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WORKING PAPER NO. 340

# On the Fundamentalness of Nonfundamentalness in DSGE Models

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September 2013

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**Abstract**
This note warns against the use of noncausal VARs as a reliable test for indeterminacy. By means of a simple example, we show that determinate models may well entail nonfundamental ARMA equilibrium reduced forms - which only (and uniquely) depend on the fundamental structural shocks -, whereas indeterminate ones may actually be sunspot-free and possess fundamental (i.e. invertible) equilibrium representations. Hence, detecting a causal representation of the data cannot be interpreted as evidence of determinacy.

**Keywords**: Indeterminacy; Noncausal VAR; Nonfundamentalness

**JEL Classification**: D84; E0; C62

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# 1 Introduction

Since the seminal work of Hansen and Sargent (1980), a rapidly growing literature has explored the importance of nonfundamentalness for the empirical evaluation of structural DSGE models (e.g. Hansen and Sargent, 1991; Lippi and Reichlin, 1993, 1994; Giannone and Reichlin, 2006, Fernańdez-Villaverde et al., 2007; Forni et al., 2009). First-order approximate solutions to these models typically admit a state space representation, which involves a VAR in terms of the observed variables. When nonfundamentalness (or noninvertibility) is present, no linear rotation of the VAR innovations is able to recover the structural shocks of the underlying model. Several methods have been proposed to detect the presence of nonfundamentalness and to circumvent its effects for estimation purposes[1].

This note explores an idea put forward by Lanne and Saikkonen (2011,2013) in the context of the analysis of noncausal (vector) autoregressive models. As one of the potential application of their work, Lanne and Saikkonen (2013) mention that "[...] checking for causality facilitates checking for [equilibrium] determinacy in that detecting a causal VAR representation of the data can be interpreted as evidence in favor of determinate equilibria". Roughly, a linear AR process is noncausal when some of the roots of the lag polynomials lie inside the unit disc. In this case, the autoregression has no linear representation only in terms of lags, and admits an infinite-order MA representation whose filter depends on negative powers of the lag operator, i.e. on future realizations of the forcing process. Since a similar dependence also occurs in the noncausal VAR model examined by Lanne and Saikkonen (2013), the authors argue on the opportunity of exploiting noncausal VARs to detect the presence of nonfundamentalness and hence indeterminacy.

The goal of this note is to warn against the use of noncausal VARs as a reliable test for indeterminacy. To this end, we draw attention to the potentially confounding treatment of nonfundamental (indeterminate) solutions to DSGE models as identical with nonfundamental VARMA-type representations of these solutions. The nonfundamental (i.e. noninvertible) representation of a determinate equilibrium model and the potential for nonfundamental (i.e. not related to the economy's fundamentals) or sunspot uncertainty under equilibrium indeterminacy are in fact different objects. Our analysis makes clear, by means of a simple example, that determinate models may well entail nonfundamental ARMA equilibrium reduced forms - which only (and uniquely) depend on the structural (fundamental) shocks -, whereas indeterminate ones may actually possess fundamental (i.e. invertible) representations.

Remarkably, both determinacy and indeterminacy may be associated with nonfundamentalness even when equilibrium reduced forms are only driven by structural (fundamental) shocks. We agree that nonfundamentalness is ultimately an issue of limited information. However, we emphasize, its origin

[1] Alessi et al. (2011) provide a comprehensive review of the related literature.

crucially hinges on the (untestable) dynamic structure of the underlying data generating process - which in turn governs the responsiveness of equilibrium paths to endogenous revisions in expectations -, rather than on the mere possibility of multiple equilibria[2]. As a consequence, checking for noncausality in the data may not be interpreted as a useful device to test for the indeterminacy hypothesis.

The structure of the note is as follows. Section 2 presents standard definitions of statistical nonfundamentalness and (equilibrium) indeterminacy, to be employed for the subsequent analysis. In section 3, we exploit a highly stylized model economy to show that indeterminacy is neither necessary nor sufficient for nonfundamentalness to arise. Section 4 concludes.

# 2 Background

Let $y_t$ be a $n$-dimensional (zero-mean) covariance stationary, square-integrable process defined on a properly filtered probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Assume $y_t$ admits the following MA representation:

$$y_t = C(L)u_t \tag{1}$$

where $u_t$ is a $q$-dimensional (zero-mean) white noise process with time-invariant (diagonal) covariance matrix $\Sigma_u$. Requiring a rational spectral density for $y_t$ implies that the entries of the (possibly infinite order) filter $C(L)$ are rational functions of the lag operator $L$.

Let $\mathcal{S}^y$ denote the subspace spanned by $\{y_{t-k}, k \geq 0\}$. The following definitions of nonfundamental representation and nonfundamental shocks for the square case ($n = q$) are borrowed from Alessi et al (2011)[3]:

**Definition 1.** *Consider the MA representation (1). Then:*
*(i) $y_t = C(L)u_t$ is fundamental if all the roots of $det\, C(z)$ are outside the complex unit circle, i.e.:*

$$det\, C(z) \neq 0 \quad \forall z \in \mathbb{C} \quad s.t. \quad |z| < 1$$

*(ii) the process $u_t$ in (1) is $y_t$-fundamental if $u_t \in \mathcal{S}^y$.*

When (1) is fundamental, it can be inverted to a (possibly infinite-order) VAR representation with one-sided lag polynomial (invertibility in the past). Hence, the shocks $u_t$ can be fully recovered from an estimated causal VAR model.

[2]This finding echoes the analysis of determinate versus indeterminate equilibrium frameworks in the recent macroeconometrics literature (e.g., Kamihigashi, 1996; Lubik and Schorfheide, 2004; Beyer and Farmer, 2007; Beyer and Farmer, 2008; Fanelli, 2012; Sorge, 2012).

[3]Notice that we consider nonfundamentalness and noninvertibility in the past as synonym, as we are interested in the possibility of recovering the $u_t$ as forecast errors of optimal recursive linear forecasts of the observables $y_t$. See Alessi et al. (2011) for a deeper discussion of this point.

Equilibrium conditions of DSGE models are typically in the form of nonlinear expectational difference equations, whose solutions can be typically cast in VAR form. Equilibrium indeterminacy (or local nonuniqueness) means that there exists an infinite number of these solutions, possibly involving driving forces which are unrelated to fundamentals (sunspots). In the latter case, solutions are typically referred to as nonfundamental in the structural macroeconomics literature (e.g. Benhabib and Farmer, 1999; Lubik and Schorfheide, 2003). More formally, consider the generic RE model written in Sims (2002)'s (log-linearized) canonical form:

$$\Gamma_0(\theta)s_t = \Gamma_1(\theta)s_{t-1} + \Psi(\theta)u_t + \Pi(\theta)\eta_t \tag{2}$$

where $s_t$ is the state vector (composed of all endogenous and possibly some exogenous variables), $u_t$ collects exogenous innovations (shocks), and $\eta_t$ are endogenous expectations revisions (forecast errors). $\Gamma_0$, $\Gamma_1$, $\Psi$ and $\Pi$ are conformable matrices holding the (deep) structural parameters $\theta$. According to Lubik and Schorfheide (2003, 2004), the full set of (stable) solutions to (2) is of the form[4]:

$$s_t = \Gamma^*(\theta)s_{t-1} + \Psi^*(\theta, \tilde{M})u_t + \Pi^*(\theta, M_\xi)\xi_t \tag{3}$$

where $\xi_t$ is a vector of conditionally mean zero sunspot shocks, which are orthogonal to $u_t$. The matrix pair $(\tilde{M}, M_\xi)$, which is needed to add back to the system the components of the equilibrium forecast errors left undetermined by the stability requirement, is unrestricted as it does not depend on the deep parameters $\theta$. Notice that the above reduced form involves two sources of nonuniqueness (indeterminacy). First, the coefficient matrix $\Psi^*$ - which governs the impact of the exogenous process on the state of the system - depends on an arbitrary reduced form matrix $\tilde{M}$. Second, equilibrium dynamics can be driven by extrinsic uncertainty, embedded into the sunspot vector $\xi_t$. Hence, even when $\xi_t = 0$ $a.s.$ $\forall t$ or $M_\xi = 0$, equilibrium indeterminacy may still be present[5]. We can summarize the foregoing argument as follows:

**Definition 2.** *The RE model (2) has a determinate (fundamental) equilibrium if* $\Pi^*$ *is empty and* $\Psi^*(\theta, \tilde{M}) = \Psi^*(\theta)$ *for any arbitrary choice of* $\tilde{M}$.

In words, the RE equilibrium is determinate if the reduced form depends on structural (fundamental) shocks only and the forecast errors are uniquely determined by the fundamental shocks (i.e. the impact of the latter on equilibrium dynamics is fully pinned down by the structure of the model itself).

[4]Existence of a stationary equilibrium requires a vector of endogenous forecast errors capable of offsetting the effect of shocks on the unstable components of the system (Sims, 2002).

[5]Broze and Szafarz (1991) label this situation as parametric indeterminacy, while Lubik and Shorfheide (2004) classify it as indeterminacy without sunspots.

# 3 Two sides of the same coin?

We discuss the relationship between nonfundamentalness and equilibrium indeterminacy in the context of the simple RE model for inflation considered by Cochrane (2011), consisting of a Fisher equation and a Taylor rule for monetary policy:

$$i_t = r + E_t[\pi_{t+1}] \tag{4}$$

$$i_t = r + \phi\pi_t + u_t, \quad \phi > 0 \tag{5}$$

where $i_t$ is the nominal interest rate, $\pi_t$ is inflation and $r$ is the constant real rate. The term $u_t = d(L)\epsilon_t$ is a stationary (possibly parametric) exogenous process (i.e. a monetary policy disturbance), where $\epsilon_t$ is i.i.d. white noise[6]. Combining these two relations the following equilibrium condition obtains:

$$E_t[\pi_{t+1}] = \phi\pi_t + u_t \tag{6}$$

Any solutions to (6) satisfies the recursive equation:

$$\pi_t = \phi\pi_{t-1} + u_{t-1} + \eta_t \tag{7}$$

where $\eta_t := \pi_t - E_{t-1}[\pi_t]$ is the endogenous revision in the expectation for inflation. When $\phi > 1$ (determinacy), the (locally) unique stationary solution is derived through forward substitutions and excluding explosive paths:

$$\pi_t = -\sum_{i=0}^{\infty} \frac{1}{\phi^{1+i}} E_t[u_{t+i}], \qquad \lim_{i\to\infty} E_t[\pi_{t+i}] < \infty \tag{8}$$

When $\phi < 1$ (indeterminacy)[7], by contrast, the endogenous forecast error is not restricted by stability requirements and hence any conditionally mean zero $\eta_t$ will deliver an RE stationary equilibrium of the form (7).

Assume that $d(L) = 1$ (hence $u_t = \epsilon_t$) and let inflation forecast errors be an arbitrary (linear) function of the fundamental shocks only (parametric indeterminacy), i.e. $\eta_t = \tilde{m}u_t$ (Lubik and Schorfheide, 2003). Then, under indeterminacy the reduced form (7) boils down to the stationary ARMA(1,1) process:

$$(1 - \phi L)\pi_t = \tilde{m}(1 + \tilde{m}^{-1})u_t \tag{9}$$

[6] A parametric time series model can be expressed as $(u, \mathcal{P}_\theta, \theta)$, where $\mathcal{P}_\theta$ is a law defined on $u = (u_t)$ and the vector $\theta$ collects the model parameters.

[7] Without loss of generality, we exclude the random walk case $\phi = 1$ from consideration. The subsequent analysis is unaffected by this simplifying assumption.

Evidently, indeterminate equilibria indexed by $|\tilde{m}| > 1$ will entail an invertible MA(1) component. That is, the indeterminate solution is nonfundamental in the macroeconomic sense (the impact of the structural shock $u_t$ on endogenous forecast errors $\eta_t$ is ambiguous), yet the reduced form shock $u_t$ are not necessarily $y_t$-nonfundamental (hence it can be recovered as forecast error from the observable $y_t$, for given suitable choice of $\tilde{m}$[8]). As a consequence, indeterminacy is not sufficient to generate (statistical) nonfundamentalness.

Assume now that $u_t$ admits the ARMA(1,1) representation, i.e. $a(L)u_t = b(L)\epsilon_t$. Then if we let:

$$a(L) = 1 - \phi L; \qquad b(L) = (1 - \tilde{m}) - (1 + \phi)L \tag{10}$$

the stationary ARMA(1,1) solution

$$\pi_t = \phi\pi_{t-1} + \epsilon_{t-1} + \tilde{m}\epsilon_t \tag{11}$$

emerges as the locally unique (determinate) solution of the RE model[9]:

$$E_t[\pi_{t+1}] = (1 + \phi)\pi_t + u_t, \quad \phi \in (0, 1) \tag{12}$$

Evidently, for any $\tilde{m} \in (-1, -\phi)$, the determinate model (12) will exhibit a noninvertible (nonfundamental) MA(1) component. Although depending only (and unambiguously) on the structural shock $\epsilon_t$, the solution is nonfundamental in the statistical sense. Hence, nonfundamentalness does not require the presence of an indeterminate equilibrium.

That nonfundamentalness is non-specific to indeterminacy is clearly not an entirely novel finding. As an increasingly popular example, the literature on news shocks implicitly demonstrates that otherwise standard DSGE models with locally unique equilibria (determinacy) generate nonfundamental VARMA representations if subject to news shocks or foresight[10]. We have cast the question more narrowly, in terms of the actual scope for indeterminacy testing via noncausal (vector) autoregressive models.

# 4 Conclusion

This short note has challenged the view, recently put forward by Lanne and Saikkonen (2011, 2013), that noncausal VAR models might be fruitfully used to test for the indeterminacy hypothesis. Though based

[8]See Lubik and Shorfheide (2003, 2004) on this point.

[9]See Cochrane (2011, Appendix B) for a proof of this statement.

[10]See, among others, Beaudry and Portier (2006), Leeper et al. (2008), Fève et al. (2009), Barsky and Sims (2011), Fujiwara et al. (2011), Leeper and Walker (2011), Fève and Jidoud (2012).

on a highly stylized specification, our results make clear that this is not necessarily the case. Structural macroeconomics and dynamic economic theory are arguably related to the emergence of nonfundamentalness. Yet, equilibrium indeterminacy in linearized DSGE models and nonfundamentalness in time series processes are quite different things. They sometimes cross each other, but never fully overlap. Our message is then one of caution in the application of Lanne and Saikkonen (2011, 2013)'s work to the econometric analysis of nonuniqueness in RE frameworks.

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