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Abstract

This paper introduces time inconsistent preferences into a moral hazard setting where the agent is risk-averse. We derive a necessary optimality condition on the consumption allocation that is different from the so-called Inverse Euler Equation of Rogerson (1985). Specifically, inverse marginal utility is not a martingale, rather it follows a partial adjustment process. If the bias for the present is sufficiently large the optimal allocation will leave the agent with the desire to borrow. We extend the analysis to the case in which the principal does not know if the agent is time consistent or not. Finally, we show that in a setting with a risk-neutral agent and limited liability everything is as if the principal faces a time consistent agent.

JEL classification: D82, D03, E21, D86.

Keywords: repeated moral hazard, time-inconsistency, $\beta\delta$-preferences.

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1 Introduction

It is common in economic models to capture impatience assuming a discount factor between any two time periods that is independent of when utility is evaluated. This assumption implies time consistency. However, this is in sharp contrast both with experimental evidence[1] (Thaler 1991) and with empirical research on the field.[2]

The present paper introduces time inconsistent preferences into the moral hazard model of Rogerson (1985): our agent has preferences that display present-bias or quasi-hyperbolic discounting as in Phelps and Polak (1968) and Laibson (1997) (Section 2).

We derive a necessary optimality condition on the consumption allocation that is different from the so-called Inverse Euler Equation of Rogerson (1985). When preferences are time consistent, agent's inverse marginal utility of consumption is a martingale: conditional on today's outcome, the expectation of tomorrow's inverse marginal utility equals today's inverse marginal utility. When, instead, agent's preferences display present-bias, conditional on today's outcome, tomorrow's marginal utility is a weighted average of current inverse marginal utility and its mean, with weights determined by the magnitude of the agent's time inconsistency. The underlying stochastic process for the inverse marginal utility is a partial adjustment process. An intuitive consequence of this result is that, differently from the time consistent model, the optimal allocation will not always leave the agent with the desire to save. We see that if the bias for the present is sufficiently large, the agent will wish to borrow.[3] We note how the desire to undo the optimal contract by transferring resources across time is less damaging if the agent present bias is large, since in that case borrowing might be impeded by the agent being credit constrained (Section 3).

While most papers in the repeated moral hazard literature focus on the trade-off between insurance and incentives,[4] recently there has been a growing literature in repeated moral hazard models with risk-neutral agents and limited liability.[5] We show that previous results do not extend to this setting. The intuition relies once again on the relevant constraints of the problem: limited liability and the incentive compatibility jointly ensure that the participation constraint is slack. Hence, differently from the model with a risk averse agent, time inconstency does not influence the optimal choice of wages (Section 5).

Finally, we explicitly consider the possibility that the principal does not know if the agent is time consistent or not and, in the latter case, if he is sophisticated (and foresees his future self-control problems) or naive (and does not). If the principal has a large opportunity cost from not having the contract signed the optimal contract does not differ with respect to the one found in the baseline case for a sophisticated agent (Section 4).

In a work that we became aware of after the completion of this paper, Yilmaz (2010) also studies a moral

[1] For comprehensive surveys, see Loewenstein and Prelec (1992) and Frederick et al. (2002).

[2] See Della Vigna (2009) for a recent survey.

[3] It is interesting to notice that a similar result is true in models with preferences that are not time-separable, as in Koehne (2009) and Grochulski and Kocherlakota (2010).

[4] Cf. Lambert (1983), Rogerson (1985), Fudenberg et al. (1990), Malcomson and Spinnewyn (1988), Rey and Salanie (1990), and the comprehensive survey by Chiappori et al. (1994).

[5] See for instance Ohlendorf and Schmitz (2012).

hazard environment with preferences that are not time consistent. His paper focusses on a two efforts two outcomes environment and finds the optimal contract using Kuhn-Tucker conditions. The present paper characterizes all its results in a more general model with n outcomes and any compact set of efforts. In particular the proof of the necessary optimality condition is based on a variational argument similar to Rogerson (1985). Moreover, differently from Yilmaz (2010) we also clarify what happens with a risk-neutral agent and limited liability.

Section 6 offers some concluding remarks.

2 Model

Our model follows closely Rogerson (1985). We consider a principal and a time inconsistent agent who foresees his future self-control problems (i.e. he is sophisticated) meeting at time 0 to agree on a contract that lasts for two more periods, 1 and 2. In each of these, the agent takes an unobservable costly action e_t, chosen in a compact set E, that induces a stochastic outcome $q^t \in \{q_1, .., q_N\}$, $t = 1, 2$. Outcomes in the two periods are stochastically indipendent. The time invariant probability distribution, which depends only on effort within period, is defined by $\Pr(q = q_i|e_t) \equiv \pi_i^{e_t}$.

Outome is observable by both the principal and the agent. In order to implement efforts (e_1, e_2) the principal offers a long term contract that specifies a vector of payments $(w_i, w_{jk})_{i=1,..,N;k=1,..,N} \in \mathbb{R}^N \times \mathbb{R}^{N^2}$, where w_i is the wage after outcome q_i in period 1 and w_{jk} is the wage after outcomes q_i in period 1 and q_k in period 2. When referring to wage as a random variable, we will denote wage in first period as w^1, unconditional wage in period 2 as w^2, while conditional on outcome q_i in period 1 as w^{i2}. The principal is risk neutral and time consistent with discount factor δ. Her objective is to minimize the long term expected cost of implementing efforts in the two periods.

The agent, on the other hand, is risk averse and time inconsistent with $\beta - \delta$ preferences and δ assumed equal to the principal's discount factor. His Bernoulli utility function is $u(w)$, which is strictly increasing and strictly concave, with inverse $h(u) = u^{-1}$. For a cost of effort $\psi(e)$ with $\psi(0) = 0$, and present bias parameter $\beta \in (0, 1)$, his expected intertemporal utility in periods 0 and 1 are:

$$\begin{aligned} U_0\left(w^1, w^2\right) &= u_0 + \beta\delta\mathbb{E}_{q^1}[u\left(w^1\right) + \delta\mathbb{E}_{q^2}[u\left(w^{i2}\right)|q^1 = q_i] - \beta\delta\psi_1\left(e^1\right) - \beta\delta^2\psi_2\left(e^2\right) \\ U_1\left(w^1, w^2\right) &= \mathbb{E}_{q^1}[u\left(w^1\right) + \beta\delta\mathbb{E}_{q^2}[u\left(w^{i2}\right)|q^1 = q_i] - \psi_1\left(e^1\right) - \beta\delta\psi_2\left(e^2\right). \end{aligned}$$

This representation of intertemporal utility has the property that the marginal rate of substitution between incomes in period 1 and 2 is $\mathbb{E}u'\left(w^1\right)/\delta\mathbb{E}u'\left(w^2\right)$ from the perspective of the agent at time 0, while $\mathbb{E}u'\left(w^1\right)/\beta\delta\mathbb{E}u'\left(w^2\right)$ from the perspective of the agent at time 1.

The exact timing of the model is:

- $t = 0$. The principal offers a long term contract to the agent that specifies payments $(w_i, w_{jk})_{i=1,..,N;k=1,..,N}$. The agent accepts or refuses the contract. We normalize the outside option of the agent to 0.

- $t = 1$. The agent chooses e^1, first period outcome is realized and payment w^1 is implemented.
- $t = 2$. The agent chooses e^2, second period outcome is realized and payment w^2 is implemented.

Note that in period 0 there is no effort and no payments are made.

In Section 3 we characterize the stochastic process of wages induced by the optimal contract.

3 Optimal long term contract with a sophisticated agent

We follow the standard approach for solving moral hazard problems with a risk averse agent taking as choice variables the levels of utilities $\{u(w_j), u(w_{jk})\}_{j,k=1,..,N} \equiv \{u_j, u_{jk}\}_{j,k=1,..,N}$. The wage corresponding to level of utility u is given by the inverse function $w = h(u)$. Since there are N outcomes per period and period 2 wage can be conditioned on the outcome of period 1, the optimal contract must choose $N + N^2$ utilities. In order to induce participation and a choice of efforts (e_1, e_2) the principal must choose $\{u_j, u_{jk}\}_{j,k=1,..,N}$ that satisfy:

$$\sum_i \pi_i^{e_1} u_i + \delta \sum_i \pi_i^{e_1} \sum_k \pi_k^{e_2} u_{ik} - \psi_1\left(e^1\right) - \delta\psi_2\left(e^2\right) \geq 0 \tag{PC}$$

$$e_1 \in \arg\max_e \sum_i \pi_i^{e} u_i + \beta\delta \sum_i \pi_i^{e} \sum_k \pi_k^{e_2} u_{ik} - \psi_1(e). \tag{IC1}$$

$$e_2 \in \arg\max_e \sum_k \pi_k^{e_2} u_{ik} - \psi_2(e) \text{ for each } i = 1,..,N. \tag{IC2}$$

Since the agent commits to a long term contract, he will decide participation using his time zero utility function, which trades-off period 1 and 2 using δ (PC).[6] On the other hand, he will choose effort in period 1 using his preferences of period 1, which trade off period 1 and 2 using $\beta\delta$ (IC1). The choice of effort in period 2 is essentially static (IC2). The principal chooses the levels of utilities in order to minimize discounted expected transfers

$$\sum_i \pi_i^{e_1} h(u_i) + \delta \sum_i \pi_i^{e_1} \sum_k \pi_k^{e_2} h(u_{ik}) \tag{1}$$

under constraints (PC), (IC2), (IC1).

In the following proposition, we derive a necessary condition on the inverse marginal utility of consumption that must be satisfied by the optimal contract.

Proposition 1 *At a cost-minimizing solution, if the principal wants to implement efforts* (e_1, e_2) *, wages must satisfy*

$$\beta \frac{1}{u'(w_j)} + (1-\beta) \sum_i \frac{\pi_i^{e_1}}{u'(w_i)} = \sum_k \frac{\pi_k^{e_2}}{u'(w_{jk})} \text{ for } j = 1,..,N. \tag{2}$$

[6] Since we are normalizing outside utility to zero and we are ruling out transfers in period 0, we divide expected discounted utility by $\beta\delta$ with no loss of generality..

In any period, conditional on realization of outcome j, future expected inverse marginal utility is a convex combination of current inverse marginal utility and its mean, weighted by the parameter β that measures the bias for the present. Note that the lower is β the less future expected compensation responds to today's information. The resulting stochastic process of inverse marginal utilities induced by the optimal contract is therefore reminiscent of a partial adjustment process, with the magnitude of adjustment driven by the time inconsistency of the agent.

An important feature of a long term contract is its effect on the desire of the agent to transfer resources across time. With a time consistent agent, Rogerson (1985) has shown that "the agent is *always* left with a desire to save some of his wage and *never* to borrow against future wages". The following proposition shows that this result fails to be true for a sufficiently present-biased agent.

Proposition 2 *Suppose that w is a cost minimizing contract under the assumption that the agent cannot borrow or save. If after period 1 the agent is allowed to borrow or save at the interest rate r such that $1 + r = 1/\delta$ then for β close to 0 the agent will want to borrow, for β close to 1 he will want to save.*

Proof. If the agent can save part of his wage between any two consecutive periods his payoff would be $\mathbb{E}_{q^1}[u(w^1) + \delta\mathbb{E}_{q^2}[u(w^{i2})\,|q^1 = q_i]$

$$V_i(s) = u(w_i - s) + \beta\delta\mathbb{E}_{q^2}\left[u\left(w^{i2} + (1+r)s\right)\middle|\, q^1 = q_i\right],$$

differentiating and evaluating at $s = 0$

$$\begin{aligned}
\left.\frac{d}{ds}V_i(s)\right|_{s=0} &= -u'(w_i) + \beta\mathbb{E}_{q^2}\left[u'\left(w^{i2}\right)\middle|\, q^1 = q_i\right] \qquad (3)\\
&= \beta\left[\frac{-u'(w_i)}{\beta} + \mathbb{E}_{q^2}[u(w^{i2})\,|q^1 = q_i]\right]\\
&= \beta\left[\mathbb{E}_{q^2}[u(w^{i2})\,|q^1 = q_i] - \frac{1}{\mathbb{E}_{q^2}\left[\frac{1}{u'(w^{i2})}\middle| q_1\right] - (1-\beta)\mathbb{E}_{q^1}\left[\frac{1}{u'(w^1)}\right]}\right]
\end{aligned}$$

where in the last line we substituted for $-u'(w_i)/\beta$ from (2), assuming $\beta > 0$. If $\beta = 1$, $dV_i(s)/ds$ is strictly positive by strict convexity of the function $1/x$. If $\beta = 0$, it is strictly negative from (3). By continuity of the solution with respect to β we can deduce that for β close to 0 the agent will want to borrow, for β close to 1 he will want to save, and that there must exist at least a $\beta \in (0, 1)$ for which the agent is indifferent between borrowing and saving. ■

If the principal cannot control agent's savings, a time consistent agent must be unable to save for the principal to achieve a cost minimizing contract. If time inconsistency enters the picture, this is true only if the agent features a small bias for the present. Conversely, if his bias for the present is sufficiently large the agent will be left with a desire to borrow. While it is extreme and unrealistic that the agent is unable to save, it might well be that he is credit constrained and therefore unable to borrow. The previous proposition documents, then, that the agent's desire to undo the contract maybe less costly for the principal if his present bias is large since in that case his desire to borrow can be impeded by the market.

4 Naive, time consistent or sophisticated agent

In this section we consider the possibility that the principal does not know if the agent is time consistent or not and in the latter case if he is sophisticated (and foresees his future self-control problems) or naive (and does not). Specifically we assume that the agent can be S(ophisticated), N(aive) or T(ime-consistent). Moreover the principal can either know agent's "type" or not. We distinguish two possible information sets: (I)$\{TC, \{S, N\}\}$ (II)$\{TC, S, N\}$. In case (I) she is able to set apart time consistent from time inconsistent agents. In case (II) she cannot distinguish the agent's type. Finally, we assume that the principal has a large opportunity cost from not having the contract signed and hence the job not executed (since we assume perfect commitment). Therefore, the principal wants the contract to be accepted regardless of her prior belief on the types' distribution. The next proposition gathers some useful results.[7]

Proposition 3 *i) A naive agent would accept the contract (characterized in Section 3) for the sophisticated, while the opposite is not true; ii) A time consistent agent would accept the contract characterized in Section 3 for the sophisticated,while the opposite is not true; iii) The expected profit from a time consistent agent is higher than that from a sophisticated agent.*

Proof. i) An optimal contract for a principal who faces a naive agent maximizes the same program solved by a principal facing a sophisticated agent except for the first period incentive compatibility constraints:

$$\text{IC for } S \quad : \quad \sum_i \frac{\partial \pi_i^e}{\partial e} u_i + \beta\delta \sum_i \frac{\partial \pi_i^e}{\partial e} \sum_k \pi_k^{e2} u_{ik} - \psi_1^{'}(e) = 0 \tag{4}$$

$$\text{IC for } N \quad : \quad \sum_i \frac{\partial \pi_i^e}{\partial e} u_i + \delta \sum_i \frac{\partial \pi_i^e}{\partial e} \sum_k \pi_k^{e2} u_{ik} - \psi_1^{'}(e) = 0. \tag{5}$$

It is simple to adapt standard techniques (see for instance Laffont and Martimort 2002, pg. 196-199) to show that $\sum_i \partial\pi_i^e/\partial e \sum_k \pi_k^{e2} u_{ik} \geq 0$. This fact togheter with $\beta < 1$ implies that a contract satisfying the IC for S makes the N agent choose an higher effort $\widetilde{e}$. Note that the perceived IC influences the probability with which the agent expects to obtain payments in the future. Then, in period 0, the N agent expects to exert effort $\widetilde{e}$, which maximizes his discounted expected utility using his time consistency delusion, and uses the relative probability distribution in the IR. This implies that, if the S agent accepts the contract, a fortiori the N agent would also accept it since:

$$\sum_i \pi_i^{\widetilde{e}} u_i + \delta \sum_i \pi_i^{\widetilde{e}} \sum_k \pi_k^{e2} u_{ik} - \psi(\widetilde{e}) \geq \sum_i \pi_i^e u_i + \delta \sum_i \pi_i^e \sum_k \pi_k^{e2} u_{ik} - \psi(e) \geq 0.$$

[7] The proofs of this section rely on the validity of the first order approach.

Similarly, it can be shown that a contract which satisfies (5) and the relative IR would not be accepted by the sophisticated agent.

ii) Using an argument similar to the one in the previous step, we notice that the contract for S is accepted by a time consistent agent. This is true since the IC constraint for the time consistent is equal to the one for the N agent.

iii) Finally, recall that when the agent is S the principal minimize (1) subject to the constraints (PC), (IC2) and (IC1). Then, by the envelope theorem we get $d\pi\left(S;\beta\right)/d\beta = \lambda\delta\sum_i \partial\pi_i^e/\partial e \sum_k \pi_k^{e2}u_{ik}$ where λ is the Lagrange multiplier attached to the period 1 IC constraint. Since λ, δ and $\sum_i \partial\pi_i^e/\partial e \sum_k \pi_k^{e2}u_{ik}$ are all positive we have that that $\pi\left(T\right) > \pi\left(S\right)$. ■

Recalling that the principal wants the contract to be accepted regardless of her prior belief on the types' distribution, as an immediate corollary of the previous analysis we get the following result:

Corollary 1 *i) If the principal is only able to set apart time consistent from time inconsistent agents (case I), she will offer the Rogerson's contract to the time consistent agent and the sophisticated agent contract to any time inconsistent agent; ii) If the principal cannot distinguish the agent's type at all (case II) she will offer the contract for the sophisticated type.*

Since, in practice, the last scenario where the principal has no clue on the agent's type seems the most realistic (case II), we expect many real life contracts to have the same characteristcs of the contract characterized in Section 3.

Remark: If the agent is naive and the principal has the same expectation on future agent's selves as the agent, in period 0 they will agree on the same contract that a time consistent agent would accept. However, this would potentially lead to breach of contract in future periods because the agent's future selves will sometimes find it convenient not to exert effort.

5 Risk neutral agent and limited liability

While most papers in the repeated moral hazard literature focus on the trade-off between insurance and incentives, recently there has been a growing literature in repeated moral hazard models focusing on the trade-off between effort incentives and rent extraction. Then it is interesting to realize that our results with a present biased agent do not extend to a setting with a risk-neutral agent and limited liability. The intuition relies on the relevant constraints of the problem. Once again, the principal maximizes the expected profits subject to the participation constraint (PC), the incentive compatibility constraints (IC2) and (IC1) and the limited liability constraints:

$$\begin{aligned} w_i &\geq 0 \; i = 1, .., N \text{ and} \\ w_{jk} &\geq 0 \; j = 1, .., N, k = 1, .., N. \end{aligned}$$

Since the agent can always choose to exert zero effort, which costs zero $\psi\left(0\right) = 0$, both in period 1 and in period 2, limited liability togheter with the incentive compatibility constraints imply that the period 0 participation constraint is always satisfied and hence does not bind at the optimal allocation. Then the difference in preferences when he takes the decision to participate (δ) and the choice of effort in period 1 ($\beta\delta$) does not influence the solution, and everything is as if the principal faces a time consistent agent whose discount factor is equal to $\beta\delta$.

6 Conclusion

In this paper, we analyzed optimal contracts in an finitely repeated moral hazard model in which the agent is time inconsistent. This research would benefit from extending our results to an infinitely repeated setting where there is a stationary representation of the optimal contract. This representation would reduce the multi-period problem to a static variational problem which could be analyzed using standard variational techniques. This is left for future research.

7 Appendix

In this appendix we prove Proposition 1 through a variational argument similar to Rogerson(1985). Suppose that $\{u_j^*, u_{jk}^*\}_{j,k=1,..,N}$ are the cost-minimizing levels of utilities to induce efforts e_1 and e_2, in periods 1 and 2. Pick some outcome j in period 1, and make the following variation for some $y \in \mathbb{R}$:

$$\begin{array}{l} u_j = u_j^* + \frac{\beta y}{\pi_j^{e1}} + (1-\beta)\, y \\ u_i = u_i^* + (1-\beta)\, y \text{ for } i \neq j \\ u_{jk} = u_{jk}^* - \frac{y}{\delta \pi_j^{e1}} \text{ for any } k = 1,..,N \\ u_{ik} = u_{ik}^* \text{ for } i \neq j \text{ and for any } k = 1,..,N. \end{array} \tag{6}$$

The following lemma shows that the new distribution of utilities satisfies the constraints of the problem.

Lemma 1 *$\{u_j, u_{jk}\}_{j,k=1,..,N}$ satisfies (PC), (IC1), (IC2) whenever $\{u_j^*, u_{jk}^*\}_{j,k=1,..,N}$ does.*

Proof. In period 2 the relative desirability of outcomes that are the result of effort in period 2 is not changed, therefore choice in period 2 is unaltered. Ignoring the cost of effort, which is unchanged by the variation, returns from effort in period 1 are given by:

$$\begin{aligned} \sum_i \pi_i^e u_i + \beta\delta \sum_i \pi_i^e \sum_k \pi_k^{e_2} u_{ik} &= \\ &= \begin{cases} \pi_j^e \left(u_j^* + \frac{\beta y}{\pi_j^{e1}} + (1-\beta)\, y \right) + \sum_{i\neq j} \pi_i^e \left(u_i^* + (1-\beta)\, y \right) \\ + \beta\delta \left(\pi_j^e \sum_k \pi_k^{e_2} \left(u_{jk}^* - \frac{y}{\delta \pi_j^{e1}} \right) + \sum_{i \neq j} \pi_i^e \sum_k \pi_k^{e_2} u_{ik}^* \right) \end{cases} \\ &= (1-\beta)\, y + \sum_i \pi_i^e u_i^* + \beta\delta \sum_i \pi_i^e \sum_k \pi_k^e u_{ik}^*. \end{aligned}$$

Therefore the difference in returns with u^* is a constant that does not depend on effort. If e_1 was chosen before, it will be chosen under the new distribution of utilities. Last, the participation constraint is unchanged:

$$\begin{aligned}
\sum_i \pi_i^{e_1} u_i + \delta \sum_i \pi_i^{e_1} \sum_k \pi_k^{e_2} u_{ik} &= \\
&= \begin{cases} \pi_j^{e_1} \left(u_j^* + \frac{\beta y}{\pi_j^{e_1}} + (1-\beta)\, y \right) + \sum_{i \neq j} \pi_i^{e_1} \left(u_i^* + (1-\beta)\, y \right) \\ + \, \delta \left(\pi_j^{e_1} \sum_k \pi_k^{e_2} \left(u_{jk}^* - \frac{y}{\delta \pi_j^{e_1}} \right) + \sum_{i \neq j} \pi_i^{e_1} \sum_k \pi_k^{e_2} u_{ik}^* \right) \end{cases} \\
&= \begin{cases} \sum_i \pi_i^{e_1} u_i^* + \delta \sum_i \pi_i^{e_1} \sum_k \pi_k^{e_2} u_{ik}^* \\ + \pi_j^{e_1} \left(\frac{\beta y}{\pi_j^{e_1}} \right) + (1-\beta)\, y + \delta \left(\pi_j^{e_1} \left(- \frac{y}{\delta \pi_j^{e_1}} \right) \right) \end{cases} \\
&= \sum_i \pi_i^{e_1} u_i^* + \delta \sum_i \pi_i^{e_1} \sum_k \pi_k^{e_2} u_{ik}^*.
\end{aligned}$$

Hence, the new distribution of utilities satisfies all the constraints. ■

Proof of Proposition 1. Write the objective function of the principal as a function of y defined in (6):

$$\begin{aligned}
\Pi(y) &= \pi_j^{e_1} h \left(u_j^* + \frac{\beta y}{\pi_j^{e_1}} + (1-\beta)\, y \right) + \sum_{i \neq j} \pi_i^{e_1} h \left(u_i^* + (1-\beta)\, y \right) \\
&\quad + \delta \left(\pi_j^{e_1} \sum_k \pi_k^{e_2} h \left(u_{jk}^* - \frac{y}{\delta \pi_j^{e_1}} \right) + \sum_{i \neq j} \pi_i^{e_1} \sum_k \pi_k^{e_2} h \left(u_{ik}^* \right) \right).
\end{aligned}$$

Since the original u^* was optimal and u^* coincides with u in $y = 0$, we must have $\Pi'(0) = 0$. Hence:

$$\Pi'(0) = \pi_j^{e_1} h'\left(u_j^*\right) \left(\frac{\beta}{\pi_j^{e_1}} + (1-\beta) \right) + (1-\beta) \sum_{i \neq j} \pi_i^{e_1} h'\left(u_i^*\right) - \sum_k \pi_k^{e_2} h'\left(u_{jk}^*\right) = 0,$$

which simplifies to

$$\beta h'\left(u_j^*\right) + (1-\beta) \sum_i \pi_i^{e_1} h'\left(u_i^*\right) = \sum_k \pi_k^{e_2} h'\left(u_{jk}^*\right).$$

Since $h = u^{-1}$, and therefore $h'(u) = 1/u'(w)$, we obtain the formula in the proposition. ■

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