lt;sup>14</sup>In particular, the employment subsidy is chosen so that  $(1 + \mu)(1 + \tau^*) = 1$ .

<sup>&</sup>lt;sup>15</sup>For the sake of precision, we should point out that the second term in equation (25) actually defines the aggregate real market capitalization. However, given the assumptions regarding the mass of wholesalers and the total outstanding shares, both normalized to 1, the aggregate market capitalization is also equivalent to the aggregate stock-price index.

As a consequence of the above conditions, and given also condition (25), the demand-side of the economy is summarized by the following aggregate resource constraints

$$Y_t = C_t + G_t = C_t + \varpi_t Y_t \tag{27}$$

$$P_t Y_t = (1 - \tau) N_t W_t + P_t D_t,$$
(28)

the labor supply

$$C_t = \frac{\delta}{1-\delta} \frac{W_t}{P_t} (1-N_t), \qquad (29)$$

and the two aggregate Euler equations

$$(\Sigma_t - 1)C_t = \gamma \delta Q_t + (1 - \gamma)E_t \{ \mathcal{F}_{t,t+1} \Pi_{t+1} \Sigma_{t+1} C_{t+1} \}$$
(30)

$$Q_t = E_t \Big\{ \mathcal{F}_{t,t+1} \Pi_{t+1} \Big[ Q_{t+1} + D_{t+1} \Big] \Big\}.$$
(31)

Equation (30) defines the dynamic path of aggregate consumption, in which an explicit role is played by the dynamics of stock prices. The latter is defined by equation (??), which is a standard pricing equation micro-founded on the consumers' optimal behavior and derives from the aggregation across firms of equation (5).

Finally, note that the benchmark set-up of infinitely-lived consumers is a special case of the one discussed here, and corresponds to a zero-replacement rate,  $\gamma = 0$ . In this case, indeed, equation (30) loses the term related to stock prices and collapses to the usual Euler equation for consumption, relating real aggregate consumption only to the long-run real interest rate:<sup>16</sup>

$$(\Sigma_t - 1)C_t = E_t \{ \mathcal{F}_{t,t+1} \Pi_{t+1} \Sigma_{t+1} C_{t+1} \}.$$

#### 2.4 Steady State and Linearization.

In the long-run, the system converges to a non-stochastic zero-inflation steady state.

In such steady state the following relations hold:<sup>17</sup>

$$(1+r)^{-1} = \tilde{\beta} \equiv \frac{\beta}{1+\psi}$$
(32)

$$\frac{D}{(1+r)Q} = 1 - \tilde{\beta},\tag{33}$$

where we defined  $\psi \equiv \gamma \frac{1-\beta(1-\gamma)}{(1-\gamma)} \frac{\Omega}{C}$ .

First-order approximation around such a steady state yields the following log-linear system for

 $<sup>^{16}</sup>$ In this case, moreover, equation (31) becomes redundant, given market completeness, and can therefore be disregarded.

<sup>&</sup>lt;sup>17</sup>For details on steady state equilibrium and linearization refer to the working paper version (Nisticò, 2005b); an appendix with all the derivations is available upon request. Note that in the long-run, the stochastic discount factor for one-period ahead stochastic payoffs converges to  $(1 + r)^{-1}$ , as implied by equation (6).

the demand-side of our model economy:<sup>18</sup>

$$y_t = c_t + g_t \tag{34}$$

$$w_t - p_t = c_t + \varphi n_t \tag{35}$$

$$c_t = \frac{1}{1+\psi} E_t c_{t+1} + \frac{\psi}{1+\psi} q_t - \frac{1}{1+\psi} (r_t - E_t \pi_{t+1} - \tilde{\rho}) - (1+\psi_\nu) E_t \Delta \nu_{t+1}$$
(36)

$$q_{t} = \tilde{\beta} E_{t} q_{t+1} + (1 - \tilde{\beta}) E_{t} d_{t+1} - (r_{t} - E_{t} \pi_{t+1} - \tilde{\rho}) + e_{t}$$
(37)

$$d_{t} = \frac{Y}{D}y_{t} - \frac{WN}{PD}(n_{t} + w_{t} - p_{t}),$$
(38)

in which  $\varphi \equiv \frac{N}{1-N} = \frac{\delta}{(1-\delta)(1-\varpi)}$  is the inverse of the (steady-state) Frisch elasticity of labor supply and we set  $g_t \equiv -\log(\frac{1-\varpi_t}{1-\varpi})$  and  $\psi_{\nu} \equiv \frac{\psi[\beta(1-\gamma)\rho_{\nu}]}{(1+\psi)[1-\beta(1-\gamma)\rho_{\nu}]}$ . Equation (36) is the linear state equation for consumption. Note that a positive turnover rate  $\gamma$ 

Equation (36) is the linear state equation for consumption. Note that a positive turnover rate  $\gamma$  affects the degree of smoothing in the inter-temporal path of aggregate consumption. An increase in financial wealth (and therefore stock prices) enlarges the wedge between stochastic discount factor and marginal rate of substitution in aggregate consumption, because it makes the difference between the average consumption of "old traders" and that of "newcomers" larger. This mechanism, therefore, implies a direct channel by which the dynamics of stock prices can feed back into the real part of the model. Indeed, a rise in stock prices at time t reflects an increase in the expected financial wealth for period t+1, as shown by equation (5). All individuals in the financial market at t will then increase their current consumption expenditures to optimally smooth their intertemporal profile. At t + 1, however, a fraction of these individuals will be replaced by newcomers who hold no equity shares, and whose consumption will therefore not be affected by the shock to the stock wealth. Consequently, the increase in stock prices affects current aggregate consumption more than the aggregate level expected for tomorrow. This makes the dynamics of aggregate financial wealth relevant for current aggregate consumption and the transmission of real and monetary shocks.

Equation (37) defines the equilibrium stock-price dynamics implied by the model. Here we assume that these dynamics are affected by non-fundamental factors, captured by the additional, stochastic disturbance:  $e_t$ .<sup>19</sup>

As to the supply-side, moving from the definition of the general price level (18) and considering that a fraction  $(1-\theta)$  of all firms revise their price at t at the common level  $P_t^o$ , and that a fraction  $\theta$  keeps the price constant at last period's general price level, we can describe inflation dynamics with a familiar New Keynesian Phillips curve:

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\tilde{\beta})}{\theta} mc_t.$$
(39)

Notice that the households' finite horizon (affecting the long-run interest rate) imply a lower weight on future inflation  $\tilde{\beta} \equiv \frac{\beta}{1+\psi}$  and a higher weight on the marginal costs compared with the standard RA case.

<sup>&</sup>lt;sup>18</sup>In what follows lower-case letters denote log-deviations from the steady state:  $x_t \equiv \log(X_t/X)$ . Note that,  $(1+r_t)$  being the gross interest rate,  $r_t$  is (to first order) the net interest rate. The log-deviation of the gross interest rate from its steady state is therefore  $r_t - \tilde{\rho}$ , where we set  $\tilde{\rho} \equiv \log(1+r) = -\log \tilde{\beta}$ .

<sup>&</sup>lt;sup>19</sup>See Smets and Wouters (2003), and Castelnuovo and Nisticò (2010) for an analogous *ad hoc* modelling choice. In particular, Castelnuovo and Nisticò (2010) show that this non-fundamental disturbance improves the empirical fit of the model.

Linearization of equation (21) yields

$$mc_t \equiv \log((1+\mu)MC_t) = w_t - p_t - a_t = (1+\varphi)(y_t - a_t) - g_t,$$
(40)

where the last equality is obtained using the labor supply (35), the resource constraint (34) and the production function.

Imposing condition (23) on equation (40), finally, we can retrieve the equation for the natural level of output

$$y_t^n = a_t + \frac{1}{1+\varphi}g_t,\tag{41}$$

and therefore link short-run real marginal costs to the output gap  $x_t \equiv y_t - y_t^n$ :

$$mc_t = (1+\varphi)x_t. \tag{42}$$

For future reference, it is useful to use the above equations (40) and (42) to obtain the following formulation for aggregate dividends:

$$d_t = y_t^n - \frac{1 + \varphi - \mu}{\mu} x_t, \tag{43}$$

which highlights that the elasticity of real dividends with respect to the output gap is decreasing in the steady state markup ( $\mu$ ) and the Frisch elasticity of labor supply (1/ $\varphi$ ).

#### 2.4.1 The Complete Linear Model.

Exploiting the results of the previous sections, the economic system described so far can be reduced to the following three equations:

$$y_t = \frac{1}{1+\psi} E_t y_{t+1} + \frac{\psi}{1+\psi} q_t - \frac{1}{1+\psi} (r_t - E_t \pi_{t+1} - \tilde{\rho}) - (1+\psi_\nu) E_t \Delta \nu_{t+1} + \frac{1+\psi - \rho_g}{1+\psi} g_t \quad (44)$$

$$q_t = \tilde{\beta} E_t q_{t+1} - \lambda E_t x_{t+1} - (r_t - E_t \pi_{t+1} - \tilde{\rho}) + (1 - \tilde{\beta}) E_t y_{t+1}^n + e_t$$
(45)

$$\pi_t = \hat{\beta} E_t \pi_{t+1} + \kappa x_t, \tag{46}$$

where we set  $\lambda \equiv (1 - \tilde{\beta}) \frac{1 + \varphi - \mu}{\mu}$  and  $\kappa \equiv \frac{(1 - \theta)(1 - \theta \tilde{\beta})}{\theta} (1 + \varphi)$ . Equation (44) defines a forward-looking IS-type relation that relates real output to its own

Equation (44) defines a forward-looking IS-type relation that relates real output to its own expected future values, a short-term real interest rate and assigns an explicit role to real stock prices in driving its dynamics, through a wealth effect.<sup>20</sup> The benchmark case of infinitely-lived consumers is a special case, featuring  $\gamma = \psi = \psi_{\nu} = 0$ , and reduces equation (44) to the usual forward-looking IS schedule:

$$y_t = E_t y_{t+1} - (r_t - E_t \pi_{t+1} - \rho) - E_t \Delta \nu_{t+1} - E_t \Delta g_{t+1}.$$
(47)

<sup>&</sup>lt;sup>20</sup>Recent empirical evidence show that wealth effects of this kind might be non-negligible. In particular, Castelnuovo and Nisticò (2010) estimate an empirical version of this model on post-war US data, and show that the magnitude of the replacement rate  $\gamma$  (governing the stock-wealth effects) is quite sizable, around .13. Such estimated value implies non-negligible wealth effects on output and inflation, with real activity responding to a 1% increase in stock prices with an increase ranging from .05 to .1%, depending on the monetary policy stance. An additional interesting result that these authors report, indeed, is that monetary policy has a non-trivial role in shaping the response of real variables to a stock-price shock, and that the Federal Reserve has historically looked at stock-price dynamics in designing monetary policy actions. We view this empirical evidence as an encouraging support to our exploration of the theoretical implications of stock-wealth effect in a DNK model.

Equation (45), finally, describes the dynamics of real stock prices, which can therefore be driven by fundamental factors (supply shocks to productivity and labor supply and demand shocks to public consumption and the stochastic discount factor) and by non-fundamental ones.

We will show below that in the face of a given swing in stock prices, the identification of what specific shock is the actual driving force plays a crucial role in defining the appropriate policy response relative to the RA setup.

## **3** Monetary Policy and Stock-Price Dynamics.

In this Section we study the interplay between stock-price dynamics and the design of monetary policy from the perspective of a Central Banking targeting price stability. In particular, absent cost-push shocks, we consider the case in which the Central Bank can fully stabilize both inflation and output at their flexible-price equilibrium levels, and derive the *Wicksellian* Monetary Policy, i.e. the interest-rate dynamics consistent with flexible prices.

We should stress that we do not address here the issue of whether or not in this setting price stability is an optimal target from the perspective of a welfare-maximizing Central Bank. The reason is that the derivation of a properly defined welfare criterion, in our framework with nonricardian agents, involves non-trivial issues related to the aggregation across cohort of the utility flows. Such derivation goes beyond the scope of this analysis, and is currently being tackled in a companion paper (see Nisticò, 2011). Nonetheless, we view the flexible-price allocation as an interesting equilibrium to study, given the importance that has been assigned in practice to price stability by many major Central Banks.

#### 3.1 Price Stability: the Wicksellian Monetary Policy.

Assuming that the monetary policy instrument is the short-term interest rate  $r_t$ , the value consistent with the flexible-price allocation is what Woodford (2003) calls the *Wicksellian Natural Rate of Interest*, henceforth  $rr_t^n$ . To assess the implications that the stock-wealth effects have in driving the Wicksellian Monetary Policy, therefore, below we compare the Natural Interest Rate implied by the model at hand with the one arising in the benchmark RA setup.

In the RA setup (implying  $\gamma = \psi = \psi_{\nu} = 0$ ), from the IS schedule of equation (47) we obtain:

$$rr_{RA,t}^{n} = \rho + E_{t}\Delta y_{t+1}^{n} - (E_{t}\Delta\nu_{t+1} + E_{t}\Delta g_{t+1}) = \rho + E_{t}\Delta a_{t+1} - E_{t}\Delta\nu_{t+1} - \frac{\varphi}{1+\varphi}E_{t}\Delta g_{t+1}, \quad (48)$$

in which  $\rho = -\log \beta$ : with no wealth effects the interest-rate response consistent with price stability requires accommodating supply shocks  $(a_t)$  while offsetting demand shocks  $(g_t \text{ and } \nu_t)$ .

Within the framework with real effects of stock prices on real activity the *Wicksellian natural* rate of interest is retrieved as the solution of the two-equation system

$$y_t^n = E_t y_{t+1}^n + \psi(q_t^n - y_t^n) - (rr_t^n - \tilde{\rho}) - (1 + \psi)(1 + \psi_\nu)E_t \Delta\nu_{t+1} + (1 + \psi - \rho_g)g_t$$
(49)

$$q_t^n = \beta E_t q_{t+1}^n - (rr_t^n - \tilde{\rho}) + (1 - \beta) E_t y_{t+1}^n + e_t.$$
(50)

Given the natural rate of output  $y_t^n$  determined by equation (41), the system above yields the potential level of stock prices  $q_t^n$  and the natural rate of interest  $rr_t^n$ .

The solution implies that the potential level of the stock-market capitalization ratio  $(q_t^n - y_t^n)$  is only affected by demand and non-fundamental shocks:

$$q_t^n - y_t^n = -\frac{1 + \psi - \rho_g}{1 + \psi - \tilde{\beta}\rho_g} g_t - (1 + \psi_\nu) \frac{(1 + \psi)(1 - \rho_\nu)}{1 + \psi - \tilde{\beta}\rho_\nu} \nu_t + \frac{1}{1 + \psi - \tilde{\beta}\rho_e} e_t.$$
 (51)

The potential level of stock prices, therefore, is decreasing in g and  $\nu$  (reflecting the negative effect on private savings that a positive innovation on both shocks induce) and increasing in non-fundamental shocks (like for example fads) that lower the equity premium.

Using equations (48) and (51), we can retrieve the final reduced form for the *Wicksellian natural* rate of interest, which highlights the excess response implied by the stock-wealth effects:

$$rr_t^n = \tilde{\rho} + E_t \Delta a_{t+1} - \Delta \nu_{t+1} - \frac{\varphi}{1+\varphi} E_t \Delta g_{t+1} + \Psi_e e_t + \Psi_g g_t + \Psi_\nu \nu_t$$
$$= \bar{\rho} + rr_{RA,t}^n + \Psi_e e_t + \Psi_g g_t + \Psi_\nu \nu_t, \quad (52)$$

in which  $\bar{\rho} \equiv \tilde{\rho} - \rho = \log(1 + \psi)$  reflects the difference in the long-run interest rate relative to the RA setup.<sup>21</sup>

Equation (52) shows that the optimal interest rate dynamics implied by a framework in which the stock market performance has real effects on output and inflation, displays three terms which are implied by the stock-wealth effects, and require a dedicated response to shocks to the equity premium  $(e_t)$  and an additional response to shocks on public expenditures and the stochastic discount factor  $(g_t \text{ and } \nu_t)$ .

As derived in the Appendix, in fact, the coefficients are the following functions of underlying structural parameters:

$$\Psi_g \equiv \frac{\psi \rho_g (1 - \beta)}{1 + \psi - \tilde{\beta} \rho_g} \tag{53}$$

$$\Psi_{\nu} \equiv (1 - \rho_{\nu}) \left[ \frac{\psi_{\nu}(1 + \psi) - \tilde{\beta}\rho_{\nu}[(1 + \psi)(1 + \psi_{\nu}) - 1]}{1 + \psi - \tilde{\beta}\rho_{\nu}} \right]$$
(54)

$$\Psi_e \equiv \frac{\psi}{1 + \psi - \tilde{\beta}\rho_e}.$$
(55)

It is straightforward to see that all three coefficients shrink to zero as the framework converges to the RA setup (in which  $\gamma = \psi = \psi_{\nu} = 0$ ).

Moreover, it is just as straightforward to see that, given the theoretical restrictions on the structural parameters,  $\Psi_g$  and  $\Psi_e$  are always positive. This implies that the interest rate dynamics consistent with the flexible-price allocation entail an over-restrictive response both to shocks to government consumption  $g_t$  and to non-fundamental financial shocks originating within the stock market,  $e_t$ , relative to the benchmark case (in which pure financial shocks are neutral and remain unaccounted for in the natural interest rate dynamics).

The over-restriction following a shock to government expenditures is the result of two competing effects: a positive impatience effect, IE(g) and a negative stock-wealth effect, SWE(g). The IE is direct: higher impatience implies a stronger effect of  $g_t$  on current output relative to the RA case

 $<sup>^{21}</sup>$ More specifically, the steady state interest rate in the RA case is lower, as a consequence of the lower impatience due to the zero-probability of exiting the market. See Nisticò (2005b) for details on the derivation.

and therefore requires an over-restriction to offset the higher inflationary pressures. Equation (49) implies therefore  $IE(g) = \psi$ . The negative SWE effect is instead indirect and works through the contraction that the increase in the interest rate induces on stock prices. From equations (49) and (51) we can derive:

$$SWE(g) = \psi \frac{\partial q_t^n}{\partial g_t} = -\psi \frac{1 + \psi - \rho_g}{1 + \psi - \tilde{\beta}\rho_q}$$

Notice that, given  $\tilde{\beta} \in (0, 1)$ , the IE is stronger and the net effect  $\Psi_g \equiv IE(g) + SWE(g)$  is therefore positive (or zero, in the limiting case of  $g_t$  following a white noise process).

As to the response to  $\nu_t$ , the interplay of impatience and stock-wealth effects implies an ambiguous sign for the response coefficient  $\Psi_{\nu}$ , depending on the specific structure of the economy. As before, the first effect is positive and direct, and has the same interpretation as IE(g): since a positive replacement rate ( $\gamma > 0$ ) reduces the degree of smoothing in the inter-temporal path of aggregate consumption, a positive innovation on  $\nu_t$  pushes current output above its natural level more than it would in the RA setup, thus asking for a higher raise in the rate of interest. From equation (49) it follows

$$IE(\nu) = (1 - \rho_{\nu})[\psi_{\nu} + \psi(1 + \psi_{\nu})].$$

The second effect is instead negative and indirect: a positive innovation on  $\nu_t$  has a negative impact on the stochastic discount factor, tends to reduce the current demand for stocks, and drives down their price and, thereby, the value of aggregate wealth. The latter effect, in a system where wealth matters for consumption decisions, has then the consequence of dragging output down, asking for an easier monetary policy compared to a system where wealth is ineffective. Equation (51) implies:

$$SWE(\nu) = \psi \frac{\partial q_t^n}{\partial \nu_t} = -\psi (1+\psi_\nu) \frac{(1+\psi)(1-\rho_\nu)}{1+\psi-\tilde{\beta}\rho_\nu}.$$

Tedious algebra shows that the latter effect dominates over the former ( $\Psi_{\nu} < 0$ ) if the following condition holds:

$$\frac{\psi}{1+\psi} > \frac{1-\tilde{\beta}\rho_{\nu}}{\tilde{\beta}\rho_{\nu}}\psi_{\nu}.$$
(56)

The above condition, derived in the Appendix, imply therefore that if the stock-wealth elasticity of consumption  $(\frac{\psi}{1+\psi})$  is strong enough relative to the direct excess-effect of the preference shock on real current consumption  $(\psi_{\nu})$ , the policy response should be under-restrictive compared to a framework in which stock prices are redundant for the equilibrium allocation  $(\psi_{\nu} = \frac{\psi}{1+\psi} = 0)$ .

#### 3.2 Implementing Price Stability: Equilibrium Determinacy.

We can use the results of the previous section to write the the linear model as a system of three equations, in terms of deviations of each variable from its frictionless level:

$$x_t = \frac{1}{1+\psi} E_t x_{t+1} + \frac{\psi}{1+\psi} s_t - \frac{1}{1+\psi} (r_t - E_t \pi_{t+1} - rr_t^n)$$
(57)

$$s_t = \tilde{\beta} E_t s_{t+1} - \lambda E_t x_{t+1} - (r_t - E_t \pi_{t+1} - r r_t^n)$$
(58)

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + \kappa x_t, \tag{59}$$

in which  $rr_t^n$  is given by equation (52), and  $s_t \equiv q_t - q_t^n$  denotes the deviation of the stock-price level from the flexible-price equilibrium: the "stock-price gap".<sup>22</sup> Notice that this system generalizes the SNK along one dimension only, by simply adding one equation (the stock-price equation) which feeds back into the IS curve, and is therefore not redundant for the equilibrium allocation.

As in the SNK model, however, the *Wicksellian* policy derived in the previous Section identifies the interest-rate dynamics *consistent* with price stability, but it cannot be regarded as a rule that a Central Bank could follow to successfully *implement* price stability.

This Section explores this issue by analyzing equilibrium determinacy when the Central Bank adopts a simple instrument rule of the (general) form

$$r_t = rr_t^n + \phi_\pi \pi_t + \phi_y x_t + \phi_q s_t. \tag{60}$$

The policy makers are then assumed to commit to a rule that has the nominal interest rate match its *Wicksellian* level at all times, unless any one of the endogenous variables deviates from its target level (the frictionless one).

We analyze the implication of such a rule for equilibrium determinacy by numerically simulating the model within a wide parameter sub-space for the policy rule's coefficients, and plotting the resulting regions of determinacy.

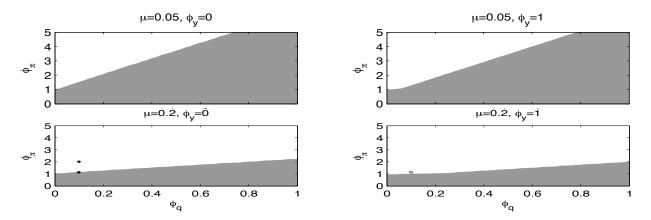
The exercise is conducted considering a quarterly parameterization, taken from widespread convention. Specifically, the steady-state net quarterly interest rate r was fixed at 0.01, implying a long-run real annualized interest rate of 4%, and the intertemporal discount factor  $\beta$  was set at 0.995; accordingly, to meet the steady-state restrictions, the turnover rate  $\gamma$  was set at 0.03.<sup>23</sup> The level of steady-state consumption was set at 80% of total output ( $\omega = 0.2$ ). As to the steady-state Frisch elasticity of labor supply,  $1/\varphi$ , there is wide controversy about the value that should be assigned to this parameter. The empirical microeconomic literature suggests values ranging from .1 to .5 (see Card, 1994, for a survey), while business cycle literature mostly uses values greater than 1 (see e.g. Cooley and Prescott, 1995). We choose a baseline value of 1 (and parameterize  $\delta$ accordingly, given  $\omega$ ) and show robustness checks over two alternative values: 2 and 1/3. Finally, the probability for firms of having to keep their price fixed for the current quarter was set at 0.75, implying an average price-duration of a year.

Hence, Figure 1 displays the determinacy regions in the parametric space  $(\phi_q, \phi_\pi)$ , for different stances towards the output gap  $(\phi_y=0, 1)$  and different degrees of competitiveness in the wholesale sector ( $\epsilon=21$ , implying a 5% net markup  $\mu$  and  $\epsilon=6$ , implying  $\mu=.2$ ). Specifically, the shaded areas indicate the regions in which the equilibrium is indeterminate.

The analysis of determinacy confirms that the Wicksellian policy  $(\phi_{\pi}=\phi_{y}=\phi_{q}=0)$  implies an indeterminate equilibrium, for all parameterizations. It also confirms though that such policy can be implemented through a credible threat to move the rate of interest if the actual allocation deviates from the potential one; i.e. if the appropriate conditions on the response coefficients are met. In particular, responding to deviations of the stock-price level from the one prevailing in the Flexible-Price equilibrium reduces the determinacy space. In other words, for a given stance towards inflation, reacting too strongly to non-zero stock-price gaps might be destabilizing because it makes

 $<sup>^{22}</sup>$ Castelnuovo and Nisticò (2010) show that the smoothed estimate of the dynamic evolution of the stock-price gap is a sensible measure of the US stock-market conditions in the post-war period.

 $<sup>^{23}</sup>$ We regard this as a rather conservative calibration. In particular, see Castelnuovo and Nisticò (2010), who provide a bayesian structural estimate of this parameter in an empirical version of the same framework.



**Figure 1:** Analysis of Equilibrium Determinacy, for different values of the market power and response to the output gap. White (shaded) regions indicate determinacy (indeterminacy). •: BG 1999, accommodative ( $\phi_{\pi} = 1.01$ ,  $\phi_q = 0.1$ ,  $\phi_y = 0$ );  $\star$ : BG 1999 aggressive ( $\phi_{\pi} = 2.0$ ,  $\phi_q = 0.1$ ,  $\phi_y = 0$ );  $\circ$  CGLW 2000 ( $\phi_{\pi} = 1.01$ ,  $\phi_q = 0.1$ ,  $\phi_y = 1$ )

the system subject to potential endogenous fluctuations. Interestingly, however, the Figure 1 also shows that the risks of indeterminacy implied by a strong reaction to stock-price gaps are the lower the higher is the market power characterizing the wholesale sector  $(1/\epsilon)$ , i.e. the higher is the average profitability of stocks over time.

To get an intuition of this result, consider the effects of an upward revision in inflation expectations on system (57)–(59). When the Central Bank adopts a policy rule satisfying the Taylor principle, such revision in expectations will induce an increase in the real interest rate and a fall in both the output gap and the inflation rate, as well as in their expectations. The impact reaction of stock prices will depend on the output-gap elasticity of dividends:  $(1 + \varphi - \mu)/\mu$ . Low enough markups (i.e. high enough  $\lambda$ ), in fact, imply that the positive effect of falling expected output gaps offsets the negative effect of rising real rates, inducing an increase in the stock-price gap  $s_t$ . In this case, if the interest rate does not react to stock prices, the dynamics of inflation and the output gap will revert to zero. Eventually, thereby, also the stock-price gap will return to its equilibrium value. However, if monetary policy reacts aggressively also to asset prices, a positive stock-price gap will imply a stronger increase in the real interest rate, a stronger fall in output gap and inflation, and a further increase in stock prices. The process thus feeds itself and ultimately leads the system away from the equilibrium.

Determinacy is restored if the policy response to inflation is aggressive enough (high  $\phi_{\pi}$ ) or if market power is high enough (high  $\mu$ ). On the one hand, indeed, if monetary policy is aggressive enough towards inflation, then the pressures towards a strong deflation prevent the interest rate from rising as much as required by rising stock prices. As a consequence, interest rates are driven mainly by the inflation rate, inflation and the output gap will revert to zero and, eventually, so will the stock-price gap. On the other hand, if the market power of listed firms is higher, their dividends are more inelastic to variations in marginal costs. Therefore, falling demand induces milder pressures towards an increase in stock prices and the evolution of interest rates is again mainly driven by the dynamics of inflation and the output gap.

This conclusion resembles, in some way, the one drawn by Bernanke and Gertler (1999), since it implies that an explicit reaction to stock-price deviations from a specific level might yield macroe-

conomic instability, the more so the less aggressive is the reaction to inflation. To further explore this point, in Figure 1 we also mark the policy rules analyzed in Bernanke and Gertler (1999) and Cecchetti et al. (2000): the policy that in Bernanke and Gertler (1999) yields what the authors call a "perverse outcome" ( $\phi_{\pi}=1.01$ ,  $\phi_q=0.1$ ,  $\phi_y=0$ ) here too produces instability, in the form of endogenous fluctuations triggered by innovations in the sunspot variable, for each level of the average markup (see the solid dot in the left panels of Figure 1); however, shifting to a more aggressive reaction to inflation ( $\phi_{\pi}=2.0$ ,  $\phi_q=0.1$ ,  $\phi_y=0$ , marked by a star in Figure 1) or adding an aggressive reaction to output ( $\phi_{\pi}=1.01$ ,  $\phi_q=0.1$ ,  $\phi_y=1$ , see the small circle in the right panels of Figure 1) as in Cecchetti et al. (2000) ensures higher macroeconomic stability.<sup>24</sup>

## 4 Simple Monetary Policy Rules and Stock-Price Dynamics.

On one hand, the results derived in the previous section may be taken as a warn against the perils of an excessive threat to control a specific stock-price level. In this section we complete the analysis by studying the macroeconomic performance of alternative "simple" and "operational" rules.

### 4.1 Simple Rules

We start by analyzing two alternative simple rules, and asses their macroeconomic performance along three dimensions: equilibrium determinacy, implied dynamic response of the economy to selected shocks, implications for macroeconomic stability. The two rules considered here both modify the basic formulation proposed by Taylor (1993) to account for an explicit consideration of stock-price dynamics in monetary policy actions.

The first one augments a Taylor-type standard rule by having the interest rate respond also to deviations of a *stock-price level* from its "natural" counterpart (the "Gap-Rule"):

$$r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_y x_t + \phi_q s_t + u_{r,t}.$$
(61)

In the second rule considered, on the other hand, the concern about stock-price dynamics takes the form of changes in the policy rate in response to deviations of the *stock-price growth rate* from a given target, assumed here to be zero (the "Growth-Rule"):

$$r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_y x_t + \phi_q \Delta q_t + u_{r,t}.$$
(62)

#### 4.1.1 Equilibrium Determinacy and Impulse-Response Analysis

The first dimension along which the two policy rules considered have importantly different implications is the determinacy of the rational expectations equilibrium, and the ability to rule out endogenous instability. As discussed in the previous section, in fact, the Gap-rule implies a shrinkage in the determinacy space, the more so the higher is the degree of competitiveness in the wholesale sector. Figure 2, in contrast, shows that a rule entailing a response to stock-price growth does not have any effect on the Taylor Principle, allowing for a stronger concern about stock-price

 $<sup>^{24}</sup>$ Bernanke and Gertler (1999) and Cecchetti et al. (2000) actually analyze policy rules responding to expected, as opposed to current, inflation. The point we make in the text, however, is robust to this specification of the policy rule. See Nisticò (2005b) for more on this issue.

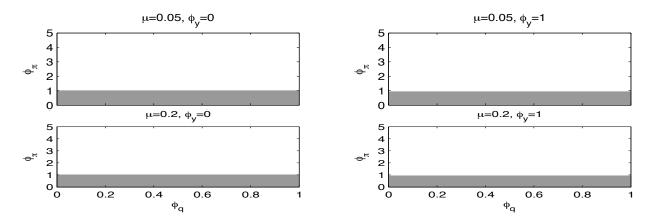


Figure 2: Analysis of Equilibrium Determinacy for the Growth-Rule, for different values of the market power and response to the output gap. White (shaded) regions indicate determinacy (indeterminacy).

dynamics without producing *in itself* endogenous instability. The intuition here is simple: if the Central Bank follows the Gap-rule, a permanent increase in the level of stock prices would imply a permanent increase in the interest rate, feeding back in the level stock prices and fueling the divergence; on the contrary, if the Central Bank follows the Growth-rule, a permanent increase in the stock-price level – which implies only a temporary increase in the stock-price growth rate – would only require a *temporary* rise in the interest rate, and therefore allow mean reversion.

The second important dimension along which to evaluate the Gap- and Growth-rule is related to the dynamic response of the economy conditional on specific shocks. Figure 3 and 4 plot the impulse-response functions relative to a productivity shock and a shock to the stochastic discount

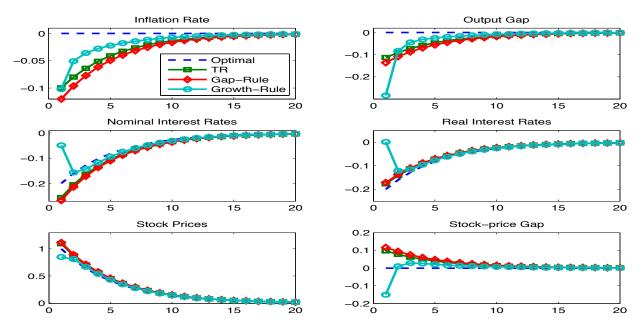


Figure 3: Dynamic response of the economy to a productivity shock.

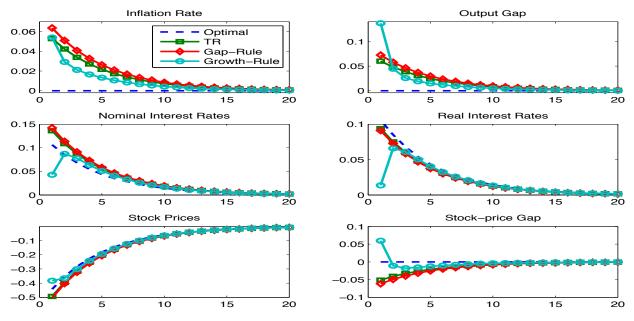


Figure 4: Dynamic response of the economy to a shock to the stochastic discount factor.

factor. The calibration of the structural parameters is the same as in the previous section. As to the elasticity of substitution among intermediate goods, we choose an intermediate value of 11, implying an average markup of 10%; as for the response coefficient, we set  $\phi_{\pi} = 2$ ,  $\phi_y = .5$ and  $\phi_q = .35$ . The plots show that adding an explicit reaction to stock-price dynamics in the form of a "Gap-Rule" affects only marginally the performance of the standard Taylor Rule, while the "Growth-Rule" has little but beneficial effects on the dynamic performance of the economy, especially for the dynamics of inflation.

#### 4.1.2 Implications for Macroeconomic Stability

Finally, we evaluate the cyclical properties of the two alternative simple rules, and their implications for macroeconomic stability.

Table 1 reports the standard deviations of selected variables implied by the alternative simple rules, and the relative gain/loss of adding an explicit response to the stock-price level and/or growth rate. The calibration of structural and policy parameters is the same as in the previous subsection; as to the calibration of the stochastic properties of structural shocks, since the aim is not to replicate the variables' moments in the data, and in order to make the results not dependent on the relative weight of the single shocks, we parameterize the persistence of all shocks at .8 (except  $u_{r,t}$  which is assumed white noise) and their standard deviation at .01. The Table shows that while adopting the Gap Rule yields minor effects on the volatility implied by the standard Taylor Rule, actively reacting to the stock-price growth rate allows the Central Bank to achieve about a 17% reduction in the volatility of inflation and a 28% reduction in that of the nominal interest rate, while only marginally affecting the standard deviation of real output and the output gap.

To explore more deeply the implications for macroeconomic stability of responding to stock prices, and the differences between the Gap- and the Growth-rule, we next use as metrics an index

	Wick sellian	Taylor Rule	Gap Rule	Growth Rule	
Output Gap	0.000	0.592	0.544	0.631	
			(0.918)	(1.066)	
Real Output	2.205	2.122	2.087	2.092	
			(0.983)	(0.986)	
Inflation Rate	0.000	0.243	0.280	0.203	
			(1.153)	(0.835)	
Stock Prices	2.103	2.395	2.421	2.119	
			(1.011)	(0.885)	
Stock-price Gap	0.000	0.578	0.521	0.488	
			(0.901)	(0.845)	
Interest Rates	0.441	0.783	0.748	0.564	
			(0.955)	(0.720)	
Real Int. Rates	0.441	0.664	0.591	0.518	
			(0.890)	(0.780)	
Note: Standard Deviations in %. Ratio to Taylor Rule in parentheses.					

Table 1: Implied Volatility under Alternative Simple Rules

of systemic stability, defined as a weighted average of the variances of inflation, the output gap and the interest rate:

$$\mathcal{L}_t \equiv var(\pi_t) + \alpha_v var(x_t) + \alpha_r var(r_t).$$
(63)

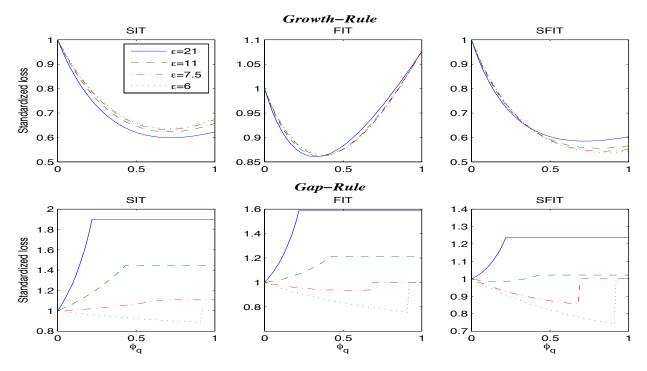
With respect to this exercise, we wish to stress that we do not mean the index (63) to be a measure of social welfare whatsoever, since we do not micro-found it on consumers' preferences. Rather, and more simply, we interpret  $\mathcal{L}_t$  as a measure of overall volatility in the economic system, which the Central Bank regards as reflecting its own preferences.<sup>25</sup>

This formulation is useful, since it allows us to encompass several monetary policy regimes that are adopted by modern Central Banks, depending on the specific value of the relative weights  $\alpha$ 's. In particular, we consider three alternative monetary policy regimes. First, a Strict Inflation Targeting regime pursued by a fully conservative Central Bank, which implies that the only concern is inflation stability, and corresponds to the case in which  $\alpha_y = \alpha_r = 0$  (henceforth SIT). The second regime is that of Flexible Inflation Targeting, in which the Central Banker is not only concerned with price stability, but it also attaches some weight to real stability (FIT,  $\alpha_y > 0$ ,  $\alpha_r = 0$ ). Finally, we also consider a "Smooth" Flexible Inflation Targeting regime, in which the Central Bank is also explicitly concerned about interest rates' volatility (SFIT,  $\alpha_y$ ,  $\alpha_r > 0$ ). As to the parameterization of the relative weights on output and interest rates, in the regimes where they are not zero, we set a value equal to 0.1, consistently with the evidence in Lippi and Neri (2007).

Under each regime, we consider an aggressive response towards inflation ( $\phi_{\pi}=2.0$ ) and output ( $\phi_{y}=0.5$ ) and four decreasing values for the degree of competitiveness in the wholesale sector,  $\epsilon$ : 21, 11, 7.5 and 6 (implying an average markup of respectively 5, 10, 15 and 20 per cent).

The top panels of Figure 5, then, plots the policy loss (63) for the case in which the Central Bank follows the simple rule (61), as a function of the response coefficient to stock prices, for the three

 $<sup>^{25}</sup>$ See Smets and Wouters (2002) and Ascari and Ropele (2004) for analogous choices, and Svensson (2002) for a discussion of this class of loss functions. See instead Nisticò (2011) for a formal analysis of the welfare implications of this framework.



**Figure 5:** Standardized loss implied by alternative *simple* rules, for different parameterizations and policy regimes. SIT: Strict Inflation Targeting; FIT: Flexible Inflation Targeting; SFIT: Smooth Flexible Inflation Targeting. Gap-Rule:  $r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_y x_t + \phi_q s_t + u_{r,t}$ ; Growth-Rule:  $r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_y x_t + \phi_q \Delta q_t + u_{r,t}$ 

different monetary policy regimes, and the four degrees of competitiveness considered. The bottom panels do the same for the case in which the Central Bank follows the Growth-rule (62). Moreover, to evaluate the gains or losses of adopting an active concern about stock prices, we normalize to 1 the value of the policy loss implied by the benchmark case of a rule featuring a zero-response to stock prices ( $\phi_q=0$ ). Finally, since the Gap-Rule eventually yields equilibrium indeterminacy, in the bottom panels endogenous instability is indicated by a flat line, at the maximum value in the simulation range.

The bottom-left panel of Figure 5 shows that under SIT, responding to deviations of the stockprice index from its potential level produces a deadweight loss compared to the benchmark Taylor Rule, unless the firms' market power (and therefore the average profitability of equity claims) is sufficiently high. In contrast, the top-left panel in the same Figure shows that a strong response to stock-price *growth* yields a stability gain as big as a 40% reduction in the loss function, relative to the benchmark rule, regardless of the market power of monopolistic firms. These results are obviously in line with the implications of Table 1: when the only target of monetary policy is price stability, the growth-rule is substantially more effective, and has the appealing side-effect of ruling out endogenous fluctuations.

Allowing for additional targets in the monetary policy loss function does not substantially change the implications above. Indeed, under both FIT and SFIT, higher values of the firms' market power (which make equity shares more profitable and increase stock-price relevance for the dynamics of real consumption) not only widen the subspace of response-coefficient values that ensure determinacy, but they also leave considerably more room for a stabilization role of reacting

	OpTR	OpLY	OpLS	OpLYS	OpGY	OpGS	OpGYS
Output Gap	0.812	0.921	0.880	1.393	0.853	0.874	0.932
		(1.134)	(1.084)	(1.715)	(1.050)	(1.076)	(1.148)
Real Output	2.152	1.547	2.079	1.340	1.978	2.116	1.946
		(0.719)	(0.966)	(0.623)	(0.919)	(0.983)	(0.904)
Inflation Rate	0.338	0.667	0.506	1.165	0.355	0.289	0.333
		(1.976)	(1.497)	(3.448)	(1.050)	(0.857)	(0.984)
Stock Prices	2.527	2.679	2.682	3.193	2.367	2.135	2.014
		(1.060)	(1.061)	(1.263)	(0.937)	(0.845)	(0.797)
Stock-price Gap	0.791	0.835	0.831	1.214	0.765	0.644	0.677
		(1.055)	(1.051)	(1.534)	(0.967)	(0.813)	(0.856)
Interest Rates	0.959	0.921	0.993	1.150	0.772	0.643	0.548
		(0.960)	(1.035)	(1.199)	(0.805)	(0.671)	(0.572)
Real Int. Rates	0.821	0.645	0.757	0.520	0.637	0.613	0.507
		(0.785)	(0.922)	(0.634)	(0.776)	(0.746)	(0.618)
Note: Standard Deviations in %. Ratio to OpTR in parentheses.							

Table 2: Implied Volatility under Alternative Operational Rules

to the *stock-price gap*. This result extends to the case of the Growth Rule: higher market power in the wholesale sector implies a higher response coefficient to stock-price growth to minimize the loss function.<sup>26</sup>

## 4.2 Operational Rules

It is worth noticing that, as argued by Galì (2003), the concept of output gap in operational policy rules is generally different from the one arising in theoretical models like the one at hand, since the latter is a function of unobservable structural shocks. As a consequence, a *strictly operational* policy rule consistent with the analysis above would be obtained by omitting any unobservable variable (like the output and stock-price gaps).

We consider several alternative operational specifications. The benchmark rule responds only to the inflation rate (OpTR):

$$r_t = \tilde{\rho} + \phi_\pi \pi_t + u_{r,t},$$

while the alternative ones augment the benchmark with an explicit response to the level or growth rate of real output and the stock-price index. In particular, we consider:

OpLY :	$r_t =  ilde{ ho} + \phi_\pi \pi_t + \phi_y y_t + u_{r,t}$	(64)
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OpLS:	$r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_q q_t + u_{r,t}$	(65)
Opus.	$T_t = \rho + \psi_\pi \pi_t + \psi_q q_t + a_{r,t}$	(00)

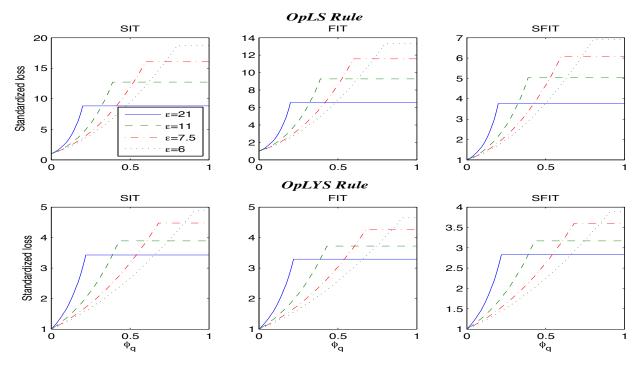
OpLYS: 
$$r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_y y_t + \phi_q q_t + u_{r,t}$$
(66)

OpGY: 
$$r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_y \Delta y_t + u_{r,t}$$
 (67)

OpGS: 
$$r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_q \Delta q_t + u_{r,t}$$
 (68)

OpGYS: 
$$r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_y \Delta y_t + \phi_q \Delta q_t + u_{r,t}.$$
 (69)

 $<sup>^{26}</sup>$ In order to assess whether the above results are in some way determined by the informational role that stock prices play with respect to future inflation and output, we replicated the exercise for the forward-looking versions of the Gap- and Growth-Rules, without finding substantial effects on the qualitative results. See Nisticò (2002b).



**Figure 6:** Standardized loss implied by alternative *operational* "level–rules", for different parameterizations and policy regimes. SIT: Strict Inflation Targeting; FIT: Flexible Inflation Targeting; SFIT: Smooth Flexible Inflation Targeting. OpLS-Rule:  $r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_q q_t + u_{r,t}$ ; OpLYS-Rule:  $r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_q q_t + u_{r,t}$ ; OpLYS-Rule:  $r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_q q_t + u_{r,t}$ 

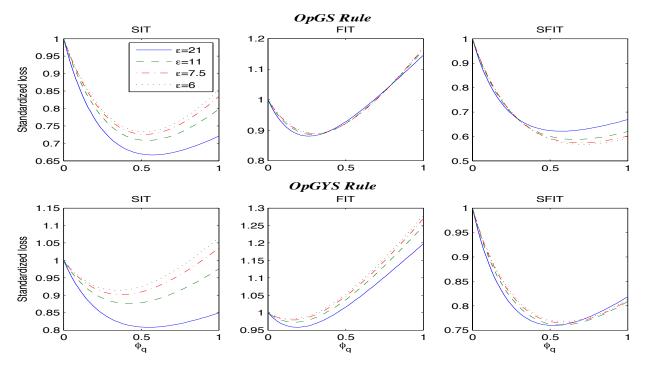
Table 2 shows the volatility implied by all the mentioned operational rules for the main variables in the system, and shows that granting a response to the stock-price growth rate and not the output growth rate (OpGS) improves the overall performance of the basic OpTR rule and performs better than any other operational rule, achieving a stronger stabilization of all variables while only marginally affecting the volatility of the output gap. The dominance of OpGS, moreover, is robust to different parameterizations of the response coefficient  $\phi_q$  and the wholesalers market power, as Figures 6 and 7 clearly show.<sup>27</sup>

## 5 Summary and Conclusions.

This paper enters the debate in the literature about the links between monetary policy and financial stability, and about the desirability that Central Banks be actively concerned about the stock market dynamics in the design of their monetary policy actions.

The analysis is carried out within a small-scale stochastic general equilibrium DNK model with heterogenous households à la Blanchard–Yaari, in which stock prices have direct wealth effects on real activity. In particular, this departure from the standard small-scale DNK model implies a theoretical rationale for considering stock-price dynamics in the design and conduct of monetary

<sup>&</sup>lt;sup>27</sup>As a robustness check, we performed the same exercise for alternative values of the steady-state Frisch elasticity of labor supply. Higher values for  $\varphi$  have effects analogous to lower values for  $\mu$ , since both affect the elasticity of dividends with respect to the output gap. However, altering the Frisch elasticity of labor supply produces some quantitative effects on the performance of the Gap- (larger) and Growth- (much smaller) rules, but does not undermine the qualitative results concerning the latter. See Nisticò (2002b) for the full set of results.



**Figure 7:** Standardized loss implied by alternative *operational* "growth-rules", for different parameterizations and policy regimes. SIT: Strict Inflation Targeting; FIT: Flexible Inflation Targeting; SFIT: Smooth Flexible Inflation Targeting. OpGS-Rule:  $r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_q \Delta q_t + u_{r,t}$ ; OpGYS-Rule:  $r_t = \tilde{\rho} + \phi_\pi \pi_t + \phi_q \Delta q_t + u_{r,t}$ 

policy. The aim of the paper is to assess what specific role (if any) stock prices play in driving monetary policy for an inflation-targeting Central Bank and what are the macroeconomic implications of adopting simple policy rules that explicitly control for stock-price dynamics.

It is shown that in the face of a given swing in stock prices, the *Wicksellian* policy response – i.e. the interest-rate dynamics consistent with price stability – depends on the shocks underlying the observed dynamics. If the driving forces are supply shocks (like to technology) then the real effects of stock prices do not require a dedicated policy response, and the "natural" interest rate dynamics are the same as the one emerging from the standard Representative-Agent setup, in which there is no *a priori* point in reacting to stock prices, since they are redundant for the equilibrium allocation. In contrast, if the driving force is a demand shock like a shock to the stochastic discount factor, government expenditure or a fad affecting the equity premium, then positive stock-wealth effects require an additional, dedicated response relative to the RA setup.

As to the implementation, it is shown that simple rules explicitly responding to the stockprice gap – defined as deviations of the stock-price index from its flexible-price level – may yield endogenous instability, the more so the less profitable (on average) are equity shares. A policy rule responding to the *growth rate* of stock prices, on the other hand, does not imply these indeterminacy risks.

The macroeconomic implications of different policy rules within different policy regimes are also derived. Common to all regimes analyzed are three results. First, loss minimization requires a (potentially large) response to stock prices, in the form of either a Gap- or a Growth-Rule. Second, economies with a higher degree of competitiveness in the productive sector entail a lower stabilizing power of a monetary policy response to *stock-price gaps*, and a higher risk that such a response yield endogenous instability. However, also these economies would find it largely beneficial in terms of inflation and interest rate volatility to augment their monetary policy rules with a response to *stock-price growth*. Third, when it comes to *strictly operational rules*, a rule which has the interest rate vary in response to the growth rate of stock prices (besides inflation) outperformes all the alternative ones considered, and can achieve potentially large stability gains for inflation and the interest rates, with the lowest price in terms of output gap volatility.

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## A Appendix

## A.1 Solving for Individual and Aggregate Consumption

Here we show how the methodology described in Piergallini (2004) can be applied to a framework with preference shocks.

The first-order conditions w.r.t. bonds and stocks reduce the budget constraint to equation (7):

$$\frac{1}{\delta} P_t C_{j,t} + E_t \left\{ \mathcal{F}_{t,t+1} (1-\gamma) \Omega_{j,t+1}^* \right\} = W_t - P_t T_t + \Omega_{j,t}^*.$$
(A.1)

Solving forward and using the definition of human wealth (8) yields:

$$\Omega_{j,t}^{*} = \frac{1}{\delta} E_t \left\{ \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k} (1-\gamma)^k P_{t+k} C_{j,t+k} \right\} - h_t^*;$$
(A.2)

as in Piergallini (2004), substituting equation (4) for the stochastic discount factor allows to take current nominal consumption out of the sum, yielding the closed-form solution:

$$\Omega_{j,t}^{*} = \frac{1}{\delta} P_t C_{j,t} E_t \left\{ \sum_{k=0}^{\infty} \beta^k (1-\gamma)^k \exp(\nu_{t+k} - \nu_t) \right\} - h_t^{*} = \frac{1}{\delta} P_t C_{j,t} \Sigma_t - h_t^{*}$$
(A.3)

where in the second equality we  $\Sigma_t \equiv E_t \left\{ \sum_{k=0}^{\infty} \beta^k (1-\gamma)^k \exp(\nu_{t+k} - \nu_t) \right\}$  collects the effects of the growth in the preference shock  $\nu$ . Rearranging finally yields equation (9) in Section 2.1.

As to aggregate consumption, using equation (14) to substitute for  $\Omega_t$  in equation (13) yields:

$$P_t C_t = \frac{1}{\Sigma_t} \bigg[ P_t C_t + E_t \big\{ \mathcal{F}_{t,t+1} \Omega_{t+1}^* \big\} + \delta(h_t^* - W_t + P_t T_t) \bigg].$$
(A.4)

Write equation (13) in nominal terms, lead it forward one period, multiply by  $\Sigma_{t+1} \mathcal{F}_{t,t+1}(1-\gamma)$  and take conditional expectations:

$$E_t \left\{ \Sigma_{t+1} \mathcal{F}_{t,t+1} (1-\gamma) P_{t+1} C_{t+1} \right\} = \delta E_t \left\{ \mathcal{F}_{t,t+1} (1-\gamma) \Omega_{t+1}^* \right\} + \delta E_t \left\{ \mathcal{F}_{t,t+1} (1-\gamma) h_{t+1}^* \right\}.$$
 (A.5)

Using the definition of human wealth to substitute out the last term implies:

$$\delta(h_t^* - W_t + P_t T_t) = (1 - \gamma) E_t \bigg\{ \Sigma_{t+1} \mathcal{F}_{t,t+1} P_{t+1} C_{t+1} \bigg\} - \delta(1 - \gamma) E_t \bigg\{ \mathcal{F}_{t,t+1} \Omega_{t+1}^* \bigg\}.$$
(A.6)

Plugging the above condition into equation (A.4) and rearranging finally yields equation (16) in the text:

$$(\Sigma_t - 1)C_t = \gamma \delta E_t \{ \mathcal{F}_{t,t+1} \Pi_{t+k} \Omega_{t+1} \} + (1 - \gamma) E_t \{ \Sigma_{t+1} \mathcal{F}_{t,t+1} \Pi_{t+k} C_{t+1} \}.$$
 (A.7)

## A.2 The Natural Rate of Interest.

From system (49)-(50) in Section 3, solving equation (49) for  $(rr_t^n - \tilde{\rho})$  yields

$$(rr_t^n - \tilde{\rho}) = E_t \Delta y_{t+1}^n - E_t \Delta g_{t+1} + \psi(q_t^n - y_t^n) - (1 + \psi)(1 + \psi_\nu)E_t \Delta \nu_{t+1} + \psi g_t$$
(A.8)

and substituting into equation (50) gives

$$q_t^n = \tilde{\beta} E_t q_{t+1}^n + (1 - \tilde{\beta}) E_t y_{t+1}^n + e_t - E_t \Delta y_{t+1}^n + E_t \Delta g_{t+1} - \psi(q_t^n - y_t^n) + (1 + \psi)(1 + \psi_\nu) E_t \Delta \nu_{t+1} - \psi g_t = \\ = \tilde{\beta} (E_t q_{t+1}^n - E_t y_{t+1}^n) + (1 + \psi) y_t^n - \psi q_t^n + E_t \Delta g_{t+1} + (1 + \psi)(1 + \psi_\nu) E_t \Delta \nu_{t+1} - \psi g_t + e_t.$$
(A.9)

The above equation takes the form of a stochastic difference equation in the capitalization ratio  $(q_t^n - y_t^n)$ :

$$q_t^n - y_t^n = \frac{\tilde{\beta}}{1+\psi} E_t \{ q_{t+1}^n - y_{t+1}^n \} + \frac{1}{(1+\psi)} E_t \Delta g_{t+1} + (1+\psi_\nu) E_t \Delta \nu_{t+1} - \frac{\psi}{(1+\psi)} g_t + \frac{1}{1+\psi} e_t.$$
(A.10)

Iterating forward, and using  $E_t \Delta v_{t+k+1} = \rho_v^k E_t \Delta v_{t+1}$ , for  $v = g, \nu$ , yields

$$q_{t}^{n} - y_{t}^{n} = \frac{1}{(1+\psi)} \sum_{k=0}^{\infty} \left(\frac{\tilde{\beta}\rho_{g}}{1+\psi}\right)^{k} (E_{t}\Delta g_{t+1} - \psi g_{t}) + (1+\psi_{\nu}) \sum_{k=0}^{\infty} \left(\frac{\tilde{\beta}\rho_{\nu}}{1+\psi}\right)^{k} E_{t}\Delta\nu_{t+1} + \frac{1}{1+\psi} \sum_{k=0}^{\infty} \left(\frac{\tilde{\beta}\rho_{e}}{1+\psi}\right)^{k} e_{t} \quad (A.11)$$

Considering  $0 < \frac{\tilde{\beta}}{1+\psi}$ ,  $\rho_v < 1$ , for  $v = g, e, \nu$  we get the final solution for the stock-market-to-GDP ratio under flexible prices:

$$q_t^n - y_t^n = \frac{1}{1 + \psi - \tilde{\beta}\rho_g} (E_t \Delta g_{t+1} - \psi g_t) + (1 + \psi)(1 + \psi_\nu) \frac{1}{1 + \psi - \tilde{\beta}\rho_\nu} E_t \Delta \nu_{t+1} + \frac{1}{1 + \psi - \tilde{\beta}\rho_e} e_t.$$
(A.12)

Plugging the above solution into equation (A.8) finally gives the reduced form for the *Wicksellian Natural Rate* of Interest:

$$rr_{t}^{n} = \tilde{\rho} + E_{t}\Delta y_{t+1}^{n} - E_{t}\Delta g_{t+1} - (1+\psi)(1+\psi_{\nu})E_{t}\Delta\nu_{t+1} + \psi g_{t} + \psi \frac{1}{1+\psi - \tilde{\beta}\rho_{g}}(E_{t}\Delta g_{t+1} - \psi g_{t}) + \psi(1+\psi)(1+\psi_{\nu})\frac{1}{1+\psi - \tilde{\beta}\rho_{\nu}}E_{t}\Delta\nu_{t+1} + \frac{\psi}{1+\psi - \tilde{\beta}\rho_{e}}e_{t}.$$
 (A.13)

Rearranging the terms conveniently and using equation (48) we can finally express the Natural Rate of Interest as:

$$rr_{t}^{n} = \bar{\rho} + rr_{RA,t}^{n} + \psi g_{t} - \psi (1 + \psi - \rho_{g}) \frac{1}{1 + \psi - \tilde{\beta}\rho_{g}} g_{t} + (1 - \rho_{\nu})[(1 + \psi)(1 + \psi_{\nu}) - 1]\nu_{t} - \psi(1 - \rho_{\nu}) \frac{(1 + \psi)(1 + \psi_{\nu})}{1 + \psi - \tilde{\beta}\rho_{\nu}} \nu_{t} + \frac{\psi}{1 + \psi - \tilde{\beta}\rho_{e}} e_{t}.$$
 (A.14)

Collecting the coefficients of the structural shocks we finally get

rr

$$_{t}^{n} = \bar{\rho} + rr_{RA,t}^{n} + \Psi_{e}e_{t} + \Psi_{g}g_{t} + \Psi_{\nu}\nu_{t}, \qquad (A.15)$$

where the following holds:

$$\Psi_e \equiv \frac{\psi}{1 + \psi - \tilde{\beta}\rho_e} > 0 \tag{A.16}$$

$$\Psi_g \equiv \psi \rho_g \frac{1 - \tilde{\beta}}{1 + \psi - \tilde{\beta} \rho_g} > 0.$$
(A.17)

As to the coefficient of the preference shock  $\nu$ , we get

$$\Psi_{\nu} \equiv (1 - \rho_{\nu})(1 + \psi)(1 + \psi_{\nu}) - (1 - \rho_{\nu}) - \psi(1 - \rho_{\nu})(1 + \psi)(1 + \psi_{\nu}) \frac{1}{1 + \psi - \tilde{\beta}\rho_{\nu}}$$
$$= (1 - \rho_{\nu})(1 + \psi)(1 + \psi_{\nu}) \left[\frac{1 - \tilde{\beta}\rho_{\nu}}{1 + \psi - \tilde{\beta}\rho_{\nu}}\right] - (1 - \rho_{\nu})$$
$$= (1 - \rho_{\nu}) \left[\frac{(1 + \psi)(1 + \psi_{\nu}) - (1 + \psi)(1 + \psi_{\nu})\tilde{\beta}\rho_{\nu} - (1 + \psi) + \tilde{\beta}\rho_{\nu}}{1 + \psi - \tilde{\beta}\rho_{\nu}}\right]. \quad (A.18)$$

Simplifying finally yields

$$\Psi_{\nu} \equiv (1 - \rho_{\nu}) \left[ \frac{\psi_{\nu}(1 + \psi) - \tilde{\beta}\rho_{\nu}[(1 + \psi)(1 + \psi_{\nu}) - 1]}{1 + \psi - \tilde{\beta}\rho_{\nu}} \right]$$
(A.19)

To determine the sign of  $\Psi_{\nu}$ , note that it is negative if and only if is the numerator. The latter condition requires

$$\frac{\psi}{1+\psi} > \frac{1-\tilde{\beta}\rho_{\nu}}{\tilde{\beta}\rho_{\nu}}\psi_{\nu}.$$
(A.20)

**Proof**:

$$(1+\psi)(1+\psi_{\nu}) - (1+\psi)(1+\psi_{\nu})\tilde{\beta}\rho_{\nu} - (1+\psi) + \tilde{\beta}\rho_{\nu} < 0$$

$$(1+\psi_{\nu}) - (1+\psi_{\nu})\tilde{\beta}\rho_{\nu} - 1 + \frac{\tilde{\beta}\rho_{\nu}}{1+\psi} < 0$$

$$(1+\psi_{\nu})(1-\tilde{\beta}\rho_{\nu}) + \frac{\tilde{\beta}\rho_{\nu}}{1+\psi} < 1$$

$$1 - \frac{\tilde{\beta}\rho_{\nu}}{1+\psi} > (1+\psi_{\nu})(1-\tilde{\beta}\rho_{\nu})$$

$$\tilde{\beta}\rho_{\nu} - \frac{\tilde{\beta}\rho_{\nu}}{1+\psi} > \psi_{\nu}(1-\tilde{\beta}\rho_{\nu})$$

$$\frac{\psi}{1+\psi} > \frac{1-\tilde{\beta}\rho_{\nu}}{\tilde{\beta}\rho_{\nu}}\psi_{\nu}.$$