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Reciprocity in the Principal Multiple Agent Model

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Abstract

This paper studies how incentives are affected by intention-based reciprocity preferences when the principal hires many agents. Our results describe the agents' psychological attitudes required to sustain a given strategy profile. We also show that hiring reciprocal agents to implement a first or a second-best contract will always benefit the principal if the strategy profile is symmetric. When instead the profile (first or second-best) is asymmetric the principal's best interest might be better served by self- interested agents. We conclude the paper by clarifying when symmetric profiles are most likely to arise.

JEL classification: C72; D03; D86.

Keywords: reciprocity, many agents, psychological games.

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1 Introduction

In the seminal paper by Mookerjee (1984), the principal-agent model of moral hazard is extended to a multiple agent setting: Production depends on the entire profile of efforts exerted by the team of agents. The principal cannot observe the effort chosen by each member of the team and designs wage schedules contingent on outputs. Therefore, optimal contracts are chosen under a system of incentive compatibility constraints which take naturally the form of Nash equilibrium conditions of the underlying game between the agents.

Given that each agent must share the marginal benefit of his effort, but he alone bears its costs, teams are affected by free riding problems which, in turn, might produce a (negative) psychological externality on agents. Therefore, teams seem to be a natural environment in which reciprocity plays an important role. Indeed, if a team member is sufficiently sensitive to reciprocity then he will reasonably have incentives to free ride if he believes that other players' intentions are bad, in the sense that they plan to exert a level of effort which is lower than the one they expect from him, and, conversely, he will have incentives to work harder if he believes that his partners' intentions are good. In this paper we address this issue and, building on the reciprocity motives previously described, we study how contracts are affected by intention-based reciprocity preferences when a self-interested principal hires reciprocal players.

More precisely, we include reciprocal agents in the Mookerjee's model so that the IC conditions appear as a *psychological game*[1]. Under the assumption that the principal chooses incentive schemes to implement agents actions as a psychological Nash equilibrium, necessary and sufficient conditions are derived for the attainability of the first best. Our results describe the psychological attitudes of the team members required to sustain a given strategy profile in equilibrium and, therefore, give useful insights for hiring and team design. However, the agents's psychological characteristics used to prove the previous results are not sufficient to attain the first best if some partners can collude. We show that additional psychological assumptions allow for the the collusion-proof implementation of the first best. Moreover, we ask whether hiring reciprocal agents would increase or decrease the net benefit of the principal. It turns out that a principal will always prefer reciprocal agents to implement a first or a second-best contract if the strategy profile is symmetric and show, by way of examples, that whenever the first or the second-best profiles are asymmetric the principal might prefer self-interested to intention-based reciprocity agents. Finally, since our previous results have underlined the important role played by symmetry for reciprocity to be in the principal's interest, we conclude the paper by clarifying when symmetric profiles are most likely to arise.

Several papers have studied the welfare properties of the optimal incentive schemes in a principal-many agents problem (Holmstrom 1982, Demski and Sappington 1984, Mookherjee 1984, Malcomson 1986), and have attempted to solve the problems arising from the fact that those incentive schemes might not implement the principal's chosen vector of actions as a unique equilibrium (Demski and Sappington 1984, Mookherjee 1982 and 1984, Ma 1988). However, the role of reciprocity was not addressed in this early literature. More recently, experimental evidence in Fehr, Gächter, and Kirchsteiger (1997) has suggested that

[1]See Geanakoplos, Pearce and Stacchetti (1989) for psychological games and equilibria.

reciprocal motives contribute to the enforcement of contracts. In this vein, Dufwenberg and Kirchsteiger (2000), Englmaier and Leider (2008) and Netzer and Schmutzler (2010) all show that efficiency is generally increased when a materialistic principal interacts with a reciprocal agent[2]. Our contribution extends the study of the role played by reciprocity to the multiple agent case.

The present paper is also related to a growing literature that studies the impact of inequity aversion to multiple agent models (Itoh 2004, Demougin and Fluet 2006, Rey Biel 2008). These articles find that when there are multiple agents who care about the final monetary distributions among each other, the principal can exploit their nature by designing interdependent contracts. However, "reciprocity and inequity aversion are distinct motives, and often intention matters more, in particular in the domain of punishing behaviour, as suggested by recent evidence" (Itoh 2004). Accordingly, in our model contracts are affected in a different way with respect to inequity aversion models since a team member may react badly to other agents' free-riding regardless of everyone's rewards. Moreover, we show that – if intentions are the driving motive – the principal is more likely to exploit it if first or second best strategy profiles are symmetric.

Finally, in De Marco and Immordino (2010) we study the impact of intention-based reciprocity preferences on the free-riding problem arising in non-stochastic partnerships. We suggest a tendency of efficient partnerships to consist of members whose sensitivity to reciprocity is – individually or jointly – sufficiently high and construct associated reciprocity based sharing rules. The analysis in this paper is motivated by this result and it extends the theoretical framework developed in our companion paper to encompass the more complex setting where a principal can write optimal contracts to incentivize the agents.

The paper is organized as follows. In Section 2.1, Mookherjee's many agent model is summarized. We introduce the psychological features of our model in Section 2.2. In Section 3 we provide sufficient and necessary conditions for the implementation of the efficient strategy profile. Section 4 shows that reciprocal agents always increase the net benefit of the principal when the material second best is symmetric but not when the material second best is asymmetric. Section 5 deals with the collusion-proof implementation of the first best In Section 6 we provide the sufficient conditions for a symmetric first-best strategy profile to arise. Section 7 concludes. All proofs are relegated to the Appendix.

2 The model

2.1 The moral hazard problem

We first introduce the moral hazard problem and to ease the comparison we borrow most assumptions and notation from Mookherjee (1984).

*Agents' Material Payoffs.*The model consists of a set of agents $N = \{1, ..., n\}$ with $n \geq 2$, a set of effort levels A_k for each agent and a disutility function $G_k : A_k \to \mathbb{R}$ for each agent. $A = \prod_{k \in N} A_k$. Moreover, we assume that $A_i = \{1, ..., m\}$ for any i. We donote with Q the

[2]Even if, as shown by Netzer and Schmutzler (2010), firms may not want to employ a reciprocal worker.

finite set of outcomes and $|Q| = \nu$. The probability distribution on Q induced by an effort (strategy) profile $a = (a_1, \ldots, a_n)$ is denoted by $\pi(a)$ where $\pi_q(a)$ is the probability of the output q given a.[3] Denote also with $J(a)$ the support of the probability distribution on $\pi(a)$, that is

$$J(a) = \{q \in Q \,|\, \pi_q(a) > 0\}.$$

Each agent k has a von-Neumann-Morgenstern utility function u_k which is additively separable in the action chosen by agent k and in the payment received

$$u_k(a_k, I^k) = V_k(I^k) - G_k(a_k),$$

where the principal's payment I^k ranges in an interval $[\underline{I}, \overline{I}]$. The agent k has a reservation utility $\underline{u}_k$ and we impose the following

Assumption 1. *Each function V_k is continuous, strictly increasing and concave over $[\underline{I}, \overline{I}]$ with $V_k(\underline{I}) - \min_{a_k \in A_k} G_k(a_k) < \underline{u}_k$. Moreover, for every $a_k \in A_k$ there exists $I \in [\underline{I}, \overline{I}]$ such that $V_k(I) - G_k(a_k) = \underline{u}_k$. Finally, we impose the normalization $V_k(\underline{I}) = 0$.*

The principal's benefit from an output profile q is denoted by $\widetilde{B}(q)$ and the principal's expected benefit is defined by

$$B(a) = \sum_{q \in Q} \pi_q(a) \widetilde{B}(q).$$

First-Best. In the first best the principal can observe the action chosen by the agents and, in light of Assumption 1 she can write contracts forcing any agent to choose any feasible action that guarantees the agent his reservation utility. If agent k was required to choose a_k, the principal would pay him a sum of

$$C^k_{FB}(a_k) = V_k^{-1}(\underline{u}_k + G_k(a_k)) = h(\underline{u}_k + G_k(a_k)) \quad \text{where } h = V_k^{-1}$$

if he chooses a_k and $\underline{I}$ otherwise. The first best cost to the principal from the effort profile a is then

$$C_{FB}(a) = \sum_{k \in N} C^k_{FB}(a_k)$$

and the *first-best effort profile* $a^* = (a_1^*, a_2^*, \ldots, a_n^*)$ is the one which maximize in A the net benefit of the principal, that is, the function $P : A \to \mathbb{R}$ defined by

$$P(a) = B(a) - C_{FB}(a) \quad \forall a \in A.$$

Second-Best. In a second best world the principal cannot observe the actions chosen by agents, hence payments may only be based on the outcomes Q. An incentive scheme for player i is therefore an ν-dimensional vector $\mathcal{I}^k = (I^k_q)_{q \in Q} \in [\underline{I}, \overline{I}]^\nu$ where the element I^k_q is the

[3] This is different from Mookherjee (1984) and Ma (1988), where there exists for each player k a finite set of possible outputs $Q_k = \{q^k_1, \ldots, q^k_{\nu_k}\}$ and $Q = \prod_{k \in N} Q_k$. Our approach is more general and simplifies notation. The reason we can adopt it is due to the fact that we do not need any assumption on the probability distribution over output pairs.

payment given to player k if the output vector q has occurred. Given the vector of incentive schemes $\mathcal{I} = (\mathcal{I}^1, \mathcal{I}^2, \dots, \mathcal{I}^n)$ and the effort profile a the principal incurs an expected cost

$$\mathcal{C}(a, \mathcal{I}) = \sum_{q \in Q} \pi_q(a) \left[\sum_{k \in N} I_q^k \right].$$

Given an incentive scheme $\mathcal{I}^k$ and the effort profile $a = (a_1, a_2, \dots, a_n)$, the expected utility of agent k is

$$E_k(a, \mathcal{I}^k) = \sum_{q \in Q} \pi_q(a) u_k(a_k, I_q^k) = \sum_{q \in Q} \pi_q(a) V_k(I_q^k) - G_k(a_k).$$

Then, given the vector of incentive schemes $\mathcal{I} = (\mathcal{I}^1, \mathcal{I}^2, \dots, \mathcal{I}^n)$, we define the game played by the agents with

$$\Gamma(\mathcal{I}) = \{N; A_1, \dots, A_n; E_1(\cdot, \mathcal{I}^1), \dots, E_n(\cdot, \mathcal{I}^n)\}.$$

We now introduce the useful definitions of second-best contract and attainable first-best profile.

Definition 1. *A second-best contract consists in a pair $(a, \mathcal{I})$ which maximizes the net benefit*

$$B(a) - \mathcal{C}(a, \mathcal{I})$$

subject to the constraints that

IC) a is a Nash equilibrium of $\Gamma(\mathcal{I})$,

IR) $E_k(a, \mathcal{I}^k) \geq \underline{u}_k$ for every $k \in N$.

Definition 2. *The first-best effort profile a^* is said to be attainable if there exist incentive schemes $\mathcal{I}$ such that $(a^*, \mathcal{I})$ satisfies (IR) and (IC) and $\mathcal{C}(a^*, \mathcal{I}) = C_{FB}(a^*)$.*

We end this section by introducing the following useful notation. First, the set of incentive schemes such that $(a, \mathcal{I})$ is a feasible contract is denoted with

$$L(a) = \{\mathcal{I} \,|\, (a, \mathcal{I}) \, \text{satisfies} \, IR \, \text{and} \, IC\,\}.$$

Then, the second-best cost is defined by

$$C_{SB}(a) = \min_{\mathcal{I} \in L(a)} C(a, \mathcal{I}) \quad \forall a \in A$$

and the correspondence of optimal incentive schemes is[4]

$$M(a) = \{\mathcal{I} \in L(a) \,|\, C(a, \mathcal{I}) = C_{SB}(a)\}.$$

[4]It can be easily checked that the second best contracts can be equivalently obtained as the pair $(a, \mathcal{I})$ such that *i)* $\mathcal{I} \in M(a)$ and *ii)* a that maximizes $B(\cdot) - C_{SB}(\cdot)$ in $\overline{A} = \{a \in A \,|\, L(a) \neq \emptyset\}$.

2.2 Reciprocity

To complete the presentation of our set-up, we now introduce the psychological features and the equilibrium concept. Following our companion work (De Marco and Immordino, 2010) we innovate with respect to the previous literature in two main respects. First, to measure kindness we deviate from the previous literature that uses the concept of player's equitable payoff (see for instance Rabin, 1993) and we use instead the agents' level of effort as a more direct measure of each agent contribution to the output of the partnership. In the traditional model of reciprocity (Rabin 1993, Dufwenberg and Kirchsteiger 2004), the strategy sets may represent choices of different nature so that the natural way to measure kindness is to look at the players' payoffs. However, when the action sets represent comparable choices (efforts), we can adopt the simpler approach to use the level of efforts as a more direct – and easier to test – measure of kindness. In this case, the reciprocity term in the psychological utility function does not depend on the material payoffs, capturing the idea that an agent may react badly to other agents free-riding regardless of everyone's rewards. Our definition of kindness, besides being better suited to many applications (partnerships, principal with many agents), has also the advantage to be completely unrelated to Pareto efficiency.

Second, to specify the psychological utility function for each player, we introduce for every pair of players a function that assigns to each combination of kindness and belief about reciprocated kindness the disutility caused to one player by the mismatch between his and the other agent's intentions. Our single-peaked preferences depend only on the distance between the kindness of k to t, and partner $k's$ belief about how kind t is to him and differently from the functional forms in previous reciprocity models our formulation does not have their characteristic "explosive" feature.

All this should become readily clear by looking at the rest of the section.

Denote by $b_{kt} \in A_t$ partner $k's$ beliefs about partner $t's$ strategy, and by $c_{ktk} \in A_k$ partner $k's$ beliefs about partner $t's$ beliefs about partner $k's$ strategy. We denote by $\chi_{kt}(a_k, b_{kt})$ the kindness of k to t and by $\lambda_{ktk}(b_{kt}, c_{ktk})$ partner $k's$ belief about how kind t is to him. The kindness terms are defined to be

$$\chi_{kt}(a_k, b_{kt}) = a_k - b_{kt}$$

and

$$\lambda_{ktk}(b_{kt}, c_{ktk}) = b_{kt} - c_{ktk},$$

so that positive kindness from k to t arises if partner k contributes to the output with an effort level larger than the one he expects from partner t. Moreover, partner k will believe that t is kind to him if the effort he expects from partner t is larger than the one he believes partner t expects from him.

For every pair of players (k, t) the reciprocity term of k with respect to t assigns to each combination of kindness χ_{kt} and belief about reciprocated kindness λ_{ktk} the disutility caused to player k by the mismatch between these two intentions. Let

$$h_{kt}(\chi_{kt}, \lambda_{ktk}) = -(\omega_k \chi_{kt} - \lambda_{ktk})^2,$$

be the functional form of such reciprocity terms. Those single-peaked preferences depend uniquely on the distance between the kindness of k to t, and partner $k's$ belief about how kind t is to him. A remarkable characteristic of the present formulation is that it is free from the explosive feature that characterize the functional forms of previous reciprocity models such as Dufwenberg and Kirchsteiger (2004) where $h_{kt}(\chi_{kt}, \lambda_{ktk}) = \chi_{kt}\lambda_{ktk}$ or Rabin (1993) where $h_{kt}(\chi_{kt}, \lambda_{ktk}) = \lambda_{ktk}(1 + \chi_{kt},)$. Indeed, in all functional forms in which h_{kt} diverges positively (negatively) with χ_{kt} and accordingly with the sign of λ_{ktk} then there is no limit to how kind (unkind) k wants to be to t.

Then, the overall (psychological) utility function of player k is defined by

$$U_k(a_k, a_{-k}, (b_{kt})_{t\neq k}, (c_{ktk})_{t\neq k}; \mathcal{I}^k) = E_k(a, \mathcal{I}^k) + \rho_k \left[\sum_{t\neq k} h_{kt}(\chi_{kt}(a_k, b_{kt}), \lambda_{ktk}(b_{kt}, c_{ktk})) \right], \quad (1)$$

and is made up by the sum of the material payoff $E_k(a, \mathcal{I}^k)$ and the reciprocity term $\rho_k \left[\sum_{t\neq k} h_{kt}(\chi_{kt}(a_k, b_{kt}), \lambda_{ktk}(b_{kt}, c_{ktk}) \right]$.

Parameters ρ_k and ω_t summarize the psychological characteristics of each player k: $\rho_k > 0$ measures the relative importance (the weight) of the psychological term with respect to the material payoff; instead, the parameter ω_k relates the relative importance of player k's intentions – towards the others – to his beliefs about other players' intentions. Each ω_k is assumed to be positive, meaning that, for a given λ_{ktk}, the optimal kindness of player k (taking into account only h_{kt}) is $\chi_{kt} = \lambda_{ktk}/\omega_k$, which reciprocates kind behavior ($\lambda_{ktk} > 0$) with kind behavior ($\chi_{kt} > 0$) and unkind behavior ($\lambda_{ktk} < 0$) with unkind behavior ($\chi_{kt} < 0$).[5] Moreover, the magnitude of ω_k affects the optimal kindness which increases in absolute value as ω_k decreases to zero and, when $\omega_k = 1$, perfectly reciprocates the believed kindness, i.e. $\chi_{kt} = \lambda_{ktk}$.

Finally, key for our results is how those parameters affect the psychological disutilities caused by deviating from the optimal kindness for each pair of players (k, t). These disutilities are related to the products $\rho_k\omega_k^2$ as follows: fix a pair of players (k, t) and the size ε of a deviation from the optimal kindness $\chi_{kt} = \lambda_{ktk}/\omega_k$, then the player k's disutility with respect to t is given by

$$-\rho_k(\omega_k(\lambda_{ktk}/\omega_k + \varepsilon) - \lambda_{ktk})^2 = -\rho_k\omega_k^2\varepsilon^2$$

and it decreases to zero as $\rho_k\omega_k^2$ decreases to zero. Therefore, $\rho_k\omega_k^2$ – from now on denoted with θ_k – measures the 'sensitivity' of partner k's reciprocity. In other words, the greater is θ_k the more sensible to the psychological reciprocity is partner k.

We are now ready to define the psychological game (derived from the game $\Gamma(\mathcal{I})$) corresponding to the psychological utility functions defined in (1)

$$\Gamma_{\rho,\omega}(\mathcal{I}) = \{N; A_1, \ldots, A_n; U_1(\cdot, \mathcal{I}^1), \ldots, U_k(\cdot, \mathcal{I}^n)\}.$$

We conclude this section by recalling the equilibrium concept that will be used.[6]

Definition 3 (Geanakoplos, Pearce and Stacchetti 1989). *A strategy profile* $(\overline{a}_1, \ldots, \overline{a}_n) \in A$ *is a psychological Nash equilibrium of* $\Gamma_{\rho,\omega}(\mathcal{I})$ *if for all* $k \in N$
i) $\overline{a}_k \in \arg\max_{a_k \in A_k} U_k(a_k, \overline{a}_{-k}, (\overline{b}_{kt})_{t\neq k}, (\overline{c}_{ktk})_{k\neq t}; \mathcal{I}^k)$,
ii) $\overline{b}_{kt} = \overline{a}_t$ *and* $\overline{c}_{ktk} = \overline{a}_k$ *for all* $t \neq k$.

[5] $\omega_k = 0$ is excluded, since regardless of $k's$ kindness, $k's$ overall utility would be always decreasing in $k's$ belief of $t's$ (positive) kindness.

[6] In the rest of the paper $\overline{a}$ will usually denote an equilibrium and a^* a symmetric equilibrium.

3 Implementation of the First Best

We start this section by redefining second-best contracts and attainable first-best profiles under the assumption that agents have intention-based reciprocity preferences.

Definition 4. *A second-best contract under reciprocity vectors (ρ, ω) consists in a pair $(\overline{a}, \mathcal{I})$ which maximizes the net benefit*

$$B(a) - \mathcal{C}(a, \mathcal{I})$$

subject to the constraints that

($P_{\rho,\omega}$-IC) *$\overline{a}$ is a psychological Nash equilibrium of $\Gamma_{\rho,\omega}(\mathcal{I})$,*

($P_{\rho,\omega}$-IR) *$U_k(\overline{a}_k, \overline{a}_{-k}, (\overline{b}_{kt})_{t\neq k}, (\overline{c}_{ktk})_{k\neq t}; \mathcal{I}^k) \geq \underline{u}_k$, $\overline{b}_{kt} = \overline{a}_t$ and $\overline{c}_{ktk} = \overline{a}_k$ for all $t \neq k$ and for every $k \in N$.*

Definition 5. *The first-best effort profile a^* is said to be attainable under reciprocity vectors (ρ, ω) if there exist incentive schemes $\mathcal{I}$ such that $(a^*, \mathcal{I})$ satisfies ($P_{\rho,\omega}$-IR) and ($P_{\rho,\omega}$-IC) and $\mathcal{C}(a^*, \mathcal{I}) = C_{FB}(a^*)$.*

Denote with

$$\nu_k(a^*) = \max_{a_k \in A_k} \left[G_k(a_k^*) - G_k(a_k)\right],$$

an upper bound to the cost saving from an unilateral deviation for agent k.

We are now ready to state our sufficient condition for the implementation of a symmetric first-best profile.

Proposition 1. *Let a^* be a symmetric first-best profile. If $\theta_k \geq \nu_k(a^*)/(n-1)$ for every player k then a^* is attainable under reciprocity vectors (ρ, ω).*

The proof of the previous proposition shows that a^* is attainable since, when agents sensitivity to reciprocity is sufficiently high, there exist an incentive scheme $\widehat{\mathcal{I}}$ that satisfy ($P_{\rho,\omega}$-IR) and ($P_{\rho,\omega}$-IC) and $\mathcal{C}(a^*, \mathcal{I}) = C_{FB}(a^*)$. Specifically, for every $k \in N$, $\widehat{\mathcal{I}}^k$ is the incentive scheme of player k defined by

$$\widehat{I}_q^k = \begin{cases} C_{FB}^k(a_k^*) & \text{if} \quad q \in J(a^*) \\ \underline{I} & \text{if} \quad q \in Q \setminus J(a^*). \end{cases} \tag{2}$$

Below, we always denote with $\widehat{\mathcal{I}}$ the incentive schemes defined by (2) for every $k \in N$ and $q \in Q$.

Some more notation is needed to state our necessary conditions for the implementation of a symmetric first-best profile. Define

$$\mathcal{M}_k(a_k^*) = \sum_{a_k \in A_k} (a_k - a_k^*)^2$$

and

$$\mathcal{G}_k(a_k^*) = \sum_{a_k \in A_k} (G_k(a_k^*) - G_k(a_k))$$

where $\mathcal{M}_k(a_k^*)$ measures the variance of the efforts with respect to a_k^*, whereas $\mathcal{G}_k(a_k^*)$ measures the mean incremental cost from unilateral deviations from a_k^* for agent k. Finally,

$$\psi_k(a_k^*) = \sum_{a_k \in A_k} \left[\sum_{q \in J(a^*)} \left[\pi_q(a_k, a_{-k}^*) - \pi_q(a^*) \right] \right],$$

measures the mean probability of reaching some outcome q outside the support $J(a^*)$, by unilaterally deviating from a_k^* for agent k.

Then, the next proposition gives a necessary condition for a symmetric strategy profile to be implemented by a psychological equilibrium.

Proposition 2. *Assume that each function V_k is strictly concave and a^* is the symmetric first best profile. If a^* is attainable under reciprocity vectors (ρ, ω), then, for every $k \in N$, it follows that*

i) $I_q^k = C_{FB}^k(a_k^*)$ *for all* $q \in J(a^*)$

ii)

$$\theta_k \geq \frac{\psi_k(a_k^*) V_k(C_{FB}^k(a_k^*)) + \mathcal{G}_k(a_k^*)}{(n-1)\mathcal{M}_k(a_k^*)}. \tag{3}$$

The next example illustrates how reciprocity sustains efficiency and puts Propositions 1 and 2 to work.

Example 1: Consider a game with two agents with strategy sets $A_1 = A_2 = \{1, 2\}$, the random output has the following support $Q = \{q_1, q_2, q_3, q_4\}$. Agents' costs of effort are defined by $G_1(1) = 5$, $G_1(2) = 10$, $G_2(1) = 4$ and $G_2(2) = 10$ while agents' reservation utilities are $\underline{u}_1 = \underline{u}_2 = 10$. Moreover, the utility functions V_k are given by the identity mappings for each $k = 1, 2$. The probability distributions $\pi(a) = (\pi_{q_1}(a), \pi_{q_2}(a), \pi_{q_3}(a), \pi_{q_4}(a))$ on Q induced by the strategy profiles $a \in A_1 \times A_2$ are given by $\pi(2,2) = (1/8, 1/4, 5/8, 0)$, $\pi(1,2) = (1/8, 3/8, 3/8, 1/8)$, $\pi(2,1) = (1/4, 1/2, 1/8, 1/8)$ and $\pi(1,1) = (1/8, 1/8, 0, 3/4)$. It can be easily calculated that $C_{FB}^1(1) = 15$, $C_{FB}^1(2) = 20$, $C_{FB}^2(1) = 14$ and $C_{FB}^2(2) = 20$. If the principal's benefits from output are given by $\widetilde{B}(q_1) = \widetilde{B}(q_2) = \widetilde{B}(q_4) = 0$ and $\widetilde{B}(q_3) = 80$, then her net benefits are $P(2,2) = (5/8)80 - 20 - 20 = 10$, $P(2,1) = (3/8)80 - 20 - 14 = -4$, $P(1,2) = (1/8)80 - 20 - 15 = -25$ and $P(1,1) = -15 - 14 = -29$. Hence, the first best would be obtained implementing $(2,2)$ at the cost $C_{FB}(2,2) = 40$. However, the first best would be attainable only if $(2,2)$ is a Nash equilibrium of the game defined by incentive schemes that give the first best cost on the support of $(2,2)$, as it is the case for $\widehat{I}_{q_1}^k = \widehat{I}_{q_2}^k = \widehat{I}_{q_3}^k = 20$ and (without loss of generality) $\widehat{I}_{q_4}^k = \underline{I} = 0$, for $k = 1, 2$. It is immediate to see that $(2,2)$ is not a Nash equilibrium for the resulting game

	$a_2 = 2$	$a_2 = 1$
$a_1 = 2$	10 , 10	7.5 , 13.5
$a_1 = 1$	12.5 , 7.5	0 , 1

Suppose now that the agents are reciprocal with reciprocity parameters $\theta_1 = 2.5$ and $\theta_2 = 3.5$. Denote with $a^* = (a_1^*, a_2^*) = (2, 2)$. Let $b_{kt}^* = a_t^*$ and $c_{ktk}^* = a_k^*$ for $t \neq k$ be the first and second order beliefs consistent with (a_1^*, a_2^*). Being $a_1^* = a_2^*$ then $\lambda_{ktk} = 0$ for $k = 1, 2$ and $t \neq k$.

$$U_k(a_k^*, a_{-k}^*, (b_{kt}^*)_{t\neq k}, (c_{ktk}^*)_{t\neq k}; \widehat{\mathcal{I}}^k) = E_k(a^*, \widehat{\mathcal{I}}^k)$$

and

$$U_k(a_k, a_{-k}^*, (b_{kt}^*)_{t\neq k}, (c_{ktk}^*)_{t\neq k}; \widehat{\mathcal{I}}^k) = E_k(a_k, a_{-k}^*, \widehat{\mathcal{I}}^k) - \rho_k \sum_{t\neq k} \omega_k^2 (a_k - b_{kt}^*)^2 \quad \text{if } a_k \neq a_k^*.$$

Hence

$$\begin{aligned} &i) \quad U_1(a_1^*, a_2^*, b_{12}^*, c_{121}^*; \widehat{\mathcal{I}}^1) = E_1(a_1^*, a_2^*; \widehat{\mathcal{I}}^1) = 10; \\ &ii) \quad U_1(a_1 = 1, a_2^*, b_{12}^*, c_{121}^*; \widehat{\mathcal{I}}^1) = E_1(a_1 = 1, a_2^*; \widehat{\mathcal{I}}^1) - \rho_1\omega_1^2 = 12.5 - \theta_1 \end{aligned}$$

while

$$\begin{aligned} &iii) \quad U_2(a_1^*, a_2^*, b_{21}^*, c_{212}^*; \widehat{\mathcal{I}}^2) = E_2(a_1^*, a_2^*; \widehat{\mathcal{I}}^2) = 10; \\ &iv) \quad U_2(a_1^*, a_2 = 1, b_{21}^*, c_{212}^*; \widehat{\mathcal{I}}^2) = E_2(a_1^*, a_2 = 1; \widehat{\mathcal{I}}^2) - \rho_2\omega_2^2 = 13.5 - \theta_2. \end{aligned}$$

Therefore if $\theta_1 \geq 2.5$ and $\theta_2 \geq 3.5$ it follows that

$$\begin{aligned} &1) \quad U_1(a_1^*, a_2^*, b_{12}^*, c_{121}^*; \widehat{\mathcal{I}}^1) \geq U_1(a_1 = 1, a_2^*, b_{12}^*, c_{121}^*; \widehat{\mathcal{I}}^1) \\ &2) \quad U_2(a_1^*, a_2^*, b_{21}^*, c_{212}^*; \widehat{\mathcal{I}}^2) \geq U_2(a_1^*, a_2 = 1, b_{21}^*, c_{212}^*; \widehat{\mathcal{I}}^2). \end{aligned}$$

Hence, a^* is a psychological Nash equilibrium of $\Gamma_{\rho,\omega}(\widehat{\mathcal{I}})$ (therefore a^* is attainable under reciprocity parameters ρ, ω) if and only if $\theta_1 \geq 2.5$ and $\theta_2 \geq 3.5$. It can be checked that in this game $\nu_1(a^*) = 5$ and $\nu_2(a^*) = 6$ hence the sufficient conditions in Proposition 1 give $\theta_1 \geq 5$ and $\theta_2 \geq 6$. However, it can be checked that $\psi_k(a_k^*) = -1/8$, $V_k(C_{FB}^k(a_k^*)) = 20$ and $\mathcal{M}_k(a_k^*) = 1$ for $k = 1, 2$. Moreover, $\mathcal{G}_1(a_1^*) = 5$ and $\mathcal{G}_2(a_2^*) = 6$. Hence, the necessary conditions (3) give back $\theta_1 \geq 2.5$ and $\theta_2 \geq 3.5$.

The next example shows that reciprocity has instead a negative effect on asymmetric profiles and in particular it renders unattainable first best contracts which were attainable in the material game.

Example 2: Consider the game presented in the previous example except that now the probability distributions over Q are given by $\pi(2,2) = (1/2, 1/2, 0, 0)$, $\pi(1,2) = (1/4, 1/4, 1/4, 1/4)$, $\pi(2,1) = (1/2, 1/2, 0, 0)$ and $\pi(1,1) = (1/4, 1/4, 1/4, 1/4)$. Again $C_{FB}^1(1) = 15$, $C_{FB}^1(2) = 20$, $C_{FB}^2(1) = 14$ and $C_{FB}^2(2) = 20$. If the principal's benefits are given by $\widetilde{B}(q_2) = \widetilde{B}(q_3) = \widetilde{B}(q_4) = 0$ and $\widetilde{B}(q_1) = 70$ then her net benefits are $P(2,2) = 35 - 40 = -5$, $P(2,1) = 35 - 34 = 1$, $P(1,2) = 17.5 - 35 = -17.5$ and $P(1,1) = 17.5 - 29 = -11.5$. Hence, the first best is obtained by implementing $(2,1)$ at the cost $C_{FB}(2,1) = 34$. Therefore, the first best is attainable only if $(2,1)$ is a Nash equilibrium of the game corresponding to incentive schemes $\widehat{\mathcal{I}}$ such that $\widehat{I}_{q_1}^1 = \widehat{I}_{q_2}^1 = 20$, and $\widehat{I}_{q_1}^2 = \widehat{I}_{q_2}^2 = 14$ and $\widehat{I}_{q_3}^1 = \widehat{I}_{q_4}^1 = \widehat{I}_{q_3}^2 = \widehat{I}_{q_4}^2 = \underline{I} = 0$. The resulting game is

	$a_2 = 2$	$a_2 = 1$
$a_1 = 2$	10 , 4	10 , 10
$a_1 = 1$	5 , -3	5 , 3

It can be checked that the asymmetric strategy profile $(2, 1)$ is a Nash equilibrium and the IC conditions are satisfied.

Now we show that, with a reciprocal player 1, the equilibrium $(2, 1)$ is destroyed and therefore the $P_{\rho,\omega}$-IC conditions are not satisfied. Denote with $a^* = (a_1^*, a_2^*) = (2, 1)$. Let $b_{kt}^* = a_t^*$ and $c_{ktk}^* = a_k^*$ for $t \neq k$ be the first and second order beliefs consistent with (a_1^*, a_2^*). Recalling that

$$U_1(a_1, a_2^*, b_{12}^*, c_{121}^*; \widehat{\mathcal{I}}^1) = E_1(a_1, a_2^*, \widehat{\mathcal{I}}^1) - \rho_1(\omega_1(a_1 - b_{12}^*) - (b_{12}^* - c_{121}^*))^2$$

then

$$i) \quad U_1(a_1 = 2, a_2^*, b_{12}^*, c_{121}^*; \widehat{\mathcal{I}}^1) = E_1(2, 1, \widehat{\mathcal{I}}^1) - \rho_1(\omega_1(2 - b_{12}^*) - (b_{12}^* - c_{121}^*))^2 = 10 - \rho_1(\omega_1(2-1) - (1-2))^2 = 10 - \rho_1(\omega_1 + 1)^2$$

$$ii) \quad U_1(a_1 = 1, a_2^*, b_{12}^*, c_{121}^*; \widehat{\mathcal{I}}^1) = E_1(1, 1, \widehat{\mathcal{I}}^1) - \rho_1(\omega_1(1 - b_{12}^*) - (b_{12}^* - c_{121}^*))^2 = 5 - \rho_1(\omega_1(1-1) - (1-2))^2 = 5 - \rho_1.$$

Consider, for example, $\varrho_1 \geq 1$ and $\omega_1 \geq 2$ then

$$U_1(a_1 = 1, a_2^*, b_{12}^*, c_{121}^*; \widehat{\mathcal{I}}^1) > U_1(a_1 = 2, a_2^*, b_{12}^*, c_{121}^*; \widehat{\mathcal{I}}^1)$$

which implies that $(2, 1)$ is not a psychological Nash equilibrium of $\Gamma_{\varrho,\omega}(\widehat{\mathcal{I}})$. This shows that a material asymmetric and efficient equilibrium might be destroyed by reciprocal agents.

4 Principal's utility in the Second Best

In this section we show that reciprocal agents always increase the net benefit of the principal when the material second best is symmetric (Proposition 3 and Example 3). However, this result does not extend to the case where the material second best is asymmetric (Example 4). The next proposition states our first result.

Proposition 3. *Let $(a^*, \mathcal{I})$ be a (material) second-best contract such that a^* is a symmetric strategy profile and at least one of the IR constraints is not binding. Then, for every pair of reciprocity vectors (ρ, ω) there exist incentive schemes $\overline{\mathcal{I}}$ such that $(a^*, \overline{\mathcal{I}})$ satisfies ($P_{\rho,\omega}$-IR), ($P_{\rho,\omega}$-IC) and*

$$B(a^*) - \mathcal{C}(a^*, \mathcal{I}) < B(a^*) - \mathcal{C}(a^*, \overline{\mathcal{I}}).$$

In words, reciprocity reduces the costs of implementing the second best, whenever the second best profile is symmetric. The next example illustrates Proposition 3.

Example 3: Consider a game with two agents with strategy sets $A_1 = A_2 = \{1, 2\}$, the random output has the following support $Q = \{q_1, q_2, q_3\}$. Agents' costs of effort are defined by $G_1(2) = G_2(2) = 1$, $G_1(1) = G_2(1) = 0$, while agents' reservation utilities are $\underline{u}_1 = \underline{u}_2 = 0$. The probability distributions over Q are given by $\pi(2, 2) = (1/2, 1/2, 0)$,

$\pi(1,2) = (1/3, 1/3, 1/3)$ and $\pi(2,1) = \pi(1,1) = (0,0,1)$. It can be easily calculated that $C^1_{FB}(1) = C^2_{FB}(1) = 0$, $C^1_{FB}(2) = C^2_{FB}(2) = 1$. Given the incentive schemes $\mathcal{I}$ where $I^k_{q_i}$ denotes the payment to player k if outcome q_i has occurred, then the material game is

	$a_2 = 2$	$a_2 = 1$
$a_1 = 2$	$(I^1_{q_1}/2) + (I^1_{q_2}/2) - 1, (I^2_{q_1}/2) + (I^2_{q_2}/2) - 1$	$I^1_{q_3} - 1, I^2_{q_3}$
$a_1 = 1$	$(I^1_{q_1}/3) + (I^1_{q_2}/3) + (I^1_{q_3}/3), (I^2_{q_1}/3) + (I^2_{q_2}/3) + (I^2_{q_3}/3) - 1$	$I^1_{q_3}, I^2_{q_3}$

Note that $(2,2)$ is a Nash equilibrium (and therefore satisfies the IC conditions) if and only if

$$\begin{aligned} &i) \quad (I^1_{q_1}/2) + (I^1_{q_2}/2) - 1 \geq \ (I^1_{q_1}/3) + (I^1_{q_2}/3) + (I^1_{q_3}/3) \\ &ii) \quad (I^2_{q_1}/2) + (I^2_{q_2}/2) - 1 \geq \ I^2_{q_3}. \end{aligned}$$

It can be easily checked that $C_{SB}(2,2) = 8$ and

$$M(2,2) = \{(I^1_{q_i})^3_{i=1}, (I^2_{q_i})^3_{i=1} \,|\, I^1_{q_1} + I^1_{q_2} = 6,\ I^2_{q_1} + I^2_{q_2} = 2,\ I^1_{q_3} = I^2_{q_3} = 0\}.$$

On the other hand $(1,2)$ is a Nash equilibrium if and only if

$$\begin{aligned} &i) \quad (I^1_{q_1}/3) + (I^1_{q_2}/3) + (I^1_{q_3}/3) \geq \ (I^1_{q_1}/2) + (I^1_{q_2}/2) - 1 \\ &ii) \quad (I^2_{q_1}/3) + (I^2_{q_2}/3) + (I^2_{q_3}/3) - 1 \geq \ I^2_{q_3}. \end{aligned}$$

Therefore $C_{SB}(1,2) = 3$ and

$$M(1,2) = \{(I^1_{q_i})^3_{i=1}, (I^2_{q_i})^3_{i=1} \,|\, I^2_{q_1} + I^2_{q_2} = 3,\ I^1_{q_1} = I^1_{q_2} = I^1_{q_3} = I^2_{q_3} = 0\}.$$

Analogous calculations show that $(2,1)$ is never a Nash equilibrium, while $C_{SB}(1,1) = 0$. Now if the principal's benefits from output are $\widetilde{B}(q_1) = \widetilde{B}(q_3) = 0$ and $\widetilde{B}(q_2) = 36$. It can be checked that the first best is to implement $(2,2)$ at the cost $C^1_{FB}(2) + C^2_{FB}(2) = 2$. While the second best is to implement $(2,2)$ at the second best cost $C_{SB}(2,2) = 8$. In fact $B(2,2) - C_{SB}(2,2) = 18 - 8 = 10$, $B(1,2) - C_{SB}(1,2) = 12 - 3 = 9$ and $B(1,1) - C_{SB}(1,1) = 0$. Suppose now that agents are reciprocal with reciprocity vectors (ρ, ω). Following the same steps as in the previous examples for the calculation of psychological payoffs, we get the following psychological equilibrium conditions for the strategy profile $(2,2)$

$$\begin{aligned} &i) \quad (I^1_{q_1}/2) + (I^1_{q_2}/2) - 1 \geq \ (I^1_{q_1}/3) + (I^1_{q_2}/3) + (I^1_{q_3}/3) - \theta_1 \\ &ii) \quad (I^2_{q_1}/2) + (I^2_{q_2}/2) - 1 \geq \ I^2_{q_3} - \theta_2. \end{aligned}$$

Such conditions tell that the strategy profile $(2,2)$ can be implemented as a psychological Nash equilibrium under the following incentive schemes

$$\{(I^1_{q_i})^3_{i=1}, (I^2_{q_i})^3_{i=1} \,|\, I^1_{q_1} + I^1_{q_2} = \max\{6(1-\theta_1), 0\},\ I^2_{q_1} + I^2_{q_2} = \max\{2(1-\theta_2), 0\},\ I^1_{q_3} = I^2_{q_3} = 0\}.$$

and therefore the second best costs under reciprocity vectors (ρ, ω) are

$$C^P_{SB}(2,2) = \max\{6(1-\theta_1), 0\} + \max\{2(1-\theta_2), 0\} < C_{SB}(2,2),$$

implying that reciprocity reduces the costs of implementation of the second best, whenever the second best profile is symmetric.

We now show by way of an example that reciprocity does not always benefit the principal if the material second best is asymmetric.

Example 4: Consider a game similar to the one in Example 3. Two agents have strategy sets $A_1 = A_2 = \{1, 2\}$. Again, the random output has support $Q = \{q_1, q_2, q_3\}$, agents' costs of effort are $G_1(2) = G_2(2) = 1$, $G_1(1) = G_2(1) = 0$, while reservation utilities are $\underline{u}_1 = \underline{u}_2 = 0$. The probability distributions over Q are now given by $\pi(2,2) = (1/2, 1/2, 0)$, $\pi(1,2) = (0, 1/2, 1/2)$ and $\pi(2,1) = \pi(1,1) = (0,0,1)$. It can be easily calculated that $C^1_{FB}(1) = C^2_{FB}(1) = 0$, $C^1_{FB}(2) = C^2_{FB}(2) = 1$. Given the incentive schemes $\mathcal{I}$ then the material game is

	$a_2 = 2$	$a_2 = 1$
$a_1 = 2$	$(I^1_{q_1}/2) + (I^1_{q_2}/2) - 1, (I^2_{q_1}/2) + (I^2_{q_2}/2) - 1$	$I^1_{q_3} - 1, I^2_{q_3}$
$a_1 = 1$	$(I^1_{q_2}/2) + (I^1_{q_3}/2), (I^2_{q_2}/2) + (I^2_{q_3}/2) - 1$	$I^1_{q_3}, I^2_{q_3}$

Note that $(2, 2)$ is a Nash equilibrium (and therefore satisfies the IC conditions) if and only if

$$\begin{aligned} i)\quad & (I^1_{q_1}/2) + (I^1_{q_2}/2) - 1 \geq (I^1_{q_2}/2) + (I^1_{q_3}/2) \\ ii)\quad & (I^2_{q_1}/2) + (I^2_{q_2}/2) - 1 \geq I^2_{q_3}. \end{aligned}$$

It can be easily checked that $C_{SB}(2, 2) = 4$ and

$$M(2,2) = \{(I^1_{q_i})^3_{i=1}, (I^2_{q_i})^3_{i=1} \mid I^1_{q_1} = 2,\ I^2_{q_1} + I^2_{q_2} = 2,\ I^1_{q_2} = I^1_{q_3} = I^2_{q_3} = 0\}.$$

On the other hand $(1, 2)$ is a Nash equilibrium if and only if

$$\begin{aligned} i)\quad & (I^1_{q_2}/2) + (I^1_{q_3}/2) \geq (I^1_{q_1}/2) + (I^1_{q_2}/2) - 1 \\ ii)\quad & (I^2_{q_2}/2) + (I^2_{q_3}/2) - 1 \geq I^2_{q_3}. \end{aligned}$$

Therefore $C_{SB}(1, 2) = 2$ and

$$M(1,2) = \{(I^1_{q_i})^3_{i=1}, (I^2_{q_i})^3_{i=1} \mid I^2_{q_2} = 2,\ I^2_{q_1} = I^1_{q_1} = I^1_{q_2} = I^1_{q_3} = I^2_{q_3} = 0\}.$$

Analogous calculations show that $(2, 1)$ is never a Nash equilibrium, while $C_{SB}(1, 1) = 0$. If the principal's benefits from output are now $\widetilde{B}(q_1) = 0$, $\widetilde{B}(q_2) = 10$ and $\widetilde{B}(q_3) = 2$. It can be checked that the first best is to implement $(1, 2)$ at the cost $C_{FB}(1, 2) = 1$. While the second best is to implement $(1, 2)$ at the second best cost $C_{SB}(1, 2) = 2$. In fact $B(2, 2) - C_{SB}(2, 2) = 5 - 4 = 1$, $B(1, 2) - C_{SB}(1, 2) = 6 - 2 = 4$ and $B(1, 1) - C_{SB}(1, 1) = 2$.

Suppose now that agents are reciprocal with reciprocity vectors (ρ, ω). Following the same steps as in the previous examples for the calculation of psychological payoffs, we get the following psychological equilibrium conditions for the strategy profile $(2, 2)$

$$\begin{aligned} i)\quad & (I^1_{q_1}/2) + (I^1_{q_2}/2) - 1 \geq (I^1_{q_2}/2) + (I^1_{q_3}/2) - \theta_1 \\ ii)\quad & (I^2_{q_1}/2) + (I^2_{q_2}/2) - 1 \geq I^2_{q_3} - \theta_2. \end{aligned}$$

For $(1,2)$, we have the following equilibrium conditions

$$\begin{array}{lll} i) & (I^1_{q_2}/2)+(I^1_{q_3}/2)-\rho_1(\omega_1+1)^2 \geq & (I^1_{q_1}/2)+(I^1_{q_2}/2)-1-\rho_1 \\ ii) & (I^2_{q_2}/2)+(I^2_{q_3}/2)-1-\rho_2(\omega_2+1)^2 \geq & I^2_{q_3}-\rho_2 \end{array}$$

Fix for example $\rho_1=\rho_2=1/2$ and $\omega_1=\omega_2=1$. Then the equilibrium conditions tell that the strategy profile $(2,2)$ can be implemented as a psychological Nash equilibrium under the following incentive schemes

$$\{(I^1_{q_i})^3_{i=1},(I^2_{q_i})^3_{i=1}\,|\,I^1_{q_1}=1,\ I^2_{q_1}+I^2_{q_2}=1,\ I^1_{q_2}=I^1_{q_3}=I^2_{q_3}=0\}.$$

Therefore the second best costs is $C^P_{SB}(2,2)=2$. The strategy profile $(1,2)$ can be implemented as a psychological Nash equilibrium under the following incentive schemes

$$\{(I^1_{q_i})^3_{i=1},(I^2_{q_i})^3_{i=1}\,|\,I^1_{q_1}=5/2,\ I^2_{q_2}=5,\ I^2_{q_1}=I^1_{q_1}=I^1_{q_2}=I^1_{q_3}=I^2_{q_3}=0\}.$$

and the second best cost is now $C^P_{SB}(1,2)=15/2$.

Therefore the second best under reciprocity vectors (ρ,ω) is obtained, in this case, by the strategy profile $(2,2)$ which yields a net benefit to the principal equal to $B(2,2)-C^P_{SB}(2,2)=5-2=3>B(1,2)-C^P_{SB}(1,2)=6-15/2=-3/2$. Since $B(1,2)-C_{SB}(1,2)=4>B(2,2)-C^P_{SB}(2,2)$, we deduce that reciprocal agents may reduce the net benefit of the principal when the material second best is asymmetric.

5 Collusion-proof Implementation

In the presence of many players, collusion-proof implementation of the efficient strategy profile is often an issue. In particular, in the (material) principal-multiple agents framework, Mookherjee (1982) investigates the effects of a principal who seeks to implement action pairs as a strong Nash equilibrium (Aumann (1959)) in the two-agent case. An equilibrium is said to be a strong Nash equilibrium if no subset of players, taking the actions of the others as fixed, can jointly deviate in a way that benefits all of them. This concept has been introduced for environments in which players can agree privately upon a joint deviation. In that case, any meaningful agreement by the whole set of players must be stable against deviations by all possible coalitions of players. More precisely,

Definition 6 (Aumann 1959)**.** *A strategy profile* $(\overline{a}_1,\ldots,\overline{a}_n)$ *is a strong Nash equilibrium of the material game* Γ *if for all subset of players* $T\subseteq N$ *and for all* $a_T\in A_T=\prod_{t\in T}A_t$ *there exists a player* $k\in T$ *such that* $u_k(\overline{a}_T,\overline{a}_{-T})\geq u_k(a_T,\overline{a}_{-T})$*, with* $\overline{a}_{-T}=(\overline{a}_t)_{t\notin T}$*.*

Building upon the work of Aumann, in our companion paper we introduced the definition of psychological strong Nash equilibrium which extends the Aumann's concept to psychological games. The definition is based on the idea that – since players commit ex-ante to a deviation – the deviants' beliefs should be consistent with the deviation itself.[7] More precisely

[7]Note that when we consider only deviations by singletons, this definition boils down to the definition of psychological Nash equilibrium. Moreover, it generalizes the Aumann's definition of strong Nash equilibrium which can be easily obtained removing the psychological term from the payoffs. In the next two sections we will introduce our results on implementation and unique implementation, respectively.

Definition 7. *A strategy profile $(\overline{a}_1, \ldots, \overline{a}_n)$ is a psychological strong Nash equilibrium of $\Gamma_{\rho,\omega}(\mathcal{I})$ if it is stable with respect to joint deviations of each coalition $T \subseteq N$: for every $a_T \in A_T$ there exists a player $k \in T$ such that*
i) $U_k(\overline{a}_k, \overline{a}_{-k}, (\overline{b}_{kt})_{t\neq k}, (\overline{c}_{ktk})_{t\neq k}; \mathcal{I}^k) \geq U_k(a_T, \overline{a}_{-T}, (b_{kt})_{t\neq k}), (c_{ktk})_{t\neq k}; \mathcal{I}^k)$, with $\overline{a}_{-T} = (\overline{a}_t)_{t\notin T}$
ii) $\overline{b}_{kt} = \overline{a}_t$ and $\overline{c}_{ktk} = \overline{a}_k$ for all $t \neq k$,
iii) $b_{kt} = \overline{a}_t$ and $c_{ktk} = \overline{a}_k$ for all $t \in N \setminus T$,
iv) $b_{kt} = a_t$ and $c_{ktk} = a_k$ for all $t \in T \setminus \{k\}$.

In De Marco and Immordino (2010), we found out that this concept allows for the unique and collusion proof implementation of the efficient (symmetric) strategy profile in a classical partnership model. Now we address the question whether the psychological strong Nash equilibrium allows for an analogous result for symmetric first best profiles in the principal-multiple agents model. Indeed, the condition used to prove Proposition 1 is not sufficient to sustain the first best if agents can collude. In the next proposition we show that there exists a set of parameters θ_k such that the symmetric first best is attainable even if proper coalitions can collude. Whether the symmetric first best is stable also with respect to joint deviations of the grand coalition (and therefore a psychological strong Nash equilibrium), it depends only on the material game, meaning that only the (first best) incentives schemes prevent from the joint deviations of the grand coalition. Indeed, as it can be easily deduced from the definition of psychological strong Nash equilibrium, in the agents' psychological utilities – corresponding to a deviation of the grand coalition from a symmetric strategy profile towards another symmetric strategy profile – the psychological terms disappear and therefore psychological utilities coincide with the material ones.

Proposition 4. *Let a^* be the symmetric first best profile. If $\theta_k \geq \nu_k(a^*)$ for every player k then a^* is attainable and stable with respect to joint deviations of each proper coalition $T \subset N$ in the psychological game $\Gamma_{\rho,\omega}(\widehat{\mathcal{I}})$.*

Finally, whenever the symmetric first best profile is sustained as a psychological strong Nash equilibrium then additional assumptions on the psychological parameters of the agents are needed to guarantee that it is unique[8].

6 A remark on symmetric first best profiles

The previous results underline the important role played by symmetry for reciprocity to be in the principal's interest. It is then natural to ask when a symmetric first-best strategy profile is likely to arise.

Given a strategy profile $a = (a_1, \ldots, a_n)$, denote with $\left\lfloor \frac{\sum_{i=1}^n a_i}{n} \right\rfloor$ the integer part of $\frac{\sum_{i=1}^n a_i}{n}$ and let $\mu(a) = (m(a), \ldots, m(a))$ be the symmetric profile in A defined by

$$m(a) = \begin{cases} \frac{\sum_{i=1}^n a_i}{n} & \text{if } \frac{\sum_{i=1}^n a_i}{n} = \left\lfloor \frac{\sum_{i=1}^n a_i}{n} \right\rfloor \\ \left\lfloor \frac{\sum_{i=1}^n a_i}{n} \right\rfloor + 1 & \text{otherwise.} \end{cases}$$

[8]Even if the first best profile is not the unique psychological strong Nash equilibrium of the game, it is possible to show that if a^* is the symmetric first best profile and if $\rho_k \left[(\omega_k + 1)^2 - (n-1)(m-1)^2\right] > \nu(a^*) \quad \forall k \in N$ then every asymmetric strategy profile is not a strong psychological equilibrium of $\Gamma_{\rho,\omega}(\widehat{\mathcal{I}})$. The proof is in the appendix.

Then, the following proposition immediately follows

Proposition 5. *Assume that,*

i) the function B is increasing in the sum of agents' efforts, i.e.,

$$\sum_{i=1}^{n} a_i > \sum_{i=1}^{n} a'_i \implies B(a) > B(a')$$

ii) $C_{FB}(\mu(a)) \le C_{FB}(a)$ for every $a \in A$.

Then there exists at least a symmetric first best profile.

The proof is obvious since $B(\mu(a^*)) - C_{FB}(\mu(a^*)) \ge B(a^*) - C_{FB}(a^*)$ which implies that if a^* is a first best profile then also $\mu(a^*)$ is a first best profile[9].

In the next lemma we provide sufficient conditions to obtain condition (ii) in Proposition 5 in the case agents are symmetric. For the sake of simplicity denote in this case $A_1 = \cdots = A_n = \{1, \ldots, m\} = S$. Moreover, for every $x \in [1, m]$ denote with $N(x) = \{s \in S \,|\, \text{such that}\, \|x - s\| \le 1\}$ the *discrete neighborood* of x. Then

Lemma 1. *Assume that $C^1_{FB}(s) = C^2_{FB}(s) = \cdots = C^n_{FB}(s) = c(s)$ for every s in S where $c(s)$ is a discretely strict convex function in S, that is, given $s', s'' \in S$ and $\alpha \in [0, 1]$ it follows that*

$$\max_{s \in N(\alpha s' + (1-\alpha)s'')} c(s) \le \alpha c(s') + (1 - \alpha)c(s''). \tag{4}$$

Then $C_{FB}(\mu(a)) \le C_{FB}(a)$ for every $a \in A$.[10]

This section has shown that symmetric first-best profiles arise naturally when the principal's benefit increases in the total amount of effort provided by the agents and the first-best cost features a convexity-like property. Note that the discretely strict convexity of C^k_{FB} follows from the analogous property for agent k disutility function G_k. Finally, notice that those assumptions are only sufficient and it is simple to find examples where the first-best profile is symmetric despite the previous assumptions are not satisfied.

7 Conclusion

In this paper, we examine the impact of intention-based reciprocity preferences on the multiple agent model. Our main result is that a principal will always prefer to hire reciprocal agents to implement a contract when the strategy profile is symmetric, while if profiles are

[9]If the strategy sets were closed convex subsets of finite dimensional spaces then condition (ii) in Proposition 5 could be replaced by C_{FB} convex and by the following property: $\sum_{i=1}^n a_i = \sum_{i=1}^n a'_i \implies C_{FB}(a) = C_{FB}(a')$.

[10]Note that the discretely strict convexity assumption used in the lemma is stronger than the *discretely convexity* assumption introduced in Miller (1991) where the *max* in (4) is replaced with *min*. Moreover, it can be easily checked that – in the continuous (strategy set) case – if we replace condition (ii) in Proposition 5 with the assumpion that agents are symmetric with a convex cost function c then we obtain again the existence of a symmetric first best profile.

asymmetric she might prefer self-interested to intention-based reciprocity agents. Moreover, we describe the agents' psychological characteristics required to sustain a given strategy profile. Finally, since our main results underline the important role played by symmetry for reciprocity to be in the principal's best interest, we show when symmetric strategy profiles are most likely to arise.

The current paper together with the companion one (De Marco and Immordino, 2010) on partnerships study natural environments for reciprocity to play a central role and are intended to be a step toward the economic analysis of teamwork and optimal team design in the presence of reciprocal agents. An important by-product of our analysis is to demonstrate that, despite their apparent complexity, intention-based reciprocity models can be useful to study economically relevant settings in a novel and simple way.

8 Appendix

Proof of Proposition 1. From the assumptions it follows that, for every player k,

$$\rho_k \sum_{t \neq k} \omega_k^2 (a_k - b_{kt}^*)^2 \geq (n-1)\rho_k \omega_k^2 \geq \nu_k(a^*) \geq G_k(a_k^*) - G_k(a_k) \quad \forall a_k \in A_k. \tag{5}$$

Moreover, let $\widehat{\mathcal{I}}^k$ the incentive scheme of player k defined by

$$\widehat{I}_q^k = \begin{cases} C_{FB}^k(a_k^*) & \text{if} \quad q \in J(a^*) \\ \underline{I} & \text{if} \quad q \in Q \setminus J(a^*) \end{cases} \tag{6}$$

In this case, we get

$$E_k(a^*, \widehat{\mathcal{I}}^k) = \sum_{q \in Q} \pi_q(a^*) V_k(C_{FB}^k(a_k^*)) - G_k(a_k^*) = V_k(C_{FB}^k(a_k^*)) - G_k(a_k^*)$$

while

$$E_k(a_k, a_{-k}^*, \widehat{\mathcal{I}}^k) = \sum_{q \in Q} \pi_q(a_k, a_{-k}^*) V_k(\widehat{I}_q^k) - G_k(a_k) =$$

$$V_k(C_{FB}^k(a_k^*)) \left[\sum_{q \in J(a^*)} \pi_q(a_k, a_{-k}^*) \right] + V_k(\underline{I}) \left[\sum_{q \notin J(a^*)} \pi_q(a_k, a_{-k}^*) \right] - G_k(a_k).$$

Note that $V_k(\underline{I}) \leq V_k(C_{FB}^k(a_k^*))$ which implies that

$$V_k(C_{FB}^k(a_k^*)) = \sum_{q \in Q} \pi_q(a^*) V_k(C_{FB}^k(a_k^*)) \geq$$

$$V_k(C_{FB}^k(a_k^*)) \left[\sum_{q \in J(a^*)} \pi_q(a_k, a_{-k}^*) \right] + V_k(\underline{I}) \left[\sum_{q \notin J(a^*)} \pi_q(a_k, a_{-k}^*) \right].$$

Hence from the previous inequality and from (5) it follows that

$$V_k(C_{FB}^k(a_k^*)) + \sum_{t \neq k} \omega_k^2 (a_k - b_{kt}^*)^2 \geq \sum_{q \in Q} \pi_q(a_k, a_{-k}^*) V_k(\widehat{I}_q^k) + G_k(a_k^*) - G_k(a_k) \quad \forall a_k \in A_k.$$

which implies that

$$E_k(a^*, \widehat{\mathcal{I}}^k) \geq E_k(a_k, a^*_{-k}, \widehat{\mathcal{I}}^k) - \rho_k \sum_{t\neq k} \omega_k^2 (a_k - b^*_{kt})^2 \quad \forall a_k \in A_k \setminus \{a^*_k\}. \tag{7}$$

If $b^*_{kt} = a^*_t$ and $c^*_{ktk} = a^*_k$ for all $t \neq k$, then

$$U_k(a^*_k, a^*_{-k}, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) = E_k(a^*, \widehat{\mathcal{I}}^k)$$

and

$$U_k(a_k, a^*_{-k}, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) = E_k(a_k, a^*_{-k}, \widehat{\mathcal{I}}^k) - \rho_k \sum_{t\neq k} \omega_k^2 (a_k - b^*_{kt})^2 \quad \forall a_k \in A_k \setminus \{a^*_k\}.$$

Since the previous arguments apply for every player k, then (7) implies that a^* is a psychological Nash equilibrium of $\Gamma_{\rho,\omega}(\widehat{\mathcal{I}})$, where each $\widehat{\mathcal{I}}^k$ is defined as in (6). Therefore $(a^*, \widehat{\mathcal{I}})$ satisfies the condition $\mathrm{P}_{\rho,\omega}$-IC.

Finally, since for every player k it follows that

$$U_k(a^*_k, a^*_{-k}, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) = E_k(a^*, \widehat{\mathcal{I}}^k) = V_k(C^k_{FB}(a^*_k)) - G_k(a^*_k) = \underline{u}_k$$

then $(a^*, \widehat{\mathcal{I}})$ satisfies the condition $\mathrm{P}_{\rho,\omega}$-IR. And the assertion follows. ■

Proof of Proposition 2. The first best a^* is attainable, then it satisfies the $P_{\rho,\omega}$-IR conditions. Notice that since a^* is a symmetric strategy profile and $P_{\rho,\omega}$-IR involves correct beliefs then the $P_{\rho,\omega}$-IR condition is equivalent to the IR condition for material payoffs. Hence, following the same steps in Proposition 5 in Mookherjee (1984) we get that, given the strict concavity of V the principal can attain the first best only if each agent is paid a constant sum with probability one under a^*. Given that the contract is optimal, agent k will be paid $C^k_{FB}(a^*_k)$ whenever the outcome belongs to $J(a^*)$.

Now, without loss of generality, we assume that the principal always gives the minimum payment $\underline{I}$ for $q \notin J(a^*)$. Let $b^*_{kt} = a^*_t$ and $c^*_{ktk} = a^*_k$ for all $t \neq k$. From the assumptions it follows that a^* satisfies the $P_{\rho,\omega}$-IC conditions under incentive schemes $\widehat{\mathcal{I}}$, then it is a psychological Nash equilibrium of $\Gamma_{\rho,\omega}(\widehat{\mathcal{I}})$. Hence, for every player k, it follows that

$$U_k(a^*_k, a^*_{-k}, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) \geq U_k(a_k, a^*_{-k}, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) \quad \forall a_k \in A_k \setminus \{a^*_k\}, \tag{8}$$

where

$$U_k(a^*_k, a^*_{-k}, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) = E_k(a^*, \widehat{\mathcal{I}}^k)$$

and

$$U_k(a_k, a^*_{-k}, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) = E_k(a_k, a^*_{-k}, \widehat{\mathcal{I}}^k) - \rho_k \sum_{t\neq k} \omega_k^2 (a_k - b^*_{kt})^2 \quad \forall a_k \in A_k \setminus \{a^*_k\}.$$

Condition (8) implies that

$$\rho_k \sum_{t\neq k} \omega_k^2 (a_k - b^*_{kt})^2 \geq E_k(a_k, a^*_{-k}, \widehat{\mathcal{I}}^k) - E_k(a^*, \widehat{\mathcal{I}}^k) =$$

$$V_k(C^k_{FB}(a^*_k))\left[\sum_{q\in J(a^*)}\left[\pi_q(a_k,a^*_{-k})-\pi_q(a^*)\right]\right]+V_k(\underline{I})\left[\sum_{q\notin J(a^*)}\pi_q(a_k,a^*_{-k})\right]+G_k(a^*_k)-G_k(a_k).$$

Being $V_k(\underline{I})=0$ and taking the sum over $a_k\in A_k$ we get

$$\theta_k(n-1)\mathcal{M}_k(a^*_k)=\rho_k\omega_k^2\sum_{a_k\in A_k}\sum_{t\neq k}(a_k-b^*_{kt})^2\geq$$

$$\sum_{a_k\in A_k}\left(V_k(C^k_{FB}(a^*_k))\left[\sum_{q\in J(a^*)}\left[\pi_q(a_k,a^*_{-k})-\pi_q(a^*)\right]\right]\right)+\sum_{a_k\in A_k}(G_k(a^*_k)-G_k(a_k))=$$

$$\psi_k(a^*_k)V_k(C^k_{FB}(a^*_k))+\mathcal{G}_k(a^*_k)$$

Hence condition (3) holds and the assertion follows. ■

Proof of Proposition 3. Assume that $(a^*,\mathcal{I})$ is a (material) second best contract with a^* symmetric strategy profile. Fix reciprocity vectors (ρ,ω). Given the incentive schemes $\mathcal{I}$ and $\epsilon_k\geq 0$ for all $k\in N$, denote with $\mathcal{I}_\epsilon=(\mathcal{I}^1_{\epsilon_1},\dots,\mathcal{I}^n_{\epsilon_n})$ the incentive schemes defined by

$$I^k_{\epsilon_k,q}=\max\{\underline{I},I^k_q-\epsilon_k\}\quad\forall k\in N,\ \forall q\in Q.$$

Denote with $g_k(\epsilon_k)=E_k(a^*,\mathcal{I}^k_{\epsilon_k})$. It easily follows that each function g_k is continuous and $g_k(0)\geq\underline{u}_k$ for every k. Denote with $N_{IR}=\{k\in N\mid g_k(0)>\underline{u}_k\}$ which is not empty by assumption. Then for every $k\in N_{IR}$ there exists κ_k such that $g_k(\epsilon_k)>\underline{u}_k$ for all $\epsilon_k<\kappa_k$. Hence the $P_{\rho,\omega}$-*IR* conditions are satisfied for every $k\in N_{IR}$ and $\epsilon_k<\kappa_k$.

For every $k\in N_{IR}$ and every $a_k\in A_k\setminus\{a^*_k\}$, let

$$h_k(\epsilon_k,a_k)=\sum_{q\in Q}\left[\pi_q(a_k,a^*_{-k})-\pi_q(a^*)\right]\left[V_k(I^k_{\epsilon_k,q})-V_k(I^k_q)\right].$$

For every $a_k\in A_k\setminus\{a^*_k\}$, $h_k(0,a_k)=0$ and $h_k(\cdot,a_k)$ is continuous. Hence there exists $\overline{\kappa}_k(a_k)$ such that

$$\sum_{t\neq k}\theta_k(a_k-a^*_k)^2\geq\sum_{q\in Q}\left[\pi_q(a_k,a^*_{-k})-\pi_q(a^*)\right]\left[V_k(I^k_{\epsilon_k,q})-V_k(I^k_q)\right]\quad\forall\epsilon_k\leq\overline{\kappa}_k(a_k).$$

Therefore, if $\overline{\kappa}_k=\min_{a_k\in A_k\setminus\{a^*_k\}}\overline{\kappa}_k(a_k)$ and $\epsilon_k\leq\overline{\kappa}_k$, it follows that

$$\sum_{t\neq k}\theta_k(a_k-a^*_k)^2\geq\sum_{q\in Q}\left[\pi_q(a_k,a^*_{-k})-\pi_q(a^*)\right]\left[V_k(I^k_{\epsilon_k,q})-V_k(I^k_q)\right]\quad\forall a_k\in A_k\setminus\{a^*_k\}.$$

Since $(a^*,\mathcal{I})$ is a (material) second best contract then from the IC condition it follows that

$$-\sum_{q\in Q}V_k(I^k_q)\left[\pi_q(a_k,a^*_{-k})-\pi_q(a^*)\right]\geq G_k(a^*_k)-G_k(a_k)\quad\forall a_k\in A_k\setminus\{a^*_k\}.$$

Hence for $\epsilon_k\leq\overline{\kappa}_k$, it follows that

$$-\sum_{q\in Q}\left[\pi_q(a_k,a^*_{-k})-\pi_q(a^*)\right]V_k(I^k_{\epsilon_k,q})+\sum_{t\neq k}\theta_k(a_k-a^*_k)^2\geq G_k(a^*_k)-G_k(a_k)\quad\forall a_k\in A_k\setminus\{a^*_k\}$$

which implies that

$$\sum_{q\in Q}\pi_q(a^*)V_k(I^k_{\epsilon_k,q})-G_k(a^*_k) \geq \sum_{q\in Q}\pi_q(a_k,a^*_{-k})V_k(I^k_{\epsilon_k,q})-G_k(a_k)-\sum_{t\neq k}\theta_k(a_k-a^*_k)^2 \quad \forall a_k\in A_k\setminus\{a^*_k\}. \tag{9}$$

If $b^*_{kt}=a^*_t$ and $c^*_{ktk}=a^*_k$ for all $t\neq k$, then

$$U_k(a^*_k,a^*_{-k},(b^*_{kt})_{t\neq k},(c^*_{ktk})_{t\neq k};\mathcal{I}^k_\epsilon)=\sum_{q\in Q}\pi_q(a^*)V_k(I^k_{\epsilon_k,q})-G_k(a^*_k)$$

and

$$U_k(a_k,a^*_{-k},(b^*_{kt})_{t\neq k},(c^*_{ktk})_{t\neq k};\mathcal{I}^k_\epsilon)=$$
$$\sum_{q\in Q}\pi_q(a_k,a^*_{-k})V_k(I^k_{\epsilon_k,q})-G_k(a_k)-\sum_{t\neq k}\theta_k(a_k-b_{k,h})^2 \quad \forall a_k\in A_k\setminus\{a^*_k\}.$$

Therefore, for every player k in N_{IR} and every $\epsilon_k\leq\overline{\kappa}_k$,

$$U_k(a^*_k,a^*_{-k},(b^*_{kt})_{t\neq k},(c^*_{ktk})_{t\neq k};\mathcal{I}^k_\epsilon)\geq U_k(a_k,a^*_{-k},(b^*_{kt})_{t\neq k},(c^*_{ktk})_{t\neq k};\mathcal{I}^k_\epsilon) \quad \forall a_k\in A_k\setminus\{a^*_k\}, \tag{10}$$

which finally implies that the $P_{\rho,\omega}$-IC conditions are satisfied for every player k in N_{IR} and every $\epsilon_k\leq\overline{\kappa}_k$.

Consider a vector $\overline{\epsilon}=(\overline{\epsilon}_1,\overline{\epsilon}_2,\ldots,\overline{\epsilon}_n)$

$$\begin{cases}\overline{\epsilon}_k=0 & \text{if} k\notin N_{IR}\\ 0<\overline{\epsilon}_k<\min\{\kappa_k,\overline{\kappa}_k\} & \text{if} k\in N_{IR}.\end{cases}$$

Denote with $\overline{\mathcal{I}}=\mathcal{I}_{\overline{\epsilon}}$. Previous arguments imply that for every $k\in N_{IR}$, the contract $(a^*,\overline{\mathcal{I}})$ satisfies $P_{\rho,\omega}$-IR and $P_{\rho,\omega}$-IC.
Now we consider players $k\notin N_{IR}$. Note that if $\epsilon_k=0$ one obviously gets that $\mathcal{I}^k_{\epsilon_k}=\mathcal{I}^k$. Being $(a^*,\mathcal{I})$ a (material) second best contract, then the contract $(a^*,\overline{\mathcal{I}})$ clearly satisfies the $P_{\rho,\omega}$-IR conditions also for every $k\notin N_{IR}$ since a^* is symmetric and the psychological term in the $P_{\rho,\omega}$-IR conditions disappears when beliefs are correct. Finally, if $k\notin N_{IR}$, it follows that

$$U_k(a^*_k,a^*_{-k},(b^*_{kt})_{t\neq k},(c^*_{ktk})_{t\neq k};\mathcal{I}^k_{\overline{\epsilon}})=E_k(a^*_k,a^*_{-k},\widehat{\mathcal{I}}^k_{\overline{\epsilon}})\geq E_k(a_k,a^*_{-k},\widehat{\mathcal{I}}^k_{\overline{\epsilon}})\geq$$
$$E_k(a_k,a^*_{-k},\widehat{\mathcal{I}}^k_{\overline{\epsilon}})-\rho_k\sum_{t\neq k}\omega^2_k(a_k-b^*_{kt})^2=U_k(a_k,a^*_{-k},(b^*_{kt})_{t\neq k},(c^*_{ktk})_{t\neq k};\mathcal{I}^k_{\overline{\epsilon}}) \quad \forall a_k\in A_k$$

which implies that the $P_{\rho,\omega}$-IR conditions hold also for every $k\notin N_{IR}$ in the contract $(a^*,\overline{\mathcal{I}})$. Finally, by construction it follows that $\overline{I}^k_q=I^k_{\overline{\varepsilon}_k,q}\leq I^k_q$ for every $k\in N$ and $q\in Q$.. Let $k\in N_{IR}$, then there exists $\widetilde{q}\in J(a^*)$ such that $I^k_{\widetilde{q}}>\underline{I}$. It follows that $\overline{I}^k_{\widetilde{q}}=I^k_{\overline{\varepsilon}_k,\widetilde{q}}<I^k_{\widetilde{q}}$, which implies

$$\mathcal{C}(a^*,\mathcal{I})>\mathcal{C}(a^*,\overline{\mathcal{I}}) \tag{11}$$

and the assertion follows. ∎

Proof of Proposition 4. From the assumptions it follows that, for every coalition T there exists a player $k\in T$ such that

$$\sum_{t\notin T}\theta_k(a_k-a^*_k)^2\geq(n-|T|))\theta_k\geq\nu_k(a^*)\geq G_k(a^*_k)-G_k(a_k) \quad \forall a_k\in A_k. \tag{12}$$

Following similar steps of the proof of Proposition 1 we get

$$E_k(a^*,\widehat{\mathcal{I}}^k) = \sum_{q\in Q} \pi_q(a^*)V_k(C^k_{FB}(a^*_k)) - G_k(a^*_k) = V_k(C^k_{FB}(a^*_k)) - G_k(a^*_k)$$

whereas

$$E_k(a_T, a^*_{-T}, \widehat{\mathcal{I}}^k) = V_k(C^k_{FB}(a^*_k)) \left[\sum_{q\in J(a^*)} \pi_q(a_T, a^*_{-T})\right] + V_k(\underline{I}) \left[\sum_{q\notin J(a^*)} \pi_q(a_T, a^*_{-T})\right] - G_k(a_k).$$

Since $V_k(\underline{I}) \leq V_k(C^k_{FB}(a^*_k))$ and (12) holds we finally get

$$V_k(C^k_{FB}(a^*_k)) + \sum_{t\notin T} \theta_k(a_k - a^*_k)^2 \geq \sum_{q\in Q} \pi_q(a_T, a^*_{-T})V_k(\widehat{I}^k_q) + G_k(a^*_k) - G_k(a_k) \quad \forall a_T \in A_T.$$

which implies that

$$E_k(a^*,\widehat{\mathcal{I}}^k) \geq E_k(a_T, a^*_{-T}, \widehat{\mathcal{I}}^k) - \theta_k \sum_{t\notin T}(a_k - a^*_k)^2 \quad \forall a_T \in A_T \setminus \{a^*_T\}. \tag{13}$$

If $b^*_{kt} = a^*_t$ and $c^*_{ktk} = a^*_k$ for all $t \neq k$, then the psychological payoff of player k consistent with a^* is

$$U_k(a^*_T, a^*_{-T}, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) = E_k(a^*, \widehat{\mathcal{I}}^k).$$

Fixed a deviation a_T of coalition T from a^*, the beliefs of player k consistent with this deviation are
1) $b_{kt} = a^*_t$ and $c_{ktk} = a^*_k$ for all $t \in N \setminus T$. In this case, $\chi_{kt} = a_k - a^*_t = a_k - a^*_k$ and $\lambda_{ktk} = a^*_t - a^*_k = 0$.
2) $b_{kt} = a_t$ and $c_{ktk} = a_k$ for all $t \in T \setminus \{k\}$. In this case, $\chi_{kt} = a_k - a_t$ and $\lambda_{ktk} = a_t - a_k$ therefore the psychological payoff of player k consistent with a deviation a_T is

$$U_k(a_T, a^*_{-T}, (b_{kt})_{t\neq k}, (c_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k)) = E_k(a_T, a^*_{-T}, \widehat{\mathcal{I}}^k) - \theta_k \sum_{t\notin T}(a_k - a^*_k)^2 -$$

$$\rho_k \sum_{t\in T\setminus\{k\}} (\omega_k + 1)^2(a_k - a_t)^2 \leq E_k(a_T, a^*_{-T}, \widehat{\mathcal{I}}^k) - \theta_k \sum_{t\notin T}(a_k - a^*_k)^2 \quad \forall a_T \in A_T \setminus \{a^*_T\}.$$

Hence,

$$U_k(a^*_T, a^*_{-T}, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) \geq U_k(a_T, a^*_{-T}, (b_{kt})_{t\neq k}, (c_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k)) \quad \forall a_T \in A_T \setminus \{a^*_T\}.$$

Since the previous arguments apply for every coalition $T \subseteq N$ then the assertion follows.
■

Proposition 6. *Let a^* be the symmetric first best profile. If*

$$\rho_k \left[(\omega_k + 1)^2 - (n-1)(m-1)^2\right] > \nu_k(a^*) \quad \forall k \in N \tag{14}$$

then every asymmetric strategy profile is not a strong psychological equilibrium of $\Gamma_{\rho,\omega}(\widehat{\mathcal{I}})$.

Proof of Proposition 6. Consider an asymmetric strategy profile $\overline{a}$, let $T = \{k \in N \mid \overline{a}_k \neq a_k^*\}$. Consider the joint deviation of coalition T towards their first best strategies a_T^*. Obviously we get $a^* = (a_T^*, \overline{a}_{-T})$. Recall that, for every player k, $V_k(C_{FB}^k) \geq 0 = V_k(\underline{I})$, moreover $\sum_{q \in J(a^*)} \pi_q(a^*) = 1$, which obviously implies that $\sum_{q \in J(a^*)} [\pi_q(\overline{a}) - \pi_q(a^*)] \leq 0$ and $\sum_{q \notin J(a^*)} [\pi_q(\overline{a}) - \pi_q(a^*)] = \sum_{q \notin J(a^*)} [\pi_q(\overline{a})]$. Then

$$V_k(C_{FB}^k) \left[\sum_{q \in J(a^*)} [\pi_q(\overline{a}) - \pi_q(a^*)] \right] + V_k(\underline{I}) \left[\sum_{q \notin J(a^*)} [\pi_q(\overline{a}) - \pi_q(a^*)] \right] \leq 0.$$

Hence, from (14), it follows that for every player k

$$\rho_k \left[(\omega_k + 1)^2 - (n-1)(m-1)^2 \right] >$$

$$V_k(C_{FB}^k) \left[\sum_{q \in J(a^*)} [\pi_q(\overline{a}) - \pi_q(a^*)] \right] + V_k(\underline{I}) \left[\sum_{q \notin J(a^*)} [\pi_q(\overline{a}) - \pi_q(a^*)] \right] +$$

$$G_k(a_k^*) - G_k(\overline{a}_k).$$

This condition implies that

$$E_k(a^*, \widehat{\mathcal{I}}^k) - \rho_k(n-1)(m-1)^2 > E_k(\overline{a}, \widehat{\mathcal{I}}^k) - \rho_k(\omega_k + 1)^2 \tag{15}$$

Now, let $k \in T$ and consider the psychological payoff of player k consistent with the strategy profile $\overline{a}$:

$$U_k(\overline{a}, (\overline{b}_{kt})_{t \neq k}, (\overline{c}_{ktk})_{t \neq k}; \widehat{\mathcal{I}}^k) = E_k(\overline{a}, \widehat{\mathcal{I}}^k) - \sum_{t \neq k} \rho_k(\omega_k(\overline{a}_k - \overline{b}_{kt}) - (\overline{b}_{kt} - \overline{c}_{ktk}))^2$$

with $\overline{b}_{kt} = \overline{a}_t$ and $\overline{c}_{ktk} = \overline{a}_k$ for all $t \neq k$, therefore

$$U_k(\overline{a}, (\overline{b}_{kt})_{t \neq k}, (\overline{c}_{ktk})_{t \neq k}; \widehat{\mathcal{I}}^k) = E_k(\overline{a}, \widehat{\mathcal{I}}^k) - \sum_{t \neq k} \rho_k(\omega_k(\overline{a}_k - \overline{a}_t) - (\overline{a}_t - \overline{a}_k))^2 =$$

$$= E_k(\overline{a}, \widehat{\mathcal{I}}^k) - \sum_{t \neq k} \rho_k((\omega_k + 1)(\overline{a}_k - \overline{a}_t))^2 \leq E_k(\overline{a}, \widehat{\mathcal{I}}^k) - \rho_k(\omega_k + 1)^2. \tag{16}$$

Consider the psychological payoff of player k consistent with the deviation a_T^* of coalition T:

$$U_k(a^*, (b_{kt}^*)_{t \neq k}, (c_{ktk}^*)_{t \neq k}; \widehat{\mathcal{I}}^k) = E_k(a^*, \widehat{\mathcal{I}}^k) - \sum_{t \neq k} \rho_k(\omega_k(a_k^* - b_{kt}^*) - (b_{kt}^* - c_{ktk}^*))^2$$

where $b_{kt}^* = a_t^*$ $(= a_k^*)$ and $c_{ktk}^* = a_k^*$ if $t \in T$ and $b_{kt}^* = \overline{a}_t$ $(= a_k^*)$ and $c_{ktk}^* = \overline{a}_k \neq a_k^*$ if $t \notin T$. Therefore

$$\sum_{t \neq k} (\omega_k(a_k^* - b_{kt}^*) - (b_{kt}^* - c_{ktk}^*))^2 = \sum_{t \in T \setminus \{k\}} (\omega_k(a_k^* - b_{kt}^*) - (b_{kt}^* - c_{ktk}^*))^2 + \sum_{t \notin T} (\omega_k(a_k^* - b_{kt}^*) - (b_{kt}^* - c_{ktk}^*))^2 =$$

$$\sum_{t\notin T}(\omega_k(a^*_k - b^*_{kt}) - (b^*_{kt} - c^*_{ktk}))^2 = \sum_{t\notin T}(-(\overline{a}_t - \overline{a}_k))^2 \leq \sum_{t\notin T}(m-1)^2 \leq (n-1)(m-1)^2$$

Hence

$$U_k(a^*, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) \geq E_k(a^*, \widehat{\mathcal{I}}^k) - \rho_k(n-1)(m-1)^2 \qquad (17)$$

Hence from (15,16,17) it follows that

$$U_k(a^*, (b^*_{kt})_{t\neq k}, (c^*_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) > U_k(\overline{a}, (\overline{b}_{kt})_{t\neq k}, (\overline{c}_{ktk})_{t\neq k}; \widehat{\mathcal{I}}^k) \quad \forall k \in T$$

and therefore $\overline{a}$ is not a psychological strong Nash equilibrium. ■

Proof of Lemma 1. Let $a = (a_1, \ldots, a_n)$ then $C_{FB}(a) = c(a_1) + c(a_2) + \cdots + c(a_n)$. Hence

$$\frac{C_{FB}(a)}{n} = \frac{c(a_1) + c(a_2) + \cdots + c(a_n)}{n} \geq \max_{s \in N\left(\frac{\sum_{i=1}^n a_i}{n}\right)} c(s) \geq c(m(a)).$$

Hence

$$C_{FB}(a) \geq nc(m(a)) = C_{FB}(\mu(a))$$

and the assertion follows. ■

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