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The Commitment Problem of Secured Lending

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Abstract

The paper investigates optimal financial contracts when investment in pledgeable assets is endogenous and not observable to financiers. In a setting with uncertainty, two inputs with different collateral value and investment unobservability, we show that a firm-bank secured credit contract is time-inconsistent: Once credit has been granted, the entrepreneur has an ex-post incentive to alter the input combination towards the input with low collateral value and higher productivity, thus jeopardizing total bank revenues. Anticipating the entrepreneur's opportunism, the bank offers a non-collateralized credit contract, thereby reducing the surplus of the venture. One way for the firm to commit to the contract terms is to purchase inputs on credit and pledge them to the supplier in case of default. Observing the input investment and having a stake in the bad state, the supplier acts as a guarantor that the input combination specified in the bank contract will be actually purchased and that the entrepreneur will stick to the contract terms. The paper concludes that: (1) Buying inputs on account facilitates the access to collateralized bank financing; (2) Firms using both trade credit and collateralized bank finance invest more in pledgeable assets than firms only using uncollateralized bank credit. Our results are robust to the possibility of collusion between entrepreneur and supplier.

JEL classification: collateral, commitment, trade credit, bank financing.

Keywords: G32, G33, K22, L14.

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Introduction

Collateral is widely used in lending. Berger and Udell (1990) and Harhoff and Korting (1998) document that nearly 70% of commercial industrial loans in the U.S., the U.K., and Germany are secured. The common explanation of the existing literature for this evidence is that collateral boosts firms' debt capacity when credit frictions arise. However, related literature argues that asset characteristics may be important determinants of firms' debt capacity. Some papers, for example, focus on the degree of asset tangibility (Almeida and Campello, 2007), while others relate the asset debt capacity to its redeployability (Williamson, 1988; Shleifer and Vishny, 1992; Marquez and Yavuz, 2011) or to the easiness of shifting its ownership to creditors in case of distress (Hart and Moore, 1994).[1] Our paper identifies a new characteristic: the investment contractibility. We argue that if the investment in a given asset is not contractible, pledging this asset as collateral does not necessarily increase external financing: Collateral might introduce in the bank lending relationship a problem of moral hazard in the form of asset substitution. We show that trade credit can be used by the entrepreneur to mitigate this problem.

We construct a model where firms produce one good using two inputs with different collateral values, facing uncertain demand. Being specialized financial intermediaries, banks offer the cheapest source of financing. If banks observed the amount of inputs purchased and thus invested, the optimal contract would be secured debt. Collateral would give the bank protection against losses in default, thereby increasing the amount of external financing and the total surplus of the lending relationship. However, with investment unobservability, upon receiving the bank loan the entrepreneur has an incentive to alter the input combination toward the input with higher productivity but lower collateral value. This jeopardizes the bank's expected revenues, by reducing the liquidation income in the default state. Anticipating that it will not break even, the bank gives up the secured contract, thus causing an efficiency loss.

One way for the firm to commit to the contract terms with the bank is to purchase a fraction of the collateralizable inputs on credit and pledge them to the supplier in case of default. Being

[1] Empirical evidence shows that asset tangibility and salability increase debt capacity (see, among others, Almeida and Campello, 2007; Campello and Giambona, 2009; Benmelech, 2009; Rampini and Viswanathan, 2010).

the provider of inputs, the supplier observes input investment. Knowing the investment level and having a stake in the default state, he implicitly guarantees that the quantity of inputs specified in the financial contracts, and thus available for liquidation to all creditors, is actually purchased. As a consequence, the benefits of collateralized bank financing are restored. It follows that, when investment is non-contractible, buying inputs on account facilitates the access to bank lending by making the secured contract available to banks and entrepreneurs. The extent of these benefits depends on input characteristics: The higher the liquidity of the pledgeable input and the higher the degree of substitutability between inputs, the larger the benefit of the joint use of collateralized bank and trade credit.

This analysis relies on the assumption that the entrepreneur is the only contracting party facing a commitment problem. However, upon granting the loan, supplier and entrepreneur could jointly agree to alter the input combination at the expense of the bank. In this case, having the supplier acting as a financier only shifts the commitment problem: from the entrepreneur to entrepreneur and supplier jointly. If the cost of such a collusive deal is not too high, an increasing fraction of inputs must be bought on credit from the supplier to make the deviation costlier and thus unprofitable for the entrepreneur. In the extreme case in which collusion is costless, banks only offer uncollateralized debt contracts. It follows that the optimal mix of trade and bank credit and the type of contract (collateralized versus uncollateralized) depends on the cost of collusion. Moreover, since a different mixture of bank and trade credit corresponds to a different input combination, the cost of collusion also affects the input combination. Specifically, a larger share of inputs is paid for in cash, through bank credit, and a lower share on account, through trade credit, and firms use technologies more intensive in tangible assets the costlier it is for entrepreneur and supplier to collude.

Our paper is related to two strands of the literature. The first one focuses on the role of collateral in lending relationships. The second one on the determinants of trade credit use.

The literature on collateral has identified several theoretical reasons for the popularity of secured lending. First, collateral reduces lender's losses in case of default (lender's risk reduction). Second, collateral mitigates asymmetric information problems, both in case of adverse selection (Bester, 1985; Chan and Kanatas, 1985; Besanko and Thakor, 1987 a, b) and in case of moral hazard, like asset substitution (Jackson and Kronman, 1979 and Smith and Warner, 1979), under-investment and inadequate effort supply (Stulz and Johnson, 1985; Chan and Thakor, 1987; Boot and Thakor, 1994). All these papers point to the idea that borrowing not only against returns but also against assets provides the lender greater protection against losses in the event of default and increases the firm's debt capacity. This conclusion is obtained in settings where projects mostly use one input and the entrepreneur pledges outside collateral.

One contribution of our paper is to extend the previous setting to allow investment in pledgeable assets and financing to be jointly and endogenously chosen. This extension leads to new economic insights that downplay the importance of collateral in lending relationships. Specifically, our conclusion that any secured bank contract becomes time inconsistent when investment is endogenous and not observable to financiers challenges the accepted view that collateral boosts the firm's debt capacity through the lender risk reduction. Moreover, in contrast with the risk-shifting literature, where collateral is shown to mitigate a problem of asset substitution when the project is financed through unsecured debt, our analysis shows that the use of collateral may itself introduce a problem of entrepreneur's opportunism in the form of ex-post asset substitution that is absent in the unsecured contract.[2]

Two assumptions are crucial to get the time-inconsistency result of the collateralized contract: a two-input-technology and investment unobservability. With only one input, the non-contractibility of investment would be immaterial, as the loan size could be used to infer the input choice. With investment observability, a commitment problem would not arise as the entrepreneur would be able to credibly commit to the ex-ante efficient investment.

This discussion raises the question of which type of bank loan better fits our story. In practice, firms largely use secured loans as opposed to financing primarily based on the firm's cash-flow. Different types of secured loans are offered by banks. Real-estate based lending or loans collateralized by movable goods (like cars, trucks, etc.) have characteristics that depart from our theoretical setting. First, the

[2]The risk-shifting literature identifies an asset substitution problem in the use of unsecured debt by assuming conflicts of interests à la and Jensen and Meckling (1976) between shareholders and creditors and shows that this problem can be mitigated using collateralized debt contract (Jackson and Kronman, 1979; and Smith and Warner, 1979). In our model, the problem of asset substitution arises only when the project is financed through secured debt, since it is the collateral, in its role of inside asset, that gives the entrepreneur the incentive to shift to a different input combination.

problem of investment unobservability is not so relevant in this case as, being registered goods, their actual purchase is certifiable to the bank. Second, the credit is generally granted directly to the seller of the asset, to the notary (for real asset), or to the leasing company (for movable goods). This implies that the entrepreneur does not have the possibility to misuse the bank loan. A secured bank loan that better fits our model is Asset Based Lending (ABL). ABL is a type of short-term financing (typically, three years maturity) used to support working capital needs. In case of ABL, the bank avoids paying screening costs and lends in exchange of some generic collateral that generally includes accounts receivable, inventories, machineries and equipment (not real estate). Since the collateral value of ABL is clearly affected by input purchases which are not easily observable by the bank, ABL is likely to be sensitive to the commitment problem analyzed in our model.[3]

Our paper is also related to the literature on trade credit. Papers in this literature have sought to explain why agents might prefer to borrow from firms rather than from financial intermediaries. The traditional explanation is that trade credit plays a non-financial role. That is, it reduces transaction costs (Ferris, 1981), allows price discrimination between customers with different creditworthiness (Brennan et al., 1988), fosters long-term relationships with customers (Summers and Wilson, 2002), and even provides a warranty for quality when customers cannot observe product characteristics (Long et al., 1993). Financial theories (Biais and Gollier, 1997; Burkart and Ellingsen, 2004, among others) claim that suppliers are as good as or better financial intermediaries than banks. In Biais and Gollier (1997) and Burkart and Ellingsen (2004) this is due to an information advantage that suppliers have over banks. Within a context of limited enforceability, Cuñat (2007) shows that suppliers can enforce debt repayment better than banks by threatening to stop the supply of intermediate goods to their customers. Fabbri and Menichini (2010) show that trade credit can be cheaper than bank credit because of the liquidation advantage of the supplier.

Our paper is mostly related to the financial theories and in particular to Biais and Gollier (1997). Like them, we assume a supplier's information advantage. However, while in Biais and Gollier (1997) such advantage concerns the borrowers' creditworthiness, in our paper it concerns the investment in the

[3]In the last two decades in the U.S. there has been a steady increases in ABL: In 1992, there were $90 billions of ABL as opposed to $326 billions (corresponding to the 22% of the total short term credit) in 2002 and $590 billions in 2008. See Udell (2005) for more information about the characteristics of ABL.

collateralized input. The implications of the supplier information advantage are very different between the two papers. In Biais and Gollier (1997), extending trade credit signals to the bank the borrower's quality and induces banks to extend credit to entrepreneurs with profitable projects that would have been rejected otherwise. In our model, by signaling that the investment in the collateralized asset has taken place as expected, trade credit makes the secured bank loan available. Thus, while collateral is crucial in our story, it plays no role in Biais and Gollier (1997).

One prediction of our model is that buying inputs on account facilitates the access to collateralized bank financing, suggesting a complementarity between bank and trade credit, which is in the spirit of some recent empirical evidence.[4] Cook (1999) documents that account payables raise the likelihood that a Russian firm obtains a bank loan. Giannetti et al. (2008) show that U.S. firms getting credit from suppliers can secure financing from relatively uninformed banks. Alphonse et al. (2006) document that the more trade credit U.S. firms use, the more indebted towards banks they are, more so for firms with a short banking relationship. Along the same lines, Gama et al. (2008) find that the use of payables allows younger and smaller firms in Spain and Portugal to increase the availability of bank financing. Finally, Garcia-Appendini (2010) documents that small, non financial U.S. firms are more likely to get bank credit if they have been granted trade credit from their suppliers. The above evidence also suggests that the complementarity hypothesis is more relevant for young and small firms with a short banking relationship. This is an interesting finding that could be explained within our theoretical framework. Young, small firms with a short banking relationship are more opaque and also might lack incentives to commit to the contract terms in lending relations (or simply they are perceived by banks to lack incentives) since the cost of deviating from the contracts (i.e., loosing reputation) is still relatively small. So, these firms are the ones that benefit most from the use of trade credit.

The paper is organized as follows. In Section 1 we present the model. In Section 2 we describe the commitment problem that plagues an exclusive entrepreneur-bank lending relationship when the project to be financed uses two inputs. In Section 3, we show that trade credit can solve the

[4] At a theoretical level, in Biais and Gollier (1997), trade credit allows credit-constrained firms to get (uncollateralized) bank loan. In Burkart and Ellingsen (2004), trade credit also increases the amount of bank credit limit, but this is a second order effect holding only for a selected group of firms.

commitment problem and characterize the properties of the optimal financing contract. In Section 4, we extend the model to allow for collusion between entrepreneur and supplier. Section 5 delivers new testable predictions on firms decisions, by exploiting cross-sectional differences in input characteristics (liquidity and substitutability) and in the cost of collusion. Section 6 discusses the robustness of our theoretical setting. Specifically, Subsection 6.1 investigates whether alternative bank contracts could solve the commitment problem. Subsection 6.2 focuses on the degree of information sharing between supplier and bank. Subsection 6.3 discusses the role of exclusivity in the lending relation. Subsection 6.4 questions the role of the supplier as an informed lender and provides alternative interpretations. Subsection 6.5 discusses the seniority rule implied by the optimal contract. Section 7 concludes. All proofs are in the Appendix.

1 Setup and model assumptions

A risk-neutral entrepreneur has an investment project that uses two inputs, called capital (K) and labor (N). Let I_K, I_N denote the amount of investment in capital and labor inputs. The amount of the input invested is converted into a verifiable state-contingent output $Y \in \{0, y\}$. The good state ($Y = y$) occurs with probability p. Uncertainty affects production through demand (i.e., production is demand-driven). At times of high demand, invested inputs produce output according to a homothetic strictly quasi-concave production function $f(I_K, I_N)$. At times of low demand, there is no output ($Y = 0$), but unused inputs have a scrap value and can be pledged as collateral to creditors. Inputs are substitutes, but a positive amount of each is essential for production. Cross partial derivatives f_{NK} are positive.[5]

The entrepreneur is a price-taker both in the input and in the output market. The output price is normalized to 1, and so is the price of the two inputs.[6]

The entrepreneur has no internal wealth, so he needs external funding from competitive banks ($L_B \geq 0$) and/or suppliers ($L_S \geq 0$). We assume that lending is exclusive: the entrepreneur may not

[5] This amounts to saying that having more of one input increases the marginal product of other input. This condition is satisfied by the most commonly used production functions (e.g., Cobb-Douglas, CES, and their transformations).

[6] This normalization is without loss of generality since we use a partial equilibrium setting.

borrow from multiple banks or suppliers at the same time.[7] Banks and suppliers play different roles. Banks lend cash. The supplier of labor provides the input, which is fully paid for in cash. The supplier of capital, however, not only sells the input, but can also act as a financier, by delaying the payment of the inputs supplied. Each party is protected by limited liability.

Cost of funds. Banks have an intermediation advantage relative to suppliers as they face a lower cost of raising funds on the market ($r_B < r_S$). This assumption is consistent with the role of banks as specialized financial intermediaries.

Collateral value. Inputs have value for creditors when repossessed in default.[8] We assume that only capital inputs can be pledged while labor has zero collateral value. Financiers are equally good in liquidating the unused capital inputs and their liquidation value in case of default is given by $C = \beta I_K$, with $0 < \beta < 1$.[9]

Information. Banks and suppliers differ in the type of information they possess. Providing the input, suppliers of capital can costlessly observe that an input transaction has taken place. Being input provision and lending simultaneous, suppliers can condition their lending to the investment. Banks cannot observe input transaction when providing credit and the cost of acquiring this information is assumed too high to make observation worthwhile.[10] Thus banks cannot condition their lending on the investment. The supplier's information advantage is commonly accepted in the theoretical literature and frequently interpreted as a natural by-product of its selling activity. Suppliers are often in the same industries as their clients, and they often visit their customers' premises. In our setting, this assumption is even more reasonable, given that the information asymmetry refers to the input purchase. Extensive anecdotal evidence supports this assumption. The most recent example is the case of Siemens, that in 2010 has created its own bank, Siemens Bank Gm-bH, mainly to provide lines

[7] In Section 6.3, we discuss the relevance of this assumption in our analysis.

[8] We assume that inputs not pledged to any creditor are valueless to the entrepreneur in case of default. In Section 6.1, we discuss the implications of having the entrepreneur seizing the inputs.

[9] This assumption allows us to highlight the commitment role of trade credit. Giving the supplier a comparative advantage in liquidating the capital input would not alter our qualitative results, as long as this advantage is not too high, i.e., $\beta_S \leq \frac{(1-p)\beta_B r_S}{(r_B - p r_S)}$. In this latter case, the liquidation advantage would make trade credit cheaper than bank credit and therefore strictly preferred. This case has been analyzed in Fabbri and Menichini (2010).

[10] Full unobservability from the bank and full observability from the suppliers are not crucial in our analysis. We could still get our results by assuming that both banks and suppliers can partially observe the inputs, as long as suppliers have an information advantage over banks.

of credit to its most important clients.

Contracts. Since there is no output in the low state, limited liability implies that repayments to banks and suppliers in the low state are zero. Financiers can still get a repayment in the default state by having the right to a share of the scrap value of unused inputs. The entrepreneur-bank contract thus specifies the loan, L_B, the repayment obligation in the high state, R_B, and the fraction of the collateral obtained in case of default, $\gamma \in [0, 1]$. That with the supplier of the capital input specifies the input purchase, I_K, the amount of credit, L_S, the repayment obligation in the high state, R_S, and the fraction of the collateral obtained in case of default, $(1 - \gamma)$.

Last, given that labor is fully paid for when purchased, the contract between entrepreneur and workers specifies the amount of labor, I_N.

Fig. 1 summarizes the sequence of events: In $t = 1$, banks and suppliers make contract offers specifying the size of the loans, L_B, L_S, the high state repayment obligations, R_B, R_S , the share of the collateral that goes to the bank and the supplier in case of default, $\gamma, (1-\gamma)$, the amount of capital input to be purchased, I_K. More specifically, banks (and suppliers) propose a set of contracts which may range from the fully secured contract, with $\gamma = 1$, to the unsecured one, with $\gamma = 0$, passing through the partially secured one with $0 < \gamma < 1$. In $t = 2$, the entrepreneur chooses among contract offers and receives credit from the bank; in $t = 3$ the investment decisions are taken, I_K, I_N, and trade credit is provided; in $t = 4$, uncertainty resolves; and in $t = 5$, repayments are made.

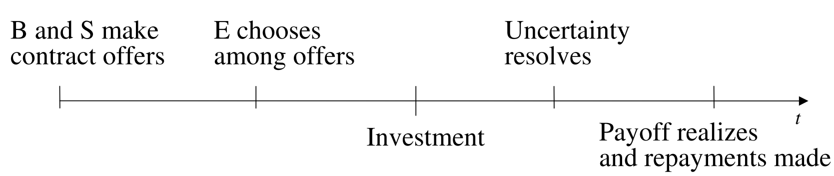

Figure 1: Time-line

2 The firm-bank contract without commitment

In this section, we show that the non-contractibility of the investment to the bank makes any entrepreneur-bank secured contract time-inconsistent and therefore not available to contracting parties. To make this point clear, we first analyze the benchmark case, where the investment is observable to the bank and therefore contractible. We derive the well-known result that secured lending is optimal since it increases the surplus of the lending relationship through a risk reduction for the lender. Then we consider the case of non-contractible investment.

Benchmark Case: Contractible Investment. In period $t = 1$, all financiers make contract offers. Since bank credit is a cheaper source of financing relative to trade credit, in period $t = 2$ firms only sign bank contracts and get financing. In period $t = 3$, firms buy and invest the inputs. The amount of inputs and financing are obtained by solving the following optimization problem ($\mathcal{P}^{\mathcal{FB}}$):

$$\begin{aligned} \max_{I_K,I_N,L_B,R_B} \Pi &= p\left[f\left(I_K,I_N\right)-R_B\right] && (1) \\ \text{s.t. } pR_B+(1-p)\,C &\geq L_B r_B, && (2) \\ L_B &\geq I_N+I_K. && (3) \end{aligned}$$

Condition (2) is the bank's participation constraint and states that banks participate to the venture if their expected returns cover at least their opportunity cost of funds. Competition among banks implies that (2) is binding. The resource constraint (3) requires that input purchase cannot exceed available funds. Solving (2) for R_B and using the resource constraint (3), the objective function (1) becomes:

$$\max_{I_K,I_N} \Pi = pf\left(I_K,I_N\right)-r_B\left(I_K+I_N\right)+(1-p)\,\beta I_K. \tag{4}$$

The solution to this problem leads to the following proposition:

Proposition 1 *When investment is contractible, the bank offers a collateralized credit contract with loan $L_B^{FB}=I_K^{FB}+I_N^{FB}$, a repayment $R_B^{FB}=\frac{1}{p}\left\{\left(I_N^{FB}+I_K^{FB}\right)r_B-(1-p)\,\beta I_K^{FB}\right\}$ in the high state, and βI_K^{FB} in the low state, with I_K^{FB}, I_N^{FB} solving the first order conditions (17) and (18) in the Appendix.*

Point A in Figure 2 displays the optimal input combination under the collateralized credit contract and represents the first-best. The input mix is stretched toward capital. The collateral value makes the actual price of capital equal to $r_B - (1-p)\beta$, thus lower than the price of labor, r_B. Notice that in our model, the input price depends on the selling price and on the financing cost, i.e. the cost of credit used to finance the input purchases. Since the selling price is set equal to one for both inputs by assumption, differences in the input price reflect only differences in the financing cost. The financing cost might differ across inputs either because of different financiers (bank versus supplier) or because of different contracts. For example, when both inputs are financed with bank credit and a collateralized contract is signed, the financing cost of the capital input is lower than the labor's financing cost, the difference being the collateral value of capital. In this case, the two inputs have different actual prices although they are both financed by the bank and the selling price is the same. In contrast, when both inputs are financed through an unsecured contract, both inputs have the same financing cost, namely r_B, and thus also the same actual price.

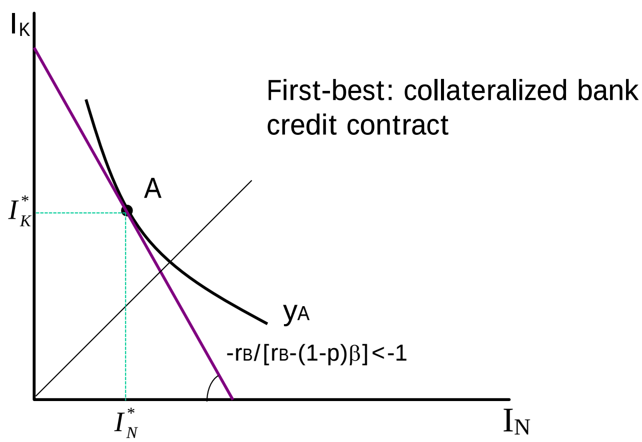

Figure 2: Contractible investment. Point A (first-best) represents the optimal input combination and the level of production under the collateralized bank credit contract.

Non-contractible investment. The result in Proposition 1 is obtained under the assumption that the entrepreneur can commit to the investment level specified in the bank contract at $t = 1$. However, if investment is unobservable, at $t = 3$, once the loan L_B^{FB} has been granted, the entrepreneur can increase profits by altering the input combination and worrying only about honoring his repayment obligations in non-defaulting states.[11] Thus, the entrepreneur re-optimizes by solving programme $\hat{\mathcal{P}}$:

$$\max_{I_K, I_N, R_B} \; pf(I_K, I_N) - pR_B \tag{5}$$

$$\text{s.t. } R_B \geq R_B^{FB}, \tag{6}$$

$$L_B^{FB} \geq I_N + I_K, \tag{7}$$

where constraint (6) requires the repayment to the bank in the high state be no less than the one promised in the secured first-best contract (i.e., R_B^{FB} in Prop. 1), while the resource constraint (7) requires that the ex-post total input expenditure be no higher than the loan obtained in the secured first-best contract (i.e., L_B^{FB} in Prop. 1).

The input combination solving the previous problem is represented by point D in Figure 3 and has a level of investment in the two inputs equal to $\hat{I}_K\left(L_B^{FB}, R_B^{FB}\right), \hat{I}_N\left(L_B^{FB}, R_B^{FB}\right)$. Point D lies to the right of point A on a higher isoquant and on a flatter isocost than the one going through point A (first-best contract). The slopes of the two isocost lines (tangent to isoquants y_A and y_D) represent the ex-ante and ex-post input price ratios, i.e., the price ratios before and after the bank loan has been received, respectively. The ex-ante input price ratio is the one implied by the collateralized credit contract (point A). Being the contract collateralized, the input price ratio is $r_B/\left[r_B - (1-p)\beta\right] > 1$ and therefore lower the higher the collateral value of the pledgeable input. Conversely, since the contract used by the bank to finance the input purchase of point D is uncollateralized, the financing cost of the two inputs is the same and equal to r_B. Therefore the ex-post input price ratio is 1. Since at the new input prices it must be still possible to afford the original contract, the new isocost line has to pass through the initial optimum (point A). By the quasi-concavity of the production function, the new input combination lies on a higher isoquant, and involves a decrease in I_K and an increase in I_N. The difference between the ex-ante and the ex-post input price ratios is the very reason why the entrepreneur can get higher profits by choosing an input combination which is different from the ex-ante efficient one.

[11] Because output is verifiable, any return from production will be claimed by creditors and the entrepreneur will get zero return if he does not repay the loan in full.

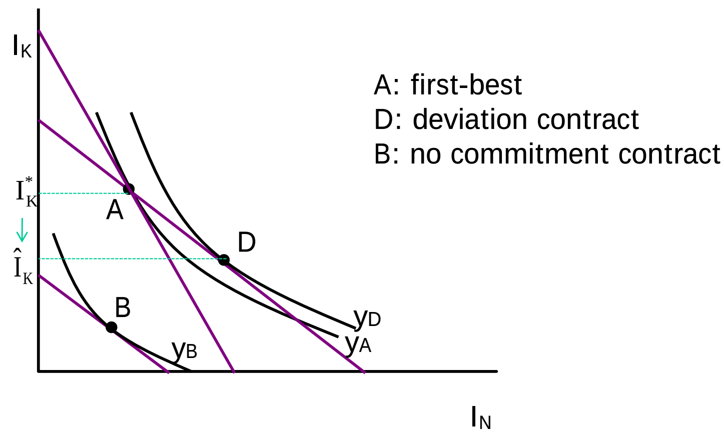

Figure 3: Contractible and non-contractible investment. Point A represents the optimal input combination when the investment is contractible. The bank offers a secured credit contract (first-best). Point B shows the optimal input combination when the investment is not contractible and the entrepreneur cannot commit to the first-best contract. The bank offers an un-collateralized credit contract. Point D is the input combination that the entrepreneur would ex-post choose after deviating from the first-best equilibrium upon receiving the bank loan. This is not an equilibrium contract since the bank does not break-even.

However, point D is not an equilibrium. Because of the decreased investment in capital inputs, in case of default the firm fails to meet its obligations. Anticipating that it will not break even, at the contracting stage the bank offers an unsecured credit contract with all the repayment obligations paid for in the good state. Setting $C = 0$ in the bank participation constraint (2), solving it for R_B and the resource constraint (3) for L_B, the objective function (1) becomes:

$$\max_{I_K, I_N} pf\left(I_K, I_N\right) - \left(I_N + I_K\right) r_B$$

which, compared to the objective function (4) of the benchmark (first-best) case, shows the loss in profits due to the inability to pledge collateral. The solution to the above maximization programme is described in Proposition 2:

Proposition 2 *When investment is non-contractible, the bank offers an unsecured credit contract lending $L_B^U = I_K^U + I_N^U < L_B^{FB}$, and getting a repayment only in non-defaulting states $R_B^U = \frac{1}{p} L_B^U r_B$. The level of investment in the collateralizable input is $I_K^U < I_K^{FB}$. There is an efficiency loss due to* *the inability to pledge inputs as collateral.*

Point B in Figure 3 is the optimal input combination when investment is not observable to the bank and the entrepreneur cannot commit to the input combination specified in the efficient contract. The new isoquant y_B lies below y_A. While the bank is indifferent between points A and B - it gets zero profit in either case - the entrepreneur's profits are strictly lower in point B. This is because the lower debt capacity, implied by the inability to pledge inputs as collateral, reduces the overall investment size and therefore the level of production. The distance between isoquants $y_A - y_B$, represents the benefits of collateral which is lost due to investment unobservability. It follows that the entrepreneur would rather commit to the investment level of the collateralized credit contract (point A). Notice also that in point B the actual prices of the two inputs are equal, since inputs are financed through a pure debt contract, where the financing cost of each input is equal to r_B. This explains why the input combination in point B implies an equal amount of capital and labor (as in point D).

So far we have shown that the fully uncollateralized contract is the equilibrium contract when investment is not observable. However, one might argue that partially collateralized contracts could also work as long as the bank can internalize the entrepreneur's deviation and change the contract terms (raising the repayment in the good state and reducing the collateral as well as the loan size) so as to break even under the investment chosen under deviation. Unfortunately, any partially collateralized contract would still be time inconsistent. So long as the ex-ante and the ex-post input price ratios are different, the entrepreneur still has an incentive to alter the input mix in favor of labor. This incentive is removed only if input prices are equal, i.e. when no assets are pledged as collateral. This is exactly what happens with a fully uncollateralized contract.

3 The commitment role of trade credit

So far we have shown that when the project needs two inputs and the investment in the pledgeable one is non-observable, any collateralized debt contract (fully and partially) is plagued by a problem of input substitution which makes this type of loan unprofitable to the bank. The unsecured debt contract removes this problem, but also eliminates the benefits of collateral.

In this section, we introduce the supplier of the collateralized input as a second financier. By observing the input transactions, he has a natural information advantage. The entrepreneur can use this advantage to restore his ability to pledge collateral to the bank.[12] In particular, he can sign a partially collateralized credit contract with the supplier. Observing the input transaction and having a stake in the default state, the supplier guarantees that the input investment is carried out as specified in the contract. This in turn induces the bank to offer a partially collateralized credit contract as well, thus mitigating the efficiency loss due to the lack of commitment.[13]

While supplier's finance allows the firm to overcome the commitment problem with the bank, this benefit comes at a cost since trade credit is more expensive than bank credit due to banks' intermediation advantage ($r_S > r_B$). To avoid the uninteresting case in which the cost of using trade credit is higher than its benefit, we introduce the following assumption:

Assumption 1 $\frac{r_B}{p} \geq \left(1 - \frac{\gamma}{p}\right) r_S + \left(\frac{\gamma}{p}\right) r_B, \forall \gamma \in [0, 1]$.

The left hand side of Assumption 1 represents the financing cost of the pledgeable input when the entrepreneur does not take trade credit and thus has only access to uncollateralized bank financing. The right hand side is the financing cost when the entrepreneur takes trade credit and has also access to collateralized bank credit. When Assumption 1 holds, the financing cost under the uncollateralized bank contract is no less than under any mix of collateralized trade and bank credit. Thus, taking trade credit is beneficial. In this case, the financing cost is a weighted average of the fund raising costs for bank (r_B) and supplier (r_S), with weights that depend on the share of inputs pledged as collateral to financiers and on the probability of default. This financing cost has r_S as upper bound. Moreover, the higher the probability of default (p) or the share of collateral accruing to the bank (γ) in case of default, the higher (the lower) the multiplier of the bank (supplier) funding cost, so the lower the average financing cost of inputs. Indeed, when default is very likely and the bank liquidates most of the firm's inputs, having access to the bank collateralized contract through the use of trade credit is

[12] In our model, the supplier acts as an informed lender. In Section 6, we discuss an alternative interpretation of the informed lender.

[13] For trade credit to work as a commitment device either the entrepreneur must be unable to resell the inputs purchased on credit in a secondary market or, if he can, the transaction costs must be sufficiently high to fully compensate the benefits of deviation.

greatly beneficial since it reduces the overall financing cost of inputs. In the extreme situation in which $\gamma = 0$, i.e., the firm uses collateralized trade credit and uncollateralized bank loan, the financing cost reaches its upper bound, i.e. r_S. In this case, Assumption 1 only depends on the model's parameter values and can be rewritten as $\frac{r_B}{p} \geq r_S$. The previous condition turns out to be a sufficient condition for Assumption 1 to be satisfied.

To find the optimal firm-bank-supplier contract, we proceed in three steps. First, we find the firm-bank-supplier contract for any generic γ. Then, we show that if the amount of trade credit is too small (γ too high), the entrepreneur could still cheat at the expense of the bank. Finally, we derive the entrepreneur's profits under deviation and introduce the entrepreneur's incentive compatibility condition that ensures that deviation is unprofitable. This condition allows us to find the incentive-compatible γ and therefore to fully characterize the optimal contract.

When bank and supplier can both provide external financing, the optimization problem is the following ($\mathcal{P}^{\gamma}$):

$$\begin{aligned} \max_{L_B, L_S, R_B, R_S I_K, I_N, \gamma} \Pi &= p\left[f\left(I_K, I_N\right) - R_B - R_S\right], & (8) \\ \text{s.t. } pR_B + (1-p)\gamma C &= L_B r_B, & (9) \\ pR_S + (1-p)(1-\gamma) C &= L_S r_S, & (10) \\ L_B + L_S &\geq I_N + I_K, & (11) \\ R_S &\geq (1-\gamma)\beta I_K & (12) \end{aligned}$$

where (29) denotes the entrepreneur's expected profits. Conditions (9) and (10) represent the participation constraints of competitive banks and suppliers, respectively. The parameter γ represents the share of the collateral accruing to the bank (and $1-\gamma$ the one accruing to the supplier). Condition (11) is the resource constraint when trade credit is also available. Last, constraint (12) requires repayments to the supplier be non-decreasing in revenues.[14]

Proposition 3 describes the solution to programme $\mathcal{P}^{\gamma}$:

Proposition 3 *The firm-bank-supplier contract has investment $I_K^*(\gamma), I_L^*(\gamma)$ solving (26) and (27) in the Appendix and displays the following properties:*

[14] This is standard in the literature (Innes, 1990).

a. *the supplier gets a secured contract with flat repayments across states: an amount* $L_S^*(\gamma) = \frac{1}{r_S}(1-\gamma)\beta I_K^*(\gamma)$ *is lent in exchange for the right to a share* $1-\gamma$ *of the collateral value of the unused inputs* $(\beta I_K^*(\gamma))$ *in the default state and to a repayment* $R_S^*(\gamma) = (1-\gamma)\beta I_K^*(\gamma)$ *in the high state; the share of inputs bought on account* $L_S^*(\gamma)/I_K^*(\gamma)$ *is decreasing in* γ;

b. *the bank gets a secured contract with increasing repayments: an amount* $L_B^*(\gamma) = I_K^*(\gamma) + I_N^*(\gamma) - \frac{1}{r_S}(1-\gamma)\beta I_K^*(\gamma)$ *is lent in exchange for the right to a share* γ *of the collateral value of the unused inputs* $(\beta I_K^*(\gamma))$ *in the default state and to a repayment* $R_B^*(\gamma) = \frac{1}{p}\left[L_B^*(\gamma) r_B - (1-p)(1-\gamma)\beta I_K^*(\gamma)\right] > \gamma\beta I_K^*(\gamma)$ *in the high state.* $L_B^*(\gamma)$ *is increasing in* γ.

c. *expected profits* $\Pi^* = pf\left(I_K^*(\gamma), I_N^*(\gamma)\right) - \left(I_K^*(\gamma) + I_N^*(\gamma)\right) r_B + \left[\frac{r_B}{r_S}(1-\gamma) + (\gamma - p)\right]\beta I_K^*(\gamma)$ *are increasing in* γ *and* β*;*

d. *asset tangibility* $\frac{I_K^*(\gamma)}{I_N^*(\gamma)}$ *is increasing in* γ.

Prop. 3 derives the properties of the optimal contract for any generic γ. This is implicitly assuming that sticking to the above contract is optimal for any γ. However, this is not always the case. The entrepreneur could increase production and thus profits by signing the collateralized credit contract with the bank, taking the bank loan agreed in the above contract, $L_B^*(\gamma)$ and then choosing a different input combination, with more labor units and less capital inputs. In evaluating the profitability of such a strategy the entrepreneur has to take into account that the supplier will refuse to sell goods on credit after observing the change in input purchase. The reason is that, involving a lower reliance on capital inputs, the supplier himself would not break even on the new input combination and the initial contract terms. As a consequence, he will not provide trade credit. Thus the entrepreneur faces a trade-off when he deviates: changing the input combination increases production, but giving up trade credit reduces financial resources thereby reducing production. In what follows, we will look for the value of γ that makes the entrepreneur indifferent between the commitment contract with collateralised bank credit and trade credit and the deviation contract with no trade credit. To this aim, we define the profits from deviation and derive the incentive compatibility constraint that must be satisfied to prevent it.

Definition 1 *Define* $\Pi^{\mathcal{D}}(\gamma) \equiv p\left[f\left(I_K^{\mathcal{D}}, L_B^*(\gamma) - I_K^{\mathcal{D}}\right) - R_B^*(\gamma)\right]$ *as the entrepreneur's expected profits after deviating from the contract specified in Prop. 3, where* $I_K^{\mathcal{D}}(L_B^*(\gamma))$ *is the level of capital chosen under deviation and satisfying programme* $\mathcal{P}^{\mathcal{D}}$ *in the Appendix.*

Given all profitable deviations, the optimal contract requires that deviating is less profitable than sticking to the ex-ante efficient contract. This is ensured by adding to programme P^{γ} the following incentive compatibility constraint:

$$\Pi^{\mathcal{D}}(\gamma) - \Pi^*(\gamma) \leq 0. \tag{13}$$

Proposition 4 *Under mild conditions, the set of bank contract offers that generates unprofitable deviation has* $0 \leq \gamma < \gamma^*$, *where* γ^* *satisfies condition 13 with equality. Since firm's expected profits* $\Pi^*(\gamma)$ *are increasing in* γ, *the entrepreneur will choose the contract with the highest possible* γ, *i.e.,* $\gamma = \gamma^*$.

Proposition 4 identifies the optimal share of collateral accruing to the bank as the upper bound of the set of contract offers that are incentive-compatible for the entrepreneur. Using γ^* in Prop. 3, we fully characterize the optimal firm-bank-supplier contract. This is the commitment contract: trade credit is used as a commitment device and its amount is the lowest possible that makes commitment credible to the bank.

Point E in Figure 4 depicts the input combination and the level of production corresponding to the commitment contract implied by Proposition 4. Point D represents the level of production obtained under deviation. Suppose that initially the entrepreneur signs the commitment contract and then decides to deviate. Upon observing the entrepreneur altering the original input provision, the supplier refuses to sell inputs on credit. This implies a contraction in trade credit, and thus in external financing, with a subsequent decline in the scale of production that makes deviating costly. By construction of the commitment contract, this decline is such that the entrepreneur is indifferent between sticking to the original contract and deviating. Graphically, point D and E lie on the same isoquant and thus involve the same production. The vertical distance between the isocost line intersecting point E and the one tangent to point D represents the amount of trade credit the entrepreneur has to give up to

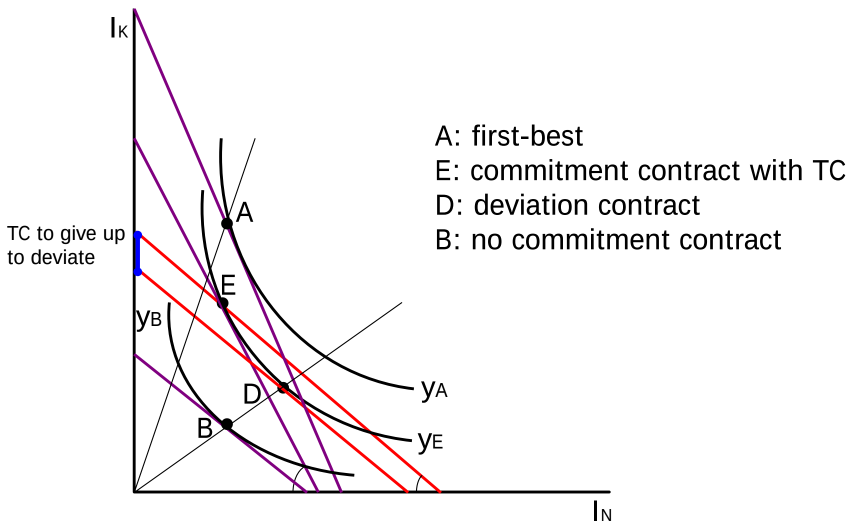

Figure 4: The commitment role of trade credit.

make the contract incentive compatible. This guarantees that point E is the equilibrium outcome, i.e., the commitment contract.

This discussion implies that whether deviating from the original contract is profitable or not depends on the amount of trade credit the entrepreneur is taking under the original contract, which corresponds to the amount of trade credit he has to give up in case of deviation. It follows that the bank can always prevent deviation by reducing its supply of financing in such a way that the amount of trade credit the entrepreneur has to give up when he deviates reduces production so much to make him prefer to stick to the original contract. With higher bank financing, and lower trade credit, the entrepreneur would still have an incentive to deviate, as the extra profits obtained through a change in input combination would more than compensate the lower investment size due to lack of trade credit.

Notice that point E lies between point A (first-best) and point B (uncollateralized bank contract). The commitment contract cannot replicate the first-best, since the cheaper bank credit is partially substituted with the more expensive trade credit. However, the commitment contract gives greater profits relative to the uncollateralized bank contract. By signaling that the bank loan will be used to purchase the inputs as specified in the bank contract, trade credit makes the collateralized bank contract available to the parties. Although the bank does not observe the firm-supplier contract, it can foresee the participation of the supplier to the venture and anticipate the commitment effect of trade credit. The commitment role of trade credit arises from the supplier providing a share of the capital inputs on credit and having the right to a share $1-\gamma^*$ of the collateral value of the same input in case of default. Both conditions have to be satisfied for the entrepreneur to have no incentive to ex-post alter the input mix.

Finally, since the commitment effect of trade credit allows the entrepreneur to access collateralized financing, point E has an input combination more intensive in capital than point B.

The previous analysis allows us to derive the following predictions:

Prediction 1. *When investment is non-contractible, trade credit facilitates the access to bank financing by making the secured contract available to bank and entrepreneur.*

Prediction 2. *Firms using trade credit and collateralized bank credit invest more intensively in pledgeable assets than firms using only uncollateralized bank credit.*

4 Firm-supplier collusion

In Section 3, we have argued that the use of trade credit allows the firm to overcome the commitment problem with the bank. This is because the supplier would always refuse to extend credit after observing the entrepreneur's deviation due to its failure to break even on the new input combination. Thus, the use of trade credit implicitly signals the bank that the entrepreneur will stick to the original contract. However, the supplier could still be willing to extend credit after observing the entrepreneur's deviation, so long as the terms of the contract were renegotiated to allow him to at least break-even. In this section, we extend the model to account for a collusive agreement between entrepreneur and supplier. Suppose that entrepreneur, bank and supplier have agreed on the contract terms described in Proposition 3, with $\gamma = \gamma^*$. Once obtained the loan from the bank, $L_B^* (\gamma^*)$, the entrepreneur may then propose the supplier an agreement to alter the input mix at the expense of the bank, i.e., reduce the investment in capital and increase the one in labor. To induce the supplier to accept, entrepreneur and supplier have to renegotiate the contract terms, i.e., loan size and repayments so as to let the supplier at least break even:[15]

$$pR_S + (1-p)(1-\gamma)\beta I_K \geq L_S(\gamma) r_S. \tag{14}$$

If agreed, the new arrangement allows to increase overall profits at the expense of the bank. Any collusive rent - the difference between the return under deviation and the return under commitment - is then shared between entrepreneur and supplier. For simplicity, we assume that any collusion rent is seized by the entrepreneur.[16]

This allows us to define the gross collusion rent and describe its properties in Proposition 5.

Definition 2 *Define $\Pi^{\mathcal{C}}(\gamma)$ as the profits from collusion for a generic γ solving programme $P^{\mathcal{C}}$ in the Appendix and $\Pi^{\mathcal{C}}(\gamma) - \Pi^*(\gamma)$ as the gross collusion rent, where $\Pi^*(\gamma)$ are the profits from commitment as defined in Prop. 3 for a generic γ.*

Proposition 5 *The gross collusion rent $\Pi^{\mathcal{C}}(\gamma) - \Pi^*(\gamma)$ is increasing in γ and is zero iff $\gamma = 0$. Thus, any firm-bank-supplier collateralized credit contract is prone to collusion between firm and supplier at the expense of the bank.*

Proposition 5 states that a collusive agreement to change ex-post the input combination causing losses to the bank is always profitable for entrepreneur and supplier. Thus, while incentive compatible in case of unilateral deviation, the commitment contract is not collusion-proof. The only way for the bank to stop parties from colluding is therefore to offer an unsecured contract ($\gamma = 0$).

However, reaching a collusive agreement may be time-consuming and thus costly.[17] To capture this cost, define $\alpha \in [0,1]$ as the fraction of the profits from deviation which is lost in reaching such an agreement and $[(1-\alpha)\Pi^{\mathcal{C}}(\gamma) - \Pi^*(\gamma)]$ as the net rent from collusion. This formulation of the collusion rent allows us to find an interior γ that stops parties from colluding. In particular, for a

[15] An alternative agreement would keep the loan size fixed ($L_S^*(\gamma)$) and would alter only the repayments. However, while this would not alter the qualitative properties of the collusion-proof contract, there is no actual reason to impose such restriction to the renegotiation process. Moreover, renegotiating all the contract terms and not only the repayments is profit-maximizing as it gives the firm the possibility of reducing its reliance on costly trade credit.

[16] This is without loss of generality as alternative distributions would not alter our qualitative results.

[17] Section 5.2 investigates the possible determinants of the cost of collusion.

small cost of collusion, the bank may always prevent it by restricting the set of contract offers to those that guarantee a non-positive collusion rent, i.e. those that satisfy the following constraint:

$$(1-\alpha)\,\Pi^{\mathcal{C}}(\gamma)-\Pi^{*}(\gamma)\leq 0 \tag{15}$$

Is the commitment contract collusion-proof, when we measure the collusion rent net of bargaining costs? More specifically, at $\gamma=\gamma^*$ (the value of γ that ensures no unilateral incentive to deviate), is constraint (15) satisfied? The answer depends on the cost of collusion, α. Let us define $\alpha^*(\gamma^*)$ as the value of α at which the the collusion proof condition (15) is satisfied with equality when $\gamma=\gamma^*$. When $\alpha\geq\alpha^*(\gamma^*)$, the value of γ that solves (15) exceeds γ^*, so that the commitment contract accommodates also collusion. When $\alpha<\alpha^*(\gamma^*)$, the value of γ that solves (15) falls short of γ^* and the commitment contract is prone to collusion. This amounts to say that the value of γ that accommodates both unilateral deviation and collusion has to solve the following global collusion proofness constraint:

$$\max\left\{(1-\alpha)\,\Pi^{\mathcal{C}}(\gamma),\Pi^{\mathcal{D}}(\gamma)\right\}-\Pi^{*}(\gamma)\leq 0. \tag{16}$$

This allows us to state the result in Proposition (6):

Proposition 6 *The set of collusion-proof bank contract offers has $\gamma=\hat{\gamma}(\alpha)\leq\gamma^*$, where $\hat{\gamma}(\alpha)$ satisfies condition 16. The properties of the firm-bank-supplier collusion proof contract are those described in Prop. 3, with $\gamma=\hat{\gamma}(\alpha)$.*

Corollary 1 *Under the collusion-proof bank-supplier contract, the share of inputs bought on credit is decreasing in the cost of collusion α, while the share of inputs bought on account, through bank credit, asset tangibility and expected profits are increasing in α.*

The above analysis shows that the properties of the optimal contract depend on the cost of collusion. Three scenarios can arise depending on the cost of collusion. First, collusion is so costly that it is never profitable: $\alpha(\gamma^*)\leq\alpha\leq 1$. The rent from collusion is lower than the rent from deviation, so that the global collusion proof condition (16) coincides with the incentive compatibility condition (13). This case corresponds to the one already analyzed in Section 3: The entrepreneur buys commitment from the supplier through trade credit and takes a partially collateralized loan from the bank. The equilibrium is point E in Fig. 5.

Second, collusion is costly but profitable: $0 < \alpha < \alpha(\gamma^*)$. The rent from collusion is greater than the rent from deviation, so that the global collusion proof condition (16) coincides with the collusion proof condition (15). The bank can reduce the scope for collusion by further reducing its participation to the venture: its stake in the bad state - the maximum share of the collateral to be liquidated - and the loan size shrink. The lower the cost of collusion, the lower the bank participation and the larger the amount of trade credit necessary to make the contract collusion proof. The properties of the optimal contract are those described in Proposition 3 with $\gamma = \hat{\gamma}(\alpha)$. The equilibrium point corresponding to this case is represented by point C in Fig. 4. This point is located between point E (commitment contract) and point B (uncollateralized bank contract), the exact position depending on the cost of collusion. The lower the cost of collusion, the further from point E and the closer to point B the new equilibrium is. Also, notice that a decrease in the cost of collusion that shifts the equilibrium away from point E towards point B, also makes the input combination more intensive in labor, since the relative (capital/labor) price ratio decreases.

Lastly, collusion is costless ($\alpha = 0$). Entrepreneur and supplier can grab the entire surplus from their agreement. In this case, the only way for the bank to break-even is by offering an uncollateralized credit contract. Having no longer a stake in the firm's low state return, the bank is paid only in the high state. This arrangement removes completely any incentive to collude. Trade credit can be still used but only to exploit the liquidation ability of the supplier, not to buy commitment. In this case, the supplier finances and liquidates the full collateral value of the capital input βI_K.[18]

[18] We are implicitly assuming here that $r_S < \frac{r_B}{p}$, i.e., Assumption 1 holds with strictly inequality when $\gamma = 0$. In this case, the equilibrium point will be still located away from point B and to its right. The entrepreneur can at least pledge some collateral to the supplier, therefore improving with respect to the case where only the uncollateralized bank credit is available (point B). If $r_S = \frac{r_B}{p}$, the entrepreneur is indifferent between taking trade credit only or a mix of trade and bank credit. The benefit of pledging inputs to the supplier is fully compensated by the higher cost of using trade credit with respect to bank credit. The equilibrium point corresponding to this case is point B.

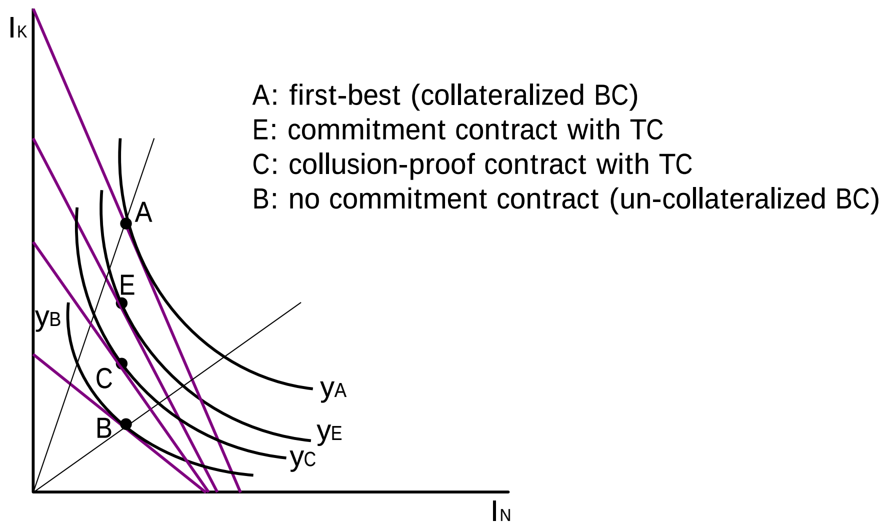

Figure 5: The collusion-proof contract.

5 Testable Predictions

This section uses the analysis developed in Sections 3 and 4 to identify new testable predictions on how firms investment and financing decisions vary across sectors and markets. Specifically, Subsection 5.1 exploits cross-sectional differences in input characteristics like liquidity and substitutability, while Subsection 5.2 focuses on differences in the cost of collusion.

5.1 The commitment problem in the cross-section of firms

In our model, trade credit solves the commitment problem between bank and entrepreneur and restores the benefits of collateral. We can define the benefits of collateral as the difference between the entrepreneur's profits under the commitment contract and the ones under the unsecured contract. Entrepreneur's profits under commitment are increasing in input liquidity, β, as shown in Prop 3. Entrepreneur's profits under the unsecured contract do not depend on input liquidity, since assets are not pledged as collateral. Thus, the benefits of trade credit are larger the higher is input liquidity. To some extent, input liquidity reflects industry characteristics. For example, standardized goods are likely to have a larger scrap value than differentiated products or services. Thus, firms using standardized products are more likely to take trade credit to access secured bank financing. This allows us to get the following prediction:

Prediction 3. *Firms buying standardized inputs are more likely to use collateralized bank and trade credit than firms buying differentiated inputs or services.*

A second important characteristic of the inputs is the degree of substitutability. The commitment problem arises because the entrepreneur can change the input combination once the bank loan has been provided. For trade credit to play a role as a commitment device, it is thus crucial that the project uses inputs that are at least partially substitutes. The higher the degree of substitutability between inputs, the more severe the commitment problem between entrepreneur and bank and more valuable trade credit is. In the extreme case where inputs are used in fixed proportions, there are no extra-profits to be gained from changing ex-post the input combination. Since, there is no incentive to deviate, the entrepreneur has access to collateralized bank financing without the need to take any trade credit. Thus, we get the following prediction:

Prediction 4. *Firms with a higher degree of substitutability between inputs are more likely to use collateralized bank and trade credit.*

5.2 Firm decisions and the cost to collude

From Prop. 6 and Corollary 1, we deduce that the properties of the financial contracts with bank and supplier depend on the cost of reaching a collusive agreement. In this section, we investigate in greater detail the relation between cost of collusion, optimal financing contracts - including not only the amount of bank and trade credit but also the specific type of contract (secured/unsecured)- and investment decisions. Fig. 6 summarizes the use of trade and bank credit and the production level for different levels of collusion costs.

In our setting, the entrepreneur buys commitment by taking trade credit. Since trade credit is more expensive than bank credit, it is optimal to take the least possible amount that guarantees a credible commitment. The first line of Figure 6 shows the share of inputs bought on account, L_S/I_K for different levels of α. If the cost of collusion is sufficiently high, $\alpha^*(\gamma^*) \leq \alpha \leq 1$, this share is equal to the scrap value of inputs β multiplied by the share of collateral accruing to the supplier

	Costly collusion $\alpha(\gamma^*) \le \alpha \le 1$	Medium-cost collusion $0 < \alpha < \alpha(\gamma^*)$	Costless collusion $\alpha = 0$
Trade credit - share of inputs bought on credit	$(1-\gamma^*)\beta$ **Independent of α**	$[1-\gamma^*(\alpha)]\beta$ **Decreasing in α**	β
Bank credit			
- share of inputs both on cash	$\gamma^* \beta$	$\gamma^*(\alpha)\beta$, **increasing in α**	$(1-\beta)$
- type on contract	**Collateralized (γ=γ*)**	**Collateralized (0<γ<γ*)**	**Uncollateralized (γ=0)**
Production level	**Point E**	**Point C**	**Close to point B**

Figure 6: Financing and investment decisions when the cost of collusion varies.

in case of default under the commitment contract, $1-\gamma^*$, thus independent of α. When the cost of collusion falls below $\alpha^*(\gamma^*)$ and keeps decreasing, the share of inputs bought on credit increases since the entrepreneur needs to increase the supplier's stake into the venture, $1-\gamma^*(\alpha)$, to credibly signal its commitment to the bank. In the extreme case of costless collusion ($\alpha = 0$), entrepreneur and supplier always cheat at the expense of the bank. Thus, the bank only offers an uncollateralized credit contract. The firm still uses trade credit but just to exploit the liquidation technology of the supplier in case of default, rather than as a commitment device. The amount of trade credit reaches its maximum, i.e. equal to the collateral value of capital inputs βI_K.

The pattern of the share of inputs paid for in cash (through bank credit) is specular to one bought on account and it is described in the second line of Fig. 6. The share is equal to the scrap value of inputs, β multiplied by the fraction of the collateral value accruing to the bank in case of default, γ. The share of input paid for in cash is constant for any $\alpha \ge \alpha^*(\gamma^*)$ and gets smaller when the cost of collusion decreases since the bank needs to reduce its stake in the venture to avoid collusion. For any positive cost of collusion, the bank offers a collateralized contract. When $\alpha = 0$, the bank only offers an uncollateralized loan that is used to finance $(1-\beta)$ of the tangible inputs and all the intangibles.

If we define the benefits of trade credit as the difference between the entrepreneur's profits under the collusion-proof contract and the profits under the no-commitment contract, we get that these benefits are larger the smaller the cost of collusion. Fig. 5 gives an indication of how profits change for different costs of collusion. In particular, for any $\alpha^*(\gamma^*) \leq \alpha \leq 1$, the equilibrium is point E, very far from point B. Here the benefits of trade credit are the highest ones. When the cost of collusion falls below $\alpha^*(\gamma^*)$, the equilibrium is represented by point C, which gets closer to point B the easier it is to collude. In the extreme situation in which collusion is costless, the benefits are low or equal zero, depending whether it is still profitable to use trade credit for its liquidation value. The equilibrium is located in B or very close to that.

Finally, notice that different production levels correspond to different degrees of input tangibility as represented in Fig. 5. For any $\alpha^*(\gamma^*) \leq \alpha \leq 1$, the degree of input tangibility is greater than 1, independent on α and represented by the slope of the expansion path going through point E. The input combination is biased towards capital inputs due to their lower financing cost implied by their collateral value. When the cost of collusion falls below $\alpha^*(\gamma^*)$, input tangibility decreases as shown by the slope of the expansion path going through point C. The lower the cost of collusion, the further downwards the equilibrium point C will be, and the lower the degree of input tangibility. In the extreme case of costless collusion, if we assume that the actual costs of trade and bank credit are equal ($r_S = \frac{r_B}{p}$), the degree of input tangibility is 1 (slope of the expansion path going through point B).

The above discussion allows us to get the following predictions:

Prediction 5. *The share of inputs bought on account is decreasing in the cost of collusion.*

Prediction 6. *The share of inputs paid for in cash, through bank credit, is increasing in the cost of collusion.*

Prediction 7. *The benefits of using trade credit are larger the costlier it is to collude.*

Prediction 8. *Firms use technologies more intensive in tangible assets the costlier it is to collude.*

The cost of reaching a collusive agreement could be interpreted as the amount of rents to be given to the supplier to induce him to collude. These rents would reflect the bargaining power of the supplier, that could depend on product characteristics. For example, if the good supplied is standardized and therefore easy to be replaced by a similar product of a competing supplier, we expect a lower rent to be distributed to the supplier and therefore a lower cost of collusion. The opposite argument holds if the product is customized, i.e. it is made to the customer's unique specification. In this case, the bargaining power of the supplier is higher and therefore a larger rent should be shared with him in case of collusion. If restated, Predictions 5-8 could then identify relations between the financing and investment decisions on the one hand and the structure of the supplier market or the product characteristics on the other hand. These predictions are still empirically unexplored.[19]

6 Discussion

In this section we discuss the robustness of our theoretical setting. Subsection 6.1 investigates whether alternative bank contracts could solve the commitment problem. Subsection 6.2 focuses on the degree of information sharing between supplier and bank. Subsection 6.3 discusses the assumption of exclusive lending relation. Subsection 6.4 questions the role of the supplier as an informed lender and provides alternative interpretations. Subsection 6.5 discusses the seniority rule implied by the optimal contract.

6.1 Are there alternative bank contracts able to solve the commitment problem?

In the previous sections, we have shown the following: First, any collateralized bank contract (either fully or partially) is plagued by a commitment problem that makes this contract unviable for contracting parties. Second, the uncollateralized contract removes this problem but also eliminates the benefits of collateralizing inputs. Third, one way to remove the commitment problem while keeping the benefits of collateralized bank loan is to involve the supplier in the lending relation. In this section, we investigate whether alternative contracts could solve the commitment problem.

In our setting, the entrepreneur has an incentive to change the input combination after the loan has been provided because the ex-ante input price ratio is different from the ex-post one, the difference being the collateral value of capital inputs. It follows that a contract where in case of default inputs are not pledged to creditors but are seized by the entrepreneur and the bank gets paid only in the good state would completely realign the entrepreneur's incentives, by making the ex-ante and ex-post

[19] The degree of bargaining power of the supplier could affect the terms at which trade credit is offered, i.e., its price, rather than the cost of collusion. The lower the competition, the higher the supplier's market power, the higher the interest rate charged on trade credit.

input prices equal. The optimization problem corresponding to this contract would be the same as the optimization problem with investment observability (see the benchmark case in Section 2). The crucial question here is whether this contract is viable or not. In practice, when a firm is unable to repay its creditors, it can call for the bankruptcy stay. Any legal system automatically requires that all assets still in place be kept away from the entrepreneur and used to repay creditors following a line of priority that depends on the specific characteristics of the contract signed. Thus, in practice firm's assets are used to repay also unsecured claims. It follows that, even if the bank contract is not secured, the assets are *de facto* used to repay the bank. To conclude, a contract stating that in case of default all assets in place are seized by the entrepreneur is not viable.[20]

Alternatively, one could argue that there is no need to use trade credit, if the bank can condition the provision of financing to the number of employees. This could indeed work as a commitment device if the deviation consisted in reducing ex-post the number of employees and the labor market regulation did not allow workers to be easily fired. However, in our setting the entrepreneur has an incentive to increase (rather than reduce) the number of employees, and this can be easily done in any labor market.

Finally, the risk-shifting literature has investigated a problem of asset substitution in a context of conflict of interest between shareholders and bondholders. This literature has also identified instruments that can help mitigating this problem. One could then wonder whether these instruments could help solving our commitment problem as well. Smith and Warner (1979), among others, show that some types of asset substitution problems that induce excess risk taking can be mitigated by adding to the debt contract covenants that constrain investment or financing decisions. In our context, debt covenants that specify the investment projects the firm is allowed to undertake, for example requiring the manager to accept only projects with a given combination of inputs, could solve the commitment problem. Debt covenants are indeed largely used, but the observed constraints place few specific limitations on the investment policy, possibly because they would be expensive to enforce.

[20] One could then claim that, as long as the entrepreneur can hide these assets and resell them before the bankruptcy procedure starts, this contract would still work. This argument would be true if the entrepreneur could indeed divert all the assets. However, if only a share of inputs were diverted to private benefits, the incentive to deviate would still remain and our results would go trough.

Rather they impose restrictions on the firm's holding of financial investment, on the disposition of assets and on the firm's merger activity.

Green (1984) shows that the risk-shifting problem can be mitigated using convertible debt. Besides the fact that there is no unanimity on the properties of convertible debt contracts, the theoretical setting and the predictions of the risk-shifting literature are very different from ours.[21] In this literature, the asset substitution problem plagues a pure (uncollateralized) debt contract and is due to conflicts of interests between shareholders and bondholders. In our model, there are no conflicts of interests between shareholders (manager) and bondholders when the project is financed with a pure debt contract. It is the use of collateral that introduces an asset substitution problem. This difference is crucial and makes the convertible debt contract not a viable instrument to solve our commitment problem.

6.2 How much information sharing between financiers?

So far, we have shown that trade credit can be used by the entrepreneur to signal his willingness to stick to the ex-ante efficient bank contract. Once the bank gets this signal, it offers a collateralized loan. Does our story imply that the bank needs to observe the amount of trade credit taken or even the full properties of the supplier-entrepreneur contract? How much information sharing between bank and supplier is needed to access a collateralized bank loan?

In our model, the bank only needs to know that there are suppliers willing to lend money to their customers. All the other relevant information, including the specific amount of trade credit offered, is inferred by the bank by solving the entrepreneur's optimization problem. The intuition is the following. In period $t = 1$, bank and supplier offer a menu of incentive compatible contracts (see the time-line in Fig. 1). Each contract offer solves the entrepreneur's optimization problem for any given combination of bank and trade credit.[22] The entrepreneur then chooses the contract that minimizes the total costs of external financing. Thus, from the choice of a given bank contract, the bank immediately infers

[21] For example, Eisdorfer (2009) shows that convertible debt, in contrast to its perceived role, can produce shareholders' risk-shifting incentives and provides evidence in line with his argument.

[22] Contracts merely satisfying the bank's participation constraint would not be incentive compatible for the entrepreneur and therefore are not offered.

how much trade credit will be taken by the entrepreneur as well as the repayment to the supplier in the good and in the bad state.

While the degree of information sharing assumed in our setting is very limited, ample anecdotal evidence suggest that, when considering a loan application, banks usually ask for financial information and for balance sheet data including data on account payables (Garcia-Appendini, 2010). In addition, banks may have access to valuable information about trade credit through credit bureaus. For example, the firm's trade credit payment history is routinely included in the credit reports provided by credit bureaus or by business information exchanges such as the Dun & Bradstreet Corporation (Kallberg and Udell 2003). This evidence suggests that in practice banks are likely to have more information about suppliers' payment practices than that implied by our model.

6.3 Exclusive lending relations

In Section 1 we have assumed that the entrepreneur has exclusive lending relations with bank and supplier. Is exclusivity a necessary condition? What happens if the entrepreneur buys inputs from several suppliers? In this case, the optimal contract implies that each supplier loan is closely tied to the amount of inputs provided by a given supplier rather than to the total amount of inputs. Thus, each supplier only observes a fraction of the input purchases, the one corresponding to his provision. Unless we assume some sort of information sharing among suppliers, none of the suppliers knows whether the optimal input combination has been purchased. Thus, the entrepreneur cannot refrain from deviating and trade credit no longer solves the commitment problem.

To better get the intuition, suppose that the optimal input combination requires 10 units of capital inputs and 8 workers. The amount of trade credit that would remove the entrepreneur's incentive to deviate is 6, and the bank is willing to finance the rest of input purchases (4 capital inputs plus 8 workers). If there is only one supplier our story goes through. Suppose there are two suppliers: one selling 6 inputs on credit and having the right to get 10% of the collateral value of these inputs back in case of default; the other one selling the 4 inputs in cash. Rather than sticking to the efficient contract, the entrepreneur could just take the 6 inputs on credit from the first supplier and use the 4 loan obtained from the bank to hire workers rather than buy the remaining 4 capital inputs. The supplier would not be aware of the deviation and he would still break-even. The bank instead would get negative expected profits, since the total collateral value would be lower than expected. Anticipating the entrepreneur's deviation, the bank would never offer a collateralized contract. With more than one supplier, trade credit no longer works as a commitment device, since the supplier providing trade credit would be unable to detect and avoid deviation.

A similar reasoning applies to the bank lending relation. With several banks, the aggregate loan size obtained from all the banks could be larger than the incentive-compatible one, giving the entrepreneur an incentive to deviate. Anticipating this, each bank would offer an uncollateralized contract and trade credit would play no role in providing commitment. Thus, as with the supplier, we need exclusivity in the bank lending relation, unless we assume a sort of information sharing among banks such that each bank knows the number of banks involved in the project financing and the amount lent by each bank.

To conclude, in our setting exclusivity of the lending relation is necessary to have trade credit working as a commitment device, unless we assume a sort of information sharing among banks and among suppliers.

6.4 Is the supplier the only informed lender?

In our analysis, we assume the costly informed lender to be the supplier and the cheaper uninformed financier to be the bank. Other interpretations are however possible. We could assume that both financiers are specialized intermediaries but with different sets of information. For example, the costly informed financier is a relationship lender (also called local lender), while the lender providing cheaper funding with no or less information is an arm's-length lender. If we follow this interpretation, our results would predict that, while local lenders can offer collateralized credit contracts, arm's-length lenders provide unsecured financing, in line with the existing evidence (Avery *et al.*, 1998; Zucherman, 1996; Frame *et al.*, 2001, 2004).

The reason why we do not follow this interpretation is related to the nature of the unobservability problem. A relationship lender is assumed to have a comparative advantage in collecting customer-specific information through multiple interactions (Boot, 2000). These features are mostly relevant when the unobservability concerns firms' characteristics, such as project quality or entrepreneurial effort. When the unobservability concerns investment, as in our setting, a specialized financial intermediary is not necessary: The relevant information is observed costlessly by the supplier by providing the input, with no need for multiple interactions. For this reason, the bank-supplier interpretation seems more natural in our framework.

Alternatively, we could interpret the informed lender as a lessor that receives a fee from the borrower for the use of capital inputs and retains the ownership on those assets. Although appealing, this interpretation is flawed as the lessor fails to solve the unobservability problem. To see why, consider the case analyzed in Section 3, in which the presence of the supplier allows the bank to offer a partially secured contract. Can the lessor play the same role as the supplier? Consider a contract for the financing of a given amount of labor units and several units of the capital input, say printers. The contract might state that a certain number of printers (say, 20) will be financed through a secured contract with the bank, while the remaining 30 through a leasing contract. Upon receiving the loan from the bank, the entrepreneur has an incentive to reduce the number of printers purchased with bank financing, say from 20 to 10. The lessor will not stop the entrepreneur from buying a lower number of printers for two reasons: First, he does not observe the entrepreneur's reduction of 10 units of printer purchases. Second, even if he does, he has no incentive to stop the entrepreneur's opportunistic behavior: Being still the owner of his 30 printers, his return in defaulting states is never jeopardized by this reduction. Anticipating the entrepreneur's deviation, the bank will never propose such an agreement. This unilateral deviation from the contracted input mix is not feasible if the informed party is the supplier: He would observe it and prevent it from happening, in order to preserve the return he is promised in defaulting states, which is a fraction of the 50 initial printers.

Thus, while under a leasing contract a profitable unilateral deviation by the entrepreneur is always possible, under a trade credit contract such deviation is never possible unless the entrepreneur colludes with the supplier. This is the very reason why trade credit can provide commitment.[23]

In the extreme case in which reaching a collusive agreement is costless, there is no secured contract

[23] Of course, if the leasing company supplies all the capital inputs and the bank finances labor purchases, the commitment problem with the bank is removed since no collateral assets are pledged to the bank.

the bank is willing to offer (see Section 4). Dealing with an informed party serves only to extract value from unused assets in default (and not to solve the commitment problem). In this case, both a leasing and a trade credit contract may serve this scope. Whether the entrepreneur will then use the supplier or the lessor as a liquidator will depend on the specific provisions of the bankruptcy codes and on the comparison between the liquidation abilities of the two financiers.[24]

6.5 Is there an optimal seniority?

In standard contract theory with multiple financiers, the creditor with a monitoring advantage should be junior so as to have a stronger incentive to monitor the project and gather the relevant information. Applying this finding to our context, one would expect the bank to be senior and the supplier junior. However, our optimal contract requires both the bank and the supplier to be senior. The reason for this discrepancy is the following. There is no need to give the supplier stronger incentive to monitor the project, given that he becomes informed simply by selling the inputs. Nevertheless he must be given contractual seniority, otherwise his information becomes irrelevant and trade credit fails to generate commitment. Indeed, by making him junior, the supplier has no liquidation rights in the bad state. Being paid only in the good state, the supplier has no longer the incentives to stop the entrepreneur from deviating from the efficient contract. Trade credit works as a commitment device only if the supplier has a stake in the bad state.

7 Conclusions

Collateral is largely used in bank lending. Prior literature has mostly analyzed how collateral affects external financing assuming investment in pledgeable assets as given. This paper investigates optimal financing contracts when investment in pledgeable assets is endogenous and not observable to financiers. We show that pledging an asset as collateral not necessarily increases the firm debt capacity. Collateral might indeed introduce in the bank lending relationship a problem of moral hazard in the form of asset substitution. Our result highlights investment contractibility as a crucial determinant of debt capacity and it helps explaining why some pledgeable assets have low debt capacity, despite

[24] A liquidation advantage of leasing relative to secured lending has been modeled by Eisfeldt and Rampini (2009).

their high resalability, and the existence of a liquid secondary market.

Our paper also prompts a role for alternative financiers with some information advantage vis-a-vis the bank. We argue that the supplier of the pledgeable input is the natural candidate to solve the contract incompleteness with the bank, since he observes whether the investment has taken place. Thus, trade credit can be used as a commitment device to restore the benefits of bank secured lending. This commitment effect is robust to the possibility of a collusive agreement between entrepreneur and supplier. More specifically, the amount of trade credit that guarantees commitment and avoids collusion is larger the cheaper it is to collude. The paper delivers new testable relations among financing decisions (including both the amount of trade and bank credit and the type of contract), input choices and the cost of collusion. We leave the empirical verification of these testable predictions to future work.

A Appendix

Proof of Proposition 1. The first-best investment in capital and labor satisfies the following FOC's:

$$p\frac{\partial f(\cdot)}{\partial I_K} = r_B - (1-p)\beta, \tag{17}$$

$$p\frac{\partial f(\cdot)}{\partial I_N} = r_B, \tag{18}$$

obtained differentiating the reduced form objective function (4) wrt I_K and I_N. These give I_K^{FB}, I_N^{FB}. By the homotheticity of the production function the optimal input ratio $\left(I_K^{FB}/I_N^{FB}\right)$ is a constant implicitly defined by the input price ratio: $r_B/\left[r_B - (1-p)\beta\right] > 1$. Using I_K^{FB}, I_N^{FB} in constraints (3) and (2) we obtain the optimal bank loan, L_B^{FB}, and the bank repayment, R_B^{FB}, respectively:

$$L_B^{FB} = I_N^{FB} + I_K^{FB}, \tag{19}$$

$$R_B^{FB} = \frac{1}{p}\left[\left(I_N^{FB} + I_K^{FB}\right) r_B - (1-p)\beta I_K^{FB}\right]. \tag{20}$$

To capture the role of collateral in lending, we carry out a comparative statics analysis on β. Using $I_K^{FB}(\beta)$ and $I_N^{FB}(\beta)$ in (17) and (18), these are satisfied as identity. Differentiating wrt β,

$$\begin{aligned} p\frac{\partial^2 f\left(I_K^{FB}(\beta), I_N^{FB}(\beta)\right)}{\partial I_K^2}\frac{\partial I_K^{FB}}{\partial \beta} + p\frac{\partial^2 f\left(I_K^{FB}(\beta), I_N^{FB}(\beta)\right)}{\partial I_K \partial I_N}\frac{\partial I_N^{FB}}{\partial \beta} - (1-p) &= 0 \\ p\frac{\partial^2 f\left(I_K^{FB}(\beta), I_N^{FB}(\beta)\right)}{\partial I_N \partial I_K}\frac{\partial I_K^{FB}}{\partial \beta} + p\frac{\partial^2 f\left(I_K^{FB}(\beta), I_N^{FB}(\beta)\right)}{\partial I_N^2}\frac{\partial I_N^{FB}}{\partial \beta} &= 0 \end{aligned}$$

whence:

$$\begin{bmatrix} \dfrac{\partial I_K^{FB}}{\partial \beta} \\ \dfrac{\partial I_N^{FB}}{\partial \beta} \end{bmatrix} = p\begin{bmatrix} f_{KK} & f_{KN} \\ f_{NK} & f_{NN} \end{bmatrix}^{-1}\begin{bmatrix} 1-p \\ 0 \end{bmatrix}.$$

Solving, we obtain:

$$\frac{\partial I_K^{FB}}{\partial \beta} = \frac{p(1-p) f_{NN}}{f_{KK} f_{NN} - f_{KN}^2} > 0,$$

$$\frac{\partial I_N^{FB}}{\partial \beta} = -\frac{p(1-p) f_{NK}}{f_{KK} f_{NN} - f_{KN}^2} > 0.$$

which shows that both I_K and I_N are increasing in the resale value of the asset. ■

Proof of Proposition 2. The solution to this problem proceeds in two steps. We first show that the firm-bank collateralized credit contract is time-inconsistent, i.e., the entrepreneur has an incentive to deviate at the expense of the bank. The input combination chosen upon been granted the loan involves a lower investment in the collateralizable input and thus insufficient low state returns to the bank to break even. We then show that the bank may prevent losses by offering an unsecured credit contract with a lower loan and a subsequent efficiency loss.

1. Time inconsistency of the fully secured (first-best) contract.

The first step consists in showing that it is profit maximizing for the firm to breach the terms of the fully collateralized contract. Thus, we need to show that the input combination chosen under deviation involves higher profits.

Consider programme $\hat{\mathcal{P}}$ faced by the firm which has obtained a loan L_B^{FB} and must repay R_B^{FB} in the good state. Solving the resource constraint (7) for I_N

$$I_N \left(L_B^{FB}, I_K\right) = L_B^{FB} - I_K \tag{21}$$

and substituting out in the objective function (5), the firm's problem becomes:

$$\max_{I_K} \; pf\left(I_K, L_B^{FB} - I_K\right) - pR_B^{FB} \tag{22}$$

whence, differentiating wrt I_K

$$p\left(\frac{\partial f(\cdot)}{\partial I_K} + \frac{\partial f(\cdot)}{\partial I_N}\frac{dI_N}{dI_K}\right) = 0 \tag{23}$$

From (21), $\frac{dI_N}{dI_K} = -1$, whence $\frac{\partial f(\cdot)}{\partial I_K} / \frac{\partial f(\cdot)}{\partial I_N} = 1$. The input price ratio is equal to 1 which is lower than the first-best/commitment input price ratio: $r_B / \left[r_B - (1-p)\beta\right]$. Thus, the relative price of capital (labor) increases (decreases). This implies that the optimal input ratio is lower than the first-best input ratio - i.e., lies on a flatter expansion path. However, we don't know which is the actual amount of the two inputs. Determining the level of the two inputs, and thus the effect on input demands of an increase in the price of input I_K and a decrease in the relative price of I_N while keeping the loan constant,[25] amounts to work out the Slutsky compensated demands for inputs I_K and I_N. Since, by the quasi-concavity of the production function, the own-price effect is non-positive, the demand for I_K decreases. Because the loan is constant and equal to L_B^{FB} and the firm uses only two inputs, the cross-price effect is non-negative, i.e., the demand for input I_N must increase. In particular, solving (23) and using (21), we get: $\hat{I}_K\left(L_B^{FB}\right) < I_K^{FB}, \hat{I}_N\left(L_B^{FB}\right) > I_N^{FB}$, with $\hat{I}_K(\cdot) + \hat{I}_N(\cdot) = L_B^{FB}$.

Last, to determine the effect that deviation has on production, notice that the new optimum lies on an isocost line that crosses the initial optimum -with the same loan L_B^{FB} the firm can afford the initial input mix- but is flatter than the first-best isocost line: the input price ratio is $1 < r_B / \left[r_B - (1-p)\beta\right]$. By the convexity of isoquants, this implies a shift on a higher isoquant and thus larger production. Since the cost component of the profit function is the same under the two cases (R_B^{FB}), profits are higher under deviation. This proves that deviating is profit-maximizing for the entrepreneur and that the initial contract is not incentive-compatible (i.e., is time-inconsistent). However, the decrease in I_K implies that in default the collateral value of the input is insufficient to repay the bank, which does

[25] This implies that at the new input prices it must be possible for the firm to afford the initial input combination.

not break even. Anticipating this, the bank offers an unsecured credit contract thereby reducing the loan provided.

2. Determination of the unsecured bank contract.

In the unsecured contract, the entrepreneur chooses I_K, I_N, R_B to maximize (1) subject to the participation constraint (2) with $C = 0$, and to the resource constraint (3). Solving (2) for R_B and using L_B from the resource constraint (3), the objective function (1) becomes:

$$\max_{I_K, I_N} pf(I_K, I_N) - (I_N + I_K) r_B.$$

The optimal input combination satisfies the following first-order conditions:

$$p\frac{\partial f(\cdot)}{\partial I_K} = r_B \tag{24}$$

$$p\frac{\partial f(\cdot)}{\partial I_N} = r_B \tag{25}$$

which give I_K^U, I_N^U. Notice that again the input price ratio is equal to 1, which implies, by the homotheticity of the production function, that the input combination lies along the same expansion path as the deviation contract. However, relative to the deviation contract, the investment in each input is now lower. This can be seen by considering that the FOC's of the secured contract, (17) and (18), coincide with those of the unsecured one, (24) and (25), when β tends to zero. Thus, evaluating the effect on the level of the investment and the loan size of switching from a secured to an unsecured contract amounts to carry out a comparative statics analysis on β as in the Proof of Proposition 1. Since by Proposition 1, both I_K and I_N are increasing in β, we deduce that $I_K^U < I_K^{FB}$, $I_N^U < I_N^{FB}$ and, by constraint (3), $I_K^U + I_N^U = L_B^U < L_B^{FB}$. Last, using L_B^U in (2) gives $R_B^U = \frac{1}{p} r_B L_B^U$.

This confirms the intuition that the amount of external funds raised under unsecured financing, L_B^U, is lower than the amount available for secured financing L_B^{FB} and used in the deviation contract. ■

Proof of Proposition 3. Consider programme $\mathcal{P}^\gamma$ defined on page 14. Substituting the binding constraints in the objective function gives

$$\max_{I_K, I_N} EP = pf(I_K, I_N) - r_B \left[I_N + I_K - (1-\gamma)\frac{\beta}{r_S} I_K \right] + [(1-p)\gamma - p(1-\gamma)] \beta I_K$$

with FOC's:

$$p\frac{\partial f(\cdot)}{\partial I_K} = r_B \left(1 - (1-\gamma)\frac{\beta}{r_S} \right) - \beta(\gamma - p) \tag{26}$$

$$p\frac{\partial f(\cdot)}{\partial I_N} = r_B \tag{27}$$

Solving (26) and (27) we obtain the investment in each input as a function of γ, i.e., $I_K^*(\gamma), I_N^*(\gamma)$. Notice the difference with the unsecured contract, where factors have the same marginal productivity, as shown by (24) and (25). Under the collateralized contract, the price of the capital input decreases, while the price of labor remains unchanged. Using (26) and (27), we get the new input price ratio as $r_B / \left[r_B \left(1 - (1-\gamma)\beta / r_S\right) + (p-\gamma)\beta \right]$. By Assumption 1, the ratio is no less than 1 and increasing in γ.[26] It follows that the higher γ, the higher the input price ratio and the higher the capital-labor ratio, which proves part *d.* of Prop. 3.

To prove part *a.* of Prop. 3, we substitute $I_K^*(\gamma)$ in constraints (10) and (12) we obtain $L_S^*(\gamma) = (1-\gamma)\beta I_K^*(\gamma)/r_S$. The share of inputs bought on credit is equal to:

$$\frac{L_S^*}{I_K^*} = \frac{\beta(1-\gamma)}{r_S},$$

and

$$\frac{\partial\left(\frac{L_S^*}{I_K^*}(\gamma)\right)}{\partial\gamma} = -\frac{\beta}{r_S} < 0,$$

which proves part *a.* of Prop. 3.

Using $L_S^*(\gamma)$ and $I_K^*(\gamma)$ in constraints (3) and (10) we get respectively $L_B^*(\gamma) = I_K^*(\gamma) + I_N^*(\gamma) - (1-\gamma)\beta I_K^*(\gamma)/r_S$, $R_B^*(\gamma) = \left[L_B^*(\gamma) r_B - (1-p)\gamma\beta I_K^*(\gamma)\right]/p$ with:

$$\begin{aligned}
\frac{\partial L_B^*(\gamma)}{\partial\gamma} &= \frac{\partial I_N^*}{\partial\gamma} + \frac{\partial I_K^*}{\partial\gamma} - \frac{\beta}{r_S}(1-\gamma)\frac{\partial I_K^*}{\partial\gamma} + \frac{\beta}{r_S} I_K^*(\gamma) \\
&= \frac{\partial I_N^*}{\partial\gamma} + \left(1 - \frac{\beta}{r_S}(1-\gamma)\right)\frac{\partial I_K^*}{\partial\gamma} + \frac{\beta}{r_S} I_K^*(\gamma)
\end{aligned}$$

To determine the sign of $\partial L_B^*(\gamma)/\partial\gamma$, we carry out a comparative statics analysis on I_K^* and I_N^* and work out $\partial I_K^*/\partial\gamma$ and $\partial I_N^*/\partial\gamma$. Using $I_K^*(\gamma), I_N^*(\gamma)$ in (26) and (27), these are satisfied as identity. Differentiating wrt γ,

$$\begin{aligned}
p\frac{\partial^2 f\left(I_K^*(\gamma), I_N^*(\gamma)\right)}{\partial I_K^2}\frac{\partial I_K^*}{\partial\gamma} + p\frac{\partial^2 f\left(I_K^*(\gamma), I_N^*(\gamma)\right)}{\partial I_K \partial I_N}\frac{\partial I_N^*}{\partial\gamma} - \beta\left(\frac{r_B}{r_S} - 1\right) &= 0 \\
p\frac{\partial^2 f\left(I_K^*(\gamma), I_N^*(\gamma)\right)}{\partial I_N \partial I_K}\frac{\partial I_K^*}{\partial\gamma} + p\frac{\partial^2 f\left(I_K^*(\gamma), I_N^*(\gamma)\right)}{\partial I_N^2}\frac{\partial I_N^*}{\partial\gamma} &= 0
\end{aligned}$$

whence

$$\begin{bmatrix} \frac{\partial I_K^*}{\partial\gamma} \\ \frac{\partial I_N^*}{\partial\gamma} \end{bmatrix} = p\begin{bmatrix} f_{KK} & f_{KN} \\ f_{NK} & f_{NN} \end{bmatrix}^{-1} \begin{bmatrix} \beta\left(\frac{r_B}{r_S} - 1\right) \\ 0 \end{bmatrix}.$$

[26] For $\gamma = 0$, this is equal to $\frac{r_B}{r_B - \frac{p}{r_S}\left(\frac{r_B}{p} - r_S\right)\beta} \geq 1$ by Assumption 1. For $\gamma = 1$, the input price ratio is equal to its commitment value: $\frac{r_B}{r_B - (1-p)\beta} > 1$.

Solving, we obtain:

$$\frac{\partial I_K^*}{\partial \gamma} = \frac{p\beta\left(\frac{r_B}{r_S}-1\right)f_{NN}}{f_{KK}f_{NN}-f_{KN}^2} > 0$$

$$\frac{\partial I_N^*}{\partial \gamma} = -\frac{p\beta\left(\frac{r_B}{r_S}-1\right)f_{NK}}{f_{KK}f_{NN}-f_{KN}^2} > 0.$$

Substituting the previous two derivatives into $\partial L_B^*(\gamma)/\partial\gamma$, we get:

$$\begin{aligned}\frac{\partial L_B^*(\gamma)}{\partial \gamma} &= -p\beta\left(\frac{r_B}{r_S}-1\right)\frac{f_{NK}}{f_{KK}f_{NN}-f_{KN}^2} + \left(1-\frac{\beta}{r_S}(1-\gamma)\right)p\beta\left(\frac{r_B}{r_S}-1\right)\frac{f_{NN}}{f_{KK}f_{NN}-f_{KN}^2} + \frac{\beta}{r_S}I_K^*(\gamma)\\ &= \frac{p\beta}{f_{KK}f_{NN}-f_{KN}^2}\left(1-\frac{r_B}{r_S}\right)\left(f_{NK}-\left(1-\frac{\beta}{r_S}(1-\gamma)\right)f_{NN}\right)+\frac{\beta}{r_S}I_K^*(\gamma)\end{aligned}$$

which, given the assumptions on the technology, is certainly positive. This concludes the proof of part *b.* of Prop. 3.

To prove part *c.* of Prop. 3, we need to show that expected profits are increasing in γ and β.

$$\begin{aligned}\Pi^*(\gamma) = p\left[f\left(I_K^*, I_N^*\right) - R_B^* - R_S^*\right] = \\ pf\left(I_K^*, I_N^*\right) - \left(I_K^* + I_N^*\right)r_B + \left[\frac{r_B}{r_S}(1-\gamma)+(\gamma-p)\right]\beta I_K^*.\end{aligned} \tag{28}$$

By the envelope theorem, they are increasing in γ:

$$\frac{\partial \Pi^*(\gamma)}{\partial\gamma} = \left(1-\frac{r_B}{r_S}\right)\beta I_K^*(\gamma) > 0$$

and convex: $\frac{\partial^2\Pi^*(\gamma)}{\partial\gamma^2} = \left(1-\frac{r_B}{r_S}\right)\beta\frac{\partial I_K^*(\gamma)}{\partial\gamma} > 0$. Moreover, using Assn. 1, they are also increasing in β :

$$\frac{\partial\Pi^*}{\partial\beta} = \left(\frac{r_B}{p} - r_S\left(1-\frac{\gamma}{p}\right) - \gamma\frac{r_B}{p}\right)\frac{p}{r_S}I_K^* > 0.$$

■

Proof of Proposition 4. To prove this, we need to solve programme $\mathcal{P}^\gamma$, defined in the proof of Proposition 3, under the no deviation condition (13), i.e. $\Pi^{\mathcal{D}}(\gamma) \leq \Pi^*(\gamma)$. We proceed in three steps. First, we derive the profits under deviation from the commitment contract as a function of γ, $\Pi^{\mathcal{D}}(\gamma)$. Second, we derive the no-deviation condition as a function of γ. Third, we show that there is a unique $\gamma = \gamma^* \in [0,1]$ that solves the no deviation condition (13).

First, $\Pi^{\mathcal{D}}(\gamma)$ is the solution to the following problem $\mathcal{P}^{\mathcal{D}}$:

$$\max_{I_K, I_N} EP = p\left[f\left(I_K, I_N\right) - R_B\right], \tag{29}$$

$$\text{s.t. } R_B \geq R_B^*(\gamma), \tag{30}$$

$$L_B^*(\gamma) \geq I_N + I_K, \tag{31}$$

Substituting constraints (30) and (31) into the objective function, we get:

$$\max_{I_K} \; p\left[f\left(I_K, L_B^*(\gamma) - I_K\right) - R_B^*(\gamma)\right] \tag{32}$$

with FOC

$$p\left(\frac{\partial f(\cdot)}{\partial I_K} + \frac{\partial f(\cdot)}{\partial I_N}\frac{dI_N}{dI_K}\right) = 0$$

Using $\frac{dI_N}{dI_K} = -1$ from the resource constraint (31), the FOC becomes:

$$\frac{\partial f(\cdot)}{\partial I_K} = \frac{\partial f(\cdot)}{\partial I_N}. \tag{33}$$

Condition (33) along with the resource constraint (31) gives

$$\begin{aligned} I_K^{\mathcal{D}} &\equiv I_K^{\mathcal{D}}\left(L_B^*(\gamma)\right), \\ I_N^{\mathcal{D}} &\equiv I_N^{\mathcal{D}}\left(L_B^*(\gamma)\right) \end{aligned}$$

and equal to each other.

Using $I_N^{\mathcal{D}}$ in the objective function gives the value function:

$$\Pi^{\mathcal{D}}(\gamma) = p\left[f\left(I_K^{\mathcal{D}}, L_B^*(\gamma) - I_K^{\mathcal{D}}\right) - R_B^*(\gamma)\right], \tag{34}$$

as in Definition 1.

Second, using the previous result, we can express the no deviation condition (13) as follows:

$$\Pi^{\mathcal{D}}(\gamma) - \Pi^*(\gamma) = p\left[f\left(I_K^{\mathcal{D}}, L_B^*(\gamma) - I_K^{\mathcal{D}}\right) - R_B^*(\gamma)\right] - p\left[f\left(I_K^*, L_B^*(\gamma) - I_K^*\right) - R_B^* - R_S^*(\gamma)\right] \leq 0$$

Third, to prove that there exists a unique $\gamma = \gamma^* \in [0,1]$ that solves the previous no deviation condition, we need to show that:

1. $\Pi^{\mathcal{D}}(0) - \Pi^*(0) < 0$;
2. $\Pi^{\mathcal{D}}(1) - \Pi^*(1) > 0$;
3. $\frac{\partial \Pi^*}{\partial \gamma}, \frac{\partial \Pi^{\mathcal{D}}}{\partial \gamma} > 0$;
4. $\frac{\partial \Pi^{\mathcal{D}}}{\partial \gamma} - \frac{\partial \Pi^*}{\partial \gamma} > 0$.

1. $\Pi^{\mathcal{D}}(0) - \Pi^*(0) < 0$.

$\Pi^*(0)$ can be worked out by considering that, when $\gamma = 0$, the firm obtains credit from the bank with an unsecured contract, and from the supplier with a fully secured contract. $\Pi^{\mathcal{D}}(0)$ instead can be obtained by considering that, by observing the firm deviating, the supplier stops financing altogether and the firm obtains only an unsecured credit contract from the bank, whose properties have been discussed in Proposition 2. Under Assumption (1), the profits obtained with the uncollateralized credit contract ($\Pi^{\mathcal{D}}(0)$) are lower than those obtained under a contract with $\gamma = 0$ and supplier secured financing ($\Pi^*(0)$), which proves our claim.

2. $\Pi^{\mathcal{D}}(1) - \Pi^*(1) > 0$.

When $\gamma = 1$, we are in the first-best contract, whose properties have been discussed in Proposition 1. By Proposition 2, profits under deviation exceed profits under commitment, which proves our second claim.

3. $\frac{\partial \Pi^*}{\partial \gamma}, \frac{\partial \Pi^{\mathcal{D}}}{\partial \gamma} > 0$.

That $\frac{\partial \Pi^*}{\partial \gamma} > 0$ has already been proved in the Proof of Proposition (3). To prove that $\frac{\partial \Pi^{\mathcal{D}}}{\partial \gamma} > 0$, differentiate (34) wrt γ, which by the envelope theorem is equal to:

$$\frac{\partial \Pi^{\mathcal{D}}}{\partial \gamma} = p\left(\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{D}}}\frac{\partial I_N^{\mathcal{D}}}{\partial L_B^*}\frac{\partial L_B^*}{\partial \gamma} - \frac{\partial R_B^*}{\partial \gamma}\right).$$

Using $L_B^*(\gamma)$ and $R_B^*(\gamma)$ from Proposition 3 and constraint (7), the above can be written as

$$\frac{\partial \Pi^{\mathcal{D}}}{\partial \gamma} = p\left[\frac{\partial f}{\partial I_N^{\mathcal{D}}}\left(\frac{\partial L_B^*(\gamma)}{\partial \gamma}\right) - \frac{1}{p}\left(\frac{\partial L_B^*(\gamma)}{\partial \gamma}r_B - (1-p)\beta I_K^*(\gamma) - (1-p)\gamma\beta\frac{\partial I_K^*}{\partial \gamma}\right)\right],$$

where $\frac{\partial L_B^*(\gamma)}{\partial \gamma} = \frac{\partial I_N^*}{\partial \gamma} + \frac{\partial I_K^*}{\partial \gamma}\left(1 - \frac{1}{r_S}(1-\gamma)\beta\right) + \frac{\beta}{r_S}I_K^*(\gamma)$. Thus, after some manipulations, and using the equality of marginal productivities under deviation (33), we get:

$$\begin{aligned}\frac{\partial \Pi^{\mathcal{D}}}{\partial \gamma} &= \left(p\frac{\partial f}{\partial I_K^{\mathcal{D}}} - r_B + (1-p)\gamma\beta\right)\frac{\partial I_K^*}{\partial \gamma} + \left(p\frac{\partial f}{\partial I_K^{\mathcal{D}}} - r_B + (1-p)r_S\right)\frac{\beta}{r_S}I_K^*(\gamma) + \\ &\quad - \left(p\frac{\partial f}{\partial I_K^{\mathcal{D}}} - r_B\right)\frac{1}{r_S}(1-\gamma)\beta\frac{\partial I_K^*}{\partial \gamma} + \left(p\frac{\partial f}{\partial I_K^{\mathcal{D}}} - r_B\right)\frac{\partial I_N^*}{\partial \gamma}.\end{aligned} \tag{35}$$

The sign of the first term is positive. This can be inferred by considering that by (26) the marginal productivity of capital under commitment is $r_B\left(1 - (1-\gamma)\frac{\beta}{r_S}\right) - \beta(\gamma - p)$. Under Assumption 1, this is no less than $r_B - \gamma(1-p)\beta$. Since under deviation the reliance on capital decreases, the marginal productivity of capital under deviation increases above its commitment level and is higher than $r_B - \gamma(1-p)\beta$. This in turn implies that the second term is also positive ($r_B - (1-p)r_S < r_B - \gamma(1-p)\beta$).

The sign of the third term can be inferred by considering that the lower reliance on capital relative to the commitment level increases its marginal productivity above this level (implicitly defined by the RHS of (26): $p\frac{\partial f}{\partial I_K^{\mathcal{D}}} > r_B\left(1 - (1-\gamma)\frac{\beta}{r_S}\right) - \beta(\gamma - p)$). However, the highest possible marginal productivity obtains in the fully unsecured contract (implicitly defined by the RHS of (24): r_B). Thus $p\frac{\partial f}{\partial I_K^{\mathcal{D}}} \leq r_B$ and the third term in brackets is negative. By the minus sign outside it, the whole third term is positive.

Last, given that by the above argument $p\frac{\partial f}{\partial I_K^{\mathcal{D}}} - r_B \leq 0$, and givent that $\frac{\partial I_N^*}{\partial \gamma} > 0$, the sign of the fourth term is negative. However, comparing the last two terms of the derivative, sufficient condition for $\frac{\partial \Pi^{\mathcal{D}}}{\partial \gamma}$ to be positive is that:

$$\left(p\frac{\partial f}{\partial I_K^{\mathcal{D}}} - r_B\right)\left(\frac{\partial I_N^*}{\partial \gamma} - \frac{1}{r_S}(1-\gamma)\beta\frac{\partial I_K^*}{\partial \gamma}\right) = \left(p\frac{\partial f}{\partial I_K^{\mathcal{D}}} - r_B\right)\frac{p\beta\left(1-\frac{r_B}{r_S}\right)}{f_{KK}f_{NN}-f_{KN}^2}\left(f_{NK} + \frac{1}{r_S}(1-\gamma)\beta f_{NN}\right) \geq 0$$

which holds if $\left(f_{NK} + \frac{1}{r_S}(1-\gamma)\beta f_{NN}\right) < 0$. Thus, under mild conditions, profits from deviation (35) are increasing in γ.

4. $\frac{\partial \Pi^{\mathcal{D}}}{\partial \gamma} - \frac{\partial \Pi^*}{\partial \gamma} > 0$, i.e. the deviation rent (13) is increasing in γ.

$$\begin{aligned}\frac{\partial \Pi^{\mathcal{D}}}{\partial \gamma} - \frac{\partial \Pi^*}{\partial \gamma} &= \left(p\frac{\partial f(\cdot)}{\partial I_K^{\mathcal{D}}} - r_B\right)\left(\frac{\partial I_N^*}{\partial \gamma} - \frac{\beta}{r_S}(1-\gamma)\frac{\partial I_K^*}{\partial \gamma}\right) + \\ &\quad + \left(p\frac{\partial f(\cdot)}{\partial I_K^{\mathcal{D}}} - r_B + (1-p)\gamma\beta\right)\frac{\partial I_K^*}{\partial \gamma} + \left(p\frac{\partial f(\cdot)}{\partial I_K^{\mathcal{D}}} - pr_S\right)\frac{\beta}{r_S}I_K^*(\gamma)\end{aligned}$$

By the same arguments used to prove that (35) is positive, the first two terms are both positive. We need to determine the sign of the third term. This can be inferred by considering that by (26) $p\frac{\partial f(\cdot)}{\partial I_K^*} = r_B\left(1-(1-\gamma)\frac{\beta}{r_S}\right) - (\gamma - p)\beta$. Since under deviation the reliance on capital decreases, the marginal productivity of capital under deviation increases above its commitment level, i.e., $p\frac{\partial f(\cdot)}{\partial I_K^{\mathcal{D}}} \geq p\frac{\partial f(\cdot)}{\partial I_K^*}$. It follows that a sufficient condition for the third term to be positive is $r_B\left(1-(1-\gamma)\frac{\beta}{r_S}\right) - (\gamma-p)\beta \geq pr_S$. Rearranging the previous inequality, we get:

$$\frac{1}{r_S}\left[(r_B - pr_S)(r_S - \beta) - \gamma\beta(r_S - r_B)\right] \geq 0.$$

Since $(r_S - \beta) > (r_S - r_B)$, sufficient condition for the inequality to hold is that $(r_B - pr_S) \geq \gamma\beta$, which is a slightly stricter condition that the one implied by Assumption 1. ■

Proof of Proposition 5. The proof proceeds in two steps. First, we derive the profits under collusion as a function of γ, i.e. $\Pi^{\mathcal{C}}(\gamma)$ and the collusion rent as the difference between profits under collusion and under commitment. Second, we show that the collusion rent is positive for any γ, meaning that any firm-bank-supplier collateralized credit contract is prone to collusion between bank and supplier at the expense of the bank.

1. Determination of profits under collusion and collusion rent.

The ex-post optimization programme with firm-supplier collusion is given by problem $\mathcal{P}^{\mathcal{C}}$:

$$\max_{I_K, I_N, R_B, R_S} \; p\left[f(I_K, I_N) - R_B - R_S\right]$$

$$\text{s.t. } R_B \geq R_B^*(\gamma), \tag{36}$$

$$L_S + L_B^*(\gamma) \geq I_N + I_K, \tag{37}$$

and to constraint (14): $pR_S + (1-p)(1-\gamma)\beta I_K \geq L_S r_S$. $L_B^*(\gamma)$ and $R_B^*(\gamma)$ are the commitment values of the bank loan and the bank high state repayment as defined in Proposition 3. Constraint (36) requires the bank repayment in the high state be no less than the one promised in the commitment contract, i.e. $R_B^*(\gamma)$. The resource constraint (37) requires the total input expenditure be no higher than the bank loan obtained in the commitment contract plus the recontracted supplier's loan. Moreover, to guarantee that in the renegotiation the supplier acts as a lender, and not as a borrower or a liquidator, we impose the non-decreasing repayment condition (12) that ensures that payments to the supplier are non-negative and non-decreasing in returns. Thus, using $R_S = (1-\gamma)\beta I_K$ in the participation constraint (14), it gives $L_S = (1-\gamma)\beta I_K / r_S$. Replacing it in the resource constraint (37), we get: $\frac{(1-\gamma)}{r_S}\beta I_K + L_B^*(\gamma) \geq I_N + I_K$. Solving this expression for I_N, we get:

$$I_N = \left(\frac{\beta(1-\gamma)}{r_S} - 1\right) I_K + L_B^*(\gamma). \tag{38}$$

Last, using R_B from constraint (36) and I_N from condition (38), the objective function becomes:

$$\max_{I_K} \; p\left[f\left(I_K, \left(\frac{\beta(1-\gamma)}{r_S} - 1\right) I_K + L_B^*(\gamma)\right) - R_B^*(\gamma) - (1-\gamma)\beta I_K\right].$$

Differentiating wrt I_K gives:

$$p\left(\frac{\partial f(\cdot)}{\partial I_K} + \frac{\partial f(\cdot)}{\partial I_N}\frac{dI_N}{dI_K}\right) - p\beta(1-\gamma) = 0. \tag{39}$$

Using $dI_N/dI_K = \frac{(1-\gamma)\beta}{r_S} - 1$ from (38), (39) becomes:

$$\left(\frac{\partial f(\cdot)}{\partial I_K} - \frac{\partial f(\cdot)}{\partial I_N}\right) = -\left(\frac{\partial f(\cdot)}{\partial I_N} - r_S\right)\frac{\beta(1-\gamma)}{r_S}. \tag{40}$$

Solving for I_K, we get $I_K^{\mathcal{C}}$, which substituted out in the resource constraint gives $I_N^{\mathcal{C}} = L_B^*(\gamma) - \left(1 - \frac{(1-\gamma)\beta}{r_S}\right) I_K^{\mathcal{C}}$. Last, substituting out in the objective function we obtain the return from collusion:

$$\Pi^{\mathcal{C}}(\gamma) \equiv p\left[f\left(I_K^{\mathcal{C}}, L_B^*(\gamma) - \left(1 - \frac{(1-\gamma)\beta}{r_S}\right) I_K^{\mathcal{C}}\right) - R_B^*(\gamma) - (1-\gamma)\beta I_K^{\mathcal{C}}\right]. \tag{41}$$

The difference between profits under collusion (41) and profits under commitment for a generic γ (28) gives the collusion rent $\Pi^{\mathcal{C}}(\gamma) - \Pi^*(\gamma)$.

2. Any firm-bank-supplier collateralized credit contract is prone to collusion.

In order to prove that any firm-bank-supplier collateralized credit contract is prone to collusion, we need to show that:

1. $\Pi^{\mathcal{C}}(0) - \Pi^*(0) = 0$;

2. $\Pi^{\mathcal{C}}(1) - \Pi^*(1) > 0$;

3. $\frac{\partial \Pi^{\mathcal{C}}}{\partial \gamma}, \frac{\partial \Pi^*}{\partial \gamma} > 0$;

4. $\frac{\partial \Pi^{\mathcal{C}}}{\partial \gamma} - \frac{\partial \Pi^*}{\partial \gamma} > 0$.

1. $\Pi^{\mathcal{C}}(0) - \Pi^*(0) = 0$.

When $\gamma = 0$, the firm obtains credit from the bank with an unsecured contract, and from the supplier with a fully secured contract. If firm and supplier collude, they recontract the terms of the credit contract at the expense of the bank. However, since the bank offers only an unsecured contract, there is no collusive agreement that can make the bank worse off. Thus, the new collusive contract is no better than the original commitment contract. It follows that when $\gamma = 0$ profits under collusion are equal to profits under commitment, which proves the claim.

2. $\Pi^{\mathcal{C}}(1) - \Pi^*(1) > 0$.

To prove this, we can refer to the argument used in the Proof of Proposition (4).

3. $\frac{\partial \Pi^*}{\partial \gamma}, \frac{\partial \Pi^{\mathcal{C}}}{\partial \gamma} > 0$.

That $\frac{\partial \Pi^*}{\partial \gamma} > 0$ has already been proved in the Proof of Proposition (3). To prove that $\frac{\partial \Pi^{\mathcal{C}}}{\partial \gamma} > 0$, we differentiate the collusion rent $\Pi^{\mathcal{C}}(\gamma)$, defined in equation (41), with respect to γ :

$$\frac{\partial \Pi^{\mathcal{C}}(\gamma)}{\partial \gamma} = p\left(\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{C}}}\frac{\partial I_N^{\mathcal{C}}}{\partial L_B^*}\frac{\partial L_B^*}{\partial \gamma} - \frac{\partial R_B^*}{\partial \gamma} + \beta I_K^{\mathcal{C}}\right).$$

Using $L_B^*(\gamma)$ and $R_B^*(\gamma)$ from the commitment problem, and $\frac{\partial I_N^{\mathcal{C}}}{\partial L_B^*} = 1$ from (38), we get:

$$\frac{\partial \Pi^{\mathcal{C}}(\gamma)}{\partial \gamma} = p\left(\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{C}}}\frac{\partial L_B^*}{\partial \gamma} - \frac{1}{p}\left(r_B\frac{\partial L_B^*}{\partial \gamma} - (1-p)\beta I_K^*(\gamma) - (1-p)\gamma\beta\frac{\partial I_K^*}{\partial \gamma}\right) + \beta I_K^{\mathcal{C}}\right),$$

where $\frac{\partial L_B^*}{\partial \gamma} = \left(1 - \frac{1}{r_S}(1-\gamma)\beta\right)\frac{\partial I_K^*(\gamma)}{\partial \gamma} + \frac{1}{r_S}\beta I_K^*(\gamma) + \frac{\partial I_N^*}{\partial \gamma}$. Thus:

$$\begin{aligned}\frac{\partial \Pi^{\mathcal{C}}(\gamma)}{\partial \gamma} &= \left(p\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{C}}} - r_B\right)\left(\frac{\partial I_N^*}{\partial \gamma} - \frac{1}{r_S}(1-\gamma)\beta\frac{\partial I_K^*}{\partial \gamma}\right) + \left(p\left(\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{C}}} - r_S\right) + r_S - r_B\right)\frac{\beta}{r_S}I_K^*(\gamma) \\ &\quad + \left(p\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{C}}} - r_B + (1-p)\beta\gamma\right)\frac{\partial I_K^*}{\partial \gamma} + p\beta I_K^{\mathcal{C}}. \end{aligned} \tag{42}$$

Notice that $p\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{C}}} < r_B$, given that under deviation the reliance on labour increases, thus reducing the marginal productivity of labour below its commitment level r_B. Since $\frac{\partial I_K^*}{\partial \gamma} > 0$ by the comparative static analysis of the proof of Proposition. 3, the sign of the first term depends then on the sign of $\frac{\partial I_N^*}{\partial \gamma}$. Sufficient condition for the first term to be positive is that $\left(f_{NK} + \frac{1}{r_S}(1-\gamma)\beta f_{NN}\right) < 0$, which is the mild condition used in point 3 of the Proof of Proposition 4.

The sign of the second term can be inferred using (40). By Assumption 1, the reliance on labour is never higher than the reliance on capital. This implies that the marginal productivity of labour is no less than the marginal productivity of capital, $\frac{\partial f(\cdot)}{\partial I_N} \geq \frac{\partial f(\cdot)}{\partial I_K}$. Using this result in (40) implies that $\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{C}}} \geq r_S$. Moreover since $r_S > r_B$, we can conclude that the second term is positive.

Finally, notice that, in point 3 of Proof of Proposition 4, we have shown that $\left(p\frac{\partial f(\cdot)}{\partial I_K^{\mathcal{D}}} - r_B + (1-p)\,\beta\gamma\right) > 0$. Comparing this term with the third term of (42), the latter is positive if

$$\left(p\frac{\partial f\left(\cdot\right)}{\partial I_N^{\mathcal{C}}} - r_B + (1-p)\,\beta\gamma\right) > \left(p\frac{\partial f\left(\cdot\right)}{\partial I_K^{\mathcal{D}}} - r_B + (1-p)\,\beta\gamma\right)$$

i.e., $\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{C}}} > \frac{\partial f(\cdot)}{\partial I_K^{\mathcal{D}}}$. However, under deviation, by (33), the marginal productivities of the factors are equal $\frac{\partial f(\cdot)}{\partial I_K^{\mathcal{D}}} = \frac{\partial f(\cdot)}{\partial I_N^{\mathcal{D}}}$. This implies that sufficient condition for the third term to be positive is that $\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{C}}} > \frac{\partial f(\cdot)}{\partial I_N^{\mathcal{D}}}$. This always holds given that the reliance on labour under collusion is strictly lower than under deviation. This can be inferred by inspecting Figure 7 and observing that the optimum when firm and supplier collude lies on a steeper expansion path than the optimum under deviation (D), more precisely on a point between points E and C, where E is the incentive-compatible contract and C is the collusion-proof contract. On any such points, the reliance on labour under collusion is strictly lower than under deviation. Thus $\frac{\partial f(\cdot)}{\partial I_N^{\mathcal{C}}} > \frac{\partial f(\cdot)}{\partial I_N^{\mathcal{D}}}$, which proves that the third term of (42) is positive. Since all the terms of (42) are positive, we deduce that $\frac{\partial \Pi^{\mathcal{C}}(\gamma)}{\partial \gamma} > 0$.

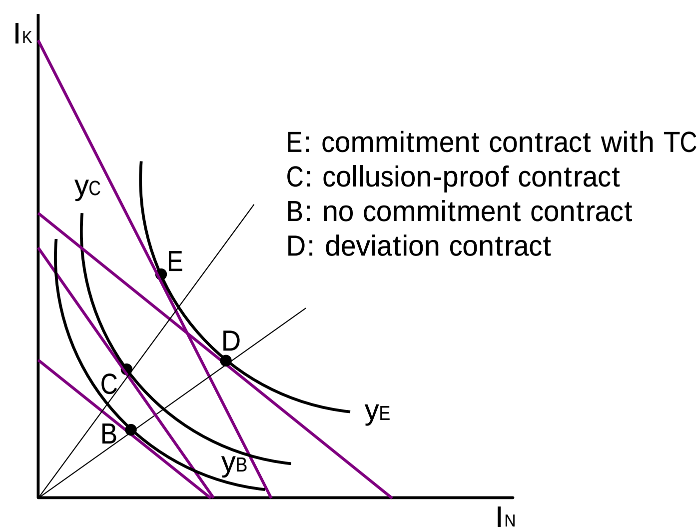

Figure 7:

4. $\frac{\partial \Pi^{\mathcal{C}}}{\partial \gamma}-\frac{\partial \Pi^{*}}{\partial \gamma}>0$.

$$\begin{aligned}\frac{\partial \Pi^{\mathcal{C}}}{\partial \gamma}-\frac{\partial \Pi^{*}}{\partial \gamma} &= \left(p\frac{\partial f}{\partial I_N^{\mathcal{C}}}-r_B+(1-p)\beta\gamma\right)\frac{\partial I_K^*}{\partial \gamma}-\left(p\frac{\partial f}{\partial I_N^{\mathcal{C}}}-r_B\right)\left(\frac{1}{r_S}(1-\gamma)\beta\frac{\partial I_K^*}{\partial \gamma}-\frac{\partial I_N^*}{\partial \gamma}\right)\\ &\quad +\frac{1}{r_S}\beta I_K^*(\gamma)\left(p\frac{\partial f}{\partial I_N^{\mathcal{C}}}-pr_S\right)+p\beta I_K^{\mathcal{C}}\end{aligned}$$

By the same arguments used in the Proof of Proposition (4), all the terms are positive. ■

Proof of Proposition 6.

The proof is the analogue of the one derived for Proposition 4 for a generic γ. In that case the value of γ was determined by the minimum exogenous level of trade credit necessary to generate commitment (γ^*). In the present case, $\hat{\gamma}(\alpha)$ obtains solving the collusion-proofness condition (16) and depends on the cost of collusion α.

To prove that when collusion is costly, there exists a contract that is collusion-proof we use the following line of analysis. By Proposition 5, we know that $\Pi^{\mathcal{C}}(\gamma)-\Pi^*(\gamma)<0$ for any $\gamma>0$ and equals zero for $\gamma=0$. Thus, the commitment contract is prone to collusion. Introducing the cost of collusion has the effect of shifting downwards the function $\Pi^{\mathcal{C}}(\gamma)$, and flattening it: $(1-\alpha)\Pi^{\mathcal{C}}(\gamma)$. Two scenarios can then arise: one in which α is so high that the function $(1-\alpha)\Pi^{\mathcal{C}}(\gamma)$ never intersects $\Pi^*(\gamma)$; and one in which α is not too high and there are values of γ that satisfy constraint (15). Define with $\bar{\alpha}(\gamma)$ the function such that $(1-\alpha)\Pi^{\mathcal{C}}(\gamma)$ and $\Pi^*(\gamma)$ have the same slope, i.e., $(1-\alpha)\frac{\partial \Pi^{\mathcal{C}}(\gamma)}{\partial \gamma}=\frac{\partial \Pi^*(\gamma)}{\partial \gamma}$, $\nabla\gamma$.

Let us analyse each scenario at a time. In scenario 1, $\alpha(\gamma)\geq\bar{\alpha}(\gamma)$: the two functions, net collusion profits and commitment profits, never converge, which implies that there is no $\gamma>0$ that solves constraint (15). It follows that constraint (15) is always slack and therefore trivially satisfied for any γ.

In scenario 2, $\alpha(\gamma)<\bar{\alpha}(\gamma)$. For any such $\alpha(\gamma)$, there exists a value of $\gamma(\alpha)$ that satisfies constraint (15). Suppose that for a particular $\alpha=\tilde{\alpha}$, there exists a $\tilde{\gamma}(\tilde{\alpha})$ that solves (15). If $\tilde{\gamma}(\tilde{\alpha})\geq\gamma^*$, the value of γ that is incentive compatible (solves (13)), then the incentive compatible contract is already collusion-proof and constraint (15) can be ignored.

Conversely, if $\tilde{\gamma}(\tilde{\alpha})<\gamma^*$, then the no deviation contract is prone to collusion and the relevant incentive constraint becomes the collusion-proofness constraint (15).

The next step is to find the threshold level of α, among those lower than $\bar{\alpha}(\gamma)$ that allows us to identify the two areas: the one in which the relevant incentive constraint is the collusion proofness (15) and the one in which the relevant incentive constraint is the no-deviation condition (13). Define $\alpha^*(\gamma^*)$ as the threshold level of α at which constraint (15) is satisfied with equality when $\gamma=\gamma^*$, i.e., $\alpha^*(\gamma^*)=1-\frac{\Pi^*(\gamma^*)}{\Pi^{\mathcal{C}}(\gamma^*)}<1$ (Recall that γ^* is also the value of γ that solves constraint (13). Thus at $\alpha=\alpha^*(\gamma^*)$, $(1-\alpha^*)\Pi^{\mathcal{C}}(\gamma^*)=\Pi^*(\gamma^*)=\Pi^{\mathcal{D}}(\gamma^*)$). It follows that for any $\alpha\geq\alpha^*(\gamma^*)$, constraint (15) is slack and the commitment contract is also collusion proof. Conversely, for any $\alpha<\alpha^*(\gamma^*)$, constraint (15) is violated and the commitment contract is not collusion proof: To prevent collusion and satisfy (15), the bank has to reduce γ below γ^*.

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