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# *Gifts, Bequests and Growth*

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## Abstract

A familiar result in the theory of private intergenerational transfers is that competitive equilibria with gifts from children to their parents are dynamically inefficient whereas they are dynamically efficient with bequests from parents to their children. This note demonstrates that if growth is endogenous, both gift and bequest economies are dynamically efficient, but gift economies grow more rapidly..



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## 1. Introduction

In neoclassical economies competitive balanced growth allocations are likely to be dynamically inefficient, i.e. there is overaccumulation of capital, if children care about their parents and provide them with gifts in their old age. In contrast, if parents leave bequests to their children, the balanced growth allocation is dynamically efficient.[1] It is well known, however, that the dynamic inefficiency phenomenon hinges on the neoclassical assumption of exogenous economic growth; if growth is endogenous, overaccumulation cannot occur.[2] Yet, as this note demonstrates, the direction of altruistically motivated intergenerational transfers remains significant for the balanced growth allocation. Both gift and bequest economies are dynamically efficient, but the growth rate of per capita income is higher in gift economies. Thus, the dynamic inefficiency of the neoclassical gift economy transforms into rapid per capita income growth.

## 2. The Economy

Consider an economy with altruistic overlapping generations. Individuals live for two periods and have one parent and $1+n$ children so that the population grows at rate $n$. Individuals obtain utility from own consumption in both periods of life and, additionally, from the consumption of their parents and their offsprings. Let $u_t = u(c_t^y, c_{t+1}^o)$ be the utility that a representative member of the generation born at time $t$ derives directly from own consumption, where $c_t^y$ and $c_{t+1}^o$ are consumption when young and old. Then his total welfare is given by:

$$v_t = u_t + \theta\, u_{t-1} + \sum_{j=1}^{\infty} \beta^j\, u_{t+j}, \tag{1}$$

where $\theta$ and $\beta$ measure the strength of altruism towards the parent and the offsprings.[3] The parameter $\beta$ satisfies $0 \leq \beta \leq \bar{\beta} < 1$, where $\bar{\beta}$ is sufficiently small so that the transversality condition holds in the presence of per capita income growth. Note that (1) implies that parents take care of the per capita utility of their offsprings.

To ensure the existence of a balanced growth path, $u$ is assumed to be of the form:

$$u(c^y, c^o) = \begin{cases} \dfrac{1}{1-\sigma} c^{y\,1-\sigma} + \rho \dfrac{1}{1-\sigma} c^{o\,1-\sigma}, & \text{for } \sigma > 0, \sigma \neq 1, \\ \log c^y + \rho \log c^o, & \text{for } \sigma = 1, \end{cases}$$

where $\rho$ discounts old age consumption and $\sigma$ denotes the inverse of the intertemporal elasticity of substitution in consumption.

In the first period of life individuals inelastically supply one unit of labor in the labor market, make gifts to their parents and receive bequests from them. In the second period of life they receive the proceeds of their savings and gifts from their children and leave them bequests. The budget constraints of a member of the generation born at time $t$ are:

$$c_t^y = (1 - q_t + b_t - s_t)\, w_t, \tag{2a}$$

$$c_{t+1}^o = (1 + r_{t+1})\, s_t\, w_t + (1+n)\,(q_{t+1} - b_{t+1})\, w_{t+1}, \tag{2b}$$

where $w_t$ and $s_t$ are the wage rate and the proportion of income saved at time $t$, and $r_{t+1}$ is the interest rate at time $t+1$. Without loss of generality, gifts and bequests are expressed as proportions of the wage rate, so that $q_t$ is the gift rate and $b_t$ is the bequest rate at time $t$.

The literature concerned with the dynamic inefficiency of the gift economy employs the so-called Nash approach to determine optimal individual choices. This means that individuals take as given the actions of their parents and their offsprings when choosing optimal values of consumption, gifts, and bequest.[4] Assuming that siblings cooperate in giving gifts to their parent[5], the first-order conditions for maximum utility are:[6]

$$u_{1,t} = (1 + r_{t+1})\, u_{2,t}, \tag{3a}$$

$$u_{1,t} \geq \theta\,(1+n)\, u_{2,t-1}, \quad \text{with } = \text{ if } q_t > 0, \tag{3b}$$

$$\beta\, u_{1,t+1} \leq (1+n)\, u_{2,t}, \quad \text{with } = \text{ if } b_{t+1} > 0, \tag{3c}$$

where $u_{1,t}$ is the derivative of $u_t$ with respect to its first argument and so forth. As Abel (1987) pointed out, intergenerational consistency requires that the first-order conditions of parents and their children do not contradict one another. Considering the first-order condition characterizing the optimal gift of a child to his parent at time $t$, given by (3b), and the first-order condition characterizing the optimal bequest from a parent to his child at time $t$, given by (3c) lagged by one period, one obtains:

$$\theta\,(1+n)\,u_{2,t-1} \leq u_{1,t} \leq \frac{1}{\beta}\,(1+n)\,u_{2,t-1}.$$

This condition requires that individuals attach at least as much importance to own consumption as other members of their dynasty do. Otherwise, individuals would try to correct the consumption plans of the members of their dynasty by means of gifts and bequests. In this way, it excludes dynastic paternalism. It implies:

$$\theta\,\beta \leq 1. \tag{4}$$

With this condition there are two restrictions on the intergenerational discount factors $\beta$ and $\theta$ to ensure finiteness of dynastic utility and intergenerational consistency. More precisely, pairs of $\theta$ and $\beta$ are restricted to the set $S = \{(\theta, \beta) \in \mathbf{R}^2_+ : 0 \leq \beta \leq \bar{\beta}, 0 \leq \theta\,\beta \leq 1\}$.

Firms hire the available labor force which equals the size of the young generation, given by $N_t$ at time $t$, and the aggregate capital stock $K_t$ to produce the homogeneous output $Y_t = F(K_t, A_t N_t)$. The technology $F$ exhibits constant returns to scale and $A_t$ measures the productivity of labor at time $t$. Marginal product pricing leads to:

$$r_t = f'(k_t), \tag{5}$$

$$w_t = A_t\,[f(k_t) - k_t\,f'(k_t)], \tag{6}$$

where $k_t \equiv K_t/A_t N_t$ and $f(k_t) \equiv F(K_t/A_t N_t, 1)$. To endogenize labor productivity, it is assumed that $A_t$ depends on cumulated investments as suggested by Arrow (1962) and Romer (1986). More precisely, labor productivity is determined by:[7]

$$A_t = \frac{1}{a}\frac{K_t}{N_t}, \tag{7}$$

where $a$ is a positive technological parameter. Note, that $A_t$ has been related to cumulated investments per worker. This ensures the existence of a balanced growth path in the presence of a growing population. Substituting (7) into (5) and (6) yields:

$$r_t = r \equiv f'(a), \tag{8}$$
$$w_t = \omega \frac{K_t}{N_t}, \quad \text{with } \omega \equiv [f(a) - af'(a)]/a. \tag{9}$$

Thus, the interest rate is constant over time and the wage rate is proportional to the capital stock per worker, with $\omega$ as the factor of proportionality. The latter can be interpreted as the external return on capital. Because of the positive externality of investments on labor productivity, the interest rate differs from the social marginal return on capital which is given by $dY_t/dK_t = r + \omega$.

Product market equilibrium obtains when aggregate investment and aggregate savings are equalized:

$$K_{t+1} = N_t\, s_t\, w_t. \tag{10}$$

This completes the model. Equations (3a,b,c), (8), (9), and (10) together define a competitive equilibrium.

## 3. Operative Transfer Motives and Growth

To determine the balanced growth rates of per capita income in case of operative gift and bequest motives (in the sense that gifts respectively bequests are determined by tangency conditions rather than corner solutions), divide (3a) by (3b) and (3c) by (3a). This yields:

$$\frac{c^o_{t+1}}{c^o_t} \leq \left(\frac{1+r}{\theta\,(1+n)}\right)^{\frac{1}{\sigma}}, \quad \text{with } = \text{ if } q_t > 0, \tag{11a}$$
$$\frac{c^y_{t+1}}{c^y_t} \geq \left(\frac{\beta\,(1+r)}{1+n}\right)^{\frac{1}{\sigma}}, \quad \text{with } = \text{ if } b_{t+1} > 0, \tag{11b}$$

where the specific form of $u$ and (8) have been considered. On a balanced growth path both young and old age consumption grow at the same rate as per capita income. Denote this rate by $g$. Equations (11a,b) then imply:

$$g_q \geq g \geq g_b,$$

where $g_q \equiv [(1+r)/\theta\,(1+n)]^{1/\sigma} - 1$ and $g_b \equiv [\beta\,(1+r)/(1+n)]^{1/\sigma} - 1$ are the balanced growth rates of per capita income when the gift respectively the bequest motive is operative. Thus, the balanced growth rates of the gift and the bequest economy define the upper and the lower bound of all possible balanced growth rates of per capita income. This leads to the following proposition.

PROPOSITION 1: $g_q \geq g_b$ with strict inequality if $\theta\,\beta < 1$.

By assumption, $\beta < 1$. Hence, gift economies grow at a strictly higher rate than bequest economies when $\theta \leq 1$, i.e. when individuals place at least as much weight on the utility they obtain from own consumption than on the utility they obtain from parental consumption. The result of rapid growth in case of an operative gift motive emerges in spite of the fact that transfers from young to old individuals serve to reduce per capita income growth since they discourage private savings and, henceforth, capital accumulation. It is not the gift motive as such which spurs growth. It is rather the fact that for the gift motive to be operative, the economy must exhibit high capital accumulation. In contrast to neoclassical economies, however, high capital accumulation leads to speedy economic growth but not, as will be demonstrated in the next section, to dynamic inefficiency, i.e. to overaccumulation.

## 4. Dynamic Efficiency of the Gift Economy

If the gift motive is operative on a balanced growth path, equations (2a,b) and (3b) imply:

$$[(1+n)\,\rho\,\theta]^{\frac{1}{\sigma}}(1+g_q)(1-q+b-s) = (1+r)s + (1+n)(1+g_q)(q-b), \quad (12)$$

where $s$, $q$, and $b$ are the balanced growth saving, gift, and bequest rates. Considering equations (9) and (10), the saving rate in case of an operative gift motive may be written as $s = (1+n)(1+g_q)/\omega$. Substituting for $s$ in (12) and considering the definition of $g_q$, straightforward manipulation yields:

$$q - b = \frac{(1+n)^{\frac{1-\sigma}{\sigma}}\,\rho^{\frac{1}{\sigma}}\,\theta^{\frac{1}{\sigma}}}{1+(1+n)^{\frac{1-\sigma}{\sigma}}\,\rho^{\frac{1}{\sigma}}\,\theta^{\frac{1}{\sigma}}} - \frac{1+r+\rho^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}}}{[1+(1+n)^{\frac{1-\sigma}{\sigma}}\,\rho^{\frac{1}{\sigma}}\,\theta^{\frac{1}{\sigma}}]\,\omega}. \quad (13)$$

If $\theta\,\beta < 1$, $b$ must be zero in a gift economy (this is a direct implication of Proposition 1). In contrast, if $\theta\,\beta = 1$, both the gift and the bequest motive can be operative on a balanced growth path. In this case equations (3b,c) coincide, implying that only the rate of the net flow of transfers, $q-b$, can be determined. Then gift economies are those economies in which the net flow of transfers is from the young to the old.

From equation (13) it can be inferred that the (net) gift motive is operative for all $\theta$'s which satisfy:

$$\theta \geq \underline{\theta} \equiv \frac{[1+r+\rho^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}}]^{\sigma}}{\rho\,(1+n)^{1-\sigma}\,\omega^{\sigma}}, \tag{14}$$

provided that they are consistent with restriction (4). This means that altruism towards parents has to be sufficiently large in order to render the gift motive operative. Considering that the growth rate of the gift economy, $g_q$, is the lower the larger is $\theta$, it follows by substituting (14) into the expression for $g_q$ that the growth factor of the gift economy, given by $(1+n)(1+g_q)$, is bounded from above as follows:

$$(1+n)(1+g_q) \leq \frac{\rho^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}}}{1+r+\rho^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}}}\,\omega.$$

Obviously, the upper bound of the growth factor is smaller than the gross social return on capital given by $1+r+\omega$. Since dynamic inefficiency would only occur, if the gross social return of capital were smaller than the growth factor [see King and Ferguson (1993)], the following result obtains:

PROPOSITION 2: Gift economies are dynamically efficient.

## 5. Concluding Remarks

In an endogenous growth framework the dynamic inefficiency of the neoclassical gift economy transforms into high per capita income growth. This has been shown by employing a specific endogenous growth framework, namely the Arrow-Romer model. However, the result can be supposed to hold also for other endogenous growth settings. This is because it is high capital accumulation which renders the gift motive operative and which spurs per capita income growth in an endogenous growth model.

## References

Abel, Andrew B. "Operative Gift and Bequest Motives." American Economic Review 77 (December 1987): 1037-1047.

Arrow, Kenneth J. "The Economic Implications of Learning by Doing." Review of Economic Studies 29 (June 1962): 155-173.

Buiter, Willem H. and Jeffrey Carmichael. "Government Debt: Comment." American Economic Review 74 (September 1984): 762-765.

Carmichael, Jeffrey. "On Barro's Theorem of Debt Neutrality: The Irrelevance of Net Wealth." American Economic Review 72 (March 1982): 202-213.

Grossman, Gene G. and Noriyuki Yanagawa. "Asset Bubbles and Endogenous Growth." Journal of Monetary Economics 32 (February 1993): 3-19.

Kimball, Miles S. "Making Sense of Two-Sided Altruism." Journal of Monetary Economics 20 (September 1987): 301-326.

King, Ian and Don Ferguson. "Dynamic Inefficiency, Endogenous Growth, and Ponzi Games." Journal of Monetary Economics 32 (February 1993): 79-104.

O'Connell, Stephen A. and Stephen P. Zeldes. "Dynamic Efficiency in the Gifts Economy." Journal of Monetary Economics 31 (June 1993): 363-379.

Romer, Paul M. "Increasing Returns and Long-Run Growth." Journal of Political Economy 94 (October 1986): 1002-1037.

Saint-Paul, Gilles. "Fiscal Policy in an Endogenous Growth Model." Quarterly Journal of Economics 106 (November 1992): 1243-1259.

## Appendix

Mathematical Notation:

$t$ = time index
$v$ = total individual welfare
$u$ = utility from own consumption
$c^y$ = young age consumption
$c^o$ = old age consumption
$\beta$ = parameter measuring altruism towards children
$\theta$ = parameter measuring altruism towards parents
$\rho$ = intertemporal discount parameter
$\sigma$ = inverse of intertemporal elasticity of substitution
$q$ = gift rate
$b$ = bequest rate
$s$ = savings rate
$Y$ = aggregate product
$F$ = technology
$K$ = aggregate capital stock
$N$ = labor force
$A$ = productivity index
$k = K/AN$
$a$ = technological parameter
$n$ = population growth rate
$g$ = productivity growth rate
$r$ = interest rate
$w$ = wage rate
$\omega$ = external return on capital

## Notes

1 See, e.g., Carmichael (1982), Abel (1987), and Kimball (1987).

2 This has been shown by Saint-Paul (1992), Grossman and Yanagawa (1993), and King and Ferguson (1993).

3 See Buiter and Carmichael (1984) and Abel (1987) for the same concept of two-sided dynastic altruism. One might argue that a dynastic utility function of the form

$$\begin{aligned} v_t &= u_t + \theta\, u_{t-1} + \beta\, v_{t+1} \\ &= u_t + \theta\, u_{t-1} + \sum_{j=1}^{\infty} \beta^j \,(u_{t+j} + \theta\, u_{t+j-1}) \end{aligned} \tag{$*$}$$

would be more satisfactory as it encompasses the fact that children care for their parents which, in turn, should enter the utility of the parents. Note, however, that $(*)$ can be written as:

$$v_t = (1+\theta\beta)\left(u_t + \frac{\theta}{1+\theta\beta}\, u_{t-1} + \sum_{j=1}^{\infty} \beta^j\, u_{t+j}\right),$$

which, since the factor $1+\theta\beta$ is behaviorally irrelevant, can be reduced to:

$$v_t = u_t + \tilde{\theta}\, u_{t-1} + \sum_{j=1}^{\infty} \beta^j\, u_{t+j},$$

with $\tilde{\theta} = \theta/(1+\theta\beta)$. From this equation it can be inferred that rather than being a conceptual issue, the difference between (1) and $(*)$ is a question of what are the relevant magnitudes of $\beta$ and $\theta$. Another specification of two-sided dynastic altruism has been proposed by Kimball (1987). It is of the form:

$$v_t = u_t + \theta\, v_{t-1} + \beta\, v_{t+1}, \tag{$**$}$$

i.e. it treats ancestors and descendants symmetrically in the sense that it considers the full welfare including altruistic concerns of both parents and children. Kimball has shown that if one imposes some restrictions on the parameters $\theta$ and $\beta$ the double recursion implicit in $(**)$ has a solution of the form:

$$v_t = \sum_{j=-\infty}^{\infty} \delta_j \, u_{t+j},$$

where the elements of the sequence $\{\delta_j\}_{j=-\infty}^{\infty}$ are strictly positive expressions of $\theta$ and $\beta$. This specification would not alter the character of the results derived in this paper since the competitive equilibrium would display similar characteristics as has been demonstrated by Kimball (1987). However, there is no logical reason that requires to treat parents and children symmetrically in the way of $(**)$, it is rather a matter of what is believed to be the appropriate representation of dynastic utility. Since I find it hard to imagine that ancestors who lived in primeval times affect the well-being of people living today, even if it is only in the indirect way as specified in $(**)$, I employ the Abel-Buiter-Carmichael specification of dynastic altruism.

[4] If, instead, parents were viewed as 'Stackelberg leaders', the problem of dynamic inefficiency in the gift economy would disappear. This has been shown by O'Connell and Zeldes (1993).

[5] Kimball (1987, p. 315) provides a justification for cooperation among siblings which relies on the notion that giving gifts to parents constitutes a repeated rather than a one-shot game. Under the alternative assumption of non-cooperation among siblings as considered by Abel (1987) qualitatively similar results would be obtained.

[6] Since the specific form of $u$ implies $u_1(0,\cdot) = \infty$ and $u_2(\cdot,0) = \infty$, corner solutions with respect to young and old age consumption can be ruled out.

[7] See Grossman and Yanagawa (1993) and King and Ferguson (1993) for a similar representation of the Arrow-Romer growth model.