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### *Spatial Competition in Credit Markets*

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**Abstract**

Using Hotelling's two-stage model of spatial competition, we develop a lending model where the equilibrium outcome may be characterized by maximal differentiation - in contrast to Hotelling's model where firms have an incentive to reduce differentiation, as long as a pure-strategy price equilibrium exists. The difference is due to the specificities of banks' activities: banks perform independent tests to assess the credit-worthiness of their loan applicants, and thereby create a nongeographic customer heterogeneity. If banks are sufficiently pessimistic about the credit-worthiness of firms, they try to minimize the risk of default by moving away from the market center.

**JEL classification:** D83, G21, L13

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## 1 Introduction

On 1 March 1999, a package of market-opening commitments for financial services between 102 members of the World Trade Organization (WTO) entered into force. This package is mainly targeted to intensify banking competition by opening formerly locally segmented markets. Similar attempts were made in Europe (e.g. the EC's Second Banking Directive of 1988) and in the United States (e.g. the Riegle-Neal Interstate Banking and Branching Efficiency Act of 1994). These measures were expected to reduce the degree of geographical specialization significantly. However, the experiences in Europe and the US have been mixed in that respect. In Europe, recent consolidation of the banking industry has tended to occur within individual countries rather than across borders, increasing local market power instead of intensifying competition (Danthine et al., 1999). Similarly, liberalizations of the statewide branching restrictions in California and New York State have not eliminated the importance of small local banks (Mishkin 1997, p. 274).

One way to deal with the phenomenon of an apparent persistence of market segmentation in the banking industry is the analysis of zero-one entry decisions by new rivals, checking whether entry into regional banking markets is blockaded by the incumbent banks. This has recently been examined by Dell'Ariccia et al. (1999).

Another way is to take the geographical dimension explicitly into consideration and analyze the reciprocal incentives of banks to move gradually into each other's regional markets instead of making a simple zero-one entry decision. This is done in our paper. Extending a lending model of Broecker

(1990), we use Hotelling's (1929) model of spatial competition and apply it to the banking sector. We show that a pure-strategy location-then-price equilibrium with maximal spatial differentiation may exist, in contrast to Hotelling's model, in which no equilibrium in pure strategies is possible (see d'Aspremont et al. (1979)).

Essentially, we use a highly simplified model to present one basic idea of why geographical space has a special importance to banking competition. Namely that banks may unintentionally deteriorate the composition of their clientele by moving closer towards each other. The driving force is imperfect information acquisition of banks. Lending exposes banks to the risk of default. So banks typically select their customers on the basis of credit-worthiness tests. Such tests generate a customer heterogeneity in the following sense. There are (i) firms that passed both banks' tests, (ii) firms that failed both banks' tests, and (iii) firms that passed the credit-worthiness test of one bank, but failed that of the rival bank. As a result, banks compete effectively only for firms that passed both tests. Conversely, firms that passed only one bank's test represent loyal customers of that bank. However, these firms provide a higher risk of default. We show that if banks are pessimistic about the firms' credit-worthiness, they have an incentive to move away from the rival bank in order to minimize the risk of unintentionally serving those customers that have been rejected by the rival bank.

Our paper is similar in spirit to that of Dell'Araccia et al. (1999) and Gehrig (1998). As in these papers, we identify market forces that are specific for the banking industry and prevent banks from entering each other's market. Analyzing a zero-one entry decision of potential entrants, Del-



l'Ariccia et al. show that adverse selection acts as a barrier to entry. In their model incumbent banks have gathered information about their clients' credit-worthiness already (learning by lending), and therefore own an informational advantage over potential entrants. If already two banks are in the market, Dell'Ariccia et al. show that the market is blockaded for a third bank. Gehrig (1998) analyzes ex ante screening incentives of a monopolistic bank threatened by entry. In his model, two banks make sequentially credit offers based on credit-worthiness tests. He finds that the second mover's pool of applicants may be so adversely selected that lending is not profitable any more. In both papers a zero-one entry decision in an asymmetric incumbent-entrant context is examined. By contrast, we consider gradual market entry by symmetric banks in geographical space.

Chiappori et al. (1995) and Economides et al. (1996) study entry decisions of banks in Salop's (1979) circular model of product differentiation. However, the locations of banks are not derived endogenously there. This, in turn, is the focus of our paper.

Related to our paper is also the work by Wong and Chan (1993) who study ex-post auditing in spatial credit market competition. Using a rather specific auditing cost function, Wong and Chan find a unique symmetric location equilibrium, which involves spatial differentiation. The key assumption in their paper is that banks use price discrimination with respect to borrowers' locations. However, to the best of our knowledge of the banking industry, banks are typically not able to do so.

Spatial competition between nonfinancial firms when customers are heterogeneous has been analyzed by de Palma et al. (1985) and Anderson et al. (1992). In their models, more loyal customers are an advantage and not a

disadvantage. The authors can therefore show that, for a sufficient degree of customer heterogeneity, suppliers choose to agglomerate in the middle of the market. A comparison with our model clearly reveals what distinguishes banking competition from competition between nonfinancial firms: it is the risk of unintentionally serving detrimental customers. This risk acts as a deglomerative force in spatial competition.

In the following section, we describe our model of spatial competition in credit markets. The price game is analyzed in Section 3. In Section 4, we study the first-stage location decisions. We discuss possible modifications of our model in Section 5, and give a conclusion in Section 6.

## 2 The model

We consider two banks,  $i = 1, 2$ , and a continuum of firms. Each firm seeks a loan to finance an investment project of a size normalized to 1. There are two types of firms,  $s \in \{a, b\}$ , differing in their return on investment: the  $b$ -type firm obtains a return of  $Y > 1$ , and the  $a$ -type firm gets a return of zero. Each bank offers credit at an interest rate  $r_i \geq 1$ . However, the banks know that firms are able to repay the credit only when their investment has been productive. Under failure, firms are protected by limited liability and repay nothing. Whether a firm is of type  $a$  or  $b$  is assumed to be unknown to firms as well as to banks. There is hence a priori no problem of adverse selection in this model. Each bank's prior belief that a firm is of type  $a$  is  $\lambda$ , where  $0 < \lambda < 1$ . In order to assess whether a firm will be able to repay the loan, the banks perform independent credit-worthiness tests. These tests are costless and stochastically independent for each firm. Each test provides

a signal,  $S \in \{A, B\}$ , about the firm's ability of loan repayment, where  $A$  denotes an unfavorable signal and  $B$  a favorable one. We assume that banks cannot observe the rival's test results. Loan applicants are therefore selected on the basis of a bank's own test result alone. Since type  $a$ 's investment project yields no positive return, each bank is willing to offer credit only to firms, for which it observes the favorable signal  $B$ . Let  $q(S|s)$  denote the conditional probability of observing signal  $S$ , given an  $s$ -type firm. As in Broecker (1990) we use the definitions:

$$\begin{aligned} q_a &\equiv q(A|a), & 1 - q_a &\equiv q(B|a) \\ q_b &\equiv q(A|b), & 1 - q_b &\equiv q(B|b) \end{aligned}$$

and assume that  $0 \leq q_b \leq q_a \leq 1$ , i.e. the test statistic is potentially informative.

Banks update their prior beliefs according to Bayes' rule. Let  $p(s|S, R)$  denote a bank's posterior belief that a firm is of type  $s$ , given it belongs to the  $(S, R)$ -group, i.e. the bank has observed signal  $S$  and the rival has observed signal  $R \in \{A, B\}$ .  $p(S, R)$  is the probability that a firm belongs to the  $(S, R)$ -group. We obtain

$$\begin{aligned} p_1 &\equiv p(b|B, A) = \frac{1}{p_4} (1 - \lambda) (1 - q_b) q_b \\ p_2 &\equiv p(b|B, B) = \frac{1}{p_3} (1 - \lambda) (1 - q_b)^2 \\ p_3 &\equiv p(B, B) = \lambda (1 - q_a)^2 + (1 - \lambda) (1 - q_b)^2 \\ p_4 &\equiv p(B, A) = \lambda (1 - q_a) q_a + (1 - \lambda) (1 - q_b) q_b \end{aligned}$$

The firms in the credit market are distributed on the real line according to a uniform density function defined over the interval  $[0, 1]$ . The banks are located at points  $z_1$  and  $1 - z_2$ . Each firm seeks at most one credit. To

obtain a credit from a bank, the firm incurs a transportation cost  $t$  per unit of distance between its own location and the bank's location. These costs arise from transactions that require a visit of the borrower to the bank, such as initial negotiations and renegotiations of the credit contract.<sup>1</sup> They include direct travel costs as well as the opportunity costs of travelling time. We assume that the firms' transportation costs are not observable by the banks. Hence, banks are neither able to make prices nor credit offers contingent on location aspects.<sup>2</sup> For simplicity, we assume further that transportation costs are only paid when the project has turned out to be successful, since firms are protected by limited liability.

There are four special cases of the model, depending on the informativeness of the banks' credit-worthiness test:

(i) The test statistic used to select customers may yield no information at all about a firm's ability to repay the credit, i.e.  $q_a = q_b$ . It follows that  $p_1 = p_2 = 1 - \lambda$ ,  $p_3 = (1 - q_b)^2$ , and  $p_4 = q_b(1 - q_b)$ .

(ii) The test may be perfectly informative, i.e.  $q_a = 1$  and  $q_b = 0$ .  $p_1$  is then not defined,  $p_2 = 1$ ,  $p_3 = 1 - \lambda$ , and  $p_4 = 0$ . Banks have perfect information about each firm's ability to repay the credit and can ensure thus to serve only  $b$ -type firms. The model reduces hence to the Hotelling model.

(iii) The test statistic may be perfectly informative with respect to  $a$ -type firms, but imperfectly informative with respect to  $b$ -type firms, i.e.

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<sup>1</sup>In a broader interpretation of the model one may also think of customers who have heterogenous preferences about the credit contracts offered by banks in different regions, for example, due to cultural and regional regulatory differences. Transportation costs are then the disutility from not signing the preferred credit contract.

<sup>2</sup>In Section 5 we discuss strategies of spatial discrimination when banks are able to observe the firms' transportation costs and locations.

$q_a = 1$  and  $q_b < 0$ . This gives  $p_1 = p_2 = 1$ ,  $p_3 = (1 - \lambda)(1 - q_b)^2$ , and  $p_4 = (1 - \lambda)(1 - q_b)q_b$ . In this case, an  $a$ -type firm sends an  $A$ -signal with certainty such that banks can infer from a  $B$ -signal with certainty that the firm will repay the credit. Hence, if banks offer credit only to firms, for which they observe a  $B$ -signal, we obtain again the Hotelling model as a special case of our lending model.

(iv) The test statistic may be perfectly informative with respect to  $b$ -type firms, and imperfectly informative with respect to  $a$ -type firms, i.e.  $q_a < 1$  and  $q_b = 0$ . This yields  $p_1 = 0$ ,  $p_2 = (1 - \lambda) / [1 - \lambda q_a (2 - q_a)]$ ,  $p_3 = 1 - 2\lambda q_a + \lambda q_a^2$ , and  $p_4 = \lambda(1 - q_a)q_a$ . In this case, a bank infers correctly from an  $A$ -signal that the firm will not be able to repay the credit. However, a  $B$ -signal does not guarantee that the firm is of type  $b$  for sure. This special case seems most suitable to describe the selection problem of banks in credit markets.

### 3 The price game

In this section, we analyze the equilibrium in the price game of two banks which are located at  $z_1 < 1/2$  and  $1 - z_2 > 1/2$ , respectively. Each bank serves two classes of customers. (1) Firms that passed only the own bank's test and are therefore loyal to that bank. (2) Firms that passed both banks' tests. The duopoly profits are given by:

$$\Pi_1 = p_4 (r_1 p_1 - 1) x_1 + p_3 (r_1 p_2 - 1) x \quad (1)$$

$$\Pi_2 = p_4 (r_2 p_1 - 1) (1 - x_2) + p_3 (r_2 p_2 - 1) (1 - x) \quad (2)$$

where  $x_1$  denotes the customer from the  $(B, A)$ -group, who is indifferent between contracting with bank 1 or contracting not at all.  $x_2$  denotes the

indifferent customer from the  $(A, B)$ -group. And finally  $x$  denotes the customer from the  $(B, B)$ -group, i.e. with positive signals from both banks, who is indifferent between contracting with bank 1 or with bank 2:

$$x = \frac{1}{2}(1 - z_2 + z_1) + \frac{1}{2t}(r_2 - r_1) \quad (3)$$

$$x_1 = z_1 + \frac{1}{t}(Y - r_1) \quad (4)$$

$$1 - x_2 = z_2 + \frac{1}{t}(Y - r_2) \quad (5)$$

The first-order conditions for maximizing profits with respect to prices are:

$$p_4 p_1 x_1 - p_4 \frac{r_1 p_1 - 1}{t} + p_3 p_2 x - \frac{1}{2} p_3 \frac{r_1 p_2 - 1}{t} = 0 \quad (6)$$

$$p_4 p_1 (1 - x_2) - p_4 \frac{r_2 p_1 - 1}{t} + p_3 p_2 (1 - x) - \frac{1}{2} p_3 \frac{r_2 p_2 - 1}{t} = 0 \quad (7)$$

with the solution:

$$r_1^* = \frac{\alpha_1 z_1 - \alpha_2 z_2 + \alpha_3}{\alpha_4} \quad (8)$$

$$r_2^* = \frac{\alpha_1 z_2 - \alpha_2 z_1 + \alpha_3}{\alpha_4} \quad (9)$$

where  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$  are positive constants. For symmetric positions, that is  $z_1 = z_2 = z$ , we have:

$$r_1^* = r_2^* = r^* = \frac{2p_4 p_1 z t + 2p_4 p_1 Y + 2p_4 + p_3 + t p_2 p_3}{4p_4 p_1 + p_3 p_2}. \quad (10)$$

The main idea of the paper is that banks may improve the composition of their clientele by moving away from each other. We are therefore interested in price equilibria for symmetric locations ( $z_1 = z_2$ ) where  $0 < x_2 < x = 1/2 < x_1 < 1$  holds (consider Figure 1, in which we plot customers' willingness to pay for a credit from each bank as functions of their location). That is, we rule out solutions in which the market is covered for loyal firms, i.e. for firms that have passed only one bank's credit-worthiness test.<sup>3</sup>

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<sup>3</sup>Note that the covered-market case is clearly an artefact of the uniform distribution.

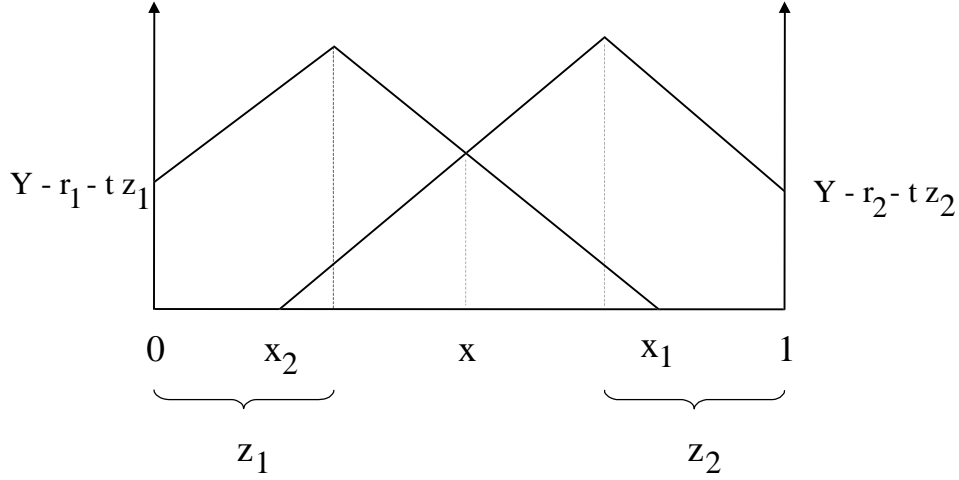


Figure 1:

The following lemma specifies parameter restrictions such that on the one hand the market for loyal firms is not covered, and on the other hand a monopoly situation is ruled out, as it is illustrated in Figure 1. Under these restrictions, the existence of a unique equilibrium in the price game is then established in Proposition 1.

**Lemma 1** *If  $Y > [2p_4 + p_3]/[2p_4p_1 + p_3p_2]$  and  $z_1 = z_2 = z < 1/2$ , then there exist unique values  $\underline{t}$  and  $\bar{t}$  with  $0 < \underline{t} < \bar{t}$  such that*

$$x = 1/2 < x_1 = 1 - x_2 < 1$$

*for all  $t \in [\underline{t}, \bar{t}]$  and  $r_1^* = r_2^* = r^*$  given by (10).*

**Proof.** See appendix.

**Proposition 1** *Under the preconditions specified in Lemma 1 and if  $z_1 = z_2 = z$  is small enough and  $t \in [\underline{t}, \bar{t}]$ , then  $r_1^*$ ,  $r_2^*$  and  $r^*$  as given by (10) is the unique equilibrium.*

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For more general but less tractable distributions this problem would vanish.

**Proof.** First, we will check whether the solution given by (10) yields positive profits.

$$\Pi_1 = p_4 (r^* p_1 - 1) x_1 + p_3 (r^* p_2 - 1) \frac{1}{2} > 0$$

if

$$r^* > \frac{2p_4 x_1 + p_3}{2p_4 x_1 p_1 + p_3 p_2} \equiv \hat{r}.$$

In fact we have

$$\begin{aligned} r^* &= \frac{2p_4 p_1 z t + 2p_4 p_1 Y + t p_2 p_3 + 2p_4 + p_3}{4p_4 p_1 + p_3 p_2} \\ &\geq \frac{2p_4 + p_3}{2p_4 p_1 + p_3 p_2} > \hat{r} \end{aligned}$$

if  $x_1 < 1$ . By symmetry, the same arguments holds for bank 2.

Next, we have to check whether one bank could gain by deviating. Investigating the monopoly profits easily reveals that, whenever  $x_1 < 1$  holds for a duopoly, a monopolist would charge a lower price than a duopolist. That rules out profitable deviation with a higher price. Finally, if  $z_1 = z_2 \equiv z$  is small enough, it is not profitable to charge a lower price to gain the whole market. This is well known from the original Hotelling game (see d'Aspremont et al. (1979)). ■

A brief inspection of the equilibrium price  $r^*$  given by (10) shows that it is increasing as banks move their locations simultaneously towards the market center. The reason is that, ceteris paribus, an increase in  $z_1 = z_2 \equiv z$  leads to more customers from the  $(B, A)$ -group for bank 1 and more customers from the  $(A, B)$ -group for bank 2, while the number of customers from the  $(B, B)$ -group remains constant for each bank. This increase in the relative amount of less preferred, but loyal customers leads, in turn, to higher prices.

Note that the price equilibrium ceases to exist as  $z_1 = z_2 \equiv z$  gets large. For sufficiently close locations, each bank has an incentive to undercut the



rival's price so that the rival loses the customer located exactly at the rival's location and - due to linear transportation costs - also all customers located further away, i.e. in the rival's "hinterland". This discontinuity in the best reply to the other bank's price precludes the existence of a price equilibrium in pure strategies, similarly as in the Hotelling model (see d'Aspremont et al. (1979)).

## 4 Location of banks

**Direct effect versus price-strategic effect.** In this section we analyze the first-stage location decision of banks. That is, we assume that banks foresee that their profits will be the equilibrium outcome of a price game taking place after their location decisions. As it is well-known from Hotelling's two-stage game, it is not possible to derive *sufficient* conditions for the location-then-price equilibrium analytically. The same problems appear in our model, and are discussed below. *Necessary* conditions for a pure-strategy equilibrium to exist in the two stage game are

$$\left. \frac{d\Pi_1(z_1, z_2, r_1^*, r_2^*)}{dz_1} \right|_{dz_2=0} = \left. \frac{d\Pi_2(z_1, z_2, r_1^*, r_2^*)}{dz_2} \right|_{dz_1=0} = 0.$$

If we restrict banks to locate in the interior of the domain of the distribution of customers, maximal differentiation is a further candidate for an equilibrium, with

$$\left. \frac{d\Pi_1(z_1, z_2, r_1^*, r_2^*)}{dz_1} \right|_{dz_2=0} < 0 \text{ and } \left. \frac{d\Pi_2(z_1, z_2, r_1^*, r_2^*)}{dz_2} \right|_{dz_1=0} < 0$$

for  $z_1 = z_2 = 0$  as necessary conditions.

We proceed by analyzing these conditions, that is we investigate each bank's incentive to move gradually closer to or further away from the rival

bank. Assuming that there exists a price equilibrium in pure strategies as characterized above, we may use the standard decomposition of the total effect resulting from a small deviation of bank 1's location on its profit (the argument for bank 2 is analogous):

$$\frac{d\Pi_1(z_1, z_2, r_1^*, r_2^*)}{dz_1} \Big|_{dz_2=0} = \underbrace{\frac{\partial\Pi_1(z_1, z_2, r_1^*, r_2^*)}{\partial z_1}}_{\text{direct effect}} + \underbrace{\frac{\partial\Pi_1(z_1, z_2, r_1^*, r_2^*)}{\partial r_2} \cdot \frac{dr_2^*}{dz_1}}_{\text{price-strategic effect}} \quad (11)$$

The change of location  $z_1$  has a direct effect on bank 1's profit. This consists of a change in the total number of bank 1's customers and a change in the composition of this clientele. To see the latter note that, as bank 1 moves towards the center of the market, the number of customers that are rejected by bank 2 but are accepted by bank 1 increases by  $\partial x_1/\partial z_1 = 1$  while the number of customers that passed both banks' tests increases only by  $\partial x/\partial z_1 = 1/2$ . The clientele contains hence more (less) firms with a higher risk of default if bank 1 increases (decreases)  $z_1$ . The sign of the overall direct effect is therefore ambiguous. Furthermore, any change in  $z_1$  affects bank 2's pricing behavior. As bank 1 moves closer towards bank 2, price competition increases. The price-strategic effect can therefore be shown to be negative. The total effect of  $z_1$  on  $\Pi_1$  is the sum of the direct and the price-strategic effects.

The main purpose of this paper is to demonstrate that geographic location plays a special role in banking competition when compared with "normal" price competition among nonfinancial firms. For the usual first-price-then-location duopoly it is known from Hotelling's (1929) famous paper that the direct effect is positive and dominates the price-strategic effect, such that suppliers have an incentive to reduce differentiation, as long as a

pure-strategy price equilibrium exists. This result has been generalized by Shilony (1981) for a wide class of customer distributions. However, the conflict between the two effects proves to be different in banking competition. We will show in Proposition 2 that in our model the total effect of  $z_1$  on  $\Pi_1$  may be negative. As a preparing step, the following lemma characterizes the parameter restrictions with respect to  $p_1, p_2, p_3, p_4$  for Proposition 2.

**Lemma 2** *Under the preconditions of Lemma 1, there exists a unique value  $\hat{p} \in [0, p_2]$  for any  $z_1 = z_2 = z$  such that  $d\Pi_1/dz_1|_{dz_2=0} < 0$  if  $p_1 < \hat{p}$  and  $p_2 < p_4/tp_3$ , and  $d\Pi_1/dz_1|_{dz_2=0} \geq 0$  otherwise.*

**Proof.** See appendix.

**Proposition 2** *There exists a non-empty set of parameters  $t, Y, p_1, p_2, p_3, p_4$  and locations  $z_1 = z_2 = z$  such that a price equilibrium exists and  $d\Pi_1/dz_1|_{dz_2=0}$  as well as  $d\Pi_2/dz_2|_{dz_1=0}$  are negative.*

**Proof.** The proposition follows from Lemma 1 and Lemma 2: Lemma 1 gives the restrictions with respect to  $t$  and  $Y$  such that a price equilibrium exists. The resulting ranges for  $t$  and  $Y$  are non-empty for any  $p_1, p_2, p_3, p_4$  and small  $z_1 = z_2 = z$ . Lemma 2 makes restrictions solely on  $p_1, p_2, p_3, p_4$ . ■

The crucial role that the parameters  $p_1$  and  $p_2$  play in determining whether banks have an incentive to move towards or away from each other is illustrated in Figure 2 and 3. We plot the change of a bank's profit resulting from a small locational change as a function of  $p_1$ , the belief that a customer that passed only the own test is creditworthy. Lemma 2 and Proposition 2 show that banks have an incentive to move away from each other, if they are sufficiently pessimistic about the creditworthiness of customers that passed

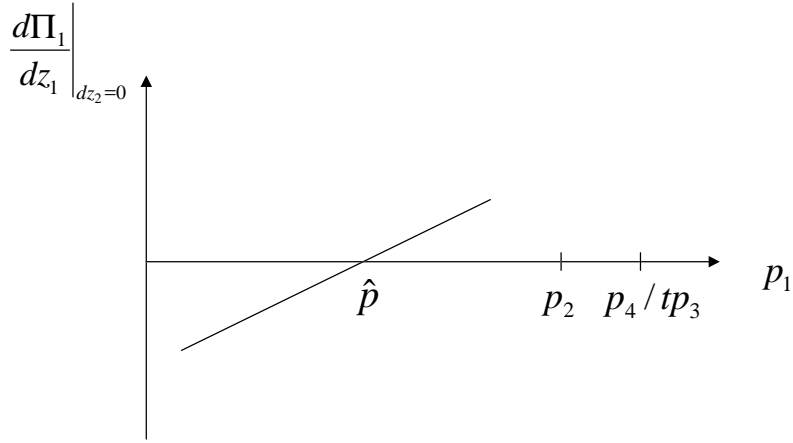


Figure 2:

only their own test, i.e.  $p_1 < \hat{p}$ , and also about customers that passed both tests, i.e.  $p_2 < p_4/tp_3$  (see Figure 2). In this case, the direct effect is too weak to overcome the price-strategic effect. Conversely, if  $p_1$  is high, the deterioration in the composition of the clientele caused by a move towards the rival bank does not matter, since banks earn a positive profit from serving both classes of customers. As a consequence, the banks are drawn into a region in which no pure- strategy price equilibrium exists.

In Figure 3, we have illustrated the situation where  $p_2 > p_4/p_3t$ , that is the loan-repayment probability of the preferred  $(B, B)$ -group is high in comparison to the size ratio of both groups. In this situation, banks are so eager to increase the number of  $(B, B)$ -firms that the composition effect is relatively unimportant, regardless of the magnitude of  $p_1$ . Again, as in the case of high  $p_1$ , banks have an incentive to move towards each other.

In the latter two cases, it is easy to see that banks will agglomerate in the market center and randomize in prices as  $Y$  becomes very large. The reason is that banks can ensure that they obtain positive profits from

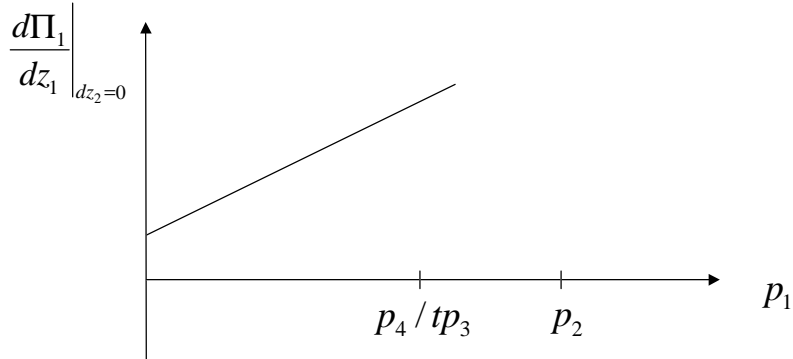


Figure 3:

serving the loyal customers that passed only the own test as  $Y$  gets large enough. As a consequence, each bank would benefit from an increase rather than a decrease in the number of these customers. However, we prefer to argue in favor of a deglomerative force of the loyal-customers effect in the banking context. As discussed above, creditworthiness tests are typically not perfectly informative with respect to bad firms. This implies that  $p_1$  and  $p_2$  are low, leading to deglomeration.

**Maximal differentiation as an equilibrium candidate.** It is interesting to note that, for  $p_1$  near by  $\hat{p}$ , there exists a  $\hat{z}$  such that  $d\Pi_1/dz_1 = d\Pi_2/dz_2 = 0$  for  $z_1 = z_2 = \hat{z}$ . However, we prove below that this does not indicate a location equilibrium. Rather, for  $z > \hat{z}$ , we have  $d\Pi_1/dz_1 > 0$  and  $d\Pi_2/dz_2 > 0$ , and hence an agglomeration force prevails, leading to a situation, in which no pure-strategy price equilibrium exists. The opposite holds for  $z < \hat{z}$ , with  $z = 0$  as a pure-strategy location-then-price equilibrium candidate.

We now demonstrate that the model could admit a unique symmetric

subgame-perfect equilibrium in pure strategies, which involves maximal differentiation. Regrettably, it is not possible to prove that this equilibrium candidate is in fact an equilibrium. The fundamental difficulty is the following: for such a proof we would have to check that no bank can gain by deviating, that is by moving closer to its competitor. That includes locations, in which no pure-strategy price equilibrium exists. Hence, we would need to compute the profits in the mixed-strategy price equilibria. These equilibria are either characterized by the incentive to exploit the loyal firms with a high price or by the incentive to capture the whole market with a low price. In the former case profits can easily be derived, similarly as in Broecker (1990). However, in the latter case the problem is similar to that analyzed by Osborne and Pitchik (1987), for which it is apparently not possible to derive the equilibrium profits analytically.<sup>4</sup>

**Proposition 3** *Suppose firms' locations are restricted to lie in the interval  $[0, 1]$ . Then the only candidate for a symmetric subgame-perfect (location-then-price) equilibrium in pure strategies is  $z_1 = z_2 = 0$  and  $r_1 = r_2 = [2p_4p_1Y + 2p_4 + p_3 + tp_2p_3]/[4p_4p_1 + p_3p_2]$  if  $p_1 < \hat{p}$  and  $p_2 < p_4/tp_3$ . Otherwise, no such equilibrium exists.*

**Proof.** We show that  $d^2\Pi_1/dz_1^2 > 0$ , what together with Lemma 2 implies the result as stated.

$$\frac{d^2\Pi_1}{dz_1^2} = (p_1p_4 + \beta_1 \cdot p_2p_3) \frac{dr_1^*}{dz_1}$$

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<sup>4</sup>We need to check deviations of one bank, given the other bank is positioned at the fringe of the distribution. This is analogous to the analysis of region T2 in Figure 2 of Osborne and Pitchik (1987).

where  $\beta_1 \equiv [7p_1p_4p_2p_3 + 8p_1^2p_4^2 + p_2^2p_3^2]/[(4p_1p_4 + p_2p_3)(3p_2p_3 + 4p_1p_4)]$  is positive. A brief inspection of (8) reveals that  $dr_1^*/dz_1 > 0$ . ■

## 5 Discussion

In the following we will discuss some possible ways to modify and generalize our model.

**Alternative transportation costs.** D'Aspremont et al. (1979) have shown that the problem of nonexistence of a pure-strategy price equilibrium in the Hotelling model can be circumvented by assuming sufficiently convex transportation costs. Similarly, this assumption would guarantee a pure-strategy price equilibrium for all locations in our banking model. As a consequence, it should be possible to prove the existence of a subgame-perfect (location-then-price) equilibrium in pure strategies. For low  $p_1$  and  $p_2$ , this equilibrium would involve maximal spatial differentiation, just as the only candidate equilibrium of our model with linear transportation costs.

Convex transportation costs are often interpreted as increasing marginal disutility in models of nongeographical product differentiation with heterogeneous customer tastes. However, in the geographical context this assumption appears as a mere expedient to ensure the existence of a pure-strategy equilibrium. When transportation costs are interpreted as disutility from time consumption, the linear case seems most plausible. In a technological sense, one typically observes weakly concave transportation costs. It is hence not surprising that in different regional economic models transportation costs are commonly assumed to be linear or concave in distance (see, for instance, von Thünen (1894), Alonso (1967), and Samuelson (1983)).

**Spatial discrimination.** We assumed that banks are not able to discriminate among loan applicants with respect to locations. This assumption seems well justified in many situations: (i) when banks cannot observe or assess the transportation costs of a firm, which include travel costs and opportunity costs of travelling time associated with the loan application and contract, and (ii) when firms can easily hide their locations, for example, by relocating unseen or choosing a correspondence address that is different from the true location.

Otherwise, spatial discrimination may be an option. One possibility is that banks decide to accept only loan applicants located within a certain zone. All applicants located beyond the border-line of that zone will be rejected, without even testing their credit-worthiness. The argument for an equilibrium in such zoning strategies would run as follows. If banks have symmetric locations with  $0 \leq z_1 < 1/2 < 1 - z_2 \leq 1$ , then in an equilibrium with  $r_1 = r_2$  bank 1 will obtain all customers of the  $(B, B)$ -group located between 0 and  $1/2$ , while all  $(B, B)$ -firms located between  $1/2$  and 1 will get credit from bank 2. Thus, bank 1 can infer from a firm that is closer located to bank 2 but willing to sign the credit contract offered by bank 1, that it does not belong to the  $(B, B)$ -group. This firm is hence less likely to repay the credit. Thus, instead of locating at the market fringe, banks could reduce the risk of default by choosing not to serve customers beyond certain border-lines.

Furthermore, banks could charge location-specific interest rates as in Wong and Chan (1993). Nevertheless, neither form of discrimination policy is commonly observed in the banking sector.



**General customer distribution.** To facilitate the analysis, we assumed a uniform customer distribution. It is possible to present the main ideas of the paper with a more general customer distribution. However, it is then difficult to ensure the uniqueness of the equilibrium in the price game.

**Two-dimensional geographical space.** In our model, banks are restricted to choose locations along a line. It is, of course, more realistic to consider a two-dimensional geographical space. Economides (1986) has analyzed the two-dimensional equivalent of Hotelling's model of spatial competition between nonfinancial firms. He was able to prove the existence of a pure-strategy price equilibrium for all symmetric locations. This leads us to conjecture that the generalization of our banking model to the two-dimensional space would eliminate the problem of possible nonexistence of a pure-strategy price equilibrium as well. However, the analysis would not become easier. In fact, one would substitute the analytical problems of finding the mixed-strategy equilibrium of the price game for equally difficult problems of solving the location game in a two-dimensional space. Moreover, we believe that the essential idea of our paper, i.e. that banks have an incentive to increase differentiation so as to minimize the risk of serving unintentionally a bad customer, would continue to hold in a two-dimensional model.

**Branches.** Location decisions of banks, as modelled here, can be interpreted as the decision of unit banks to move literally their unique position in space. As long as the market in the hinterland of each bank is covered, an alternative interpretation is that banks enlarge or reduce the size of their branch network.

## 6 Conclusion

This paper examines the nature of competition in credit markets in a geographic space. Credit markets are characterized by imperfect information, so banks typically select their customers on the basis of credit-worthiness tests. We have introduced these features, imperfect information and information acquisition activities, into Hotelling's model of spatial competition. The information acquisition activities of banks generate a non geographic customer heterogeneity. While a nongeographic customer heterogeneity may induce nonfinancial suppliers to reduce differentiation (de Palma et al. (1985) and Anderson et al. (1992)), we have shown that it may introduce the reverse incentives in the context of banking competition, that is the possibility of unintentionally serving a bad customer generates a deglomerative force. The crucial difference with spatial competition between nonfinancial firms lies in the fact that loyal customers can be detrimental in the banking context due to a high risk of default, while they are beneficial for nonfinancial firms due to a high willingness to pay.

## Appendix

**Proof of Lemma 1.** For  $r_1 = r_2$ ,  $x_1 = 1 - x_2$  holds. Using  $r_1^* = r_2^* = r^*$  (10), simple calculations yield  $x_1 \geq x = \frac{1}{2}$  if  $t \leq \bar{t}$  and  $x_1 \leq \frac{1}{2}$  if  $t \geq \underline{t}$ , where

$$\begin{aligned}\bar{t} &= 2 \frac{Y(2p_4p_1 + p_3p_2) - 2p_4 - p_3}{4p_4p_1(1-z) + p_3p_2(3-2z)} \\ \underline{t} &= \frac{Y(2p_4p_1 + p_3p_2) - 2p_4 - p_3}{2p_4p_1(2-z) + p_3p_2(2-z)}\end{aligned}$$

Finally, the condition  $Y > [2p_4 + p_3]/[2p_4p_1 + p_3p_2]$  ensures that  $0 < \underline{t} < \bar{t}$ . ■

**Proof of Lemma 2.** Consider bank 1 (the proof for bank 2 is analogous). Taking the partial derivatives  $\partial\Pi_1/\partial z_1$  and  $\partial\Pi_1/\partial r_2$  and substituting into (11) yields

$$\left. \frac{d\Pi_1}{dz_1} \right|_{dz_2=0} = p_4(r_1^*p_1 - 1) + \frac{1}{2t}p_3(r_1^*p_2 - 1) \left( t + \frac{dr_2^*}{dz_1} \right) \quad (12)$$

Using  $dr_2^*/dz_1$  from (9) and substituting into (12) yields

$$\left. \frac{d\Pi_1}{dz_1} \right|_{dz_2=0} = p_4(r_1^*p_1 - 1) + p_3(r_1^*p_2 - 1) \cdot \beta_1$$

where  $\beta_1 \equiv [7p_1p_4p_2p_3 + 8p_1^2p_4^2 + p_2^2p_3^2]/[(4p_1p_4 + p_2p_3)(3p_2p_3 + 4p_1p_4)]$  is positive and increasing in  $p_1$ .

In the following we proceed in three steps (i)-(iii). First (i), we will show for  $p_1 = 0$  that  $d\Pi_1/dz_1|_{dz_2=0} < 0$  if and only if  $p_2 < p_4/tp_3$ . The next step (ii) is to show for  $p_1 = p_2$  that  $d\Pi_1/dz_1|_{dz_2=0} > 0$ . Since  $d\Pi_1/dz_1|_{dz_2=0}$  is continuous in  $p_1$ , this proves that there exists a value for  $p_1 \in [0, p_2[$ , denoted by  $\hat{p}$  such that  $d\Pi_1/dz_1|_{dz_2=0} = 0$  if  $p_1 = \hat{p}$ . Finally (iii), we will show that  $\hat{p}$  is unique.

(i) We use  $r_1^* = r^*$  from (10) and obtain for  $p_1 = 0$  that

$$\left. \frac{d\Pi_1}{dz_1} \right|_{dz_2=0} = -p_4 + \frac{1}{3}p_3 \left( \frac{2p_4 + p_3 + tp_2p_3}{p_3p_2} p_2 - 1 \right).$$

For  $p_1 = 0$ , one easily verifies that  $\text{sign}(d\Pi_1/dz_1|_{dz_2=0}) = \text{sign}(p_2 - p_4/tp_3)$ .

(ii) For  $p_1 = p_2$ , we get

$$\left. \frac{d\Pi_1}{dz_1} \right|_{dz_2=0} = (r_1^*p_2 - 1)(p_4 + p_3 \cdot \beta_1).$$

Using the restriction on  $Y$  in Lemma 1 and  $r_1^* = r^*$ ,  $(r_1^*p_2 - 1)$  can be shown to be positive.

(iii) Finally, we will show that for  $p_2 < p_4/tp_3$  there exists a unique  $\hat{p}$ , such that  $\text{sign}(d\Pi_1/dz_1|_{dz_2=0}) = \text{sign}(p_1 - \hat{p})$ . By rearranging terms, we can write

$$\begin{aligned} d\Pi_1/dz_1|_{dz_2=0} &\stackrel{\geq}{\leq} 0 \Leftrightarrow \\ \frac{1 - r_1^*p_1}{r_1^*p_2 - 1} &\stackrel{\leq}{\geq} \frac{p_3}{p_4} \cdot \beta_1 \end{aligned} \quad (13)$$

where  $\beta_1$  and hence the RHS of (13) is increasing in  $p_1$ . Thus, we need to show that the LHS is decreasing in  $p_1$ . For this, we use  $r_1^* = r^*$  from (10) and take the derivative of the LHS with respect to  $p_1$ .

We obtain that  $dLHS/dp_1 < 0$  if

$$Y \cdot \beta_2 + z \cdot \beta_3 - 4p_4^2p_2 - 2p_4p_2p_3 + p_2^3t^2p_3^2 + tp_2^2p_3^2 > 0 \quad (14)$$

where  $\beta_2$  and  $\beta_3$  are positive terms. From Lemma 1 we know that  $Y > [2p_4 + p_3]/[2p_4p_1 + p_3p_2]$ . To prove the inequality, we may set  $z = 0$  and  $Y = [2p_4 + p_3]/[2p_4p_1 + p_3p_2]$ . Furthermore, we use the substitutions  $p_1 = \gamma_1p_2$  and  $p_2 = \gamma_2p_4/tp_3$ , where  $0 < \gamma_1 < 1$  and  $0 < \gamma_2 < 1$ . That finally allows us to verify inequality (14). ■

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