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The Veto Mechanism in Atomic Differential Information Economies

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Abstract

We establish new characterizations of Walrasian expectations equilibria based on the veto mechanism in the framework of differential information economies with a finite number of states of nature and a measure space of agents that may have atoms. We show that it is enough to consider the veto power of a single coalition, consisting of the entire set of agents, to obtain the Aubin private core. Moreover, we investigate on the veto power of arbitrary small and big coalitions, providing an extension to mixed markets of the well known Schmeidler [20] and Vind's [22] results in terms of Aubin private core allocations.

Acknowledgements: Some results of this paper are extrapolated from a chapter of my PhD thesis written during my vising at University of Vigo and at University of Salamanca, in March 2008, thanks to the kind invitation respectively of Professor Carlos Herv´es-Beloso and Professor Emma Moreno-Garc´ıa. Therefore, they come from very useful and delightful discussions with both Carlos and Emma, whom I sincerely thank for their friendly help.

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1 Introduction

The aim of this paper is to investigate on the veto mechanism in differential information economies with a finite number of states of nature and a measure space of agents that may have atoms, when some restrictions on admissible coalitions is imposed. From a mathematical point of view, an atom is a subset of the space of agents with strictly positive measure containing no proper subsets with strictly positive measure and it is typically used to represent an economic individual concentrating in his hand a large initial ownership compared with the total market endowment. This situation is known as monopoly, or more generally, oligopoly. Even if the initial endowment is spread over a continuum of small traders, it could be the case that some of them decide to act only together, as a single individual, without the possibility to form proper subgroups. This scenario, still represented via atoms, includes cartels, syndicates and other form of institutional agreements. It is well known that the presence of non negligible traders causes a lack of perfect competition and consequently a failure in the Core-Walras Equivalence Theorem. Nonetheless, it is sometimes possible to extend the core equivalence theorem to mixed markets (see [10], [19] and [21] among others for contributions in this direction), but the cost in terms of assumptions in not negligible, since it is needed that large traders lose their market power becoming competitors. This is guaranteed assuming, as Shitovitz suggested ([21]; see also [4] for an extension to infinite dimensional commodity spaces and [19] for an extension to differential information economies), that there are at least two large traders of the same type, meaning that they have the same initial endowment and same preferences. In order to characterize competitive equilibrium allocations without imposing any additional conditions on the atomic sector, we consider Aubin approach to core analysis (see [1]), according to which agents may participate by using only a fraction of their initial resources when forming a coalition. This new pondered veto concept was introduced by Aubin [1] in complete information economies with a finite number of agents and commodities, in order to characterize the set of competitive equilibria, when the ordinary core seems to be too large to coincide with it. Later, Noguchi [18] proved that even in the presence of atoms, the Aubin core provides a characterization of competitive equilibria. The Aubin pondered veto concept and the equivalence with the set of Walrasian equilibria was extended in the framework of atomic differential information economies with a finite number of states of nature by Graziano and Meo [9] (see also [19] in which the free disposal condition is avoid and [3] for the case of atomic economies with public goods). Under uncertainty and with asymmetrically informed agents, keeping the main idea of Aubin that agents in a blocking coalition may use only a fraction of their initial endowment, it is also required that members of a blocking coalition can only use their own private information. This is due to the fact that agents in forming a coalition have no opportunities to share information. It is worth of noting that in differential information economies different notions of Aubin core can be considered depending on the information sharing rules and on possibilities of communication among agents. Even if in this paper we focus on the Aubin private core, we provide conditions ensuring that in some cases the information sharing rules and the communication opportunities play no role.

Checking whether a given allocation belongs to the Aubin private core seems to require to look upon the whole set of possible coalitions in order to test whether any groups of agents, by employing a rate of their own initial endowment and by using their own private information, can improve upon such allocation. Therefore, this seems to be hard to check, unless the economy is very small. As pointed out by Hervés-Beloso and Moreno-García in [13], it may be difficult to argue that coalition formation is costless and free: *"The fact that agents are organized in some way, and perhaps they are not entirely free, may result in high formation costs, commitments and constraints, which make difficult to assume that the veto mechanism works freely and spontaneously"*. For this reason, it is usually assumed that only a subset of the set of all possible coalitions in an economy is considered to be really formed. The papers [13], [14] and [15] go in this direction (see also [16] for an infinite dimensional commodity space setting). They obtain a characterization of Walrasian equilibria by using the veto power which differs substantially from the equivalences obtained by Debreu and Scarf in [5] and Aubin in [1]. Indeed, on the one hand Debreu-Scarf and Aubin enlarge the set of blocking coalitions: the former by replicating the economy, the latter by allowing the participation of the agents with any rate of their endowments. On the other hand, Hervés Beloso and Moreno García in [13], [14] and [15] consider the veto power of just one coalition, namely the grand coalition, by enlarging the possible redistribution of endowments. In other words, they consider the veto power of a single coalition in infinitely many economies obtained by perturbing the original initial endowments. In this paper, we extend their result in terms of Aubin private core by showing that the Aubin private core coincides with the set of those allocations which are not privately blocked by any generalized coalition with full support, that is whose support equals the set of all agents. Our result differs from [15] in two main aspects: first we slack the assumption of "finitely many agents" by considering the general case of differential information mixed markets. Second, we do not need to construct a family of economies perturbing agents' initial endowment, since we show that it is enough to consider the pondered veto power of a single coalition in just one economy.

Going on our analysis on the implications that restrictions on the measure of a blocking coalition may have on the Aubin private core, we extend Schmeidler and Vind's Theorems to the set of feasible allocations not privately blocked by any generalized coalition in mixed differential information economies. Schmeidler and Vind's Theorems gave a new interpretation of the Core-Walras equivalence theorem obtained by Aumann in 1964 [2] for atomless economies. Schmeidler [20] showed that it is enough to consider the veto power of arbitrary small coalitions to get the core, and Vind [22] completed the scenario by showing that any allocation that is non blocked by arbitrary large coalitions is in the core. Behind their results the hypothesis that the economy is atomless is crucial. The aim of this paper is to provide conditions guaranteeing that in mixed economies with asymmetrically informed agents, given any positive number α, less than the measure of the grand coalition, an allocation outside the Aubin private core can be blocked by a generalized coalition whose support has measure smaller than α (extension of Schmeidler's theorem) and by a generalized coalition whose support has measure equals to α (extension of Vind's theorem). If the economy is atomless, Schmeidler and Vind's Theorems come easily from Lyapunov convexity theorem; but, if there are some large traders, it could be not possible to reduce the measure of a blocking coalition as much as we want. That's why in the case of mixed differential information economies a restriction on the real number α is needed. Indeed, in atomless economy the Aubin private core does not change whatever restriction on the measure of a blocking coalition is imposed; while in mixed markets an allocation outside the Aubin private core can be privately blocked only by generalized coalitions whose support has measure smaller (or equal) to any α greater than the measure of the atomic sector. Whenever α is smaller that the measure of the atomic sector, we need to make negligible the veto power of large traders in order to manage the measure of a blocking coalition. We show in Example 4.1 that for our purpose the presence of at least two atoms of the same type, according to Shitovitz's assumption, may not be enough, but a stronger hypothesis on the atomic sector T_1 is needed. We prove that if there are countably many large traders of the same type, even in a mixed market, an allocation x outside the Aubin private core is privately blocked by a generalized coalition whose support has arbitrarily small measure (Theorem 4.3) and by a generalized coalition whose support has a certain measure smaller than the measure of the atomless sector T_0 (Theorem 4.4). To this end the allocation x must satisfy what we call the "equal treatment property on the atomic sector", according to which identical large traders are equally treated under x. We also illustrate some examples to underline the necessity of the hypotheses used, and as a consequence of all these equivalences, we establish a list of new characterizations of Walrasian expecations allocations based on the veto mechanism.

The paper is organized as follows. We first present the theoretical model and state main definitions; then, in Section 3, we investigate on the veto mechanism of the grand coalition while Section 4 contains extensions to mixed markets of Schmeidler and Vind's results in terms of Aubin private core allocations. Proofs are collected in the Appendix.

2 The model and the main definitions

In this section we illustrate the theoretical framework for studying exchange economies with uncertainty and asymmetrically informed agents. First, we formally present the basic model describing briefly each component of it and then we focus on the key solution concepts that we will use throughout our analysis.

2.1 The model

We consider a Radner-type exchange economy $\mathcal{E}$ with differential information, modeled by the following collection

$$\mathcal{E} = \Big\{(\Omega, \mathcal{F});\ (T, \mathcal{T}, \mu);\ I\!\!R^\ell_+;\ (\mathcal{F}_t, q_t, u_t, e_t)_{t\in T}\Big\}$$

where:

1. $(\Omega, \mathcal{F})$ is a measurable space describing the exogenous uncertainty; Ω is the finite set denoting the possible states of nature (i.e., $\Omega = \{\omega_1, \dots, \omega_k\}$) and $\mathcal{F}$ is the field of all the events.

2. $(T, \mathcal{T}, \mu)$ is a complete, finite measure space, where: T is the set of agents and $\mathcal{T}$ is the σ-field of all eligible coalitions, whose economic weight on the market is given by the measure μ. An arbitrary finite measure space of agents makes us deal simultaneously with the case of discrete economies, non-atomic economies as well as economies that may have atoms. Indeed, discrete economies are covered by a finite set T with a counting measure μ. Atomless economies are analyzed by assuming that $(T, \mathcal{T}, \mu)$ is the Lebesgue measure space with $T = [0, 1]$. Finally, mixed markets are those for which T is composed by two sets: T_0 and T_1, where T_0 is the atomless sector and T_1 the set of atoms[1]. We will refer to T_0 as the set of "small" traders and to T_1 as the set of "large" traders.[2] For this reason, $\mathcal{E}$ is called differential information mixed economy (or mixed market). The disjoint union of atoms in T_1 formalizes the presence of agents concentrating in their hands an initial ownership of some commodity in a large amount compared with the total availability on the market (oligopoly), or groups of traders deciding to act only together without the possibility to form proper subgroups [3] (cartels, syndicates).

3. $\mathbb{R}^{\ell}_{+}$ is the commodity space.

4. $\left(\mathcal{F}_t, q_t, u_t, e_t\right)_{t \in T}$ is the set of agents' characteristics. Each economic individual $t \in T$ is indeed characterized by:

   - a private information described by $\mathcal{F}_t$ which is a partition of Ω. The interpretation is as usual: if $\omega \in \Omega$ is the state of nature that is going to be realized, agent t observes the unique element of $\mathcal{F}_t$ which contains ω. With an abuse of notation, we still denote by $\mathcal{F}_t$ the field generated by $\mathcal{F}_t$. Since Ω is finite, there exists a finite collection $\{\mathcal{F}_i\}_{i \in I}$ of fields on Ω, such that

   $$\{\mathcal{F}_t \ : \ t \in T\} = \{\mathcal{F}_i \ : \ i \in I\}.$$

   Denoted by

   $$I_i = \{t \in T \ : \ \mathcal{F}_t = \mathcal{F}_i\}$$

   the information set of type $i \in I$, that is the set of agents with the same information $\mathcal{F}_i$, we assume that for each $i \in I$, $I_i \in \mathcal{T}$, with $\mu(I_i) > 0$, and $(I_i)_{i \in I}$ forms a partition of T.

   - q_t is a probability measure on $\mathcal{F}$, which represents agent t's prior belief regarding nature.
   - a state-dependent utility function representing agent t's preferences:

   $$\begin{array}{rccc} u_t : & \Omega \times \mathbb{R}^{\ell}_{+} & \rightarrow & \mathbb{R} \\ & (\omega, x) & \rightarrow & u_t(\omega, x). \end{array}$$

[1]An atom is a subset with strictly positive mass containing no proper subsets of strictly positive mass. Recall that every measure space has at most countable many disjoint atoms. Hence, the measure space T can be decomposed into a countable union of atoms T_1 and an atomless part T_0 (see [12] p. 45).

[2]This terminology is, in particular, motivated when T is a separable metric space. Indeed, in this case, T_0 is the set of traders $t \in T$ for which $\mu(t) = 0$, while T_1 is the set of traders such that $\mu(t) > 0$ (see [12]).

[3]For further details on mixed markets we refer an interesting reader to [21].

We assume that for all $t \in T$ and $\omega \in \Omega$, the function $u_t(\omega, \cdot) : \mathbb{R}^\ell_+ \to \mathbb{R}$ is continuous and monotone, and for all $\omega \in \Omega$, the mapping $(t, x) \mapsto u_t(x, \omega)$ is $\mathcal{T} \otimes \mathcal{B}$-measurable, where $\mathcal{B}$ is the σ-field of *Borel* subsets of $\mathbb{R}^\ell_+$.

- an initial endowment of physical resources, also contingent to the states of nature, which is given by a $\mathcal{F}_t$-measurable function $e_t : \Omega \to \mathbb{R}^\ell_+$.

Two agents t and s are said to be **of the same type** if they have the same economic characteristics, that is they have the same private information, same prior, utility function and initial endowment, i.e., $\big(\mathcal{F}_t, q_t, u_t, e_t\big) = \big(\mathcal{F}_s, q_s, u_s, e_s\big)$.

2.2 Main notions and basic relations

We now recall the main equilibrium concepts we will analyze in the paper and state the basic relations arising among them.

An **allocation** for the economy $\mathcal{E}$ is a function[4] $x : T \times \Omega \to \mathbb{R}^\ell_+$ such that (i) $x(\cdot, \omega)$ is μ-integrable on T for all $\omega \in \Omega$ and (ii) $x(t, \cdot)$ is $\mathcal{F}_t$-measurable for almost all $t \in T$. Condition (ii) is interpreted as informational feasibility of the allocation x, while the meaning of condition (i) depends on the space of agents T. An allocation x is said to be **feasible** if for each state of nature the total consumption does not exceed the total endowment, that is

$$\int_T x_t(\omega)\ d\mu \leq \int_T e_t(\omega)\ d\mu, \qquad \text{for all } \omega \in \Omega. \tag{1}$$

The free disposal condition expressed by (1) is usually required to ensure the existence of competitive allocations supported by non negative prices (see [6]), but on the other hand it does not guarantee the incentive compatibility of equilibria (see [7]). We denote by $F(\mathcal{E})$ the set of all feasible allocations for the economy $\mathcal{E}$.

For each $t \in T$, we denote by

$$M_t = \Big\{ x_t : \Omega \to \mathbb{R}^\ell_+ \ : \ x_t(\cdot) \text{ is } \mathcal{F}_t - \text{measurable} \Big\}$$

the consumption set of agent $t \in T$. Since M_t takes into account the information constraints, not only it is smaller than the commodity space, but it also differs from agent to agent.

Although the economy is static we give a two period interpretation as follows: in the first period, traders make contracts based on their private information which are contingent on the realized state of nature. Consumption takes place in the second period, once the state of nature is realized. Since, at the time of contracting agents receive no additional informational signal, the ex ante expected utility is the appropriate measure of an agent's well being. For any function

[4] We will often denote $x(t, \omega)$ by $x_t(\omega)$.

$x : \Omega \to I\!R^\ell_+$ we denote by $h_t(x)$ the ex-ante expected utility of trader t from x, that is

$$h_t(x) = \sum_{\omega \in \Omega} u_t(\omega, x(\omega)) q_t(\omega).$$

The competitive equilibrium concept assumes that a **price vector** p is a non-zero state dependent function $p : \Omega \to I\!R^\ell_+$ exogenously given for each commodity in each state of nature. Consumers, taking prices as given, exchange their endowments trying to maximize their welfare. For each $t \in T$ and each price vector p, the **budget set** of agent t is made by all functions measurable with respect to t's private information.

$$\mathcal{B}_t(p) = \left\{ x \in M_t \;:\; \sum_{\omega \in \Omega} p(\omega) \cdot x(\omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot e_t(\omega) \right\}.$$

Definition 2.1. *A price vector p and a feasible allocation x are said to be a* **Walrasian expectations equilibrium** *for the economy $\mathcal{E}$ if for almost all $t \in T$, $x_t(\cdot)$ maximizes the ex ante expected utility h_t on $\mathcal{B}_t(p)$ and*

$$\sum_{\omega \in \Omega} p(\omega) \cdot \int_T x_t(\omega) \, d\mu = \sum_{\omega \in \Omega} p(\omega) \cdot \int_T e_t(\omega) \, d\mu.$$

The budget set of each agent t, through M_t, takes into account not only physical but also informational constraints, since agents can consume only the commodity bundles that they are able to distinguish. This implies that an increase in the information available to agent t will be reflected in a refinement of his algebra $\mathcal{F}_t$ which enlarges his budget set. Therefore, under a Walrasian expectations equilibrium, agents better informed are, in general, better off. A **Walrasian expectations allocation** is a feasible allocation x for which there exists a price vector p such that (p, x) is a Walrasian expectations equilibrium. We denote by $\mathcal{W}(\mathcal{E})$ the set of all Walrasian expectations allocations. This is a non cooperative notion since each agent maximizes his ex-ante expected utility function over his budget set without taking care of actions of the other market participants. On the other hand, according to the core notion, agents can cooperate within coalitions in order to improve their welfare. It is well known that in differential information economies, core notions depend on the information sharing rules and on possibility of communication among agents. Consequently, different core notions are analyzed: the coarse core, the fine core and the private core. We first recall the definition of private core due to [24].

Definition 2.2. *A feasible allocation x is said to be a* **private core allocation** *for the economy $\mathcal{E}$ if there do not exist a coalition S and an alternative allocation $y : T \times \Omega \to I\!R^\ell_+$ such that*

$$\begin{array}{ll} (i) & \mu(S) > 0, \\ (ii) & y_t(\cdot) \; is \; \mathcal{F}_t{-}measurable \; for \; almost \; all \; t \in S \\ (iii) & h_t(y_t) > h_t(x_t) \quad for \; almost \; all \; t \in S \\ (iv) & \int_S y_t(\omega) \, d\mu \, \leq \, \int_S e_t(\omega) \, d\mu \qquad for \; all \; \omega \in \Omega. \end{array}$$

If there exist S and y which satisfy conditions $(i)-(iv)$, then we will say that the coalition S privately blocks x via y, or that x is improved upon by coalition S via y. According to [24], the private core of the economy $\mathcal{E}$, denoted by $C(\mathcal{E})$, is the set of feasible allocations that are not privately blocked by any coalition. In other words, a feasible allocation x is in the private core if no coalition of agents can redistribute their initial endowments among themselves using their own private information and making each of its member better off. A feasible allocation x is said to be **Pareto optimal** or **efficient** if it is not privately blocked by the whole coalition of agents T. We denote by $PO(\mathcal{E})$ the set of efficient allocations of $\mathcal{E}$. Clearly, any private core allocation is Pareto optimal, i.e., $C(\mathcal{E}) \subseteq PO(\mathcal{E})$.

According to the **coarse core** notion, due to [23], agents in a blocking coalition can only use their common information, i.e., condition (ii) above must be replaced by

$$(ii_c) \quad y_t(\cdot) \text{ is } \bigwedge_{t\in S} \mathcal{F}_t\text{−measurable for almost all } t \in S.$$

On the other hand, in the **fine core**, also introduce in [23], information is pooled among members of a blocking coalition, hence (ii) should be replaced by

$$(ii_f) \quad y_t(\cdot) \text{ is } \bigvee_{t\in S} \mathcal{F}_t\text{−measurable for almost all } t \in S.$$

Let us denote by $C_c(\mathcal{E})$ and $C_f(\mathcal{E})$ respectively the coarse and the fine core, and observe that since for any $S \in \mathcal{T}$ and $t \in S$, $\bigwedge_{t\in S} \mathcal{F}_t \subseteq \mathcal{F}_t \subseteq \bigvee_{t\in S} \mathcal{F}_t$, we can easily conclude that

$$C_f(\mathcal{E}) \subseteq C(\mathcal{E}) \subseteq C_c(\mathcal{E}).$$

In this paper, we will focus on the private core for more than one reason. Differently from the coarse and the fine core, which have some problems associated with the existence and incentive compatibility, the private core presents desirable properties: it is non empty under standard continuity and convexity assumptions on utility functions; and it perceives the differences of (quality of) information among traders. Precisely, an agent who is well informed and is expected to cooperate with a non well informed agent, does not prefer pooling information nor using common knowledge, because otherwise he cannot take advantage of his finer private information. Moreover, it is shown that in atomless economies under some economic reasonable assumptions, the private core coincides with the set of Walrasian expectations allocations.

Even if the coarse core, the fine core and the private core are three different cooperative solution concepts, we now provide conditions ensuring that in some cases the information sharing rule and the communication possibilities among agents play no role, that is $C_f(\mathcal{E}) = C(\mathcal{E}) = C_c(\mathcal{E})$.

Proposition 2.1. *Let $\mathcal{E}$ be a differential information economy with common prior, i.e., $q_t = q$ for all $t \in T$. Assume that for all $t \in T$, the initial endowment e_t and the utility function u_t are state independent, i.e., $e_t(\omega) = e_t$ and $u_t(\omega, \cdot) = u_t(\cdot)$ for all $\omega \in \Omega$, and that $u_t(\cdot)$ is concave. Then,*

$$C_f(\mathcal{E}) = C(\mathcal{E}) = C_c(\mathcal{E}).$$

The above result also provides a new characterization of Walrasian expectations equilibria in those economies in which the Core-Walras equivalence theorem holds. The Core-Walras equivalence theorem states that competitive equilibria are the only allocations such that no group of agents is able to achieve a preferred outcome for its members using only its initial aggregate resources. According to the competitive equilibrium concept, agents are price takers, meaning implicitly that none of them can affect prices and the aggregate demand. Therefore, the natural mathematical model for representing perfect competition is to use a continuum of agents as Aumann proposed, since changes in individual behavior is negligible. But competition in the real economic activity is far from being perfectly competitive. For this reason we allow the presence of non negligible individuals represented by atoms, which causes a lack of perfect competition and hence the failure of the equivalence between core and competitive equilibrium allocations (see [21] for several examples). Nonetheless, it is sometimes possible to extend the core equivalence theorem to mixed markets (see [10], [19] and [21] among others for contributions in this direction), but the cost in terms of assumptions in not negligible, since it is needed that large traders lose their markets power becoming competitors. This is guaranteed assuming, as Shitovitz suggested (see [21]), that there are at least two large traders of the same type, meaning that they have the same initial endowment, same utility function, same private information and prior. In order to characterize competitive equilibrium allocations without imposing any additional conditions on the atomic sector, we consider Aubin approach to core analysis (see [1]) and define the so called Aubin private core.

Definition 2.3. *Let $\mathcal{A}$ be the set of all measurable functions $\gamma : T \to [0,1]$, whose support S_γ has positive measure, that is,*

$$\mu(S_\gamma) = \mu\left(\{t \in T : \ \gamma(t) > 0\}\right) > 0.$$

We call an element γ in $\mathcal{A}$ a **generalized coalition**.

Definition 2.4. *A generalized coalition $\gamma \in \mathcal{A}$ privately blocks an allocation $x : T \times \Omega \to I\!R^\ell_+$, if there exists an allocation $y : T \times \Omega \to I\!R^\ell_+$ such that*

$$\begin{array}{ll} (i) & y_t(\cdot) \ \ is \ \mathcal{F}_t{-}measurable \ for \ almost \ all \ t \in S_\gamma \\ (ii) & h_t(y_t) > h_t(x_t) \quad for \ almost \ all \ t \in S_\gamma \\ (iii) & \displaystyle\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \le \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \quad for \ all \ \omega \in \Omega. \end{array}$$

The **Aubin private core** of the economy $\mathcal{E}$, denoted by $C_A(\mathcal{E})$, is the set of all feasible allocations which are not privately blocked by any generalized coalition. In other words, a feasible allocation belongs to the Aubin private core if it is not possible for agents to redistribute their initial resources among themselves using their own private[5] information (see (i)) and obtaining a strictly preferred bundle (see (ii)). According to the usual interpretation, each member t of S_γ

[5] This requirement justifies the adjective "private" for the Aubin core.

may participate employing only a part $\gamma(t)$ of his initial endowment. Thus, the feasibility over the coalition S_γ takes into account for these shares as condition (iii) expresses.

As for the core, different notions of Aubin core can be defined in differential information economies depending on the information sharing rules and on possibility of communication among agents. Therefore by replacing suitably condition (i) above we can define the coarse and the fine Aubin core and notice that similar comments, as Proposition 2.1, done for the core, still hold for the Aubin core.

By allowing agents to employ only a rate of their initial resources in forming a coalition, the number of possible blocking coalitions enlarges; consequently the Aubin private core is included in the core. Indeed, if x is privately blocked (in the usual sense) by a coalition S, then x is privately blocked (in the Aubin sense) by the generalized coalition $\gamma : T \to [0,1]$, defined by $\gamma(\cdot) = \chi_S(\cdot)$. Notice that since $\gamma(t)$ is zero for all $t \in T \setminus S_\gamma$, condition (iii) in Definition 2.4 can be replaced by the integral over the whole space of agents T, that is

$$\int_T \gamma(t) y_t(\omega)\, d\mu \leq \int_T \gamma(t) e_t(\omega)\, d\mu \qquad \text{for all } \omega \in \Omega.$$

We close this section by investigating on the basic relationships among the main equilibrium concepts illustrated above. We have already observed that the Aubin private core is contained into the private core; furthermore it is well known that, under standard assumptions, a Walrasian expectations allocation exists and it is efficient. Therefore, the next proposition, stating that any Walrasian expectations allocation is not privately blocked by any generalized coalition, implies the non emptiness of the Aubin private core, and hence the existence of a private core allocation in differential information mixed markets.

Proposition 2.2. *Any Walrasian expectations allocation is in the Aubin private core, that is*

$$\mathcal{W}(\mathcal{E}) \subseteq C_A(\mathcal{E}).$$

In conclusion, in a mixed differential information economy the following relations hold true

$$\mathcal{W}(\mathcal{E}) \subseteq C_A(\mathcal{E}) \subseteq C(\mathcal{E}) \subseteq PO(\mathcal{E}).$$

3 The Veto Mechanism of the Grand Coalition

Checking whether a given allocation belongs to the Aubin private core seems to require to look upon the whole set of possible coalitions in order to test whether any groups of agents, by employing a rate of their own initial endowment and by using their own private information, can improve upon such allocation. Therefore, this seems to be hard to check, unless the economy is very small. As pointed out by Hervés-Beloso and Moreno-García in [13], it may be difficult to argue that coalition formation is costless and free: *"The fact that agents are organized in some way, and perhaps they are not entirely free, may result in high formation costs, commitments and constraints, which make difficult to assume that the veto mechanism works freely and spontaneously"*. For this reason, it is usually assumed that only a subset of the set of all possible coalitions in an economy is considered to be really formed.

The papers [13], [14] and [15] go in this direction (see also [16] for an infinite dimensional commodity space setting). They obtain a characterization of Walrasian equilibria by using the veto power which differs substantially from the equivalences obtained by Debreu and Scarf in [5] and Aubin in [1]. Indeed, on the one hand Debreu-Scarf and Aubin enlarge the set of blocking coalitions: the former by replicating the economy, the latter by allowing the participation of the agents with any rate of their endowments. On the other hand, Hervés Beloso and Moreno García in [13], [14] and [15] consider the veto power of just one coalition, namely the grand coalition, by enlarging the possible redistribution of endowments. In other words, they consider the veto power of a single coalition in infinitely many economies obtained by perturbing the original initial endowments. In this section, we extend their result in terms of Aubin private core by showing that the Aubin private core coincides with the set of those allocations which are not privately blocked by any generalized coalition with full support, that is whose support equals the set of all agents T. Our result differs from [15] in two main aspects: first we slack the assumption of "finitely many agents" by considering the general case of differential information mixed markets. Second, we do not need to construct a family of economies perturbing agents' initial endowment, since we show that it is enough to consider the pondered veto power of a single coalition only in the "original" mixed economy.

To this end, let $T - C_A(\mathcal{E})$ be the set of all feasible allocations which are not privately blocked by a generalized coalition γ, with full support[6]. Formally, an allocation x belongs to $T - C_A(\mathcal{E})$ if there do not exist a generalized coalition γ and an allocation $y : T \times \Omega \to I\!R^{\ell}_{+}$ such that

$$
\begin{array}{ll}
(i) & \gamma(t) > 0 \quad \text{for almost all } t \in T \\
(ii) & y_t(\cdot) \ \text{ is } \mathcal{F}_t\text{−measurable for almost all } t \in T \\
(iii) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in T \\
(iv) & \displaystyle\int_T \gamma(t) y_t(\omega)\, d\mu \le \int_T \gamma(t) e_t(\omega)\, d\mu \quad \text{for all } \omega \in \Omega.
\end{array}
$$

Clearly any allocation $x \in T - C_A(\mathcal{E})$ is efficient, but the converse may not be true. Moreover, since any Aubin private core allocation cannot be blocked by any generalized coalition, a fortiori it is neither privately blocked by a generalized coalition with full support; i.e., $C_A(\mathcal{E}) \subseteq T - C_A(\mathcal{E})$. We now prove that whenever agents have a positive amount of each commodity in each state of nature, also the converse inclusion is true.

[6]A generalized coalition γ has full support if $\gamma(t) > 0$ for almost all $t \in T$, or equivalently, $\mu(\operatorname{supp}\gamma) = \mu(\{t \in T : \gamma(t) > 0\}) = \mu(T)$.

Theorem 3.1. *In a mixed differential information economy $\mathcal{E}$ in which all agents have a positive amount of each commodity in each state of nature (i.e., for every $\omega \in \Omega$ and almost all $t \in T$, $e(t,\omega) \gg 0$), an allocation x does not belong to the Aubin private core if and only if it is privately blocked by a generalized coalition γ with full support, that is $C_A(\mathcal{E}) = T - C_A(\mathcal{E})$.*

The problem of coalition formation has captured the attention of many; in particular the restricted veto mechanism which allows only a subset $\mathcal{S}$ of the set of all possible coalitions in an economy to be the set of admissible coalitions. If only some coalitions, those belonging to $\mathcal{S}$, can be formed, it is possible to define the $\mathcal{S}$-core and study the implications that this assumption has with regard to the veto mechanism. Similarly, we may consider only generalized coalitions with support in $\mathcal{S}$, and define the $\mathcal{S}$- Aubin core, i.e., $\mathcal{S} - C_A(\mathcal{E})$. Notice that whatever subset $\mathcal{S}$ of admissible coalitions may be, the Aubin core is contained in $\mathcal{S} - C_A(\mathcal{E})$, since by reducing the number of possible blocking coalitions we enlarge the set of feasible allocations not privately blocked. According to our Theorem 3.1, the private Aubin core does not change if we impose that only the grand coalition can privately block an allocation, and consequently it coincides with the $\mathcal{S}$-Aubin core for every subset $\mathcal{S}$ containing the set of all agents.

Corollary 3.1. *In a mixed differential information economy $\mathcal{E}$ in which all agents have a positive amount of each commodity in each state of nature (i.e., for every $\omega \in \Omega$ and almost all $t \in T$, $e(t,\omega) \gg 0$),*

$$C_A(\mathcal{E}) = \mathcal{S} - C_A(\mathcal{E}) \quad for\ any\ \mathcal{S} \in \mathcal{T}\ such\ that\ T \in \mathcal{S}.$$

The above result is an extension of Theorem 3.3 in [13] to differential information economies, but first we work with a general mixed market while in [13] only a finite number of agents is considered; second we do not need convexity, but we assume that all agents have strictly positive endowments. This assumption enables us to prove that if a generalized coalition blocks an allocation, that coalition can block that allocation by disposing a strictly positive amount of its resources (see Lemma 5.1 in the Appendix which is crucial in the proof of Theorem 3.1). The subset $\mathcal{S}$ of admissible coalitions can be also the set of arbitrarily big coalition, that is for any $\alpha \in (0, \mu(T)]$ we may define $\mathcal{S}$ to be the set of coalitions with measure at least equals to α, i.e.,

$$\mathcal{S}^+_\alpha = \{S \in \mathcal{T}\ :\ \mu(S) \geq \alpha\}.$$

Clearly, for any $\alpha \in (0, \mu(T)]$, the set of all agents T belongs to $\mathcal{S}^+_\alpha$; therefore from Corollary 3.1 we obtain another characterization of the Aubin private core in terms of veto power of arbitrarily large generalized coalitions.

Corollary 3.2. *In a mixed differential information economy $\mathcal{E}$ in which all agents have a positive amount of each commodity in each state of nature (i.e., for every $\omega \in \Omega$ and almost all $t \in T$, $e(t,\omega) \gg 0$),*

$$C_A(\mathcal{E}) = \mathcal{S}^+_\alpha - C_A(\mathcal{E}) \quad for\ any\ \alpha \in (0, \mu(T)].$$

The presence of large traders as well as uncertainty and information asymmetry may determine imperfections or deviations from the perfect competition scheme, for example the failure of the Core-Walras equivalence theorem. Allowing agents to participate in a coalition with a share of his resources restores this classical result (see [8] for more details). The equivalence between the Aubin core and the set of Walrasian equilibrium allocations has been proved in different context (see for example [1], [3], [9], [17], [18], and [19] among others).

Theorem 3.2. *[The Aubin Core-Walras Equivalence Theorem] Let $\mathcal{E}$ be a differential information economy in which for every $\omega \in \Omega$ and almost all $t \in T$, $e(t, \omega) \gg 0$, $u_t(\cdot, x)$ is strictly monotone and quasi-concave. Then, the Aubin private core coincides with the set of Walrasian expecations allocations.*

In conclusion, we summarized all the above results in a list of equivalences stated in the following corollary.

Corollary 3.3. *Let $\mathcal{E}$ be a differential information economy in which for every $\omega \in \Omega$ and almost all $t \in T$, $e(t, \omega) \gg 0$, $u_t(\cdot, x)$ is strictly monotone and quasi-concave. Let x be a feasible allocation for $\mathcal{E}$. Then, the following statements are equivalent.*

1. *The allocation x is a Walrasian expectations allocation, i.e., $x \in W(\mathcal{E})$.*
2. *The allocation x is non privately blocked by any generalized coalition, i.e., $x \in C_A(\mathcal{E})$.*
3. *The allocation x is non privately blocked by any generalized coalition with full support, i.e., $x \in T - C_A(\mathcal{E})$.*
4. *For any $\mathcal{S} \in \mathcal{T}$ such that $T \in \mathcal{S}$, $x \in \mathcal{S} - C_A(\mathcal{E})$.*
5. *Given any $\alpha \in (0, \mu(T)]$, the allocation x is non privately blocked by any generalized coalition whose support has measure bigger than α, i.e., $x \in \mathcal{S}_\alpha^+ - C_A(\mathcal{E})$ for any $\alpha \in (0, \mu(T)]$.*

This result can be viewed as an extension of Corollary 4.1 in [16] in which only a finite number of agents has been considered and additional assumptions have been used, as for example the hypothesis $(A.5)$ in [16] according to which for each agent t the utility function $u_t(\cdot, z)$ is $\mathcal{F}_t$-measurable for any $z \in I\!R_+^\ell$. The measurability assumption of utility is usually used for incentive compatibility issues not treated in this article, since, as demonstrated in [7], equilibria with free disposal may not be incentive compatible. It is worth noting that the above equivalences hold under mild assumptions that do not guarantee the Core-Walras equivalence theorem in mixed markets (see [21] and [19] for an extension to economies with asymmetrically informed agents). Hence, we have obtained different characterizations of Walrasian expectations allocations even in those economies in which the private core is too big to cover the set of equilibria.

4 The veto power of small and big coalitions

In 1964 Aumann [2] showed that in atomless economies competitive allocations are those feasible allocations which are not blocked by any coalition of agents. In his result, the veto power of infinitely many coalitions is crucial. Indeed, in finite economies, where the number of blocking coalition is finite, the Core-Walras equivalence theorem is only asymptotic. Eight years later, three notes in the same issue of Econometrica gave new interpretation of Aumann's theorem. Schmeidler [20] showed that it is enough to consider the veto power of arbitrary small coalitions to get the core. Grodal [11] showed that the set of blocking coalitions can be further restricted by considering only those consisting of finitely many arbitrary small set of agents with similar characteristics. Vind [22] completed the scenario, by showing that any allocation that is not blocked by arbitrary large coalitions is in the core. In this section we continue to investigate on the implications that restrictions on the measure of a blocking coalition may have on the Aubin private core. Basically, we try to extend Schmeidler and Vind's Theorems to the set of feasible allocations not privately blocked by any generalized coalition in mixed differential information economies. To this end, let α be a real positive number in the interval $(0, \mu(T)]$, and define the following sets.

Let $\mathcal{S}_\alpha^- - C_A(\mathcal{E})$ be the set of all feasible allocations which are not privately blocked by any generalized coalition γ, whose support S_γ has measure smaller than α, that is, $\mu(S_\gamma) < \alpha$.

Let $\mathcal{S}_\alpha - C_A(\mathcal{E})$ be the set of all feasible allocations which are not privately blocked by any generalized coalition γ, whose support S_γ has measure equal to α, that is, $\mu(S_\gamma) = \alpha$.

When imposing a restriction on the measure of a blocking coalition we reduce the number of possible groups of agents which can block and consequently the set of non blocked allocations enlarges. Therefore, the Aubin private core is included in both sets defined above, that is

$$C_A(\mathcal{E}) \subseteq \mathcal{S}_\alpha^- - C_A(\mathcal{E}) \qquad \text{and} \qquad C_A(\mathcal{E}) \subseteq \mathcal{S}_\alpha - C_A(\mathcal{E}) \quad \text{for any } \alpha \in (0, \mu(T)].$$

We want to find conditions under which the above inclusions are equivalences and hence the blocking power of arbitrary small and big generalized coalitions is enough to obtain the Aubin private core; that is,

$$C_A(\mathcal{E}) = \mathcal{S}_\alpha^- - C_A(\mathcal{E}) \quad \text{for any } \alpha \in (0, \mu(T)] \tag{2}$$

$$C_A(\mathcal{E}) = \mathcal{S}_\alpha - C_A(\mathcal{E}) \quad \text{for any } \alpha \in (0, \mu(T)]. \tag{3}$$

If $\alpha = \mu(T)$, Theorem 3.1 already guarantees the above equivalences; hence we look at $\alpha < \mu(T)$ case only. Notice that (2) and (3) can be considered, respectively, extensions of Schmeidler and Vind's theorems in terms of Aubin private core to differential information economies. Indeed, (2) states that, given any positive number α, less than the measure of the grand coalition $\mu(T)$, an allocation outside the Aubin private core can be blocked by a generalized coalition γ, whose support S_γ has measure smaller than α. In other words, any allocation which is not blocked by “small” generalized coalitions is in the Aubin private core. Similarly, (3) states that, it is enough to consider the blocking power of arbitrary large generalized coalitions, in order to obtain the Aubin private core. Indeed, given any positive number α, less than the measure of the grand coalition $\mu(T)$, an allocation outside the Aubin private core can be blocked by a generalized coalition γ, whose support S_γ has measure equal to α. It is worth noting that if the economy $\mathcal{E}$ is atomless, (2) and (3) are immediate consequences of the Core-Walras equivalence theorem and Proposition 3.1 in [15] (see also Lemma 3 in [19] for a further extension); therefore our aim is to prove them in those economies satisfying so mild assumptions that the core may be strictly bigger than the set of equilibria.

If μ is atomless, (2) and (3) are not a big iusse since they basically follow from Lyapunov theorem; but, if there are some large traders, it could be not possible to reduce the measure of a blocking coalition as much as we want. That's why in the case of mixed differential information economies a restriction on the real number α is needed. The theorems below state that in atomless economy the Aubin private core does not change whatever restriction on the measure of a blocking coalition is imposed; while in mixed markets an allocation outside the Aubin private core can be privately blocked only by generalized coalitions whose support has measure smaller (equal) to any α greater than the measure of the atomic sector T_1.

Theorem 4.1. *Let $\mathcal{E}$ be a differential information economy. If $\mathcal{E}$ is atomless, i.e., $T_1 = \emptyset$, then*

$$C_A(\mathcal{E}) = \mathcal{S}_\alpha^- - C_A(\mathcal{E}) \quad for\ any\ \alpha \in (0, \mu(T)),$$

otherwise

$$C_A(\mathcal{E}) = \mathcal{S}_\alpha^- - C_A(\mathcal{E}) \quad for\ any\ \alpha \in (\mu(T_1), \mu(T)).$$

Theorem 4.2. *Let $\mathcal{E}$ be a differential information economy in which all agents have a positive amount of each commodity in each state of nature (i.e., for every $\omega \in \Omega$ and almost all $t \in T,\ e(t,\omega) \gg 0$). If $\mathcal{E}$ is atomless, i.e., $T_1 = \emptyset$, then*

$$C_A(\mathcal{E}) = \mathcal{S}_\alpha - C_A(\mathcal{E}) \quad for\ any\ \alpha \in (0, \mu(T)),$$

otherwise

$$C_A(\mathcal{E}) = \mathcal{S}_\alpha - C_A(\mathcal{E}) \quad for\ any\ \alpha \in (\mu(T_1), \mu(T)).$$

Thanks to the above results we can add two points in the list of equivalences of Corollary 3.3, that is Walrasian expectations allocations are the only feasible allocations which cannot be privately blocked by a generalized coalition with arbitrary support provided that conditions of Theorems 4.1 and 4.2 are satisfied. We now want to underline the importance of such conditions in order to get the equivalences discussed above. To this end, we illustrate an example of an economy in which there exists an allocation that cannot be blocked by a generalized coalition whose support has measure smaller or equal to a certain α, but which is outside the Aubin core. Notice that in the following economy there are two atoms, and the real number α is smaller than $\mu(T_1)$; therefore it does not contradict the above results.

Example 4.1. Consider a mixed economy with two goods, $T = \left[0, \frac{1}{3}\right] \cup A_1 \cup A_2$, such that $\mu(A_1) = \mu(A_2) = \frac{1}{3}$. The initial endowment of small traders is $\left(\frac{1}{2}, \frac{1}{2}\right)$, while $\left(\frac{3}{4}, \frac{1}{4}\right)$ is the initial endowment of both atoms. The utility function of each agent $t \in T$ is given by the function $u_t(x_t, y_t) = x_t + y_t$. Consider the following feasible allocation

$$(x_t, y_t) = (2, 1) \quad \text{for all } t \in T_0 \quad \text{and} \quad (x_{A_1}, y_{A_1}) = (x_{A_2}, y_{A_2}) = (0, 0)\,,$$

and notice that it is not in the core and a fortiori it is outside the Aubin core, since it it blocks by the coalition $A_1 \cup A_2$ via the initial endowment. We now show that $(x, y) \in \mathcal{S}_\alpha - C_A(\mathcal{E})$ for any $\alpha \in \left(0, \frac{1}{3}\right)$. Indeed assume, by the way of contradiction, that for some $\alpha \in \left(0, \frac{1}{3}\right)$ there exists a generalized coalition γ with support S_γ such that $\mu(S_\gamma) = \alpha$ which Aubin-blocks the allocation (x, y). Clearly, since $\alpha < \frac{1}{3}$, then $A_1 \notin S_\gamma$, otherwise $\frac{1}{3} > \alpha = \mu(S_\gamma) \geq \mu(A_1) = \frac{1}{3}$ which is an absurd. Similarly $A_2 \notin S_\gamma$. Therefore, $S_\gamma \subseteq \left[0, \frac{1}{3}\right]$ and there exists (z_t, w_t) such that $z_t + w_t > 3$ for almost all $t \in S_\gamma$ and

$$\int_{S_\gamma} \gamma(t) z_t \, d\mu \leq \int_{S_\gamma} \gamma(t) \frac{1}{2} \, d\mu$$

$$\int_{S_\gamma} \gamma(t) w_t \, d\mu \leq \int_{S_\gamma} \gamma(t) \frac{1}{2} \, d\mu.$$

By adding, we get the following contradiction

$$3 \int_{S_\gamma} \gamma(t) \, d\mu < \int_{S_\gamma} \gamma(t)[z_t + w_t] \, d\mu \leq \int_{S_\gamma} \gamma(t) \left[\frac{1}{2} + \frac{1}{2}\right] \, d\mu = \int_{S_\gamma} \gamma(t) \, d\mu.$$

With similar arguments, we can show that $(x, y) \in \mathcal{S}_\alpha^- - C_A(\mathcal{E})$ for any $\alpha \in \left(0, \frac{1}{3}\right]$.

In the above economy the generalized coalition γ does not play any role, hence Example 4.1 can be used also to show that in mixed economy even the core changes if restrictions on possible blocking coalitions are imposed. The failure of equivalences stated in Theorems 4.1 and 4.2 consists in the difficulty to manage the measure of large traders which cannot allow us to reduce the measure of a blocking coalition as much as we want. To overcome this problem we need an additional assumption which makes negligible the veto power of large traders. Shitovitz in [21], in order to satisfy a similar necessity, requires that atoms are at least two and of the same type, and he shows that this situation engenders such intense competition among large traders that they lose their market power becoming competitors, so that the Core-Walras equivalence theorem, which typically fails in presence of atoms, can be restored. To manage the measure of a blocking coalition the presence of at least two atoms of the same type, according to Shitovitz's assumption, may not be enough, as shown in Example 4.1 where two identical large traders are considered. Hence Example 4.1 suggests that for our goal a stronger assumption on the atomic sector T_1 is needed. We show below that if there are countably many large traders of the same type, even in a mixed market, an allocation x outside the Aubin private core is privately blocked by a generalized coalition whose support has arbitrarily small measure (Theorem 4.3) and by a generalized coalition whose support has a certain measure smaller than the measure of the atomless sector T_0 (Theorem

4.4). To this end the allocation x must satisfy what we call the "equal treatment property on the atomic sector", according to which identical large traders are equally treated under x. Precisely, we assume that x belongs to the following set

$$ET(\mathcal{E}) = \{x \in F(\mathcal{E}) \,:\, h_A(x_A) = h_B(x_B) \text{ if } A \in T_1 \text{ and } B \in T_1 \text{ are of the same type}\}.$$

We are now ready to state the following theorems.

Theorem 4.3. *Let $\mathcal{E}$ be a mixed differential information economy with countably many atoms of the same type such that $u_t(\omega, \cdot)$ is concave for all $t \in T_1$ and all ω. Let x be an allocation satisfying the equal treatment property on the atomic sector, i.e., $x \in ET(\mathcal{E})$. Then, for any $\alpha \in (0, \mu(T))$, x is in the Aubin private core if and only if $x \in \mathcal{S}_\alpha^- - C_A(\mathcal{E})$.*

Theorem 4.4. *Let $\mathcal{E}$ be a mixed differential information economy with countably many atoms of the same type such that for all $t \in T_1$ and all ω $u_t(\omega, \cdot)$ is concave; and for every $\omega \in \Omega$ and almost all $t \in T,\ e(t, \omega) \gg 0$. Let x be an allocation satisfying the equal treatment property on the atomic sector, i.e., $x \in ET(\mathcal{E})$. Then, for any $\alpha \in (0, \mu(T_0)]$, x is in the Aubin private core if and only if $x \in \mathcal{S}_\alpha - C_A(\mathcal{E})$.*

The following example shows that the requirement that the allocation x satisfies the equal treatment property on the atomic sector is necessary to get the desirable equivalences. Indeed, it illustrates an economy with countably many identical atoms in which for some $\alpha \in (0, \mu(T))$ there exists an allocation x such that

$$x \in \mathcal{S}_\alpha^- - C_A(\mathcal{E}) \setminus C_A(\mathcal{E}) \quad \text{and} \quad x \in \mathcal{S}_\alpha - C_A(\mathcal{E}) \setminus C_A(\mathcal{E}).$$

This example does not contradict Theorems 4.3 and 4.4, since under the allocation x the identical atoms get different level of utility, i.e., $x \notin ET(\mathcal{E})$.

Example 4.2. Consider a mixed economy $\mathcal{E}$ with two goods, and $T = \left[0, \frac{1}{2}\right] \cup \{A_n\}_{n \in I\!N}$, where $\mu(A_n) = \frac{1}{3^n}$ for any $n \in I\!N$. Moreover,

$$\begin{aligned} e_t &= (1,2) \quad \text{for all } t \in T_0 = \left[0, \frac{1}{2}\right], \\ e_t &= (2,1) \quad \text{for all } t \in T_1 \qquad \text{and} \\ u_t &= x_t + y_t \quad \text{for all } t \in T. \end{aligned}$$

Consider the following feasible allocation

$$\begin{aligned} (x_t, y_t) &= \left(1, \frac{5}{2}\right) \quad \text{for all } t \in T_0 \\ (x_t, y_t) &= \left(\left(\frac{3}{2}\right)^n, \frac{1}{2}\right) \quad \text{for all } t \in T_1. \end{aligned}$$

Notice that (x,y) is feasible and $(x,y) \notin ET(\mathcal{E})$. Moreover, it is outside the core and a fortiori $(x,y) \notin C_A(\mathcal{E})$. Indeed, it is blocks by the atoml A_1 via the initial endowment. On the other hand, we now prove that for any $\alpha < \frac{1}{9} < \mu(T_0)$, the allocation $(x,y) \in \mathcal{S}_\alpha - C_A(\mathcal{E})$. Assume by absurd that for some $\alpha < \frac{1}{9}$ there exists a generalized coalition γ with support S_γ such that $\mu(S_\gamma) = \alpha$ and which blocks in the Aubin sense (x,y) via an alternative allocation (z,w). Clearly, $A_1 \notin S_\gamma$, otherwise $\frac{1}{9} > \alpha = \mu(S_\gamma) \geq \mu(A_1) = \frac{1}{3}$, which is an absurd and similarly $A_2 \notin S_\gamma$. Hence, define $I = \{n \in I\!N \ : A_n \in S_\gamma\}$, we have that

$$
\begin{aligned}
&(1) \qquad z_t + w_t > \frac{7}{2} \quad \text{for almost all } t \in S_\gamma \cap T_0 \\
&(2) \qquad z_{A_n} + w_{A_n} > \left(\frac{3}{2}\right)^n + \frac{1}{2} \quad \text{for all } n \in I \\
&(3) \qquad \int_{S_\gamma \cap T_0} \gamma(t) z_t \, d\mu + \sum_{n \in I} \gamma_{A_n} \frac{z_{A_n}}{3^n} \leq \int_{S_\gamma \cap T_0} \gamma(t) \, d\mu + \sum_{n \in I} \gamma_{A_n} \frac{2}{3^n} \\
&(4) \qquad \int_{S_\gamma \cap T_0} \gamma(t) w_t \, d\mu + \sum_{n \in I} \gamma_{A_n} \frac{w_{A_n}}{3^n} \leq \int_{S_\gamma \cap T_0} 2\gamma(t) \, d\mu + \sum_{n \in I} \gamma_{A_n} \frac{1}{3^n}.
\end{aligned}
$$

Multiply (1) by $\gamma(t)$ for any $t \in S_\gamma \cap T_0$ and (2) by $\frac{\gamma_{A_n}}{3^n}$ for any $n \in I$; then by adding and finally by integrating we get that

$$\int_{S_\gamma \cap T_0} \gamma(t)[z_t + w_t] \, d\mu + \sum_{n \in I} \gamma_{A_n} \frac{z_{A_n} + w_{A_n}}{3^n} > \int_{S_\gamma \cap T_0} \frac{7}{2} \gamma(t) \, d\mu + \sum_{n \in I} \gamma_{A_n} [\frac{1}{2} \frac{1}{3^n} + \frac{1}{2^n}],$$

and hence by adding (3) and (4) it follows that

$$\int_{S_\gamma \cap T_0} 3\gamma(t) \, d\mu + \sum_{n \in I} \gamma_{A_n} \frac{3}{3^n} > \int_{S_\gamma \cap T_0} \frac{7}{2} \gamma(t) \, d\mu + \sum_{n \in I} \gamma_{A_n} [\frac{1}{2} \frac{1}{3^n} + \frac{1}{2^n}];$$

that is

$$\frac{1}{2} \int_{S_\gamma \cap T_0} \gamma(t) + \sum_{n \in I} \gamma_{A_n} [\frac{1}{2^n} - \frac{5}{2} \frac{1}{3^n}] < 0,$$

which is an absurd, since for any $n \geq 3$, and a fortiori for any[7] $n \in I$, $[\frac{1}{2^n} - \frac{5}{2}\frac{1}{3^n}] > 0$. Therefore, $(x,y) \in \mathcal{S}_\alpha - C_A(\mathcal{E})$, but notice that $(x,y) \notin C_A(\mathcal{E})$. Similarly we can prove that for any $\alpha < \frac{1}{9}$, $(x,y) \in \mathcal{S}_\alpha^- - C_A(\mathcal{E}) \setminus C_A(\mathcal{E})$.

It is worth of noting that the possibility for agents to use only a rate of their initial resources in forming a coalition can be dropped, so that Example 4.2 also shows that in mixed markets even with countably many atoms of the same type restrictions on the measure of a blocking coalition deeply impact on the private core. In other words, Schmeidler and Vind's theorem cannot be extended under these conditions since there may exist an allocation outside the core which cannot

[7]Recall that $A_1 \notin S_\gamma$ and $A_2 \notin S_\gamma$

be blocked by arbitrary small coalitions , i.e., $C(\mathcal{E}) \subsetneq \mathcal{S}_\alpha^- - C(\mathcal{E})$ and by a coalition of a given measure, i.e., $C(\mathcal{E}) \subsetneq \mathcal{S}_\alpha - C(\mathcal{E})$.

In the statement of Theorem 4.4, the real number α has to satisfy an additional condition: allocations outside the Aubin private core are blocked by generalized coalition whose support has measure equal to a certain α, which must be smaller than the measure of the atomless sector. The following example has the aim to underline the importance of this condition. In fact, it shows that in an economy with coutably many identical large traders, given a certain α bigger than the measure of the atomless sector $\mu(T_0)$, there exists an allocation x satisfying the equal treatment property on the atomic sector, i.e., $x \in ET(\mathcal{E})$, which is outside the Aubin core but such that $x \in \mathcal{S}_\alpha - C_A(\mathcal{E})$.

Example 4.3. Consider a mixed economy $\mathcal{E}$ with two goods, and $T = \left[0, \frac{1}{81}\right] \cup \{A_n\}_{n \in I\!N} \cup B$, where $\mu(A_n) = \frac{1}{3^n}$ for any $n \in I\!N$ and $\mu(B) = \frac{79}{162}$. Moreover,

$$\begin{aligned} e_t &= \left(40, \frac{61}{5}\right) \quad \text{for all } t \in T_0 = \left[0, \frac{1}{81}\right], \\ e_t &= \left(\frac{1}{2}, \frac{1}{20}\right) \quad \text{for all } t \in T_1 \qquad \text{and} \\ u_t &= x_t + y_t \quad \text{for all } t \in T. \end{aligned}$$

Consider the following feasible allocation

$$\begin{aligned} (x_t, y_t) &= (0,0) \quad \text{for all } t \in T_0 \\ (x_t, y_t) &= \left(1, \frac{81}{400}\right) \quad \text{for all } t \in T_1. \end{aligned}$$

Notice that $(x,y) \in ET(\mathcal{E})$, but it is outside the core and a fortiori $(x,y) \notin C_A(\mathcal{E})$. Indeed, it is blocks by the atomless sector T_0 via the initial endowment. On the other hand, we now prove that for any $\alpha \in \left(\frac{5}{27}, \frac{1}{3}\right)$, the allocation (x,y) belogs to $\mathcal{S}_\alpha - C_A(\mathcal{E})$. Notice that $\alpha > \frac{5}{27} > \frac{1}{81} = \mu(T_0)$. Assume by absurd that there exists a generalized coalition γ with support S_γ such that $\mu(S_\gamma) = \alpha$ and which blocks in the Aubin sense (x,y) via an alternative allocation (z,w). Clearly, $A_1 \notin S_\gamma$, otherwise $\frac{1}{3} > \alpha = \mu(S_\gamma) \geq \mu(A_1) = \frac{1}{3}$, which is an aburd and similarly $B \notin S_\gamma$. Moreover, observe that there does not exist a coalition $C \subseteq T$ such that $\mu(C) = \alpha$ and $A_1 \notin C$ and $B \notin C$. Indeed for any $C \subseteq T$ with $A_1 \notin C$ and $B \notin C$ we have that

$$\frac{5}{27} < \alpha = \mu(C) = \mu(C \cup T_0) + \mu(C \cup T_1) \leq \frac{1}{81} + \sum_{n=2}^{\infty} \frac{1}{3^n} = \frac{1}{81} + \frac{1}{6} = \frac{29}{162} < \frac{5}{27}.$$

Therefore, $(x,y) \in \mathcal{S}_\alpha - C_A(\mathcal{E}) \setminus C_A(\mathcal{E})$.

Again notice that in the example above the possibility for agents to use only a part of their endowment in forming a blocking coalition can be dropped, so that it can be rewritten in terms of core allocations.

5 Appendix

5.1 Proofs of Section 2

PROOF of Proposition 2.1: Since $C_f(\mathcal{E}) \subseteq C(\mathcal{E}) \subseteq C_c(\mathcal{E})$, we just need to show that $C_c(\mathcal{E}) \subseteq C_f(\mathcal{E})$. To this end, let x be a coarse core allocation and assume on the contrary that $x \notin C_f(\mathcal{E})$. This means that there exist a coalition S and an alternative allocation $y : T \times \Omega \to I\!R^{\ell}_{+}$ such that

$$\begin{array}{ll} (i) & \mu(S) > 0, \\ (ii_f) & y_t(\cdot) \text{ is } \bigvee_{t\in S} \mathcal{F}_t\text{–measurable for almost all } t \in S \\ (iii) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in S \\ (iv) & \int_S y_t(\omega)\, d\mu \le \int_S e_t(\omega)\, d\mu \qquad \text{for all } \omega \in \Omega. \end{array}$$

Now, consider the constant (i.e., state independent) allocation z_t given as follows: for each $t \in S$

$$z_t = \sum_{\omega\in\Omega} y_t(\omega) q(\omega),$$

and notice that $z_t(\cdot)$ is $\bigwedge_{t\in S} \mathcal{F}_t$ measurable for almost all $t \in S$. Moreover, condition (iii) and concavity of u_t imply that for almost all $t \in S$

$$\begin{aligned} h_t(z_t) &= \sum_{\omega\in\Omega} u_t(z_t(\omega)) q(\omega) = \\ \sum_{\omega\in\Omega} u_t\left(\sum_{\omega\in\Omega} y_t(\omega) q(\omega)\right) q(\omega) &\ge \sum_{\omega\in\Omega}\left[\sum_{\omega\in\Omega} u_t(y_t(\omega)) q(\omega)\right] q(\omega) = \\ \sum_{\omega\in\Omega} h_t(y_t) q(\omega) &= h_t(y_t) > h_t(x_t). \end{aligned}$$

Finally, from (iv) it follows that for any $\omega \in \Omega$

$$\begin{aligned} \int_S z_t(\omega)\, d\mu &= \int_S \sum_{\omega\in\Omega} y_t(\omega) q(\omega)\, d\mu = \\ \sum_{\omega\in\Omega}\left[\int_S y_t(\omega)\, d\mu\right] q(\omega) &\le \sum_{\omega\in\Omega}\left[\int_S e_t(\omega)\, d\mu\right] q(\omega) = \\ \sum_{\omega\in\Omega}\left[\int_S e_t\, d\mu\right] q(\omega) &= \int_S e_t(\omega)\, d\mu. \end{aligned}$$

This completes the proof. □

PROOF of Proposition 2.2: Let (p, x) be a Walrasian expectations equilibrium and assume, on the contrary, that there exist a generalized coalition γ, with support S_γ, and an allocation $y : T \times \Omega \to I\!R^\ell_+$, such that

$$\begin{array}{ll} (1) & y_t(\cdot) \text{ is } \mathcal{F}_t-\text{measurable for almost all } t \in S_\gamma \\ (2) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in S_\gamma \\ (3) & \displaystyle\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \leq \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \quad \text{for all } \omega \in \Omega. \end{array}$$

Since $x \in \mathcal{W}(\mathcal{E})$, condition (2) implies that for almost all $t \in S_\gamma$, $y_t \notin B_t(p)$, that is $\sum_{\omega\in\Omega} p(\omega) \cdot y_t(\omega) > \sum_{\omega\in\Omega} p(\omega) \cdot e_t(\omega)$. Now, since $\gamma(t) > 0$ for all $t \in S_\gamma$, then

$$\sum_{\omega\in\Omega} p(\omega) \cdot \gamma(t) y_t(\omega) > \sum_{\omega\in\Omega} p(\omega) \cdot \gamma(t) e_t(\omega) \quad \text{for almost all } t \in S_\gamma,$$

and hence $\sum_{\omega\in\Omega} p(\omega) \cdot \left[\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu - \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu\right] > 0$, which contradicts (3). □

5.2 Proofs of Section 3

In order to prove Theorem 3.1, the following lemmata are needed. The first lemma states that if an allocation x is privately blocked by a generalized coalition γ with support S_γ; that coalition blocks x by disposing a strictly positive amount of its resources.

Lemma 5.1. *Let $\mathcal{E}$ be a mixed differential information economy in which all agents have a positive amount of each commodity in each state of nature (i.e., for every $\omega \in \Omega$ and almost all $t \in T$, $e(t, \omega) \gg 0$). If an allocation x is privately blocked by a generalized coalition γ, with support S_γ, via an alternative allocation y, then there exists a subcoalition C of S_γ such that $y_t(\omega) \gg 0$ for almost all t in C and all ω in Ω.*

PROOF: Let x be privately blocked in the Aubin sense by a generalized coalition γ with support S_γ via an alternative allocation y, that is

$$\begin{array}{ll} (1) & y_t(\cdot) \text{ is } \mathcal{F}_t-\text{measurable} \quad \text{for almost all } t \in S_\gamma, \\ (2) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in S_\gamma \\ (3) & \displaystyle\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \leq \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \quad \text{for all } \omega \in \Omega. \end{array}$$

Thanks to the continuity assumption of utility function, there exist $\varepsilon > 0$ and a subcoalition C of S_γ, with positive measure, such that

$$h_t(\varepsilon y_t) > h_t(x_t) \qquad \text{for almost all } t \in C.$$

Notice that for all $\omega \in \Omega$,

$$\begin{aligned}\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu = \int_{S_\gamma \setminus C} \gamma(t) y_t(\omega)\, d\mu \quad + \quad \int_C \frac{\gamma(t)}{\varepsilon} \varepsilon y_t(\omega)\, d\mu \ \le \\ \le \int_{S_\gamma \setminus C} \gamma(t) e_t(\omega)\, d\mu \quad + \quad \int_C \frac{\gamma(t)}{\varepsilon} \varepsilon e_t(\omega)\, d\mu \ = \\ \int_{S_\gamma \setminus C} \gamma(t) e_t(\omega)\, d\mu \quad + \quad \int_C \frac{\gamma(t)}{\varepsilon} [e_t(\omega) - (1-\varepsilon) e_t(\omega)]\, d\mu;\end{aligned}$$

hence, for all $\omega \in \Omega$

$$\int_{S_\gamma \setminus C} \gamma(t) y_t(\omega)\, d\mu + \int_C \frac{\gamma(t)}{\varepsilon} [\varepsilon y_t(\omega) + (1-\varepsilon) e_t(\omega)]\, d\mu \ \le \int_{S_\gamma \setminus C} \gamma(t) e_t(\omega)\, d\mu + \int_C \frac{\gamma(t)}{\varepsilon} e_t(\omega)\, d\mu,$$

or equivalently

$$\int_{S_\gamma} \hat{\gamma}(t) z_t(\omega)\, d\mu \le \int_{S_\gamma} \hat{\gamma}(t) e_t(\omega)\, d\mu,$$

where

$$z_t(\omega) = \begin{cases} y_t(\omega) & \text{if } t \in S_\gamma \setminus C \\ \varepsilon y_t(\omega) + (1-\varepsilon) e_t(\omega) & \text{if } t \in C \end{cases}$$

and

$$\hat{\gamma}(t) = \begin{cases} \gamma(t) & \text{if } t \in S_\gamma \setminus C \\ \frac{\gamma(t)}{\varepsilon} & \text{if } t \in C. \end{cases}$$

Clearly $z_t(\cdot)$ is $\mathcal{F}_t$-measurable for almost all $t \in S$. Moreover, since for every $\omega \in \Omega$ and almost all $t \in T$, $e(t,\omega) \gg 0$, we have that $z_t(\omega) \gg 0$ for almost all $t \in C$ and all $\omega \in \Omega$; and $h_t(z_t) > h_t(x_t)$ for almost all $t \in S$. The function $\hat{\gamma}$ can be normalized in order to represent a generalized coalition which privately blocks x via the allocation z, with $z_t(\omega) \gg 0$ for all $\omega \in \Omega$ and for almost all $t \in C \subseteq S_\gamma$. □

Lemma 5.2. *Let $\mathcal{E}$ be a mixed differential information economy in which all agents have a positive amount of each commodity in each state of nature (i.e., for every $\omega \in \Omega$ and almost all $t \in T$, $e(t,\omega) \gg 0$). If an allocation x does not belong to the Aubin private core then it is privately blocked by a generalized coalition γ of support S_γ via an allocation y such that*

$$\int_{S_\gamma} \gamma(t)[e_t(\omega) - y_t(\omega)]\, d\mu \gg 0 \qquad \textit{for all } \omega \in \Omega.$$

PROOF: Since $x \notin C_A(\mathcal{E})$, then there exist a generalized coalition γ of support S_γ and an alternative allocation y such that

(1) $\quad y_t(\cdot)$ is $\mathcal{F}_t-$measurable $\quad$ for almost all $t \in S_\gamma$,

(2) $\quad h_t(y_t) > h_t(x_t) \quad$ for almost all $t \in S_\gamma$

(3) $\quad \displaystyle\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \le \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \quad$ for all $\omega \in \Omega$.

By the previous lemma, there exists a subset C of S_γ such that $y_t(\omega) \gg 0$ for almost all $t \in C$ and all $\omega \in \Omega$. Moreover, from the continuity assumption, it follows that there exist a real number $\varepsilon \in (0, 1)$ and a subset B of C, with positive measure, such that

$$h_t(\varepsilon y_t) > h_t(x_t) \qquad \text{for almost all } t \in B,$$

and clearly $\varepsilon y_t(\omega) \ll y_t(\omega)$ for almost all $t \in B$ and for all $\omega \in \Omega$.

Define for all $\omega \in \Omega$ and all $t \in S_\gamma$ the following allocation z

$$z_t(\omega) = \begin{cases} y_t(\omega) & \text{if } t \in S_\gamma \setminus B \\ \varepsilon y_t(\omega) & \text{if } t \in B, \end{cases}$$

and notice that for almost all $t \in S_\gamma$, $z_t(\cdot)$ is $\mathcal{F}_t$-measurable and $h_t(z_t) > h_t(x_t)$. Moreover, since $\gamma(t)$ and $y_t(\omega)$ are strictly positive for almost all $t \in B$ and all $\omega \in \Omega$, it follows that for all $\omega \in \Omega$,

$$\begin{aligned} \int_{S_\gamma} \gamma(t) z_t(\omega)\, d\mu \;&=\; \int_{S_\gamma \setminus B} \gamma(t) y_t(\omega)\, d\mu + \int_B \gamma(t) \varepsilon y_t(\omega)\, d\mu \ll \\ \ll \int_{S_\gamma \setminus B} \gamma(t) y_t(\omega)\, d\mu \;&+\; \int_B \gamma(t) y_t(\omega)\, d\mu = \int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \leq \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu. \end{aligned}$$

Therefore, x is privately blocked in the Aubin sense by the generalized coalition γ via the allocation z, and

$$\int_{S_\gamma} \gamma(t) z_t(\omega)\, d\mu \ll \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \quad \text{for all } \omega \in \Omega.$$

□

We are now ready to prove Theorem 3.1.

PROOF of Theorem 3.1: Obviously, if an allocation x is privately blocked by a generalized coalition with full support then it is not in the Aubin private core, i.e., $C_A(\mathcal{E}) \subseteq T - C_A(\mathcal{E})$.

Conversely, let $x \in T - C_A(\mathcal{E})$ and assume that $x \notin C_A(\mathcal{E})$. Then, there exist a generalized coalition $\gamma : T \to [0, 1]$ and an allocation $y : S_\gamma \times \Omega \to I\!R^\ell_+$ such that

$$\begin{array}{ll} (i) & y_t(\cdot) \text{ is } \mathcal{F}_t\text{-measurable for almost all } t \in S_\gamma, \\ (ii) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in S_\gamma, \\ (iii) & \displaystyle\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \leq \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \quad \text{for all } \omega \in \Omega. \end{array}$$

where $S_\gamma = \{t \in T : \ \gamma(t) > 0\}$.

From Lemma 5.2 we may assume that

$$(iii) \quad \int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \ll \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \quad \text{for all } \omega \in \Omega,$$

or equivalently

$$(iii) \quad \int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu + d(\omega) = \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \quad \text{for all } \omega \in \Omega,$$

where $d(\omega) \gg 0$ for all $\omega \in \Omega$. Given a vector δ with all equal and positive components (i.e., $\delta = (\delta, \ldots, \delta)$ and $\delta > 0$), define the following allocation $\hat{y} : T \times \Omega \to I\!R^\ell_+$:

$$\hat{y}_t(\omega) = \begin{cases} y_t(\omega) & \text{if } t \in S_\gamma \\ x_t(\omega) + \delta & \text{if } t \in T \setminus S_\gamma, \end{cases}$$

and we notice that $\hat{y}_t(\cdot)$ is $\mathcal{F}_t$-measurable for almost all $t \in T$ and $h_t(\hat{y}_t) > h_t(x_t)$ for almost all $t \in T$. For any state of nature ω and for any $\delta \gg 0$, there exists a positive real number $\epsilon_\delta(\omega) \in (0, 1)$, "small enough"[8] that for any $\omega \in \Omega$,

$$\epsilon_\delta(\omega) \int_{T \setminus S_\gamma} [x_t(\omega) + \delta - e_t(\omega)]\, d\mu \leq d(\omega).$$

Define[9] $\bar{\epsilon}_\delta = \min_{\omega \in \Omega} \epsilon_\delta(\omega)$, and notice that $\bar{\epsilon}_\delta \neq 0$ and for all $\omega \in \Omega$,

$$\bar{\epsilon}_\delta \int_{T \setminus S_\gamma} [x_t(\omega) + \delta - e_t(\omega)]\, d\mu \leq d(\omega).$$

Let $\hat{\gamma} : T \to [0, 1]$ be defined as follows

$$\hat{\gamma}(t) = \begin{cases} \gamma(t) & \text{if } t \in S_\gamma \\ \bar{\epsilon}_\delta & \text{if } t \in T \setminus S_\gamma, \end{cases}$$

then, for any state ω,

$$\begin{aligned} \int_T \hat{\gamma}(t) \hat{y}_t(\omega)\, d\mu &= \int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu + \int_{T \setminus S_\gamma} \bar{\epsilon}_\delta [x_t(\omega) + \delta]\, d\mu \leq \\ &\leq \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu - d(\omega) + \int_{T \setminus S_\gamma} \bar{\epsilon}_\delta e_t(\omega)\, d\mu + d(\omega) = \\ &= \int_T \hat{\gamma}(t) e_t(\omega)\, d\mu. \end{aligned}$$

We have built a generalized coalition $\hat{\gamma}$, with full support, which privately blocks x via the allocation $\hat{y}$. This completes the proof. □

[8] We use the subscription δ to stress the dependence of ϵ with respect to the vector δ.

[9] Trivially, such a minimum exists since Ω is finite.

5.3 Proof of Section 4

PROOF of Theorem 4.1: We have already noted that for any $\alpha \in (0, \mu(T))$, $C_A(\mathcal{E}) \subseteq \mathcal{S}_\alpha^- - C_A(\mathcal{E})$; therefore we just need to prove the converse. First consider the case that $\mathcal{E}$ is atomless. Assume on the contrary that for some $\alpha \in (0, \mu(T))$ there exists an allocation x in $\mathcal{S}_\alpha^- - C_A(\mathcal{E})$ which is outside the Aubin private core. This means that there exist a generalized coalition $\gamma : T \to [0,1]$ and an allocation $y : S_\gamma \times \Omega \to I\!R_+^\ell$ such that

$$\begin{array}{ll} (i) & y_t(\cdot) \text{ is } \mathcal{F}_t\text{-measurable for almost all } t \in S_\gamma \\ (ii) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in S_\gamma \\ (iii) & \displaystyle\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \le \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \quad \text{for all } \omega \in \Omega, \end{array}$$

where $S_\gamma = \{t \in T : \ \gamma(t) > 0\}$ is the support of γ.

Notice that $\mu(S_\gamma) \ge \alpha$, otherwise $x \notin \mathcal{S}_\alpha^- - C_A(\mathcal{E})$, which is a contradiction. Define the measure ν over measurable subsets of S_γ as follows: for any $B \in \mathcal{T}$, such that $B \subseteq S_\gamma$

$$\nu(B) = \left(\int_B \gamma(t)[e_t(\omega) - y_t(\omega)]\, d\mu;\ \mu(B) \right).$$

Since

$$\begin{array}{rcl} \nu & \text{is} & \text{non atomic} \\ \nu(S_\gamma) & = & \left(\displaystyle\int_{S_\gamma} \gamma(t)[e_t(\omega) - y_t(\omega)]\, d\mu;\ \mu(S_\gamma) \right) \\ \nu(\emptyset) & = & (0,0), \end{array}$$

given $\beta < \frac{\alpha}{\mu(S_\gamma)} \le 1$, by Lyapunov convexity theorem, there exists a measurable subset B_β of S_γ, such that $\nu(B_\beta) = \beta \nu(S_\gamma)$, that is

$$\begin{array}{rcl} \displaystyle\int_{B_\beta} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu & = & \beta \displaystyle\int_{S_\gamma} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \le 0 \quad \text{for all } \omega \in \Omega, \quad \text{and} \\ \mu(B_\beta) & = & \beta \mu(S_\gamma) < \alpha. \end{array}$$

Thus, the allocation x is privately blocked by the generalized coalition γ', defined as $\gamma'(\cdot) = \gamma(\cdot)\chi_{B_\beta}(\cdot)$, whose support B_β has measure smaller than α. This contradicts the assumption that x belongs to $\mathcal{S}_\alpha^- - C_A(\mathcal{E})$.

Let $\mathcal{E}$ now be a mixed differential information economy. Then, we want to show that

$$C_A(\mathcal{E}) = \mathcal{S}_\alpha^- - C_A(\mathcal{E}) \qquad \text{for all } \alpha \in (\mu(T_1), \mu(T)).$$

As observed before one inclusion is obvious. To prove the converse inclusion, let us proceede by the way of contradiction. Assume that for some α in $(\mu(T_1), \mu(T))$ there exists an allocation x outside the Aubin private core such that $x \in \mathcal{S}_\alpha^- - C_A(\mathcal{E})$. Let γ be the generalized coalition which privately blocks x via an alternative allocation y, that is

$$
\begin{array}{ll}
(i) & y_t(\cdot) \text{ is } \mathcal{F}_t - \text{measurable for almost all } t \in S_\gamma, \\
(ii) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in S_\gamma, \\
(iii) & \displaystyle\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \le \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \qquad \text{for all } \omega \in \Omega,
\end{array}
$$

where $S_\gamma = \{t \in T \,:\, \gamma(t) > 0\}$. Since $x \in \mathcal{S}_\alpha^- - C_A(\mathcal{E})$, it follows that $\mu(S_\gamma) \ge \alpha > \mu(T_1)$ and hence[10] $\mu(S_\gamma \cap T_0) > 0$. Moreover,

$$\mu(S_\gamma \cap T_0) = \mu(S_\gamma) - \mu(S_\gamma \cap T_1) \ge \alpha - \mu(S_\gamma \cap T_1) > 0.$$

Therefore, there exists $\epsilon \in (0,1)$ such that $\epsilon\mu(S_\gamma \cap T_0) < \alpha - \mu(S_\gamma \cap T_1)$. Applying Lyapunov convexity theorem to $S_\gamma \cap T_0$ we find $B_\epsilon \in \mathcal{T}$ such that $B_\epsilon \subseteq S_\gamma \cap T_0$; $\mu(B_\epsilon) = \epsilon\mu(S_\gamma \cap T_0)$ and

$$\int_{B_\epsilon} \gamma(t)\,[y_t(\omega) - e_t(\omega)]\, d\mu = \epsilon \int_{S_\gamma \cap T_0} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \quad \text{for all } \omega \in \Omega.$$

Define

$$\tilde{\gamma}(t) = \begin{cases} \gamma(t) & \text{if } t \in B_\epsilon \\ \epsilon\gamma(t) & \text{if } t \in S_\gamma \cap T_1 \\ 0 & \text{otherwise,} \end{cases}$$

and notice that the support of $\tilde{\gamma}$, denoted by $S_{\tilde{\gamma}}$, is $B_\epsilon \cup (S_\gamma \cap T_1)$ with measure smaller than α:

$$\mu(S_{\tilde{\gamma}}) = \mu(B_\epsilon \cup (S_\gamma \cap T_1)) = \mu(B_\epsilon) + \mu(S_\gamma \cap T_1) = \epsilon\mu(S_\gamma \cap T_0) + \mu(S_\gamma \cap T_1) < \alpha.$$

Since $S_{\tilde{\gamma}} \subseteq S_\gamma$, then for almost all $t \in S_{\tilde{\gamma}}$ $y(\cdot)$ is still $\mathcal{F}_t$-measurable and preferred to x. Finally, for all $\omega \in \Omega$

$$
\begin{aligned}
\int_{S_{\tilde{\gamma}}} \tilde{\gamma}(t)[y_t(\omega) - e_t(\omega)]\, d\mu &= \int_{B_\epsilon} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu + \epsilon \int_{S_\gamma \cap T_1} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \\
&= \epsilon \int_{S_\gamma \cap T_0} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu + \epsilon \int_{S_\gamma \cap T_1} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \\
&= \epsilon \int_{S_\gamma} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \le 0.
\end{aligned}
$$

This is an absurd because $x \in \mathcal{S}_\alpha^- - C_A(\mathcal{E})$. □

[10] If $\mu(S_\gamma \cap T_0) = 0$, then $\mu(S_\gamma) = \mu(S_\gamma \cap T_1) \le \mu(T_1) < \alpha$. This is an absurd since $\mu(S_\gamma) \ge \alpha$.

PROOF of Theorem 4.2: We have already noted that for any $\alpha \in (0, \mu(T))$, $C_A(\mathcal{E}) \subseteq \mathcal{S}_\alpha - C_A(\mathcal{E})$; therefore we just need to prove the converse. To this end, consider first the case that $\mathcal{E}$ is atomless and assume, by the way of contradiction, that for some $\alpha \in (0, \mu(T))$ there exists an allocation x in $\mathcal{S}_\alpha - C_A(\mathcal{E})$ which is outside the Aubin private core. This means that there exist a generalized coalition $\gamma : T \to [0,1]$ with support S_γ, and an allocation $y : S_\gamma \times \Omega \to I\!R^\ell_+$ such that

$$\begin{array}{ll} (i) & y_t(\cdot) \text{ is } \mathcal{F}_t\text{−measurable for almost all } t \in S_\gamma, \\ (ii) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in S_\gamma, \text{ and} \\ (iii) & \displaystyle\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \leq \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \quad \text{for all } \omega \in \Omega. \end{array}$$

Since $x \in \mathcal{S}_\alpha - C_A(\mathcal{E})$, $\mu(S_\gamma) \neq \alpha$. Define the measure ν over measurable subsets of S_γ as follows: for any $B \in \mathcal{T}$, such that $B \subseteq S_\gamma$

$$\nu(B) = \left(\int_B \gamma(t)[e_t(\omega) - y_t(\omega)]\, d\mu;\ \mu(B) \right).$$

Since

$$\begin{array}{rcl} \nu & \text{is} & \text{non atomic} \\ \nu(S_\gamma) & = & \left(\displaystyle\int_{S_\gamma} \gamma(t)[e_t(\omega) - y_t(\omega)]\, d\mu;\ \mu(S_\gamma) \right) \\ \nu(\emptyset) & = & (0,0), \end{array}$$

then, for any $\epsilon \in (0,1)$, by Lyapunov convexity theorem, there exists a measurable subset B_ϵ of S_γ, such that

$$\nu(B_\epsilon) = \left(\epsilon \int_{S_\gamma} \gamma(t)[e_t(\omega) - y_t(\omega)]\, d\mu;\ \epsilon\mu(S_\gamma) \right).$$

Define the following generalized coalition $\hat{\gamma}$ whose support is B_ϵ :

$$\hat{\gamma} : T \to [0,1]$$

$$\hat{\gamma}(t) = \begin{cases} \gamma(t) & \text{if } t \in B_\epsilon \\ 0 & \text{if } t \in T \backslash B_\epsilon. \end{cases}$$

Then, for all $\omega \in \Omega$,

$$\int_{B_\epsilon} \hat{\gamma}(t)[y_t(\omega) - e_t(\omega)]\, d\mu \;=\; \epsilon \int_{S_\gamma} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \leq 0 \quad \text{for all } \omega \in \Omega.$$

Thanks to the generality of ϵ in $(0,1)$, $\alpha = \epsilon\mu(S_\gamma) < \mu(S_\gamma)$, the result is proved for any $\alpha < \mu(S_\gamma)$. If $\alpha > \mu(S_\gamma)$, being $\mathcal{E}$ is atomless, there exists a coalition C containing S_γ such that $\mu(C) = \alpha$. Since $S_\gamma \subseteq C$, we can follow the proof of Theorem 3.1 replacing C by T and complete the proof for the atomless case.

Let $\mathcal{E}$ now be a mixed differential information economy. Then, we want to show that

$$\mathcal{S}_\alpha - C_A(\mathcal{E}) = C_A(\mathcal{E}) \qquad \text{for all } \alpha \in (\mu(T_1), \mu(T)).$$

Again, as observed before one inclusion is obvious. Let α be in $(\mu(T_1), \mu(T))$ and let $x \in \mathcal{S}_\alpha - C_A(\mathcal{E})$; we need to show that $x \in C_A(\mathcal{E})$. Assume on the contrary that $x \notin C_A(\mathcal{E})$, hence by Theorem 3.1 it is privately blocked by a generalized coalition with full support, i.e., $x \notin T - C_A(\mathcal{E})$. This means that there exist a generalized coalition γ and an alternative allocation y such that

$$\begin{array}{ll} (i) & y_t(\cdot) \text{ is } \mathcal{F}_t - \text{measurable for almost all } t \in T, \\ (ii) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in T, \\ (iii) & \displaystyle\int_T \gamma(t) y_t(\omega)\, d\mu \leq \int_T \gamma(t) e_t(\omega)\, d\mu \qquad \text{for all } \omega \in \Omega, \end{array}$$

Observe that $\mu(T_0) > \alpha - \mu(T_1) > 0$, and therefore there exists $\epsilon \in (0,1)$ such that $\epsilon\mu(T_0) = \alpha - \mu(T_1)$. Lyapunov convexity theorem ensures the existence of a measurable subset B of T_0 such that $\mu(B) = \epsilon\mu(T_0)$ and

$$\int_B \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu = \epsilon \int_{T_0} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \quad \text{for any } \omega \in \Omega.$$

Consider the coalition $B \cup T_1$ and the function

$$\tilde{\gamma}(t) = \begin{cases} \gamma(t) & \text{if } t \in B \\ \epsilon\gamma(t) & \text{if } t \in T_1 \\ 0 & \text{otherwise.} \end{cases}$$

Notice that the alternative allocation y and the generalized coalition $\tilde{\gamma}$ are such that

$$\begin{array}{ll} (1) & \mu(B \cup T_1) = \mu(B) + \mu(T_1) = \epsilon\mu(T_0) + \mu(T_1) = \alpha \\ (2) & y_t(\cdot) \text{ is } \mathcal{F}_t - \text{measurable for almost all } t \in B \cup T_1, \\ (3) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in B \cup T_1, \text{ and finally for all } \omega \in \Omega \end{array}$$

$$(4) \quad \int_{B \cup T_1} \tilde{\gamma}(t)[y_t(\omega) - e_t(\omega)]\, d\mu = \epsilon \int_{T_0} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu + \epsilon \int_{T_1} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \leq 0.$$

This is a contradiction since $x \in \mathcal{S}_\alpha - C_A(\mathcal{E})$. □

Observe that the assumption that all agents have a positive amount of each commodity in each state of nature only needs to apply Theorem 3.1.

PROOF of Theorem 4.3: One inclusion is obvious, since $C_A(\mathcal{E}) \subseteq \mathcal{S}_\alpha^- - C_A(\mathcal{E})$ for any $\alpha \in (0, \mu(T))$. Therefore, we just need to prove the converse. To this end, assume on the contrary that for some $\alpha \in (0, \mu(T))$ there exists an allocation x satisfying the equal treatment property on the atomic sector (i.e., $x \in ET(\mathcal{E})$) such that $x \in \mathcal{S}_\alpha^- - C_A(\mathcal{E}) \setminus C_A(\mathcal{E})$. This implies that there exist a generalized coalition γ and an alternative allocation y such that

$$\begin{array}{ll} (i) & y_t(\cdot) \text{ is } \mathcal{F}_t - \text{measurable for almost all } t \in S_\gamma, \\ (ii) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in S_\gamma, \\ (iii) & \displaystyle\int_{S_\gamma} \gamma(t) y_t(\omega)\, d\mu \leq \int_{S_\gamma} \gamma(t) e_t(\omega)\, d\mu \qquad \text{for all } \omega \in \Omega, \end{array}$$

where $S_\gamma = \{t \in T \ : \ \gamma(t) > 0\}$ is such that $\mu(S_\gamma) \geq \alpha$, since $x \in \mathcal{S}_\alpha^- - C_A(\mathcal{E})$. Assume without loss of generality[11] that $\mu(S_\gamma \cap T_0) > 0$ and let $\beta \in (0,1)$ be such that $\beta\mu(S_\gamma \cap T_0) < \alpha$. Lyapunov convexity theorem applied to $S_\gamma \cap T_0$ implies the existence of a measurable subset B of $S_\gamma \cap T_0$ such that $\mu(B) = \beta\mu(S_\gamma \cap T_0)$ and

$$\int_B \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu = \beta \int_{S_\gamma \cap T_0} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \quad \text{for all } \omega \in \Omega.$$

Let $I = \{i \in I\!N \ : \ \mu(S_\gamma \cap A_i) > 0\}$ and notice that $I \neq \emptyset$. Indeed, if $I = \emptyset$, then $\mu(S_\gamma \cap T_0) = \mu(S_\gamma)$ and the generalized coalition $\tilde{\gamma}$, defined as $\tilde{\gamma}(t) = \gamma(t)$ if $t \in B$ and $\tilde{\gamma}(t) = 0$ otherwise, blocks x via y. This is an absurd since $\mu(supp_{\tilde{\gamma}}) = \mu(B) = \beta\mu(S_\gamma \cap T_0) < \alpha$. Hence $I \neq \emptyset$. Now, notice that since $\sum_{n \in I\!N} \mu(A_n) = \mu(T_1)$ and $\mu(A_n) \geq 0$ for any n, it follows that $\lim_{n \to \infty} \mu(A_n) = 0$. This implies that there exists an atom C whose measure is smaller than $\alpha - \beta\mu(S_\gamma \cap T_0)$. Define the measure $\hat{\mu}$ on $\mathcal{T}$ such that $\hat{\mu}_{|T \setminus (S_\gamma \cap T_1)} = \mu$ and $\hat{\mu}(A_i) = \gamma(A_i)\mu(A_i)$ for any $i \in I$. Notice that $\hat{\mu}(S_\gamma \cap T_1) = \sum_{i \in I} \gamma(A_i)\mu(A_i)$ and $\int_{S_\gamma \cap T_1} y_t(\cdot)\, d\hat{\mu} = \sum_{i \in I} \gamma(A_i) y_{A_i}(\cdot)\mu(A_i)$. Moreover, define

$$\tilde{\gamma}(t) = \begin{cases} \gamma(t) & \text{if } t \in B \\ \beta \frac{\hat{\mu}(S_\gamma \cap T_1)}{\mu(C)} & \text{if } t = C \\ 0 & \text{otherwise, and} \end{cases}$$

$$\tilde{y}_t(\cdot) = \begin{cases} y_t(\cdot) & \text{if } t \in B \\ \frac{1}{\hat{\mu}(S_\gamma \cap T_1)} \int_{S_\gamma \cap T_1} y_t(\cdot)\, d\hat{\mu} & \text{if } t = C. \end{cases}$$

Since all large agents have the same private information, $\tilde{y}_t(\cdot)$ is $\mathcal{F}_t$-measurable for almost all $t \in B \cup C$. Moreover, $\tilde{\gamma}$ is a generalized coalition whose support $S_{\tilde{\gamma}} = B \cup C$ is such that

$$\mu(S_{\tilde{\gamma}}) = \mu(B) + \mu(C) = \beta\mu(S_\gamma \cap T_0) + \mu(C) < \alpha.$$

Since $x \in ET(\mathcal{E})$, it follows that $h_C(x_C) = h_t(x_t)$ for almost all $t \in S_\gamma \cap T_1$. Furthermore from (ii) and from Jensen's inequality, since atoms have the same utility function and prior, it

[11] The proof can be adopted also if $\mu(S_\gamma \cap T_0) = 0$.

follows that $h_t(\tilde{y}_t) > h_t(x_t)$ for almost all $t \in B \cup C$. Finally, since atoms have also the same initial endowment, from (iii) we have that for all $\omega \in \Omega$,

$$\begin{aligned}
\int_{S_{\tilde{\gamma}}} \tilde{\gamma}(t)[\tilde{y}_t(\omega) - e_t(\omega)]\, d\mu &= \int_B \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu + \tilde{\gamma}(C)[\tilde{y}_C(\omega) - e_C(\omega)]\mu(C) = \\
\int_B \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu &+ \beta \frac{\hat{\mu}(S_\gamma \cap T_1)}{\mu(C)} \left[\frac{1}{\hat{\mu}(S_\gamma \cap T_1)} \int_{S_\gamma \cap T_1} y_t(\omega)\, d\hat{\mu} - e_C(\omega) \right] \mu(C) = \\
\beta \int_{S_\gamma \cap T_0} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu &+ \beta \sum_{i \in I} \gamma(A_i)[y_{A_i}(\omega) - e_{A_i}(\omega)]\mu(A_i) \\
&= \beta \int_{S_\gamma} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \leq 0.
\end{aligned}$$

The function $\tilde{\gamma}$ can be normalized to represents a generalized coalition blocking x via $\tilde{y}$ and this is an absurd since $x \in \mathcal{S}_\alpha^- - C_A(\mathcal{E})$. □

PROOF of Theorem 4.4: One inclusion is obvious, since $C_A(\mathcal{E}) \subseteq \mathcal{S}_\alpha - C_A(\mathcal{E})$ for any $\alpha \in (0, \mu(T))$. Therefore, we just need to prove the converse. Assume on the contrary that for some $\alpha \in (0, \mu(T_0)]$ there exists an allocation $x \in ET(\mathcal{E}) \cap \mathcal{S}_\alpha - C_A(\mathcal{E})$ such that $x \notin C_A(\mathcal{E})$, and hence from Theorem 3.1, $x \notin T - C_A(\mathcal{E})$. This implies that there exist a generalized coalition γ and an alternative allocation y such that

$$\begin{array}{ll}
(i) & y_t(\cdot) \text{ is } \mathcal{F}_t - \text{measurable for almost all } t \in T, \\
(ii) & h_t(y_t) > h_t(x_t) \quad \text{for almost all } t \in T, \\
(iii) & \int_T \gamma(t) y_t(\omega)\, d\mu \leq \int_T \gamma(t) e_t(\omega)\, d\mu \qquad \text{for all } \omega \in \Omega.
\end{array}$$

Now, notice that since $\sum_{n \in I\!N} \mu(A_n) = \mu(T_1)$ and $\mu(A_n) \geq 0$ for any n, it follows that $\lim_{n \to \infty} \mu(A_n) = 0$. This implies that there exists an atom C whose measure is smaller than α. Notice that $\mu(T_0) + \mu(C) \geq \alpha + \mu(C) > \alpha \Rightarrow \mu(T_0) > \alpha - \mu(C) > 0$. Hence there exists $\epsilon \in (0, 1)$ such that $\epsilon\mu(T_0) = \alpha - \mu(C)$. By Lyapunov convexity theorem there exists $B \subseteq T_0$ measurable such that $\mu(B) = \epsilon\mu(T_0)$ and for all $\omega \in \Omega$

$$\int_B \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu = \epsilon \int_{T_0} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu.$$

Define the measure $\hat{\mu}$ on $\mathcal{T}$ such that $\hat{\mu}_{|T_0} = \mu$ and $\hat{\mu}(A_n) = \gamma(A_n)\mu(A_n)$ for any n. Notice that $\hat{\mu}(T_1) = \sum_n \gamma(A_n)\mu(A_n)$ and $\int_{T_1} y_t(\cdot)\, d\hat{\mu} = \sum_n \gamma(A_n) y_{A_n}(\cdot)\mu(A_n)$. Moreover, define

$$\tilde{\gamma}(t) = \begin{cases} \gamma(t) & \text{if } t \in B \\ \epsilon \frac{\hat{\mu}(T_1)}{\mu(C)} & \text{if } t = C \\ 0 & \text{otherwise, and} \end{cases}$$

$$\tilde{y}_t(\cdot) = \begin{cases} y_t(\cdot) & \text{if } t \in B \\ \frac{1}{\hat{\mu}(T_1)} \int_{T_1} y_t(\cdot)\, d\hat{\mu} & \text{if } t = C. \end{cases}$$

Since all large agents have the same private information, $\tilde{y}_t(\cdot)$ is $\mathcal{F}_t$-measurable for almost all $t \in B \cup C$. Moreover, $\tilde{\gamma}$ is a generalized coalition whose support $S_{\tilde{\gamma}} = B \cup C$ is such that

$$\mu(S_{\tilde{\gamma}}) = \mu(B) + \mu(C) = \epsilon\mu(T_0) + \mu(C) = \alpha.$$

Since $x \in ET(\mathcal{E})$, it follows that $h_C(x_C) = h_t(x_t)$ for almost all $t \in T_1$. Furthermore from (ii) and from Jensen's inequality, since atoms have the same utility function and prior, it follows that $h_t(\tilde{y}_t) > h_t(x_t)$ for almost all $t \in B \cup C$. Finally, since atoms have also the same initial endowment, from (iii) we have that for all $\omega \in \Omega$,

$$\begin{aligned}
\int_{S_{\tilde{\gamma}}} \tilde{\gamma}(t)[\tilde{y}_t(\omega) - e_t(\omega)]\, d\mu \quad &= \quad \int_B \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu + \tilde{\gamma}(C)[\tilde{y}_C(\omega) - e_C(\omega)]\mu(C) = \\
\int_B \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \quad &+ \quad \epsilon\frac{\hat{\mu}(T_1)}{\mu(C)}\left[\frac{1}{\hat{\mu}(T_1)}\int_{T_1} y_t(\omega)\, d\hat{\mu} - e_C(\omega)\right]\mu(C) = \\
\epsilon\int_{T_0} \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \quad &+ \quad \epsilon\sum_n \gamma(A_n)[y_{A_n}(\omega) - e_{A_n}(\omega)]\mu(A_n) \\
&= \quad \epsilon\int_T \gamma(t)[y_t(\omega) - e_t(\omega)]\, d\mu \leq 0.
\end{aligned}$$

The function $\tilde{\gamma}$ can be normalized to represents a generalized coalition blocking x via $\tilde{y}$ and this is an absurd since $x \in \mathcal{S}_\alpha - C_A(\mathcal{E})$. □

Observe that if $\alpha > \mu(T_1)$ the additional assumption on large traders is not needed as shown in Theorem 4.2; moreover the assumption that all agents have a positive amount of each commodity in each state of nature only needs to apply Theorem 3.1.

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