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Does Near-Rationality Matter in First-Order Approximate Solutions? A Perturbation Approach

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Does Near-Rationality Matter in First-Order Approximate Solutions? A Perturbation Approach

Frank Hespeler[*] and Marco M. Sorge[**]

Abstract
This paper studies first-order approximate solutions to near-rational dynamic stochastic models. Under near-rationality, subjective beliefs are distorted away from rational expectations via a change of measure process which fulfils some regularity conditions. As a main result, we show that equilibrium indeterminacy may arise even when the martingale representation of beliefs distortion depends on the economy's fundamentals solely. This provides theoretical support to the modeling assumptions of Woodford [American Economic Review 100, 274-333 (2010)].

Keywords: Near-Rationality; Perturbation methods; Equilibrium indeterminacy

JEL Classification: D84; E0; C62; C63

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1 Introduction

The modeling of expectations as drivers of forward-looking behavior of decisionmakers has always been a key issue in macroeconomic analysis. Since Lucas (1976), dynamic stochastic models under rational expectations (RE) have emerged as the reference framework to address policy-related and other questions of interest in macroeconomics.

The RE benchmark has been severely criticized on the ground of its implausibly strong implications, and other theories of expectations have emerged in the literature, which attempt to relax the RE postulate that forecasters' probability beliefs coincide with the model-implied ones[1]. Recently, Woodford (2010) has explored the implications for monetary policy design of the assumption of *near-rational expectations*, i.e. of subjective beliefs that are distorted away from the predictions of the underlying (forecast) model via a change of measure process which fulfills some regularity conditions. Essentially, under near-rationality the alternative probability measure driving beliefs formation is absolutely continuous with respect to the model-consistent one, and the relative discrepancy between subjective and objective conditional probabilities is non-zero almost surely and can be sustained in equilibrium.

From a modeling perspective, Woodford (2010) introduces distorted beliefs in an already approximated linear-quadratic environment under RE. This assumption has been criticized by Benigno and Paciello (2010), on the ground that distorted beliefs would not appear in linearly approximated equilibrium relationships (the AS equation) of an otherwise standard New Keynesian model, as they would rather vanish within the approximation process. The main goal of the present paper is to shed some light on this debate by exploiting a perturbation approach to solution of near-rational expectations models in the sense of Woodford (2010). Specifically, our research question is whether distorted beliefs matter in first-order approximations to the policy functions. In principle, the answer seems to depend on whether the martingale representation of the distortion in beliefs is a function of the economy's fundamentals (i.e. exogenous states) or is rather state-independent. Perhaps surprisingly, we show that distorted beliefs have the potential to induce indeterminate solutions even when they depend on fundamentals solely, as in Woodford (2010). In fact, history-dependent distortions in beliefs may involve stochastic influence factors (in the form of sunspot information) or rather introduce coefficient instability in first-order approximations while also increasing the degree of backward dependence. Overall, our results do not confirm the conjecture of Benigno and Paciello (2010) that linearly perturbed solutions are not sensitive to the near-rationality hypothesis.

The paper is organized as follows. The second section presents the general class of dynamic, discrete-

[1] This literature is too broad to be fully referenced here. For a survey on the learning approach vis-à-vis RE, see Evans and Honkapojha (2001).

time models we are interested in. In section 3 and 4 we derive first-order approximations to the policy functions for the models at issue, and discuss existence and uniqueness of solutions. The last section offers some concluding remarks.

2 The class of models

Let the following nonlinear stochastic vector difference equation:

$$\hat{E}_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \tag{1}$$

describe the equilibrium conditions of a given dynamic stochastic general equilibrium (DSGE) model, where expectations $\hat{E}_t$ are not necessarily rational. Here, the $n_x \times 1$ vector x_t collects predetermined (or state) variables, while the $n_y \times 1$ vector y_t denotes nonpredetermined (or control) variables. The initial conditions x_0 are given.

As in Schmitt-Grohé and Uribe (2004), we partition the state vector x_t into endogenous predetermined state variables x_t^1 and exogenous state variables x_t^2, with the latter following the law of motion:

$$x_{t+1}^2 = \Lambda x_t^2 + \sigma \epsilon_{t+1} \tag{2}$$

where it is assumed, without loss of generality, that both x_t^2 and ϵ_t are $n_\epsilon \times 1$ vectors. We also posit that the innovations ϵ_t are independently and identically distributed on a bounded support, with mean zero and variance/covariance matrix I, and that all the roots of Λ lie within the unit circle. The parameter $\sigma \in \Re^+$ is used to scale the amount of uncertainty stemming from the structural shocks ϵ_t.

Following Hansen and Sargent (2005) and Woodford (2010), distorted beliefs are constructed on the basis of martingale representations. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$ be a properly filtered probability space. Subjective expectations, denoted with $\hat{E}_t := \hat{E}(\cdot \mid \mathcal{F}_t)$, are associated with a distorted probability measure $\hat{\mathcal{P}}$ which is (i) absolutely continuous with respect to the objective measure $\mathcal{P}$ over any finite horizon, and (ii) such that (distorted) conditional expectations of any $\mathcal{F}_t$-measurable (square integrable) random variable w_t are expressed as:

$$\hat{E}(w_{t+j} \mid \mathcal{F}_t) = E\left(\frac{\mathcal{M}_{t+j}}{\mathcal{M}_t} w_{t+j} \mid \mathcal{F}_t\right) \quad \forall j \geq 0 \tag{3}$$

where E_t is the expectations operator associated with the true (model-implied) probability measure $\mathcal{P}$, and $\{\mathcal{M}_t\}_{t=0}^{\infty}$ - with $\mathcal{M}_0 = 1$ - is a non-negative $\mathcal{F}_t$-measurable martingale. Define the scalar process

$m_{t+1} = \mathcal{M}_{t+1}/\mathcal{M}_t$ if $\mathcal{M}_t > 0$ and $m_{t+1} = 1$ if $\mathcal{M}_t = 0$[2], then it holds:

$$m_t \geq 0 \quad a.s., \quad E_t(m_{t+1}) = 1 \quad \forall t \tag{4}$$

We also require that m_t be bounded from above[3].

From a modelling perspective, system (1) may result from optimal decisionmaking on the basis of internally consistent beliefs which satisfy the aforementioned regularity conditions (e.g. Adam and Woodford, 2012). Remarkably, the departure from the RE setting only implies that the otherwise standard structure of preferences (objective functions) of decisionmakers, which generate the equilibrium temporary map (1), are constructed on a distorted probability measure via the martingale representation. This approach is in line with the extant theory of approximately correct beliefs (e.g. Woodford, 2010), where a degree of correspondence between distorted and model-implied probabilities is specified, without explicitly modeling the expectations formation process. For the purpose of our analysis, we need not make any assumptions on the relative distance between conditional probabilities, as we only require that this discrepancy be consistent with the equilibrium system (1). In particular, we do not assume that preferences (of the model's agents and/or of policymakers) also consist of some (additive) relative entropy measure, reflecting the agents' distrust in the model-implied probability distribution or rather the policymakers' concerns for robustness against model misspecification (e.g. Hansen and Sargent, 2005; Woodford, 2010).

One main implication of the aforementioned characterization of beliefs distortion is that the class of models (1) can be equivalently written as a nonlinear stochastic system:

$$E_t z(y_{t+1}, y_t, x_{t+1}, x_t, m_{t+1}, m_t) = 0 \tag{5}$$

where the function z maps $\Re^{2n_y+2n_x} \times \Re^+ \times \Re^+$ into $\Re^{n+1}$, with $n = n_y + n_x$. The $(n+1)$-th equation corresponds to the constraint (4).

Time-invariant, analytic solutions to (5) are interpreted as a function of the state vector x_t, the distortion factor m_t and of the scaling parameter σ, i.e.:

$$y_t = g(x_t, m_t, \sigma) \tag{6}$$

and

$$x_{t+1} = h(x_t, m_t, \sigma) + \eta\sigma\epsilon_{t+1} \tag{7}$$

[2]See Woodford (2010) and Adam and Woodford (2012) for further discussion.

[3]This implies that the distortion factor is restricted to a bounded support. See Kim et al. (2008) for a discussion of the assumptions on the distributional properties of forcing variables in equilibrium systems like (1).

where the matrix $\eta := [0, I]$ is of order $n_x \times n_\epsilon$. The goal is to compute a local approximation of the functions g and h around the nonstochastic steady state $(\bar{y}, \bar{x}, \bar{m})$ of the equilibrium system (5), implicitly defined by:

$$z(\bar{y}, \bar{y}, \bar{x}, \bar{x}, \bar{m}, \bar{m}) = 0, \quad \sigma = 0 \tag{8}$$

and assuming that there exists an arbitrarily small neighborhood of $(\bar{y}, \bar{x}, \bar{m})$ for which the solution functions (g, h) are unique and on which (z, g, h) are sufficiently smooth.

For the purpose of the analysis, we consider two benchmark cases for the distortion factor m_t: first, it is assumed independent of the process ϵ_t at all times; second, it is assumed a function of the history $s^t = \sigma(x_t, x_{t-1}, \ldots)$ of state variables to that point, i.e. $m_t = m(s^t)$. Remarkably, the amount of uncertainty in the model economy is not fully controlled by the scalar σ: even with $\sigma = 0$, there might still be uncertainty stemming from the sequence $\{m_t\}$. The following Lemma provides some fairly intuitive properties of the nonstochastic steady state (NSSS):

Lemma 1. $\bar{m} = 1$ *and NSSS vectors* $(\bar{y}, \bar{x})$ *are unaltered with respect to the RE setting, under either way of specifying the* m_t *process, i.e.* $\bar{y} = g(\bar{x}, 1, 0)$ *and* $\bar{x} = h(\bar{x}, 1, 0)$*.*

3 Independent distortion factor

Let the distortion factor m_t be independent of the model's state, and define

$$Z := E_t z\left(g\left(h(x, m, \sigma) + \eta\sigma\epsilon', m', \sigma\right), g(x, m, \sigma), h(x, m, \sigma) + \eta\sigma\epsilon', x, m', m\right) \tag{9}$$

$$Z = Z(x, m, m', \sigma)$$

where a prime superscript is used to indicate $t+1$-dated variables. Evidently, derivatives of Z of any order satisfy:

$$Z_{x^k m^q m'^p \sigma^j}(x, m, m', \sigma) = 0 \qquad \forall x, m, m', \sigma, k, q, p, j \tag{10}$$

We are looking for first-order approximations to the (g, h) functions of the form:

$$g(x, m, \sigma) = \bar{y} + g_x(\bar{x}, 1, 0)(x - \bar{x}) + g_m(\bar{x}, 1, 0)(m - 1) + g_\sigma(\bar{x}, 1, 0)\sigma \tag{11}$$

$$h(x, m, \sigma) = \bar{x} + h_x(\bar{x}, 1, 0)(x - \bar{x}) + h_m(\bar{x}, 1, 0)(m - 1) + h_\sigma(\bar{x}, 1, 0)\sigma \tag{12}$$

The unknown coefficients are identified by virtue of (10), from which it follows that:

$$Z_\sigma(\bar{x}, 1, 1, 0) = 0, \quad Z_x(\bar{x}, 1, 1, 0) = 0, \quad Z_m(\bar{x}, 1, 1, 0) = 0 \tag{13}$$

The following result is straightforward:

Lemma 2. *In the first-order approximation, the unknowns* $(g_\sigma, h_\sigma, g_x, h_x)$ *are unaltered with respect to the RE framework.*

If the homogenous mapping $Z_\sigma(g_\sigma, h_\sigma)$ is invertible, the unique solution is $g_\sigma = h_\sigma = 0$[4]. Hence, the scale of structural uncertainty impinges neither on the the control nor on the state variables. Moreover, the elements of (g_x, h_x), which satisfy:

$$Z_x(\bar{x}, 1, 1, 0) = [z_{y'} g_x + z_{x'}]\, h_x + z_y g_x + z_x = 0 \tag{14}$$

are readily derived using the algorithm presented in Schmitt-Grohé and Uribe (2004).

Finally, to find (g_m, h_m) we differentiate (9) with respect to m and m' to obtain the systems:

$$[z_{y'} g_x + z_{x'} \quad z_y][h_m \quad g_m]^T = -z_m \tag{15}$$

and

$$z_{y'} g_m = -z_{m'} \tag{16}$$

Let $z_{y'}^+$ denote the Moore-Penrose pseudoinverse of the matrix $z_{y'}$[5]. Provided that $z_{y'} z_{y'}^+ z_{m'} = z_{m'}$, the complete set of solutions to the overdetermined system (16) is in the form:

$$g_m = -z_{y'}^+ z_{m'} + [I - z_{y'}^+ z_{y'}]\xi \tag{17}$$

for an arbitrary $n_y \times 1$ vector ξ. The solution for g_m is unique when $rank(z_{y'}) = n_y$. With this solution (set) at hand, the (possibly non-unique) elements of h_m are obtained from (15)'s upper part[6].

Remarkably, from (15) and (16) it follows that the existence of the zero solution $h_m = g_m = 0$ requires both z_m and $z_{m'}$ to be zero vectors. This is never the case, as long as distorted conditional expectations

[4] See Lan and Meyer-Gohde (2012) for a general discussion of the validity of the nonsingularity assumption.

[5] The pseudoinverse $z_{y'}^+$ can be accurately computed via the singular value decomposition $z_{y'} = UDV$ (Golub and Van Loan, 1996; Ben-Israel and Greville, 2003). In this case, $z_{y'}^+ = UD^+V$. Software packages such as GAUSS and MATLAB easily implement singular value decomposition procedures. Notice that, if the $n \times n_y$ matrix $z_{y'}$ has full rank, then $z_{y'}^+ = (z_{y'}^T z_{y'})^{-1} z_{y'}^T$.

[6] Since differentiating the $(n+1) - th$ equation $(E_t m_{t+1} - 1)$ of Z delivers a zero line in any partial derivative of z other than $z_{m'}$, we can derive a square $n \times n$ system from (15).

enter the expectational difference equation (5). As a consequence, the first-order approximations to the decision rules (11) and (12) under near-rationality will typically not coincide with their RE counterparts. Given the importance of the indeterminacy issue for policy analysis (e.g. Beyer and Farmer, 2008), this result calls for investigating conditions for (local) existence and uniqueness of stable first-order approximate solutions.

3.1 Local existence and uniqueness of equilibrium

In order to test for the existence of a solution in the first-order approximation of the non-linear system (6)-(7) and the distortion process m_t we gather the approximation into the system

$$\begin{pmatrix} \mathbf{I} & 0 & 0 \\ 0 & \mathbf{I} & 0 \\ -g_x & -g_m & \mathbf{I} \end{pmatrix} \begin{pmatrix} x' - \bar{x} \\ m' - 1 \\ y' - \bar{y} \end{pmatrix} = \begin{pmatrix} h_x & h_m & 0 \\ 0 & \mathbf{0} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x - \bar{x} \\ m - 1 \\ y - \bar{y} \end{pmatrix} \tag{18}$$

Exploiting the fact that the coefficient matrix on the left-hand side of (18) is invertible and using the concept of the Schur (or Jordan) decomposition delivers a system of the form

$$\mathbf{R} \begin{pmatrix} x' - \bar{x} \\ m' - 1 \\ y' - \bar{y} \end{pmatrix} = \mathbf{CR} \begin{pmatrix} x - \bar{x} \\ m - 1 \\ y - \bar{y} \end{pmatrix}; \qquad \mathbf{R}^H\mathbf{CR} = \begin{pmatrix} h_x & h_m & 0 \\ 0 & \mathbf{0} & 0 \\ g_x h_x & g_x h_m & 0 \end{pmatrix} \tag{19}$$

in which the eigenvalues are ordered in ascending order along the main diagonal of the left-hand upper block of the upper-triangular (block-diagonal) matrix $\mathbf{C}$ which is bordered by zero blocks[7]. Stability in (19) rests therefore entirely on the eigenvalues of the left-hand upper block of $\mathbf{R}^H\mathbf{CR}$.

Using the results from Hespeler (2008), the system (19) has a convergent solution if (i) its forward-looking part, i.e. potentially the entire x_t^1 and m_t, can be stabilized, i.e. the expectational errors of any future periods balance the shock terms associated within this period. In addition, potential expectational errors in the backward-looking system, i.e. x_t^2, (iia) need to be a function of the ones in the forward-looking part or (iib) need to be explained as a function of the structural shocks (or of sunspot shocks). The characteristics of the gross growth rate m_t of the martingale process in the distortion in expectations formation provide additional limitations for the existence of a solution to system (19). The memory of the martingale process implies that its gross growth rate does not follow any serial correlation pattern throughout time. Hence, the associated eigenvalues are zero and the series is a stationary one. Accordingly, the gross growth rate of the martingale process does not add to instability, but belongs to

[7] For the case of the Jordan decomposition $\mathbf{R}^{-1}$ would be used instead of the complex conjugate of $\mathbf{R}$.

the stabilizing subset of the system's forward-looking part.

By combining this conclusion with the result that the control y depends exclusively on x and m it becomes apparent that the only potentially unstable part of the system is the forward-looking part of x, i.e. x^1. Consequently, assuming unique solutions for all marginal derivatives included in the coefficient matrices of (18), according to Blanchard and Kahn (1982) the system has a unique solution, if and only if the row dimension of the vector x^1 is equal to the number of unstable eigenvalues in h_x. In addition, since $\mathbf{C}$ remains unchanged by postmultiplication by $\mathbf{R}^{-1}$, the question whether g_m and h_m are unique does not impact on the stationarity of (18). On the other hand, for non-unique solutions of g_m and h_m, the system delivers a multiplicity of stable solutions depending on the degrees of freedom in the solution space for g_m and h_m.

4 History-dependent distortion factor

Let now the gross growth rate m_t of the martingale process depend on the state $s_t = (x_t, x_{t-1}, \ldots)^T$, i.e. $m_t = m(s_t)$. Since $m_t = m(h(x_{t-1}, m(s_{t-1}), \sigma) + \eta\sigma\epsilon, \ldots)$, system (5) is equivalent to:

$$E_t \tilde{z}(y_{t+1}, y_t, x_{t+1}, s_t) = 0$$

for some composite function $\tilde{z}$. The equivalent for (9) can be stated as

$$\begin{aligned} \widetilde{Z} &:= E_t \tilde{z}\left(g\left(h(x, m(x,s), \sigma) + \eta\sigma\epsilon', m(x, s_{t-1}), \sigma\right), g(x, m(s), \sigma), h(x, m(s), \sigma) + \eta\sigma\epsilon', s\right) \\ &= \widetilde{Z}(s, \sigma). \end{aligned} \tag{20}$$

Hence, $\widetilde{Z}_{x^k_{t-i}, \sigma^j} = 0 \quad \forall k, j \wedge i \in \{0, ..., t\}$ follows for the derivatives of any potential order. This involves $t+1$ conditions for the first-order derivatives $\widetilde{Z}_{x_{t-i}}(\bar{s}, 0) = 0 \quad \forall i \in \{0, ..., t\}$ and $\widetilde{Z}_{\sigma}(\bar{s}, 0) = 0$. Using the arguments of section 3 we can establish that any unique solution requires $(g_\sigma \ h_\sigma) = (0 \ 0)$, and find solutions for the unknowns (g_x, h_x) and (g_m, h_m) similar to those in section 3. However, none of the latter vectors can be unique as the second vector is the solution to a mixed quadratic equation, on which the first one depends.

4.1 Local existence and uniqueness of equilibrium

Building on the previous results we test for the existence of a solution following the methodology of section 3.1, while reducing the $(t-1)$-th order difference equations resulting from the linearization (11), (12) and the process $m_t = m(h(x_{t-1}, m(s_{t-1}), \sigma) + \eta\sigma\epsilon, \ldots)$ to first-order difference equations by stacking lags in the vector of variables $(x,m,y)^T$ into a vector of dimension $(t-1)\times(dim(x)+dim(m)+dim(y))$. Solving for the vector $d\omega=(dx,dm,dy)^T$ and appending identity equations to the system delivers finally

$$\begin{pmatrix} d\omega_t \\ \vdots \\ d\omega_1 \end{pmatrix} = \mathbf{B} \begin{pmatrix} d\omega_{t-1} \\ \vdots \\ d\omega_0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} \mathbf{D} & \mathbf{F}_{t-2} & \dots & \mathbf{F}_0 \\ \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \dots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{I} & \mathbf{0} \end{pmatrix}, \tag{21}$$

where

$$\mathbf{D} = \begin{pmatrix} h_x + h_m h_x & 0 & 0 \\ m_x & 0 & 0 \\ (g_x + g_m g_x)(h_x + h_m m_x) + g_m m_x & 0 & 0 \end{pmatrix}$$

and

$$\mathbf{F}_i = \begin{pmatrix} h_m h_x & 0 & 0 \\ m_{x_i} & 0 & 0 \\ (g_x + g_m g_x) h_m m_x + g_m m_x & 0 & 0 \end{pmatrix}.$$

Defining $\tilde{\omega}_t = (d\omega_t, \dots, d\omega_1)^T$ and using the Schur Decomposition $\mathbf{G}=\mathbf{V}\mathbf{B}\mathbf{V}^H$ equation (21) changes to

$$\mathbf{V}\tilde{\omega}_t = \mathbf{G}\mathbf{V}\tilde{\omega}_{t-1}. \tag{22}$$

Similarly to section 3.1, the stability of the system depends on the eigenvalues of the upper-triangular matrix $\mathbf{G}$. The structure of $\mathbf{G}$ implies that the solution is entirely driven by the complete history of the state vector s_t. Since condition (i) applies again, the existence of a stable solution requires that shocks be compensated through expectational errors in the forward-looking part of the current state, i.e. x_t^1. Similarly, for the existence of a convergent solution any error in the backward-looking part of the state x_t^2 needs to be a function of either the errors in the forward-looking state or the shock terms (condition (iia) or (iib)). The question whether any existing solution is an unique one depends again on the uniqueness of the solutions for the elements of $\mathbf{B}$ as well as on the condition that the number of unstable eigenvalues in $\mathbf{G}$ matches the dimension of the vector x_t^1. However, as argued above, the elements of the matrix $\mathbf{B}$ are not unique. Hence, no unique convergent solution exists for system (21).

Remarkably, the distortion in beliefs $m(s_t)$ does matter in the first-order approximation because the degrees of freedom within the solution space for the derivatives m_{xt-i} may exert a systematic influence on the policy functions, and/or $m(s_t)$ influences the linear representation by determining the length of the minimal state vector s_t. In both cases, however, the source of beliefs distortion does not involve any independent (nonstochastic) information set: when relevant in the dynamic structure of first-order solutions, it rather introduces stochastic influence factors (in the form of sunspot information) and/or increases the degree of backward dependence, altering the stability properties of the underlying model.

5 Conclusion

The paper has analyzed first-order approximate solutions to dynamic stochastic models under the near-rationality hypothesis, in the sense of Woodford (2010). Using a perturbation approach, we have shown that distorted beliefs do matter in first-order approximations to the policy functions, even when they depend on the economy's fundamentals solely. As a main implication, the near-rationality hypothesis represents a non-negligible source of equilibrium indeterminacy and also emphasizes the potential for coefficient instability in the dynamic structure of linear approximations. This finding can be thought of as providing some theoretical support to the modeling assumptions of Woodford (2010).

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