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On Vind's Theorem for an Economy with Atoms and Infinitely Many Commodities

Anuj Bhowmik[*] and Maria Gabriella Graziano[**]

Abstract

We extend Vind's classical theorem on the measure of blocking coalitions valid in finite dimensional atomless economies (see [29]), to include the possibility of infinitely many commodities as well as the presence of atoms. The commodity space is assumed to be an ordered Banach space which has possibly the empty positive cone. The lack of interior points is compensated by an additional assumption of a cone of arbitrage that allows us to use the Lyapunov's convexity theorem in its weak form. The measure space of agents involves both negligible and non negligible traders. The extension is proved in the general class of Aubin coalitions for which a suitable version of Grodal's result ([17]) is also formulated. Our results wish to point out the relevance of cone conditions dealing with blocking coalitions of arbitrary measure or weight.

JEL Classification: D51, C71

Keywords: Coalitions, Aubin coalitions, core, cone conditions

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1 Introduction

Vind's theorem on the measure of blocking coalitions (see [29]) states that whenever an allocation f of a pure exchange economy with an atomless measure space of agents (T, Σ, μ)[1] does not belong to the core of the market, then for any $\alpha \in (0, \mu(T))$ there is a coalition S whose measure is exactly equal to α such that f is blocked by S[2]. When it combines with the core-equivalence theorem, Vind's theorem shows that the only allocations against which there is no coalition of weight $\alpha \in (0, \mu(T))$ proposing a deviation, are the competitive ones. Applications of this result cover different problems of interest for the study of the core and related cooperative solution concepts. Its proof relies on the validity of Lyapunov's convexity theorem for the range of a finite dimensional vector measure. Hence, no similar simple conclusion holds true as soon as there are atoms in the economy (as stressed in [29]), or when infinitely many commodities are taken into account.

Models of pure exchange economy with infinite dimensional commodity spaces arise naturally, among others, in modeling allocation problems over an infinite time horizon, in economies with uncertainty and in economies with commodity differentiation (see [23] for a discussion of applications and related literature). In infinite dimensional commodity spaces, Lyapunov's convexity theorem fails to be true or it holds true in the so called "weak form": the closure of the range of a vector measure with values in an infinite dimensional commodity space is convex. Consequently, the validity of Vind's theorem is not plainly guaranteed. Moreover, another main technical problem arising in the case of markets with infinitely many commodities is related to the possibility that in many concrete examples the positive cone of such spaces lacks interior points. Such deficiencies may also become crucial in an extension of the Vind's result to an economy with a Banach lattice as the commodity space. This emerges from the recent extensions of Vind's theorem to infinite dimensional commodity spaces listed below:

- the main result in [19], which is relative to the space of bounded sequences l_∞, relies heavily on the structure of the commodity space and on the assumption of myopic utility functions. It covers results in [17] and [27] for the case of countable many commodities. This extension does not face, of course, the problem of an empty positive cone;

- in the extension proved by Evren and Hüsseinov [12], the commodity space $\mathbb{B}$ is an ordered Banach space whose positive cone is assumed to be non-empty. Besides the difficulties related to the use of non-ordered preferences, the assumption of a non-empty norm interior, jointly with a survival condition, is crucial to overcome the problem of a weak version of Lyapunov's convexity theorem. On the other hand, a key ingredient is also given by the Lyapunov's theorem in its strong version for the underlying Lebesgue measure μ;

- one of the key results in [6] is a Vind-type theorem in a framework whose commodity space $\mathbb{B}$ is a Banach lattice having the empty interior: in their model, equal treatment allocations are employed to avoid the difficulties coming from the emptiness of the positive cone. Thus, a positive extension is obtained using the relatively simple form of the integral vector measures. And indeed, in the further extension provided by the authors in [8], the equal treatment property is ruled out and the assumption of a non-empty interior of the positive cone of the commodity space becomes again necessary;

[1] Typically, the continuum of agents $[0, 1]$ with the Lebesgue measure is this kind of economy.
[2] For related results, refer to [17] and [27].

- in recent extensions of Schmeidler's theorem to economies with infinitely many commodities proved in [21] (see also [16]), the authors overcome the difficulties related to the failure of Lyapunov's theorem by imposing a stronger assumption on the measure space of agents: they assume that this space is saturated, a condition that is both necessary and sufficient for the validity of Lyapunov's theorem in its strong version. Due to the strong version, the result can be proved using the classical finite dimensional proof. Obviously, it is difficult to imagine a similar approach dealing with the measure of arbitrary big coalitions. A case in which, again, an interior-like assumption must be imposed on the commodity space (see [22]). Moreover, saturated measure spaces come out to be atomless, and thus, useless for a treatment of markets including non-negligible traders.

In this paper, we consider a pure exchange economy with an infinite dimensional commodity space $I\!B$ whose positive cone has the empty interior. Our measure space of agents (T, Σ, μ) is not necessarily non-atomic. We follow the point of view according to which the atomless model corresponds to an ideal extreme case, since the competition in real economic exchange is far from being perfect[3]. As a consequence, the blocking procedure in our model involves generalized (or Aubin) coalitions defining the so called Aubin core (see [24], [15], [4], [3]). An Aubin coalition is defined as a simple measurable function $\gamma : (T, \Sigma, \mu) \to [0, 1]$, where for a trader $t \in T$ the number $\gamma(t)$ is interpreted as the rate of participation of t in the coalition itself. Under the identification of an ordinary coalition S with its characteristic function χ_S, the aggregated weight of S on the market can be expressed as $\mu(S) = \int_T \chi_S \, d\mu$. Thus, it is natural to define the weight of an Aubin coalition γ on the market by the integral $\widetilde{\mu}(\gamma) = \int_T \gamma \, d\mu$. Hence, we answer positively to the question of whether it is true that for any allocation f not in the Aubin core of a mixed market, a blocking coalition γ of weight equal to α, for any $\alpha \in (0, \mu(T))$, exists. Therefore, we can conclude that, with infinitely many commodities and atoms, any given allocation outside the Aubin core can be improved by a coalition of arbitrary small or large weight. Since Aubin coalitions permit to extend the core equivalence theorem to our general framework, the above result on the arbitrary weight of blocking coalitions also supports competitive equilibria. From a technical point of view, we consider the problem at a double level of generality: with respect to the measure space and to the commodity space. So, the following difficulties arise:

- the positive cone of the commodity space has not interior points;
- the underlying weigh-measure μ may have atoms;
- the relevant vector measures, when restricted to the atomless part of the market, only satisfy the Lyapunov's theorem in its weak form.

We show that, with a properness assumption suitably formulated, all these difficulties can be overcome. This condition is adapted from [10] and [26] and requires a lattice structure on the commodity space to establish the positive result. As it is usual in the case of an infinite dimensional commodity space with the empty positive interior, the open convex cone inherited from the properness condition replaces the role of the positive cone of the commodity space which has an interior point in its positive cone (see [1]). The same properness condition allows the use of the weak Lyapunov convexity

[3]This is for example the case when individual traders are present in the market who concentrate in their hands an initial ownership of some commodities which is large if compared with the total market endowment. Even in the case that the initial endowment is spread over a continuum of small traders, perfect competition may be violated if traders combine in non negligible large coalitions. In this situation, the remaining ocean of negligible traders, which is also present on the market, may not become effective (see [13], [28]).

property of vector measures in our results. At the same time, the presence of atoms for the measure μ does not affect the results in the "Aubin" sense, since, by means of properness, we construct a correspondence between the Aubin core of the mixed market and the core of an atomless associated market. For an allocation in this atomless market, we show that to be in the core is equivalent to have a net trade set disjoint from the properness cone. We remark that the useful role of properness condition in connection with this type of results was already mentioned in [10]. We notice also that the cone condition in the case of atomless markets allows us to extend the Vind's theorem in [12] in its standard formulation (i.e. for ordinary coalitions) to the general case of an infinite dimensional commodity space with possibly the empty interior. As a further result, we formulate and prove a suitable version of Grodal's theorem for an economy with atoms and generalized coalitions. In our Vind's and Grodal's theorem, we show that small agents in the blocking generalized coalition of a fixed measure behave as the same as in ordinary coalition, which means they use their full initial endowments. Moreover, as a by-product of the correspondence mentioned above, we obtain an Aubin core equivalence theorem improving the one proposed by [24], since preference relations are assumed to be convex only on the atomic sector.

Finally, we discuss the following main extensions of the result:

- to the case of production;
- to the case of preferences that are non-convex for all traders;
- to the case of locally convex topological vector space without a lattice structure;
- to the case of asymmetric information economies.

We close observing that results dealing with the measure of ordinary blocking coalitions in mixed market are provided by [12], while for the measure of the support of generalized blocking coalitions by [25], in this last case assuming finitely many commodities.

2 The model

We denote by $\mathbb{B}$ an ordered Banach space. The positive and the negative cone of $\mathbb{B}$ are denoted by $\mathbb{B}_+$ and $\mathbb{B}_-$, respectively. Consider an economy $\mathscr{E} = \big\{(T, \Sigma, \mu), \mathbb{B}_+, \{\succ_t\}_{t\in T}, \{\omega(t)\}_{t\in T}\big\}$ with the following specifications:

(1) (T, Σ, μ) is a complete, finite and positive *measure space of agents*;
(2) $\mathbb{B}_+$ is the *consumption set* of any agent $t \in T$;
(3) $\succ_t$ is the *preference relation* of agent $t \in T$;
(4) $\omega(t)$ is the *initial endowment density* of agent $t \in T$. The function $\omega : T \to \mathbb{B}_+$ is taken as Bochner integrable.

Thus, the economy is an oligopolistic market model: this corresponds to the allowance of atoms in the measure μ, where a μ-atom is a measurable set A of positive measure such that for any measurable subset B of A, exactly one of these equalities $\mu(B) = 0$ and $\mu(A \setminus B) = 0$ must hold. Given that μ is positive and finite, the set T can be expressed as $T = T_0 \cup T_1$, where T_0 is the atomless sector of the market while T_1 is the countable union of disjoint μ-atoms. Let $\mathscr{A} = \{A_1, A_2, \cdots\}$ be the set of μ-atoms in T. The elements of T_0 are called *small agents* and those in $\mathscr{A}$ are called *large agents*. The economy is termed as *atomless* whenever $T = T_0$. Some standard assumptions are given below and will be implicitly assumed in the rest of the paper.

$(A.1)$ *Strict monotonicity*: If $x,\, y \in I\!B_+$ and $x > y$[4], then $x \succ_t y$ for all $t \in T$;

$(A.2)$ *Continuity*: For each $x \in I\!B_+$, the set $\{y \in I\!B_+ : y \succ_t x\}$ is norm open in $I\!B_+$ for all $t \in T$;

$(A.3)$ *Measurability*: For each $x \in I\!B_+$, the set $\{(t, y) \in T \times I\!B_+ : y \succ_t x\}$ is $\Sigma \otimes \mathscr{B}(I\!B_+)$-measurable, where $\mathscr{B}(I\!B_+)$ denotes the Borel σ-algebra on $I\!B_+$.

An *assignment* is a Bochner integrable function from (T, Σ, μ) into $I\!B_+$. An assignment f is said to be an *allocation*[5] if $\int_T f d\mu = \int_T \omega d\mu$. An element of Σ with positive measure is interpreted as an *ordinary coalition* or simply, a *coalition* of agents. Each $S \in \Sigma$ can be regarded as a function $\chi_S : T \to \{0, 1\}$, defined by $\chi_S(t) = 0$, if $t \notin S$; and $\chi_S(t) = 1$, if $t \in S$. Here, $\chi_S(t)$ means the degree of membership of agent $t \in T$ to the coalition S. Following this interpretation for an ordinary coalition, it is natural to introduce a family of *generalized coalitions* as follows (see [24]). For any function $x : T \to \mathbb{R}$, define $S_x = \{t \in T : x(t) \neq 0\}$ and let

$$\mathscr{G} = \{\gamma : T \to [0, 1] : \gamma \text{ is simple, measurable and } \mu(S_\gamma) > 0\}.$$

An element of $\mathscr{G}$ is called a *Aubin* or *generalized coalition* and the set S_γ is termed as the *support* of γ. If $\gamma \in \mathscr{G}$, then $\gamma(t)$ represents the share of resources employed by agent t. By identifying $S \in \Sigma$ with χ_S, one obtains $\Sigma \subseteq \mathscr{G}$. The *weight of an Aubin coalition* γ, denoted by $\widetilde{\mu}(\gamma)$, is given by $\widetilde{\mu}(\gamma) = \int_T \gamma\, d\mu$. For any ordinary coalition, this weight simply coincides with the measure of the coalition itself[6]. Given two assignments x and y and a coalition γ, we write $x \succ_{S_\gamma} y$ to mean that $x(t) \succ_t y(t)$ for μ-almost all $t \in S_\gamma$. The *Aubin core* of $\mathscr{E}$, denoted by $\mathscr{C}^A(\mathscr{E})$, is the set of all allocations f for which there exist no $\gamma \in \mathscr{G}$ and assignment g such that $g \succ_{S_\gamma} f$ and

$$\int_T \gamma g \, d\mu = \int_T \gamma\omega \, d\mu.$$

If $\mathscr{G}$ is replaced with Σ in the previous definition, the corresponding set of allocations is called the *core* of $\mathscr{E}$, denoted by $\mathscr{C}(\mathscr{E})$.

3 Atomless Economies

In this section, using the cone condition similar to that in [26], we extend the Vind's theorem proved in [12] and [29] to the case of atomless economies whose commodity space possibly has the empty interior. As in [12] and [29], we restrict our attention to ordinary coalitions formation.

We start proving that for atomless markets the core and the Aubin core are indistinguishable. Applying the cone condition, we show that this equality holds true without additional assumptions on the commodity space, in particular without separability (see Remark 3.2). It easily follows from the definitions that $\mathscr{C}^A(\mathscr{E}) \subseteq \mathscr{C}(\mathscr{E})$. We obtain the opposite inclusion in the next proposition considering first the case in which the positive cone of the commodity space has non-empty interior.

Proposition 3.1 *Let $T = T_0$ and* $\text{int}I\!B_+ \neq \emptyset$. *If $\int_S \omega \, d\mu \in \text{int}\, I\!B_+$ for all coalition S, then $\mathscr{C}^A(\mathscr{E}) = \mathscr{C}(\mathscr{E})$.*

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[4] i.e. $x - y \in I\!B_+ \setminus \{0\}$

[5] Notice that we don't require free disposal in the feasibility condition.

[6] It is worthwhile to observe that the function $\widetilde{\mu} : \mathscr{G} \to [0, 1]$ is the fuzzy measure extending the function $\mu : \Sigma \to [0, 1]$.

Proof It only remains to prove that $\mathscr{C}(\mathscr{E}) \subseteq \mathscr{C}^A(\mathscr{E})$. Let $f \in \mathscr{C}(\mathscr{E}) \setminus \mathscr{C}^A(\mathscr{E})$. Then there exist a $\gamma \in \mathscr{G}$ and an assignment g such that $g \succ_{S_\gamma} f$ and

$$\int_T \gamma g \, d\mu = \int_T \gamma\omega \, d\mu.$$

For each $r \in \mathbb{Q} \cap (0,1)$, define $S_r = \{t \in S_\gamma : rg(t) \succ_t f(t)\}$. It is clear that $S_r \in \Sigma$. It follows from the strict monotonicity and continuity of $\succ_t$ that $S_r \subseteq S_{r'}$ for all $r, r' \in \mathbb{Q} \cap (0,1)$ satisfying $r < r'$ and

$$\mu\left(S_\gamma \setminus \bigcup_{r \in \mathbb{Q} \cap (0,1)} S_r\right) = 0.$$

For all $r \in \mathbb{Q} \cap (0,1)$, define a function $g_r : T \to [0,1]$ by

$$g_r(t) := \begin{cases} rg(t), & \text{if } t \in S_r; \\ g(t), & \text{otherwise.} \end{cases}$$

Obviously, $g_r \succ_{S_\gamma} f$ and

$$\int_{S_\gamma} \gamma g_r \, d\mu = r \int_{S_\gamma} \gamma g \, d\mu + (1-r) \int_{S_\gamma \setminus S_r} \gamma g \, d\mu.$$

Applying $\int_{S_\gamma} \gamma g \, d\mu = \int_{S_\gamma} \gamma\omega \, d\mu$, one obtains

$$\int_{S_\gamma} \gamma\omega \, d\mu - \int_{S_\gamma} \gamma g_r \, d\mu = (1-r)\left(\int_{S_\gamma} \gamma\omega \, d\mu - \int_{S_\gamma \setminus S_r} \gamma g \, d\mu\right).$$

Since γ is simple and $\int_S \omega \, d\mu \in \operatorname{int} \mathbb{B}_+$ for all coalition S, one concludes $\int_{S_\gamma} \gamma\omega \, d\mu \in \operatorname{int} \mathbb{B}_+$. By absolute continuity of the Bochner integral, one can choose an $r \in \mathbb{Q} \cap (0,1)$ sufficiently close to 1 such that

$$\int_{S_\gamma} \gamma\omega \, d\mu - \int_{S_\gamma} \gamma g_r \, d\mu \in \operatorname{int}\mathbb{B}_+.$$

Put,

$$Z = \left\{\left(\int_{S_\gamma} \lambda g_r \, d\mu, \int_{S_\gamma} \lambda\omega \, d\mu\right) : \lambda : T \to [0,1] \text{ is measurable}\right\}.$$

By [3, Proposition 2] and the infinite dimensional version of the Lyapunov's theorem (see [11] for example), one obtains

$$Z = \operatorname{cl}\left\{\left(\int_B g_r \, d\mu, \int_B \omega \, d\mu\right) : \, B \in \Sigma, B \subseteq S_\gamma\right\}.$$

Since $(\int_{S_\gamma} \gamma g_r \, d\mu, \int_{S_\gamma} \gamma\omega \, d\mu) \in Z$, there exists a sub-coalition B of S_γ such that

$$\int_B \omega \, d\mu - \int_B g_r \, d\mu \in \operatorname{int}\mathbb{B}_+.$$

This completes the proof. □

Remark 3.2 Let us denote by $\mathbb{B}^*$ the norm dual of $\mathbb{B}$, representing the price space. Given an allocation f and a price $p \in \mathbb{B}^*_+ \setminus \{0\}$, the pair (f, p) is said to be a *competitive equilibrium* (and f is called a *competitive allocation*) if for μ-almost all $t \in T$, $f(t)$ is a maximal element of $\succ_t$ in t's budget set $B_p(t) = \{z \in \mathbb{B}_+ : p \cdot z \le p \cdot \omega(t)\}$. The set of competitive allocations is denoted by $\mathscr{W}(\mathscr{E})$. It is well known that $\mathscr{W}(\mathscr{E}) \subseteq \mathscr{C}^A(\mathscr{E}) \subseteq \mathscr{C}(\mathscr{E})$. On the other hand, it was proved in [26, Theorem 4.1] that under the assumptions of Proposition 3.1 and the separability of $\mathbb{B}$, the equality $\mathscr{W}(\mathscr{E}) = \mathscr{C}(\mathscr{E})$ must hold. Thus, one concludes $\mathscr{W}(\mathscr{E}) = \mathscr{C}^A(\mathscr{E}) = \mathscr{C}(\mathscr{E})$. It is worth to point out that to get such equivalence, coalitional resource availability assumption can be weakened to only total resource availability, that is, $\int_T \omega\, d\mu \in \operatorname{int} \mathbb{B}_+$. We prove the statement of Proposition 3.1 independently by competitive allocations, since this paper only focuses on properties of core allocations, as we want to apply results in a situation where core equivalences may not be true (see Section 4).

We now introduce the additional assumptions which are needed to prove our main result.

$(A.4)$ *Convexity*: For all $(t, x) \in T_1 \times \mathbb{B}_+$, the set $\{y \in \mathbb{B}_+ : y \succ_t x\}$ is convex;

$(A.5)$ *Cone Condition*: There exist a vector $v \in \mathbb{B}_+ \setminus \{0\}$ and an open convex solid neighborhood U of 0 such that

- $(A.5.1)$ if C is the open cone spanned by $v + U$, that is, if $C = \bigcup\{\alpha(v + U) : \alpha > 0\}$, then $y \in (C + x) \cap \mathbb{B}_+$ implies $y \succ_t x$ for all $(t, x) \in T \times \mathbb{B}_+$;
- $(A.5.2)$ if $\delta_1, \cdots, \delta_n$ are positive numbers with $\sum_{i=1}^n \delta_i = 1$, $x_i \in \mathbb{B}_+$ and $x_i \notin \delta_i U$ for $i = 1, \cdots, n$, then $\sum_{i=1}^n x_i \notin U$;
- $(A.5.3)$ $\displaystyle\int_S \omega\, d\mu \in C$ for each coalition S.

Remark 3.3 Assumption $(A.4)$ is standard in mixed market models, refer to [10] and [28]. The assumption $(A.5)$, known as *cone condition*, represents the main hypothesis that allow us to extend Vind's theorem to an infinite dimensional commodity space whose positive cone may have the empty interior. This assumption is decomposed into three conditions, which are thoroughly summarized below.

(i) The bundle v satisfying $(A.5.1)$ is termed as an *extremely desirable bundle with respect to* U. Note that $(A.5.1)$ was introduced in [32] and goes back to Mas-Colell. It is commonly adopted to prove the equivalence theorem in the infinite-dimensional framework. The geometric version we formulate in $(A.5.1)$ is equivalent to the following requirement on preference relations (see [26, p. 319]): for each $(t, x) \in T \times \mathbb{B}_+$,

$$\alpha > 0, \quad z \le x + \alpha v, \quad z \in \alpha U \Rightarrow x + \alpha v - z \succ_t x. \tag{1}$$

A well known fact is that (A.5.1) is automatically satisfied when $\operatorname{int}\mathbb{B}_+ \neq \emptyset$ and $(A.1)$ is satisfied. This remark allows us to interpret the cone C as a set of "arbitrage", that is, as a collection of net trade vectors that are universally desirable (see [9] for this interpretation). The openness and convexity of the set of arbitrage allow us to apply the separation theorem in the infinite dimensional set up, to support optimal allocations. Furthermore, the openness of the set of arbitrage and $(A.3)$ can be employed to use a weak form of the Lyapunov convexity theorem.

(ii) The condition $(A.5.2)$, known as the *additivity condition*, was introduced in [26] to prove the equivalence theorem for infinite-dimensional atomless economies. The necessity of this condition stronger than $(A.5.1)$ to extend Aumann's core-equivalence result to an economy with a separable Banach lattice as the commodity space and an atomless measure space of agents, refer to [1, Exercise 3.8]. It is important to note that condition $(A.5.2)$ only concerns the 0-neighborhood U connected to the extremely desirable bundle v, not all the neighborhoods of 0 in the commodity space. In Banach lattices that are also Abstract Lebesgue spaces, the balls centered at the origin are neighborhoods of 0 satisfying $(A.5.2)$ (see [5, 10, 26]).

(iii) The condition introduced in $(A.5.3)$ represents the main difference between the cone condition $(A.5)$ and the cone condition commonly adopted in an economy with an atomless measure space of agents. It is mainly technical and allows us to show that: (a) core allocations in an atomless market can be completely characterized by means of the set of arbitrage; (b) core allocations of the mixed market can be considered as constant intra-atoms in a sense to be specified later. Note that, in general, if the total initial endowment $\bar{\omega}$ is an extremely desirable bundle for the market, then condition $(A.5.3)$ is satisfied when $\omega(t) = \lambda(t)\bar{\omega}$ for some measurable function $\lambda : T \to [0, 1]$.

The next lemma is an infinite-dimensional extension of Lyapunov's convexity theorem, which was essentially used in [8, 12]. To this end, let $\Sigma_S = \{A \in \Sigma : A \subseteq S\}$ for all coalition S of $\mathscr{E}$.

Lemma 3.4 *Let f, g be two assignments and S be a coalition of $\mathscr{E}$. Then, the set*

$$H = \mathrm{cl}\left\{ \left(\mu(B), \int_B f d\mu, \int_B g d\mu \right) : B \in \Sigma_S \right\}$$

is a convex subset of $\mathbb{R} \times \mathbb{B}^2$. Moreover, for any $0 < \delta < 1$, there is a sequence $\{S_n : n \geq 1\} \subseteq \Sigma_S$ such that $\mu(S_n) = \delta\mu(S)$ for all $n \geq 1$,

$$\lim_{n\to\infty} \int_{S_n} f d\mu = \delta \int_S f d\mu \quad \text{and} \quad \lim_{n\to\infty} \int_{S_n} g d\mu = \delta \int_S g d\mu.$$

Proof See [8, Lemma 3.3]. □

For any allocation f, define the *aggregate net-trade set* as

$$\mathscr{N}(f) = \bigcup \left\{ \int_S (g - \omega)\, d\mu : g \succ_S f, S \in \Sigma, \mu(S) > 0 \right\}.$$

Lemma 3.5 *Assume $(A.5)$ and that $S = \bigcup\{S_i : 1 \leq i \leq m\}$, where $\mu(S_i) = \eta$ for some $\eta > 0$ and all $1 \leq i \leq m$. Suppose that $g : S \to \mathbb{B}_+$ is a function such that $g(t) = e_i$, if $t \in S_i$. If $g \succ_S f$ and $\int_S (g - \hat{\omega}) d\mu \in -C$, where*

$$\hat{\omega} = \sum_{i=1}^{m} \left(\frac{1}{\eta} \int_{S_i} \omega d\mu \right) \chi_{S_i},$$

then f is blocked by S.

Proof See [26, Claim 6.1, pp. 322-323]. □

Lemma 3.6 *Assume $T = T_0$ and that the commodity space $\mathbb{B}$ is a Banach lattice. Under $(A.5)$, an allocation $f \in \mathscr{C}(\mathscr{E})$ if and only if $\mathscr{N}(f) \cap -C = \emptyset$.*

Proof Let $f \in \mathscr{C}(\mathscr{E})$ and assume that $\mathscr{N}(f) \cap -C \neq \emptyset$. Thus, there exist a coalition S and an assignment g such that $g \succ_S f$ and

$$\int_S g\, d\mu - \int_S \omega\, d\mu \in -C.$$

Since $g : S \to I\!B_+$ is Bochner integrable, there exists a sequence $\{g_k : k \geq 1\}$, $g_k : S \to I\!B_+$, of simple measurable functions converging pointwise to g for μ-almost all $t \in S$ and $\lim_{k\to\infty} \int_S \|g - g_k\|\, d\mu = 0$. Define

$$S_k = \{t \in S : g_m(t) \succ_t f(t) \text{ for all } m \geq k\}.$$

Note that $\{S_k : k \geq 1\}$ is an increasing sequence of sets and $\lim_{k\to\infty} \mu(S \setminus S_k) = 0$. Assume that there exist $y_k^1, \cdots, y_k^{m_k} \in I\!B_+$ and mutually disjoint sets $T_k^1, \cdots, T_k^{m_k}$ in Σ_{S_k} such that $g_k = \sum_{i=1}^{m_k} y_k^i \chi_{T_k^i}$ and $\mu(T_k^i) = \lambda > 0$ for all $1 \leq i \leq m_k$. It follows from the definition of S_k that $y_k^i \neq 0$ and

$$S_k = \bigcup \{T_k^i : 1 \leq i \leq m_k\}.$$

Put

$$\omega_k = \sum_{i=1}^{m_k} \left(\frac{1}{\lambda} \int_{T_k^i} \omega\, d\mu \right) \chi_{T_k^i}.$$

It is easy to verify that

$$\lim_{k\to\infty} \int_{S_k} (g_k - \omega_k)\, d\mu = \int_S (g - \omega)\, d\mu \in -C.$$

Since C is open, there exists a $k \geq 1$ such that $\int_{S_k} (g_k - \omega_k)\, d\mu \in -C$. Applying Lemma 3.5, one can show that f is blocked by the coalition S_k. Thus, one concludes $\mathscr{N}(f) \cap -C = \emptyset$.

Conversely, suppose $\mathscr{N}(f) \cap -C = \emptyset$ for an allocation f and that $f \notin \mathscr{C}(\mathscr{E})$. Thus, there are a coalition S and an assignment g such that $g \succ_S f$ and $\int_S g\, d\mu = \int_S \omega\, d\mu$. Let B a sub-coalition of S such that $rg \succ_B f$ for some $r \in (0, 1)$. Since C is a cone and $(A.5.3)$ is satisfied, one has $\int_B (1 - r)\omega\, d\mu \in C$. Thus, one can find an $\varepsilon > 0$ such that

$$-\int_B (1 - r)\omega\, d\mu + B(0, \varepsilon) \subseteq -C.$$

By Lemma 3.4, there exists a sub-coalition G of $S \setminus B$ such that

$$\left\| \int_G g\, d\mu - \int_{S\setminus B} rg\, d\mu \right\| < \frac{\varepsilon}{2} \quad \text{and} \quad \left\| \int_G \omega\, d\mu - \int_{S\setminus B} r\omega\, d\mu \right\| < \frac{\varepsilon}{2}.$$

Define $H = G \cup B$ and an assignment h by letting $h(t) = rg(t)$, if $t \in B$; and $h(t) = g(t)$, otherwise. Obviously, $h \succ_H f$ and it is easy to verify that

$$\left\| \int_H h\, d\mu - \int_H \omega\, d\mu + \int_B (1 - r)\omega\, d\mu \right\| < \varepsilon$$

Thus, $\int_H h\, d\mu - \int_H \omega\, d\mu \in -C$, which is a contradiction. □

Remark 3.7 Let $I\!B$ be an ordered Banach space with $\text{int} I\!B_+ \neq \emptyset$. Under the assumption that $\int_S \omega\, d\mu \in \text{int} I\!B_+$ for all coalition S, the fact that $\mathscr{N}(f) \cap -I\!B_+ = \emptyset$ for an allocation f implies

$f \in \mathscr{C}(\mathscr{E})$ can be established analogously. Note that in the proof of Lemma 3.6, the lattice structure, $(A.5.1)$ and $(A.5.2)$ are only required to show that if $f \in \mathscr{C}(\mathscr{E})$ then $\mathscr{N}(f) \cap -C = \emptyset$. However, the fact that $f \in \mathscr{C}(\mathscr{E})$ then $\mathscr{N}(f) \cap -\mathbb{B}_+ = \emptyset$ is straightforward and does not require any conditions. Thus, under the assumption that $\int_S \omega \, d\mu \in \operatorname{int}\mathbb{B}_+$ for all coalition S, one can established that an allocation $f \in \mathscr{C}(\mathscr{E})$ if and only if $\mathscr{N}(f) \cap -\mathbb{B}_+ = \emptyset$.

The next theorem is an extension of Vind's theorem ([29]) to an atomless economy with a Banach lattice as the commodity space. The proof can be compared with that of Theorem 1 in [12].

Theorem 3.8 *Assume $T = T_0$ and that the commodity space $\mathbb{B}$ is a Banach lattice. Let $f \notin \mathscr{C}(\mathscr{E})$ be an allocation and $\alpha \in (0, \mu(T))$. If (A.5) is satisfied, then there exists a coalition E blocking f such that $\mu(E) = \alpha$.*

Proof Suppose that f is an allocation such that $f \notin \mathscr{C}(\mathscr{E})$ and let $\alpha \in (0, \mu(T))$. By Lemma 3.6, there exists a coalition E and an assignment g such that $g \succ_E f$ and

$$\int_E g \, d\mu - \int_E \omega \, d\mu \in -C.$$

The rest of the proof is decomposed into two cases.

Case 1. $\alpha < \mu(E)$. Applying an argument similar to the initial part of the proof of Lemma 3.6, one can find a sub-coalition R of E and a simple measurable function $h : R \to \mathbb{B}_+$ such that $h \succ_R f$, $\mu(R) > \alpha$ and

$$\int_R h \, d\mu - \int_R \hat{\omega} \, d\mu \in -C,$$

where $h = \sum_{i=1}^m y_i \chi_{T_i}$, $R = \bigcup\{T_i : 1 \le i \le m\}$, $\mu(T_i) = \lambda > 0$ for all $1 \le i \le m$ and

$$\hat{\omega} = \sum_{i=1}^m \left(\frac{1}{\lambda} \int_{T_i} \omega \, d\mu \right) \chi_{T_i}.$$

Note that $\int_R \omega \, d\mu = \int_R \hat{\omega} \, d\mu$, and by Lemma 3.4, one has a sequence $\{R_n : n \ge 1\}$ of sub-coalitions of R such that $\mu(R_n) = \alpha$ for all $n \ge 1$,

$$\lim_{n\to\infty} \int_{R_n} h \, d\mu = \frac{\alpha}{\mu(R)} \int_R h \, d\mu \quad \text{and} \quad \lim_{n\to\infty} \int_{R_n} \omega \, d\mu = \frac{\alpha}{\mu(R)} \int_R \omega \, d\mu.$$

Consequently,

$$\lim_{n\to\infty} \int_{R_n} (h - \omega) \, d\mu = \frac{\alpha}{\mu(R)} \int_R (h - \omega) \, d\mu \in -C.$$

So, one has $\int_{R_n} (h - \omega) \, d\mu \in -C$ for all sufficiently large n. Fix such an $n \ge 1$. Thus, by Lemma 3.5, one can show that f is blocked by the coalition R_n.

Case 2. $\alpha \ge \mu(E)$. Take some $0 < r < 1$ such that

$$r < 1 - \frac{\alpha - \mu(E)}{\mu(T \setminus E)}.$$

Since $\int_E (\omega - g) \, d\mu \in C$, one has $\int_E r(\omega - g) \, d\mu \in C$. Thus, there is an $\varepsilon > 0$ such that

$$\int_E r(\omega - g) \, d\mu + B(0, \varepsilon) \subseteq C.$$

It follows from the strict monotonicity of preferences that $\int_E f\,d\mu \in \mathrm{cl}\int_E \Gamma_f\,d\mu$, where the correspondence $\Gamma_f : E \rightrightarrows I\!\!B_+$ is defined by

$$\Gamma_f(t) = \{x \in I\!\!B_+ : x \succ_t f(t)\}.$$

By [31, Theorem 6.2], $\mathrm{cl}\int_E \Gamma_f\,d\mu$ is convex and thus,

$$r\int_E g\,d\mu + (1-r)\int_E f\,d\mu \in \mathrm{cl}\int_E \Gamma_f\,d\mu$$

So, there exists an assignment h such that $h \succ_E f$ and

$$\left\| -\int_E h\,d\mu + r\int_E g\,d\mu + (1-r)\int_E f\,d\mu \right\| < \varepsilon,$$

which further implies

$$v = -\int_E h\,d\mu + r\int_E g\,d\mu + (1-r)\int_E f\,d\mu + r\int_E (\omega - g)\,d\mu \in C.$$

Choose an $\varepsilon' > 0$ such that $-v + B(0,\varepsilon') \subseteq -C$. By Lemma 3.4, there is a sub-coalition B of $T \setminus E$ such that $\mu(B) = (1-r)\mu(T\setminus E)$ and

$$\left\| \int_B f\,d\mu - (1-r)\int_{T\setminus E} f\,d\mu \right\| < \frac{\varepsilon'}{3} \quad \text{and} \quad \left\| \int_B \omega\,d\mu - (1-r)\int_{T\setminus E} \omega\,d\mu \right\| < \frac{\varepsilon'}{3}.$$

Put $H = E \cup B$ and note that $\alpha < \mu(H)$. Define an assignment y by letting

$$y(t) := \begin{cases} h(t), & \text{if } t \in E; \\ f(t) + \frac{z}{\mu(B)}, & \text{otherwise,} \end{cases}$$

where z is a non-zero element of $I\!\!B_+$ with $\|z\| < \frac{\varepsilon'}{3}$. Obviously, one has $y \succ_H f$. Using the feasibility of f, it can be simply verified that

$$\left\| \int_H y\,d\mu - \int_H \omega\,d\mu + v \right\| < \varepsilon'.$$

Thus, $\int_H y\,d\mu - \int_H \omega\,d\mu \in -C$. Since $\alpha < \mu(H)$, by invoking the arguments of *Case 1*, one can find a sub-coalition R of H such that f is blocked by R and $\mu(R) = \alpha$. □

4 The main results

In this section, we prove an extension of Vind's theorem (see [29]) to a mixed economy with a Banach lattice as the commodity space. Thus, our result is satisfied regardless to the number of commodities and the presence of atoms. We state this theorem below.

Theorem 4.1 *Suppose the assumptions* $(A.4)$-$(A.5)$ *are satisfied and that* $I\!\!B_+$ *is a Banach lattice. Let* $f \notin \mathscr{C}^A(\mathscr{E})$ *be an allocation and* $\alpha \in (0, \mu(T))$. *Then, there exists a* $\gamma \in \mathscr{G}$ *and an assignment* g *such that* $g \succ_{S_\gamma} f$, $\widetilde{\mu}(\gamma) = \alpha$ *and*

$$\int_T \gamma y\,d\mu = \int_T \gamma\omega\,d\mu.$$

To prove Theorem 4.1, we associate $\mathscr{E}$ with an atomless economy $\mathscr{E}^*$. To this end, suppose that $(T_1^*, \Sigma_{T_1}^*, \mu_{T_1}^*)$ is an atomless positive measure space such that $T_0 \cap T_1^* = \emptyset$, where each agent A_i one-to-one corresponds to a measurable subset A_i^* of T_1^* with $\mu^*(A_i^*) = \mu(A_i)$ and $T_1^* = \bigcup\{A_i^* : i \geq 1\}$. Define $T^* = T_0 \cup T_1^*$ with the σ-algebra

$$\Sigma^* = \Sigma_{T_0} \oplus \Sigma_{T_1}^* = \{A \cup B : A \in \Sigma_{T_0}, B \in \Sigma_{T_1}^*\}$$

and the measure $\mu^* : \Sigma^* \to \mathbb{R}_+$ by

$$\mu^*(B) = \mu_{T_0}(B \cap T_0) + \mu_{T_1}^*(B \cap T_1),$$

where μ_{T_0} is the restriction of μ to Σ_{T_0}. The space of agents of $\mathscr{E}^*$ is (T^*, Σ^*, μ^*) and the consumption set for each agent $t \in T^*$ is $\mathbb{B}_+$. Assuming that every small agent $t \in A_i^*$ is of the same type as A_i, the initial endowment and preference of each agent $t \in T^*$ are defined by

$$\begin{cases} \omega^*(t) = \omega(t), & \succ_t^* = \succ_t, & \text{if } t \in T_0; \\ \omega^*(t) = \omega_i = \omega(A_i), & \succ_t^* = \succ_i = \succ_{A_i}, & \text{if } t \in A_i^*,\ i \geq 1. \end{cases}$$

For an assignment f in $\mathscr{E}$, let $f^* = \psi(f)$ be an assignment in $\mathscr{E}^*$ defined by

$$f^*(t) := \begin{cases} f(t), & \text{if } t \in T_0; \\ f(A_i), & \text{if } t \in A_i^*,\ i \geq 1. \end{cases}$$

Conversely, for an assignment f^* in $\mathscr{E}^*$, define an assignment $f = \varphi(f^*)$ in $\mathscr{E}$ by

$$f(t) := \begin{cases} f^*(t), & \text{if } t \in T_0; \\ \dfrac{1}{\mu^*(A_i^*)} \displaystyle\int_{A_i^*} f^* \, d\mu^*, & \text{if } t = A_i,\ i \geq 1. \end{cases}$$

We now present some technical Lemmas and Proposition which are needed to prove Theorem 4.1. Denote by $\mathscr{C}^*(\mathscr{E}^*)$ the core of $\mathscr{E}^*$. The following fact is given in the proof of Lemma 3.3 in [10].

Lemma 4.2 *Let S^* be a sub-coalition of A_i^* in $\mathscr{E}^*$. Suppose that g^* is an assignment such that $g^*(t) \succ_i f(A_i)$, for all $t \in S^*$. Under (A.4),*

$$\frac{1}{\mu(S^*)} \int_{S^*} g^* d\mu^* \succ_i f(A_i).$$

Moreover, a similar result also holds if "$\succ_i$" is replaced with "$\succeq_i$".

Lemma 4.3 *Let $f^* \in \mathscr{C}^*(\mathscr{E}^*)$ and (A.4) be satisfied. Then $f(A_i) \sim_i f^*(t)$*[7] *μ-almost all $t \in A_i^*$, $i \geq 1$, where $f = \varphi(f^*)$.*

Proof Define

$$W_i^* = \{t \in A_i^* : f(A_i) \succ_i f^*(t)\}.$$

Assume that $\mu^*(W_i^*) > 0$ for some $i \geq 1$. Then there are an $r \in (0,1)$ and a sub-coalition R_i^* of W_i^* such that $rf(A_i) \succ_i f^*(t)$ for all $t \in R_i^*$. Let $\delta \in (0,1]$ be defined by $\delta = \frac{\mu^*(R_i^*)}{\mu^*(A_i^*)}$. Since $\mu(A_i)\omega_i \in C$ and C is a cone, there exists an $\varepsilon > 0$ such that

$$-\delta(1-r)\mu(A_i)\omega_i + B(0,\varepsilon) \subseteq -C.$$

[7] $\sim_i$ is the indifference relation associated with $\succ_i$

By Lemma 3.4, one can find a sub-coalition S^* of $T^* \setminus A_i^*$ such that

$$\left\| \int_{S^*} f^* \, d\mu^* - \int_{T^* \setminus A_i^*} r\delta f^* \, d\mu^* \right\| < \frac{\varepsilon}{3} \quad \text{and} \quad \left\| \int_{S^*} \omega^* \, d\mu^* - \int_{T^* \setminus A_i^*} r\delta \omega^* \, d\mu^* \right\| < \frac{\varepsilon}{3}.$$

Let v be a non-zero element of $\mathbb{B}_+$ such that $\|v\| < \frac{\varepsilon}{3}$ and $H^* = S^* \cup R_i^*$. Define an allocation h^* by letting

$$h^*(t) := \begin{cases} rf(A_i), & \text{if } t \in R_i^*; \\ f^*(t) + \frac{v}{\mu^*(S^*)}, & \text{otherwise.} \end{cases}$$

Then $h^* \succ_{H^*} f^*$. Further, one can verify that

$$\int_{H^*} h^* \, d\mu^* = v + \int_{S^*} f^* \, d\mu^* + r\delta \int_{A_i^*} f^* \, d\mu^*$$

and

$$\int_{H^*} \omega^* d\mu^* = \int_{S^*} \omega^* \, d\mu^* + \delta \int_{A_i^*} \omega^* \, d\mu^*.$$

It can be checked that

$$\delta(1-r)\mu(A_i)\omega_i + \int_{H^*} h^* \, d\mu^* - \int_{H^*} \omega^* \, d\mu^* \in B(0, \varepsilon)$$

This implies that $\int_{H^*}(h^* - \omega^*)d\mu^* \in -C$, which contradicts with Lemma 3.6. Thus, $\mu^*(W_i^*) = 0$ and $f^*(t) \succeq_i f(A_i)$ μ-almost all $t \in A_i^*$. Let

$$D_i^* = \{t \in A_i^* : f^*(t) \succ_i f(A_i)\}$$

and assume $\mu^*(D_i^*) > 0$. By Lemma 4.2, one has

$$\frac{1}{\mu^*(D_i^*)} \int_{D_i^*} f^* \, d\mu \succ_i f(A_i).$$

Then if $\mu^*(D_i^*) = \mu^*(A_i^*)$ we have a contradiction. If $\mu^*(A_i^* \setminus D_i^*) > 0$, again by Lemma 4.2, we can write

$$\frac{1}{\mu^*(A_i^* \setminus D_i^*)} \int_{A_i^* \setminus D_i^*} f^* \, d\mu \succeq_i f(A_i).$$

Let $\beta = \frac{\mu^*(D_i^*)}{\mu^*(A_i^*)}$. Then

$$f(A_i) = \beta \frac{1}{\mu^*(D_i^*)} \int_{D_i^*} f^* \, d\mu^* + (1-\beta) \frac{1}{\mu^*(A_i^* \setminus D_i^*)} \int_{A_i^* \setminus D_i^*} f^* \, d\mu^* \succ_i f(A_i),$$

which is a contradiction. □

Proposition 4.4 *Under* (A.4)-(A.5)*, the following implications hold true:*

(i) $f \in \mathscr{C}^A(\mathscr{E}) \Rightarrow f^* = \psi(f) \in \mathscr{C}^*(\mathscr{E}^*)$.

(ii) $f^* \in \mathscr{C}^*(\mathscr{E}^*) \Rightarrow f = \varphi(f^*) \in \mathscr{C}^A(\mathscr{E})$.

Proof (i) Let $f \in \mathscr{C}^A(\mathscr{E})$ and assume that $f^* = \psi(f) \notin \mathscr{C}^*(\mathscr{E}^*)$. Thus, there exist a coalition S^* and an assignment g^* such that f^* is blocked by S^* via g^* in $\mathscr{E}^*$. Put $I = \{i : \mu^*(A_i^* \cap S^*) > 0\}$. The rest of the proof is decomposed into two cases:

Case 1. $I \neq \emptyset$. In this case,

$$\int_{S^*\cap T_0} g^* d\mu^* + \sum_{i\in I} \int_{S^*\cap A_i^*} g^* d\mu^* = \int_{S^*\cap T_0} \omega^* d\mu^* + \sum_{i\in I} \int_{S^*\cap A_i^*} \omega^* d\mu^*.$$

For each $i \in I$, choose some $\alpha_i \in (0,1]$ such that $\mu^*(S^* \cap A_i^*) = \alpha_i \mu(A_i)$ and let

$$g_i = \frac{1}{\mu^*(S^* \cap A_i^*)} \int_{S^*\cap A_i^*} g^* d\mu^*.$$

Obviously, $g_i \succ_i f(A_i)$ for all $i \in I$ and

$$\int_{S^*\cap T_0} g^* d\mu^* + \sum_{i\in I} \alpha_i g_i \mu(A_i) = \int_{S^*\cap T_0} \omega^* d\mu^* + \sum_{i\in I} \alpha_i \omega_i \mu(A_i).$$

Define an assignment $g : T \to I\!\!B_+$ by

$$g(t) := \begin{cases} g^*(t), & \text{if } t \in S^* \cap T_0; \\ g_i, & \text{if } t = A_i,\ i \in I; \\ f(t), & \text{otherwise,} \end{cases}$$

and a generalized coalition $\gamma : T \to [0,1]$ by

$$\gamma(t) := \begin{cases} 1, & \text{if } t \in S^* \cap T_0; \\ \alpha_i, & \text{if } t = A_i,\ i \in I; \\ 0, & \text{otherwise.} \end{cases}$$

Then, one has $g \succ_{S_\gamma} f$ and $\int_T \gamma g d\mu = \int_T \gamma \omega d\mu$, which is a contradiction to that fact that $f \in \mathscr{C}^A(\mathscr{E})$.

Case 2. $I = \emptyset$. Similar to *Case 1*, one can show that f is blocked by a generalized coalition γ via g, where the assignment $g : T \to I\!\!B_+$ is defined by

$$g(t) := \begin{cases} g^*(t), & \text{if } t \in S^* \cap T_0; \\ f(t), & \text{otherwise,} \end{cases}$$

and the generalized coalition $\gamma : T \to [0,1]$ is defined by

$$\gamma(t) := \begin{cases} 1, & \text{if } t \in S^* \cap T_0; \\ 0, & \text{otherwise.} \end{cases}$$

(ii) Let $f^* \in \mathscr{C}^*(\mathscr{E}^*)$ and assume that $f = \varphi(f^*) \notin \mathscr{C}^A(\mathscr{E})$. Thus, there are an element $\gamma \in \mathscr{G}$ and an assignment g in $\mathscr{E}$ such that $g \succ_{S_\gamma} f$ and

$$\int_T \gamma g \, d\mu = \int_T \gamma \omega \, d\mu.$$

Let $I = \{i : A_i \in S_\gamma\}$. The rest of the proof is decomposed into two cases:

Case 1. $I \neq \emptyset$. In this case, one has $g(A_i) \succ_i f(A_i)$ for all $i \in I$ and

$$\int_{S\cap T_0} \gamma g \, d\mu + \sum_{i\in I} \gamma(A_i)\mu(A_i)g(A_i) = \int_{S\cap T_0} \gamma\omega \, d\mu + \sum_{i\in I} \gamma(A_i)\mu(A_i)\omega_i.$$

Fix an element $i \in I$. Let $r \in (0,1)$ be such that $rg(A_i) \succ_i f(A_i)$. Since $(A.5.3)$ is satisfied and C is a cone, one has $(1-r)\gamma(A_i)\mu(A_i)\omega_i \in C$. Thus, there exists an $\varepsilon > 0$ such that

$$-(1-r)\gamma(A_i)\mu(A_i)\omega_i + B(0,\varepsilon) \subseteq -C.$$

Applying an argument similar to that in the proof of Proposition 3.1, one can find a sub-coalition G of $S \cap T_0$ such that

$$\left\| \int_G g \, d\mu - \int_{S\cap T_0} r\gamma g \, d\mu \right\| < \frac{\varepsilon}{2} \quad \text{and} \quad \left\| \int_G \omega \, d\mu - \int_{S\cap T_0} r\gamma\omega \, d\mu \right\| < \frac{\varepsilon}{2}.$$

Define

$$\begin{cases} G_j^* \subseteq A_j^* : r\gamma(A_j)\mu^*(A_j^*) = \mu^*(G_j^*), & \text{if } j \in I, j \neq i; \\ G_i^* \subseteq A_i^* : \gamma(A_i)\mu^*(A_i^*) = \mu^*(G_i^*), & \text{if } j = i, \end{cases}$$

and $H^* = \bigcup \{G_j^* : j \in I\} \cup G$. Define an assignment h^* such that $h^*(t) = rg(A_i)$, if $t \in G_i^*$; and $h^*(t) = g^*(t)$, otherwise, where $g^* = \psi(g)$. It follows from Lemma 4.3 that $h^* \succ_{H^*}^* f^*$. It is easy to verify that

$$(1-r)\gamma(A_i)\mu(A_i)\omega_i + \int_{H^*} h^* \, d\mu^* - \int_{H^*} \omega^* \, d\mu^* \in B(0,\varepsilon).$$

Thus, $\int_{H^*}(h^* - \omega^*) \, d\mu^* \in -C$, which is a contradiction to Lemma 3.6.

Case 2. $I = \emptyset$. In this case,

$$\int_{S\cap T_0} \gamma g \, d\mu = \int_{S\cap T_0} \gamma\omega \, d\mu$$

and $g \succ_{S\cap T_0} f$. Let $D \subseteq S \cap T_0$ and $r \in (0,1)$ be such that $rg \succ_D f$. Since γ is simple and C is a cone, one obtains $\int_D (1-r)\gamma\omega \, d\mu \in C$. Thus, there exists an $\varepsilon > 0$ such that

$$-\int_D (1-r)\gamma\omega \, d\mu + B(0,\varepsilon) \subseteq -C.$$

Let G be a sub-coalition of $(S \cap T_0) \setminus D$ such that

$$\left\| \int_G g \, d\mu - \int_{(S\cap T_0)\setminus D} r\gamma g \, d\mu \right\| < \frac{\varepsilon}{4} \quad \text{and} \quad \left\| \int_G \omega \, d\mu - \int_{(S\cap T_0)\setminus D} r\gamma\omega \, d\mu \right\| < \frac{\varepsilon}{4},$$

and $G^{'}$ be a sub-coalition of D such that

$$\left\| \int_{G^{'}} rg \, d\mu - \int_D r\gamma g \, d\mu \right\| < \frac{\varepsilon}{4} \quad \text{and} \quad \left\| \int_{G^{'}} \omega \, d\mu - \int_D \gamma\omega \, d\mu \right\| < \frac{\varepsilon}{4}.$$

Define $H^* = G \cup G^{'}$ and an allocation h^* in $\mathscr{E}^*$ by $h^*(t) = g(t)$, if $t \in G$; and $h^*(t) = rg(t)$, otherwise. Clearly, $h^* \succ_{H^*}^* f^*$. It can be checked that

$$\int_D (1-r)\gamma\omega \, d\mu + \int_{H^*} h^* \, d\mu^* - \int_{H^*} \omega \, d\mu^* \in B(0,\varepsilon).$$

This implies that $\int_{H^*}(h^* - \omega^*)\, d\mu^* \in -C$, which is a contradiction. □

Proof of Theorem 4.1 Let f be an allocation of $\mathscr{E}$ that does not belong to $\mathscr{C}^A(\mathscr{E})$ and $\alpha \in (0, \mu(T))$. By Proposition 4.4, one has $f^* = \psi(f) \notin \mathscr{C}^*(\mathscr{E}^*)$. Applying Theorem 3.8, one can find a coalition S^* and an assignment g^* in $\mathscr{E}^*$ such that $\mu^*(S^*) = \alpha$ and f^* is blocked by the coalition S^* via g^*. Put $I = \{i : \mu^*(A_i^* \cap S^*) > 0\}$. The rest of the proof is decomposed into two cases:

Case 1. $I \neq \emptyset$. By invoking the argument similar to *Case 1* in the proof of Proposition 4.4(i), one can show that f is blocked by a $\gamma \in \mathscr{G}$ via an assignment g, where the assignment $g : T \to \mathbb{B}_+$ is defined by

$$g(t) := \begin{cases} g^*(t), & \text{if } t \in S^* \cap T_0; \\ g_i, & \text{if } t = A_i,\ i \in I; \\ f(t), & \text{otherwise,} \end{cases}$$

and the generalized coalition $\gamma : T \to [0, 1]$ is defined by

$$\gamma(t) := \begin{cases} 1, & \text{if } t \in S^* \cap T_0; \\ \alpha_i, & \text{if } t = A_i,\ i \in I; \\ 0, & \text{otherwise,} \end{cases}$$

where

$$g_i = \frac{1}{\mu^*(S^* \cap A_i^*)} \int_{S^* \cap A_i^*} g^* d\mu^* \quad \text{and} \quad \alpha_i = \frac{\mu^*(S^* \cap A_i^*)}{\mu(A_i)}.$$

Note that

$$\int_T \gamma\, d\mu = \mu(S^* \cap T_0) + \sum_{i \in I} \int_{A_i} \alpha_i d\mu = \mu^*(S^*) = \alpha.$$

Case 2. $I = \emptyset$. Similar to *Case 1*, one can show that f is blocked by a $\gamma \in \mathscr{G}$ via g, where the function $g : T \to \mathbb{B}_+$ is defined by

$$g(t) := \begin{cases} g^*(t), & \text{if } t \in S^* \cap T_0; \\ f(t), & \text{otherwise,} \end{cases}$$

and the generalized coalition $\gamma : T \to [0, 1]$ is defined by

$$\gamma(t) := \begin{cases} 1, & \text{if } t \in S^* \cap T_0; \\ 0, & \text{otherwise.} \end{cases}$$

Note that $\int_T \gamma\, d\mu = \mu^*(S^* \cap T_0) = \mu^*(S^*) = \alpha$. □

Remark 4.5 It is clear from the proof of Theorem 4.1 that for an $\varepsilon > 0$, there exists a generalized coalition γ such that f is blocked by γ, $\tilde{\mu}(\gamma) = \varepsilon$ and $\gamma(t) = 1$ if $t \in S_\gamma \cap T_0$. Thus, as in the case of atomless economies, non-atomic agents in S_γ use their full initial endowments. However, the atomic agents in γ only use parts of their initial endowments and the share α_i for an atomic agent A_i depends on the size of γ. So, Theorem 4.1 can be treated as an extension of that in an atomless economy.

Given an $\varepsilon > 0$ and an atomless economy, the blocking coalition S in the result of [17] is chosen as a union of finitely many disjoint sub-coalitions, each of which has measure and diameter less than ε. A coalition whose measure and diameter less than ε intuitively means that the coalition consists of relatively "few" agents, and that the agents in the coalition resemble one another in chosen characteristics. Extensions of this result are given in Bhowmik and Cao [8], Evren and Hüsseinov [12], and Hervés-Beloso et al. [19] in atomless economies with commodity spaces are either ℓ_∞ or ordered Banach spaces having interior points in their positive cones. Next, an extension of Grodal's result to an economy with a mixed measure space of agents and a Banach lattice as the commodity space is presented. To this end, we say that two generalized coalition γ_1 and γ_2 are *disjoint* if $(\gamma_1 \wedge \gamma_2)(t) = \min\{\gamma_1(t), \gamma_2(t)\} = 0$ for all $t \in T$. This means that $S_{\gamma_1} \cap S_{\gamma_1} = \emptyset$.

Theorem 4.6 *Assume* (*A*.4)-(*A*.5)*. Let* T_0 *be endowed with a pseudometric which makes* T_0 *a separable topological space such that* $\mathscr{B}(T_0) \subseteq \Sigma$ *and* $f \notin \mathscr{C}^A(\mathscr{E})$*. For any* $\varepsilon, \delta > 0$*, there exist a generalized coalition* γ *with* $\tilde{\mu}(\gamma) \leq \varepsilon$ *and a finite collection* $\{\gamma_1, \cdots, \gamma_n\}$ *of pairwise disjoint generalized coalitions such that the diameter of* γ_j *smaller than* δ *and* $S_{\gamma_j} \subseteq T_0$ *for all* $j = 1, \cdots, n$*,* f *is blocked by* γ *and*

$$\gamma = \begin{cases} \sum_{j=1}^n \gamma_j + \sum_{i \in I} \alpha_i \chi_{A_i}, & \text{if } I \neq \emptyset; \\ \sum_{j=1}^n \gamma_j, & \text{if } I = \emptyset, \end{cases}$$

where $I = \{i : A_i \in S_\gamma\}$ *and* $\alpha_i \in (0, 1]$ *if* $i \in I$*.*

Proof By Proposition 4.4, one claims $f^* \notin \mathscr{C}^*(\mathscr{E}^*)$. Thus, as in Theorem 3.8, there are a coalition S^* and a simple measurable function y^* defined on S^* such that $\mu^*(S^*) \leq \varepsilon$ and $\int_{S^*} (y^* - \omega^*) d\mu^* \in -C$. Suppose that $\{t_j : j \geq 1\}$ is a dense subset of T_0. For all $j \geq 1$, let

$$B_j^* = (S^* \cap T_0) \cap B\left(t_j, \frac{\delta}{2}\right).$$

Take

$$R_1^* = B_1^* \text{ and } R_j^* = B_j^* \setminus \bigcup\{B_s^* : 1 \leq s < j\}$$

for all $j \geq 2$. Put

$$C_n^* = \bigcup\{A_i^* : 1 \leq i \leq n\} \cap (S^* \setminus T_0) \quad \text{and} \quad S_n^* = \bigcup\{R_j^* : 1 \leq j \leq n\} \cup C_n^*$$

for all $n \geq 1$. It follows that $\lim_{n \to \infty} \mu^*(S^* \setminus S_n^*) = 0$. Thus, for sufficiently large n, one has $\int_{S_n^*} (y^* - \omega^*) d\mu^* \in -C$. Fix such an $n \geq 1$. Applying Lemma 3.6, one can derive that f^* is blocked by S_n^* by some assignment g^*. Put $I = \{i : \mu^*(A_i^* \cap S_n^*) > 0\}$. Similar to Theorem 4.1, it can be easily verified that f is blocked by some $\gamma \in \mathscr{G}$ with $\tilde{\mu}(\gamma) \leq \varepsilon$, where γ is defined for $I \neq \emptyset$ as

$$\gamma(t) = \begin{cases} 1, & \text{if } t \in S_n^* \cap T_0; \\ \alpha_i, & \text{if } t = A_i,\ i \in I; \\ 0, & \text{otherwise,} \end{cases}$$

and for $I = \emptyset$ as

$$\gamma(t) = \begin{cases} 1, & \text{if } t \in S_n^* \cap T_0; \\ 0, & \text{otherwise,} \end{cases}$$

where

$$\alpha_i = \frac{\mu^*(A_i^* \cap S_n^*)}{\mu(A_i)}.$$

For all $1 \le j \le n$, define $\gamma_j : T \to [0,1]$ by $\gamma_j = \chi_{R_j^*}$. Then

$$\gamma = \sum_{j=1}^{n} \gamma_j + \sum_{i \in I} \alpha_i \chi_{A_i}.$$

Clearly, $\{\gamma_1, \cdots, \gamma_n\}$ is a finite collection of pairwise disjoint generalized coalitions and $S_{\gamma_i} \subseteq T_0$ for all $1 \le j \le n$. The diameter γ_i is (compare [14])

$$\Delta(\gamma_i) = \sup\left\{\min\{\beta, \xi\}\|a - b\| : \lambda_a^\beta, \lambda_b^\xi \text{ are fuzzy points of } \gamma_i\right\}$$

Since $\beta = 0$ if $a \notin S_{\gamma_i}$ and $\xi = 0$ if $b \notin S_{\gamma_i}$, one obtains

$$\Delta(\gamma_i) = \sup\left\{\min\{\beta, \gamma\}\|a - b\| : \lambda_a^\beta, \lambda_b^\gamma \text{ are fuzzy points of } \gamma_i \text{ and } a, b \in S_{\gamma_i}\right\}.$$

So, by taking $\beta = \gamma = 1$, one has

$$\Delta(\gamma_i) = \sup\left\{\|a - b\| : a, b \in S_{\gamma_i}\right\} = \Delta(S_{\gamma_i}) < \delta.$$

Thus, the proof has been completed. □

Remark 4.7 Given a $\delta > 0$, there is a mixed measure space whose atoms contain open balls with diameters less than δ. For example, let $T = [0,1] \cup (1, 1+2\delta]$, where $[0,1]$ is endowed with the Lebesgue measure and $(1, 1+2\delta]$ is an atom. Note that $[0,1]$ is a metric space with the usual metric and $1+\delta$ has an $\frac{\delta}{2}$-neighborhood contained in $(1, 1+2\delta]$. As in Grodal [17] , each sub-coalition γ_j of $\gamma|_{T_0}$ is chosen as the set of agents sharing their full initial endowments and the diameter of γ_j is exactly the same as that of S_{γ_j}. Consequently, agents in γ_j have δ-similar characteristics in the ordinary sense. Since Grodal's idea is to take only finitely many different δ-similar sub-coalitions of agents, we take neighborhoods containing either δ-similar non-atomic agents or a single atom, not a neighborhood of points contained in an atom. Since each large agent can be treated as δ-similar to itself for any $\delta > 0$, our approach for taking a neighborhood containing a single atom does not violate Grodal's requirements. As a particular case, if the economy is atomless then it is clear from our proof that each γ_j and γ are just ordinary coalitions. In addition, if one confines his attention to an economy with countably many agents, then Theorem 4.1 and Theorem 4.6 tell us that only finitely many large agents are enough to block an allocation not in the core if they share only parts of their endowments.

Remark 4.8 It can be checked that the lattice structure is only needed to prove Lemma 3.6. Careful examinations of the proofs of Theorem 4.1 and Theorem 4.6 provide the use of Lemma 3.6 in their proofs. Thus, if $\mathbb{B}_+$ is an ordered Banach space with $\operatorname{int}\mathbb{B}_+ \neq \emptyset$, results similar to Theorem 4.1 and Theorem 4.6 can be proved under the assumption $(A.4)$ and the assumption $\int_S \omega \, d\mu \in \operatorname{int}\mathbb{B}_+$ for all coalition S. The proof of these results are analogous and simpler than the corresponding one with Banach lattice as the commodity space.

5 Concluding remarks

We close with further remarks dealing with possible extensions and applications of our results.

Remark 5.1 Suppose that $I\!\!B_+$ is a separable Banach lattice. The Aubin core equivalence theorem has been proved in [24] assuming convex preference relations satisfying the cone condition introduced in [26], i.e. under our assumptions $(A.5.1)$ and $(A.5.2)$. Assume that $f \in \mathscr{C}^A(\mathscr{E})$. By Proposition 4.4, $f^* = \psi(f) \in \mathscr{C}^*(\mathscr{E}^*)$. Thus, f^* can be decentralized by a price $p \in I\!\!B^*_+$, refer to [26]. It is easy to verify that p supports f as a competitive allocation and the Aubin-core equivalence holds true without the convexity assumption on the atomless sector as it is natural to expect.

Remark 5.2 For any $\alpha \in (0, \mu(T))$, define $\mathscr{G}_\alpha = \{\gamma \in \mathscr{G} : \widetilde{\mu}(\gamma) = \alpha\}$. It follows from Theorem 4.1 that the Aubin core defined by coalitions of $\mathscr{G}_\alpha$ coincides with $\mathscr{C}^A(\mathscr{E})$ for any α. This equivalence implies in particular, due to Remark 5.1, the equivalence with competitive equilibria under the restriction on (generalized) coalitions formation.

Remark 5.3 Pesce [25] provides results on the measure of supports of generalized blocking coalitions in a mixed market with finitely many commodities and asymmetric information. She imposes some restriction on the measure of the support. Such restrictions can be avoided when all the atoms are of the same type and share the same convex preference order. Although our paper focuses on the weight of a generalized coalition, we claim that similar results could be provided in our framework, i.e. with an infinite dimensional commodity space under the cone condition.

Remark 5.4 The production sector has been incorporated into the mixed model in [24], proving that under cone conditions for preferences and the production sets, the Aubin core Walras equivalence holds true. The correspondence between the Aubin core of a finite economy (i.e. $T_0 = \emptyset$ and T_1 finite) and the core of an associated atomless economy with finitely many types has been proved in [15]. We believe that similar arguments can be proved here in the presence of production in order to show that if a coalition γ blocks an allocation (f, y) then the same allocation can be blocked by a generalized coalition of arbitrary weight on the market. The production sets should be convex.

Remark 5.5 The case of preferences that are not convex at all (i.e. non convex preferences for the atomic as well as for the atomless sector) can be incorporated in our results by means of a suitable notion of generalized coalition and Aubin core. This can be done by means of the strong Aubin core introduced in [20] and studied in [15]. In their models, a generalized coalition is defined as a bundle of coalitions $\gamma \equiv (\gamma_1, \ldots, \gamma_m)$ in which each trader $t \in T$ takes part with the shares $\gamma_j(t)$, $j = 1, \cdots, m$ of his endowment. Define the weight of a coalition as the measure $\widetilde{\mu}(\gamma) = \sum_j \int_T \gamma_j \, d\mu$. By invoking our approaches, one can prove that the strong Aubin core coincides with the core of the atomless associated economy, deriving the result on the measure of γ.

Remark 5.6 In the recent paper [21], it is proved that vector measures with values in a Banach space $I\!\!B$ satisfy the Lyapunov convexity property in its strong form, i.e. without closure, when the underlying measure space (T, Σ, μ) is saturated. In this situation and assuming that the commodity space has a non-empty positive interior, our result can be derived using finite dimensional arguments. On the other hand, the assumption of a saturated measure space implies non-atomicity. Hence, the set of atoms is empty and thus, the Aubin core coincides with the core of the market (refer to Proposition 3.1) and Vind's theorem holds true in its standard formulation. Consider a mixed market such that only the restriction of μ on T_0 is saturated. In this case, it can be verified that Theorem 4.1 holds true assuming only for atoms that $\mu(A_i)\omega_i \in int\mathbb{B}_+$ (a condition already used in [10]) and $\text{int}\mathbb{B}_+ \neq \emptyset$. The same remark holds true in the general case, but clearly conditions $(A.5.1)$ and $(A.5.2)$ become still necessary.

Remark 5.7 Basile and Graziano [4] introduces the concept of Aubin core in a finite economy with an ordered locally convex topological vector space as the commodity space, where it is studied in connection with non linear price equilibria introduced by [2]. In the case of commodity spaces without a vector lattice structure, the properness assumption is formulated in [4] and [2] in order to use the so called disaggregated approach, i.e. to apply the separation theorem in the product space $\mathbb{B}^m$, where m is the number of agents. This approach is not useful in this paper, as it does not provide us with the arbitrage cone capable to replace the positive cone. However, under the properness condition and the assumptions in [4] (see also [2]), we claim that one can proceed according to the following steps: Let f be an allocation not in the Aubin core and it is blocked by an assignment g. Assume that we can choose $u > 0$ in $\mathbb{B}$ such that the order interval $[-u, u]$ contains $f(t), \omega(t), g(t)$ for μ-almost all $t \in T$ (this is true in the case of an economy with finitely many types of traders). Then, the ordered vector subspace $\mathbb{B}_u = \bigcup_{\lambda>0} \lambda[-u, u]$ equipped with the order topology is Archimedean and has u as an order unit. Its order topology is normable: there is a norm on $\mathbb{B}_u$ that generates the order topology whose closed unit ball is precisely the order interval $[-u, u]$ and u is an interior point of its positive cone. Since f is not in the Aubin core of the economy restricted to $\mathbb{B}_u$ and the results of the previous section hold true for an ordered Banach space with non-empty norm interior, one can derive a result similar to Theorem 4.1 in this settings.

Remark 5.8 An extension of Lemma 3.5 to an asymmetric information economy where the feasibility is defined as free disposal can be found in the proof of Theorem 3.3 in [5]. In addition, the initial part of Lemma 3.6 depends on Lemma 3.5 and it is also true under asymmetric information, refer to Theorem 3.4 in [5]. Thus, with simple modifications of the arguments used in the proofs of our results, one can easily extend our main results to an asymmetric information economy without the free disposal feasibility condition. However, it is unclear to the authors whether a result similar to Lemma 3.6 can be obtained in the case of asymmetric information without free disposal. Since our main results rely heavily on Lemma 3.6, we pose results similar to our main theorems in an asymmetric information economy without the free disposal feasibility condition as open questions.

Remark 5.9 Recently, Bhowmik and Cao [7] extends the characterization theorem of Walrasian allocations in terms of robustly efficient allocations in [18] to a mixed economy with an ordered separable Banach space whose positive cone has an interior point. It can be checked that Vind's theorem and the equivalence theorem are the main tools to prove their characterization theorem. In addition, to avoid convex preferences for non-atomic agents, they established Theorem 3.7. Thus, the equivalence theorem in [26], our Vind's theorem for an atomless economy and convex preferences for non-atomic agents allow us to get a characterization theorem similar to that in [7] in a mixed economy with a separable Banach lattice as the commodity space.

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