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WORKING PAPER NO. 402

Indeterminacy, Misspecification and Forecastability: Good Luck in Bad Policy?

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April 2015

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Indeterminacy, Misspecification and Forecastability: Good Luck in Bad Policy?

Luca Fanelli[*] and Marco M. Sorge[**]

Abstract

A recent debate in the forecasting literature revolves around the inability of macroecono-metric models to improve on simple univariate predictors, since the onset of the so-called Great Moderation. This paper explores the consequences of equilibrium indeterminacy for quantitative forecasting through standard reduced form forecast models. Exploiting U.S. data on both the Great Moderation and the preceding era, we first present evidence that (i) higher (absolute) forecastability obtains in the former rather than the latter period for all models considered, and that (ii) the decline in volatility and persistence captured by a .nite-order VAR system across the two samples is not associated with inferior (absolute or relative) predictive accuracy. Then, using a small-scale New Keynesian monetary DSGE model as laboratory, we generate arti.cial datasets under either equilibrium regime and investigate numerically whether (relative) forecastability is improved in the presence of indeterminacy. It is argued that forecasting under indeterminacy with e.g. unrestricted VAR models entails misspecification issues that are generally more severe than those one typically faces under determinacy. Irrespective of the occurrence of non-fundamental (sunspot) noise, for certain values of the arbitrary parameters governing solution multiplicity, the pseudo out-of-sample VAR-based forecasts of in.ation and output growth can outperform simple univariate predictors. For other values of these parameters, by contrast, the opposite occurs. In general, it is not possible to establish a one-to-one relationship between indeterminacy and superior forecastability, even when sunspot shocks play no role in generating the data. Overall, our analysis points towards a 'good luck in bad policy' explanation of the (relative) higher forecastability of macroeconometric models prior to the Great Moderation period.

Keywords: DSGE, Forecasting, Indeterminacy, Misspecification, VAR system

JEL Classification: C53, C62, E17

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1 Introduction

A recent debate in the (inflation) forecasting literature revolves around the inability of elaborate macroeconometric models to improve on simple univariate predictors, since the onset of the so-called Great Moderation. Contributions in the field include, but are not limited to, Atkeson and Ohanian (2001), Orphanides and van Norden (2005), Faust and Wright (2009), Tulip (2009), Rossi and Sekhposyan (2010), Christoffel *et al.* (2010), Edge and Gurkaynak (2010), Trehan (2015). While shown to be robust across a large variety of models – e.g. activity-based Phillips curves (Stock and Watson, 2007) and factor-augmented autoregressions (D'Agostino *et al.*, 2007) –, this finding has been largely associated with the emergence of weakly persistent inflation dynamics, as mostly dominated by transitory rather than permanent components (e.g. Stock and Watson, 2007).

A few authors have investigated this phenomenon in the context of dynamic stochastic general equilibrium (DSGE) models. Benati and Surico (2008) exploit a small-scale monetary New Keynesian model to explore the extent to which the persistence and predictability of inflation vary with the parameters of the monetary rule, and conclude that a more aggressive policy stance towards inflation caused the decline in inflation predictability. In the same vein, Fujiwara and Hirose (2014) suggest that forecast difficulties in the Great Moderation period can be potentially associated with the occurrence of equilibrium determinacy. More specifically, Fujiwara and Hirose (2014) argue that the relatively low volatility of macroeconomic aggregates during the Great Moderation episode, ascribed to active monetary policy behavior (e.g., Clarida *et al.*, 2000; Lubik and Schorfheide, 2004; Boivin and Giannoni, 2006; Castelnuovo and Fanelli, 2015), insulated the economy from nonfundamental shocks and hence prevented excessive business cycle fluctuations. The resulting reduction of the persistence and volatility of inflation and output turned out to penalize the forecastability of macroeconometric models. Conversely, the superior forecastability documented on the Great Inflation period would be, according to this argument, a by-product of equilibrium indeterminacy induced by the 'passive' monetary policy conduct of the Fed, which led to higher persistence and volatility of macroeconomic variables.[1]

Indeterminacy in DSGE models is generically associated with a richer correlation structure –

[1]Kolasa *et al.* (2012) also examine the relative accuracy of DSGE model-based forecasts vis-à-vis professional ones using real-time data. The authors find the DSGE model to be relatively successful in forecasting the U.S. economy. for almost all the variables and horizons under investigation.

e.g., a stronger degree of endogenous persistence – of resulting equilibrium representations.[2] At its very core, Fujiwara and Hirose's (2014) exercise involves generating artificially data for output and inflation from a standard (calibrated) DSGE model, without restricting the parameter space to the determinacy region. That is, the DSGE model is assumed to be the actual data generating process, and then forecasts from univariate autoregressive models are derived using the time-series simulated for the endogenous variables under determinacy and indeterminacy. The authors' conclusions then hinge on the enhanced autocovariance patterns of simulated data under indeterminacy, which result in superior predictive power of the indeterminate version of the underlying DSGE model, provided the degree of uncertainty about sunspots shocks is not too large.[3]

Against this background, this paper aims at exploring the relationship between indeterminacy in monetary DSGE models and (relative) forecastability of conventional reduced form predictors, and understand whether such relationship helps to explain the deteriorating forecast performance observed on the Great Moderation relative to the preceding era. More specifically, we take an econometric approach and ask two distinct though presumably intimately related questions: first, is indeterminacy *per se* bound to favor data predictability *in absolute terms*? And second, can the declining *relative* forecastability across the two historical periods be unambiguously framed in the context of equilibrium indeterminacy?

Our findings point towards a qualified negative answer to both these questions. In fact, we argue that forecasting inflation and output growth, or any other variables of interest, is a non-trivial exercise when the data generating process belongs to the class of indeterminate equilibria associated with a monetary DSGE model. Appropriate forecasting under indeterminacy requires estimating the arbitrary auxiliary parameters that index solution multiplicity, as well as identifying the sunspot shocks that may characterize the dynamics of the system in addition to fundamental shocks. Making inference on both these features of indeterminate equilibria is a complicate task even when the investigator specifies the correct statistical model for the data, e.g. Lubik and Schorfheide (2003, 2004), Fanelli (2012) and Castelnuovo and Fanelli (2015). More generally, in the presence of multiple solutions, practitioners who forecast inflation and output growth by univariate autoregressive (AR) processes or vector autoregressive (VAR) systems, face misspecification issues that are comparatively more serious than those he/she would face under determinacy. As a consequence, the relative predictive accuracy of multivariate mod-

[2] This argument is not new to the macroeconometrics literature, see e.g. Broze and Szafarz (1991), Lubik and Schorfheide (2003, 2004).

[3] The authors warn against drawing general conclusions from their analysis, upon observing that highly volatile sunspot-driven (nonfundamental) dynamics – which can only arise under indeterminacy – may actually thicken the veil of uncertainty surrounding the forecasting exercise and thereby worsen the relative accuracy of forecasts.

els across the two regimes is likely to hinge on the actual degree of misspecification in all the models the forecaster includes in her battery.

As a first step of our analysis, we carry out a simple empirical exercise on U.S. quarterly data compiled on the time span preceding the Great Moderation, conventionally denoted 'Great Inflation' period (1954-1984), as well as on the Great Moderation one (1984-2008). We exploit a three-variate VAR for inflation, output growth and the policy interest rate to forecast the first two variables on both regimes, and then evaluate both *absolute* and *relative* forecast accuracy, using univariate ARs and random walk (RW) predictors as benchmarks. We establish a number of basic facts regarding the indeterminacy-forecastability nexus. First, for all models considered, the absolute accuracy of inflation forecasts – as measured by the root-mean-square errors (RMSEs) of the pseudo out-of-sample forecasts – slightly increases across the samples, notwithstanding the decline of volatility of macroeconomic variables. Roughly the same occurs for output growth. This result holds true even though the persistence of the two variables, as measured by the estimated largest root of the VAR companion matrix, declines substantially across the two regimes. In this sense, macroeconomic time series have become more easier to forecast in absolute terms, as already pointed out by Stock and Watson (2007). Second, the VAR-based forecast performance relative to the parsimonious benchmarks is not found to deteriorate along the move from the Great Inflation to the Great Moderation period, although a simple stationary AR(1) process outpredicts the multivariate model in both samples. While confirming the existence of a wedge between better data predictability and ability of multivariate models to improve on univariate predictors, our results point out that enhanced variables persistence – as possibly originating from, e.g., equilbrium indeterminacy (Lubik and Schorfheide, 2004) – need not be associated with lower forecast uncertainty in absolute terms, nor does it necessarily worsen the relative accuracy of more elaborate macroeconometric forecasting models.

We next attempt to interpret this evidence through the lens of a structural DSGE framework capturing the main stylized facts of the U.S. business cycle. Using the small-scale New Keynesian monetary model investigated in Benati and Surico (2008, 2009) as laboratory, we generate artificial datasets under determinacy and indeterminacy and exploit a reduced form VAR system to assess both *absolute* and *relative* inflation and output growth forecasts' accuracy in either equilibrium regime. Under determinacy, monetary policy is designed to react aggressively to inflation and prevents self-fulfilling sunspots, while passive policy stance leads to equilibrium nonuniqueness. Our analysis shows that in the presence of indeterminate equilibria, irrespective of the presence of sunspot noise, for certain values of the arbitrary parameters that govern solution multiplicity, the pseudo out-of-sample forecasts of inflation and output growth obtained with finite-order VARs exhibit smaller average RMSEs, compared to the forecasts computed with simple univariate AR(1) or RW predictors. For other values of these parameters, by contrast, the opposite occurs. Hence, a stark asymmetry emerges between our empirical and numerical findings in the presence of indeterminacy, in that stronger (absolute and relative) predictive ability of multivariate models proves to depend quite heavily on the selected equilibrium in the indeterminate set, which is assumed to produce the data. When the employed time-series models are inherently misspecified with respect to this data generating process, persistent dynamics induced by indeterminate equilibria do not necessarily lead to superior forecastability. In particular, our approach shows that 'more stable' environments (determinacy) generally enhance forecastability, while the situation is mixed in dynamically more involved situations. Maintaining that our chosen New Keynesian monetary DSGE model represents a reasonable *prima facie* approximation of the post-WWII U.S. business cycle, we read this asymmetry as pointing to a 'good luck in bad policy' explanation of the (*relative*) higher forecastability of macroeconometric models prior to the Great Moderation period.

As mentioned, our paper is conceptually related to Benati and Surico (2008) and Fujiwara and Hirose (2014), yet it differs from these works in several respects. First, Benati and Surico (2008) refer to a DSGE-based measure of inflation predictability, whereas we appeal to reduced form forecasting models and hence employ a standard measure of forecast uncertainty (i.e. the RMSEs). Remarkably, Benati and Surico (2014)'s conclusions are substantially based on the idea that more (DSGE-based) persistence leads to superior (DSGE-based) predictability. Our results are a natural complement to this argument, as they help clarify the subtle link between indeterminacy (determinacy) of DSGE equilibria and their dynamic (regime-specific) properties on the one hand, and the predictive ability of possibly misspecified forecast models, on the other hand. Second, while following Fujiwara and Hirose (2014) in not restricting attention to unique equilibrium models, our analysis is broad in scope as we are interested in exploring the role of indeterminacy in favoring multivariate forecast models vis-à-vis univariate predictors of macroeconomic time series. Third, from an operational perspective, we run a more comprehensive simulation experiment that fully exploits the time-series representation of equilibria under indeterminacy. In particular, our findings are not constrained by the choice of a particular solution – the orthogonal one (Lubik and Schorfheide, 2003) – from the indeterminate set.

Finally, it is worth remarking that in this paper we are not concerned with the forecast performance of monetary DSGE models vis-à-vis conventional forecasting tools such as univariate time-series models or naive forecasts. We refer to e.g. Giacomini (2015) for a critical review of how theory-based models like DSGE models perform in forecasting macroeconomic variables, see also Gürkaynak *et al.* (2013). In this respect, we are sympathetic with Wickens's (2014) observation that DSGE frameworks would only outperform purely backward-looking time-series forecasting models were the theory-implied cross-equation restrictions their structure places on data to prove empirically valid. Unfortunately, testing the correct specification and the determinacy/indeterminacy of DSGE models before forecasting the variables of interest is not what practitioners typically do.

The remainder of this paper is organized as follows. Section 2 provides the main idea of the paper by discussing connections between equilibrium (in)determinacy, misspecification and forecastability in the context of a simple linear rational expectations model. Section 3 presents some empirical facts on the forecast accuracy of time-series models on U.S. quarterly data over both the Great Inflation and Great Moderation periods. Section 4 introduces the reference small-scale New Keynesian structural model and Section 5 reports its state-space representation under either equilibrium regime. Section 6 summarizes the results of our simulation experiments and interpret these results in light of the empirical facts from Section 3. Section 7 offers a few concluding remarks. A technical Appendix focuses on the derivation of the model's equilibria.

2 Background

Does indeterminacy imply superior forecast performance of conventional time-series models, relative to the case of determinacy? To gain insight into this issue, we introduce a simple univariate linear rational expectations model, already used in Lubik and Schorfheide (2004), Lubik and Surico (2010) and Fujiwara and Hirose (2014) for illustrative purposes.

Let x_t be a (scalar) endogenous variable defined on a properly filtered probability space, whose dynamics are governed by the following forward-looking equation

$$x_t = \frac{1}{\theta} E_t x_{t+1} + \omega_t, \quad \omega_t \sim iidN(0, \sigma_\omega^2) \tag{1}$$

where $E_t x_{t+1} := E(x_{t+1} \mid \mathcal{F}_t)$, $\mathcal{F}_t$ represents the conditioning information set at time t, ω_t is a fundamental shock, and θ is a structural parameter. Initial conditions are kept fixed. As is known, any solution to (1) satisfies the recursive equation:

$$x_t = \theta x_{t-1} - \theta \omega_{t-1} + \eta_t \tag{2}$$

where $\eta_t := x_t - E_{t-1} x_t$ is the endogenous expectation error. When $\theta > 1$ (determinacy), the (locally) unique non-explosive solution is given by

$$x_t = \omega_t \tag{3}$$

implying that x_t follows white noise dynamics and θ is not identifiable. When $\theta < 1$ (indeterminacy),[4] by contrast, the endogenous forecast error is not constrained by stability requirements, hence any covariance-stationary martingale difference η_t will deliver an RE stationary equilibrium of the form (2). In this case, θ is identifiable.

The forecast error η_t can be expressed as a linear combination of the model's fundamental disturbance and a conditionally mean-zero sunspot shock, i.e. $\eta_t = \tilde{M}\omega_t + s_t$ (Lubik and Schorfheide, 2003), where $\tilde{M}$ is an arbitrary parameter unrelated to θ and σ_ω^2. For simplicity, we assume s_t obeys a martingale difference sequence (MDS) with respect to $\mathcal{F}_t$ ($E_t s_{t+1} = 0$), with variance σ_s^2. The full set of solutions under indeterminacy is described by the following ARMA(1,1)-type process

$$x_t = \theta x_{t-1} + \tilde{M}\omega_t - \theta\omega_{t-1} + s_t \tag{4}$$

Simple inspection of Eq.s (3) and (4) reveals that the content of indeterminacy for the forecaster is essentially twofold. First, dynamic properties of the model's equilibrium are richer under indeterminacy as opposed to the determinate (pure noise) case. For $\tilde{M} \neq 1$ and generic σ_s^2, Eq. (4) gives rise to a large variety of equilibria.[5] While inducing a richer lag structure and hence persistence in x_t, indeterminacy also implies serial correlation in the composite error term. As a result, the presence of pure beliefs shocks as well as of an arbitrary response to the fundamental shock have the potential to induce higher volatility in data generated under Eq. (4).[6] Overall, answering the question of how this feature impacts on data forecastability under indeterminacy is not a trivial task. Second, indeterminacy involves richly parameterized time-series representations of equilibria, which are to be estimated for forecasting purposes. A relevant question is how significant the potential for dynamically misspecified forecasting models is when the data are generated according to Eq. (4), even in sunspot-free environments.[7]

[4]Without loss of generality, we abstract from the random walk case, $\theta = 1$, bacause our focus is on stable (asymptotically stationary) solutions.

[5]For $\tilde{M} = 1$ and $\sigma_s^2 = 0$, and despite $\theta < 1$, Eq. (4) collapses to a Minimum State Variable (MSV) solution which is observationally equivalent to the determinate equilibrium in Eq. (3).

[6]In principle, volatility in x_t may be further amplified by endogenous expectations revisions which are arbitrarily related to fundamentals, whereas the converse might occur for a suitable choice of the reduced form parameter $\tilde{M}$. Moreover, different dynamic structures of the underlying model, e.g. those featuring lagged expectations, may allow for serially correlate sunspots to arise in equilibrium (e.g. Sorge, 2012).

[7]Notice that while determinacy involves a one-to-one mapping between the endogenous variable x_t and the structural shock ω_t, indeterminacy may generate non-invertibility, i.e. the reduced form (4) might not be inverted to a (possibly infinite-order) autoregressive representation with one-sided lag polynomial (invertibility in the past). More generally, both determinacy and indeterminacy may be associated with nonfundamentalness even when equilibrium reduced forms are only driven by structural (fundamental) shocks. While non-invertibility may hinder the possibility of fully recovering the shock ω_t from an estimated causal AR model of x_t, this issue is immaterial for forecasting purposes, as the MA representation in (4) is naturally chosen to be fundamental. This

Common intuition suggests that x_t in Eq. (3) is less forecastable (predictable) than x_t in Eq. (4) because the variable's dynamics are purely white noise under determinacy, while it exhibits some persistence under indeterminacy which can potentially be exploited for forecasting purposes. As we will illustrate shortly, forecast accuracy under indeterminacy – both in absolute and *relative* terms – is likely to depend on the type/degree of dynamic misspecification of the employed forecast model.

To see this, we consider the extreme case of a forecaster endowed with data $x_1, x_2, ..., x_T$, and wishing to predict the future path of x_t by using the simple AR(1) model:

$$x_t = \beta x_{t-1} + u_t \ , \ u_t \sim WN(0, \sigma_u^2) \ , \quad |\beta| < 1 \ , \ \ t = 1, ..., T. \tag{5}$$

which we assume not to be theory-based. That is, the forecaster is not aware of the regime-dependent nature of the underlying data generating process (DGP), and also not concerned with the potential misspecification of the employed forecast model. If Eq. (3) is the true DGP, this amounts to overspecify the response dynamics including an irrelevant predictor, whereas an omitted-variable bias arises when Eq. (4) generates the data, irrespective of whether $\sigma_s^2 = 0$ or $\sigma_s^2 \neq 0$. Hence, our naive forecaster is either failing to impose relevant restrictions on the lag structure of the underlying (forecasting) model, or rather forcing the moving average part of the model's solution into the error process. From the point of view of the estimation of the parameters β and σ_u^2, the misspecification implied by the AR(1) model is almost irrelevant under determinacy, but can be severe under indeterminacy. Which are the consequences on the forecast performance of the AR(1) model?

We answer this question exploiting the following numerical experiment. First, N =1000 artificial samples of length $T = 500$ are generated from the linear rational expectations model in Eq. (1) under determinacy and indeterminacy, respectively. In the first case, for each of the 1000 simulations we generate 600 synthetic observations from Eq. (3), setting the variance of the fundamental shock to σ_ω^2:=0.5; the first 100 observations are then discarded. In the second case, for each of the 1000 simulations we generate 600 synthetic observations from the ARMA(1,1)-type process in Eq. (4) by calibrating the structural parameters to θ:=0.95 and σ_ω^2:=0.5, and selecting the indeterminacy parameter $\tilde{M}$ from the set $\mathcal{M}$:={1, 1.01, 0.98, 0.85, 0.80, 0.015}; with no loss of generality with respect to our argument, the sunspot shock is set to zero (σ_s^2:=0). Also in this case, the first 100 observations are discarded. The initial values x_0 and ω_0 are fixed to zero. On each simulated dataset, we use the first $T - P$ observations to estimate the parameters β and σ_u^2 of the AR(1) model through OLS, and the last P observations to evaluate its forecast accuracy. Absolute forecastability is measured by the average (across simulations) RMSEs.

argument fully generalizes to VAR-based forecasting (e.g. Alessi *et al.*, 2011).

The forecast performance of the AR(1) model under indeterminacy *relative* to determinacy is assessed by the ratio of the average (across simulations) RMSEs obtained in the two cases. But as is known, absolute evaluation of forecast performance is not likely to be informative, and the issue of evaluating the forecast performance of a given model is best answered by using relative evaluation methods that use a benchmark. To quantify the extent of the misspecifciation of the AR(1) forecast model estimated with OLS, we consider a 'theory-based' optimal benchmark, represented by a forecaster who perfectly knows the data generating process, i.e. whether she is forecasting future paths for x_t under determinacy or indeterminacy, as well as the true values of the parameters, namely σ_ω^2 if the data are generated under determinacy, and θ, σ_ω^2 and $\tilde{M}$ if the data are generated under indeterminacy.[8] In our simple example, the indeterminate (stable) solution has purely backward dynamics, and the variance of (rational) forecast errors grows with the forecasting horizon at any given date t in which predictions are made. Hence, this benchmark provides a theoretical lower-bound on the forecasting performance of less-than-rational forecasting models, e.g. those that are not in the form of the model's indeterminate solution. The forecast performance of the AR(1) model *relative* to this 'theory-based' benchmark is assessed by the ratio of the average (across simulations) RMSEs obtained in the two regimes.

Results for T=500, P=1, P=8 and P=16 are reported in Table 1. The first column of Table 1 collects the absolute (average) RMSEs obtained with the AR(1) model under determinacy and in the six indeterminacy cases. The second column reports the ratio between the average (across simulations) RMSEs obtained under the indeterminacy scenarios on the average (across simulations) RMSEs obtained under determinacy. Finally, the last column reports the forecast performance of the AR(1) model *relative* to the 'theory-based' optimal benchmark.

Focusing on the second and third columns of Table 1, we observe that for any considered forecast evaluation window, indeterminacy does not necessarily imply superior forecastability. The average RMSEs obtained under indeterminacy may be lower or higher than the average RMSEs obtained under determinacy, depending on the values of $\tilde{M}$. As expected, the forecast performance under the MSV equilibrium is the same as under the determinate solution. But for $\tilde{M}$=1.01 and $\tilde{M}$=0.98, which are values for which the indeterminate equilibrium is relatively 'close' to the MSV solution, the performance of the AR(1) model may change (albeit slightly) across the two regimes. For values of $\tilde{M}$ that are substantially far from the value that leads to the MSV solution (e.g. $\tilde{M}$=0.015), the forecast performance can be markedly worse.

The forth column of Table 1 confirms our conjecture about the misspecification of the AR(1)

[8] This benchamark forecasting model coincides with the actual law of motion of the economy, and empirical forecasts of future endogenous variables will necessarily coincide with model-consistent ones. In fact, the h-step ahead 'theory-based' optimal forecasts will be $E_{T-P}x_{T-P+h} = 0$ under determinacy, and $E_{T-P}x_{T-P+h} = \theta(x_{T-P+h-1} - \omega_{T-P+h-1})$ for $h = 1$ and $E_{T-P}x_{T-P+h} = \theta E_{T-P}x_{T-P+h-1}$ for $h \geq 2$ under indeterminacy.

model with respect to the data generating process in either equilibrium regime. Under determinacy, the misspecification of the AR(1) model bears no consequences on the property of the OLS estimators of β and σ_u^2, and therefore does not affect its forecast performance *relative* to the benchmark. Indeed, the chosen univariate predictor has essentially the same average (across simulation) RMSEs as the 'theory-based' benchmark predictor. The same happens under the MSV indeterminate solution. The picture changes, by contrast, for the other type of indeterminate equilibria, for which we observe a non-negligible (though not substantial) forecast error associated with the misspecified AR(1) predictor.

The simple simulation experiment discussed in this section allows us to conclude that: (i) it is not necessarily true that forecastability, as measured by the RMSEs associated with a simple univariate model, improves if the data generating process belongs to the class of indeterminate equilibria generated by a linear expectational model like Eq. (1); (ii) the endogenous persistence implied by the indeterminate equilibria in Eq. (4) may, or may not, help forecasters predict the future path of x_t, depending on the forecast model at hand and the magnitude of the auxiliary parameters that govern model's indeterminacy; (iii) it is certainly true that a very high degree of uncertainty resulting from sunspot shocks can reduce forecastability of time-series models, but the forecast accuracy of these models under indeterminacy can be inferior relative to determinacy also in the absence of sunspot shocks.

It can be argued that the findings obtained in this section are specific to the highly stylized univariate model under investigation. From Section 4 onwards, we show that the argument can be generalized to more realistic model-based forecasting frameworks, and fruitfully used to interpret a few empirical facts, to which we turn next.

3 Empirical facts

To shed light on the role of indeterminacy for data forecastability, we implement a simple empirical exercise by which we explore the forecast performance of a set of conventionally employed time-series models on U.S. data. We consider quarterly data, sample 1954Q4-2008Q2, on three observable variables collected in the vector $y_t := (\Delta o_t, \pi_t, R_t)'$, where output o_t is the log of the real GDP, the inflation rate π_t is the quarterly growth rate of the GDP deflator, and the short-term nominal interest rate R_t is proxied as the effective Federal funds rate expressed in quarterly terms (averages of monthly values).[9]

[9] The source of the data is the Federal Reserve Bank of St. Louis' web site. Our choice of the sample's chosen length is due to data availability (in particular, of the effective Federal Funds rate), as well as our intention to avoid dealing with the 'zero-lower bound' phase began in December 2008, which triggered a series of non-standard policy moves by the Federal Reserve whose effects are hardly captured by our standard New Keynesian model.

Following most of the literature on the Great Moderation, we divide the post-WWII U.S. era in two periods, roughly corresponding to the Great Inflation and the Great Moderation samples. More specifically, our Great Inflation sample covers the period 1954Q4-1984Q2 ($T = 119$), while our Great Moderation sample covers the period 1985Q1-2008Q2 ($T = 94$). This choice is consistent with e.g. D'Agostino *et al.* (2007) and Castelnuovo and Fanelli (2015).

For either of the two regimes, we apply the following models:

(a) Reduced form VAR systems for the observable variables $y_t{:=}(\Delta o_t, \pi_t, R_t)'$;

(b) univariate AR(1) models for π_t and Δo_t;

(c) univariate RW models for π_t and Δo_t.

The VAR systems in (a) is our reference time-series model through which we investigate forecastability of inflation and output, while the models in (b) and (c) serve as benchmarks (e.g., Atkeson and Ohanian, 2001). Owing to their flexibility, reduced form VAR frameworks have naturally lent themselves for forecasting since their inception. At the same time, theory-based structural VAR methods have been widely used to identify the driving force(s) behind the Great Moderation, see e.g. Stock and Watson (2002), Primiceri (2005), Gambetti *et al.* (2008), Benati and Surico (2009).

Our VAR specification includes a constant and its lag order is selected combining the Schwarz's criterion with a LM-type vector test for uncorrelated residuals, considering 1 up to 3 lags. In particular, the selected lag order is the first lag chosen by the Schwarz's criterion such that the hypothesis of uncorrelated VAR disturbances is not rejected. The AR(1) predictor in (b) also includes a constant. On both regimes, we use the first $T - P$ observations to estimate the models in (a) and (b) through OLS, and the last P observations to compute pseudo-out-of-sample forecasts and compute the RMSEs for π_t and Δo_t. Thus, the *absolute* forecast performance of the VAR systems is measured by the RMSEs. Instead, the VAR forecast performance *relative* to the benchmarks in (b)-(c) is computed by the ratio of the corresponding RMSEs. As a measure of persistence, we employ the (absolute value of the) estimated largest root of the VAR companion matrix for the model in (a), and the estimated autoregressive coefficients for the univariate model in (b). Alternative measures of persistence in the multivariate framework have been recently proposed by Cogley and Sargent (2005) and Cogley *et al.* (2010). While Cogley and Sargent's (2005) measure, based on the normalized spectrum at frequency zero, frames naturally in the frequency-domain approach, Cogley's *et al.* (2010) R^2-like measure of persistence seems particularly suited for the case of VAR systems with drifting-parameters. Given our time-domain approach and the idea of using fixed-parameters VARs on the two subsamples, the largest root of the VAR companion matrix appears a tenable summary measure of the overall persistence of the variables in $y_t := (\Delta o_t, \pi_t, R_t)'$.[10]

Results for the evaluation window $P = 8$ (eight quarters) are summarized in Table 2. The picture that emerges from Table 2 is the following. First, we notice a substantial reduction of the variance of the model's residuals across the two regimes; focusing on the VAR residuals, the magnitude of the estimated variances of inflation and output growth on the Great Moderation sample is roughly half of the magnitude of the estimated variances of the two aggregates on the Great Inflation sample. A similar reduction of volatility is observed with the estimated AR(1) models. Second, our measure of persistence reduces substantially in the move from the Great Inflation (0.9632) to the Great Moderation (0.8545).

Is the observed decline in volatility and persistence associated with inferior VAR forecastability? The answer seems to be 'no'. Table 2 shows that in *absolute* terms, the VAR-based RMSEs associated with inflation and output growth are substantially smaller on the Great Moderation period compared to the Great Inflation period. This also holds for the AR(1)-based and (though less markedly) for RW-based absolute RMSEs. This evidence, with supports what already observed in Stock and Watson (2007) using completely different models and econometric techniques, suggests that from the point of view of VAR practitioners, inflation has become 'easier' to forecast in the sense that the risk of inflation forecasts, as measured by RMSEs, has fallen. The same is observed for output growth. Such result is somehow expected: business cycle fluctuations have become considerably more stable during the Great Moderation period relative to the preceding era, hence more predictable by VARs.

Things are slightly different when focusing on *relative* forecast performance. We observe that the VAR-based forecasts of inflation and output growth are outperformed by the AR(1) models on both regimes. As concerns inflation, the *relative* forecast performance is substantially similar in the two regimes: the ratio of the RMSEs obtained with the VAR on the RMSEs obtained with the AR(1) is 1.3384 on the Great Inflation sample, and 1.2546 on the Great Moderation sample. As concerns output growth, the *relative* forecast accuracy seems to be superior during the Great Moderation: the ratio of the RMSEs computed with the VAR on the RMSEs computed with the AR(1) is 1.4660 on the Great Inflation sample, and is 1.0059 on the Great Moderation sample. These evidences seem to reflect a long-standing tradition that puzzles macroeconomic forecasting: multivariate models may struggle to significantly improve on univariate ones. Moreover, the VAR-based forecasts of inflation and output growth are outpredicted by the RW models on the Great Inflation sample, but not on the Great Moderation

[10]See also Koop *et al.* (1996) and Fanelli and Paruolo (2010) for a comprehensive treatment of measures of shock persistence in multivariate models like VARs. Instead, a detailed analysis of the persistence of U.S. inflation may be found in e.g. Pivetta and Reis (2007), where alternative measures of persistence are discussed for univariate models. Fuhrer (2010) also explores the notion of inflation persistence in macroeconomic theory.

sample. This result is explained by the high persistence of time-series during the Great Inflation.

Overall, using a VAR for y_t:=$(\Delta o_t, \pi_t, R_t)'$ and the chosen lag order specification rule, we do not observe a deteriorating forecast performance on the Great Moderation period compared to the preceding era, neither in *absolute* nor in *relative* terms. Table 2 clearly suggests that the decline in volatility and persistence captured by the VAR system in the move from the Great Inflation sample to the Great Moderation sample is not associated with inferior *relative* forecast accuracy. This result is not necessarily at odds with what reported and documented in the literature on the deteriorating performance of macroeconomic forecast models on the Great Moderation. Note, indeed, that we are just focusing on a VAR model in which the information set is limited to inflation, output growth and the short term interest rate, ignoring activity-based Phillips Curve-type forecasts models, the role of monetary and real aggregates as predictors of inflation, among others.

In the next sections we use a New Keynesian monetary DSGE structural model as the data generating process, and investigate whether the empirical facts reported in Table 2 can be reconciled with absolute and relative VAR-based forecast accuracy associated with simulated data.

4 Structural model

We consider Benati and Surico's (2009) small-scale DSGE model, given by the three equations:

$$\tilde{o}_t = \gamma E_t \tilde{o}_{t+1} + (1-\gamma)\tilde{o}_{t-1} - \delta(R_t - E_t \pi_{t+1}) + \omega_{\tilde{y},t} \tag{6}$$

$$\pi_t = \frac{\beta}{1+\beta\alpha} E_t \pi_{t+1} + \frac{\alpha}{1+\beta\alpha}\pi_{t-1} + \kappa \tilde{o}_t + \omega_{\pi,t} \tag{7}$$

$$R_t = \rho R_{t-1} + (1-\rho)(\varphi_\pi \pi_t + \varphi_{\tilde{y}} \tilde{o}_t) + \omega_{R,t} \tag{8}$$

where

$$\omega_{x,t} = \rho_x \omega_{x,t-1} + \varepsilon_{x,t} \quad , \text{-}1<\rho_x<1 \ \ , \varepsilon_{x,t} \sim \mathrm{WN}(0, \sigma_x^2) \ \ , \ \ x = \tilde{y}, \pi, R \tag{9}$$

and expectations are conditional on the information set $\mathcal{F}_t$, i.e. $E_t\cdot := E(\cdot \mid \mathcal{F}_t)$. The variables $\tilde{o}_t$, π_t, and R_t stand for the output gap, inflation, and the nominal interest rate, respectively; γ is the weight of the forward-looking component in the intertemporal IS curve; α is price setters' extent of indexation to past inflation; δ is households' intertemporal elasticity of substitution; κ is the slope of the Phillips curve; ρ, φ_π, and $\varphi_{\tilde{o}}$ are the interest rate smoothing coefficient, the long-run coefficient on inflation, and that on the output gap in the monetary policy rule, respectively; finally, $\omega_{\tilde{o},t}$, $\omega_{\pi,t}$ and $\omega_{R,t}$ in eq. (9) are the mutually independent, autoregressive of order one disturbances and $\varepsilon_{\tilde{o},t}$, $\varepsilon_{\pi,t}$ and $\varepsilon_{R,t}$ are the structural (fundamental) shocks.

This or similar small-scale models have successfully been employed to conduct empirical analysis concerning the U.S. economy. Clarida *et al.* (2000), Lubik and Schorfheide (2004) and Castelnuovo and Fanelli (2015) have investigated the influence of systematic monetary policy over the U.S. macroeconomic dynamics; Boivin and Giannoni (2006), Benati and Surico (2009), and Lubik and Surico (2010) have replicated the U.S. Great Moderation, Benati (2008) and Benati and Surico (2009) have investigated the drivers of the U.S. inflation persistence; Castelnuovo and Surico (2010) have replicated the VAR dynamics conditional on a monetary policy shock in different sub-samples.

The output gap in Eq. (6) is defined by $\tilde{o}:=o_t - o_t^n$, where o_t is output and o_t^n is the natural rate of output. We complete the structural model specification by postulating that o_t^n is driven by a technology shock and follows the Random Walk process

$$o_t^n = o_{t-1}^n + \varepsilon_{o^n,t} \quad , \quad \varepsilon_{o^n,t} \sim \mathrm{WN}(0, \sigma^2_{o^n}). \tag{10}$$

Eq. (10) will be used to define the measurement equation associated with the state-space equilibrium representation of our DSGE model, see the Appendix.

5 Equilibria

We compact the structural system composed by Eq.s (6)-(9) in the general representation

$$\Gamma_0 X_t = \Gamma_f E_t X_{t+1} + \Gamma_b X_{t-1} + \omega_t \tag{11}$$

$$\omega_t = \Xi\omega_{t-1} + \varepsilon_t \ , \ \ \varepsilon_t \sim \mathrm{WN}(0, \Sigma_\varepsilon) \tag{12}$$

$$\Xi:=dg(\rho_{\tilde{o}}, \rho_\pi, \rho_R) \quad , \quad \Sigma_\varepsilon:=dg(\sigma^2_{\tilde{o}}, \sigma^2_\pi, \sigma^2_R)$$

where $X_t:=(\tilde{o}_t, \pi_t, R_t)'$, $\omega_t:=(\omega_{\tilde{o},t}, \omega_{\pi,t}, \omega_{R,t})'$, $\varepsilon_t:=(\varepsilon_{\tilde{o},t}, \varepsilon_{\pi,t}, \varepsilon_{R,t})'$ and

$$\Gamma_0:=\begin{pmatrix} 1 & 0 & \delta \\ -\kappa & 1 & 0 \\ -(1-\rho)\varphi_{\tilde{y}} & -(1-\rho)\varphi_\pi & 1 \end{pmatrix}, \Gamma_f:=\begin{pmatrix} \gamma & \delta & 0 \\ 0 & \frac{\beta}{1+\beta\alpha} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_b:=\begin{pmatrix} 1-\gamma & 0 & 0 \\ 0 & \frac{\alpha}{1+\beta\alpha} & 0 \\ 0 & 0 & \rho \end{pmatrix}.$$

Let $\theta:=(\gamma, \delta, \beta, \alpha, \kappa, \rho, \varphi_{\tilde{o}}, \varphi_\pi, \rho_{\tilde{o}}, \rho_\pi, \rho_R, \sigma^2_{\tilde{o}}, \sigma^2_\pi, \sigma^2_R)'$ be the vector of structural parameters. The elements of the matrices Γ_0, Γ_f, Γ_b and Ξ depend nonlinearly on θ and, without loss of generality, the matrix $\Gamma_0^\Xi:=(\Gamma_0 + \Xi\Gamma_f)$ is assumed to be non-singular. The space of all theoretically admissible values of θ is denoted by $\mathcal{P}$ and X_0 and X_{-1} are fixed initial conditions.

A solution to system (11)-(12) is any stochastic process $\{X_t^*\}_{t=0}^\infty$ such that, for $\theta_s \in \mathcal{P}$, $E_t X_{t+1}^* = E(X_{t+1}^* \mid \mathcal{F}_t)$ exists and it solves the system (11)-(12) at any t, for fixed initial conditions. A reduced form solution is a member of the solution set whose time-series representation is such that X_t can be expressed as a function of ε_t, lags of X_t and ε_t and, possibly, other arbitrary MDSs with respect to $\mathcal{F}_t$, possibly independent of ε_t, called 'sunspot shocks' (e.g. Broze and Szafarz, 1991).

As is known, determinacy/indeterminacy is a system property that depends on all elements in θ, see Lubik and Schorfheide (2004) and Fanelli (2012). The solution properties of the system of Euler equations (11)-(12) depend on whether θ lies in the determinacy or indeterminacy region of the parameter space. Thus, the theoretically admissible parameter space $\mathcal{P}_\theta$ is decomposed into two disjoint subspaces, the determinacy region, $\mathcal{P}_\theta^D$, and its complement $\mathcal{P}_\theta^I{:=}\mathcal{P}_\theta\backslash\mathcal{P}_\theta^D$. We assume that for each $\theta_s \in \mathcal{P}_\theta$, an asymptotically stationary (stable) reduced form solution to system (11)-(12) exists, hence the case of non-stationary possibly 'explosive' (unstable) indeterminacy is automatically ruled out. Since we consider only stationary solutions, $\mathcal{P}_\theta^I$ contains only values of θ for which multiple stable solutions arise.

A technical discussion of the equilibria associated with the DSGE system (11)-(12) is reported in the Appendix, where the interested reader is referred to. Next we summarize the state-space representation of the DSGE model in the two scenarios.

Under determinacy, the so-called ABCD form (Fernández-Villaverde *et al.* 2007; Ravenna, 2007) of the determinate equilibrium is given by the system

$$\begin{aligned} \underset{2n\times 1}{x_t^d} &= \underset{2n\times 2n}{A^d(\theta)}\ \underset{2n\times 1}{x_{t-1}^d} + \underset{2n\times(n+b)}{B(\theta)}\ \underset{(n+b)\times 1}{u_t^d} \\ \underset{p\times 1}{y_t} &= \underset{p\times 2n}{C(\theta)}\ \underset{2n\times 1}{x_{t-1}^d} + \underset{2n\times(n+b)}{D(\theta)}\ \underset{(n+b)\times 1}{u_t^d} \end{aligned} \tag{13}$$

where $x_t^d{:=}(X_t', X_{t-1}')'$ is the state vector, n is the dimension of the state vector X_t in Eq. (11) ($n = 3$ in our specific case), y_t is the vector of observable variables, which in our specific case is given by $y_t{:=}(\Delta o_t, \pi_t, R_t)'$, Δo_t being output growth, $A^d(\theta)$, $B(\theta)$, $C(\theta)$ and $D(\theta)$ are conformable matrices whose elements depend nonlinearly on θ, and $u_t^d{:=}(\varepsilon_t', v_t')'$ is the vector containing all system innovations, i.e. the fundamental shocks ε_t and the innovations associated with the measurement system, v_t, if any. The superscript 'd' stands for 'determinacy'.

Under indeterminacy, instead, the ABCD representation of the equilibria is given by

$$\begin{aligned} \underset{3n\times 1}{x_t^{in}} &= \underset{3n\times 3n}{A^{in}(\theta)}\ \underset{3n\times 1}{x_{t-1}^{in}} + \underset{3n\times(4n+b)}{B(\theta,\tilde{m})}\ \underset{(4n+b)\times 1}{u_t^{in}} \\ \underset{p\times 1}{y_t} &= \underset{p\times 3n}{C(\theta)}\ \underset{3n\times 1}{x_{t-1}^{in}} + \underset{3n\times(4n+b)}{D(\theta,\tilde{m})}\ \underset{(4n+b)\times 1}{u_t^{in}} \end{aligned} \tag{14}$$

where the superscript 'in' stands for 'indeterminacy'. Here, the state vector $x_t^{in}{:=}(X_t', X_{t-1}', X_{t-2}')'$ involves an additional lag compared to the case of determinacy, while $y_t{:=}(\Delta o_t, \pi_t, R_t)'$ is still our vector of observable variables. Notably, the 'B' and 'D' matrices in Eq. (14) depend not only on the structural parameters θ, but also on a vector of auxiliary parameters, unrelated to θ, that we collect in the vector $\tilde{m}$. u_t^{in}:=$(e_t', v_t')'$ is the vector containing all system innovations, i.e. the shocks e_t and the innovations associated with the measurement system, v_t, if any. Notably, the sub-vector e_t does not contain only the fundamental shocks ε_t, but also additional stochastic terms, collected in the vector ζ_t. More precisely, e_t:=$(\varepsilon_t', \zeta_t')'$, where ζ_t is a vector that has the same dimesion as ε_t and features a number $n_2 \leq$dim(ε_t) of possibly non-zero stochastic terms independent on ε_t; the remaining dim(ε_t)-n_2 elements of ζ_t are equal to zero, see the Appendix.

Thus, while the determinate equilibrium in system (13) depends only on the state variables and the structural parameters θ, the class of indeterminate equilibria summarized by system (14) features (i) a higher lag order, (ii) a set of auxiliary parameters in addition to the structural parameters ($\tilde{m}$), and (iii) additional shocks unrelated to the fundamental shocks (the non-zero elements of ζ_t). As shown in the Appendix, the 'parametric indeterminacy' that springs from $\tilde{m}$ characterizes the moving average part of the VARMA-type reduced form solution for X_t. Such parameters index solution multiplicity and may arbitrarily amplify or dampen the fluctuations of the variables in y_t other those implied by the fundamental shocks. The 'stochastic indeterminacy' stems from the non-zero sunspot shocks which enter the vector ζ_t. These shocks may arbitrarily alter the dynamics and volatility of the system, see Lubik and Schorfheide (2003, 2004) and Lubik and Surico (2010) for discussions. A special case of interest is obtained when $\tilde{m}$=$vec(I_{n_2})$ and no sunspot shocks drive the reduced form; in this case, despite θ lies in the indeterminacy region of the parameter space, the equilibrium collapses to a MSV solution which admits the same time-series representation as the determinate reduced form solution.

A note of caution about the relationship between persistence and determinacy/indeterminacy in DSGE models is in order. As already noticed in Section 3, persistence in the observables y_t can be measured in different ways. Suppose we measure persistence by the largest root of the companion matrix associated with a finite-order VAR representation for y_t, and assume for simplicity a 'purely forward-looking' DSGE model, which corresponds to $\Gamma_b = 0_{n\times n}$ and $\Xi = 0_{n\times n}$ in system (11)-(12). In this case, it is possible to show that system (13) is such that $x_t^d = X_t$ and y_t depends on the innovations u_t^d alone, while system (14) features VARMA(1,1)-type dynamics. Hence, in the 'purely forward-looking' case, determinacy implies that the observables y_t are driven by pure noise and have no persistence, while indeterminacy implies more persistence. In more general and comples situations, however, and as our simulation studies will show, it is not possible to claim that the persistence of y_t under indeterminacy will be generally higher than the persistence of y_t under determinacy.

The richness and variety of equilibria implied by our reference New-Keynesian model can partly be understood by the graphs in Figure 1. Figure 1 plots simulated paths of inflation of length T=150, obtained using the structural model in Eq.s (6)-(10) as data generating process under determinacy and indeterminacy. The red line in Figure 1 is generated under determinacy. The blue line is generated under indeterminacy setting the indeterminacy parameter to $\tilde{m}$=1 and not including sunspot shocks, which implies a MSV solution; the green line is generated under indeterminacy by setting the indeterminacy parameter to $\tilde{m}$=1.01 and not including sunpsot shocks; finally, the purple line is generated with $\tilde{m}$=1.01, setting the variance of the sunspot shock to σ_s^2=2. The appealing feature of Figure 1 is that even in the simple case in which the degree of indeterminacy of the system is only one, there exists a continuum of indeterminate equilibria and corresponding inflation paths indexed by the extra parameters $\tilde{m}$ and σ_s^2, which are all consistent with the specified structural model. The variety of indeterminate equilibria suggest that forecasting inflation with potentially misspecified time-series models are likely not to lead to unambiguous resulting about data forecastability.

The simulation experiments we discuss in the next section evaluate the forecast performance (and persistence) of VARs for y_t:=$(\Delta o_t, \pi_t, R_t)'$ when the data are generated under the null of the DSGE model's parameters belonging to the determinacy and indeterminacy regions, respectively.

6 Simulation experiment

In this section, we run a set of Monte Carlo experiments to assess the forecast performance of the time-series models in (a), (b) and (c) as detailed in Section 3, when the data are produced by system (6)-(10) under determinacy and indeterminacy, respectively. Specifically, we generate N=1000 synthetic datasets in each regime, assuming Gaussian fundamental shocks. To match the features of the empirical forecast exercise, we consider periods of unequal lengths in the two cases: T=94 observations under determinacy, and T=119 observations under the various indeterminacy scenarios we consider.

Data under determinacy are generated from system (13) for $\theta = \breve{\theta} \in \mathcal{P}_\theta^D$, where $\breve{\theta}$ is calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2009), column 'After the Volcker Stabilization'. Hence, we mimic the scenario documented by Benati and Surico (2009) for the Great Moderation period, namely a structural system characterized by an 'active' monetary policy able to prevent self-fulling inflation expectations. We label such a scenario the Great Moderation-type data generating process. The calibrated $\theta = \breve{\theta}$ used in the simulation experiment under determinacy is reported for ease of exposition in the right column of Table 3.

Data under indeterminacy are generated from system (14) for $\theta = \breve{\theta} \in \mathcal{P}_\theta^I$, where $\breve{\theta}$ is calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico

(2001), column 'Before October 1979'. In this case, we mimic the scenario documented by Benati and Surico (2009) for the Great Inflation period, namely a structural system characterized by a 'passive' monetary policy that does not react aggressively enough to inflation shocks, inducing multiple (stable) equilibria. In this framework, the degree of indeterminacy of the system is one (i.e. n_2=1), hence we also need to calibrate the indeterminacy (scalar) parameter $\tilde{m}$ and the variance of the (univariate) sunspot shock $\zeta_t = s_t$ (which is also assumed Gaussian). The calibrated $\theta = \breve{\theta}$ used in the simulation experiment under indeterminacy is again reported in the left column of Table 3, along with the indeterminacy parameters, summarized at the bottom.

The differences in the values assumed by the structural parameters θ in the two scenarios are highlighted in bold in Table 3. It can be noticed that the main divergence across the two data generating processes essentially lies in the conduct of monetary policy, namely in the long run response of the policy rate to output gap and inflation shocks.

On each simulated dataset, we apply the models (a), (b) and (c), where, as in the empirical forecast exercise of Section 3, the VAR for y_t:=$(\Delta o_t, \pi_t, R_t)'$ is the reference system and the univariate models in (b) and (c) serve as benchmarks. The VAR lag order is selected using Schwarz's criterion, considering 1 up to 3 lags. Observe that all three models are misspecified by construction with respect to the true data generating processes in Eq.s (13) and (14).

The first $T - P$ observations are used to estimate the models in (a) and (b) by OLS, and the last P observations to compute forecasts for π_t and Δo_t and associated RMSEs. In particular, the *absolute* VAR forecast performance is measured by the average (across simulations) RMSEs, while the *relative* VAR forecast performance with respect to the benchmarks in (b)-(c) is computed by the ratio of the corresponding average (across simulations) RMSEs. In line with the empirical exercise of Section 3, our measure of persistence is the modulus of the estimated largest root of the VAR companion matrix; we refer to the estimated autoregressive coefficient for the AR(1) models. Results are summarized in Table 4 for the evaluation windows P=8 (eight quarters).

Several interesting findings emerge from the picture. First, in *absolute* terms, forecast uncertainty of VAR-based forecasts – as measured by their RMSEs – is generally lower under determinacy compared to the case of indeterminacy, for both inflation and output growth.[11] This

[11] As expected, the (average) RMSEs associated with the VAR-based forecasts obtained under a MSV indeterminate equilibrium is very close to the (average) RMSE obtained under Great Moderation-type scenario. It is worth remarking that differently from the simple univariate case discussed in Section 2, the MSV solution in this case has the same time-series representation as the determinate solution but is characterized by different values for the structural parameters θ. Hence, the average RMSEs associated with the VAR model under the MSV data generating process, need not to coincide numerically with the average RMSEs associated with the VAR under the determinate data generating process.

finding is consistent with the entries of Table 2 reporting empirical results, and lines up with our a-priori conjecture: the forecast ability of possibly misspecified macroeconometric models is superior in relatively stable environments in which only fundamental shocks drive fluctuations, and deteriorates in environments in which additional (intrinsic and extrinsic) sources of business cycle fluctuations other than fundamental shocks, not accounted by the forecast model, are at work.

Second, persistence tends to increase as the model's reduced form solution is 'far' from the MSV solution. In general, however, for particular values of the indeterminacy parameters, the largest root of the approximating VAR can be smaller under indeterminacy relative to the case of determinacy. Hence, our simulation results seem to suggest the richer correlation structure and stronger degree of endogenous persistence featured by indeterminate equilibria need not be associated with superior forecast performance *relative* to the case of determinacy. In fact, a severe model misspecification of the forecast model relative to the true DGP – which typically involves quite complex cross-equation restrictions under indeterminacy – may adversely affect forecastability, which is a conditional property, even when the variables of interest feature higher persistence.[12] Apparently, the forecast performance of VARs under the Great Inflation-type scenario tends to deteriorate, for fixed indeterminacy parameter, as the uncertainty resulting from sunspot shocks (σ_s^2) increases.

Third, when considering *relative* forecast accuracy, we observe that the VAR-based forecasts are substantially similar to that of the AR(1)-based forecasts under determinacy, but may be inferior or superior to the AR(1)-based forecasts under indeterminacy, depending on the values taken by the indeterminacy parameters $\tilde{m}$ and σ_s^2. For some values of $\tilde{m}$ and σ_s^2, the VAR forecasts are inferior to that of AR(1) models. For other values, the converse occurs. A similar property characterizes the VAR-based forecasts relative to the RW's. In general, it is not possible to claim that the relative performance of the VAR model is superior or inferior to that of our univariate benchmarks under indeterminacy.

Assuming that the New Keynesian DSGE model as expressed in Eq.s (6)-(10) represents a good *prima facie* approximation of the post-WWII U.S. business cycle, we read these findings as pointing to a 'good luck in bad policy' explanation of the (relative) higher forecastability of macroeconometric models prior to the Great Moderation period.

[12] Along the same line, Canova and Gambetti (2010) emphasize that, conditional on lags of the endogenous variables, past expectations Granger-cause current values of the latter under ideterminacy but not under determinacy, and hence – irrespective of higher persistence – the omission of (proxies for) such expectations from the forecast model may result in larger prediction errors in the former regime.

7 Concluding remarks

This paper has analyzed the consequences of indeterminacy for quantitative forecasting through popular reduced form time-series models. Our simulation as well as empirical findings suggest that forecastability under indeterminacy may be severely undermined by model misspecification issues, even when extrinsic uncertainty (i.e. sunspot noise) plays no role in generating the data. Although indeterminacy need not imply superior forecastability - however measured - of (potentially misspecified) macroeconometric models, it may well be the case that a particular multivariate model estimated on data generated under indeterminate equilibria produces better forecasts than univariate predictors, regarless of the level of persistence and volatility that characterizes the observed time-series.

How should a practitioner address forecasting when she believes that a DSGE model describes the observed economy? Of course, forecasts should be computed from the DSGE model. But in these cases, understanding which regime observed samples belong to and the forecaster himself operates in, should be regarded as a necessary stage in the process of developing forecasting models suitable for that specific regime. As a minimal requirement, the auxiliary components that characterize business cycle fluctuations under indeterminacy should be properly identified and incorporated in the forecast model to improve on forecast accuracy.

With this in mind, we argue that forecasting ought to be confronted with misspecification and/or identifiability issues which typically arise when theoretical models are to be meaningfully taken to the data. As emphasized in Fanelli (2012) and Castelnuovo and Fanelli (2015), the potential for dynamic misspecification – i.e. the omission of lags, expectational leads and/or relevant variables with respect to the actual data generating process – and the strength of local identification of the underlying model's parameters (as well as sunspot-related ones) may well drive an incorrect assessment of determinacy versus indeterminacy, see Mavroeidis (2005, 2010). Lack of identifiability *across* model structures may also generate empirical difficulties in discriminating between determinate equilibrium frameworks driven by exogenous shocks featuring a richer dynamic structure (e.g. MA shocks) and indeterminate equilibrium ones subject to nonfundamental – or arbitrarily related to fundamentals – sunspots (e.g., Beyer and Farmer, 2007, 2008; Sorge, 2012). Overall, for DSGE model-based forecasting purposes we believe there remains the need to tackle a misspecification-robust approach to testing whether a determinate or an indeterminate equilibrium is supported by the data.

Appendix: DSGE equilibria and their representation

In this Appendix, we discuss the solutions associated with the DSGE model compacted in Eq.s (11)-(12). To keep exposition as general as possible, throughout this Appendix we denote with n the dimension of the state vector X_t in Eq. (11) (notice that $n = 3$ in our specific case). Moreover, we use the notations '$A(\theta)$' and '$A{:=}A(\theta)$' to indicate that the elements of the matrix A depend nonlinearly on the structural parameters θ, hence in our setup $\Gamma_0{:=}\Gamma_0(\theta)$, $\Gamma_f{:=}\Gamma_f(\theta)$, $\Gamma_b{:=}\Gamma_b(\theta)$, $\Xi{:=}\Xi(\theta)$ and $\Gamma_0^{\Xi}{:=}\Gamma_0^{\Xi}(\theta)$. We call 'stable' a matrix that has all eigenvalues inside the unit disk and 'unstable' a matrix that has at least one eigenvalue outside the unit disk. Thus, denoted with $\lambda_{\max}(\cdot)$ the absolute value of the largest eigenvalue of the matrix in the argument, we have $\lambda_{\max}(A(\theta)) < 1$ for stable matrices and $\lambda_{\max}(A(\theta)) > 1$ for unstable ones. We also consider the partition $\theta{:=}(\theta_s', \theta_\varepsilon')'$, where θ_ε contains the non-zero elements of $vech(\Sigma_\varepsilon)$ and θ_s all remaining elements. The 'true' value of θ, $\theta_0{:=}(\theta_{0,s}', \theta_{0,\varepsilon}')'$, is assumed to be an interior point of $\mathcal{P}$. The corresponding partition of the parameter space is given by $\mathcal{P}{:=}\mathcal{P}_{\theta_s} \times \mathcal{P}_{\theta_\varepsilon}$. This partition is important because the determinacy/indeterminacy of the system depends only on the values taken by θ_s.

A detailed derivation of the time-series representation of the reduced form solutions associated with the New-Keynesian DSGE system (11)-(12) is reported in Castelnuovo and Fanelli (2015). Using the Binder and Pesaran's (1995) solution method, they show that uniqueness/multiplicity of solutions is governed by the stability/instability of the the matrix $G(\theta_s){:=}(\Gamma_0^{\Xi} - \Gamma_f\Phi_1)^{-1}\Gamma_f$, where Φ_1 stems from the solution of a quadratic matrix equation.

Determinacy

For values of θ_s such that the matrix $G(\theta_s){:=}(\Gamma_0^{\Xi} - \Gamma_f\Phi_1)$ is stable, i.e. $\lambda_{\max}(G(\theta_s)){<}1$, then $\theta_s \in \mathcal{P}_{\theta_s}^{D}$ and the reduced form solution to system (11)-(12) can be represented in the form

$$(I_n - \Phi_1(\theta_s)L - \Phi_2(\theta_s)L^2)X_t = u_t \quad , \quad u_t{:=}\Upsilon(\theta_s)^{-1}\varepsilon_t \tag{15}$$

where L is the lag operator ($L^j X_t = X_{t-j}$), $\Phi_1(\theta_s)$, $\Phi_2(\theta_s)$ and $\Upsilon(\theta_s)$ are 3×3 matrices whose elements depend nonlinearly on θ_s and embody the cross-equation restrictions implied by the small New-Keynesian model. The matrices $\Phi_1(\theta_s)$ and $\Phi_2(\theta_s)$ in Eq. (15) are obtained as the unique solution to the second-order quadratic matrix equation

$$\mathring{\Phi}{=}(\mathring{\Gamma}_0 - \mathring{\Gamma}_f\mathring{\Phi})^{-1}\mathring{\Gamma}_b \tag{16}$$

where $\mathring{\Gamma}_f$, $\mathring{\Gamma}_0$, $\mathring{\Gamma}_b$ and the stable matrix $\mathring{\Phi}$ are respectively given by

$$\mathring{\Gamma}_0{:=}\begin{pmatrix} \Gamma_0^{\Xi} & 0_{n\times n} \\ 0_{n\times n} & I_n \end{pmatrix} \;,\; \mathring{\Gamma}_f{:=}\begin{pmatrix} \Gamma_f & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} \end{pmatrix} ,\; \mathring{\Gamma}_b{:=}\begin{pmatrix} \Gamma_{b,1}^{\Xi} & \Gamma_{b,2}^{\Xi} \\ I_n & 0_{n\times n} \end{pmatrix} ,\; \mathring{\Phi}{:=}\begin{pmatrix} \Phi_1 & \Phi_2 \\ I_3 & 0_{3\times 3} \end{pmatrix},$$

and $\Gamma_{b,1}^{\Xi}{:=}(\Gamma_b + \Xi\Gamma_0)$, $\Gamma_{b,2}^{\Xi}{:=}{-}\Xi\Gamma_b$ and $\Upsilon(\theta){:=}(\Gamma_0 - \Gamma_f\Phi_1(\theta))$. The matrix $\Phi_1{:=}\Phi_1(\theta_s)$ is the one that enters the definition of $G(\theta_s)$.

A convenient representation of the equilibrium in Eq. (15) is given by

$$\underset{x_t^d}{\begin{pmatrix} X_t \\ X_{t-1} \end{pmatrix}} = \underset{A^d(\theta_s)}{\begin{pmatrix} \check{\Phi}_1 & \check{\Phi}_2 \\ I_n & 0_{n\times n} \end{pmatrix}} \underset{x_{t-1}^d}{\begin{pmatrix} X_{t-1} \\ X_{t-2} \end{pmatrix}} + \underset{G^d(\theta_s)}{\begin{pmatrix} \check{\Upsilon}^{-1} \\ 0_{n\times n} \end{pmatrix}} \varepsilon_t \tag{17}$$

where $\check{\Phi}_1 = \Phi_1(\theta_s)$, $\check{\Phi}_2 = \Phi_2(\theta_s)$, $\check{\Upsilon} = \Upsilon(\theta_s)$, the matrices $A^d(\theta_s)$ and $G^d(\theta_s)$ are $2n \times 2n$ and $2n \times n$, respectively, and the superscript 'd' stands for 'determinacy'. Let $y_t{:=}(y_{1,t}, y_{2,t}, \cdots, y_{p,t})'$ be the $p \times 1$ vector of observable variables.

When all variables in X_t are observed, $y_t = X_t$, the state system (17) along with the measurement system: $y_t = Hx_t^d$, $H{:=}(I_n : 0_{n\times n})$, give rise to a VAR representation for y_t (X_t) with coefficients that depend on θ through the CER in Eq. (16). In general, however, not all variables in X_t belong to the forecaster's information set. In this case, the measurement system will take the form

$$y_t = Hx_t^d + Qv_t \tag{18}$$

where H is a $p \times 2n$ matrix, v_t a $b \times 1$ vector ($b \leq p$) of measurement errors with covariance matrix Σ_v, and Q is a $p \times b$ selection matrix. For the specific structural model we consider in the paper, the counterpart of the measurement system (18) is given by

$$\underset{y_t}{\begin{pmatrix} \Delta o_t \\ \pi_t \\ R_t \end{pmatrix}} = \underset{H}{\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}} \underset{x_t^d}{\begin{pmatrix} \tilde{o}_t \\ \pi_t \\ R_t \\ \tilde{o}_{t-1} \\ \pi_{t-1} \\ R_{t-1} \end{pmatrix}} + \underset{Q}{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}} \underset{v_t}{\varepsilon_{o^n,t}}$$

and is obtained by exploiting the RW assumption in Eq. (10). Let $u_t{:=}(\varepsilon_t', v_t')'$ be the $(n+b)$-dimensional vector containing all system innovations. By substituting Eq. (17) into Eq. (18) and using some algebra, one obtains the ABCD form (Fernández-Villaverde *et al.* 2007; Ravenna, 2007) of the determinate equilibrium:

$$\begin{aligned} \underset{2n\times 1}{x_t^d} &= \underset{2n\times 2n}{A^d(\theta_s)}\ \underset{2n\times 1}{x_{t-1}^d} + \underset{2n\times(n+b)}{B(\theta_s)}\ \underset{(n+b)\times 1}{u_t} \\ \underset{p\times 1}{y_t} &= \underset{p\times 2n}{C(\theta_s)}\ \underset{2n\times 1}{x_{t-1}^d} + \underset{2n\times(n+b)}{D(\theta_s)}\ \underset{(n+b)\times 1}{u_t} \end{aligned} \tag{19}$$

where $B(\theta_s){:=}(G^d(\theta_s) : 0_{2n\times b})$, $C(\theta_s){:=}HA^d(\theta_s)$ and $D(\theta_s){:=}(HG^d(\theta_s) : Q)$.

Provided a suitable set of minimality and identification conditions are met (Komunjer and Ng, 2011), system (19) can be used as the data generating process implied by our reference New-Keynesian DSGE model under determinacy. Replacing θ_s with θ it corresponds to system (13) in Section 5 of the paper.

Indeterminacy

For values of θ_s such that the matrix $G(\theta_s){:=}(\Gamma_0^{\Xi}{-}\Gamma_f\Phi_1)^{-1}\Gamma_f$ is unstable, i.e. $\lambda_{\max}(G(\theta_s)){>}1$, then $\theta_s \in \mathcal{P}_{\theta_s}^I$ and the class of reduced form solutions associated with the New-Keynesian system (11)-(12) becomes more involved from a dynamic standpoint. When $\lambda_{\max}(G(\theta_s)){>}1$, the matrix $G(\theta_s)$ can be decomposed in the form

$$G(\theta_s){=}P(\theta_s)\begin{pmatrix} \Lambda_1 & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & \Lambda_2 \end{pmatrix}P^{-1}(\theta_s)$$

where $P(\theta_s)$ is a $n \times n$ non-singular matrix, Λ_1 is the $n_1 \times n_1$ $(n_1 < n)$ Jordan normal block that collects the eigenvalues of $G(\theta_s)$ that lie inside the unit disk, and Λ_2 is the $n_2 \times n_2$ $(n_2 \leq n)$ Jordan normal block that collects the eigenvalues of $G(\theta_s)$ that lie outside the unit disk. Observe that $n_1 + n_2{=}n$, where n_2 determines the 'degree of multiplicity' of solutions.

The reduced form solutions can be given the VARMA-type representation:

$$(I_n - \Pi(\theta_s)L)(I_n - \Phi_1(\theta_s)L - \Phi_2(\theta_s)L^2)X_t = (\Psi(\theta_s,\tilde{m}) - \Pi(\theta_s)L)V(\theta_s,\tilde{m})^{-1}\varepsilon_t + \tau_t \quad (20)$$

$$\tau_t{:=}(\Psi(\theta_s,\tilde{m}) - \Pi(\theta_s)L)V(\theta_s,\tilde{m})^{-1}P(\theta_s)\zeta_t + P(\theta_s)\zeta_t \quad (21)$$

where the matrices $\Phi_1(\theta_s)$ and $\Phi_2(\theta_s)$ are defined as in the case of determinacy, see Eq. (16), while the matrices $\Pi(\theta_s)$, $\Psi(\theta_s,\tilde{m})$ and $V(\theta_s,\tilde{m})$ are respectively given by

$$\Pi(\theta_s){:=}P(\theta_s)\begin{pmatrix} 0_{n_1\times n_1} & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & \Lambda_2^{-1} \end{pmatrix}P^{-1}(\theta_s) \quad , \quad \Psi(\theta_s,\tilde{m}){:=}P(\theta_s)\begin{pmatrix} I_{n_1} & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & \tilde{M} \end{pmatrix}P^{-1}(\theta_s)$$

$$V(\theta_s,\tilde{m}){:=}(\Gamma_0(\theta_s) - \Gamma_f(\theta_s)\Phi_1(\theta_s)) - \Xi(\theta_s)\Gamma_f(\theta_s)(I_n - \Psi(\theta_s,\tilde{m})).$$

The $n_2 \times n_2$ sub-matrix $\tilde{M}$ of $\Psi(\theta_s,\tilde{m})$ contains a set of arbitrary auxiliary parameters that do not depend on θ_s; for ease of reference, we collect these parameters in the vector $\tilde{m}{:=}vec(\tilde{M})$. The 'additional' vector moving average term τ_t depends on an extra source of random fluctuations potentially independent on the fundamental disturbances ε_t, i.e. on the $n \times 1$ vector $\zeta_t{:=}(0'_{n_1\times 1}, s'_t)'$, where s_t is a $n_2 \times 1$ MDS which collects the 'sunspot shocks' featured by the system. We assume, without any loss of generality, that s_t has a time-invariant covariance matrix Σ_s. The sunspot shocks might be also absent form the reduced form solution, i.e. $\Sigma_s = 0_{n_2\times n_2}$

implying $\zeta_t=0_{n\times 1}$ a.s. (and $\tau_t=0_{n\times 1}$ a.s.). This situation will be denoted 'indeterminacy without sunspots'.

A convenient representation of the class of indeterminate equilibria described by Eq.s (20)-(21) is given by the system

$$\underset{x_t^{in}}{\begin{pmatrix} X_t \\ X_{t-1} \\ X_{t-2} \end{pmatrix}} = \underset{A^{in}(\theta_s)}{\begin{pmatrix} -(\check{\Phi}_1+\check{\Pi}) & (\check{\Phi}_1\check{\Pi}-\check{\Phi}_2) & \check{\Phi}_2\check{\Pi} \\ I_n & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & I_n & 0_{n\times n} \end{pmatrix}} \underset{x_{t-1}^{in}}{\begin{pmatrix} X_{t-1} \\ X_{t-2} \\ X_{t-3} \end{pmatrix}} + \underset{G^{in}(\theta_s,\tilde{m})}{\begin{pmatrix} \check{K}_1 & \check{K}_2 \\ 0_{n\times 2n} & 0_{n\times 2n} \\ 0_{n\times 2n} & 0_{n\times 2n} \end{pmatrix}} \underset{\varepsilon_t^0}{\begin{pmatrix} e_t \\ e_{t-1} \end{pmatrix}} \tag{22}$$

where $\check{\Phi}_1 = \Phi_1(\theta_s)$, $\check{\Phi}_2 = \Phi_2(\theta_s)$, $\check{\Pi} = \Pi(\theta_s), \check{\Psi} = \Psi(\theta_s,\tilde{m})$, $\check{V} = V(\theta_s,\tilde{m})$, $\check{K}_1:=[\check{\Psi}\check{V}^{-1} : (\check{\Psi}\check{V}^{-1}+I_n)\check{P}]$, $K_2:=[\check{\Pi}\check{V}^{-1} : \check{\Pi}\check{V}^{-1}\check{P}]$ are $n\times 2n$ matrices, $e_t:=(\varepsilon_t', \zeta_t')'$ is a $2n\times 1$ vector that collects the fundamental and sunspot shocks of the system, the matrices $A^{in}(\theta_s)$ and $G^{in}(\theta_s,\tilde{m})$ are $3n\times 3n$ and $3n\times 4n$, respectively, and the superscript 'in' stands for 'indeterminacy'.

Given the $p\times 1$ vector of observables y_t, the associated measurement system is given by

$$y_t = Hx_t^{in} + Qv_t \tag{23}$$

and, a part from their dimensions, the matrices H, Q and the vector v_t have the same meaning as in Eq. (18). For the specific structural model we consider in the paper, the counterpart of the measurement system (23) is given by

$$\underset{y_t}{\begin{pmatrix} \Delta o_t \\ \pi_t \\ R_t \end{pmatrix}} = \underset{H}{\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}} \underset{x_t^{in}}{\begin{pmatrix} \tilde{o}_t \\ \pi_t \\ R_t \\ \tilde{o}_{t-1} \\ \pi_{t-1} \\ R_{t-1} \\ \tilde{o}_{t-2} \\ \pi_{t-2} \\ R_{t-2} \end{pmatrix}} + \underset{Q}{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}} \underset{v_t}{\varepsilon_{o^n,t}}.$$

Upon defining the $(4n+b)$-dimensional vector $u_t:=(e_t, v_t')'$ which contains the complete set of innovations, the ABCD form of the indeterminate equilibria reads

$$\begin{aligned} \underset{3n\times 1}{x_t^{in}} &= \underset{3n\times 3n}{A^{in}(\theta_s)}\ \underset{3n\times 1}{x_{t-1}^{in}} + \underset{3n\times(4n+b)}{B(\theta_s,\tilde{m})}\ \underset{(4n+b)\times 1}{u_t} \\ \underset{p\times 1}{y_t} &= \underset{p\times 3n}{C(\theta_s)}\ \underset{3n\times 1}{x_{t-1}^{in}} + \underset{3n\times(4n+b)}{D(\theta_s,\tilde{m})}\ \underset{(4n+b)\times 1}{u_t} \end{aligned} \tag{24}$$

where $B(\theta_s,\tilde{m}):=(G^{in}(\theta_s,\tilde{m}) : 0_{2n\times b})$, $C(\theta_s):=HA^{in}(\theta_s)$ and $D(\theta_s,\tilde{m}):=(HG^{in}(\theta_s,\tilde{m}) : Q)$.

Also in this case, provided a suitable set of minimality and identification conditions are met, system (24) can be used as the data generating process implied by our reference New-Keynesian DSGE model under indeterminacy. Replacing θ_s with θ it corresponds to system (14) in Section 5 of the paper.

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TABLES

Table 1. RMSEs for x_t computed from data simulated from the linear rational expectations model in Eq. (1) under determinacy and indeterminacy.

Equilibrium regime	AR(1)	Relative to DET.	Relative to BECH.
T=500 Evaluation window: $P = 1$			
DETERMINACY	0.5561	-	1
INDETERMINACY. MSV	0.5561	1	1
INDETERMINACY, $\tilde{M} = 1.01$	0.5627	1.012	1.0114
INDETERMINACY, $\tilde{M} = 0.98$	0.5468	0.983	0.9829
INDETERMINACY, $\tilde{M} = 0.85$	0.6890	0.996	1.1046
INDETERMINACY, $\tilde{M} = 0.80$	0.5564	1.0005	1.249
INDETERMINACY, $\tilde{M} = 0.015$	0.5273	0.948	63.18
T=500 Evaluation window: $P = 8$			
DETERMINACY	0.6791	-	1
INDETERMINACY, MSV	0.6791	1	1
INDETERMINACY, $\tilde{M} = 1.01$	0.6866	1.0110	1.0009
INDETERMINACY, $\tilde{M} = 0.98$	0.6662	0.9810	1.0008
INDETERMINACY, $\tilde{M} = 0.85$	0.6545	0.9638	1.0932
INDETERMINACY, $\tilde{M} = 0.80$	0.6799	1.0011	1.1639
INDETERMINACY, $\tilde{M} = 0.015$	1.1526	1.6972	1.1360
T=500 Evaluation window: $P = 16$			
DETERMINACY	0.6919	-	1
INDETERMINACY, MSV	0.6919	1	1
INDETERMINACY, $\tilde{M} = 1.01$	0.6995	1.011	1.0008
INDETERMINACY, $\tilde{M} = 0.98$	0.6785	0.9810	1.0002
INDETERMINACY, $\tilde{M} = 0.85$	0.5393	0.9695	1.0604
INDETERMINACY, $\tilde{M} = 0.80$	0.6256	0.9592	1.1046
INDETERMINACY, $\tilde{M} = 0.015$	1.4076	2.0344	1.0554

NOTES: Results are based on N=1000 simulations. Data under determinacy are generated from Eq. (3) with σ_{ω}^2=0.5. Data under indeterminacy are generated from Eq. (4) with θ:=0.95, σ_{ω}^2:=0.5 and $\sigma_s = 0$, for different values of $\tilde{M}$ and fixed initial conditions. 'AR(1)': average (across simulations) absolute RMSEs obtained with the model in Eq. (5). The first $T - P$ observations are used to estimate the model and the remaining P observations to compute the RMSEs. 'Relative to DET.': ratio between the average RMSE obtained with the AR(1) model under indeterminacy and the average RMSEs obtained with the AR(1) model under determinacy. 'Relative to BENCH.': ratio between the average AR(1)-based RMSE and the average RMSEs obtained under a 'theory-based' benchmark,.

Table 2. Absolute and relative RMSE for inflation and output growth computed on the Great Inflation and Great Moderation samples with time-series models

Great Inflation: 1954Q4-1984Q2	$T = 119$		Evaluation window:	$P = 8$	
RW (c)	Absolute	Relative to RW (b), (a)/(c)	Relative to AR (c), (a)/(b)	Variance	Persistence
inflation	0.3289	-	1.0794		
output growth	0.6643	-	0.6350		
AR(1) (b)					
inflation	0.3047	0.9264	-	0.3743	0.8490
output growth	1.0462	1.5749	-	1.0935	0.2796
VAR (a) 3 lags					
inflation	0.4078	1.2340	1.3384	0.3552	0.9632
output growth	1.5337	2.3087	1.4660	0.9876	
Great Moderation: 1985Q1-2008Q2	$T = 94$		Evaluation window:	$P = 8$	
RW (c)					
inflation	0.3034	-	1.4999		
output growth	0.4951	-	1.0412		
AR(1) (b)					
inflation	0.2023	0.6667	-	0.1876	0.6145
output growth	0.4755	0.9604	-	0.4989	0.1905
VAR (a) 3 lags					
inflation	0.2538	0.8365	1.2546	0.1754	0.8545
output growth	0.4783	0.9661	1.0059	0.4775	

NOTES: Forecasts are computed using the first $T-P$ obsertations of the sample to estimate the model and the last remaining P observations to compute one-step ahead forecasts (fixed scheme) of inflation and output growth P times and the corresponding RMSEs. 'RW (c)' stands for univariate random walk; 'AR(1) (b)' stands for univariate autoregressive model of order one which includes a constant; 'VAR (a)' denotes a three-variate VAR system for $y_t := (\Delta o_t, \pi_t, R_t)'$ which includes a constant and whose lag order is selected using Schwarz's (SC) information criterion combined with a LM-type vector test for uncorrelated VAR disturbances, considering 1 up to 3 lags. Δo_t is multiplied by 100. 'Variance' reports the estimated variance of residuals for π_t and Δo_t by the AR(1) and VAR models'. 'Persistence' reports the estimated autoregressive coefficient for the AR(1) model and the absolute value of the estimated largest root of the VAR companion matrix.

Table 3 Values of the structural parameters of the New Keynesian model in Eq.s (6)-(9) used in the simulation experiments of Section 6.

		Great Inflation data generating process	Great Moderation data generating process
Parameter	Interpretation	$\breve{\theta}$	$\breve{\theta}$
γ	IS: forward looking term	0.744	0.744
δ	IS: inter. elast. of substitution	0.124	0.124
α	NKPC: indexation past inflation	0.059	0.059
κ	NKPC: slope	0.044	0.044
ρ	Rule, smoothing term	**0.595**	**0.834**
$\varphi_{\tilde{o}}$	Rule, reaction to output gap	**0.527**	**1.146**
φ_{π}	Rule, reaction to inflation	**0.821**	**1.749**
$\rho_{\tilde{o}}$	Output gap shock, persistence	0.796	0.796
ρ_{π}	Inflation shock, persistence	0.418	0.418
ρ_R	Policy rate shock, persistence	0.404	0.404
$\sigma^2_{\tilde{o}}$	IS: shock variance	0.055	0.055
σ^2_{π}	NKPC: shock variance	0.391	0.391
σ^2_R	Policy rule: shock variance	0.492	0.492
$\sigma^2_{o^n}$	Natural rate of output: shock variance	0.25	0.25
$\tilde{m}$,	Indeterminacy parameters	1; 1.01; 0.98	-
σ^2_s	Variance of sunspot shock	0; 2; 5	-

NOTES: $\breve{\theta}$ under determinacy is calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2009), column 'After the Volcker Stabilization'. $\breve{\theta}$ under indeterminacy is calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2009), column 'Before October 1979'. In bold the parameter values that change across the two regimes.

Table 4. Absolute and relative (average) RMSE for inflation and output growth computed from data simulated from the New-Keynesian DSGE model in Eq.s (6)-(10) under determinacy and indeterminacy on different sample lenghts.

	Evaluation window: $P = 8$			
	Absolute (a)	Relative to RW (a/c)	Relative to AR (a/b)	VAR persistence
		$T = 94$		
DETERMINACY				
inflation	1.1714	0.7655	1.0094	0.8496
output growth	1.3580	0.7778	1.0048	
		$T = 119$		
INDETERMINACY: MSV solution				
inflation	1.2793	0.7413	1.0029	0.8214
output growth	1.3373	0.7498	1.0001	
INDETERMINACY: $\tilde{m}$=1.01, σ_s^2=0				
inflation	2.6674	0.8118	0.9647	0.8454
output growth	5.5618	0.8018	0.9942	
INDETERMINACY: $\tilde{m}$=0.98, σ_s^2=0				
inflation	2.4867	0.8467	0.9835	0.8238
output growth	5.5395	0.8014	0.9954	
INDETERMINACY: $\tilde{m}$=1.01, σ_s^2=2				
inflation	5.5149	1.0265	1.0064	0.9710
output growth	6.2177	0.8284	0.9995	
INDETERMINACY: $\tilde{m}$=1.01, σ_s^2=5				
inflation	7.7233	1.0369	1.0170	0.9729
output growth	7.0334	0.8538	0.9998	

NOTES: Results are based on N=1000 simulations. Data under determinacy are generated by simulating system (13) for $\theta = \breve{\theta} \in \mathcal{P}_\theta^D$, where $\breve{\theta}$ is calibrated as in the right column of Table 2. Data under indeterminacy are generating by simulating system (14) for $\theta = \breve{\theta} \in \mathcal{P}_\theta^I$, where $\breve{\theta}$ and the indeterminacy parameters $\tilde{m}$ and σ_s^2 are calibrated as in the left column of Table 2. Forecasts are computed using $T - P$ observations to estimate the model and the last P observations to evaluate forecasts (fixed scheme) and the corresponding RMSEs. (a) denotes the three-variate VAR system for $y_t := (\Delta o_t, \pi_t, R_t)'$ whose lag order is selected using Schwarz's (SC) information criterion, considering 1 up to 4 lags; 'RW' stands for univariate random walk, i.e. model (c); 'AR(1)' stands for univariate autoregressive model of order one, i.e. model (b). 'VAR persistence reportes the absolute value of the largest estimated root of the VAR companion matrix'.

FIGURES

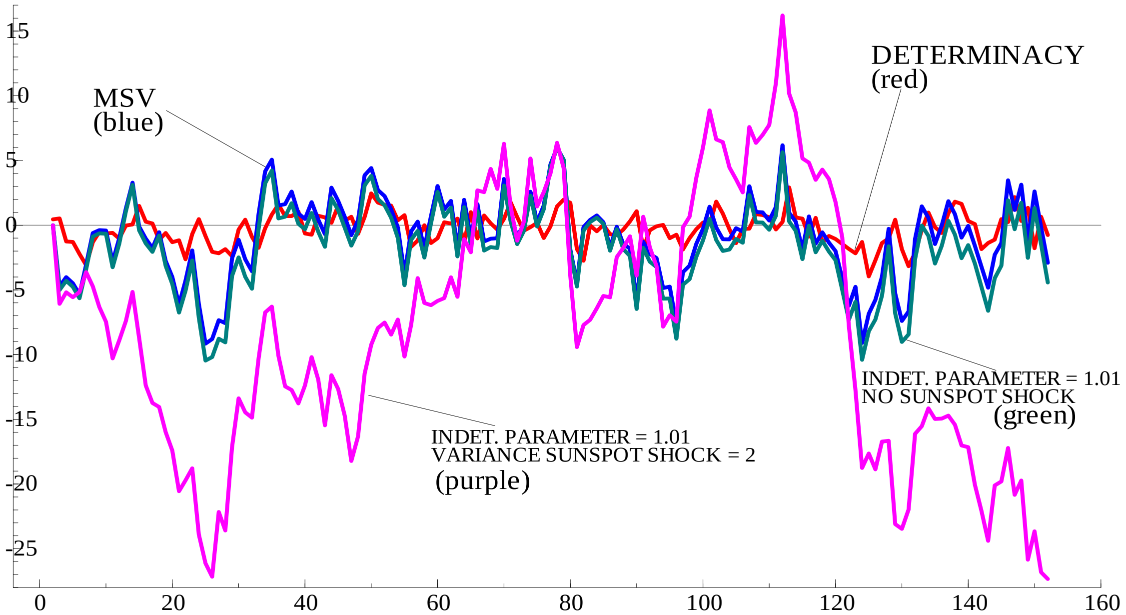

Figure 1. Simulated inflation paths ($T = 150$) obtained using the New-Keynesian DSGE model in Eq.s (6)-(10) as data generating process under determinacy (red line) and indeterminacy (blue, green and purple lines). The red line is obtained by simulating system (13) for $\theta = \breve{\theta} \in \mathcal{P}_{\theta}^{D}$, where $\breve{\theta}$ is calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2001), column 'After the Volcker Stabilization'. The other lines are obtained from system (14) for $\theta = \breve{\theta} \in \mathcal{P}_{\theta}^{D}$, where $\breve{\theta}$ is calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2009), column 'Before October 1979', setting the indeterminacy parameter $\tilde{m}$ and variance of the sunspot shock to different values (see main text).