WORKING PAPER NO. 438

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Emilio Calvano and Michele Polo

April 2016

This version November 2017

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS – UNIVERSITY OF NAPLES
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unina.it
ISSN: 2240-9696

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Emilio Calvano* and Michele Polo**

Abstract

Why do `Free' and `Pay' content cohabit in practically all media markets? We develop a model in which two identical broadcasters compete for viewers and advertisers that leads to endogenous differentiation. We show that differentiation does not require heterogeneous agents. Instead, we relate it to the `two-sided' nature of these markets. The asymmetric outcome is driven by the property that business models form strategic substitutes: if one station goes towards the 'pay' business model the rival has stronger incentives to choose the 'free' business model and viceversa. We propose a simple and natural property of the advertising technology that enhances strategic substitutability guaranteeing differentiation. In regime of competition there is a misallocation of advertising messages and therefore a waste of viewer attention. We show that a multi-station monopolist does not necessarily maintain differentiation and never offers content for free.

Acknowledgements: We thank Bruno Jullien, Martin Peitz, Augusto Preta, Markus Reisinger, Patrick Rey, John Sutton, Tommaso Valletti, Helen Weeds and seminar participants at the 2014 Earie Conference in Milan, the 2014 Workshop on Media Economics in Naples, the 2015 MACCI Conference on Competition Policy in Mannheim and the 2015 CRESSE Conference in Crete.

* University of Bologna and CSEF.

** Bocconi University, IEFE and IGIER

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1 Introduction

We often observe the coexistence within a media market of outlets that follow opposite business models. For instance Free-to-Air (FTA) broadcasters distribute content free of charge and depend entirely on advertising revenue, whereas Pay-TV broadcasters rely mostly on subscription fees. Likewise, among the news media some outlets collect revenues solely from ads while others put their content behind paywalls. Why do these opposite funding regimes, with either advertisers or consumers footing the bill, so frequently coexist? Can these striking differences in business models be traced back to competition among firms?

In this paper we argue that a "principle of differentiation" driven by strategic considerations helps to account for these asymmetric outcomes. We design a model in which, under certain conditions, two identical broadcasting stations with the same set of potential viewers and advertisers elect opposite pricing structures (business models), each raising most of its revenues from distinct sides of the market. We shut down preference-driven differentiation to separate our story from classic differentiation results. We do so assuming all agents are homogeneous thus sharing the same preferences. Instead we relate differentiation to the 'two-sided' nature of these markets.

We show that the key property leading to the asymmetric equilibrium is a strong form of strategic substitutability. Loosely speaking, if one station supplies more advertising and decreases or eliminates subscription fees (i.e., shifts towards the FTA model), it increases its competitor's incentive to raise fees and reduce advertising (i.e., to move towards the Pay-TV model), and vice-versa. To understand what drives this property notice that in media markets consumers and advertisers typically satisfy their needs for content and advertising on multiple outlets (what the literature calls "multi-homing"). This means that if one broadcaster moves towards an FTA model and the competitor mimics this move, the two stations, catering the exact same viewers, turn out to be substitute means of delivering advertising messages to the same audience. Such an overlap induces competition for advertising dollars in the form of lower ad prices. Moreover, the move towards FTA, and in particular the resulting increase in the amount of advertising aired, increases viewers' willingness to pay for ad-free content. Both effects make it more attractive for the competitor to shift the other way, towards the Pay-TV model. Strategic substitutability can also be illustrated the other way around. If a station moves towards Pay-TV pricing, setting a positive subscription fee and not supplying advertising space, the other station becomes advertisers' only medium for reaching viewers, making the FTA option more attractive.

We argue that this reasoning is sound if and only if the revenue potential of both the market for viewers and the market for ads is positive and balanced. If one side is 'too attractive' then the asymmetric equilibrium breaks down. Indeed we show that the extent of differentiation (i.e. the 'distance' between the equilibrium business models) is hump shaped in the revenue potential of one side relative to the other.

Strategic substitutability needs to be 'strong enough' for an asymmetric outcome to always exist. We provide a mathematically simple, rather weak and intuitive sufficient condition for existence that can be traced back to a property of the technological process that describes how advertising works. Such property, namely strict log-concavity, captures a fact well taken in the industry which is that concentrating the messages of an advertiser on a smaller number of outlets (one in our model) maximally increases the reach of his advertising campaign.

From a normative viewpoint we document an allocative inefficiency stemming from competition which occurs whenever both stations are active on the advertising side of the market. We show that under strict log-concavity competition may lead all advertisers to broadcast on all stations resulting in a lower overall industry surplus.

We use our model to discuss the exercise of market power by contrasting the duopoly outcome with the outcome implemented by a hypothetical monopoly owner of both stations. We show that under the same conditions that guarantee existence in duopoly, a monopolist does not have preferences over the business models of the individual stations. Furthermore there is no outcome in which a monopolist offers content free of charge. So the 'Free-to-Air' business model arises only in the competitive setting. In this sense the incentives to differentiate are stronger in competition.

We also provide a thorough discussion of a richer model accounting for preference heterogeneity and in particular viewers having a different taste for marginal quality. We argue that this reinforces the baseline logic leading to differentiation. Finally some policy lessons are drawn.

Relation to the literature. Our paper naturally relates to the literature on endogenous product differentiation in traditional markets as a means to relax price competition.[1] In these models ex-ante symmetric firms differentiate their products (for instance, offering high- and low-quality versions) to cater to different types of consumer. Likewise, in our

[1] The classical references are Hotelling (1929) and d'Aspremont et al. (1979) for endogenous differentiation by variety and Shaked and Sutton (1982) for endogenous differentiation by quality.

model one firm supplies high-quality (for instance ad-free) paid content to viewers and another firm supplies low-quality (ad-supported) free content to viewers. However, the classic results rely heavily on the heterogeneity of consumer tastes, which is necessary for screening purposes. What distinguishes this paper is the fact that heterogeneity within either market side is not essential to the asymmetric outcome. Indeed, we obtain differentiation assuming throughout the paper homogeneous viewers and homogeneous advertisers. What is essential is the presence of two separate types of agents which is one of the defining features of two-sided markets.

This paper contributes to a thriving literature on differentiation in media markets. Peitz and Valletti (2008) and Anderson, Foros and Kind (2016) have studied media outlets' choice of genre and content, extending the classic differentiation frameworks to two-sided outlets in the context, respectively, of single- and multi-homing consumers. We do not explicitly differentiate according to content; however, in our framework one can consider the quality of airtime to be better, the lower the amount of advertising breaks. A few recent works focus specifically on business models. Weeds (2013), among other things, provides an alternative case for the thesis that Pay-TVs and FTAs cohabit, in a framework akin to Shaked and Sutton (1982) with exclusive and heterogeneous consumers. The drivers of differentiation in our paper are different (and we speculate complementary) to hers. Kind, Nielssen and Sorgard (2009) link symmetric business models to the extent of content differentiation among firms. Content substitutability, they contend, makes it harder to extract rents though subscriptions and thus fosters FTA. Another related work is Dietl, Lange and Lin (2012), which takes the nature of the operators as given (one free, one pay) and draws implications on the quantity of ads. Unlike these papers, we posit generalized multi-homing agents and obtain asymmetric rather than symmetric outcomes.

The paper also contributes to the broader literature on the exercise of market power and the effect of competition in two-sided markets. So far, theories of "price skewness" in this literature have focused on the reasons why *all* the platforms in a given market may tilt their pricing structure to one side or the other. By now there is a well-established understanding of symmetric business model equilibria characterized by asymmetric price structures, with all platforms cross-subsidizing the same side at the expense of the other. Which side is favored, then, depends on the relative elasticity and the strength of indirect network externalities, the established result in two-sided markets (Rochet and Tirole (2006), Armstrong (2006), Bolt and Tienman (2008) and Schmalensee (2011), Spiegel (2013)). This offers a good explanation for one fundamental feature of two-sided markets, namely the unbalanced price structures, but it neglects another key feature, the coexistence of opposing price structures. To our knowledge, only Ambrus and Argenziano (2010) study in a general setting the case of asymmetric network equilibria with single-homing consumers. They show that asymmetric networks arise endogenously in equilibrium: each one relatively cheaper and larger on one side. Their argument depends on heterogeneity among consumers in how much they value the network good and is thus different from but complementary to ours, which relies on multi-homing. Finally, our paper is related to a recent strand of theoretical and empirical work that revisits some classic results in media economics (for instance Anderson and Coate (2005), Crampes, Haritchabalet and Jullien (2009)), allowing consumers to satisfy their content needs on multiple platforms: Anderson, Foros and Kind (2016), Ambrus, Calvano and Reisinger (2016) and Athey, Calvano and Gans (2016). We share with these works the idea that multi-homing viewers are less valuable, as they can be served by advertisers via different operators, so the associated rents are competed away.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 illustrates the equilibrium and comparative statics in duopoly. Section 4 provides the monopoly benchmark. Section 5 informally discusses the case with viewers having heterogeneous preferences for content quality. Section 6 provides policy lessons. All proofs are in Appendix.

2 The model

Two broadcasting stations, indexed by $i = 1, 2$, offer their content and advertising services to a continuum of identical viewers and identical advertisers (mass one). We allow agents to patronize more than one station if they wish so. We follow the literature and refer to this behavior as 'multi-homing' (as opposed to 'single-homing' referring to that of patronizing only one). To simplify the notation we do not index viewers nor advertisers.

Viewer preferences and choices. Viewers choose which stations to patronize, if any. Conditional on their subscription choice, they must allocate to the stations a finite endowment of time which for simplicity we assume equal to 2 slots.[2] Let $v_i \geq 0$ be the amount of time spent watching station i (referred to as 'viewing time'). Each slot is assumed of length 1 so $v_i \in \{0, 1, 2\}$. Let a_i denote the quantity of advertising on station i which is assumed nonnegative and weakly lower than some arbitrary real $\overline{a}$. The function

[2]Finiteness can also be justified on the grounds that content (movies, episodes of a TV-series, entertainment) require a minimum allotting of time to be consumed.

$U(v_1, v_2, a_1, a_2)$ denotes the gross utility from watching. We decompose the utility as the sum of the gross utility of single-homing on station i and j given by some function $u(\cdot)$ and a function $g(\cdot)$ capturing the fact that content substitutability reduces the overall utility for those who multi-home:

$$U(v_1, v_2, a_1, a_2) := \sum_{i=1,2} u(v_i, a_i) - g(v_1, v_2). \tag{2.1}$$

Assume that u, g (and hence U) are twice continuously differentiable in all arguments. We assume that $u(v_i, a_i) \geq 0$ with $u(0, a_i) = 0$. That is a 'free' subscription is always welcome. $g(\cdot) \geq 0$ and is symmetric in (v_1, v_2). Furthermore:

(Love for Content with Diminishing Returns) $\frac{\partial u}{\partial v_i} > 0$ and $\frac{\partial^2 u}{\partial v_i^2} < 0$.

(Love for variety) $u(2, a_i) \leq U(1, 1, a_i, a_j)$.

(Advertising Aversion) $\frac{\partial u}{\partial a_i} = -v_i$.

(Content Substitutability) $g(v_1, v_2) = 0$ if and only if $v_1 v_2 = 0$.

These assumptions capture two key features of this market. First, content is preferred to advertising: willingness to pay decreases in a_i and it does so in proportion to the viewing time. So a_i can be though as a vertical (quality) dimension of differentiation. Second viewers have taste for variety: spreading attention on different outlets always increases utility. This naturally implies a tendency to multi-home. We also allow differentiation along a horizontal (variety) dimension captured by our content substitutability assumption. Of course this plays a role only if both stations are actually consumed. The utility of not subscribing to any station is normalized to zero. All viewers get a payoff equal to the utility minus all fees paid. In appendix A we provide an illustration of these assumptions with a quadratic utility function in the spirit of Levitan and Shubik (1980).

Advertising technology and preferences. Advertisers wish to inform viewers. Informing one viewer is worth k. Their payoff is equal to k times the number of viewers informed.

How do viewers get informed? Throughout the paper we need to distinguish between the quantity of advertisement on station i allocated to a *given* individual advertiser denoted m_i, and the aggregate quantity of advertising a_i. Clearly in equilibrium we require the allocation to individual advertisers to integrate to a_i across all advertisers. $\phi : [0, \overline{a}] \to [0, 1]$ denotes a function that maps advertising exposure $e_i := m_i v_i$ to the probability that a given viewer with viewing time v_i is informed *through station* i. If v_i is identical across all viewers then $\phi(e_i)$ reads also as the fraction of the population informed through station i. It is assumed to be twice continuously differentiable, strictly increasing and strictly concave, with $\phi(0) = 0$ and $\phi(\overline{a}) < 1$. As a multi-homing viewer can be informed through either of the two stations, the probability of being informed *at least once on some station* is denoted Φ and assumed equal to one minus the probability that the viewer is not informed on either station. That is:

$$\Phi(e_i, e_j) := 1 - (1 - \phi(e_i))(1 - \phi(e_j)). \tag{2.2}$$

This formulation captures the fact that advertising on i and j are substitute means to inform a multi-homing viewer. On the contrary if the viewer single-homes, say on station i, then $e_j = 0$ and $\Phi = \phi(e_i)$. In this case i becomes a competitive bottleneck to reach this viewer.

Stations. The stations' profit is equal to the sum of subscription and advertising revenues. Stations choose the quantity of ads $a_i \in [0, \overline{a}]$, the subscription fee $f_i \geq 0$ and (a continuum of) advertising contracts (one for each advertiser) which specify a quantity $m_i \geq 0$ in exchange for a payment t_i.

Observe that we do not allow subscription fees to be contingent on viewing choices $\{v_i, v_j\}$, that is on the viewing time actually spent on each station as we deem it unrealistic. For instance, this rules out equilibrium outcomes in which viewers are basically *paid* to watch commercials ($f_i < 0$ and a_i large). Indeed, in this case one would expect them to just grab the subsidy and choose $v_i = 0$.

Timing and Equilibrium. At stage 1 the stations simultaneously choose the quantity of advertising.[3] At stage 2 the stations simultaneously choose the subscription fees and post the advertising contracts. At stage 3, viewers and advertisers observe the stations' offers and choose which station(s) to patronize (if any) and viewers allocate their attention. The equilibrium concept is Subgame Perfect Nash Equilibria.[4]

[3]Our timing implicitly assume that the aggregate quantity of ads is fixed when advertisers and viewers make their choices. The assumption captures the idea that content production and the program schedules (and therefore the quantity of commercial breaks) are set in advance. In the United States, for instance, broadcasters and advertisers meet on a seasonal basis at an "upfront" event to sell commercials on the networks' upcoming programs whose length is predetermined. Unsold airtime, if any, is filled with tune-ins.

[4]We break indifferences as follows: If a viewer / advertiser is indifferent between subscribing / accepting

3 Competition leads to differentiation

Viewers' choices and equilibrium subscription fees. Let $\mathbb{I}_i(v_i > 0)$ be an indicator function equal to 1 if its argument holds true and zero otherwise. The viewer problem is:

$$\max_{v_i, v_j} U(v_1, v_2, a_1, a_2) - \sum_{i=1,2} f_i \mathbb{I}_i(v_i > 0). \tag{3.1}$$

$$\text{subject to } v_i \in \{0, 1, 2\} \text{ for } i = 1, 2 \text{ and } v_1 + v_2 \leq 2$$

The above formulation encompasses two problems. First, given strategies (f_i, a_i) and (f_j, a_j), viewers need to choose which (if any) stations to subscribe to. Second, given subscription choices, they choose how to allocate time. Love for content and love for variety immediately imply that the optimal time allocation is equal to $(v_i = 2, v_j = 0)$, $(v_i = 0, v_j = 2)$ and $(v_i = 1, v_j = 1)$ conditional on single-homing on i, single-homing on j and multi-homing respectively. Given this, consider subscription choices. A key object in what follows is the incremental utility that the viewer would get if she were to subscribe to station i given that she has already subscribed to $j \neq i$:

$$\Delta U_i(a_i, a_j) = U(1, 1, a_i, a_j) - u(2, a_j) \geq 0 \tag{3.2}$$

Observe that if i chooses a fee smaller or equal than the incremental utility then all viewers necessarily subscribe to i and this holds true for all f_j. We now claim that each station charging a subscription fee equal to its incremental value (inducing all viewers to multi-home) is the unique equilibrium subscription fee. This is what Anderson, Foros and Kind (2016) in a closely related setting refer to as the 'incremental pricing principle'. Intuitively, station i cannot raise f_i unilaterally above its incremental utility without losing the viewers in the subgame that follows. Similarly no station can unilaterally lower its fee below the incremental value without leaving money on the table. In summary, letting $f_i(a_i, a_j)$ denote the unique Subgame Perfect subscription fee then competition to get the viewer on board leads to (a formal proof is provided in appendix B.2):

$$f_i(a_i, a_j) := \Delta U_i(a_i, a_j) \quad \text{for } i = 1, 2. \tag{3.3}$$

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a contract or not, she/he opts to subscribe / accept. If a station is indifferent between a fee /contract which induces no viewer / advertiser participation and a fee which induces some viewer / advertiser participation, it always chooses the latter.

Note that when station j reduces its advertising quantity, it lowers the incremental utility ΔU_i, thereby forcing i to reduce its subscription fee.

Equilibrium advertising contracts. Recall that stations can in principle discriminate advertisers by tailoring the contracts at the single advertiser level. So we split the analysis in two parts. First, we derive the equilibrium fee t_i given an arbitrary feasible allocation m_i. Second we show that in equilibrium each station $i = 1, 2$ necessarily propose the same m_i to all its advertisers.

Let $\Delta\Phi_i$, referred to as the *incremental* probability of station i, denote the increase in the probability of informing a multi-homing viewer (who allocates her time spending one slot on both stations) if the advertiser were to purchase m_i ads on i *in addition* to m_j ads on j. Hence, $\Delta\Phi_i$ is equal to the expected probability that a multi-homing viewer is informed through i but not through j:

$$\Delta\Phi_i(m_i, m_j) := \Phi(m_i, m_j) - \Phi(0, m_j) \tag{3.4}$$

$$= \phi(m_i)(1 - \phi(m_j)) \geq 0. \tag{3.5}$$

By an argument analogous to the one presented above, in equilibrium t_i and t_j must be equal to k times the respective incremental probabilities:

$$t_i(m_i, m_j) = k \cdot \Delta\Phi_i(m_i, m_j) \quad \text{for } i = 1, 2. \tag{3.6}$$

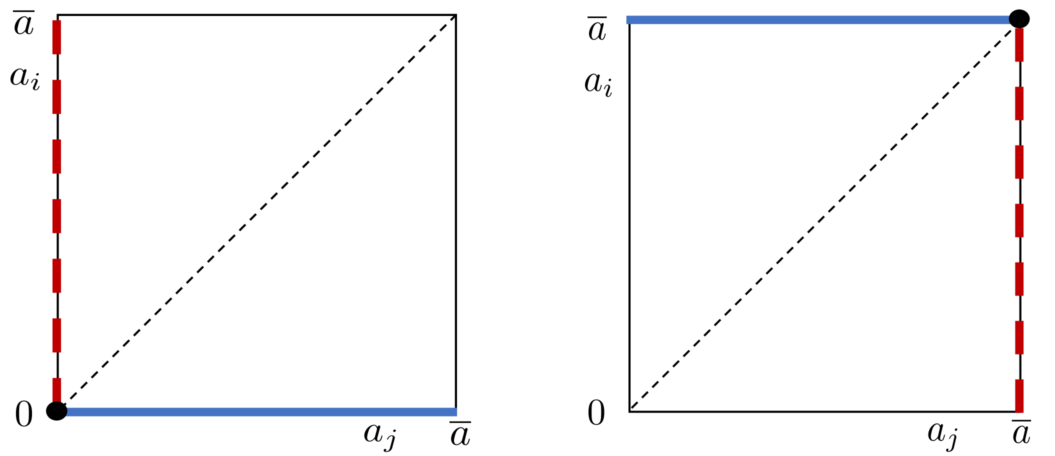
How do stations allocate the aggregate quantity a_i across advertisers? Diminishing returns (ϕ strictly concave) imply that the incremental probabilities are strictly concave in m_i and therefore that in equilibrium stations 'spread' their advertising quantity across all advertisers. As there is a unit mass of advertisers then this means that in equilibrium $m_i = \frac{a_i}{1} = a_i$ for all advertisers. The following claim takes stock.

Claim 1 *With competing stations, given any pair of first-stage choices $\{a_i, a_j\}$, each station i offers the same contract to all advertisers $\{t_i, m_i\}$ and all advertisers accept all contracts. The contracts have the feature $m_i = a_i$ and $t_i = k \cdot \Delta\Phi_i(m_i, m_j)$ for $i = 1, 2$.*

Equilibrium quantity. Given a_j,the problem for station i is:

$$\max_{a_i \in [0, \overline{a}]} \pi_i := \pi(a_i, a_j, k) = \Delta U_i(a_i, a_j) + k \cdot \Delta\Phi_i(a_i, a_j). \tag{3.7}$$

k parametrizes the relative profitability of the two sides of the market. When choosing the quantity of advertising, the stations trade-off revenues from subscription for revenues



(a) $k \leq \overline{k}$

(b) $k \geq \overline{k}$

Figure 1: i's best responses (solid line - blue) and j's (dotted line - red)

from advertising. Indeed if the profit maximizing quantity lies in the interior of the choice set, it is characterized by the familiar first order condition equating marginal revenues on opposite sides:

$$\frac{\partial \pi_i}{\partial a_i} = \frac{\partial \Delta U_i}{\partial a_i} + k \frac{\partial \Delta \Phi_i}{\partial a_i} = 0. \tag{3.8}$$

A Subgame Perfect Nash Equilibrium is basically a vector of fees and quantities satisfying (3.3), (3.6) and solving problem (3.7) for $i = 1, 2$. As asymmetric equilibria always come in pairs, without loss of generality we restrict attention to equilibria with $a_1 \geq a_2$. So an asymmetric equilibrium is an equilibrium in which 1 supplies more ads $a_1^\star > a_2^\star \geq 0$ and hence has lower subscription prices $0 \leq f_1^\star < f_2^\star$ and larger advertising fees $t_1^\star > t_2^\star \geq 0$.

Clearly if $k = 0$, that is informing viewers is worthless, then the game has a trivial unique symmetric equilibrium in which both stations forego advertising altogether: $a_1^\star = a_2^\star = 0$ and set the same subscription fee. Figure 1 (a) depicts this situation showing the shape of the best responses and their intersection in the origin. On the contrary, if informing viewers is arbitrarily profitable (that is k is sufficiently large) then the game has another straightforward symmetric equilibrium which is unique in which both stations advertise as much as feasible $a_1^\star = a_2^\star = \overline{a}$ and set the same subscription fee (Figure 1 (b)). It follows that an asymmetric equilibrium can obtain in loose terms only for 'intermediate' values of k. The next claim formalizes this requirement providing a *necessary* condition on k for an asymmetric outcome to exist (proof in appendix B.3).

Claim 2 *An asymmetric equilibrium exists only if:*

$$\underline{k} := \left(\frac{\partial\phi(0)}{\partial e_i}\right)^{-1} < k < \left(\frac{\partial\phi\left(\overline{a}\right)}{\partial e_i}\left(1-\phi(\overline{a})\right)\right)^{-1} := \overline{k} \tag{3.9}$$

From now on assume that $k \in (\underline{k}, \overline{k})$. A fundamental property of the game which is key in driving the asymmetric outcome is that quantities form strategic substitutes, i.e. the optimal quantity of firm i decreases with the quantity of firm j. In other words, the best responses are negatively sloped. Loosely speaking if one station goes towards the 'pay' business model (that is decreases quantity a_i and increases subscription fee f_i) then the other station has a stronger incentive to move in the opposite direction and go towards the 'free' business model (that is increasing advertising and decreasing subscription fee) and vice-versa. Intuitively, if a competitor supplies a larger quantity of ads, the marginal returns of advertising for a station go down as the probability that the viewer is not informed by the competitor and that can be potentially informed on the station shrinks. To see this via an extreme example note that if a_j is such that $\phi(a_j)$ is close to 1 then $\Delta\Phi_i$ is close to zero and so i has basically no incentive to trade-off subscription revenues for advertising ones and the optimal a_i is close to 0.

The fact that the best responses are negatively sloped (as opposed to positively sloped) makes it *possible* that they cross at least once *away* from the diagonal. Strategic substitutability by itself, however, does not imply that an asymmetric equilibrium exists.[5] Necessary Condition (3.9) merely says that reaction functions are not flat guaranteeing that the best response to 0 is strictly larger than that to $\overline{a}$. Intuitively strategic substitutability needs also to be 'strong enough' to guarantee existence, in a sense we make precise below.

The next proposition proposes a sufficient condition, strict log-concavity of $1-\phi$ that we argue being intuitive and natural in our setting. As we will show, it can be traced back to a readily interpretable property of the 'communication technology' embedded in the function $\phi(e)$ which we expect to naturally hold in media markets. Symmetric equilibria are not ruled out. However as will be clear from the discussion that follows, log-concavity has the additional benefit of making these symmetric outcomes unstable guaranteeing at

[5]We refer the reader to Amir, Garcia and Knauff (2010) for an excellent discussion of asymmetric outcomes in static games with global strategic substitutes. They also provide sufficient conditions for asymmetric outcomes (diagonal non concavities) that are different than the ones offered in this paper.

the same time existence and uniqueness of a stable asymmetric one.

Proposition 1 *Suppose that* $1 - \phi(e)$ *is strictly log-concave in exposure and* k *falls in the interval characterized by* (3.9)*. Then an asymmetric equilibrium exists and is unique.*

Proof in appendix B.4. Why is a log-concave technology sufficient for existence? $1-\phi$ strictly log concave implies that the function Φ has the following property (formal proof in appendix B.4):

$$\Phi(e_1 + e_2, 0) > \Phi(e_1, e_2) \quad \text{for all } e_1, e_2 > 0. \tag{3.10}$$

It says that multi-homers are easier to inform 'concentrating' advertising on one station rather than spreading it around. It captures a point that is well taken in the industry which is that multi-station advertising campaigns are wasteful as individual stations cannot predict which ads viewers have seen on other outlets. This leads to lower reach, that is some individuals not being exposed on either station.[6] Log-concavity encapsulates the idea that strategic substitutability is 'strong enough' implying the (local) property that the best response function is very steep (slope lower than -1) when it crosses the diagonal. This immediately guarantees existence of an asymmetric outcome. To build geometric intuition figure 2 shows the qualitative shape of the best response of firm i (solid blue) and j (dotted red) in two notable cases discussed below. Observe in both diagrams that in a left neighborhood of the crossing point, i's best response (solid line) must lie above the inverse of j (dotted line) due to the slope being larger than -1. But then the two lines must eventually cross again giving rise to an asymmetric equilibrium. The same property implies that such asymmetric equilibrium is stable while the symmetric equilibrium (which always exists) is not stable for a wide range of best response dynamics. In this sense the symmetric outcome is less compelling.[7,8]

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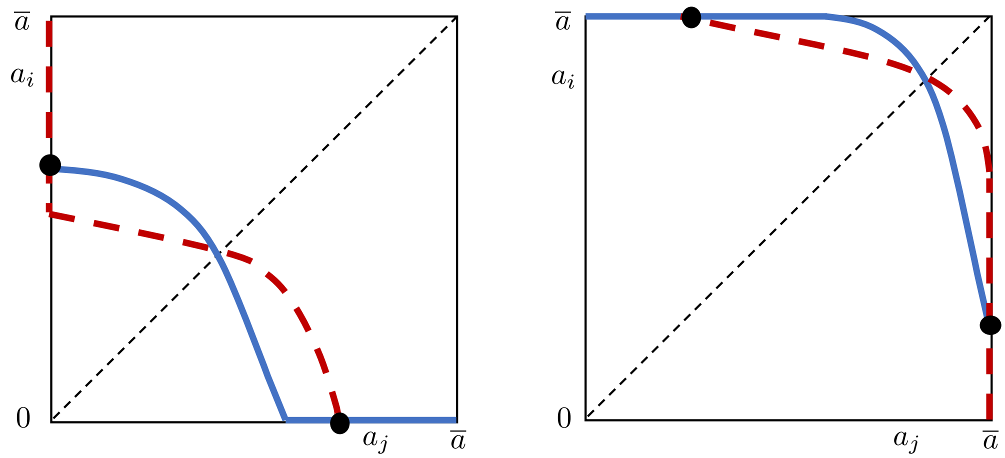
[6]Tying ads to content and synchronizing airings are simple strategies that tv-stations, newspapers and websites use to enhance reach leading to (3.10). To see this with a simple illustration suppose each station has two units of content each requiring one unit of attention and supplies only one advertising message tied to each piece of content. Suppose there are two advertisers each purchasing two messages. Viewers consume one random piece of content on each station. If advertisers concentrate all messages on one station then all consumers eventually are exposed. If advertisers purchase one message on each outlet then on average a quarter of consumers are not informed.

[7]In fact with continuous best responses, a symmetric pure strategy equilibrium always exist. See Vives (1990) footnote 7 and theorem 4.2 (iii.) for a formal argument in a related context

[8]In oligopoly models, stability is often used as a selection criterium for a number of reasons. For instance stable equilibria are 'more compelling' in that they allow to think of the static equilibrium as the rest point of some dynamic adjustment process which captures some learning or bounded rationality of



(a) Case $k \leq \hat{k}$: one station supplies. (b) Case $k > \hat{k}$: Both stations supply.

Figure 2: Asymmetric equilibria

How does the asymmetric equilibrium look like? The following proposition completes the description of the equilibrium providing a full characterization of the quantities (proof in appendix B.5).

Proposition 2 *Suppose* $1 - \phi$ *is strictly log-concave. Then there are thresholds* $\tilde{k}$ *and* $\hat{k}$ *with* $\underline{k} < \tilde{k} \leq \hat{k} < \overline{k}$ *such that:*

(i.a) If $(\underline{k}, \tilde{k}]$ *then* $a_2^\star = 0$ *and* $a_1^\star \in (0, \overline{a})$

(i.b) If $(\tilde{k}, \hat{k}]$ *then* $a_2^\star = 0$ *and* $a_1^\star = \overline{a}$,

(ii.) If $(\hat{k}, \overline{k}]$ *then* $a_2^\star > 0$ *and* $a_1^\star = \overline{a}$.

Depending on the relative profitability of k we can have basically one of two regimes. If k is 'small' then only one station is active on the advertising side of the market (figure 2(a))). Furthermore if $a_1^\star < \overline{a}$ then 1 choses the quantity that equates marginal returns on opposite side of the market:

$$1 = k \cdot \frac{\partial \phi(a_1^\star)}{\partial a_1}. \tag{3.11}$$

the players. 'Stability guaranteeing' assumptions are also needed to ensure 'natural' comparative statics results such as output going down with marginal costs (Dixit (1986)). See Vives (2001) chapter 2 for a definition and discussion of this property)

If k is 'large' then both stations are active with station 1 at capacity and $a_2^\star$ equating revenues on opposite sides of the market figure 2(b):

$$1 = k \cdot \frac{\partial \phi(a_2^\star)}{\partial a_2}(1 - \phi(\overline{a})). \tag{3.12}$$

Intuitively the threshold between the two regimes, denoted $\hat{k}$, is such that station 2 is just indifferent between supplying one unit or not when the rival station supplies at capacity $\overline{a}$. Geometrically, log-concavity does not restrict the solid line to be concave. However proposition 2 says that it cannot be 'too convex' by which we mean that i's best reponse never crosses j's (dotted line) in the interior of the strategy space $[0, \overline{a}]^2$.

Applying the implicit function theorem to (3.11) and (3.12) allows to establish that whenever $a_i^\star$ is interior then it must be continuous and monotone increasing in k. This coupled with proposition 2 implies that differentiation is maximally enhanced when the revenue potential of the two sides is most balanced.

Proposition 3 *The extent of differentiation, as measured by $a_1^\star - a_2^\star$ is continuous and hump-shaped in the relative profitability of advertising k.*

Specifically $a_1^\star - a_2^\star$ is equal to zero for $k \notin [\underline{k}, \overline{k}]$, increasing for $k < \tilde{k}$, decreasing for $k > \hat{k}$ and flat otherwise.

Relationship with classic results on product differentiation. The result on asymmetric business models relates to the theoretical literature on endogenous product differentiation. Following Hotelling (1929), these studies typically analyze equilibrium oligopoly models with a product-choice stage preceding price competition, as in D'Aspremont et al. (1979) and Shaked and Sutton (1982). Product differentiation emerges in equilibrium, with ex-ante identical firms supplying different products. It is 'strategic' in that it is driven by the need to relax price competition rather than by the need to cater to demand. These differentiation results depend crucially on the assumption that consumers have different tastes for either quality (vertical differentiation) or variety (horizontal differentiation). So consumers with different characteristics (or of different "types") patronize different firms in equilibrium. Heterogeneity is key in that it allows firms to set positive mark ups. While consumer and advertiser heterogeneity is certainly an important feature of media markets, an important insight of our analysis is that heterogeneity is not essential to sustain an asymmetric equilibrium outcome. So our model contributes providing a principle of differentiation which does not rely on preferences. Indeed in our setting viewers and advertisers are all alike and in equilibrium consume the same bundle of products. However having two groups of agents implies that each firm has two different sources of profits corresponding to the two sides. In analogy to the classic result, differentiation is 'strategic' as choosing to raise revenues from the opposite side as one's rival allows to relax price competition on that side and vice-versa. However the mechanism is different: it is heterogeneity across rather than within sides of the market that leads to different business models.[9] In this sense our result is not just the simple extension of a familiar result to two-sided platforms. Instead, it highlights in a multi-sided environment an additional and specific source of differentiation that cannot arise in one-sided settings.

The analysis maintained the assumption that the stations have substitutable but otherwise 'symmetric' content. Clearly assuming that distaste for advertising is correlated with taste for content quality in other dimensions, the stations will have an additional incentive to differentiate along these other dimensions with the Pay-TV opting for 'premium content.' The analysis of the interactions between business model choice and content choice is left to future work.

4 Monopoly

To better understand how competition works we contrast the duopoly outcome with that arising when a monopolist controls both stations.

It is reasonable to assume that a multi-station monopolist can also offer a bundle. That is a 'bouquet of stations' to viewers and a 'multi-station campaign' to advertisers. To account for this we allow the monopolist to offer bundles in addition to individual subscriptions. That is, at stage 2 he may set an additional transfer, denoted t_{12}, that the advertiser needs to pay to be allotted (m_1, m_2) ads and a viewer subscription fee, denoted f_{12} for a bouquet including both stations. As the monopolist problem is symmetric in (a_1, a_2), we maintain without loss of generality that $a_1 \geq a_2$.

Conditional on purchasing a subscription, viewers' time allocation does not depend on the ownership structure so nothing changes: love for variety and love for content imply that multi-homing viewers allocate one time slot to each station and single-homing viewers both slots to the same station.

Consider now the optimal subscription fees. In our simple setting pure bundling is an obvious course of action as it allows to capture *all* the surplus in the industry. As the

[9] We are indebted to Helen Weeds and Patrick Rey for raising this issue and helping us develop this argument.

monopolist cannot improve on what he can achieve through pure bundling, f_1 and f_2 are optimally set arbitrarily high. The optimal bundle fee leaves viewers indifferent:

$$f_{12}(a_1, a_2) = U(1, 1, a_1, a_2). \tag{4.1}$$

Consider the advertising contracts. Given an arbitrary advertiser and an arbitrary allocation (m_i, m_j) the optimal price necessarily leaves such advertiser indifferent between accepting or not:

$$t_{12}(m_1, m_2) = k \cdot \Phi(m_1, m_2). \tag{4.2}$$

Once more, pure bundling allows to capture all the surplus of the advertiser so t_1 and t_2 are optimally set arbitrarily high.

We now turn to the optimal allocation across advertisers of the inventory (a_i, a_j). Diminishing returns from advertising (ϕ strictly concave) pin down the total quantity $m_i + m_j$ offered to individual advertisers which needs to be spread equally. So $m_i + m_j = a_i + a_j$. How is this quantity allocated across stations? Under strict log-concavity we know (4.2) is maximized when all the advertising messages are concentrated on one station. So the optimal allocation (and hence contract) at the individual advertiser level is either $(m_i = a_i + a_j, m_j = 0)$ or $(m_i = 0, m_j = a_i + a_j)$. How many of these contracts can the monopolist provide? Given that total supply is predetermined and equal to a_i and a_j he can offer up to $a_i/(a_i + a_j)$ advertisers a contract $\{m_i = a_i + a_j,\ m_j = 0,\ t_{12} = k \cdot \phi(a_i + a_j)\}$ and up to $a_j/(a_i + a_j)$ advertisers a contract $\{m_i = 0,\ m_j = a_i + a_j, t_{12} = k \cdot \phi(a_i + a_j)\}$. So contrary to what we found in the previous section different advertisers are offered different contracts in equilibrium whenever $a_i, a_j > 0$.

Claim 3 *Given a pair of first-stage choices $\{a_i, a_j\}$, a monopoly owner offers two contracts $\{m_i', m_j', t_{12}'\}$ and $\{m_i'', m_j'', t_{12}''\}$ to two different arbitrary subsets of advertisers of mass $\frac{a_i}{a_i+a_j}$ and $\frac{a_j}{a_i+a_j}$ respectively and all advertisers accept. If $1 - \phi$ is strictly log-concave the contracts have the feature that $m_i' = m_j'' = a_i + a_j,\ m_j' = m_i'' = 0$ and $t_{12}' = t_{12}'' = k \cdot \phi\,(a_i + a_j)$.*

Observe that in equilibrium the price of both advertising bundles is the same and all advertisers are indifferent. Under log-concavity the total advertising revenues are equal to the price of the bundle times the mass 1 of advertisers.

Plugging the equilibrium fees, the problem of the monopolist is:

$$\max_{(a_1, a_2) \in [0, \overline{a}]^2} \pi_m(a_1, a_2) := U(1, 1, a_1, a_2) + k \cdot \phi\,(a_i + a_j) \tag{4.3}$$

Inspection of (4.3) already allows to draw a number of lessons on the nature of the problem. First, in analogy to canonical models one would conjecture that market power goes hand-in-hand with higher prices. Indeed, subscription and advertising fees increase when the market is monopolized (in a sense that we make precise below). Second, profits in (4.3) depend on the total quantity of advertising $a_1 + a_2$. The cost (in terms of lost viewer surplus) of an extra unit of advertising is 1 regardless of whether it comes from an increase in a_i or a_j whereas the benefit in terms of increased advertising surplus depends on total inventory $a_i + a_j$. It follows that the monopolist does not have preferences over the business models of the individual stations. Furthermore this is an obvious source of multiplicity as the following proposition remarks.

Proposition 4 *Suppose that $1-\phi(e)$ is strictly log-concave in exposure and $k \in (\underline{k}, \overline{k})$. Then a continuum of equilibria exist. That is a pair (a_1^m, a_2^m) is part of an equilibrium if and only if $a_1^m + a_2^m = a_{12}^m$ where a_{12}^m, is the unique quantity equating marginal returns on opposite side of the market:*

$$-1 + k \cdot \frac{\partial \phi(a_{12}^m)}{\partial a_i} = 0. \tag{4.4}$$

Does monopoly lead to quantity restrictions? The following formal statement basically says that market power leads to no change in quantities and to higher subscription fee if $k \in (\underline{k}, \hat{k})$ (or equivalently $a_2^\star = 0$), whereas hardly compares otherwise.

Proposition 5 *Suppose that $1-\phi(e)$ is strictly log-concave in exposure.*

(i.) If $(\underline{k}, \hat{k}]$ then $a_1^\star > a_2^\star = 0$, $a_{12}^m = a_1^\star + a_2^\star$, $f_{12}^m > f_1^\star + f_2^\star$ and $t_{12}^m = t_1^\star + t_2^\star$.

(ii.) If $k \in (\hat{k}, \overline{k}]$ then $a_1^\star > a_2^\star > 0$ and $a_{12}^m \gtreqless a_1^\star + a_2^\star$, then $f_{12}^m \gtreqless f_1^\star + f_2^\star$ and $t_{12}^m \gtreqless t_1^\star + t_2^\star$.

We address these two cases in turn. To appreciate why the duopoly and monopoly outcome coincide in case *(i.)* notice that given $a_2^\star = 0$ the profit of firm 1 in duopoly can be decomposed as the total surplus π_m minus the advertiser and the viewer outside option. That is the value of single-homing on 2:

$$\begin{aligned} a_1^\star = \arg\max_{[a_1 \in 0, \overline{a}]} \pi(a_1, 0) &= \Delta U_1(a_1, 0, 1, 1) + k \cdot \Delta\Phi_1\,(a_1{,}0) \\ &= \pi_m(a_1, 0) - u(2, 0) - k \cdot \phi(0). \end{aligned} \tag{4.5}$$

Crucially firm 1 has no control over the outside options which depend on 2's strategy only. Then, given $a_2 = 0$ the value of a_1 that maximizes $\pi(a_1, 0)$ coincides with a_{12}^m and the result follows. In summary while on the one hand individual station choices have an externality on the rival's payoff through their effect on the outside option, these externalities only affect the profit level and not the marginal returns. Therefore while this setting differs from that of competition, the incentives are remarkably similar. In case (ii.) in duopoly both stations are active on the advertising side of the market. All advertisers multi-home and purchase a quantity of messages $m_1^\star = a_1^\star$ and $m_2^\star = a_2^\star$. In light of our earlier discussion on the role of technological log-concavities in shaping the advertising surplus, it is not surprising that a monopolist, which can control the *joint* allocation, would instead concentrate all messages on one station. By doing so, the monopolist achieves a higher surplus through higher efficiency. Also, and most importantly for the question at hand, this changes also its incentives at the margin leading to a different total quantity and hence allocation. We thus contribute to the debate on the effects of market power on the amount of advertising in the marketplace (Anderson and Coate (2005); Anderson, Foros and Kind (2016), D'Annuzio and Russo (2017)) uncovering a different motive, linked to the technology, for why competitive pressure may lead to a distortion in the level of advertising.

By a revealed preference argument the monopolist obtains larger profits than the sum of the duopoly ones and this also implies that the sum of the fees are always larger in monopoly than in duopoly. Moreover viewers always have to pay for access. So, in contrast with the duopoly outcome, under no condition a monopolist chooses a business model entailing free access to content. There is therefore a sense in which market power threatens the existence of free content which in many jurisdictions is considered a legitimate public policy goal.

The fact that monopoly profits are equal to the total surplus in the industry allows to use our model to also tackle normative issues. In particular it allows to use the monopoly outcome as a normative benchmark for the competitive one. As showed in section 3, if $k > \hat{k}$, competitive pressure induces an outcome in which advertisers purchase advertising messages on *both* stations. Under log-concavity, allocative efficiency requires instead that advertisers concentrate their effort on one station (see claim 3). Hence competition leads to a 'misallocation' of advertising messages. More precisely by 'misallocation' we mean two things. First, given a total quantity, a planner can always enhance total surplus reallocating messages across advertisers. Second, as this affects marginal returns from supplying advertising, it also implies that the market provision $(a_1^\star + a_2^\star)$ does not necessarily maximize consumer surplus.

Proposition 6 *Suppose that* $1 - \phi(e)$ *is strictly log-concave and* $k > \hat{k}$ *(or equivalently* $a_1^\star > a_2^\star > 0$*) then competition leads to a misallocation of advertising messages.*

5 Heterogeneous preferences and viewer sorting

To emphasize the role of 'two sidedness' in shaping the asymmetric outcome, the model proposed shuts down the classic demand-driven incentives to differentiate assuming that all agents share the same preferences. In Calvano and Polo (2016) we analyze a richer model allowing for a continuum of heterogeneous viewers with idiosyncratic taste for quality (in particular different marginal dis-utility from advertising).[10,11] Under conditions analogous to those presented in Proposition 1, an asymmetric equilibrium exists with one station raising money only from subscription fees (i.e. a Pay-TV) and a second station raising money solely from the advertising side of the market (i.e. a FTA).

How does preference heterogeneity affect incentives? With heterogeneous viewers the choice to subscribe can differ across viewers, with some of them single-homing and others accessing both stations. An important (and realistic) equilibrium feature in this richer setting is that the two stations cater to different subsets of viewers with the FTA serving *all* viewers and the Pay-TV serving only *some* with strong distaste for advertising / taste for quality. So in contrast with the analysis in section 3, the FTA serves a mixture of single-homing and multi-homing viewers. A key insight from previous work is that demand composition matters for profits and hence for incentives. The reason being that single-homing viewers are more valuable than multi-homing ones for advertising purposes as the outlet who caters them becomes a bottleneck monopolizing and monetizing accordingly their attention. So in the richer model stations have preferences over the demand *level* (how many?) and its *composition* (single- vs multi-homers). In the rest of this section we provide some broad intuition why the new effects in play there actually reinforce and

[10]We posit a utility function in the spirit of Levitan and Shubik (1980):

$$U(a_1, a_2, v_1, v_2; \theta) = \sum_{i=1}^{2} \left[\theta(1 - a_i)v_i - \frac{2 - \sigma}{2} v_i^2 \right] - \sigma v_1 v_2, \tag{5.1}$$

with $v_i \in [0, 1]$ denoting viewing time on station i, θ denoting idiosyncratic marginal utility from exposure to content and $\sigma \in [0, 1)$ measuring the degree of substitutability between stations (contents).

[11]Another ingredient we did not consider is advertisers' heterogeneity, say, in the expected profit of informing. Athey et al (2017) show that equilibrium sorting of advertisers across outlets arises in a setting with exogenous viewer demand and log-concave technology. Those advertisers whose opportunity cost of not informing viewer is highest multi-home while low value advertisers single-home on the outlet with the larger number of viewers in relative terms. We speculate that the externalities discussed above leading to negatively sloped response function with the amount of advertising on one station reducing the incremental probability on the other would carry over to this richer setting. A full-fledged analysis of this case is left for future work.

enrich the baseline logic presented. The reader is referred to Calvano and Polo (2016) for the analysis.

Station i's marginal returns to increasing one's strategy (by which we mean increasing advertising and reducing subscription fees) decrease with station j's strategy (and vice-versa) even in this richer setup. First, consider the case $a_i > a_j$ and $f_i < f_j$. That is the case when station i is closer to a free-to-air business model and station j to a pay business model. Now consider what happens when firm j further increases f_j and decreases a_j, that is it strengthens its 'pay' nature further reducing its audience. Some of the previously shared viewers are now exclusively served by firm i, as individuals formerly at the margin between single-homing and multi-homing now strictly prefer to single-home.[12] This selection is favorable to firm i, whose incentives to provide advertising and thus to move in the opposite direction, other things held constant, increase. This is what we refer to as 'composition effect' since the impact of the 'pay' station's strategy on the 'free' station's incentives is due to its effect on the composition of the rival's viewer base. Second, consider the impact on firm j's incentives of firm i strengthening its FTA nature by increasing a_i. In addition to the effects already highlighted in section 2 and driving j to reduce a_j, [13] demand for j's subscription increases, as the former marginal viewer now strictly prefer to subscribe to j. This effect, which we refer to as 'level effect' further pushes j towards setting a higher f_j thus sharpening its 'pay' nature.[14]

In summary, elastic demand due to heterogeneous preferences preserves and reinforces the strategic substitutability property which is key for differentiation.

[12]This new effect adds up to the one we already found in the benchmark model. When a_j is reduced the incremental probability of informing viewers on station i increases, leading this station to increase the quantity of advertising a_i.

[13]An increase in a_i raises the incremental utility of station j leading to a lower response a_j.

[14]Calvano and Polo (2016) also allow viewers to choose the viewing time on a continuous set rather than a discrete grid. Then we identify additional effects working at the intensive margin that go in the same direction of what we already observe in the benchmark model. Single-homers spend more time on the patronized station than multi-homers. Hence, when f_j is raised inducing some multi-homers to watch only station i, this latter gains *exclusive* viewers that spend *more time* watching its programs. The willingness to pay of advertisers is further enhanced. Conversely, when station i increases its quantity of advertising a_i multi-homing viewers of station j spend *more time* watching its programs, with a higher willingness to pay that allows to raise f_j.

6 Policy implications: business models and the relevant market

An established practice in antitrust and media regulation is to treat operators that have opposite business models as belonging to different relevant markets. For instance, in the merger cases BSkyB/Kirch Pay-TV[15] and News Corporation/Premiere[16] the European Commission has ruled that FTA and Pay-TV operators belong to separate product markets. The German Bundeskartellamt reached similar conclusions examining the Springer/ProSieben/Sat1 case. The common view is that a Pay-TV broadcaster deals only with viewers, whereas an FTA deals only with advertisers, with no overlapping or competitive constraints. This argument is then extended also to the case when a Pay-TV raises most of its revenues from subscription but offers also some advertising. To the best of our knowledge BSkyB/ITV is the only case in which an authority (the UK Competition Commission) has taken a different position, recognizing that " in two-sided markets suppliers can compete with one another at different price points, given the ability to generate revenues in two separate markets. For instance, FTA services may compete directly for viewers with pay services, with higher viewing figures indirectly generating higher advertising revenues."(UK Competition Commission (2007), par. 4.6)

There are two complementary arguments for the traditional approach, as is maintained by Filistrucchi et al. (2014). First, until recently antitrust authorities and regulators had not been willing to recognize the two-sided nature of many media markets, although two-sidedness has started to influence their practice in other areas, such as credit cards. Moreover, from the one-sided perspective, there is a tradition of requiring a positive price (the so-called "trade relationship") as a prerequisite for market interaction and antitrust concerns.[17]

[15]See case COMP/JV.37, BSkyB/Kirch Pay TV (Mar. 21 2000). The merger involved BSkyB, whose main activity was pay-TV broadcasting in the UK, and KirchPayTV Gmbh offering pay-TV services in Germany and Austria. The Commission distinguished two product markets, one for pay-TV and one for interactive digital TV, according to the nature of the business model, without considering the advertising and viewer sides of the market.

[16]18 See case COMP/M.5121, 2008 O.J. (C 219) 2. The concentration involved the acquisition of a 25% stake in Premiere, a pay-TV operator active in Germany and Austria, by News Corporation, a large international media company active in the pay-TV segment. The Commission considered the pay-TV services only, expressing some concern for vertical relationships but ignoring the impact of FTA operators.

[17]In KinderStart v. Google the Court of the Northern District of California, for instance, held that there is "no authority indicating that antitrust law concerns itself with competition in the provision of

If stations adopting different business models belong to different relevant markets, in the logic of the SSNIP test we should observe that once we pool together the two activities no raise in prices or change in strategies should derive, since this excercise would simply add together two independent lines of business. We showed instead that moving to a monopoly adversely affects prices and, depending on the parameters, also affects the total quantity/quality of contents and how such quantity is allocated across stations and advertisers. The two stations, in duopoly, deeply affect each other strategic choices. In this sense observing the adoption of very different business models does not imply that strategic interaction is weak, but rather that it is strong leading stations to adopt different business models to relax and therefore restrain competition. This result directly implies that the two stations should belong to the same relevant market, even when in a duopoly they would follow opposite business models as a FTA and a Pay-TV.

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A Illustrating the assumptions with quadratic utility

Consider the following quadratic utility function is in the spirit of Levitan and Shubik (1980):

$$u(a_i, v_i) = (\gamma - a_i)v_i - \frac{\beta}{2}v_i^2 \tag{A.1}$$

$$g(v_i, v_j) = \frac{v_1 v_2}{2} \tag{A.2}$$

and check the parameter restrictions that make the six assumptions holding.

(Love for variety and diminishing returns): $\frac{\partial u}{\partial v_i} = \gamma - a_i - \beta v_i > 0$ for $a_i \in [0, \overline{a}]$ and $v_i \in \{0, 1, 2\}$ requires $\beta < \frac{\gamma - \overline{a}}{2}$; $\frac{\partial^2 u}{\partial v_i^2} = -\beta < 0$ Summing up we need:

$$0 < \beta < \frac{\gamma - \overline{a}}{2} \tag{A.3}$$

that implies $\gamma > \overline{a}$.

(Love for variety): $u(2, a_i) \leq U(1, 1, a_i, a_j)$ corresponds to

$$2(\gamma - a_i) - 2\beta \leq 2\gamma - a_i - a_j - \frac{\beta}{2} - \frac{1}{2} \tag{A.4}$$

that simplifies to

$$a_j \leq a_i + \beta - \frac{1}{2}. \tag{A.5}$$

This condition holds for any $(a_i, a_j) \in [0, \overline{a}]^2$ if

$$\beta \geq \overline{a} + \frac{1}{2}. \tag{A.6}$$

(Advertising aversion): $\frac{\partial u}{\partial a_i} = -v_i$

(Content substitutability): Since $g(v_i, v_j) = v_1 v_2$ the assumption on content substitutability is always satisfied.

To sum up if

$$\gamma > \overline{a} \tag{A.7}$$

$$\overline{a} + \frac{1}{2} \leq \beta < \frac{\gamma - \overline{a}}{2}, \tag{A.8}$$

the four assumptions are satisfied. For example, if $\gamma = 3,25$, $\beta = 1,25$ and $\overline{a} = 0,5$ the assumptions are satisfied.

B Mathematical Proofs

B.1 Intermediate and instrumental results

The following results will be used throughout the appendix. Consider the following family of optimization problems parametrized by a_j and k where $\Delta U_i(a_i, a_j)$ is defined in (3.2):

$$b(a_j, k) := \arg \max_{a_i \in [0,\overline{a}]} \pi(a_i, a_j) = \Delta U_i(a_i, a_j) + k \cdot \Delta \Phi_i(a_i, a_j). \tag{B.1}$$

Claim 4 *$b : [0, \overline{a}] \times \mathbb{R} \to [0, \overline{a}]$ is well-defined, single-valued, continuous, nondecreasing in k and nonincreasing in a_j. If $b(a_j, k) \in (0, \overline{a})$ then b is increasing in k.*

Proof: The objective function is the sum of a linear and a strictly concave function and so is strictly concave. By Berge's maximum theorem the solution correspondence of the problem, denoted $b(a_j, k)$, is non-empty, single valued and continuous. The objective function is strictly submodular in (a_i, a_j) and supermodular in (a_i, k) (strictly so for $k > 0$). Then $b(a_j, k)$ is nonincreasing in a_j and nondecreasing in k by Topkis (1978) Theorem 6.3. For the case where b is interior then the stronger result follows by noting the property of increasing marginal returns $\frac{\partial^2 \pi_i}{\partial a_i \partial k} > 0$ and applying Edlin and Shannon (1998) theorem 1. $\square$

Claim 5 *Suppose the assumptions of proposition 1 hold. If $(a_1^\star, a_2^\star)$ belongs to the equilibrium set then $(a_1^\star, a_2^\star) \notin (0, \overline{a})^2$. So in equilibrium either (i) $a_2^\star = 0$ or (ii.) $a_1^\star = \overline{a}$. That is the unique asymmetric equilibrium cannot be 'interior.'*

Proof: We proceed by contradiction. A stable interior asymmetric equilibrium is any (a_1, a_2) with $a_1 \geq a_2$ such that

$$\overline{a} > a_1 > a_2 > 0 \tag{B.2}$$

$$-1 + k \frac{\partial \phi}{\partial a_1}(1 - \phi(a_2)) = 0 \tag{B.3}$$

$$-1 + k \frac{\partial \phi}{\partial a_2}(1 - \phi(a_1)) = 0 \tag{B.4}$$

$$\frac{\partial^2 \phi}{\partial a_1 \partial a_1}(1 - \phi(a_2)) \leq 0 \tag{B.5}$$

$$\frac{\partial^2 \phi}{\partial a_2 \partial a_2}(1 - \phi(a_1)) \leq 0 \tag{B.6}$$

$$\frac{\partial^2 \phi}{\partial a_1 \partial a_1}(1 - \phi(a_2)) \frac{\partial^2 \phi}{\partial a_2 \partial a_2}(1 - \phi(a_1)) + \frac{\partial \phi}{\partial a_1} \frac{\partial \phi}{\partial a_2} > 0 \tag{B.7}$$

Consider now the following auxiliary problem:

$$\max_{a_1, a_2 \in [0, \overline{a}]} U(1, 1, a_1, a_2) + k \cdot \Phi(a_1, a_2) \tag{B.8}$$

Note that any (a_1, a_2) satisfying (B.2) through (B.7) must also be an interior local maximum of (B.8) (the first and second order sufficient conditions for an interior local maximum of (B.8) are (B.3) through (B.7)). Consider now the locus $\{(a_1, a_2) : a_1 > a_2 \text{ and } a_1 + a_2 = a_1^\star + a_2^\star\}$. We have that U is constant along this locus while by property (3.10), Φ increases which contradicts the working hypothesis $(a_1^\star, a_2^\star)$ being a local maximum of (B.8). It follows that there can be no (a_1, a_2) satisfying (B.2) through (B.7) and therefore no interior equilibrium. $\square$

B.2 Equilibrium subscription fee

Love for content and substitutability imply that:

$$u(2, a_i) + u(2, a_j) > u(1, a_i) + u(1, a_j) > U(1, 1, a_i, a_j) > 0 \tag{B.9}$$

which in turn implies $\Delta U_i < u(2, a_i)$. Offers $f_i > u(2, a_i)$ are always rejected by all viewers so are weakly dominated by $f_i = 0$. By assumption all stations when indifferent choose $f_i = 0$, so we can restrict attention to equilibrium candidates in which $f_i \leq u(2, a_i)$ and $f_j \leq u(2, a_j)$.

Let us derive the optimal fee for station i for different values of the rival station's subscription fee. Consider two cases: if $0 \leq f_j \leq \Delta U_j < u(2, a_j)$ the viewers choose station j regardless of their choice on station i. If $0 \leq f_i \leq \Delta U_i$ then the viewers will subscribe also to station i, whereas by setting $\Delta U_i < f_i \leq u(2, a_i)$ station i would not induce the viewers to add i to j. Notice that in this case the viewers single-homes. Choosing only station j gives a net utility $u(2, a_j) - f_j \geq u(2, a_j) + u(2, a_i) - U(1, 1, a_i, a_j)$ whereas single-homing on i would give $u(2, a_i) - f_i \leq u(2, a_j) + u(2, a_i) - U(1, 1, a_i, a_j)$. Hence, single-homing on j is the viewers' optimal choice. We conclude that if $0 \leq f_j \leq \Delta U_j$ then setting $f_i = \Delta U_i$ is the optimal choice for station i.

Consider next the case $\Delta U_j < f_j \leq u(2, a_j)$. In this case the viewers choose station j stand alone but not if they already subscribe for station i. If this latter sets $f_i \leq \Delta U_i$ the viewers choose it (and do not subscribe for station j) whereas setting a fee $\Delta U_i < f_i \leq u(2, a_i)$ makes viewers opting for single-homing on the more convenient station. Then the optimal response to f_j is to set $f_i = f_j + u(2, a_i) - u(2, a_j) - \varepsilon$ bringing all viewers onboard, an instance of Bertrand competition.

Putting together the optimal replies in the two regions, since $f_i < \Delta U_i$ cannot be optimal the only equilibrium involves each station setting the subscription fee equal to its incremental utility: $f_i^*(a_i, a_j) = \Delta U_i(a_i, a_j)$.

B.3 Proof of claim 2

Suppose for some $\tilde{k}$ we have $b(0, \tilde{k}) = 0$. Then by claim 4 (monotonicity) $b(a_j, \tilde{k}) = 0$ for all $a_j > 0$. The unique equilibrium is $a_i^\star = a_j^\star = 0$. Notice that $b(0, k) = 0$ if and only if $\frac{\partial \pi(0,0,k)}{\partial a_i} = -1 + k\frac{\partial \phi(0)}{\partial e_i} \leq 0$ which is equivalent to $k \leq \left[\frac{\partial \phi(0)}{\partial e_i}\right]^{-1}$. Similarly suppose that for some $\tilde{k}$ we have $b(\overline{a}, \tilde{k}) = \overline{a}$ then by claim 2 $b(a_j, k) = \overline{a}$ for all $a_j \leq \overline{a}$. The unique equilibrium when $k = \tilde{k}$ is $a_i^\star = a_j^\star = \overline{a}$. Notice that $b(\overline{a}, k) = \overline{a}$ if and only if $\frac{\partial \pi(\overline{a},\overline{a},k)}{\partial a_i} = -1 + k\frac{\partial \phi(\overline{a})}{\partial e_i}(1 - \phi(\overline{a})) \geq 0$ which is equivalent to $k \geq \left[\frac{\partial \phi(\overline{a})}{\partial e_i}(1 - \phi(\overline{a}))\right]^{-1}$. So for k outside the interval (3.9) an asymmetric equilibrium cannot exist $\square$.

B.4 Proof of property (3.10) and Proposition 1

Log-concavity implies (3.10)**.** We show that if $1-\phi(x)$ is strictly log concave in x and $\phi(0) = 0$ then condition (3.10) holds. To simplify the exposition let $g(x) := log(1-\phi(x))$. By definition strict log-concavity is equivalent to

$$g(tx + (1-t)y) > tg(x) + (1-t)g(y) \quad \text{for all} \quad 0 < t < 1. \tag{B.10}$$

$y = 0$ implies $g(0) = 0$ so $x > 0$ the above reduces to:

$$g(tx) > tg(x) \quad \text{for all} \quad 0 < t < 1, \quad x > 0 \tag{B.11}$$

Let $t_x = x/(x+y)$ and $t_y = x/(x+y)$. Applying (B.11) twice with weights t_x and t_y we have

$$g(t_x(x+y)) = g(x) > t_x g(x+y) \quad \text{and} \quad g(t_y(x+y)) = g(y) > t_y g(x+y) \tag{B.12}$$

Observe that $t_x + t_y = x/(x+y) + y/(x+y) = 1$. Using this fact with (B.12) we obtain

$$g(x) + g(y) > (t_x + t_y)g(x+y) = g(x+y). \tag{B.13}$$

Finally applying the definition of g, Φ, the properties of the $log(x)$, for all $x, y > 0$ we have:

$$\log(1-\phi(x)) + \log(1-\phi(x)) > \log(1-\phi(x+y)) \tag{B.14}$$

$$\Leftrightarrow \log((1-\phi(x))(1-\phi(y))) > \log(1-\phi(x+y)) \tag{B.15}$$

$$\Leftrightarrow (1-\phi(x))(1-\phi(y)) > (1-\phi(x+y)) \tag{B.16}$$

$$\Leftrightarrow \phi(x+y) > 1-(1-\phi(x))(1-\phi(y)) \tag{B.17}$$

$$\Leftrightarrow \Phi(x+y,0) > \Phi(x,y) \tag{B.18}$$

Proof of Proposition 1: Since ϕ is concave, $\frac{\partial\phi(0)}{\partial e_i} > \frac{\partial\phi(\overline{a})}{\partial e_i}(1-\phi(\overline{a}))$ and therefore the interval (3.9) is non-empty. For the rest of the proof suppose (3.9) is satisfied. Since b is continuous in a_j and maps a compact set A into itself there is at least a fixed point, denoted $a^\star$, corresponding by definition to a symmetric equilibrium. $a_i^* = a_j^* = a^*$. Moreover, as b in nonincreasing in a_j then $a^\star$ is necessarily unique. In addition recall that (3.9) is equivalent to $b(\overline{a}, k) < \overline{a}$ and $b(0,k) > 0$. So $a^\star \in (0, \overline{a})$ and is characterized by the first order condition (3.8). By the implicit function theorem around the fixed point:

$$\frac{\partial b(a^\star, k)}{\partial a_j} = \frac{\left(\frac{\partial\phi(a^*)}{\partial e_i}\right)^2}{\frac{\partial^2\phi(a^*)}{\partial e_i^2}(1-\phi(a^\star))}. \tag{B.19}$$

$1-\phi(x)$ strictly log-concave is equivalent (by definition) to the second derivative of $\log(1-\phi(x))$ being strictly negative. Differentiating leads to the equivalent condition $\left(\frac{\partial\phi(x)}{\partial x}\right)^2 > -\frac{\partial^2\phi(x)}{\partial x^2}(1-\phi(x))$ for all x. It follows that $1-\phi$ strictly log-concave implies that (B.19) is < -1 (notice that this means that the symmetric equilibrium is unstable).
(B.19) implies that $b(a_j)$ is strictly monotone around $a^\star$ and thus its inverse b^{-1} exists and has slope $\in (-1, 0)$. In a left neighborhood of $a^\star$, denoted $a^\star_-$ we have.

$$b(a^\star_-) > b^{-1}(a^\star_-) \tag{B.20}$$

That is: i's best response is above the inverse of j's best response for a_i smaller and close to $a^\star$. To show existence we now argue that there is a value of $a \in [0, a^\star)$ such that i's best response must lie (weakly) below j's inverse and so by continuity an asymmetric equilibrium exists. The only difficulty is that b being weakly monotone in a_j implies that

b^{-1} is a convex valued correspondence:

$$b^{-1}(x) := \{a \in [0, \overline{a}] : b(a) = x\}. \tag{B.21}$$

So we are looking for a value of a such that:

$$a < a^\star \tag{B.22}$$
$$b(a) \leq \sup\{b^{-1}(a)\} \tag{B.23}$$

To this end consider the candidate $a = b(\overline{a})$. Clearly $\overline{a} > a^\star$ implies $b(\overline{a}) < a^\star$ by monotonicity of b around $a^\star$ so (B.22) holds. Next observe that by definition (B.21) $\overline{a} \in b^{-1}(b(\overline{a}))$. So $\sup\{b^{-1}(b(a))\} = \overline{a}$ which by definition is greater or equal than $b(a)$ so (B.23) holds and therefore at least an asymmetric equilibrium exists.
By Claim 5 the asymmetric equilibrium can be one of three types: (i) $(a_1^\star < \overline{a}, a_2^\star = 0)$; (ii.) $(a_1^\star = \overline{a}, a_2^\star = 0)$; (iii.) $(a_1^\star = \overline{a}, a_2^\star > 0)$. These three cases are mutually exclusive. To see this suppose there is an equilibrium of type (i.) then $b(0) < \overline{a}$ so (ii) cannot occur and $b(0) < \overline{a}$ so by monotonicity of $b(a > 0) < \overline{a}$ ruling out (iii). By an analogous argument if there is an asymmetirc equilibrium of type (ii.) then (i.) and (iii.) cannot exists and if there is an equilibrium of type (iii.) then (ii.) and (i.) cannot exits. Uniqueness follows from claim 4 (monotonicity of best response).

B.5 Proof of Proposition 2

Consider first this instrumental result:

Claim 6 *Suppose that the assumptions of proposition 1 hold and consider the asymmetric equilibrium. (i.) There is a unique threshold level of k, denoted $\hat{k}$, such that $a_2^\star > 0$ if and only if $k > \hat{k}$ and $\hat{k} \in (\underline{k}, \overline{k})$. (ii.) There is a unique threshold level of k denoted $\tilde{k}$ such that $a_1^\star = \overline{a}$ if and only if $k \geq \tilde{k}$ and $\tilde{k} \in (\underline{k}, \overline{k})$. (iii.) $\tilde{k} \leq \hat{k}$.*

For future reference let:

$$\underline{k} := \{k \geq 0 : 1 = \frac{\partial \phi(0)}{\partial a} k\} \tag{B.24}$$
$$\overline{k} := \{k \geq 0 : 1 = \frac{\partial \phi(\overline{a})}{\partial a}(1 - \phi(\overline{a}))k\} \tag{B.25}$$
$$\tilde{k} := \{k \geq 0 : 1 = \frac{\partial \phi(\overline{a})}{\partial a} k\} \tag{B.26}$$
$$\hat{k} := \{k \geq 0 : 1 = \frac{\partial \phi(0)}{\partial a}(1 - \phi(\overline{a}))k\} \tag{B.27}$$

By strict monotonicity of ϕ it follows that $\hat{k}, \tilde{k} \in (\underline{k}, \overline{k})$ so the above thresholds are well defined. Consider part (i.) of the claim. If $k > \hat{k}$ then firm 2 first order condition satisfies for all a_1:

$$1 < \frac{\partial \phi(0)}{\partial a}(1 - \phi(a_1))k \tag{B.28}$$

which implies that $a_2^\star = 0$ cannot be part of an equilibrium. If $k \leq \hat{k}$ then by claim 5 there are only 2 equilibrium candidates: $(a_1^\star \leq \overline{a}, a_2^\star = 0)$ and $(a_1^\star = \overline{a}, a_2^\star > 0)$. To rule out the latter note that firm 2 first order condition evaluated at $a_2 = 0$ satisfies:

$$1 \geq \frac{\partial \phi(0)}{\partial a}(1 - \phi(\overline{a}))k \quad \Rightarrow \quad 1 > \frac{\partial \phi(a_2 > 0)}{\partial a}(1 - \phi(\overline{a}))k \tag{B.29}$$

and so that $a_2^\star = 0$, a contradiction. As an asymmetric equilibrium always exists it follows that for all $k \leq \hat{k}$ it must be that $a_2^\star = 0$.

Consider part (iii.). To see that $\tilde{k} \leq \hat{k}$, suppose that this were not the case and consider an arbitrary value of k belonging to$(\hat{k}, \tilde{k})$. By (i.), in the unique asymmetric outcome $a_2^\star > 0$. By definition $k < \tilde{k}$ implies $1 > \frac{\partial \phi(a)}{\partial a}k > \frac{\partial \phi(a)}{\partial a}(1 - \phi(a_2^\star))k$ so station 1 best response should be below $\overline{a}$. However by claim 5 an interior equilibrium cannot exist, so $\hat{k} < k < \tilde{k}$ leads to a contradiction.

Finally consider part (ii.). Given $a_2^\star = 0$, if $k < \tilde{k}$ then firm 1 first order condition can be satisfied only if $a_1 < \overline{a}$ so in equilibrium $a_1^\star < \overline{a}$. Now suppose that $k \in [\tilde{k}, \hat{k}]$. By monotonicity $1 < \frac{\partial \phi(\frac{1}{2}\overline{a})}{\partial a}k$ so $a_1^\star = \overline{a}$. finally if $k > \hat{k}$ then $a_2^\star > 0$ and so by claim 5 it must be that $a_1^\star = \overline{a}$. □

Proof of proposition 2: Suppose $k \leq \hat{k}$. Then $a_2^\star = 0$. By claim 4 the solution $a_1^\star$ is continuous. Furthermore the difference $a_1^\star - a_2^\star$ is constant and equal to 0 if $k \in (0, \underline{k}]$ since $a_1^\star = a_2^\star = 0$ by definition of $\underline{k}$. It is strictly increasing in $k \in (\underline{k}, \tilde{k})$ by claim 4 since $a_2^\star = 0$ and $a_1^\star < \overline{a}$ and increasing in k. It is constant and equal to $\overline{a}$ for $k \in \left[\tilde{k}, \hat{k}\right]$. Suppose $k > \hat{k}$. Recall that $a_1^\star = \overline{a}$ for all k in this range. $a_2^\star$ is greater than 0 and lower than $\overline{a}$ in this range and hence continuous and strictly increasing in $k \in (\hat{k}, \overline{k})$ by claim 4 with $a_2^\star = \overline{a}$ for $k = \overline{k}$ and $\lim_{k \to \tilde{k}} a_2^\star = \overline{a}$.

B.6 Proof of Proposition 4

If $1 - \phi(e)$ is log-concave, according to Claim 3 it is optimal to concentrate all the messages of an advertisers on either of the two stations, allocating the advertisers in share such that the total number of messages on the two stations is $a = a_1 + a_2$. The monopolist problem is then (4.3). The objective function is the sum of a linear and a stricty concave function in a_i and a_j. Moreover

$$\frac{\partial \pi_m(a_1^m, a_2^m, k)}{\partial a_i} = k\frac{\partial \phi\left(a_1^m + a_2^m\right)}{\partial a_i} - 1 = 0 \tag{B.30}$$

$$\frac{\partial \pi_m^2(a_1^m, a_2^m, k)}{\partial a_i^2} = \frac{\partial \pi_m^2(a_1^m, a_2^m, k)}{\partial a_i \partial a_j} = k\frac{\partial^2 \phi\left(a_1^m + a_2^m\right)}{\partial a_i^2} < 0 \tag{B.31}$$

for $i = 1, 2$. Hence, all that matters is the aggregate level of advertising a_{12}^m, and the monopolist profits are flat at $a_i + a_j = a$. The first order conditions then identify the optimal level of aggregate advertising a_{12}^m that is the optimal allocation of advertising, and any combination $a_i^m a_j^m \geq 0$ with $a_i^m + a_j^m = a_{12}^m$ gives the same profits.

B.7 Proof of Proposition 5

Consider the first order conditions in the duopoly and monopoly cases:

$$\frac{\partial \pi_i(a_i^*, a_j^*, k)}{\partial a_i} = -1 + k\frac{\partial \phi\left(a_i^*\right)}{\partial e_i}\left(1 - \phi\left(a_j^*\right)\right) = 0 \tag{B.32}$$

$$\frac{\partial \pi(a_i^m, a_j^m, k)}{\partial a_i} = -1 + k\frac{\partial \phi(a_i^m + a_j^m)}{\partial e_i} = 0$$

that imply:

$$\frac{\partial \phi\left(a_i^*\right)}{\partial a_i}\left(1 - \phi\left(a_j^*\right)\right) = \frac{\partial \phi\left(a_j^*\right)}{\partial a_i}\left(1 - \phi\left(a_i^*\right)\right) = \frac{\partial \phi\left(a_i^m + a_j^m\right)}{\partial a_i}. \tag{B.33}$$

If in the duopoly equilibrium $a_i^* > a_j^* = 0$ from the equality (B.33) we get:

$$\frac{\partial \phi\left(a_i^*\right)}{\partial a_i} = \frac{\partial \phi\left(a_i^m + a_j^m\right)}{\partial a_i}. \tag{B.34}$$

Hence, $a_i^m + a_j^m = a_i^*$. If, instead, $a_i^* > a_j^* > 0$ we can have $a_i^m + a_j^m = a_{12}^m \gtreqless a_i^* + a_j^*$.
In order to compare in the different cases the subscription and advertising fees in the duopoly and monopoly equilibria we can proceed as follows. First recall that substitutability implies:

$$U(1, 1, a_1, a_2) < u(1, a_1) + u(1, a_2). \tag{B.35}$$

Then:

$$f_1^\star + f_2^\star = 2U(1,1,a_1^*,a_2^*) - u(2,a_1^*) - u(2,a_2^*) \qquad \text{by (3.3)} \tag{B.36}$$
$$< 2U(1,1,a_1^*,a_2^*) - u(1,a_1^*) - u(1,a_2^*) \qquad \text{by (love for content)} \tag{B.37}$$
$$\leq 2U(1,1,a_1^*,a_2^*) - U(1,1,a_1^*,a_2^*) \qquad \text{by (B.35)} \tag{B.38}$$
$$= U(1,1,a_1^*,a_2^*). \tag{B.39}$$

The total subscription fees of the monopolist, instead, are:

$$f_{12}^m = U(1,1,a_1^m,a_2^m) \tag{B.40}$$

Let:

$$\Delta a = a_{12}^m - a_1^* + a_2^*. \tag{B.41}$$

Since in the monopoly solution the profits depend on the total quantity of advertising and not on its allocation in the two stations, it is convenient to select, for a given value of Δa, the following monopoly allocation:

$$a_1^m = a_1^*, \;\; a_2^m = a_2^* + \Delta a. \tag{B.42}$$

Hence, we can write $f_{12}^m = U(1,1,a_1^*,a_2^* + \Delta a)$. Drawing from the previous inequalities, then,

$$f_1^\star + f_2^\star < U(1,1,a_1^*,a_2^*) < U(1,1,a_1^*,a_2^* + \Delta a) = f_{12}^m \tag{B.43}$$

if $\Delta a \leq 0$ whereas we cannot sign the inequality if $\Delta a > 0$.
Turning to the advertising fees:

$$t_1^\star + t_2^\star = k\phi\left(a_1^*\right) + k\phi\left(a_2^*\right) - 2k\phi\left(a_1^*\right)\phi\left(a_2^*\right) \qquad \text{by (3.6)} \tag{B.44}$$
$$< k\phi\left(a_1^*\right) + k\phi\left(a_2^*\right) - k\phi\left(a_1^*\right)\phi\left(a_2^*\right) = \Phi\left(a_1^*,a_2^*\right) \tag{B.45}$$
$$< k\Phi\left(a_1^* + a_2^*, 0\right) \qquad \text{by log-concavity} \tag{B.46}$$
$$\tag{B.47}$$

Since

$$t_{12}^m = k\Phi\left(a_1^m + a_2^m, 0\right), \tag{B.48}$$

if $\Delta a = 0$ we have

$$t_1^\star + t_2^\star = k\Phi\left(a_1^* + a_2^*, 0\right) = k\Phi\left(a_1^m + a_2^m, 0\right) = t_{12}^m. \tag{B.49}$$

If $\Delta a > 0$ we have

$$t_1^\star + t_2^\star < k\Phi\left(a_1^* + a_2^*, 0\right) < k\Phi\left(a_1^m + a_2^m, 0\right) = t_{12}^m \tag{B.50}$$

whereas if $\Delta a < 0$ the inequality can go either way.