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Abstract

This paper studies the optimal fiscal treatment of addictive goods (cigarettes, drugs, fatty foods, alcohol, gambling etc.). It shows that, when agents have private information about their productivity levels and their degree of rationality, the Atkinson and Stiglitz result of optimal uniform commodity taxation does not hold: addictive and non-addictive goods should be taxed at different rates. Depending on the direction of redistribution, the addictive good should be taxed more or less than the non-addictive good. Differential commodity taxation is not driven by the planner's paternalism, but only by incentive considerations. A tax authority which fully respects consumers' sovereignty taxes the consumption of addictive and non-addictive goods at different rates to improve screening of types and increase income redistribution.

JEL classification: A12, D91, E21, H55.

Keywords: Bounded Rationality, Optimal Taxation, Minimal Paternalism, Multidimensional Screening.

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1 Introduction

A central assumption of neoclassical economic theory is that individuals are *unitary* — i.e., have consistent goals and preferences — and *perfectly rational* — i.e., make utility-maximizing decisions, given their constraints and available information. In other words, economic agents know exactly their objectives and how to pursue them. In recent years, however, a solid empirical and experimental evidence has spurred debate about this traditional way of modeling individuals' decision-making process. In some circumstances, in fact, individuals seem to display a *limited* (or bounded) rationality, and their choices do not always reflect a utility-maximizing attitude. The emerging of the so-called *behavioral economics* has increased the explanatory power of neoclassical economic models by introducing more realistic psychological foundations about individuals' decision-making.[1]

Understanding whether individuals' rationality is "perfect" or "bounded" is important, especially because different assumptions on agents' behavior might deliver conflicting policy prescriptions. With perfectly rational individuals, in fact, the two welfare theorems guarantee that, in absence of market failures, competitive markets achieve the maximum level of welfare; hence, the economic role of the government is necessarily limited. On the other hand, when agents have limited rationality, the government can play a more active role: welfare gains could be achieved by opportune paternalistic interventions, provided that the planner knows better than agents what is good for them. Hence, on the basis of the planner's superior information set, individuals' behavior could be modified in virtuous directions. Examples of such interventions are prohibitionist policies — i.e., the planner restricts people's choice set (a form of *strong* paternalism) — or *minimal* (or *libertarian*) policies — i.e., the government influences people's choices in welfare-promoting directions, but with minimal losses for those who are perfectly rational (Sunstein and Thaler, 2003 and 2008).

Paternalism in presence of bounded rationality raises, however, questions about the excessive role of the government and, in particular, the opportunity that agents' life is regulated pervasively by someone else. According to Saint-Paul (2011), for instance, paternalistic interventions intended to protect individuals against their irrationality diminish agents' freedom, by imposing unnecessary restrictions on private preferences and choices. Moreover, if the interventions of the paternalistic state are justified on the grounds that individuals' behavior would otherwise be irresponsible, nothing guarantees the responsibility of the state (Glaeser, 2006).

An example of how different assumptions on agents' degree of rationality generate opposite policy prescriptions is given by the fiscal treatment of addictive goods — i.e., taxes on the consumption of cigarettes,

[1] Examples of behavioral anomalies documented in the empirical and experimental literature are: *cognitive dissonance* (people discard signals which reveal that their choices might have been erroneous), *loss-aversion* (the disparity between the strong aversion to losses relative to a reference point and the weaker desire for gains of equivalent magnitude), *time inconsistency* (people evaluate alternatives depending on the date on which they make their choice), *mental budgeting* (people ascribe different income sources to different kinds of expenditure), *intrinsic motivation* (people undertake costly actions even is absence of monetary or extrinsic rewards), *framing effects* (the way a problem is formulated affects decisions), *etc.* For a review, see Camerer et al (2003).

drugs, fatty foods, alcohol, gambling etc.: the so-called *sin taxes*.

To explain addiction, the economic literature has proposed models with perfect or with bounded rationality. In the first case (Becker and Murphy, 1988), addicted agents are forward-looking utility maximizers who perfectly understand the harmful nature of the sin good. Therefore, if an agent values the utility from consumption more than the costs created by addiction, it is coherent with the perfect rationality paradigm to observe addiction, and nothing justifies a special fiscal treatment of addictive goods compared to any other good. Hence, if the authority can levy an optimal non-linear tax on labor income, and the hypothesis of the Atkinson-Stiglitz theorem (1976) are satisfied, all goods (including addictive goods) should be taxed at the same rate, which can be normalized to zero.[2]

Alternatively, addiction might be explained with a model of limited rationality, in the form of time-inconsistent (or present-biased) preferences (Gruber and Köszegi, 2004, O'Donoghue and Rabin, 2003 and 2009). By consuming the addictive good, individuals pursue immediate gratification in ways that do not correspond to their long-run well-being.[3] Time inconsistency induce agents to over-consume the addictive product today, and regret about it later on. Because of this regret, and only if paternalism is socially accepted, the fiscal policy might represent a way to correct the behavior of addicted consumers. The introduction of a sin tax represents a form of minimal paternalism: by taxing the consumption of addictive goods, the planner changes his relative price: hence, consumers' choices are altered: a higher price represents, for addicted agents with limited rationality, a commitment device that helps them to reduce consumption levels. However, the possibility of consuming the good for those who are fully rational is not eliminated.

Since the two explanations of addiction have very different implications, a policy maker which bases his actions on the wrong approach risks to impose substantial welfare losses to society. If agents were rational addicted, paternalistic interventions inefficiently distort their choices. If, on the contrary, addiction is ascribed to self-control issues, a utilitarian planner who does not intervene fails to correct agents' behavior and imposes an additional costs on their future utilities. On the basis of these considerations, one may be tempted to conclude that positive sin taxes are justified only if agents display bounded rationality and only if paternalism is accepted by society.

The objective of this paper is to show, instead, that the optimal fiscal treatment of sin goods should not depend neither on the agents' degree of rationality nor on the planner's paternalistic desire of overcome consumers' sovereignty, but only on the information available to the tax authority about individuals' innate characteristics. In other words, positive sin taxes optimally emerge in a second-best framework with a planner that fully respects agents' choices: it is sufficient that some agents have limited rationality, and are privately informed about it, to justify the taxation of addictive goods. These results are demonstrated within

[2] This is true if the negative externalities created by addiction are ignored; if these externalities are taken into account, the optimal sin tax should be positive. However, this result would not depend neither on the planner's attitude (paternalistic or not) nor on agents' rationality. In fact, including externalities in the picture would only reinforce the main point of this work: efficiency considerations, and not paternalism, are enough to justify positive sin taxes.

[3] The role played by bounded rationality in explaining addiction is analyzed in several studies of attitudes towards smoking. Loewenstein et al. (2003), for instance, illustrates that the lack of self-control is the main cause for becoming a smoker.

a simple model of optimal taxation, in which agents are heterogeneous with respect to two characteristics: their productivity level and their preferences for immediate gratification. In particular, individuals with such preferences display a “self-control problem”: they have a short-term desire for the addictive good which will be regretted in the future. As a consequence, they purchase more addictive good compared to agents with standard exponential preferences, other things being equal. The tax authority is assumed to be utilitarian and raises money to meet an exogenous revenue requirement and to redistribute income.[4] Commodity taxes and labor income taxes are the policy instruments available to achieve these objectives. When productivity is agents' private information (but their degree of rationality is observable), the benchmark is represented by the Atkinson-Stiglitz theorem: provided that preferences between consumption and leisure are weakly separable, differential commodity taxation (both linear and non-linear) is not necessary, if the tax authority can levy an optimal (in the sense of Mirrlees, 1975) non-linear tax on labor income. This is because using an additional tax instrument — in this case, commodity taxes — is useful only if it improves screening of types. In particular, this instrument should “hurt” the mimicker more than the mimicked, in order to relax an otherwise binding incentive constraint. Differential commodity taxation cannot accomplish this objective: the separability assumption ensures that the mimicker and the mimicked (agents with the same degree of rationality but different productivity levels) have identical preferences over consumption. Hence, a commodity tax does not improve screening, and the redistributive objective can be achieved only with an appropriate design of the non-linear income tax. Hence, the presence of agents with observable bounded rationality is not enough to justify differential taxation between addictive and non-addictive goods.

However, when heterogeneity in preferences for immediate gratification is agents' private information, the Atkinson-Stiglitz theorem does not hold, and differential commodity taxation becomes optimal. This is because, even with the separability assumption, the mimicker and the mimicked do not share the same preferences for the addictive goods. When an exponential agent tries to mimic an agent with preferences for immediate gratification by consuming the same bundle, he does not achieve the same utility level of the mimicked, given that differences in the degree of rationality induce different preferences for the addictive good. Hence, to “hurt” the mimicker, it is optimal to impose small and different taxes on the addictive and the non-addictive good: in this way, the bundle of the mimicked becomes less attractive for the mimicker, and the incentive constraint is relaxed, allowing the planner to increase redistribution. Hence, sin taxes are optimal not for the paternalistic attitude of the planner, but only because allow to relax the incentive constraints in the multidimensional screening problem.[5]

This work contributes to two strands of literature; first, to the literature on the optimal tax treatment of addictive goods in presence of bounded rationality. O'Donoghue and Rabin (2009) determine optimal

[4] Redistribution is not in contrast with utilitarism, as long as government's interventions are not paternalistic — i.e., it does not prescribe individuals' actions. By redistributing income, a planner only changes, under the veil of ignorance, people's opportunities. (Saint-Paul, 2011).

[5] Of course, if the planner, besides the redistributive concern, wishes to modify agents' behavior, the optimal sin tax would be higher relative to the libertarian case studied here. The sin tax, in fact, would include also a commitment device component that discourages consumption of the addictive good.

commodity taxes on addictive and non-addictive goods when individuals have observable preferences for immediate gratification. They show that taxing more addictive goods relative to non-addictive goods increases welfare, provided that the tax proceeds are returned in a lump-sum fashion to consumers. Such taxes are useful for two reasons: from the one hand, they counteract the over-consumption of the addictive good. From the other hand, sin taxes redistribute income from individuals with bounded rationality to agents with perfect rationality. Gruber and Köszegi (2001) show that, if agents have time inconsistent preferences *à la* Laibson (1997), positive sin taxes represent a commitment device that allows consumers with self-control issues to internalize the future costs of addiction. Gul and Pesendorfer (2007) compare, in a model of temptation, the welfare effects of a sin taxes and a prohibitionist policy. The second policy is shown to be always Pareto-superior to the first one. This is because a tax increases the cost of consuming the sin good, but it does not eliminate the possibility for the addicted agent to consume it anyway. By banning the addictive good, on the contrary, the tempting alternative is eliminated.

These studies might be improved in three directions. First, all works limit exogenously the set of fiscal instruments available to the planner: as in the standard Ramsey (1927) model of taxation, taxes (including labor income taxes) are assumed to be linear. Second, informational issues between the taxpayer and the tax authority are ignored. Third, the desirability of sin taxes follows only from planner's paternalism. This paper, instead, does not rely on planner's paternalism to justify sin taxes. Moreover, results are derived in a model which explicitly allows for non-linear income taxes and asymmetric information between parties.

This work also contribute to the literature on optimal mixed taxation (commodity and labor income taxes), which tries to understand why the widespread use of differential indirect taxation contrasts with the uniform taxation result of the Atkinson-Stiglitz theorem. Several explanations have been proposed to justify the discrepancy: violation of the separability assumption (Browning and Meghir, 1991), tax evasion (Boadway *et al.,* 1994), uncertainty (Cremer and Gahvari, 1995) or multidimensional asymmetric information between the tax authority and taxpayers. In fact, if individuals have private information not only on their productivity (as in Atkinson-Stiglitz) but also in wealth (Cremer *et al.,* 2001) or preferences (Blomquist and Christiansen 2004, Saez 2002, Boadway *et al.*, 2002, Boadway and Pestieau 2003, among others), commodity taxes represents a useful screening device that supplements the optimal non-linear income tax. This paper proposes as an additional motivation for commodity taxation: the existence of heterogeneity in agents' degree of rationality.

The rest of the paper is organized as follows. Section 2 presents the model. Sections 3 and 4 study a simplified problem, in which there is only one source of asymmetric information. Both cases of linear and non-linear commodity taxes are considered. Section 5 characterizes the optimal tax system when both productivity and the degree of preferences for immediate gratification are agents' private information. Section 6 concludes. All proofs are in the appendix.

2 The Model

Economic Environment Consider a consumption model in the spirit of Mirrlees (1976) and Atkinson and Stiglitz (1976). There are three goods, an *addictive/sin* good (A), a *non-addictive* good (B), and a *numeraire/composite* good (C). All goods are produced with a linear production function that uses only labor as input. Marginal costs of production are constant and normalized to one. Markets are competitive.

Preferences Following Strotz (1956), Phelps and Pollacks (1968) and Laibson (1997), agents' intertemporal preferences are described by

$$W = U_1 + \beta \sum_{\tau=2}^{\overline{\tau}} \delta^{\tau-1} U_\tau,$$

where U_τ, for $\tau = 1, ..., \overline{\tau}$, denotes instantaneous utility in period τ. The term $\beta \in (0,1)$ denotes the short-term subjective discount factor, while $\delta \in (0,1]$ is the long-term one. This formulation implies that, from today's perspective, the discount factor between period one and two is $\beta\delta$, while that between period τ and $\tau+1$ is δ; in other words, the discount factor first declines with time, and stays constant thereafter. Notice that, for $\beta = 1$, the model reduces to a traditional one with exponential discounting. For simplicity, set $\delta = 1$.

The instantaneous utility function U_τ is increasing and concave, and it is separable between leisure and consumption,

$$U_\tau(x_{A,\tau}, x_{B,\tau}, x_{C,\tau}, L_\tau) = u_\tau(x_{A,\tau}, x_{B,\tau}, x_{C,\tau}) - l_\tau(L_\tau),$$

where L_τ denotes labor supply and $x_{k,\tau}$ consumption of good $k = A, B, C$ at time τ. The sub-utility function $u_\tau(.)$ is increasing and concave in all arguments, while the disutility of effort $l_\tau(.)$ is increasing and convex.

The sin nature of good A implies that, as in O'Donoghue and Rabin (2009), its consumption increases the agent's utility at time τ (like any other good) but reduces his utility at time $\tau+1$ because, for instance, it creates health damages. These costs are summarized by the increasing and convex function $c_\tau(x_{A,\tau-1})$, where $x_{A,\tau-1}$ denotes consumption of the good A in the previous period.[6] Hence, the instantaneous utility function at time τ becomes

$$U_\tau(.) = u_\tau(x_{A,\tau}, x_{B,\tau}, x_{C,\tau}) - l_\tau(L_\tau) - c_\tau(x_{A,\tau-1}). \quad (1)$$

To simplify, let all functions be time-invariant. Moreover, assume also that individuals cannot borrow or save. This assumption allows to isolate the intertemporal distortions induced by the consumption of sin goods from the intertemporal distortions in saving behavior.

[6] An alternative way to model addiction would be the Becker and Murphy (1988) framework, in which current utility and future health costs are a function of current consumption levels of the addictive good, as well as the stock of past consumption. Although more complicated, the Becker and Murphy framework would give the same qualitative result of this simplified version of addiction.

The structure of intertemporal preferences, together with the assumption of delayed costs associated to the consumption of A, implies that each agent faces, at every τ, a series of independent decisions. In particular, each individual solves

$$\max_{L,\{x_k\}_{k=A,B,C}} U(.) = u(x_A, x_B, x_C) - l(L) - \beta c(x_A),$$

Finally, denote with $Y = \theta L$ gross labor income, given by the product of agent's innate productivity, θ, and his labor supply, L. Wage is normalized to one.

Heterogeneity Agents differ with respect to two inalterable and uncorrelated characteristics: *productivity* and *preferences for immediate gratification* — i.e., their attitude toward present consumption relative to future consumption —, which is interpreted as a proxy for the agent's degree of rationality. Productivity can take two values, θ^1 and θ^2, with $\theta^2 > \theta^1$. Agents' behavioral type depends on the value of the short-term discount factor β, which can be either β^1 (the agent has preferences for immediate gratification) or β^2 (the agent is an exponential discounter). Let $\beta^1 < \beta^2$. A superscript i denotes productivity type while j denotes the behavioral type.

Hence, four types (represented in Figure 1) coexist in the economy: **type (1;1)** individuals are low-skilled agents with preferences for immediate gratification; **type (2;1)** agents are high-skilled with preferences with immediate gratification. **Type (1;2)** are low-skilled exponential discounters, while **type (2;2)** agents are high-skilled exponential discounters. The probabilities of being of type (i, j) is π^{ij}, with $\sum_{i,j} \pi^{ij} = 1$. The probability distribution is common knowledge. Let x_k^{ij} be the consumption level of good k of an individual of type (i, j). Denote also Y^{ij} and L^{ij} as agent's gross income and labor supply, respectively.

Tax Authority The government taxes labor income and the consumption of the three commodities to achieve two objectives: $(i.)$ raise an exogenous amount $\bar{R}$; $(ii.)$ redistribute resources, either from poor to rich, or from exponential to time inconsistent agents (or vice-versa) or both. The taxes levied depend on the information structure available to the tax authority. First-best taxation and lump-sum tax/transfers conditioned on ability or behavioral type are ruled out by assumption. Hence, depending on the information about consumption levels, $\bar{R}$ can be raised with

1. a non-linear tax function $T(x_A^{ij}, x_B^{ij}, x_C^{ij}, Y^i)$, which depends on individuals' observable consumption levels, x_k^{ij}, and labor income, Y^{ij}. Agents' disposable income is $I^{ij} = Y^{ij} - T(x_A^{ij}, x_B^{ij}, x_C^{ij}, Y^{ij})$.

2. a non-linear tax on labor income, $T(Y^{ij})$, and linear commodity taxes on good A and B, denoted t_k, when consumption levels are not observable by the tax authority, but anonymous transaction are (Guesnerie, 1995). Notice that good C is set untaxed. Agents' disposable income is $I^{ij} = Y^{ij} - T(Y^{ij})$.

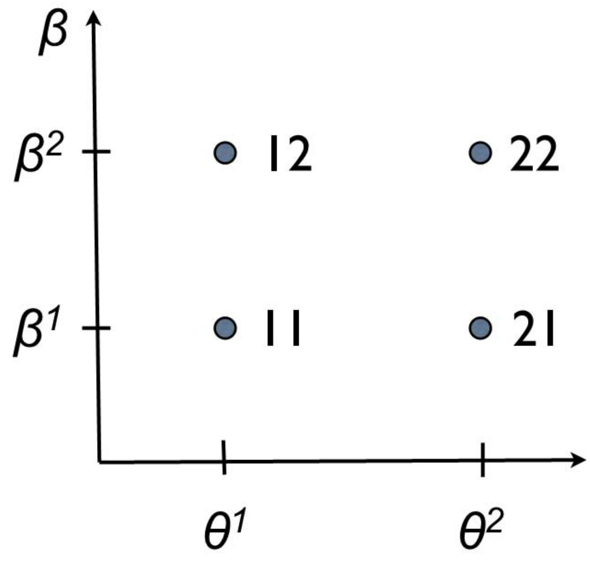

Figure 1: Distribution of types

The problem of the government is to select the tax schedule that maximize the utilitarian social welfare function,

$$\Phi = \sum_{i,j} \alpha^{ij} U^{ij}(.),$$

where $U^{ij}(.)$ is the utility function of an individual of type (i, j) and α^{ij} is the social weight attached to him. Since tax authority is interested in redistributing income from rich to poor, let $\alpha^{1j} > \alpha^{2j}$, for given j. To avoid renegotiation issues, assume that the tax authority can credibly commit to keep the tax schedule chosen at time τ until the last period $\overline{\tau}$.

3 Optimal Nonlinear Taxation

This section assumes that consumption levels are observable and that a non-linear tax function $T(x_A^{ij}, x_B^{ij}, x_C^{ij}, Y^i)$ can be levied by the tax authority.

Consider, first, consumers' problem. An individual of type (i, j) solves

$$\max_{\{x_k\}, Y^i} \quad v^{ij}(x_A^{ij}, x_B^{ij}, x_C^{ij}; Y^{ij}, \beta^j) = u(x_A^{ij}, x_B^{ij}, x_C^{ij}) - l\left(\frac{Y^{ij}}{\theta^i}\right) - \beta^j c(x_A^{ij}),$$

subject to

$$Y^{ij} \geq x_A^{ij} + x_B^{ij} + x_C^{ij} + T(x_A^{ij}, x_B^{ij}, x_C^{ij}, Y^{ij}).$$

Standard conditions for utility maximization with respect to x_k^{ij} imply (in a interior solution)[7]

$$MRS_{A,k}^{ij} \equiv \frac{\partial v^{ij}/\partial x_A}{\partial v^{ij}/\partial x_k} = \frac{1+(T_A^{'})_{ij}}{1+(T_k^{'})_{ij}}, \tag{2}$$

$$MRS_{B,C}^{ij} \equiv \frac{\partial v^{ij}/\partial x_B}{\partial v^{ij}/\partial x_C} = \frac{1+(T_B^{'})_{ij}}{1+(T_C^{'})_{ij}}, \tag{3}$$

where $(T_A^{'})_{ij}$ and $(T_k^{'})_{ij}$ are marginal commodity taxes, respectively, on good A and on good $k = B, C$, levied on an agent of type (i, j). Hence, when $(T_A^{'})_{ij} = (T_k^{'})_{ij}$ and $(T_B^{'})_{ij} = (T_C^{'})_{ij}$, $\forall i, j$, — i.e., uniform commodity taxation — all agents face the first-best trade-offs, $MRS_{A,k}^{ij} = 1$ and $MRS_{B,C}^{ij} = 1$. It follows that differential non-linear commodity taxes are needed if and only if a Pareto constrained efficient allocation implies $MRS_{A,k}^{ij} \neq 1$ and $MRS_{B,C}^{ij} \neq 1$, $\forall i, j$.

Notice that the demand function of the addictive good — x_A^{ij} — increases with β, for given θ: agents with preferences for immediate gratification underestimate the future cost of consuming more addictive good today relative to an agent with no such preferences. Hence, $x_A^{i1} > x_A^{i2}$ and, from the budget constraint, $x_k^{i2} > x_k^{i1}$, for $k = B, C$.

Before considering the double heterogeneity case, the following two subsections present, as a benchmark, the problem with one source of asymmetric information: in subsection 3.1, agents have only private information on θ; in subsection 3.2, there is only private information on β. The objective is to disentangle the effects introduced by the two sources of asymmetric information.

3.1 Productivity is Private Information

When productivity levels and labor supply are agents' private information, while preferences for immediate gratification — i.e., the parameter β^j — are observed by the tax authority, the population consists of two identifiable behavioral types: type $i1$ is characterized by $\{(\theta^1, \beta^1), (\theta^2, \beta^1)\}$ and type $i2$ is characterized by $\{(\theta^1, \beta^2), (\theta^2, \beta^2)\}$.

As pointed out by Stiglitz (1982), the government can implicitly choose the optimal tax function $T(x_A^{ij}, x_B^{ij}, x_C^{ij}, Y^{ij})$ by choosing quantities x_k^{ij}, for $k = A, B, C$ and pre-tax income Y^{ij}. Hence, the planner's problem can be stated as

$$\max_{\{x_k^{ij}\}_k, Y^{ij}} \quad \sum_{i,j} \alpha^{ij} v^{ij}(x_A^{ij}, x_B^{ij}, x_C^{ij}, \frac{Y^{ij}}{\theta^i}; \beta^j)$$

subject to

$$\sum_{i,j} \pi^{ij}(Y^{ij} - x_A^{ij} - x_B^{ij} - x_C^{ij}) = \bar{R}, \tag{4}$$

[7] First-order conditions on Y^i can be used to determine the structure of the optimal income tax. Being optimal commodity taxation the focus of this work, and being the characterization of the labor income tax rather standard, these conditions are not immediately relevant for our purposes, and are suppress. The tax authority is assumed to be able to tax labor income optimally.

$$v^{21}(x_A^{21}, x_B^{21}, x_C^{21}, \tfrac{Y^{21}}{\theta^2}; \beta^1) \geq \widetilde{v}^{21}(x_A^{11}, x_B^{11}, x_C^{11}, \tfrac{Y^{11}}{\theta^2}; \beta^1), \tag{5}$$

$$v^{22}(x_A^{22}, x_B^{22}, x_C^{22}, \tfrac{Y^{22}}{\theta^2}; \beta^2) \geq \widetilde{v}^{22}(x_A^{12}, x_B^{12}, x_C^{12}, \tfrac{Y^{12}}{\theta^2}; \beta^1), \tag{6}$$

where (4) is the government's budget constraint and (5) and (6) are the self-selection constraints. Being the behavioral type common knowledge, the only relevant incentive constraints are those from high productive agents towards low productive individuals with the same β^j. Hence, the planner needs to design an allocation such that an high-productive agent weakly prefers the bundle $(x_A^{2j}, x_B^{2j}, x_C^{2j}, Y^2)$ designed for him to that intended for a low-productive individual with the same behavioral type. Solving the problem yields the following Proposition, which describes the structure of optimal commodity tax.[8]

Proposition 1 *With observable heterogeneity on preferences for immediate gratification and asymmetric information on productivity levels, optimal non-linear commodity taxes are zero.*

The Proposition shows that, when productivity levels are agents' private information, but there is observable heterogeneity with respect to preferences for immediate gratification, sin taxes are not needed — i.e., the Atkinson-Stiglitz theorem remains valid —, and the redistributive objective can be achieved only with a (optimal) non-linear tax on labor income. To see why, first notice that the separability assumption ensures that the preferences over consumption of the mimicker (an individual of type (θ^2, β^j)) are the same of the mimicked (an individual of type (θ^1, β^j) with the same j). Hence, since the mimicker has to consume the same bundle of the mimicked, their marginal rate of substitution are also the same. From equilibrium conditions (2) and (3), the optimality of uniform commodity taxation follows immediately.

Hence, the Proposition shows that observable differences in the degree of rationality do not justify sin taxes. Differential commodity taxation does not provide additional information to the tax authority about the hidden characteristics of the taxpayer; in other words, it does not reduce the incentive for an individual of type (θ^2, β^j) to mimic a type (θ^1, β^j). A labor income tax is sufficient to achieve this objective.

Notice that assuming that the planner weighs more poor agents' utility relative to rich — i.e., $\alpha^{1j} > \alpha^{2j}$ — guarantees that the only binding incentive constraints are the "downward" ones — i.e., from the more productive agents to the less productive agents.[9] If, instead, one assumes *reverse redistribution* (Stiglitz, 1982) — i.e., $\alpha^{2j} > \alpha^{1j}$ — the binding incentive constraints would be the "upward" ones, from less productive to more productive agents with the same β^j. However, reverse redistribution would alter only the structure of the optimal income tax, but not the result of Proposition 1.

[8] As anticipated, the structure of the optimal income tax replicates Mirrlees (1976). In order to relax the incentive constraints, the labor supply of low-income agents is distorted downward: hence, the labor income tax is characterized by $(T_Y^{'})_{1j} > 0$ and $(T_Y^{'})_{2j} = 0$, for all j. A small reduction of type (θ^1, β^j)'s labor supply has no impact on his utility and the resource constraint. However, it relaxes the incentive constraints, and allows the planner to increase redistribution.

[9] Downward incentive constraints also bind when social weights are equal to the proportion of agents in the economy — i.e., $\alpha^{ij} = \pi^{ij}$, $\forall i, j$.

3.2 Preferences are Private Information

When the tax authority observes agents' productivity level θ^i but not their degree of rationality, β^j, the population can be divided into two identifiable productivity types: type $1j$ is characterized by $\left\{(\theta^1, \beta^1), (\theta^1, \beta^2)\right\}$ and type $2j$ is characterized by $\left\{(\theta^2, \beta^1), (\theta^2, \beta^2)\right\}$. Hence, the planner solves

$$\max_{\left\{x_k^{ij}\right\}_k, Y^{ij}} \quad \sum_{i,j} \alpha^{ij} v^{ij}(x_A^{ij}, x_B^{ij}, x_C^{ij}, \frac{Y^{ij}}{\theta^i}; \beta^j)$$

subject to (4) and

$$v^{12}(x_A^{12}, x_B^{12}, x_C^{12}, \tfrac{Y^{12}}{\theta^1}; \beta^2) \geq \widetilde{v}^{12}(x_A^{11}, x_B^{11}, x_C^{11}, \tfrac{Y^{11}}{\theta^1}; \beta^2) \qquad (\lambda_1) \tag{7}$$

$$v^{22}(x_A^{22}, x_B^{22}, x_C^{22}, \tfrac{Y^{22}}{\theta^2}; \beta^2) \geq \widetilde{v}^{22}(x_A^{21}, x_B^{21}, x_C^{21}, \tfrac{Y^{21}}{\theta^2}; \beta^2) \qquad (\lambda_2) \tag{8}$$

$$v^{11}(x_A^{11}, x_B^{11}, x_C^{11}, \tfrac{Y^{11}}{\theta^1}; \beta^1) \geq \widetilde{v}^{11}(x_A^{12}, x_B^{12}, x_C^{12}, \tfrac{Y^{12}}{\theta^1}; \beta^1) \qquad (\lambda_3) \tag{9}$$

$$v^{21}(x_A^{21}, x_B^{21}, x_C^{21}, \tfrac{Y^{21}}{\theta^2}; \beta^1) \geq \widetilde{v}^{21}(x_A^{22}, x_B^{22}, x_C^{22}, \tfrac{Y^{22}}{\theta^2}; \beta^1) \qquad (\lambda_4) \tag{10}$$

where (7), (8), (9) and (10) are the self-selection constraints. Notice that, in this case, the incentive constraints might bind in both directions, irrespective from the weights in the social welfare function. Incentive compatibility requires that, in the optimal allocation, and for any i, (a) an agent with exponential discounting — i.e., $j = 2$ — weakly prefers the bundle $(x_A^{i2}, x_B^{i2}, x_C^{i2}, Y^{i2})$ designed for him to that intended for an agent with behavioral type β^1 and the same productivity level; (b) an agent with preferences for immediate gratification — i.e., $j = 1$ — weakly prefers the bundle $(x_A^{i1}, x_B^{i1}, x_C^{i1}, Y^{i1})$ designed for him to that intended for an exponential agent.

Depending on the pattern of the binding constraints, solving the maximization problem yields the following results.

Proposition 2 *With observable heterogeneity on productivity levels and asymmetric information on preferences for immediate gratification,*

(i.) if $\lambda_3 = \lambda_4 = 0$, optimal non-linear commodity taxes are positive for low types — i.e., (θ^i, β^1) — and zero for high types (θ^i, β^2). Moreover, $(T_A^{'})_{i1} < (T_B^{'})_{i1}$;

(ii.) if $\lambda_1 = \lambda_2 = 0$, optimal non-linear commodity taxes are positive for high types — i.e., (θ^i, β^2) — and zero for low types (θ^i, β^1). Moreover, $(T_A^{'})_{i2} > (T_B^{'})_{i2}$.

The Proposition establishes that, when preferences for immediate gratification are agents' private information, the optimal tax structure involves positive and different marginal taxes on consumption levels. This is because the separability assumption can not be invoked to establish the Atkinson-Stiglitz theorem. In fact, although the mimicker and mimicked should purchase the same bundle, their marginal rates of substitution do not coincide when the former consumes the bundle intended for the latter, given that future health costs are discounted at different rates. However, the identity of the agent whom choices are distorted with respect to the first-best, as well as the structure of optimal non-uniform taxes, depend on the pattern of binding incentive constraints or, in other words, on the direction of redistribution. In particular, part (*i.*) of the Proposition studies the case of redistribution going from time consistent to time inconsistent agents, while case (*ii.*) studies the opposite case.

When the binding incentive constraints are the downward ones (from exponential to time inconsistent — part (*i.*)) the mimicker's marginal rate of substitution between good A and good k, calculated in correspondence of the bundle of the mimicked, x_A^{i1}, is lower than the marginal rate of substitution of the mimicked in the same point. In fact, a type $(i, 2)$, in order to mimic an addicted type $(i, 1)$, should *increase* his consumption of good A relative to a truth-teller with the same behavioral type. But, being his health costs discounted by the factor $\beta^2 > \beta^1$, it follows that his marginal utility of consuming x_A^{i1} is lower than the marginal utility of the mimicked agent. Then, standard results in incentive theory should apply: choices of high types remains the same of the first-best, while the choices of the the low types need to be distorted, in order to make them less attractive for a (potential) mimicker. This objective can be achieved with an appropriate design of taxes on type $i1'$s consumption levels. In particular, the optimal marginal tax on the addictive good A should be lower than the marginal tax on the non-addictive good B. This is because the rational mimicker who pretends to be an addicted agent, in order to increase his consumption of A, has to reduce his consumption of B: hence, by taxing relatively more the good the mimicker likes the most, the planner imposes on him a high cost of mimicking. In this way, the tax authority can induce truth-telling by reducing, in the allocation of the mimicked, the consumption of the good that a non-addicted agent prefers (relatively) more. By doing so, the incentive constraint becomes slack, information rents of the high type are reduced, and resources available for redistribution are increased.

On the other hand, when the binding incentive constraints are those upward (from time inconsistent to exponential — part (*ii.*)), consumption choices of a type $(i, 1)$ are not distorted relative to the first-best allocation, while those of a type $(i, 2)$ are. To understand why, notice that a mimicker of type $(i, 1)$ should *decrease* his consumption of A in order to mimic an agent of type $(i, 2)$. Hence, giving that the short-term discount factor of the mimicker is lower than that of the mimicked, his marginal utility of consuming A at x_A^{i2} is higher compared to a truth-teller. In this case, the tax on the addictive good for the mimicked agent should be higher than the tax levied on the non-addictive goods: to hurt the mimicker more than the mimicked, the tax should be higher on the good that he prefers relatively more — i.e., good A.

4 Optimal Liner Taxation

When consumption levels are no longer observable, but anonymous transactions are, the tax authority can only tax consumption of A and B in a linear way (the numeraire C is assumed to be untaxed). Define t_A and t_B such taxes. Before tax income $Y^{ij} = \theta^i L^{ij}$ is observable, but neither θ^i nor L^{ij} are. Define consumers' prices $q_k = 1 + t_k$, for $k = A, B$, and recall that I^{ij} represents disposable income of an agent with productivity level i and behavioral type j.

Since the distribution of types is discrete, the income tax schedule is given by the points (I^{ij}, T^{ij}), which represent individuals' choices. Consider, first, consumers' problem: an individual of type (i, j) solves

$$\max_{\{x_k^{ij}\}, Y^{ij}} \quad u(x_A^{ij}, x_B^{ij}, x_C^{ij}) - l\left(\frac{Y^{ij}}{\theta^i}\right) - \beta^j c(x_A^{ij}),$$

subject to the budget constraint

$$I^{ij} \equiv Y^{ij} - T^{ij}(.) \geq q_A x_A^{ij} + q_B x_B^{ij} + x_C^{ij}. \tag{11}$$

Standard conditions for utility maximization imply, in an interior solution,

$$MRS_{A,B}^{ij} \equiv \frac{\partial v^{ij}/\partial x_A}{\partial v^{ij}/\partial x_B} = \frac{q_A}{q_B} \tag{12}$$

which, together with (11), defines consumers' demand functions for A and B, $x_A^{ij} = x_A^{ij}(q_A, q_B, I^{ij}, \beta^j)$ and $x_B^{ij} = x_B^{ij}(q_A, q_B, I^{ij}, \beta^j)$. Let

$$V^{ij}(q_A, q_B, I^{ij}; \beta^j) \equiv \max_{\{x_k^{ij}\}} \left\{ u(x_A^{ij}, x_B^{ij}, x_C^{ij}) \middle| \, I^{ij} = q_A x_A^{ij} + q_B x_B^{ij} + x_C^{ij} \right\}$$

be the indirect utility function of an agent of type (i, j) associated only with the sub-utility function $u(.)$.

Condition (12) implies that, when $q_A = q_B$ — i.e., uniform commodity taxation — agents face the first-best trade-offs, $MRS_{A,B}^{ij} = 1$. Hence, differential linear commodity taxation is needed if and only if a Pareto constrained efficient allocation implies that $MRS_{A,B}^{ij} \neq 1, \forall i, j$.

4.1 Productivity is Private Information

Assume, like in subsection 3.1, that productivity levels and labor supply are agents' private information, while agents' preferences for immediate gratification — i.e., the parameter β^j — is observed by the tax authority. Hence, the population can be divided into two identifiable behavioral types: type $i1$ is characterized by $\{(\theta^1, \beta^1), (\theta^2, \beta^1)\}$ and type $i2$ is characterized by $\{(\theta^1, \beta^2), (\theta^2, \beta^2)\}$.

The government chooses implicitly the optimal tax function $T^{ij}(.)$ by choosing before-tax income Y^{ij}, and choose directly the two tax rates by choosing q_A and q_B. The planner's problem can be stated as

$$\max_{\{I^{ij}\}_{ij}, Y^{ij}, q_A, q_B} \quad \sum_{i,j} \alpha^{ij} \left[V^{ij}(q_A, q_B, I^{ij}; \beta^j) - l\left(\frac{Y^{ij}}{\theta^i}\right) \right]$$

subject to

$$\sum_{i,j} \pi^{ij} (Y^i - I^{ij} + (q_A - 1) x_A^{ij}(q_A, q_B, I^{ij}, \beta^j) + (q_B - 1) x_B^{ij}(q_A, q_B, I^{ij}, \beta^j)) = \bar{R}, \qquad (\mu) \tag{13}$$

and

$$V^{21}(q_A, q_B, I^{21}; \beta^1) - l\left(\frac{Y^{21}}{\theta^2}\right) \geq \widetilde{V}^{21}(q_A, q_B, I^{11}; \beta^1) - l\left(\frac{Y^{11}}{\theta^2}\right), \qquad (\lambda_1) \tag{14}$$

$$V^{22}(q_A, q_B, I^{22}; \beta^2) - l\left(\frac{Y^{22}}{\theta^2}\right) \geq \widetilde{V}^{22}(q_A, q_B, I^{12}; \beta^2) - l\left(\frac{Y^{12}}{\theta^2}\right), \qquad (\lambda_2) \tag{15}$$

where (13) is the government's budget constraint and (14) and (15) are the self-selection constraints, which ensure that, for any observable j, the utility of a type $(2, j)$ at $(I^{2j}, T^{2j}(.))$ — i.e., V^{2j} — should be at least equal to the utility obtained by choosing $(I^{1j}, T^{1j}(.))$ — i.e., $\widetilde{V}^{2j}$. Again, the assumption $\alpha^{1j} > \alpha^{2j}$ guarantees that the binding incentive constraints are those from high productive individuals to low productive individuals with the same behavioral type j.

Solving the maximization problem yields the following system of first-order conditions, which defines optimal taxes t_A and t_B

$$\sum_{i,j} \pi^{ij} \left(\sum_{k=A,B} (q_k - 1) \frac{\partial \widehat{x}_k^{ij}}{\partial q_A} \right) = \sum_j \frac{\lambda_j}{\mu} (x_A^{1j} - \widetilde{x}_A^{2j}) \frac{\partial \widetilde{V}^{2j}}{\partial I^{1j}} = 0, \tag{16}$$

$$\sum_{i,j} \pi^{ij} \left(\sum_{k=A,B} (q_k - 1) \frac{\partial \widehat{x}_k^{ij}}{\partial q_B} \right) = \sum_j \frac{\lambda_j}{\mu} (x_B^{1j} - \widetilde{x}_B^{2j}) \frac{\partial \widetilde{V}^{2j}}{\partial I^{1j}} = 0, \tag{17}$$

where $\widehat{x}_k^{ij}$ represents compensated demands[10] of good $k = A, B$, and $\widetilde{x}_k^{2j}$ denotes the consumption level of k of an individual with productivity θ^2 and behavioral type j who is mimicking an agent with productivity θ^1 and the same behavioral type.

Proposition 3 *With observable heterogeneity on productivity levels and asymmetric information on preferences for immediate gratification, optimal linear commodity taxes are zero — i.e.,* $q_A = q_B = 1$.

The Proposition shows that, also in the more realistic case of unobservable consumption levels and linear commodity taxation, the results of Proposition 1 remain valid — i.e., uniform commodity taxation is optimal. This is not surprising: if a non-linear commodity tax can not improve screening of types, linear taxes can not do better. To understand why conditions (16) and (17) are equal to zero, notice that a fully rational agent with high productivity who is mimicking a poor agent with the same behavioral type has not only the same disposable income I^{1j} (although different labor supplies) but also, because of separability, the same consumption levels of A and B. In other words, $x_A^{1j} - \widetilde{x}_A^{2j} = 0$ and $x_B^{1j} - \widetilde{x}_B^{2j} = 0$ must hold in the optimal second-best allocation. Hence, given that the left-hand sides of (16) and (17) are zero if only if both q_A and q_B are equal to one, it follows that $t_A = t_B = 0$.

To sum up, when differences in agents' degree of rationality are observable (or trivially, when all agents have the same short-term discount factor), efficiency requires that addictive goods should not be taxed, neither linearly nor non-linearly. Hence, with this informational structure, only paternalistic considerations, as in O'Donoghue and Rabin (2009), justify the existence of sin taxes.

[10] The compensation takes the form of disposable income.

4.2 Preferences are Private Information

When the behavioral type is agents' private information, but productivity is observable, the planner solves

$$\max_{\{I^{ij}\}_{ij}, Y^{ij}, q_A, q_B} \quad \sum_{i,j} \alpha^{ij} \left[V^{ij}(q_A, q_B, I^{ij}; \beta^j) - l\left(\frac{Y^{ij}}{\theta^i}\right)\right],$$

subject to the budget constraint (13) and

$$V^{12}(q_A, q_B, I^{12}; \beta^2) - l\left(\tfrac{Y^{12}}{\theta^1}\right) - \beta^2 c(x_A^{12}) \geq \widetilde{V}^{12}(q_A, q_B, I^{11}; \beta^2) - l\left(\tfrac{Y^{11}}{\theta^1}\right) - \beta^2 c(x_A^{11}), \qquad (\lambda_{12}) \qquad (18)$$

$$V^{22}(q_A, q_B, I^{22}; \beta^2) - l\left(\tfrac{Y^{22}}{\theta^2}\right) - \beta^2 c(x_A^{22}) \geq \widetilde{V}^{22}(q_A, q_B, I^{21}; \beta^2) - l\left(\tfrac{Y^{21}}{\theta^2}\right) - \beta^2 c(x_A^{21}), \qquad (\lambda_{22}) \qquad (19)$$

$$V^{11}(q_A, q_B, I^{11}; \beta^1) - l\left(\tfrac{Y^{11}}{\theta^1}\right) - \beta^1 c(x_A^{11}) \geq \widetilde{V}^{11}(q_A, q_B, I^{12}; \beta^1) - l\left(\tfrac{Y^{12}}{\theta^1}\right) - \beta^1 c(x_A^{12}), \qquad (\lambda_{11}) \qquad (20)$$

$$V^{21}(q_A, q_B, I^{21}; \beta^1) - l\left(\tfrac{Y^{21}}{\theta^2}\right) - \beta^j c(x_A^{21}) \geq \widetilde{V}^{21}(q_A, q_B, I^{22}; \beta^1) - l\left(\tfrac{Y^{22}}{\theta^2}\right) - \beta^1 c(x_A^{22}), \qquad (\lambda_{21}) \qquad (21)$$

where (18), (19), (20) and (21) are the self-selection constraints, which have an intuitive interpretation.

The maximization problem yields the following system of first-order conditions

$$\sum_{i,j} \pi^{ij} \left(\sum_{k=A,B} (q_k - 1) \frac{\partial \widehat{x}_k^{ij}}{\partial q_A} \right) = \sum_{i,j} \frac{\lambda_{ij}}{\mu} (x_A^{ij} - \widetilde{x}_A^{ij}) \frac{\partial \widetilde{V}^{ij}}{\partial I}, \qquad (22)$$

$$\sum_{i,j} \pi^{ij} \left(\sum_{k=A,B} (q_k - 1) \frac{\partial \widehat{x}_k^{ij}}{\partial q_B} \right) = \sum_{i,j} \frac{\lambda_{ij}}{\mu} (x_B^{ij} - \widetilde{x}_B^{ij}) \frac{\partial \widetilde{V}^{2j}}{\partial I}. \qquad (23)$$

Proposition 4 *With observable heterogeneity on productivity levels and asymmetric information on preferences for immediate gratification, differential commodity taxation is always optimal. Moreover,*

(i) when $\lambda_{11} = \lambda_{21} = 0$, $t_A < t_B$;

(ii) when $\lambda_{12} = \lambda_{22} = 0$, $t_A > t_B$.

The Proposition shows that introducing asymmetric information of agents' degree of rationality has three implications for the design of the optimal system of linear commodity taxes: first, differential taxation is optimal; second, optimal sin taxes are different from zero, meaning that the separability assumption does not ensure anymore that the mimicker and the mimicked are consuming the same amount of A and B. Third, the structure of these taxes — i.e., which good should be taxed more — ultimately depends on the interplay between screening and redistributive considerations or, in other words, on the pattern of binding incentive constraints.

When the binding incentive constraints are the downward ones (point $(i.)$), the mimicker is an exponential agent with disposable income I^{i2}, aware of the costs implied by addiction, who consumes less good A (and more B) relative to an agent with the same disposable income but with preferences for immediate gratification. Hence, when he claims to be a behavioral type β^1, he receives disposable income I^{i1}, but his preferences are such that his (unobservable) consumption levels — $\widetilde{x}_A^{2j}$ and $\widetilde{x}_B^{2j}$ — are, respectively, lower and higher than the consumption levels of a true β^1 type. It follows that the term on the right-hand side of (22) is positive, while that on the right-hand side of (23) is negative; hence, the left-hand sides imply

$$t_A \frac{\partial \widehat{x}_A^{ij}}{\partial q_A} + t_B \frac{\partial \widehat{x}_B^{ij}}{\partial q_A} \quad > \quad 0, \tag{24}$$

$$t_A \frac{\partial \widehat{x}_A^{ij}}{\partial q_B} + t_B \frac{\partial \widehat{x}_B^{ij}}{\partial q_B} \quad < \quad 0. \tag{25}$$

Using the negativity of own price effects, and the fact that good A and B are net substitutes, it is immediate to see that optimal taxes must satisfy $t_B > t_A$. Furthermore, notice that the left-hand sides of (24) and (25) represent the reduction in compensated demands due to the introduction of taxes on A and B. Hence, the reduction must be higher for the good that the mimicker likes the most. By taxing relatively good B, the government relaxes the binding incentive constraints (18) and (19), and increases resources for redistribution.

To see it, define two new quantities: $\underline{x}_A$, the extra consumption of good A of an addicted individual compared to fully rational agents with the same productivity, and $\bar{x}_A$, the consumption of the addictive good net of the component due to preferences for immediate gratification. Hence, for given i,j

$$\bar{x}_A = x_A^{ij} - \underline{x}_A. \tag{26}$$

Being the component $\bar{x}_A$ common to the mimicker and the mimicked with the same i, the sub-utility $V(.)$ is the same for addicted and fully rational agents. Evidently, $\underline{x}_A = 0$ for the latter type. Moreover, using (26), and assuming that commodity taxes are zero, the budget constraint can be rewritten as

$$I^{ij} - \underline{x}_A = x_B^{ij} - \bar{x}_A.$$

Figure 2 illustrates the budget constraint of a type $i2$ mimicking a type $i1$ and the constraint for a true addicted agent, as well as the utility-maximizing choices of x_B^{ij} and $\bar{x}_A$ of the mimicker and the mimicked. From the graph, it is immediate to see that $\widetilde{\bar{x}}_A - \bar{x}_A < \underline{x}_A$ (where $\widetilde{\bar{x}}_A$ is the mimicker's consumption level of good A); hence,

$$\widetilde{x}_A^{i2} - x_A^{i1} + \underline{x}_A < \underline{x}_A \Longrightarrow \widetilde{x}_A^{i2} < x_A^{i1}, \tag{27}$$

meaning that the fully rational mimicker purchases less A relative to the mimicked, and more good B.[11]

Therefore, to show that taxing more good B reduces the incentive to misreport their behavioral type, imagine that, starting from a zero-tax situation, the planner decides to increase the tax on B by Δt_B; the utility level of type ij agents would decrease[12] by x_B^{ij}. However, if the tax authority adjusts, at the same time, the income tax schedule such that $\Delta I^{ij} = -\Delta t_B x_B^{ij}$, the utility levels of type $i1$ and $i2$ agents and the government's budget constraint will remain unchanged. But for a mimicker of type $i2$ who declares a disposable income I^{i1}, these changes reduce his total utility, because the increase of disposable income

[11] From Figure 2, it is immediate to see that the opposite relationship holds for good B — i.e., $\widetilde{x}_B^{i1} > x_B^{i1}$.

[12] Using $V^{ij}(q_A, q_B, I^{ij}; \beta^j)$, the envelope theorem ensures that $\partial V^{ij}/\partial q_B = -x_B^{ij}$.

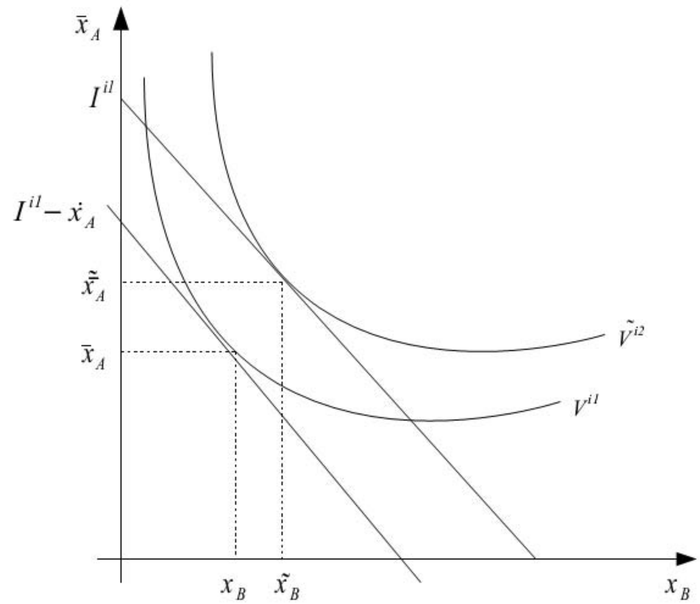

Figure 2: Mimicker and mimicked choices: case $\lambda_{11} = \lambda_{21} = 0$.

which follows the introduction of t_B is not enough to compensate the new tax, since his preferences for good B are different from a true $i1$ type. Therefore, taxing good B is a way to reduce the incentives of a perfectly rational agent to mimic an addicted agent with the same i.

The intuitions and the logic are easily reversed in the opposite case — i.e., when the binding incentive constraints are the upward ones (point (*ii.*)). The mimicker is now an addicted agent with disposable income I^{i1} who pretend to be a fully rational agent with disposable income I^{i2} who prefer to consume good A instead of B; hence, when he claims to be a behavioral type β^2, his (unobservable) consumption levels $\widetilde{x}_A^{1j}$ and $\widetilde{x}_B^{1j}$ are, respectively, higher and lower than those of a true β^2 type. From conditions (22) and (23),

$$\begin{aligned} t_A \frac{\partial \widehat{x}_A^{ij}}{\partial q_A} + t_B \frac{\partial \widehat{x}_B^{ij}}{\partial q_A} &< 0, \\ t_A \frac{\partial \widehat{x}_A^{ij}}{\partial q_B} + t_B \frac{\partial \widehat{x}_B^{ij}}{\partial q_B} &> 0, \end{aligned}$$

which imply that $t_A > t_B$. The good that the mimicker likes relatively more is now good A: hence, the tax on this good must be higher than that on B.

Besides the pattern of pattern of binding incentive constraints, conditions (22) and (23) show that optimal commodity taxes depend also on other economic quantities. First, the tax on good k is negatively related to its own compensated elasticity of demand with respect to q_k, provided that compensated cross-price effects are small. Like in the standard Ramsey model of commodity taxation, in order to minimize distortions on consumers' choices, goods with high elasticity with respect to price should be taxed less heavily, other things being equal.

Second, taxes are related to the term $\left|x_k^{ij} - \widetilde{x}_k^{ij}\right|$ — i.e., mimicker's difference in demand of good k compared to the mimicked agent. Being this difference increasing with $\beta^2 - \beta^1$ — a measure of the degree of bounded rationality of type $i, 1$ agents — our results implies that the more severe the self-control problem is, the higher the tax on the good preferred by mimickers should be. However, recalling part $(i.)$ of Proposition 4, more "irrationality" does not necessarily mean that the addictive good should the one with the higher tax rate.

In conclusion, the optimality of differential commodity taxation arises only because agents' degree of rationality is private information, and not because the planner is trying to overcome agents' limited rationality.

5 Optimal Taxes with Multidimensional Asymmetric Information

This section characterizes the optimal tax structure when both productivity and the behavioral type are taxpayers' private information. In the following, we focus on the more realistic case that individuals' labor supply and consumption levels are unobservable. However, anonymous transactions and gross income can be observed by the tax authority. Hence, a non-linear tax on labor income and linear commodity taxes can be levied. To save notation, the four types are denoted now with $m = 1, 2, 3, 4$. In particular,

- Type 1 is characterized by (θ^1, β^1);
- Type 2 is characterized by (θ^1, β^2);
- Type 3 is characterized by (θ^2, β^1);
- Type 4 is characterized by (θ^2, β^2).

Individuals' problem parallels that of section 4, except that demand functions for good A and B of type m are denoted $x_A^m = x_A^m(q_A, q_B, I^m, \beta^m)$ and $x_B^m = x_B^m(q_A, q_B, I^m, \beta^m)$.

The government chooses indirectly the income tax function $T(Y^m)$ by choosing before-tax income Y^m, and chooses directly the two tax rates by choosing post-tax prices q_A and q_B. Hence, his problem $\wp$ can be stated as

$$\max_{\{I^m\}, Y^m, q_A, q_B} \quad \sum_m \alpha_m \left[V^m(q_A, q_B, I^m; \beta^m) - l\left(\frac{Y^m}{\theta^m}\right)\right],$$

subject to

$$\sum_m \pi_m (Y^m - I^m + (q_A - 1) x_A^m(q_A, q_B, I^m, \beta^m) + (q_B - 1) x_B^m(q_A, q_B, I^m, \beta^m)) = \bar{R} \qquad (\mu) \qquad (28)$$

$$V^m(q_A, q_B, I^m; \beta^m) - l\left(\frac{Y^m}{\theta^m}\right) \geq \widetilde{V}^{mr}(q_A, q_B, I^r; \beta^m) - l\left(\frac{Y^r}{\theta^m}\right) \qquad (\lambda_{mr}) \tag{29}$$

where (28) is government's budget constraint and (29) are the incentive constraints: being types private information, the utility obtained by each individual m (the left-hand side of (29)) should at least be equal to the utility level he would obtain by mimicking an individual of type $r \neq m$ (the right-hand side of (29)). Denoting by μ and λ_{mr} the Lagrange multipliers associated to these constraints, problem $\wp$ yields the following Proposition.

Proposition 5 *With unobservable heterogeneity on productivity levels and preferences for immediate gratification, optimal linear commodity taxes are characterized by*

$$\sum_m \alpha_m \left(\sum_{k=A,B} (q_k - 1)\frac{\partial \widehat{x}_k^m}{\partial q_A}\right) = \sum_{m,r} \frac{\lambda_{mr}}{\mu}(x_A^m - \widetilde{x}_A^{mr})\frac{\partial \widetilde{V}^{mr}(q_A, q_B, I^r; \beta^m)}{\partial I}, \tag{30}$$

$$\sum_m \alpha_m \left(\sum_{k=A,B} (q_k - 1)\frac{\partial \widehat{x}_k^m}{\partial q_B}\right) = \sum_{m,r} \frac{\lambda_{mr}}{\mu}(x_B^m - \widetilde{x}_B^{mr})\frac{\partial \widetilde{V}^{mr}(q_A, q_B, I^r; \beta^m)}{\partial I}, \tag{31}$$

for $m, r = 1, 2, 3, 4$; $r \neq m$, and where $\widehat{x}_k^m$ denotes type m's compensated demand of good k while $\widetilde{x}_k^{mr}$ indicates consumption of good k of an agent of type m who is mimicking a type r.

The first step is to determine when the left-hand sides of conditions (30) and (31) are zero, implying that both q_A and q_B are equal to one — i.e., uniform commodity taxation. This happens if, for all binding incentive constraints — i.e., for all m and r such that $\lambda_{mr} > 0$ — the demand function for good $k = A, B$ of the mimicker and the mimicked types coincide. Recalling the analysis of Section 4, this happens only when there is private information on productivity levels but not on preferences — i.e., only when the relevant incentive constraints are those from high productive to the low productive agents with the same behavioral type.[13] Hence, in this case, one obtains an extension of the Atkinson-Stiglitz theorem.

Corollary 1 *If, for all m and r, at the solution of the problem $\wp$ we have that $\lambda_{mr}(x_A^{mr} - \widetilde{x}_A^{mr}) = 0$ and $\lambda_{mr}(x_B^{mr} - \widetilde{x}_B^{mr}) = 0$, optimal taxes are zero: $t_A = t_B = 0$.*

The result illustrated in the Corollary is expected: recall that a commodity tax, in this framework, is useful when it helps the tax authority to discriminate between types, and to relax an otherwise binding incentive constraint. However, with basically one source of asymmetric information and separability between utility from consumption and disutility of effort, disposable income and consumption levels of a type m mimicking a type r are exactly the same. Therefore, commodity taxes do not represent a useful screening device for the tax authority, which raises resources only through the income tax.

For any other pattern of binding incentive constraints, however, consumption levels x_k^{mr} and $\widetilde{x}_k^{mr}$ do not necessarily coincide, and differential commodity taxation becomes optimal. However, the multidimensional

[13] This condition is also trivially satisfied when all agents in the population have the same behavioral type.

case does not represent a simple extension of the single-screening models presented above. The reason is that it is not clear which are the relevant incentive constraints. Moreover, the double heterogeneity introduces the possibility of "double" mimicking, which was not possible in the previous sections. Agents with high productivity and perfect foresight may want to mimic agents with low productivity and preferences for immediate gratification: in this case, the optimal tax structure should take into account also the nature of the addictive good — i.e., the tax on A should be related to the sign of income elasticity, like in the standard (many households) Ramsey model. For instance, if A is inferior, then the difference $x_A^1 - \widetilde{x}_A^{41}$ is positive, implying that a rich time consistent agent consumes less addictive good compared to a poor agents with bounded rationality. Hence, in this case, results and intuitions derived in previous sections can be easily extended: good B should be taxed more than good A.[14] However, if A is a normal good, then it could be possible that the mimicker, despite his perfect rationality, is consuming more addictive good than the mimicked, thus changing the structure of the optimal tax. in this case, good A should be taxed more than B. Moreover, from (30) and (31), it follows that, when cross-price effects are small, taxes on A and B are inversely proportional to their compensated elasticity of demand with respect to prices, as in the single heterogeneity case.

Although these complications, three clear results emerge from the multidimensional analysis: first, addictive goods need to be taxed; second, the more mimickers consume the good, the higher the optimal tax should be; third, the necessity of commodity taxes arises only because of the double asymmetry of information, and not because of a paternalistic motive (either *strong* or *libertarian*) of the planner, which is fully respecting consumers' freedom of consuming the desired amount of the addictive good.

6 Conclusions

This paper has examined a problem of optimal taxation of addictive goods when consumers are heterogenous in their productivity and degree of rationality, measured as the preferences for the immediate pleasure that the consumption of an addictive good creates. When these characteristics are unobservable by the tax authority, this paper has shown that, contrary to previous works on this topic (O'Donoghue and Rabin, 2009), taxes on addictive goods (sin taxes) can be justified also when the planner's objective is not the desire to protect individuals against their irrationality (*paternalism*), but only income redistribution. Hence, the existence of sin taxes do not necessarily represent an intrusion on agents' preferences and freedom of choice (as pointed out by Saint-Paul, 2011), but only an instrument that helps the tax authority to screen agents when there are multiple sources of asymmetric information.

To make this point, the paper has presented a version of the Atkinson and Stiglitz (1976) model of optimal direct/indirect taxation. It has shown, first, that observable heterogeneity in agents' level of rationality does not change the Atkinson-Stiglitz result: consumption of the addictive and the non-addictive good should

[14] The opposite holds when low-productive time-inconsistent agents mimic high-produtive exponential.

go untaxed, and redistribution occurs only via the income tax. However, it has also shown that differential commodity taxation becomes optimal when agents have private information on their degree of rationality, even if their productivity is observable. This is because the separability assumption can not be invoked; in fact, irrespective on the pattern of binding incentive constraints, different levels of rationality imply that the mimicker and the mimicked are not consuming the same amount of the addictive and the non-addictive good. Hence, the structure of commodity taxes should depend on the interplay between screening and redistributive considerations. More precisely, in the incentive compatible second-best allocation, the consumption level of the good that a potential mimicker likes the most should be distorted downward. Hence, if redistribution goes from agents with perfect rationality to individuals with limited rationality, this result implies that the non-addictive good should be taxed more than the addictive good.

When the two sources of asymmetric information are considered at the same time, the results of the single-heterogeneity case can not easily be extended, because it is not clear which incentive constrains are binding. However, two results emerge from the multidimensional analysis: first, addictive goods need to be taxed; second, the more mimickers consume the addictive good, the higher the tax must be.

This analysis suggests that policy makers should not justify their decision of imposing sin taxes on the basis of their better responsibility or their superior information set, but only on the desire of maximizing efficiency and increase redistribution of income.

A Appendix

Proof of Proposition 1

Denoting with μ, λ_1 and λ_2 the Lagrange multipliers associated to constraints (4), (5) and (6), the Langrangean of the problem is

$$\begin{aligned}\Lambda &= \sum_{i,j}\alpha^{ij}\left(u^{ij}(x_A^{ij},x_B^{ij},x_C^{ij},)-l\left(\frac{Y^{ij}}{\theta^i}\right)-\beta^j c(x_A^{ij})\right)+\lambda_1\left(\sum_{i,j}\pi^{ij}(Y^i-x_A^{ij}-x_B^{ij}-x_C^{ij})-\bar{R}\right)\\ &\quad+\lambda_1\left(u^{21}(x_A^{21},x_B^{21},x_C^{21},)-l\left(\frac{Y^{21}}{\theta^2}\right)-\beta^1 c^{21}(x_A^{21})-\widetilde{u}^{21}(x_A^{11},x_B^{11},x_C^{11},)-\widetilde{l}\left(\frac{Y^{11}}{\theta^2}\right)+\beta^1\widetilde{c}^{21}(x_A^{11})\right)\\ &\quad+\lambda_2\left(u^{22}(x_A^{22},x_B^{22},x_C^{22},)-l\left(\frac{Y^{22}}{\theta^2}\right)-\beta^2 c^{22}(x_A^{22})-\widetilde{u}^{22}(x_A^{12},x_B^{12},x_C^{12},)-\widetilde{l}\left(\frac{Y^{12}}{\theta^2}\right)+\beta^2\widetilde{c}^{22}(x_A^{12})\right)\end{aligned}$$

where the expressions $v^{ij}(x_A^{ij},x_B^{ij},x_C^{ij},\frac{Y^i}{\theta^i};\beta^j)$ have been made explicit.

The problem yields the following first-order conditions with respect to consumption levels, for $k=B,C$ and $j=1,2$

$$\begin{aligned} x_A^{2j}: &\quad \alpha^{2j}\left(\frac{\partial u^{2j}}{\partial x_A}-\beta^j\frac{\partial c^{2j}}{\partial x_A}\right)-\pi^{2j}\mu+\lambda_j\left(\frac{\partial u^{2j}}{\partial x_A}-\beta^2\frac{\partial c^{2j}}{\partial x_A}\right)=0,\\ x_A^{1j}: &\quad \alpha^{1j}\left(\frac{\partial u^{1j}}{\partial x_A}-\beta^j\frac{\partial c^{1j}}{\partial x_A}\right)-\pi^{1j}\mu-\lambda_j\left(\frac{\partial \widetilde{u}^{1j}}{\partial x_A}-\beta^j\frac{\partial \widetilde{c}^{1j}}{\partial x_A}\right)=0,\\ x_k^{2j}: &\quad \alpha^{2j}\frac{\partial u^{2j}}{\partial x_k}-\pi^{2j}\mu+\lambda_j\frac{\partial u^{2j}}{\partial x_k}=0,\\ x_k^{1j}: &\quad \alpha^{1j}\frac{\partial u^{1j}}{\partial x_k}-\pi^{1j}\mu-\lambda_j\frac{\partial \widetilde{u}^{2j}}{\partial x_k}=0.\end{aligned}$$

Rearranging these expressions, marginal rate of substitutions of individuals of type (θ^2,β^j) are

$$\begin{aligned} MRS^{2j}_{x_A^{2j},x_k^{2j}} &\equiv \frac{\frac{\partial u^{2j}}{\partial x_A}-\beta^j\frac{\partial c^{2j}}{\partial x_A}}{\frac{\partial u^{2j}}{\partial x_k}}=1,\\ MRS^{2j}_{x_B^{2j},x_C^{2j}} &\equiv \frac{\frac{\partial u^{2j}}{\partial x_B}}{\frac{\partial u^{2j}}{\partial x_C}}=1,\end{aligned}$$

implying that $(T_A^{'})_{2j}=(T_k^{'})_{2j}$, $\forall k$, and $(T_B^{'})_{2j}=(T_C^{'})_{2j}$ $\forall j$ — i.e., uniform commodity taxation is optimal for individuals of type (θ^2,β^j). For individuals of type (θ^1,β^j), marginal rate of substitutions are

$$MRS^{1j}_{x_A^{1j},x_k^{1j}} \equiv \frac{\frac{\partial u^{1j}}{\partial x_A}-\beta^j\frac{\partial c^{1j}}{\partial x_A}}{\frac{\partial u^{1j}}{\partial x_k}}=\frac{\alpha^{ij}\mu+\lambda_j\left(\frac{\partial \widetilde{u}^{2j}}{\partial x_A}-\beta^j\frac{\partial \widetilde{c}^{2j}}{\partial x_A}\right)}{\alpha^{1j}\mu+\lambda_j\frac{\partial \widetilde{u}^{2j}}{\partial x_k}}, \tag{32}$$

$$MRS^{1j}_{x_B^{1j},x_C^{1j}} \equiv \frac{\frac{\partial u^{1j}}{\partial x_B}}{\frac{\partial u^{1j}}{\partial x_C}}=\frac{\alpha^{1j}\mu+\lambda_j\frac{\partial u^{1j}}{\partial x_B}}{\alpha^{1j}\mu+\lambda_j\frac{\partial \widetilde{u}^{2j}}{\partial x_C}}. \tag{33}$$

Define

$$\widetilde{MRS}^{2j}_{x_A^{1j},x_k^{1j}} \equiv \frac{\frac{\partial \widetilde{u}^{2j}}{\partial x_A}-\beta^j\frac{\partial \widetilde{c}^{2j}}{\partial x_A}}{\frac{\partial \widetilde{u}^{2j}}{\partial x_k}} \tag{34}$$

as the marginal rate of substitution between good A and good k of individuals of type (θ^2, β^j) mimicking individuals of type (θ^1, β^j), for given j. Moreover, let

$$\widetilde{MRS}^{2j}_{x_B^{1j}, x_C^{1j}} \equiv \frac{\frac{\partial u^{1j}}{\partial x_B}}{\frac{\partial \widetilde{u}^{2j}}{\partial x_C}}$$

be the marginal rate of substitution between goods B and C of individuals of type (θ^2, β^j) mimicking individuals of type (θ^1, β^j), for given j. Remember that a mimicker should consume the same bundle of the individuals he is mimicking. Hence, (32) and (33) can be rewritten as

$$\begin{aligned} MRS^{1j}_{x_A^{1j}, x_k^{1j}} &\equiv \frac{\phi + \widetilde{MRS}^{2j}_{x_A^{1j}, x_k^{1j}}}{\phi + 1}, \\ MRS^{i1}_{x_B^{i1}, x_C^{i1}} &= \frac{\varphi + \widetilde{MRS}^{i2}_{x_B^{i1}, x_C^{i1}}}{\varphi + 1} \end{aligned}$$

where $\phi = \frac{\alpha^{1j}\mu}{\lambda_j \frac{\partial \widetilde{u}^{2j}}{\partial x_k}}$ and $\varphi = \frac{\alpha^{i1}\mu}{\lambda_i \frac{\partial \widetilde{u}^{i2}}{\partial x_C}}$ are constant terms. Notice that the mimicker and mimicked agents have the same disposable income, I^{1j}, $j = 1, 2$, but different labor supplies. However, because of separability, demand functions of the three goods are the same. Hence, $\widetilde{MRS}^{2j}_{x_A^{1j}, x_k^{1j}} = MRS^{1j}_{x_A^{1j}, x_k^{1j}}$, $\widetilde{MRS}^{i2}_{x_B^{i1}, x_C^{i1}} = MRS^{i1}_{x_B^{i1}, x_C^{i1}}$, and

$$\begin{aligned} MRS^{1j}_{x_A^{1j}, x_k^{1j}} &= 1, \\ MRS^{1j}_{x_B^{1j}, x_C^{1j}} &= 1, \end{aligned}$$

implying that, also for individuals of type (θ^1, β^j), conditions $(T'_A)_{1j} = (T'_k)_{1j}$ and $(T'_B)_{1j} = (T'_C)_{1j}$ have to be satisfied $\forall k, j$. ■

Proof of Proposition 2

The Langrangean of the problem is

$$\begin{aligned} \Lambda = &\sum_{i,j} \alpha^{ij} \left(u^{ij}(x_A^{ij}, x_B^{ij}, x_C^{ij},) - l\left(\frac{Y^{ij}}{\theta^i}\right) - \beta^j c(x_A^{ij}) \right) + \mu \left(\sum_{i,j} \pi^{ij} (Y^{ij} - x_A^{ij} - x_B^{ij} - x_C^{ij}) - \bar{R} \right) \\ &+ \lambda_1 \left(u^{12}(x_A^{12}, x_B^{12}, x_C^{12},) - l\left(\tfrac{Y^{12}}{\theta^1}\right) - \beta^2 c^{12}(x_A^{12}) - \widetilde{u}^{12}(x_A^{11}, x_B^{11}, x_C^{11},) + l\left(\tfrac{Y^{12}}{\theta^1}\right) + \beta^2 \widetilde{c}^{12}(x_A^{11}) \right) \\ &+ \lambda_2 \left(u^{22}(x_A^{22}, x_B^{22}, x_C^{22},) - l\left(\tfrac{Y^{22}}{\theta^2}\right) - \beta^2 c^{22}(x_A^{22}) - \widetilde{u}^{22}(x_A^{21}, x_B^{21}, x_C^{21},) + l\left(\tfrac{Y^{21}}{\theta^2}\right) + \beta^2 \widetilde{c}^{22}(x_A^{21}) \right) \\ &+ \lambda_3 \left(u^{11}(x_A^{11}, x_B^{11}, x_C^{11},) - l\left(\tfrac{Y^{11}}{\theta^1}\right) - \beta^1 c^{11}(x_A^{11}) - \widetilde{u}^{12}(x_A^{12}, x_B^{12}, x_C^{12},) + l\left(\tfrac{Y^{12}}{\theta^1}\right) + \beta^1 \widetilde{c}^{12}(x_A^{12}) \right) \\ &+ \lambda_4 \left(u^{21}(x_A^{21}, x_B^{21}, x_C^{21},) - l\left(\tfrac{Y^{21}}{\theta^2}\right) - \beta^1 c^{21}(x_A^{21}) - \widetilde{u}^{21}(x_A^{22}, x_B^{22}, x_C^{22},) + l\left(\tfrac{Y^{22}}{\theta^2}\right) + \beta^1 \widetilde{c}^{22}(x_A^{22}) \right), \end{aligned}$$

where the expressions $v^{ij}(x_A^{ij}, x_B^{ij}, x_C^{ij}, \frac{Y^i}{\theta^i}; \beta^j)$ have been made explicit.

Maximizing with respect to consumption levels, for $k = B, C$, one obtains the following first-order conditions

$$
\begin{aligned}
x_A^{22}: &\quad \alpha^{22}\left(\frac{\partial u^{22}}{\partial x_A}-\beta^2\frac{\partial c^{22}}{\partial x_A}\right)-\pi^{22}\mu+\lambda_2\left(\frac{\partial u^{22}}{\partial x_A}-\beta^2\frac{\partial c^{22}}{\partial x_A}\right)-\lambda_4\left(\frac{\partial \widetilde{u}^{21}}{\partial x_A}-\beta^1\frac{\partial \widetilde{c}^{21}}{\partial x_A}\right)=0,\\
x_A^{12}: &\quad \alpha^{12}\left(\frac{\partial u^{12}}{\partial x_A}-\beta^2\frac{\partial c^{12}}{\partial x_A}\right)-\pi^{12}\mu+\lambda_1\left(\frac{\partial u^{12}}{\partial x_A}-\beta^2\frac{\partial c^{12}}{\partial x_A}\right)-\lambda_3\left(\frac{\partial \widetilde{u}^{12}}{\partial x_A}-\beta^1\frac{\partial \widetilde{c}^{12}}{\partial x_A}\right)=0,\\
x_A^{21}: &\quad \alpha^{21}\left(\frac{\partial u^{21}}{\partial x_A}-\beta^1\frac{\partial c^{21}}{\partial x_A}\right)-\pi^{21}\mu-\lambda_2\left(\frac{\partial \widetilde{u}^{22}}{\partial x_A}-\beta^2\frac{\partial \widetilde{c}^{22}}{\partial x_A}\right)+\lambda_4\left(\frac{\partial u^{21}}{\partial x_A}-\beta^1\frac{\partial c^{21}}{\partial x_A}\right)=0,\\
x_A^{11}: &\quad \alpha^{11}\left(\frac{\partial u^{11}}{\partial x_A}-\beta^1\frac{\partial c^{11}}{\partial x_A}\right)-\pi^{11}\mu-\lambda_1\left(\frac{\partial \widetilde{u}^{12}}{\partial x_A}-\beta^2\frac{\partial \widetilde{c}^{12}}{\partial x_A}\right)+\lambda_3\left(\frac{\partial u^{11}}{\partial x_A}-\beta^1\frac{\partial c^{11}}{\partial x_A}\right)=0,\\
x_k^{22}: &\quad \alpha^{22}\frac{\partial u^{22}}{\partial x_k}-\pi^{22}\mu+\lambda_2\frac{\partial u^{22}}{\partial x_k}-\lambda_4\frac{\partial \widetilde{u}^{21}}{\partial x_k}=0,\\
x_k^{12}: &\quad \alpha^{12}\frac{\partial u^{12}}{\partial x_k}-\pi^{12}\mu+\lambda_1\frac{\partial u^{12}}{\partial x_k}-\lambda_3\frac{\partial \widetilde{u}^{12}}{\partial x_k}=0,\\
x_k^{i1}: &\quad \alpha^{i1}\frac{\partial u^{i1}}{\partial x_k}-\pi^{i1}\mu-\lambda_j\frac{\partial \widetilde{u}^{i2}}{\partial x_k}=0,\\
x_k^{i1}: &\quad \alpha^{i1}\frac{\partial u^{i1}}{\partial x_k}-\pi^{i1}\mu-\lambda_j\frac{\partial \widetilde{u}^{i2}}{\partial x_k}=0.
\end{aligned}
$$

Assume, first, that the binding incentive constraints are (7) and (8) — i.e., $\lambda_3 = \lambda_4 = 0$. Hence, rearranging these expressions, marginal rate of substitutions of individuals of type (θ^i, β^2) are

$$
\begin{aligned}
MRS^{i2}_{x_A^{i2},x_k^{i2}} &\equiv \frac{\frac{\partial u^{i2}}{\partial x_A}-\beta^2\frac{\partial c^{i2}}{\partial x_A}}{\frac{\partial u^{i2}}{\partial x_k}}=1,\\
MRS^{i2}_{x_B^{i2},x_C^{i2}} &\equiv \frac{\frac{\partial u^{i2}}{\partial x_B}}{\frac{\partial u^{i2}}{\partial x_C}}=1,
\end{aligned}
$$

implying, using (2) and (3), that $(T_A^{'})_{i2} = (T_k^{'})_{i2}$, $\forall k, i$ — i.e., optimal commodity taxation is uniform for individuals of type (θ^i, β^2).

For individuals of type (θ^1, β^j), marginal rate of substitutions are

$$
MRS^{i1}_{x_A^{i1},x_k^{i1}} \equiv \frac{\frac{\partial u^{i1}}{\partial x_A}-\beta^1\frac{\partial c^{i1}}{\partial x_A}}{\frac{\partial u^{i1}}{\partial x_k}} = \frac{\alpha^{i1}\mu+\lambda_i\left(\frac{\partial \widetilde{u}^{i2}}{\partial x_A}-\beta^2\frac{\partial \widetilde{c}^{i2}}{\partial x_A}\right)}{\alpha^{i1}\mu+\lambda_i\frac{\partial \widetilde{u}^{i2}}{\partial x_k}}, \tag{35}
$$

$$
MRS^{i1}_{x_B^{i1},x_C^{i1}} \equiv \frac{\frac{\partial u^{i1}}{\partial x_B}}{\frac{\partial u^{i1}}{\partial x_C}} = \frac{\alpha^{i1}\mu+\lambda_i\frac{\partial \widetilde{u}^{i2}}{\partial x_B}}{\alpha^{i1}\mu+\lambda_i\frac{\partial \widetilde{u}^{i2}}{\partial x_C}}. \tag{36}
$$

Define

$$
\widetilde{MRS}^{i2}_{x_A^{i1},x_k^{i1}} \equiv \frac{\frac{\partial \widetilde{u}^{i2}}{\partial x_A}-\beta^2\frac{\partial \widetilde{c}^{i2}}{\partial x_A}}{\frac{\partial \widetilde{u}^{i2}}{\partial x_k}}
$$

as the marginal rate of substitution between good A and good k for individuals of type (θ^i, β^2) mimicking types (θ^i, β^1), for given i. Moreover, let

$$
\widetilde{MRS}^{i2}_{x_B^{i1},x_C^{i1}} \equiv \frac{\frac{\partial \widetilde{u}^{i2}}{\partial x_B}}{\frac{\partial \widetilde{u}^{i2}}{\partial x_C}}
$$

be the marginal rate of substitution between good B and good C for individuals of type (θ^i, β^2) mimicking individuals of type (θ^i, β^1), for a given i. Remember that a mimicker should consume the same bundle of the mimicked individual.

Hence, (35) and (36) can be rewritten as

$$\begin{aligned} MRS^{i1}_{x_A^{i1}, x_k^{i1}} &= \frac{\hat{\phi} + \widetilde{MRS}^{i2}_{x_A^{i1}, x_k^{i1}}}{\phi + 1}, \\ MRS^{i1}_{x_B^{i1}, x_C^{i1}} &= \frac{\hat{\varphi} + \widetilde{MRS}^{i2}_{x_B^{i1}, x_C^{i1}}}{\varphi + 1} \end{aligned}$$

where $\hat{\phi} = \frac{\alpha_{i1}\mu}{\lambda_i \frac{\partial \tilde{u}^{i2}}{\partial x_k}}$ and $\hat{\varphi} = \frac{\alpha_{i1}\mu}{\lambda_i \frac{\partial \tilde{u}^{i2}}{\partial x_C}}$ are constant terms.

Contrary to Proposition 1, although the mimicker and mimicked have the same disposable income and the same labor supply, marginal rates of substitution are no longer the same when the bundle intended for the mimicked is consumed by the mimicker. In fact, having preferences for immediate gratification, an agent with $\beta = \beta^1$ enjoys more utility by consuming the amount x_A^{i1} of the addictive good relative to a mimicker of type $\beta = \beta^2$ who consumes the same amount. To see it, rewrite the marginal rate of substitution $\widetilde{MRS}^{i2}_{x_A^{i1}, x_k^{i1}}$ as follows

$$\begin{aligned} \widetilde{MRS}^{i2}_{x_A^{i1}, x_k^{i1}} &= \frac{\frac{\partial \tilde{u}^{i2}}{\partial x_A} - \beta^1 \frac{\partial \tilde{c}^{i2}}{\partial x_A} + \beta^1 \frac{\partial \tilde{c}^{i2}}{\partial x_A} - \beta^2 \frac{\partial \tilde{c}^{i2}}{\partial x_A}}{\frac{\partial \tilde{u}^{i2}}{\partial x_k}} \\ &= MRS^{i1}_{x_A^{i1}, x_k^{i1}} - \frac{\left(\beta^2 - \beta^1\right)}{\frac{\partial \tilde{u}^{i2}}{\partial x_k}} < MRS^{i1}_{x_A^{i1}, x_k^{i1}}, \end{aligned}$$

which implies that $\widetilde{MRS}^{i2}_{x_A^{i1}, x_k^{i1}} < MRS^{i1}_{x_A^{i1}, x_k^{i1}}$ and $\widetilde{MRS}^{i2}_{x_B^{i1}, x_C^{i1}} \neq MRS^{i1}_{x_B^{i1}, x_C^{i1}}$. Summing up,

$$\begin{aligned} MRS^{i1}_{x_A^{i1}, x_k^{i1}} &= \frac{\hat{\phi} + \widetilde{MRS}^{i2}_{x_A^{i1}, x_k^{i1}}}{\hat{\phi} + 1} < 1, \\ MRS^{i1}_{x_B^{i1}, x_C^{i1}} &= \frac{\hat{\varphi} + \widetilde{MRS}^{i2}_{x_B^{i1}, x_C^{i1}}}{\hat{\varphi} + 1} \neq 1. \end{aligned}$$

Using the conditions for utility maximization, it is immediate to see that optimal marginal taxes for a individuals with $\beta = \beta^1$ are such that $(T_A^{'})_{i1} < (T_k^{'})_{i1}$ and $(T_B^{'})_{i1} \neq (T_C^{'})_{i1}$.

Assume instead that the binding incentive constraints are (9) and (10) — i.e., $\lambda_1 = \lambda_2 = 0$. Rearranging the first-order conditions above, marginal rate of substitutions for individuals of type (θ^i, β^1) are

$$\begin{aligned} MRS^{i1}_{x_A^{i2}, x_k^{i2}} &\equiv \frac{\frac{\partial u^{i1}}{\partial x_A} - \beta^1 \frac{\partial c^{i1}}{\partial x_A}}{\frac{\partial u^{i1}}{\partial x_k}} = 1, \\ MRS^{i1}_{x_B^{i2}, x_C^{i2}} &\equiv \frac{\frac{\partial u^{i1}}{\partial x_B}}{\frac{\partial u^{i1}}{\partial x_C}} = 1, \end{aligned}$$

implying that $(T_A^{'})_{i1} = (T_k^{'})_{i1}$, $\forall k, i$ — i.e., optimal commodity taxation is uniform for types (θ^i, β^1).

For agents of type (θ^2, β^j), marginal rate of substitutions are

$$MRS^{i2}_{x^{i1}_A, x^{i1}_k} = \frac{\overline{\phi} + \widetilde{MRS}^{i1}_{x^{i2}_A, x^{i2}_k}}{\overline{\phi} + 1}, \tag{37}$$

$$MRS^{i2}_{x^{i1}_B, x^{i1}_C} = \frac{\overline{\varphi} + \widetilde{MRS}^{i1}_{x^{i2}_B, x^{i2}_C}}{\overline{\varphi} + 1}. \tag{38}$$

where $\overline{\phi} = \frac{\alpha^{i2}\mu}{\lambda_i \frac{\partial \widetilde{u}^{i2}}{\partial x_k}}$ and $\overline{\varphi} = \frac{\alpha^{i2}\mu}{\lambda_i \frac{\partial \widetilde{u}^{i2}}{\partial x_C}}$ are constant terms, while the term $\widetilde{MRS}^{i1}_{x^{i1}_A, x^{i1}_k}$ (*resp.* $\widetilde{MRS}^{i1}_{x^{i2}_B, x^{i2}_C}$) is the marginal rate of substitution between good A and good k (*resp.* between good B and C) for individuals of type (θ^i, β^1) mimicking individuals of type (θ^i, β^2), for given i. Notice that, using the separability assumption and the fact the mimicker and the mimicked have to consume the same bundle, the marginal rate of substitution $\widetilde{MRS}^{i1}_{x^{i2}_A, x^{i2}_k}$ can be rewritten as follows

$$\begin{aligned} \widetilde{MRS}^{i1}_{x^{i2}_A, x^{i2}_k} &= \frac{\frac{\partial \widetilde{u}^{i1}}{\partial x_A} - \beta^2 \frac{\partial \widetilde{c}^{i1}}{\partial x_A} + \beta^2 \frac{\partial \widetilde{c}^{i1}}{\partial x_A} - \beta^1 \frac{\partial \widetilde{c}^{i1}}{\partial x_A}}{\frac{\partial \widetilde{u}^{i1}}{\partial x_k}} \\ &= MRS^{i2}_{x^{i1}_A, x^{i1}_k} + \frac{\left(\beta^2 - \beta^1\right)}{\frac{\partial \widetilde{u}^{ij}}{\partial x_k}} > MRS^{i2}_{x^{i1}_A, x^{i1}_k}, \end{aligned}$$

implying that $\widetilde{MRS}^{i1}_{x^{i2}_A, x^{i2}_k} > MRS^{i2}_{x^{i1}_A, x^{i1}_k} > 1$. Hence, $(T^{'}_A)_{i2} > (T^{'}_k)_{i2}$. ■

Proof of Proposition 3

The Langrangean of the problem is

$$\begin{aligned} \Lambda \;=\; & \sum_{i,j} \alpha^{ij} \left(V^{ij}(q_A, q_B, I^i; \beta^j) - l\left(\frac{Y^{ij}}{\theta^i}\right)\right) + \mu \left(\sum_{i,j} \pi^{ij} (Y^{ij} - I^{ij} + (q_A - 1) x^{ij}_A(.) + (q_B - 1) x^{ij}_B(.)) - \bar{R} \right) \\ & + \lambda_1 \left(V^{21}(q_A, q_B, I^{21}; \beta^1) - l\left(\frac{Y^{21}}{\theta^2}\right) - \widetilde{V}^{21}(q_A, q_B, I^{11}; \beta^1) + l\left(\frac{Y^{11}}{\theta^2}\right)\right) \\ & + \lambda_2 \left(V^{22}(q_A, q_B, I^{22}; \beta^2) - l\left(\frac{Y^{22}}{\theta^2}\right) - \widetilde{V}^{22}(q_A, q_B, I^{12}; \beta^2) + l\left(\frac{Y^{12}}{\theta^2}\right)\right), \end{aligned}$$

where $x^{ij}_k(q_A, q_B, I^i; \beta^j)$, $k = A, B$, are the Marshallian demand functions of good k. The first-order conditions of the planner's problem, with respect to q_A, q_B, I^{1j} and I^{2j}, are

$$I^{1j}: \quad \sum_j \alpha^{1j} \frac{\partial V^{1j}}{\partial I} + \mu \sum_j \pi^{1j} \left(-1 + (q_A - 1)\frac{\partial x^{1j}_A}{\partial I} + (q_B - 1)\frac{\partial x^{1j}_B}{\partial I}\right) - \sum_j \lambda_j \frac{\partial \widetilde{V}^{2j}}{\partial I} = 0, \tag{39}$$

$$I^{2j}: \quad \sum_j \alpha^{2j} \frac{\partial V^{2j}}{\partial I} + \mu \sum_j \pi^{2j} \left(-1 + (q_A - 1)\frac{\partial x^{2j}_A}{\partial I} + (q_B - 1)\frac{\partial x^{2j}_B}{\partial I}\right) + \sum_j \lambda_j \frac{\partial V^{2j}}{\partial I} = 0, \tag{40}$$

$$q_A: \quad \sum_{i,j} \alpha^{ij} \frac{\partial V^{ij}}{\partial q_A} + \mu \sum_{i,j} \pi^{ij} \left(\sum_k (q_k - 1)\frac{\partial x^{ij}_k}{\partial q_A} + x^{ij}_A \right) + \sum_j \lambda_j \left(\frac{\partial V^{2j}}{\partial q_A} - \frac{\partial \widetilde{V}^{2j}}{\partial q_A}\right) = 0, \tag{41}$$

$$q_B: \quad \sum_{i,j} \alpha^{ij} \frac{\partial V^{ij}}{\partial q_B} + \mu \sum_{i,j} \pi^{ij} \left(\sum_k (q_k - 1)\frac{\partial x^{ij}_k}{\partial q_B} + x^{ij}_B \right) + \sum_j \lambda_j \left(\frac{\partial V^{2j}}{\partial q_B} - \frac{\partial \widetilde{V}^{2j}}{\partial q_B}\right) = 0. \tag{42}$$

Recall that, by Roy's identity, $\forall k$,

$$x_k^{ij} = -\frac{\partial V^{ij}/\partial q_k}{\partial V^{ij}/\partial I^i}, \tag{43}$$

and that the Slutsky decomposition ensures that

$$\frac{\partial \widehat{x}_k^{ij}(q_A, q_B, I^{ij})}{\partial q_A} = \frac{\partial x_k^{ij}(q_A, q_B, I^{ij})}{\partial q_A} + x_k^{ij}(q_A, q_B, I^i)\frac{\partial x_k^{ij}(q_A, q_B, I^{ij})}{\partial I}, \tag{44}$$

$$\frac{\partial \widehat{x}_k^{ij}(q_A, q_B, I^{ij})}{\partial q_B} = \frac{\partial x_k^{ij}(q_A, q_B, I^{ij})}{\partial q_B} + x_k^{ij}(q_A, q_B, I^{ij})\frac{\partial x_k^{ij}(q_A, q_B, I^{ij})}{\partial I}, \tag{45}$$

where $\widehat{x}_k^{ij}$ denotes compensated demand of good k. Hence, multiplying (39) by $\sum_j x_A^{1j}$ and (40) by $\sum_j x_A^{2j}$, and replacing the result obtained into (41),

$$\sum_{i,j} \pi_{ij} \left(\sum_k (q_k - 1) \frac{\partial \widehat{x}_k^{ij}}{\partial q_A} \right) = \sum_j \frac{\lambda_j}{\mu} (x_A^{1j} - \widetilde{x}_A^{2j}) \frac{\partial \widetilde{V}^{2j}}{\partial I}, \tag{46}$$

where definitions (43) and (44) have been used. Notice that

$$\widetilde{x}_A^{2j} = -\frac{\partial \widetilde{V}^{2j}/\partial q_A}{\partial \widetilde{V}^{2j}/\partial I}$$

denotes consumption of good A for an individual with productivity θ^2 and behavioral type j who is mimicking an agent with productivity θ^1 and behavioral type j.

Moreover, multiplying (39) by $\sum_j x_B^{1j}$ and (40) by $\sum_j x_B^{2j}$ and replacing into (42), one obtains

$$\sum_{i,j} \pi_{ij} \left(\sum_k (q_k - 1) \frac{\partial \widehat{x}_k^{ij}}{\partial q_B} \right) = \sum_j \frac{\lambda_j}{\mu} (x_B^{1j} - \widetilde{x}_B^{2j}) \frac{\partial \widetilde{V}^{2j}}{\partial I}. \tag{47}$$

Having the same disposable income (although different labor supplies), and because of the separability assumption, the mimicker and the mimicked agent have the same consumption levels; hence $x_A^{1j} - \widetilde{x}_A^{2j} = 0$ and $x_B^{1j} - \widetilde{x}_B^{2j} = 0$. It follows that the left-hand side of conditions (46) and (47) are zero: this is true only if both q_A and q_B are equal to one, implying that $t_A = t_B = 0$. ■

Proof of Proposition 4

The Langrangean of the problem is

$$\begin{aligned}
\Lambda = & \sum_{i,j} \alpha^{ij} \left(V^{ij}(q_A, q_B, I^i; \beta^j) - l\left(\frac{Y^i}{\theta^i}\right) \right) + \mu \left(\sum_{i,j} \pi^{ij} (Y^i - I^i + (q_A - 1)x_A^{ij} + (q_B - 1)x_B^{ij}) - \bar{R} \right) \\
& + \lambda_{12} \left(V^{12}(q_A, q_B, I^1; \beta^2) - l\left(\tfrac{Y^{12}}{\theta^1}\right) - \beta^2 c(x_A^{12}) - \widetilde{V}^{12}(q_A, q_B, I^1; \beta^2) + l\left(\tfrac{Y^{11}}{\theta^1}\right) + \beta^2 c(x_A^{11}) \right) \\
& + \lambda_{22} \left(V^{22}(q_A, q_B, I^2; \beta^2) - l\left(\tfrac{Y^{22}}{\theta^2}\right) - \beta^2 c(x_A^{22}) - \widetilde{V}^{22}(q_A, q_B, I^2; \beta^2) + l\left(\tfrac{Y^{21}}{\theta^2}\right) + \beta^2 c(x_A^{21}) \right) \\
& + \lambda_{11} \left(V^{11}(q_A, q_B, I^{11}; \beta^1) - l\left(\tfrac{Y^{11}}{\theta^1}\right) - \beta^1 c(x_A^{11}) - \widetilde{V}^{11}(q_A, q_B, I^{12}; \beta^1) + l\left(\tfrac{Y^{12}}{\theta^1}\right) + \beta^1 c(x_A^{12}) \right) \\
& + \lambda_{21} \left(V^{21}(q_A, q_B, I^{21}; \beta^1) - l\left(\tfrac{Y^{21}}{\theta^2}\right) - \beta^j c(x_A^{21}) - \widetilde{V}^{21}(q_A, q_B, I^{22}; \beta^1) + l\left(\tfrac{Y^{22}}{\theta^2}\right) + \beta^1 c(x_A^{22}) \right).
\end{aligned}$$

The first-order conditions with respect to q_A, q_B, I^{1j} and I^{2j} are

$$I^{1j}: \quad \sum_j \alpha^{1j} \frac{\partial V^{1j}}{\partial I} + \mu \sum_j \pi^{1j} \left(-1 + (q_A - 1) \frac{\partial x_A^{1j}}{\partial I} + (q_B - 1) \frac{\partial x_B^{1j}}{\partial I} \right) + \lambda_{1j} \left(\frac{\partial \widetilde{V}^{1j}}{\partial I} - \frac{\partial \widetilde{V}^{1j}}{\partial I} \right) = 0, \quad (48)$$

$$I^{2j}: \quad \sum_j \alpha^{2j} \frac{\partial V^{2j}}{\partial I} + \mu \sum_j \pi^{2j} \left(-1 + (q_A - 1) \frac{\partial x_A^{2j}}{\partial I} + (q_B - 1) \frac{\partial x_B^{2j}}{\partial I} \right) + \lambda_{2j} \left(\frac{\partial \widetilde{V}^{2j}}{\partial I} - \frac{\partial \widetilde{V}^{2j}}{\partial I} \right) = 0, \quad (49)$$

$$q_A: \quad \sum_{i,j} \alpha^{ij} \frac{\partial V^{ij}}{\partial q_A} + \mu \sum_{i,j} \pi^{ij} \left(\sum_k (q_k - 1) \frac{\partial x_k^{ij}}{\partial q_A} + x_A^{ij} \right) + \sum_{i,j} \lambda_{ij} \left(\frac{\partial V^{ij}}{\partial q_A} - \frac{\partial \widetilde{V}^{ij}}{\partial q_A} \right) = 0, \quad (50)$$

$$q_B: \quad \sum_{i,j} \alpha^{ij} \frac{\partial V^{ij}}{\partial q_B} + \mu \sum_{i,j} \pi^{ij} \left(\sum_k (q_k - 1) \frac{\partial x_k^{ij}}{\partial q_B} + x_B^{ij} \right) + \sum_{i,j} \lambda_{ij} \left(\frac{\partial V^{ij}}{\partial q_B} - \frac{\partial \widetilde{V}^{ij}}{\partial q_B} \right) = 0. \quad (51)$$

Multiplying (48) by $\sum_j x_A^{1j}$ and (49) by $\sum_j x_A^{2j}$, and replacing into (50), yields

$$\sum_{i,j} \pi^{ij} \left(\sum_k (q_k - 1) \frac{\partial \widehat{x}_k^{ij}}{\partial q_A} \right) = \sum_{i,j} \left[\frac{\lambda_{ij}}{\mu} (x_A^{ij} - \widetilde{x}_A^{ij}) \frac{\partial \widetilde{V}^{ij}}{\partial I^i} + x_A^{i1} \left(\frac{\partial V^{ij}}{\partial I} - \frac{\partial \widetilde{V}^{ij}}{\partial I} \right) \right], \quad (52)$$

where definitions (43) and (44) have been used. The term $\widehat{x}_k^{ij}$ denotes compensated demand of good k, where the compensation takes the form of disposable income. The term $\widetilde{x}_A^{i2}$ indicates consumption of good A of an agent of type (θ^i, β^2) who is mimicking an individual with(θ^i, β^1). Using the fact that, by separability, marginal utility of income are the same for mimicker and mimics, the expression $\frac{\partial V^{ij}}{\partial I} - \frac{\partial \widetilde{V}^{ij}}{\partial I}$ in (52) is equal to zero. Hence,

$$\sum_{i,j} \pi^{ij} \left(\sum_k (q_k - 1) \frac{\partial \widehat{x}_k^{ij}}{\partial q_A} \right) = \sum_{i,j} \left(\frac{\lambda_{ij}}{\mu} (x_A^{ij} - \widetilde{x}_A^{ij}) \frac{\partial \widetilde{V}^{ij}}{\partial I} \right). \quad (53)$$

Moreover, multiplying (48) by $\sum_i x_B^{i2}$ and (49) by $\sum_i x_A^{i2}$, and replacing both conditions into (51), one obtains

$$\sum_{i,j} \pi^{ij} \left(\sum_k (q_k - 1) \frac{\partial \widehat{x}_k^{ij}}{\partial q_B} \right) = \sum_{i,j} \left(\frac{\lambda_{ij}}{\mu} (x_B^{ij} - \widetilde{x}_B^{ij}) \frac{\partial \widetilde{V}^{ij}}{\partial I} \right). \quad (54)$$

Assume first that $\lambda_{11} = \lambda_{21} = 0$. Since the consumption of good A of an agent with preferences for immediate gratification is higher than that of an exponential agent — i.e., a mimicker should increase his consumption of A to mimic a type $j = 1$ agent — the individual's budget constraint implies that a mimicker's consumption of B should be lower; it follows that the right-hand side of (54) has to be positive. Hence, to solve at the same time equations (53) and (54), it must be that

$$t_A \frac{\partial \widehat{x}_A^{ij}}{\partial q_A} + t_B \frac{\partial \widehat{x}_B^{ij}}{\partial q_A} > 0$$

and

$$t_A \frac{\partial \widehat{x}_A^{ij}}{\partial q_B} + t_B \frac{\partial \widehat{x}_B^{ij}}{\partial q_B} < 0$$

using the negativity of own-substitution effects — i.e., $\frac{\partial \widehat{x}_k^{ij}}{\partial q_k} \geq 0, \forall k$ — the symmetry of the Slutsky matrix i.e., $\frac{\partial \widehat{x}_k^{ij}}{\partial q_h} = \frac{\partial \widehat{x}_k^{ij}}{\partial q_h}$ for $k \neq h$ — (Varian, 1992) and the fact that A and B are net substitutes, it follows that conditions (53) and (54) are satisfied if only if

$$t_A < t_B.$$

Hence, good A must be taxed less than B. Similar reasoning shows that, when $\lambda_{12} = \lambda_{22} = 0$,

$$t_A > t_B. \blacksquare$$

Proof of Proposition 5

The Langrangean of the problem is

$$\begin{aligned}\Lambda \quad = \quad & \sum_m \alpha^m \left(V^m(q_A, q_B, I^m; \beta^m) - l\left(\frac{Y^m}{\theta^m}\right)\right) + \mu \left(\sum_m \pi^m (Y^m - I^m + (q_A - 1)x_A^m(q_A, q_B, I^m, \beta^m) \right. \\ & \left. + (q_B - 1)x_B^m(q_A, q_B, I^m, \beta^m)) - \bar{R} \right) \\ & + \sum_{m,r} \lambda_{mr} \left(V^m(q_A, q_B, I^m; \beta^m) - l\left(\frac{Y^m}{\theta^m}\right) - \widetilde{V}^{mr}(q_A, q_B, I^r; \beta^m) - l\left(\frac{Y^r}{\theta^m}\right)\right)\end{aligned}$$

The first-order conditions with respect to q_A, q_B and I^m are

$$I^m: \quad \alpha_m \frac{\partial V^m}{\partial I} + \mu\pi_m \left(-1 + (q_A - 1)\frac{\partial x_A^m}{\partial I} + (q_B - 1)\frac{\partial x_B^m}{\partial I}\right) + \sum_r \lambda_{mr} \left(\frac{\partial V^m}{\partial I} - \frac{\partial \widetilde{V}^{mr}}{\partial I}\right) = 0, \tag{55}$$

$$q_A: \quad \sum_m \alpha_m \frac{\partial V^m}{\partial q_A} + \mu \sum_m \pi_m \left(\sum_k (q_k - 1)\frac{\partial x_k^m}{\partial q_A} + x_A^m \right) + \sum_{m,r} \lambda_{mr} \left(\frac{\partial V^m}{\partial q_A} - \frac{\partial \widetilde{V}^{mr}}{\partial q_A}\right) = 0, \tag{56}$$

$$q_B: \quad \sum_m \alpha_m \frac{\partial V^m}{\partial q_B} + \mu \sum_m \pi_m \left(\sum_k (q_k - 1)\frac{\partial x_k^m}{\partial q_B} + x_A^m \right) + \sum_{m,r} \lambda_{mr} \left(\frac{\partial V^m}{\partial q_B} - \frac{\partial \widetilde{V}^{mr}}{\partial q_B}\right) = 0 \tag{57}$$

Using the Slutsky decomposition,

$$\frac{\partial \widehat{x}_k^m}{\partial q_A} \equiv \frac{\partial x_k^m}{\partial q_A} + \frac{\partial x_k^m}{\partial I} x_k^m, \qquad k = A, B,$$

as well as Roy's Identity,

$$x_k^m = \frac{\partial V^m / \partial q_k}{\partial V^m / \partial I},$$

conditions (55) and (56) can be written as

$$\sum_m \alpha_m \left(\sum_{k=A,B} (q_k - 1)\frac{\partial \widehat{x}_k^m}{\partial q_A} \right) = \sum_m \sum_r \frac{\lambda_{mr}}{\mu} (x_A^{mr} - \widetilde{x}_A^{mr}) \frac{\partial \widetilde{V}^{mr}(q_A, q_B, I^r; \beta^m)}{\partial I},$$

Repeating the same steps for (55) and (57),

$$\sum_m \alpha_m \left(\sum_{k=A,B} (q_k - 1)\frac{\partial \widehat{x}_k^m}{\partial q_B} \right) = \sum_m \sum_r \frac{\lambda_{mr}}{\mu} (x_B^{mr} - \widetilde{x}_B^{mr}) \frac{\partial \widetilde{V}^{mr}(q_A, q_B, I^r; \beta^m)}{\partial I}. \blacksquare$$

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