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Asset Market Structure and Growth

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Abstract

In this paper we illustrate the possible normative relevance of the links between human capital and financial assets via an example related to growth. Human capital investments occur in a risky environment, in that they are subject to aggregate uncertainty. Agents are heterogenous in their income streams, and this generates different risk attitudes and the scope for trading in financial assets. In this environment, human capital is a non-marketable asset that interacts with the existing financial structure in transferring wealth over time.
When the financial structure is complete, growth is indeterminate because *individual* allocations between human capital and a tradable asset are indeterminate. When the financial structure is incomplete, the growth rate depends on the payoff structure of the assets. An issue of optimality for the structure of asset returns is raised.

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1 Introduction

The issue of risk-sharing matters for investment in human capital.[1] Because future labor incomes are uncertain, human capital bears risks. Some risks are physical: death, accidents, illness. Other risks are economic, and result from the uncertainties affecting the effectiveness of education and training. In a sense, the formation of human capital corresponds to the accumulation of assets yielding uncertain outcomes. However, these assets have two distinguishing features. First, they are non-marketable and the investment in these assets must respect individual-specific feasibility constraints.[2] As a result, human capital can only be transferred onward and in finite amounts. Second, as opposed to financial assets, investments in human capital are likely to produce direct effects on productivity and output.

As other assets, human capital investments interact with the existing financial structure: they depend on the overall asset structure and simultaneously have an effect on the use of alternative financial instruments. The interaction of human capital with the financial structure has received attention in positive analysis. In particular, it has been recently explored as a possible source of deviations of observed market outcomes from those predicted by the theory of financial markets. It has been shown, for instance, that the presence of human capital-specific risk may explain both the equity premium (Weil [11]) and the international portfolio puzzles (Bottazzi et al. [**?**]). The aim of this paper is rather that of showing how the mere fact that human capital does interfere with the existing asset structure may easily give raise to normative issues.

The core of our argument runs as follows. Human capital, though non-tradable, might act as substitute for some tradable asset in transferring wealth, thus causing possible indeterminacies when the financial structure corresponds to a full set of Arrow securities. Conversely, when financial markets are incomplete, the presence of human capital may lead to allocations that mimic those reached when financial markets are complete. In such cases, the structure of returns on financial assets may have real effects in that they shape the individual incentives towards undertaking education and training, activities that are likely to affect the production possibilities of the economy.

We illustrate our argument via an example related to growth. We construct an overlapping generations model in which the outcome of investments in human capital is subject to (aggregate) risk. When young, agents are uncertain about the future income they derive from their investment in human capital. In our example, the population is heterogeneous in terms of income, and this is sufficient to generate different attitudes towards risk for different agents. In such a framework, each individual gains from trading (ex-ante) financial assets with agents belonging to different income classes. If productivity is positively affected by aggregate human capital investments, a spillover is created that acts as the engine of growth (Lucas [9]). We show that when the financial structure is complete, growth is indeterminate because *individual* allocations between

[1] See Drèze[2] on the issue of human capital and risk-bearing.
[2] Given, for instance, by time availability and learning ability.

human capital and a tradable asset are indeterminate. When the number of available financial assets is smaller than the dimension of the uncertainty, *the rate of growth depends upon the structure of asset returns*, thus leaving room for policy intervention.

It is well established from the theory of incomplete markets that Pareto efficiency of equilibrium allocations is not preserved when the asset market is incomplete. Moreover, the allocation is in this case affected by the asset structure (Geanakoplos and Polemarchakis [**?**], Geanakoplos [3]). Demange and Laroque[**?**], among others, characterize the structure of asset payoffs that leads to Pareto optimal allocations when markets are incomplete. They consider a static exchange economy where individual endowments are risky, information is symmetric, the distribution of the risk is known and individuals trade ex-ante. Our example is only loosely related to this literature. The introduction of human capital may allow for the effective completeness of the market, so that the asset structure may be irrelevant for consumption allocations even when a full set of Arrow securities is not available. We show, though, that dependence of consumption allocations on the asset structure is restored when it is introduced a process of endogenous growth generated by knowledge spillovers. The financial policy in this case matters for the determination of individual investment in human capital, and then for growth and consumption allocations. We study the optimality of the asset structure in such a framework.

There is substantial agreement on the relevance of financial markets for the growth performance of economies. Financial systems permit a more efficient allocation of resources and mobilize a higher amount of savings through better hedging, diversification and pooling of risk. However, while an abundant theoretical literature exists studying different channels through which the existence of financial markets may have a positive effect on growth, there is not much work highlighting the relation between the financial structure and the growth performance of an economy (Levine[8]). In this paper, we take a small step in this direction. In our model, financial markets enhance growth because they allow the *specialization of heterogenous agents in the use of different assets*, as alternative tools to transfer wealth over time. Relatively rich agents specialize in financial assets, relatively poor agents specialize in human capital investments, and aggregate investments in human capital increase with the extent of this specialization. We show that gains from specialization can be fully exploited provided that the financial structure is *not complete* and that asset returns are sufficiently high in states where human capital investments are effective. Moreover, in the presence of an optimal structure of asset returns, the individual constraints that characterize human capital investments must be binding for some agent. Depending on the distribution of income, either poor agents could not invest less than desired, or rich agents could not invest less.

The remainder of the paper is organized as follows. In section 2 we present the structure of our argument in a static framework. In section 3, we develop an overlapping generations model and study the impact of the financial structure on consumption allocations and investments in human capital. In section 4 we characterize the asset structure that leads to the Pareto optimum. Section 5 illustrates some policy implications. The concluding remarks follow.

2 Structure of the problem

We present here the basic argument that will be used in the application to growth in the next sections. There are one good, I individuals (differentiated by their utilities or endowments) and two periods. In the second period, individual endowments are subject to uncertainty: they may assume different values, according to the state that is realized. Information is perfect. Individuals choose consumption plans maximizing a well-behaved Von Neumann-Morgenstern utility function. Agents do not discount the future.[3] Denoting individuals by a superscript, $i, i = 1, ..., I$, and time and states of nature (respectively, $t, t = 1, 2$ and $s, s = 1, ..., S$) by subscripts, the problem of individual i writes as follows

$$\max_{c_1^i, c_{2,1}^i, \ldots c_{2,S}^i} E(u^i) = u^i(c_1^i) + \sum_{s=1}^{S} \pi_s u^i(c_{2,s}^i), \tag{1}$$

where π_s is the probability of state s ($\sum_{s=1}^{S} \pi_s = 1$), and $\partial u^i(.)/\partial c^i > 0$, $\partial u^i(.)/\partial c^i < 0$ for all i.

In the first period, agent $i, i = 1, ..., I$, is born with endowment ω_1^i and receives by nature a second period endowment $\omega_{2,s}^i$ if state s realizes. Each agent can transfer first period endowments to period 2 via a storage technology, z^i. This storage technology is individual-specific and subject to uncertainty as well. The second period endowment for agent i in state s is given by $\lambda_s^i(\omega_{2,s}^i, z^i)$ if $\omega_{2,s}^i$ is received by nature and z^i is stored in the first life period. The set of constraints for the problem of individual i writes as follows:

$$c_1^i = \omega_1^i - z^i, \tag{2}$$

$$c_{2,s}^i = \lambda_s^i(\omega_{2,s}^i, z^i) \quad s = 1, ..., S, \tag{3}$$

$$0 \leq z^i \leq \omega_1^i. \tag{4}$$

The storage technology works as a non-marketable asset in transferring wealth across time. In such a framework we introduce J tradable inside assets and the individual constraints are modified in the following way:

[3] The presence of a discount rate does not play any role in our argument. It is therefore normalized to unity for the sake of saving notation.

$$c_1^i = \omega_1^i - z^i - \sum_{j=1}^{J} q_j y_j^i, \tag{5}$$

$$c_{2,s}^i = \lambda_s^i(\omega_{2,s}^i, z^i) + r^j y_j^i \quad s = 1, ..., S, \; j = 1, ..., J \tag{6}$$

The market-clearing conditions impose

$$\sum_{i=1}^{I} y_j^i = 0 \quad j = 1, ..., J. \tag{7}$$

When markets for a full set of Arrow securities are open $J = S$ and the asset markets are complete (7) must be satisfied for all agents $i = 1, ..., I$

$$\frac{\partial u^i}{\partial c_1^i} \left(\sum_{j=1}^{J} q_j \frac{\partial \lambda_s^i}{\partial z^i} - 1 \right) = 0, \tag{8}$$

$$-\frac{\partial u^i}{\partial c_1^i} q_j + \pi_s \frac{\partial u^i}{\partial c_{2,s}^i} = 0 \quad s = 1, ..., S. \tag{9}$$

The individual maximization problem requires $\sum_{j=1}^{J} q_j \frac{\partial \lambda_s^i}{\partial z^i} = 1, s \in S$ whenever agent i is willing to use both the storage z^i and some of the Arrow securities to smooth consumption. This means that at the optimum, each individual must find the storage technology z^i and some combination of the assets y_j^i to be perfect substitute in transferring resources for consumption to period 1. The (augmented) payoff matrix faced by agent i when choosing z^i appears as follows

$$R^i = \begin{bmatrix} -1 & -q_1 & -q_2 & . & -q_J \\ \frac{\partial \lambda_1^i}{\partial z^i} & 1 & 0 & . & 0 \\ \frac{\partial \lambda_2^i}{\partial z^i} & 0 & 1 & . & 0 \\ . & . & . & . & . \\ \frac{\partial \lambda_S^i}{\partial z^i} & 0 & 0 & . & 1 \end{bmatrix} \tag{10}$$

By (8), at the optimum the choice of z^i must satisfy $\sum_{j=1}^{J} q_j \frac{\partial \lambda_s^i}{\partial z^i} = 1$. Multiplying the column corresponding to asset y_j by $\frac{\partial \lambda_s^i}{\partial z^i}$, and summing up all the columns representing the payoff structure of financial assets, one obtain the column for z. The payoff structure of the storage technology is necessarily collinear to that of a combination of financial assets. At the individual level, the choice of z^i is therefore indeterminate. Since markets are complete, individual consumption allocations are instead determinate and Pareto optimal.

Consider now the case where $J < S$ financial assets in zero net supply are present, i.e. the case of incomplete assets market. It is well-known (e.g., Geanakoplos [3]) that in this setup individual allocations are generally no more independent of asset payoffs. Denoting by $r_{j,s}$ the payoff of asset j in state s, individual budget constraints rewrite as follows

$$c_1^i = \omega_1^i - z^i - \sum_{j=1}^{J} q_s y_j^i \tag{11}$$

$$c_{2,s}^i = \lambda_s^i(\omega_{2,s}^i, z^i) + \sum_{j=1}^{J} r_{j,s} y_j^i \quad s = 1, ..., S \tag{12}$$

If any agent i happens to be constrained in its choice of z^i by the requirement that $0 \leq z^i \leq \omega_1^i$, consumption allocations would not be Pareto efficient because of market incompleteness. Only allowing for a full set of Arrow securities all agents could be in the position to efficiently smooth their consumption. In this case, the incompleteness of the financial structure has real effects. We say then that markets are *effectively incomplete*. However, if *all* individuals maximize their utility by choosing a positive level of z compatible with their first period endowment, then the storage technology perfectly substitutes the missing tradable asset in transferring wealth over time. We say in this case that markets are *effectively complete*, because consumption allocations would be unchanged compared with the case where a full set of Arrow securities is available. In spite of this, the individual choice of z^i need not be indeterminate. Furthermore, even if asset returns are irrelevant for consumption allocations, they may matter at the individual level for the use of the storage z^i and the financial assets y_j^i as alternative tools for consumption smoothing.

The above considerations may have consequences in common economic problems characterized by uncertainty. In the following sections, we give an intuition of this through an application to a standard growth framework. In an overlapping generation economy, investments in the storage technology are interpreted as (risky) human capital investments. Allowing for an endogenous growth process generated by knowledge spillovers from human capital, growth increases as aggregate human capital investments increase. However, the pace of human capital accumulation is determinate only if *individual* investments in human capital are determinate. As a result, starting from a complete asset markets case, only eliminating one asset can restore economic policy.

3 Financial structure

3.1 The model

A continuum of agents of measure 1 live for two periods, 1 and 2, and at each time t, $t = 0, 1, ...$, a new generation is born. Population is constant. In the economy there is only one (non-storable) consumption good produced by a continuum of perfectly competitive firms using one production factor, H, human capital. Production, Y, is linear in H. Individuals' human capital differs in terms of its productivity. Output at time t also depends upon a time-dependent, common productivity term F_t.

When *young*, agents are born with one unit of human capital and one unit of available time. Agents may be either of type 1 or of type 2, according with the productivity of their human capital, δ^1 or δ^2.[4] Without loss of generality, we assume that $\delta^1 > \delta^2$ and we call, consistently, type-1 individuals "rich", and individuals of type 2 "poor". In each generation, a share n, $n \in (0,1)$, of agents is born of type 1, and a share $1-n$ is born of type 2. So, agents differ in their "genetic abilities", which are handed down, from generation to generation, along the same dynasty as, for instance, in Grandstein *et al.*.[6].

When *old*, individuals' human capital endowments depend upon how much they invested in education when young; *i.e.*, on the fraction of time they devoted to learning. Human capital investments, however, yield uncertain outcomes. Uncertainty in our model originates from the possibility that human capital investments may *fail on aggregate*. Each generation, at period t, faces some probability of receiving a type of education which ends up being unproductive. The reasons for this eventuality are many. Aggregate risk, for instance, may arise when individual human capital investments are complementary among themselves or with risky innovations.[5] Which is the cause for aggregate risk does matter for our results. We thus keep things simple by assuming that aggregate risk is derived from the presence of pure aggregate shocks.[6] If a shock occurs at time t, education is "unsuccessful" and does not affect the amount of human capital available to individuals when old, at time $t+1$. So, two states may be realized in each period, either state 1 or state 2, according as human capital investments are successful or not. The probability at which state 1 (resp. 2) occurs is known, constant across time, and equal to π (resp., $1-\pi$). When the investment in human capital fails (state 2), productivity differences across agents are also likely to be smoothed. We consistently assume that, in case of success (state 1), human capital endowments of old agents of type i at time t equal the fraction of time invested in education when young, s_{t-1}^i, and their productivity is given by δ^i. In case of failure (state 2), instead, human capital endowments are independent of the investment undertaken when young, equal across agents, and equally productive. We denote by δ the amount of output that can be produced out of this endowment.[7] Aggregate output at time t in

[4] As will be clear, this is the productivity of human capital in the first life period and in the second period only if education investments are successful.

[5] See also Acemoglu and Zilibotti (1999) for a framework of analysis where aggregate risk derives from risky investment projects requiring a mimimum size.

[6] There is evidence that aggregate uncertainty may matter for human capital accumulation. Jacoby and Skoufias (1997), for instance, show that unanticipated aggregate shocks (rainfalls) significantly affect children' s school attendance in rural India.

[7] The following results do not depend on the value of δ that can be chosen to be arbitrarily small. This justifies the interpretation that when state 2 occurs, human capital investment fails.

expected terms, Y_t^e, is therefore given by

$$
\begin{aligned}
Y_t^e &= F_t\left[n\delta^1(1-s_t^1)+(1-n)\delta^2(1-s_t^2)+\right. \\
&\left.+\pi(n\delta^1 s_{t-1}^1+(1-n)\delta^2 s_{t-1}^2)+(1-\pi)\delta\right].
\end{aligned}
\tag{13}
$$

Assume, for the moment, that the common productivity term F_t is increased between time t and time $t+1$ by a commonly known, exogenous factor $k_t > 1$, so that[8]

$$F_{t+1} = k_t F_t. \tag{14}$$

At the beginning of their youth, agents choose their consumption plans, facing an uncertain environment in period 2. In the following analysis, we use the subscripts j, t, s, to distinguish, respectively, the life period (age) of each agent $j = 1, 2$, the time period t, and the state $s = 1, 2$. We assume further that preferences are equal across agents and logarithmic in consumption.[9] The problem for an individual of type $i, i = 1, 2$ born at time t is the following

$$\max_{c_{1t}^i, c_{2t+1,1}^i, c_{2t+1,2}^i} \ln(c_{1t}^i) + \pi\ln(c_{2t+1,1}^i) + (1-\pi)\ln(c_{2t+1,2}^i) \qquad i = 1, 2 \tag{15}$$

subject to

$$c_{1t}^i = F_t\delta^i(1-s_t^i) \tag{16}$$

$$c_{2t+1,1}^i = k_t F_t \delta^i s_t^i \tag{17}$$

$$c_{2t+1,2}^i = k_t F_t \delta \tag{18}$$

$$0 \le s_t^i \le 1 \tag{19}$$

The individual problem collapses to that of choosing how much to invest in human capital, (choosing s_t^i), under constraint (19), namely, the requirement that this investment cannot be negative and cannot exceed the amount of available time (fixed to unity for all agents). In other terms, constraint (19) summarizes

[8] In a later section the growth factor k_t will become endogenous, determined by human capital investments.

[9] All the following results hold under a general CRRA representation for utility functions. The logarithmic specification is adopted for analytical convenience.

the features that distinguish human capital investments with respect to financial investments: human capital is non-marketable and its accumulation must satisfy individual-specific feasibility requirements.

The solution of problem (15)-(19) yields an equal fraction of time devoted in accumulating human capital for each agent in each period: $s_t^i = \frac{\pi}{1+\pi}, i = 1, 2$, for all t. Aggregate human capital in expected terms is thus constant over time, and aggregate expected output, as expressed in (13), grows at the rate $k_t - 1$ because of common productivity growth. Investing in human capital is in this case the only way to allocate consumption across time. Due to income heterogeneity, the introduction of asset markets would permit agents to better smooth their consumption. As it will become clear in the next sections, rich agents, when young, would gain from lending to poor agents.

3.2 Asset markets

3.2.1 Two assets case

Consider the introduction of a full set of Arrow securities: y_1 and y_2. Individual budget constraints rewrite in analogy with (5)-(6) and market-clearing imposes

$$ny_{1t}^1 + (1-n)y_{1t}^2 = 0 \tag{20}$$

$$ny_{2t}^1 + (1-n)y_{2t}^2 = 0. \tag{21}$$

The first order conditions for utility maximization with respect to s_t^i and y_{1t}^i are mutually compatible if and only if the following relations are satisfied together

$$q_{1t} = \frac{1}{k_t} \tag{22}$$

$$F_t\delta^i\left(1 - s_t^i\right) - q_{1t}y_{1t}^i - q_{2t}y_{2t}^i = \frac{k_tF_t\delta^i s_t^i + y_{1t}^i}{k_t\pi}. \tag{23}$$

No equilibrium with $q_{1t} \neq \frac{1}{k_t}$ is possible: both type of agents would be willing to exploit arbitrage opportunities between asset 1 and human capital. Summing-up across agents the first-order conditions with respect to y_{2t}^i and using market clearing ((20) and (21)), the price for asset 2 is obtained as

$$q_{2t} = \frac{(1-\pi)}{1+\pi}\frac{\overline{\delta}}{k_t\delta}, \tag{24}$$

where $\overline{\delta} \equiv n\delta^1 + (1-n)\delta^2$ is average individual-specific productivity. After substitution of (24) into first-order conditions, the following equilibrium relation between s_t^i and y_{1t}^i is derived

$$s_t^i = \frac{\pi}{2} + \frac{\pi(1-\pi)\overline{\delta}}{2\delta^i} - \frac{y_{1t}^i}{k_tF_t\delta^i}. \tag{25}$$

Human capital and asset y_1 are perfect substitutes from the individual viewpoint. Because $q_{1t} = \frac{1}{k_t}$ human capital and asset y_1 are equally efficient tools in transferring wealth onwards if state 1 realizes. Agent i, $i = 1, 2$, is thus indifferent between any combination between s_t^i and y_{1t}^i which is compatible with (25). It follows that the *individual* distribution between *human capital* and financial investments is *indeterminate* when a full set of Arrow securities is present.

The market for asset y_1 must clear, and this creates the following relation concerning human capital investments across agents

$$n\delta^1 s_t^1 + (1-n)\,\delta^2 s_t^2 = \overline{\delta}\frac{\pi}{1+\pi}, \tag{26}$$

which must hold together with the requirement that $0 \leq s_t^i \leq 1$, $i = 1, 2$.[10] Condition (26) fixes the value for aggregate output in expected terms (13). The same condition also ensures that aggregate output changes over time only because of changes in the common productivity term F_t, as in the case where financial assets are not available. However, because of the indeterminacy in individual human capital investments, any sequence $\left\{s_t^1, s_t^2 : 0 \leq s_t^i \leq 1,\ i = 1, 2,\ t \geq 0\right\}$ which is compatible with (26) may realize. This means that, while aggregate output is determinate, aggregate human capital investments are not. Condition (26) in fact does not fix any particular value for aggregate human capital investments, $ns_t^1 + (1-n)\,s^2$, because the value of both s_t^1 and s_t^2 is indeterminate at each time period t. Hence, in the presence of a full set of Arrow securities, *human capital investments are indeterminate* not only at the individual level, but also *in the aggregate.*

Individual consumption allocations are instead determinate and given by:

$$c_{1t}^{i*} = \frac{F_t}{2}\left[\delta^i + \frac{(1-\pi)}{1+\pi}\overline{\delta}\right] \quad i = 1, 2 \tag{27}$$

$$c_{2t+1,1}^{i*} = \frac{k_t F_t \pi}{2}\left[\delta^i + \frac{(1-\pi)}{1+\pi}\overline{\delta}\right] \quad i = 1, 2 \tag{28}$$

$$c_{2t+1,2}^{i*} = k_t F_t \delta\left[\frac{(1+\pi)\delta^i + (1-\pi)\overline{\delta}}{2\overline{\delta}}\right] \quad i = 1, 2 \tag{29}$$

Since markets are complete, allocations (27)-(29) are necessarily independent of asset payoffs.

3.2.2 One asset case

Consider now the case where only one marketable asset is available. This asset, y, pays off r_1 in state 1 and $r_2 \neq 0$ if state 2 is realized. Again, individual budget

[10] It is checked that when (26) is satisfied at most one type of agent can be constrained by the requirement that $0 \leq s_t^i \leq 1$.

constraints are modified in analogy with (11)-(12). The fact that one marketable asset is missing from a full set of Arrow securities matters for consumption allocations only if any agent is constrained in his (her) choice by the requirement that $0 \leq s_t^i \leq 1$. In this case, markets are effectively incomplete, and the market equilibrium lacks allocative efficiency. However, if agents happen not to be constrained in their maximization problem, markets are effectively complete even if one financial asset is missing. Human capital is in this case an efficient substitute for the missing tradable asset. Equilibrium consumption allocations are Pareto efficient, independent of asset payoffs, and expressed as in (27)-(29). What we want to remark is that in this case, as opposed with the case where two assets are present, *individual* human capital investments are determinate.

The interior solution for the individual problem is characterized as follows

$$q_t^* = \frac{1}{k_t} r_1 + \frac{\overline{\delta}\,(1-\pi)}{\delta \kappa_t\,(1+\pi)} r_2 \tag{30}$$

$$s_t^{1*} = \frac{\pi}{2} + \frac{\pi(1-\pi)\overline{\delta}}{2\delta^1(1+\pi)} - \frac{(1-n)\,(1+\pi)\left(\delta^1 - \delta^2\right)\delta}{2\delta^1\overline{\delta}} \frac{r_1}{r_2} \tag{31}$$

$$s_t^{2*} = \frac{\pi}{2} + \frac{\pi(1-\pi)\overline{\delta}}{2\delta^2(1+\pi)} + \frac{n\,(1+\pi)\left(\delta^1 - \delta^2\right)\delta}{2\delta^2\overline{\delta}} \frac{r_1}{r_2} \tag{32}$$

$$y_t^{1*} = \frac{\delta \kappa_t F_t (1-n)\left(\delta^1 - \delta^2\right)(1+\pi)}{2\overline{\delta} r_2} \tag{33}$$

$$y_t^{2*} = -\frac{\delta \kappa_t F_t n\left(\delta^1 - \delta^2\right)(1+\pi)}{2\overline{\delta} r_2}. \tag{34}$$

Some remarks concerning expressions (30)-(34) are in order. First, the price of the financial asset, and equilibrium values for s_t^i and y_t^i depend upon the returns on the financial asset. While the asset price and equilibrium values for trades in y depend upon growth rates $k_t - 1$, investments in human capital are independent of them, and constant over time. In this case, we can consistently drop the time index and write $s_t^{i*} = s^{i*}$, $i = 1, 2$. Second, rich agents always buy asset y, while the poor are sellers. Since in case of failure (state 2) human capital productivity and then income is made equal for both, the rich are relatively more averse to state 2 compared with the poor. They are therefore those that are willing to buy asset y, in that this is the unique means for transferring wealth to the second period of life in case state 2 realizes. Finally, it is to note that an interior solution for the individual problem of both type of agents (namely, $0 < s^{i*} < 1$, $i = 1, 2$) exists as long as r^1/r^2 and δ^1/δ^2 are not too high.

Positive values for r^1/r^2 exist such that the solution is in the interior whenever the "income gap" $\frac{\delta^1}{\delta^2}$ is lower than a given threshold, i.e., whenever

$$\frac{\delta^1}{\delta^2} < 1 + \frac{2}{\pi n (1-\pi)}, \tag{35}$$

an assumption that we take as satisfied henceforth.

3.3 Endogenous growth

Assume now that the common productivity term F_t is not anymore determined by an exogenous process. Imagine that, as is often assumed in the growth literature, productivity at time t depends upon the stock of accumulated knowledge. Accordingly, productivity increases over time in proportion with aggregate investments in human capital undertaken by the old generation. The growth rate of the economy at period t, g_t, is thus given by

$$g_t = \frac{F_{t+1} - F_t}{F_t} = s_t, \tag{36}$$

where

$$s_t = n s_t^1 + (1-n) s_t^2 \tag{37}$$

Human capital investments of young agents of generation t, on aggregate, produce an externality on the productivity realized at time $t+1$ (knowledge spillover).[11] Each young agent, though, being atomistic, does not take into account the influence of his human capital on the productivity that will be realized during his second period of life. The individual problem writes therefore as in the previous cases, but the term $k_t F_t$ is now to be replaced by F_{t+1}^e, where the superscript e denotes expectations. Productivity realized at time $t+1$ depends on aggregate human capital investments undertaken at period t. At a perfect foresight equilibrium, plans and conjectures are mutually compatible, so that $F_{t+1}^e = F_{t+1} = F_t(1+s_t)$.

When knowledge spillovers operate, the growth rate of the economy is driven by investments in human capital. The financial structure of the economy, when shaping individual choices between alternative forms of investment, also produces additional real effects through the growth rate. Consider the case where a full set of Arrow securities is available. Under this scenario, since human capital and financial assets are perfect substitutes, individual, and then aggregate investments in human capital are indeterminate. In such a case, the growth rate between each time period will also be indeterminate. Conversely, if only one asset is present, human capital investments need not be indeterminate. Consider

[11] It is commonly argued in modern growth theory that human capital accumulation may produce externalities that matter for growth (see, for instance, Lucas (1988)).

the case where agents are not constrained by the requirement that $0 \leq s^{i*} \leq 1$. In such a case, the expression for total human capital investments is obtained as[12]

$$s^* = \frac{\pi}{2} + \frac{\pi(1-\pi)\overline{\delta}\widetilde{\delta}}{2\delta^1\delta^2(1+\pi)} + \frac{n(1-n)\,\delta\,(1+\pi)\left(\delta^1-\delta^2\right)^2}{2\delta^1\delta^2\overline{\delta}}\frac{r_1}{r_2}, \tag{38}$$

where $\widetilde{\delta} \equiv n\delta^2 + (1-n)\delta^2$. Aggregate human capital investments are affected by asset returns. Notice that what matters for human capital accumulation is only the payoff ratio, r^1/r^2, which gives a measure of the relative efficiency of the asset in transferring wealth in the two states. Consistently, henceforth we normalize $r_2 = 1$, and set $r \equiv r_1$. By inspection of expressions (31) and (32), one checks that, as r increases, rich (poor) individuals use less (more) human capital in transferring wealth to the second period. Human capital investments react in opposite way to r according as agents happen to be short or long of asset y. The higher is r, the higher is the amount of output that poor agents have to pay back to the rich in period 2, when state 1 realizes. This brings about a lower supply of y on the part of the poor, a higher asset price q, and a more intense use of human capital as a tool to redistribute wealth from the first to the second life period. As for the rich, their behavior is symmetric: when r increases, their demand for y is raised, and their use of human capital is reduced, because it has become a less efficient asset in comparison with y. So, as r becomes higher, the use of human capital increases at an aggregate level because *agents become more specialized in their use of alternative assets.* The higher r, the higher the relative use of human capital on the part of the poor. Since poor agents always invest a higher fraction of available time in accumulating human capital compared with the rich, aggregate human capital investments increase.

From inspection of expressions (27)-(29) for individual consumptions, after substituting k_t with $1+s^*$, it is immediate that asset payoffs affect consumption allocations through the growth rate. Once a process of endogenous growth is introduced through knowledge spillovers, consumption allocations are no more independent of asset returns even when agents are not constrained by the requirement that $0 \leq s_t^i \leq 1$, $i = 1, 2$. The introduction of an externality generated by human capital poses a general issue of optimality of the asset structure.

4 Efficiency

Assume that the asset structure is under the control of economic policy. Consider a planner that takes into account the presence of the externality and that acts only by choosing the number of financial asset markets to be opened and the asset returns.[13] How a Pareto efficient asset structure would be charaterized

[12] The economy is constantly on a determinate balanced growth path (no transition dynamics), so that the time index can be omitted.

[13] So, we abstract from lump-sum redistribution.

for this economy? We showed in the previous section that the asset structure may affect both the allocation of individual consumption across their life periods (*allocative efficiency*) and the growth rate of the economy (*growth efficiency*). In this section, we characterize the asset structure that maximizes growth without compromising allocative efficiency. We also show that there exist cases in which growth efficiency may contrast with allocative efficiency. Whenever this trade off is present, it is optimally solved by the optimal asset structure.

Take the case where a full set of Arrow securities is available. By the argument presented in section 3.2.1. when all financial assets are present, the growth rate is necessarily indeterminate. The presence of a full set of Arrow securities therefore guarantees allocative efficiency at the expense of the control over growth efficiency.

Consider one financial asset only. Recall that, as long as all individuals are not constrained by the feasibility requirement $0 < s^{i*} < 1$, $i = 1, 2$, allocative efficiency is preserved: markets are effectively complete. The growth rate depend on aggregate human capital investments which in turn are affected by asset payoffs as illustrated in (38). As a result, as long as asset returns are compatible with individual human capital investments not being constrained by (19), higher values for r entail Pareto improvements (this is checked from (38)). Growth efficiency in this case does not contrast with allocative efficiency. As r increases, rich (poor) agents reduce (increase) their investment in human capital, until one type of agent become constrained at either $s_t^1 = 0$ or $s_t^2 = 1$. Denote by r^1 and r^2 the highest payoff ratio compatible with, respectively, s_t^1 and s_t^1 not being constrained by (19), i.e.: $r^1 \equiv \max\left[r : s^{1*} \geq 0\right]$, $r^2 \equiv \max\left[r : s^{2*} \leq 1\right]$. Denote by r^c the asset return ratio prevailing at a candidate Pareto optimum. It follows from the argument sketched above that $r^c \geq \min\left[r^1, r^2\right]$; the reverse would mean that the asset structure is Pareto dominated by either r^1 or r^2.

If $r^1 \leq r^2$, then at an optimal asset structure rich agents would make investments in human capital that are null, but would finance abundant use of human capital on the part of the poor. At the opposite, if $r^1 \geq r^2$, poor agents at the optimum would spend all their youth in accumulating human capital, and rich agents would make a "small", but positive investment in human capital.[14] The values for r^1 and r^2 are obtained, respectively, by setting $s^{1*} = 0$ and $s^{2*} = 1$ from (31) and (32)

$$r^1 = \frac{\overline{\delta}\pi\left[\delta^1(1+\pi) + \overline{\delta}(1-\pi)\right]}{\delta(\delta^1 - \delta^2)(1-n)(1+\pi)^2} \tag{39}$$

$$r^2 = \frac{\overline{\delta}\left[2\delta^2 + \pi\left(1-\pi\right)\left(\delta^2 - \overline{\delta}\right)\right]}{\delta(\delta^1 - \delta^2)n((1+\pi)^2} \tag{40}$$

[14]It is a matter of simple algebra to show that $s^1 > 0$ when poor agents are constrained at $s^2 = 1$.

Optimal asset returns depend on the fundamentals of the economy. It is immediate that

$$r^1 \gtreqless r^2 \Leftrightarrow \frac{\delta^1}{\delta^2} \gtreqless \frac{(1-n)}{n\pi}. \qquad (41)$$

If the economy is characterized by a sufficiently small income-gap δ^1/δ^2, at the optimum rich agents would not invest in human capital, while, if the income-gap is large enough, poor agents would make the maximum investment in human capital. We then call the first type of economy a "relatively equal" economy, and the second type a "relatively unequal" economy.

So, an efficient asset structure requires the payoff ratio to be at least equal to $\min\left[r^1, r^2\right]$. This is the value at which growth efficiency is maximized without contrasting with allocative efficiency. Will it be higher? If yes, at the prevailing asset payoffs, some agents will find themselves constrained by the requirement $0 \leq s_t^i \leq 1$, and any possible gain in terms of growth comes at the expense of allocative efficiency: markets become effectively incomplete. This eventuality cannot be excluded. When knowledge spillovers are present, individuals agents always underinvest in human capital. If setting $r > \min\left[r^1, r^2\right]$ leads to higher aggregate human capital investments, faster growth may compensate for the allocative inefficiency and yield Pareto superior allocations.[15] We can summarize the above findings in the following proposition.

Proposition 1 *At an optimal asset structure, if the economy is relatively equal* ($\frac{\delta^1}{\delta^2} < \frac{(1-n)}{n\pi}$)*, then* $r \geq r^1$*, and type 1 (rich) agents invest* $s^1 = 0$*; if the economy is relatively unequal* ($\frac{\delta^1}{\delta^2} > \frac{(1-n)}{n\pi}$)*, then* $r \geq r^2$ *and type 2 (poor) agents invest* $s^2 = 1$.

The set of asset structures that are Pareto dominated depends on the fundamentals of the economy, and in particular on the extent of income inequality. In general, undominated asset structures must pay sufficiently high returns in the state where human capital investments are effective. Only if this is the case, agents with different income becomes sufficiently specialized in the use of alternative means for transferring wealth to the future (human capital or the financial asset). Human capital investments, on aggregate, are more abundant when the extent of specialization is higher.

The optimal asset structure may require that some agents are constrained in their human capital investment decisions. Depending on the distribution of income, either the poor may be willing to devote to education more than the available time, or the rich may be willing to make a negative investment in human capital. In other words, at the optimum, feasibility requirements peculiar to human capital investments might be binding, and markets could be

[15] Few can be said concerning aggregate investments in human capital when $r > \min\left[r^1, r^2\right]$. Unreported simulations show that, depending on parameters values, investments for the unconstrained agents may increase or decrease compared with the case where $r = \min\left[r^1, r^2\right]$. Furthermore, since investments in human capital depend in this case upon expected growth rates, multiple equilibria may arise.

effectively incomplete. Of course, in such cases, the optimal asset structure must compensate the loss of allocative efficiency associated with missing markets with a sufficiently strong growth.

Table 1. Simulated values for investment and utility levels

r	s	$E(u^1)$	$E(u^2)$
$\min\left[r^1, r^2\right] = r^1 = 25.6$	.4833	-1.857	-2.807
26	.4837	-1.856	-2.806
30	.4851	-1.847	-2.808
35	.4879	-1.842	-2.81
40	.4886	-1.840	-2.812
45	.4891	-1.839	-2.813
50	.4893	-1.839	-2.814
55	.4893	-1.84	-2.816

$\delta^1 = 1.9$, δ^2=1, δ=.1, π=.5, n=.5.

As the numerical example in Table 1 shows, there may exist values for r above $\min\left[r^1, r^2\right]$ where the expected welfare of both type of agents is higher compared with the case of effectively complete markets (where $r \leq \min\left[r^1, r^2\right]$).[16] This is a typical illustration of the second best principle. Sub optimality is driven there by two sources: the lack of complete asset markets and the externality associated with human capital investments. Solving the allocative inefficiency by setting $r \leq \min\left[r^1, r^2\right]$ and allowing markets to be effectively complete might increase the extent of underinvestment, leading to Pareto dominated outcomes.[17]

5 Discussion and policy implications

We have illustrated a rational for Pareto improving policy actions. Policy makers, however, hardly possess all the relevant information to implement an optimal asset structure which takes into account criteria of both allocative and growth efficiency. Furthermore, in real world situations, state-contingent asset payoffs are hardly implementable or enforceable by economic policy. Asset trades may nonetheless be affected by alternative policy tools, which alter the outcome of economic decisions taken by heterogenous agents. We discuss in the following the role of fiscal policy. We show how public intergenerational redistribution may interact with the financial structure in shaping the incentives to accumulate human capital through the same channel that has been highlighted in the previous section.

[16] The optimal asset structure for the example in Table 1 would assign a value of r close to 26, which is slightly above $\min\left[r^1, r^2\right] = 25.6$.

[17] Of course, the presence of an externality may be viewed in the Coasian tradition as a further case of missing markets. In this sense, the appropriate definition of effectively complete markets should take into account the lack of appropriate markets aimed at dealing with the externality.

Denote by τ_t a lump-sum transfer that at time t is levied on each of the agents belonging to the young generation, and distributed to each of those belonging to the old generation. At each time period, the government budget is necessarily balanced. The transfer scheme evolves according with the state of productivity, so that $\tau_t = F_t\tau$.

If we maintain that the feasibility requirement $0 \leq s_t^i \leq 1$ is satisfied when only one financial asset is available, agents dispose of all the needed tools to efficiently smooth consumption, and markets are effectively complete. The budget constraints rewrite as

$$c_{1t}^i = F_t\delta^i(1 - s_t^i) - F_t\tau - q_t y_t^i \quad i = 1,2 \tag{42}$$

$$c_{2t+1,1}^i = kF_t\delta^i s_t^i + kF_t\tau + ry_t^i \quad i = 1,2 \tag{43}$$

$$c_{2t+1,2}^i = kF_t\delta + kF_t\tau + y_t^i \quad i = 1,2. \tag{44}$$

The lump-sum transfer has different effects on the use human capital and the financial asset y. As for s, its use is reduced for all agents, at given trades in y. The reason is clear. The presence of τ transfers wealth onwards and to state 1, acting this way as a substitute for human capital. How the exchange of y is affected by τ, at given s, is less clear. The fiscal transfer increases the willingness to sell y and reduces that of buying it. The price of y is consistently reduced to $\widehat{q} = \frac{r}{k} + \frac{\overline{\delta}(1-\pi)}{(\delta+\tau)k_t(1+\pi)}$. As for asset trades, direct computations show that they are unambiguously increased to $\widehat{y_t^i} = \frac{\delta+\tau}{\delta}y_t^{i*}$, where the values of y_t^{i*} are given in (33) and (34). The increase in the supply of y more than compensates the reduction in its demand. Since the fiscal transfer is the same for all agents, its presence has a relatively stronger impact on poor agents, who are those that sell the asset. But this also means that the poor have more finance for their human capital investments, and the rich, correspondingly less. Namely, a more intense use of the financial asset increases the extent of specialization of rich and poor in the use of y and s.

The transfer scheme improves upon consumption allocations at given growth rate $k-1$. Under the requirement that $0 \leq s_t^i \leq 1$ markets are effectively complete and allocative efficiency is preserved. Furthermore, the presence of the transfer τ improves *intertemporal* efficiency. The total amount of resources available for consumption across life periods rises. Consumption allocations are easily found by direct computations. As for the first life period and state 1 of the second period, agents are in the position to replicate the consumption pattern described in (27) and (28). In state 2 of the second period, instead, consumption is higher (δ in (29) is replaced by $\delta+\tau$). Agents offset the reallocation of wealth between period 1 and state 1 in period 2 operated by τ through the use of human capital. The fiscal transfer, though, has the same effect as that of raising δ in state 2. This can be understood looking at the budget constraints in (42)-(44).

When the growth rate of the economy is determined by human capital spillovers, the presence of a system of intergenerational redistribution has the further effect of changing consumption allocations through human capital accumulation. The effect of τ on aggregate human capital investments is shaped by two contrasting forces. On the one hand, investments in human capital are reduced for all agents because of the direct "substitution" effect: the fiscal transfer is a direct substitute to human capital in transferring wealth onwards. On the other, the presence of τ enhances the use of y, raising aggregate human capital investments through the indirect "specialization" effect. The interplay of these contrasting forces are visible in the expression of equilibrium human capital investments, derived as

$$\widehat{s} = s^{*} + \frac{n\left(1-n\right)\left(1+\pi\right)\left(\delta^{1}-\delta^{2}\right)^{2} r - 2\overline{\delta}}{2\delta^{1}\delta^{2}\overline{\delta}}\tau, \tag{45}$$

where s^{*} is given in (38). The higher the extent of inequality (the higher is the difference $\delta^{1}-\delta^{2}$) and the more efficient is the asset in transferring wealth to state 1 (the higher r), the stronger is the specialization effect, and the more likely the eventuality that the transfer scheme raises human capital investments. Public transfers might both improve intertemporal efficiency and speed up growth.

The above example illustrates how fiscal policy and intergenerational transfers may interact with the asset structure in shaping the incentives to accumulation through a channel that so far has probably received insufficient attention. Pension systems or other forms of public intergenerational transfers affect individual behaviour both via a direct transfer of wealth across life periods and via indirect effects induced by a changed perception of risk on the part of heterogenous agents. Depending on the asset structure, and other fundamentals of the economy, the indirect effects of fiscal policy that acts through the financial market might offset those that directly affect individual intertemporal decisions.

We conclude by stressing the role of income inequality in our argument. In our framework, trade in the financial asset emerges only when agents differ in their income streams. Moreover, as long as inequality is not too big (*i.e.* as long as condition (35) is met) a more unequal context translates into a higher need of trading in the financial asset, and then into a greater extent of human capital accumulation. It is the specialization of rich and poor agents in the use of different assets that brings about this result. Our argument has implications for the debate concerning the relation between inequality and growth. So far, the link between financial markets and human capital accumulation has been generally described as working through the existence of non-convexities (see, e.g., Galor and Tsiddon[4], Perotti[10]). Investments require a minimum size, so that the growth rate is higher the higher the fraction of individuals that are above a wealth threshold during their youth. Financial markets relax the budget constraints of the poor, thus promoting growth. Following this argument, in many of the existing analysis, the relation between growth and income distribution turns out to be non-linear, in accordance with the Kuznets hypothesis.

Our story suggests that income distribution might affect growth through an unexplored channel. The existence of financial markets creates different incentives to accumulate human capital across agents. The more so, the more agents differ in their income holdings. Letting to invest more in their education those who judge human capital to be the best investment alternative leads to higher human capital accumulation on aggregate.

6 Conclusions

In this paper we raise a point concerning the normative consequences of the interaction between human capital and financial assets as alternative tools to transfer wealth in a risky environment. In our example, when a full set of Arrow securities is present, individual, and then aggregate investments in human capital are indeterminate. When one financial asset is missing, yet, the investment in human capital is determinate and is affected by asset payoffs. Allowing for externalities generated by aggregate human capital investment, growth rates are necessarily indeterminate when a full set of Arrow securities is present, while it is determinate if one financial asset is missing. The asset payoffs in this second case affects the rate of growth, and then consumption allocations. Optimality of the asset structure arises as a natural issue. Policy action requires the absence of one financial assets from a full set of Arrow securities: only in this case the growth rate can be under control. Optimal asset payoffs could then be structured in such a way that a group of agents is constrained in their human capital investment decisions. Relatively poor agents, at the optimum, should specialize in human capital investments, rich agents would finance human capital of the poor through investments in the financial asset. Depending on the initial distribution of income, either the poor may be willing to invest more than feasible in human capital, or the rich may be willing to make a negative investment. Of course, this contrast with the result that, in absence of externalities, the unconstrained equilibrium is Pareto superior to cases where agents end up being constrained by the feasibility requirements that are peculiar to human capital investments.

This paper may contribute to fill some gaps existing in two unrelated fields of literature. On the one hand, it gives an illustration of the possible applied relevance of the issue concerning the optimality of the asset structure. In our opinion, this issue has been so far too much confined abstract analysis (see, e.g., Demange and Laroque[**?**]). On the other, it poses some new issues in the analysis of the relation between financial markets and growth. There is substantial agreement that few work has been done so far trying to relate the characteristics of the asset structure to the growth performance of countries (Levine[8]).

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