CSEF
CENTRO STUDI IN ECONOMIA E FINANZA
CENTRE FOR STUDIES IN ECONOMICS AND FINANCE

WORKING PAPER NO. 47

Credibility and Commitment of Monetary Policy in Open Economies

Marcello D'Amato and Riccardo Martina

September 2000

DIPARTIMENTO DI SCIENZE ECONOMICHE - UNIVERSITÀ DEGLI STUDI DI SALERNO
Via Ponte Don Melillo - 84084 FISCIANO (SA)
Tel. 089-96 3167/3168 - Fax 089-96 3169 – e-mail: csef@unisa.it

CSEF WORKING PAPER NO. 47

Credibility and Commitment of Monetary Policy in Open Economies

Marcello D'Amato* **and Riccardo Martina****

Abstract

In this paper we study the delegation of monetary policy to independent central bankers in a two country world with monetary spill-overs. The paper shows that under imperfect commitment and private information of the Central Bankers about their objectives the optimal degree of commitment depends on the correlation structure of the shock hitting the economies. When the correlation of the shocks across countries is negative, as when the variance to output depends mainly on shocks to the terms of trade, there exist strategic complementarity in the optimal degree of commitment. When the correlation of shocks is positive (common technological or demand shocks) there exist strategic substitutability. These result may provide rationale for the simultaneous increasing attention to the institutional solution to the credibility problem in monetary policy in most advanced countries in the last decades.

Keywords: monetary policy delegation, central bankers, private information, strategic interaction.

JEL Classification: E58, D82, F42.

Acknowledgements: Thanks are due to Sandro Brusco, Marco Celentani and Vincenzo Galasso for useful comments.We also thank partecipants at the 1999 annual meeting of the EEA in Santiago de Compostela. The usual disclaimers apply.

* CSEF, Università di Salerno.

** Università di Napoli Federico II.

Table of contents

1 Introduction

One of the likely consequences of the foundation of EMU among several European countries will be the increasing importance of strategic interdependence in monetary policies pursued by large actors like United States, European Union and Japan. Starting from the contribution by Hamada (1974), different authors (see, for example, Canzoneri and Henderson, 1988, 1991, Rogoff, 1985b) have analyzed strategic interdependence in monetary policymaking in open economies.

In the presence of monetary policy interdependence across countries, the optimal design of the institution in charge for stabilization policies may have different characteristics than in the case of closed economy as analyzed, for example, in Rogoff (1985a) in Persson and Tabellini (1993), Walsh (1994). Persson and Tabellini (1995), for example, extend to interdependent open economies the analysis of optimal design of monetary institutions relying on the contractual approach. They analyze a two country political-monetary policy game and show that a cooperative outcome in the stabilization game can be implemented by a decentralized contracting scheme such that the central banker' compensation in each country is made contingent on the stabilization policy in both countries. The feasibility of such contracts is largely discussed by the authors although not definitely established (see Persson and Tabellini, 1995, p.44-45).

An alternative route, suboptimal when compared to the contracting approach and to the inflation targeting approach (Svensson, 1997), but much less demanding in terms of the contracting structure, originally pursued by Rogoff (1985a) and extended by many others to different contexts is to look at the relationship between "society" and the "central banker" as a relationship where the type of the latter agent is a commitment device in the hands of the former. The main issue in extending the analysis of the institutions adopted to solve the problem of credibility of monetary policy to the case of open economies is related to the effects of openness as a discipline device on the Cb's behavior, (Rogoff 1985b). The analysis of the relationship between openness and inflation is analyzed in Romer (1993) in a straightforward extension of the model by Barro and Gordon (1983) to the case of open economies and tested on a cross country of data, over a period between 1973 and 1988. In his paper Romer considers a model for an open economy in the presence of dynamic inconsistency of optimal monetary policy, showing that increased openness reduces equilibrium inflation. Although testing this prediction of the credibility approach turns out to be robust across different sub-sample of the data set, no evidence is found that the prediction holds for the most developed countries. This result is interpreted by Romer

as evidence that "the most highly developed countries may have found some means of overcoming the problem of dynamic inconsistency of optimal monetary policy". Though the empirical evidence and the interpretation provided in Romer (1993) are rather convincing [1], some questions on the ability of the credibility approach to the analysis of monetary institutions in interdependent open economies are left without explanation. Why some countries seem to have relied on the institutional solution whereas other countries do not?

If the credibility approach predicts that, coeteris paribus, the degree of openness in a given economy is a substitute and possibly an alternative mechanism compared to institutional reforms to achieve monetary discipline, then why is it the case that in the last thirty years an increasing degree of openness among the most developed countries has been accompanied by an increasing attention to the institutional solution to the credibility problem? More generally, how the strategic interrelationship at the monetary policy stage shapes the design of monetary institutions?

The purpose of this paper is to tackle these questions in the framework of the precommitment approach (Rogoff, 1985a) to the credibility problem in open economies with strategic interdependence at the monetary policy stage.Within this more specific framework the questions raised above can be qualified: since it is possible to show that in the standard precommitment model the prediction is that a larger degree of openness is a substitute for commitment of monetary policy to an independent central bank because of the self built in check mechanism analyzed in Rogoff (1985b) and since the stylized facts point in opposite directions (a larger degree of openness has been accompanied in the last twenty years by increasing attention to the institutional solution), should we dismiss a credibility argument as the basis for Cb's independence? In other words: can we use a credibility argument to explain the observation of simultaneous increasing attention to the problem of Cb's independence and implementation of institutional reforms in the most advanced countries in the last decades? Can we use the credibility approach to justify why the institutional solution failed to emerge in many countries?

In the case of open economies the increasing degree of openness is not a good candidate to explain why the institutional solution has emerged in recent years in most developed countries and we need to look in different directions[2]. We take the view that the strategic interdependence at the insti-

[1] For a different interpretation see Terra (1998) and the reply by Romer (1998).

[2] The incresed variability of inflation in the western economies after the oil shocks may be also considered as one of the facts that has triggered institutional reforms in those countries. However, if the oil shock is considered as a measure for an increased variability of the economic environment in western economies and since the cost of achieving credibility is larger the larger the variance of the shock to output, the standard model predicts

tutional design level may be a good candidate for trying to provide a possible answer to the questions above within the credibility framework. The nature of strategic interdependence and the incentives to precommit monetary policy in open economies, may depend on more fundamental determinants like the nature and the synchronization of the shocks hitting the economies, i.e. the likelihood that different economies will be in similar states (booms or slumps), on the variability of the environment surrounding and possibly trying to force, to the extent allowed by the commitment technology, the action of the Central banker (Cb henceforth).

To analyze these issues we consider a simple model of two economies where stabilization policies are interdependent. We adopt the credibility framework but assume a less than perfect commitment technology at the institutional design stage. This elements will allow us to study how the incentives to delegate monetary policies to an independent Cb are modified in the case of open economies. The issue of a less then perfect commitment technology in the case of delegating monetary policy to a Cb has been previously studied in the literature (see Cukierman, 1992). Day to day monetary policymaking is subject to strong pressures of different nature: academic researchers may influence the view about the costs and benefits of stabilization of the shocks in different directions. Unions may push for stabilization of employment shocks. Firms and institutions in the financial sector may want to push for stabilizing shocks to the price of stocks. Governments may want to push for a more or less activist policy according to the state of the economy because of electoral reasons. Exporting industries may want to push for a depreciation of the real exchange rate to stabilizing an adverse shock to the world relative demand for the outside good. These all factors make the commitment of the monetary policy an imperfect one since the perceived cost - benefit trade off of a given stabilization policy will depend on contingencies that cannot be contractible upon at the delegation stage and, moreover, the amount of pressures is likely to be contingent on the state of the economy.

We also take the view that the nature and the amount of pressures that shapes Cbs objective at the time when their action is called for are not

that the optimal degree of commitment would be decreasing in it. The implication would be that the ultimate source for understanding the attempt to solve the credibility problem by committing agencies in charge of monetary policies in most advanced countries is a simultaneus shift in the society to higher preferences for price stability. The distribution of benefits and costs of inflation within advanced western societies may indeed have evolved towards a larger weight given to inflation and therefore can be considered a significant explanation for the institutional reforms. However important this explanation for institutional reforms in western countries may be, we still think that the exploration of other explanations based on the idea of precommitment is also worth pursuing .

independent across countries. The weight that a given "society", at a given point in time and for a given institutional arrangement, is willing to assign to inflation relative to output stabilization is likely to be related to the way in which the benefits and the costs of surprise inflation affect different agents in the society at that time. Increasing homogeneity in social structure and institutions across countries entails increasing similarity in the environment surrounding the action of the Cb. One important example we will emphasize throughout the discussion is the influence that academic scholars may have on policymaking about the convenience and the feasibility of the stabilization policies. After II World War different academic views of the benefits and cost of (surprise) inflation have influenced monetary policy making through the analysis of marginal benefits and cost of stabilization policies influencing the perception of the relevant trade off when the Cbs' action has been called for. These academic views, since based on a common scientific ground, have been correlated across countries, (consider different attitudes towards activist policies and fine tuning during the 60s,70s and 80s).

The last feature that we will try to capture in our model is that these pressures and attempts to influence the policy reply to unexpected shocks in each country are private information to the Cbs at the time when the policy is implemented. Therefore we also assume that a less than perfect commitment technology involves a certain degree of opaqueness in the objectives of the agency in charge for the stabilization policy so that private information is an important ingredient for describing the behavior and the strategies of the Cb. By introducing this hypothesis we take the view that the impact of the institutional and political environment surrounding the action of the Cb in each country is not perfectly transparent when looked at from the point of view of the external observer.

The analysis of strategic issues in the institutional approach to delegation of monetary policy in the context of open economies is not completely new. Dolado, Griffith and Padilla (1994) analyzed a similar commitment problem. Our model, however, differs from their analysis under many respects: they model the decision of the governments at the institutional design stage as being taken *after* the observation of a (common) shock hitting the economy, whereas, following Rogoff (1985a), we model the government's decision so that the institutional arrangement has to be taken ex ante, i.e. *before* the shocks are observed. They also assume perfect commitment and myopic private agents and as a consequence the credibility issue is discarded in their analysis. Differently from Dolado et al. (1994), and consistently with the questions raised above, we set our analysis using a model for open economies where the wage-price setting private agents have rational expectations, thus leading to a time consistency problem in policymaking.

Our work is also related to Eijfinger et al. (1997) where they study the optimal degree of central banking conservativeness extending the Rogoff (1985a) analysis to the case of an open economy. Differently from our work, however, their focus is on the case of a small open economy in the absence of strategic interdependence across countries, both in the stabilization, and in the delegation stage of the monetary policy game.

The paper is organized as follows: in section 2 we briefly describe the economic model, the payoff functions of the agents and the timing of the game. In section 3 we solve the commitment game under the hypothesis of common knowledge of the preference functions of the players. In section 4 we solve the commitment game by introducing private information of the policymakers on their preferences. Section 5 concludes.

2 The macroeconomic environment

The macroeconomic model used to describe the open economy environment faced by the policymakers in their decision problem draws largely on the one proposed by Persson and Tabellini (1996) which, in turn, is a simplified version of the workhorse sticky price open economy model introduced by Rogoff (1985b) and thoroughly described by Canzoneri and Henderson (1988, 1991)[3]. There are two countries, the domestic country, whose economic variables and institutional parameters are indicated with subscripts i, and the foreign country, whose variables are indicated with subscripts j, each producing one good. Monetary policy is subject to a credibility problem but is also effective in stabilizing the economy because of wage-price stickiness in the short run. Apart from the political environment, to be described later on, the two countries are perfectly symmetric. The building blocks of the model are the following ones (all of the variables are measured in rate of changes).

The real exchange rate is given by

$$z = s + q_j - q_i \tag{1}$$

where s denotes the nominal exchange rate, q_i and q_j the producer price inflation in the domestic and foreign country respectively. The consumer

[3] Obstfeld and Rogoff (1995) laid the microfoundations for a full fledged dynamic model of two countries, where the exchange rate and current account are allowed for much richer dynamics than in the model used here. Starting from this contribution further developments have been provided in the literature on macroeconomic interdependence. See, for example, Corsetti and Pesenti (1999) and Benigno (1998). However, all of these papers, mainly due to technical difficulties, share the limitation that agents' expectations are assumed to be exogenous, leaving aside any issue related to the credibility problem which is the main focus of the present paper.

price inflation in country i is given by

$$p_i = q_i + \beta z \tag{2}$$

where β represents the share in the consumer basket of the good produced abroad. Monetary policy is neutral in the long run and the following version of the quantity theory holds: $q_i = m_i$ and $q_j = m_j$.

Output growth is determined by the following expectations augmented Phillips-curve

$$x_i = \gamma(m_i - m_i^e) + \varepsilon_i \tag{3}$$

where the superscripts m_i and m_j indicate the money supply to be chosen by Cbs after the shock hits the economy, m^e indicates expectations that private agents hold about the money supply process, $\gamma > 0$ is the output elasticity to unexpected money supply and finally, ε_i and ε_j represent the shocks hitting the economy, distributed according to the density function $f(\varepsilon_i, \varepsilon_j)$ defined on the support $S = [-\overline{\varepsilon}, \overline{\varepsilon}] \times [-\overline{\varepsilon}, \overline{\varepsilon}] \subset R^2$. Moreover, $E(\varepsilon_i) = E(\varepsilon_j) = 0$, $E(\varepsilon_i^2) = E(\varepsilon_j^2) = \sigma_\epsilon^2$, $E(\varepsilon_i \varepsilon_j) = Cov(\varepsilon_i, \varepsilon_j) = \rho_\varepsilon \sigma_\varepsilon^2$. Therefore we allow output shocks to be correlated across countries, i.e. we consider reduced form shocks to economic activity arising both from demand shocks (different from unexpected money supply) and supply shocks. Being reduced form shocks the sign of the correlation is left undetermined, depending on the prevailing fundamental source of variability in the output process in the two countries. For example if the main source of variability is due to common level of demand or technological shocks the correlation is likely to be positive, if the main source of variability is due to shocks to the terms of trade, then the correlation is likely to be negative.

The equilibrium condition in the goods market leads to the following equilibrium level for the real exchange rate

$$z = \delta(x_i - x_j) \tag{4}$$

where $\delta > 0$ is the inverse relative demand elasticity of outside good.

The reduced form equilibrium level of the real exchange rate and of the CPI inflation rate are

$$z = \delta\gamma[m_i - m_i^e - (m_j - m_j^e)] + \delta(\varepsilon_i - \varepsilon_j) \tag{5}$$

$$p_i(m_i, m_j) = m_i + \beta\delta\gamma[m_i - m_i^e - (m_j - m_j^e)] + \beta\delta(\varepsilon_i - \varepsilon_j) \tag{6}$$

The real exchange rate works as a transmission mechanism of supply shocks (and unexpected money supply change) from one country to the other,

leading to economic and strategic interdependence in monetary policymaking.

>From equation (6) the basic economics of monetary spillover across countries can be easily described. Let us consider, for example, a positive shock hitting the domestic country. It triggers a real exchange rate depreciation required by the clearing of the trade balance, this will increase the domestic CPI inflation rate and will reduce the foreign one. The same transmission mechanism leads to spillovers across monetary policy variables: an (unexpected) increase of money supply in the domestic country will determine an increase of the same percentage in the domestic producer price inflation rate, a larger effect on the domestic CPI inflation, because of depreciation of the domestic currency, and a reduction of the foreign CPI inflation rate. In other words any spillover works through the terms of trade effect on foreign CPI inflation and will drive the results of the analysis below.

2.1 Preferences of the players and the timing of the game

We move, now, to the description of the preferences of the policymakers and the timing of the commitment game. The preferences of the Government and the Central Banker in country i are the following ones:

$$E(W_i^G) = \int_\Omega \{-\frac{1}{2}[p_i(\varepsilon_i,\varepsilon_j)]^2 + \psi_i^G(1-\overline{k}_i\epsilon_i)x_i(\varepsilon_i,\varepsilon_j)\}dG(\varepsilon_i,\varepsilon_j,k_i,k_j) \tag{7}$$

$$W_i = -\frac{1}{2}[p_i(m_i,m_j)]^2 + \psi_i(1-k_i\varepsilon_i)x_i(m_i) \tag{8}$$

Where $\psi_i^G > 0$, $\psi_i > 0$, $k_i > 0$, $\overline{k}_i > 0$, will be described in the following. $G(\varepsilon_i,\varepsilon_j,k_i,k_j)$ is the joint distribution function for the random variables ε_i, ε_j, k_i, k_j, with density $g(\varepsilon_i,\varepsilon_j,k_i,k_j) = f(\varepsilon_i,\varepsilon_j)h(k_i,k_j)$.The integration is taken over $\Omega = S \times K$, with $S = [-\overline{\varepsilon},\overline{\varepsilon}]\times[-\overline{\varepsilon},\overline{\varepsilon}] \subset \Re^2$ defined as the support for $f(\varepsilon_i,\varepsilon_j)$ and $K \subseteq \Re^2$ defined as the support for $h(k_i,k_j)$.

This specification of the payoff functions contains a linear surprise inflation term to indicate that the time inconsistency problem arises from the temptation that the policymaker has to exploit the benefits from surprising agents (as, for example, in Barro and Gordon,1983, Vickers, 1985, Giavazzi and Pagano, 1988, Cukierman, 1993), rather than from the stabilization of the variance of output. Without any loss of generality we will restrict our

analysis to the case of positive average inflation rates, i.e. $\psi_i \in [0, \bar{\psi}]$, there fore $\psi_i^G \in [\underline{\psi}^G, \bar{\psi}^G]$, with $\underline{\psi}^G > 0$. ψ_i^G represents the preferences that the government in country i has for surprise inflation, represents the preferences that the Cb in country i has been assigned in evaluating inflation surprise[4]. Notice that the choice of ψ_i has two effects on the expected welfare of the government: it defines the average (predictable component) of the CPI inflation rate and it also the magnitude of the slope of the marginal benefit from unexpected increase in money supply. A lower ψ_i decreases the inflationary bias of the Cb, at the cost of a reduced elasticity of the money supply to the magnitude of the shock. Moreover, the benefits from surprising agents also depend on the state of the economy, i.e., upon the size of the supply shock, ϵ_i, the larger the positive (negative) shock the lower (larger) the degree of temptation for surprising agents.

Finally, and crucially for our analysis, the dependence of the marginal benefit from the state of the economy is measured by $\overline{k}_i$ and k_i in the government's and in the Cb's objective function respectively. $\overline{k}_i$ is a parameter measuring the effect of the ϵ_i shock on the marginal benefit from surprise inflation perceived by the government. A larger $\overline{k}_i$ raises the marginal benefit from surprise inflation in bad times ($\epsilon_i < 0$) and lowers it in good times ($\epsilon_i > 0$). $\overline{k}_i$ can be thought of as a public signal about the marginal benefit from stabilization policy, observed by the two government at the time when they build up the institution in order to deal with the credibility problem. k_i plays a similar role in the payoff function to be assigned to the bankers and is interpreted as a correlated private signal the Cbs observe before choosing their monetary stance in the stabilization game[5]. k_i and k_j are distributed according to $h(k_i, k_j)$ assumed to be jointly normal with $E(k_i) = \overline{k}_i$ and $Var(k_i) = \sigma_k^2$, $E(k_i\ k_j) \neq 0$, $E(k_i\epsilon_i) = 0$ and $E(k_i\epsilon_j) = 0$. For consistency, we require that the marginal benefit from surprise inflation when averaging across states k_i, is always larger than zero, that is, $\min(1 - \overline{k}_i\varepsilon_i) > 0$, for any ϵ_i i.e. $\overline{k}_i < 1/\overline{\varepsilon}_i$.

>From the point of view of the game structure described above, k_i is a

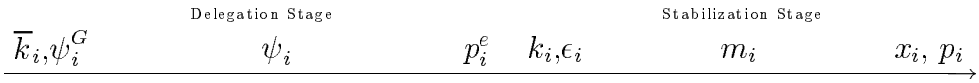
[4]Notice that in the specification for the pay-off function adopted here ψ measures the policymaker's degree of temptation in exploiting surprise inflation and is inversely related to the more usual "degree of conservativeness", which measures the penalty to be assigned to CPI inflation in the criterium function. Other interpretations also present in the literature relate ψ_i to the degree of dependence of the Cb from political bodies.

[5]By differentiating the CB's pay-off function with respect to m_i we get: $dW_i = -(1 + a)p_i\frac{\partial p_i}{\partial m_i}dm_i + \psi_i(1 - \overline{k}_i\varepsilon_i)dm_i$, the first term being the inflation marginal cost, the second one being the marginal benefit of an increase in money supply. It can be easily seen that the larger $\overline{k}_i$, the larger the sensitivity of marginal benefit to the shock, i.e. the larger the preferences for flexibility.

private signal received by the Cb on the preferences for surprise inflation in different states of the world at the time when its action is called for. As far as the economic interpretation this parameter allows us to describe an institutional environment in which the government has a limited ability of the government to commit monetary policy. In other words, in making its choices about ψ_i, the government has no control on k_i. Therefore the game is an imperfect commitment game where the government cannot assign a completely arbitrary payoff function to the Cb[6]. In the time interval between the delegation stage and the stabilization policy is implemented, the parameter affecting the perceived marginal benefit from surprise inflation in the payoff function of the Cb may change, due to a changed view in the society about the effectiveness and the convenience of stabilizing a given shock.

Therefore, summarizing, the institutional setting we are trying to capture can be described as follows: society, as represented for example by the median voter aggregator, i.e. the government, is characterized, by two parameters ψ_i^G and a public signal $\overline{k}_i$ on the relative marginal benefit over two alternative policy targets, it commits monetary policy to an institution, the Cb, by assigning it an objective function, defined by ψ_i. When the institution has to implement its policy, its perceived marginal benefit may change according to the private signal k_i. The timing of the game is as follows: 1) two governments are elected, ψ_i^G and $\overline{k}_i$, are realized. The two governments simultaneously select preferences for their central bankers, ψ_i, and those preferences are observed by private agents and by the foreign Cb; 2) Private agents, as price-wage setters, formulate expectations about the inflation process, conditional on their information; 3) k_i, and supply shocks ε_i, are independently drawn. The ε_i shocks to the level of economic activity in both countries are common knowledge, while Cb in country i privately observes the k_i parameter and formulates a posterior belief about k_j using the conditional density function $h_{k|k_i}(k \mid k_i)$; 4) the two central bankers choose monetary policies according to the assigned payoff functions conditional on their information set; 5) macroeconomic outcomes are realized. This timing structure is represented as follows:

[6] In discussing the main aspects of central bank independence Cukierman (1993), analyses the role of partial commitments in designing monetary policy institutions. He formalises his ideas about this issue by assuming a payoff function for the Cb which is quite similar to the one adopted here. See Cukierman (1993), p. 357. See also Muscatelli (1998) p.536 and ss. for a similar specification and for a discussion of the arguments in defense of its adoption.



In the next section we will solve the commitment game for the case when the k_i parameter is common knowledge among the Cbs as a benchmark. Then in section 4 we will analyze the model as described above.

3 The commitment of monetary policy in open economies under complete information

In this section we solve, as a benchmark, the game for the case in which the Cbs have common knowledge about their types k_i.

By backward induction we solve first the stabilization game played by the Cbs. Domestic and foreign Cbs maximize (8). The first order condition for Cb in country i is

$$\left\{m_i + a[m_i - m_i^e - (m_j - m_j^e)] + b(\varepsilon_i - \varepsilon_j)\right\}(1+a) = \gamma\psi_i(1 - k_i\varepsilon_i) \quad (9)$$

where $0 < a = \beta\delta\gamma < 1$, $0 < b = \beta\delta < 1$.[7] The LHS of (9) has to be interpreted as the marginal cost of an increase in m_i in terms of inflation, the RHS of (9) as the marginal benefit of an increase in m_i in terms of surprise inflation. Notice that m_i and m_j are *strategic complements* in the Cbs' payoff functions, i.e. $\partial^2(W_i)/\partial m_i \partial m_j > 0$. The economics of strategic complementarity is easily understood: for example, by increasing money supply, given private agents' expectations, country i will increase output and inflation at home and by depreciating the real exchange rate, will decrease output and CPI inflation abroad. This implies that the marginal cost of inflation abroad decreases, while the marginal benefit of surprise inflation increases: this induces the foreign Cb to increase money supply to balance net benefits [8].

[7] Since a depends on the share of the foreign good in the consumer's basket (β), and differently from b measures the predictable component of the money supply process, we will label it as our measure for the degree of openness.

[8] Strategic complementarity in money supplies implies the result, common in this literature, see Canzoneri and Henderson (1991), of under provision of stabilisation by the two policymakers. Strategic complementarity leads to competitive depreciation after a negative shock.

Solving the system (9), the Nash equilibrium money supplies are given by

$$m_i = \frac{\gamma\psi_i}{1+a} - \frac{1}{1+2a}\left[\gamma k_i \varepsilon_i \psi_i + \gamma \frac{a}{1+a} k_j \varepsilon_j \psi_j + b(\varepsilon_i - \varepsilon_j)\right] \qquad (10)$$

This result is synthesized in the following

Lemma 1 *For any ψ_i, there exists a unique Nash equilibrium in the monetary stabilization game, given by (10). Best reply functions in money supplies satisfy the condition for strategic complementarity. The equilibrium level for the predictable component of the money supply in each country (inflationary bias) is inversely related to the degree of openness.*

The term $\gamma\psi_i/(1+a)$ represents the inflationary bias in the case of open economies. It is lower than in the case of a closed economy,(i.e. for $a = 0$), and tracks the effect of the self built in check mechanism characterizing the non cooperative monetary regime as analyzed in Rogoff (1985b) and the analogous prediction developed and tested in Romer (1993)[9]. The terms in the square parenthesis measure the stabilization component in the money supply. Consider for example the case $k_i > 0$, $\varepsilon_i > 0$. The term $k_i\varepsilon_i\psi_i$ in (10) measures the effect of a reduction in the marginal benefit from surprise inflation due to a positive shock in country i. The term $\frac{a}{1+a}k_j\varepsilon_j\psi_j$ measures the reduction in country i money supply after a positive shock in country j, due to *strategic complementarity* in the money supply. The term $b\varepsilon_i$ measures the reduction in money supply due to a direct terms of trade effect of a positive shock to the level of economic activity. For example, a positive shock triggers a real exchange rate depreciation, required to clear the trade balance, this leads to an increase in CPI inflation rate that the Cb is willing to partially stabilize. The term $b\varepsilon_j$ measures the same effect triggered by a, say, positive shock abroad.

By substituting (10), into (3) and (6) we get the following reduced form for the inflation and output processes

$$p_i(\psi_i) = \frac{1 - k_i\varepsilon_i}{1+a}\gamma\psi_i \qquad (11)$$

and

[9] This represents the cornerstone of the point made in Rogoff (1985b) about how international monetary cooperation may rise "the rate of wage inflation because wage setters recognise that a non cooperative regime contains a built-in check on each central bank's incentive to inflate." The reason for this barrier to be effective is that "when a central bank expands its money supply unilaterally, it causes its country real exchange rate to depreciate" thereby increasing the inflation costs. See Rogoff (1985b) p. 200.

$$x_i(\psi_i, \psi_j) = -\frac{1}{1+2a}\left[\gamma^2 k_i \varepsilon_i \psi_i + \gamma^2 \frac{a}{1+a} k_j \varepsilon_j \psi_j + \gamma b(\varepsilon_i - \varepsilon_j)\right] + \varepsilon_i \quad (12)$$

Notice that a feature of the Nash equilibrium in money supply is that the equilibrium outcome for the inflation process in each country is completely insulated from the shock in the foreign country, while the output process is not.[10] Moreover, as in Romer (1993), the predictable component of the inflation process in each country is inversely related to the degree of openness.

We can now solve for the optimal degree of commitment that the governments will choose to adopt by delegating monetary policy to a Cb with suitable institutional objectives. By substituting (11) and (12) into (7), the strategy of the Government i satisfies

$$\begin{aligned} \psi_i^* \quad = \quad & Arg \max_{\psi_i} E(W_i^G) = \\ & \int_\Omega \{-\frac{1}{2}(p_i(\psi_i)^2 + \\ & \psi_i^G(1 - \overline{k}_i \epsilon_i) x(\psi_i, \psi_j)\} dG(k_i, k_j, \varepsilon_i, \varepsilon_j) \end{aligned} \quad (13)$$

By taking expectations and maximizing with respect to ψ_i we get the following solution for the Cb's preferences

$$\psi_i^* = \frac{(1+a)^2}{1+2a} \frac{\overline{k}_i^2 \sigma_\epsilon^2}{1 + (\sigma_k^2 + \overline{k}_i^2)\sigma_\varepsilon^2} \psi_i^G \quad (14)$$

One important feature exhibited by the equilibrium strategy at the institutional design stage is that there is no strategic interdependence between the government's choices, since from (13) it is clear that Government i has a dominant strategy in the appointment of its Cb. Setting $a = 0$ we get the Rogoff (1985a) result of a conservative Cb in the case of a closed economy. As expected there exists an incentive to precommit monetary policy in a closed economy, i.e. $\frac{\overline{k}_i^2 \sigma_\epsilon^2}{1+(\sigma_k^2+\overline{k}_i^2)\sigma_\varepsilon^2} < 1$. In the case of an open economy, instead, the "built-in check mechanism" on the incentive to inflate, due to the real exchange rate effect, turns out to reduce the optimal degree of commitment, since $\frac{(1+a)^2}{1+2a} > 1$. Intuitively this result holds for the same reason

[10] Notice that this feature does not hinge on the linear quadratic form of the objective functions adopted in this paper. Solving a similar model with the alternative duoble quadratic functional form also used in the literature, under the hypothesis of rational expectations of the agents leads to the same result.

why cooperation in monetary policy can be counterproductive, as found in Rogoff (1985b).

We may ask whether it is possible, that the "built-in-check" effect could be so strong to counteract the incentives to precommitment, ($\psi_i^* > \psi_i^G$). A necessary and sufficient condition for avoiding this possibility is given by $\frac{(1+a)^2}{1+2a}\frac{\overline{k}^2\sigma_\epsilon^2}{1+(\sigma_k^2+\overline{k}^2)\sigma_\varepsilon^2} < 1$. The results, holds under the hypothesis of the model, i.e. for $0 < a < 1$ and $\bar{k} < 1/\bar{\epsilon}$.

These results are synthesized in the following

Lemma 2 *(Conservative Cb result) In a two country world, the monetary policy delegation game with imperfect commitment and complete information has a unique perfect equilibrium in dominant strategies where the government of country i, appoints a Cb whose preferences satisfy (14). There exist incentive to precommit monetary policy ($\psi_i^* < \psi_i^G$). The optimal degree of commitment is lower than in the case of a closed economy.*

Proof. Given Lemma 1, substitute (11) and (12) into (13) and maximize to get (14). The proof that for $a = 0$, the Cb is more conservative than in the case when $a > 0$ is immediate, set $a = 0$ into (14) and get the result. Proving that $\psi_i^* < \psi_i^G$, is also straightforward: the inequality $\frac{(1+a)^2}{1+2a}\frac{\overline{k}^2\sigma_\epsilon^2}{1+(\sigma_k^2+\overline{k}^2)\sigma_\varepsilon^2} < 1$ can be rewritten as $\frac{a^2}{1+2a}\overline{k}^2\sigma_\epsilon^2 < (1+\sigma_k^2\sigma_\varepsilon^2)$, and since, for $a < 1$, $\frac{a^2}{1+2a} < 1$, a sufficient condition for the inequality to hold is $\overline{k}^2\sigma_\epsilon^2 < 1$. This is satisfied since, the condition $1 - \overline{k}\varepsilon_i > 0$ and the symmetry of the support $[-\overline{\varepsilon}_i, \overline{\varepsilon}_i]$ imply $1 > \mid \overline{k}\varepsilon_i \mid$. By squaring both terms and by taking expectations with respect to ε, we get the sufficient condition $\overline{k}^2\sigma_\epsilon^2 < 1$. ■

As expected, $d\psi_i^*/d\sigma_\varepsilon^2 > 0$ i.e. the optimal degree of commitment is decreasing in the variance to the output shock. Since $d\psi_i^*/d\overline{k}_i > 0$, we obtain that the larger the average preferences for flexibility the lower will be the Cb's optimal degree of commitment. Moreover, $d\psi_i^*/d\sigma_k^2 < 0$, i.e. the larger the variance of the signal on the marginal benefit of inflation, the stronger the incentive to precommit. In our interpretation for the k_i parameter, the higher the variability in the institutional and economic environment surrounding the action of the Cb, the larger the incentive to precommit. The intuition for this property of the model is that a larger variability in the signal perceived by the banker leads to a higher variance of inflation without affecting the marginal benefit from surprise inflation perceived by the government at the institutional design stage. As a consequence, the marginal incentive for the government to precommit increases.[11]

[11] To check some of the predictions of the present model we computed the correlation

Finally, $d\psi_i^*/da > 0$, i.e. the larger the degree of openness of the economy the lower will be the Cb's optimal degree of conservativeness, in other words, the more effective the self "built-in check" mechanism is, the lower the incentive for the government to commit.

The substitutability between commitment and the self discipline imposed to the CB by the exchange rate depreciation following an increase in money supply is an intuitive and quite general result. In particular it does not depend on the hypothesis of imperfect commitment. Therefore, since the degree of openness is predicted to be a substitute for commitment in the government's eyes, if the credibility approach captures an important aspect of the institutional reforms in central banking we should expect, coeteris paribus, not to observe increasing attention to institutional reforms pointing to a larger degree of independence and strict monetary targets over periods characterized by increasing degree of openness. In this very simple two country model increasing variability in the output shocks (say after the oil shock) and increasing degree of openness would call for a release of the commitment in both countries. The only comparative statics result on which we can rely to justify the increasing attention to the institutional solution in the last decades is the increasing variability in the socio-political environment σ_k^2 where the Cb's action takes place. This may be factual for some countries but not for others and it would leave us without any explanation about why the most developed countries addressed the credibility problem by focusing on institutional solutions at the same time.

The next section addresses these issues in more detail, by analyzing the effects of introducing in the model the hypothesis that the signal on the (marginal) benefits of a given stabilization policy observed by each Cb at the time when policy is implemented is their private information.

---

between the index for the degree of dependence (cbdep) and the degree of openness (open) and index of political instability (inst) for different subsets of countries in the data set used in Romer (1993). When computed on the sample of 114 countries the results are as follows: corr(cbdep, openness)=0.029; corr(cbdep, inst)=0.42. This results do not provide evidence for the predictions of the commitment model as defined above. However, when the same index are computed for the 25 most advanced countries (ranked according to the real gdp in 1980) the results are as follows corr(cbdep, openness)= 0.71 and corr(cbdep, inst)=-0.13. Therefore, for a subset of countries that seem to have solved the credibility problem relying on institutional solution, the substitutability between openness and commitment and the commitment being increasing in the measure of instability seem to be confirmed. We thank David Romer for kindly providing the data.

4 The institutional solution with imperfect commitment and private information

As argued by Romer (1993) there is a group of approximately fifteen to twenty highly developed country that have largely solved the problem of dynamic inconsistency of optimal monetary policy. The solution of the time inconsistency problem has been achieved mainly by granting independence to Cbs and by assigning the institutions an important weight to inflation stabilization. However, as we have seen in the previous section, under standard assumptions, a simple credibility argument should predict that the optimal level of precommitment to an independent central banker is inversely related to the degree of openness. Does this mean that the credibility approach fails in a major prediction? Moreover, as argued in the introduction historical evidence on recent reforms in monetary institutions and central bankers' attitude towards inflation in the past decades has shown that the emergence of an inflation aversion has spilled over across countries, the model solved under the case of complete information yields a dominant strategies equilibrium at the delegation stage that does not allow for any spillover at the institutional design stage. Is this informal evidence inconsistent with the predictions of the credibility approach to institutional reforms of central banking?

In this section we solve the simple model outlined in section 2 where the credibility problem is still the main ingredient. However, as described before, we model the environment in which the Cb's action takes place as informationally opaque to the external observer, i.e. k_i is private information to the policymakers at the stabilization stage. Our analysis will allow for strategic interdependence at the institutional design level implied by the interdependence of monetary policy reply, imperfect commitment and private information. This will allow us to address the issue of the interdependence at the institutional design stage across countries and to suggest conditions under which the credibility theory may be able to predict conditions under which opening countries to trade may increase the incentive to precommit.

In the following we solve the two sided private information stabilization game with incomplete information for the equilibrium money supplies and the reduced form for the output and inflation processes in the two countries are obtained. By folding backward these latter in the governments' pay-off functions and by taking expectations, given the two governments' information set, taking into account private agents expectations, we will solve the first stage of the game for the optimal degree of commitment of each government as a subgame perfect Nash equilibrium.

4.1 The stabilization game under private information

The stabilization stage game takes now the form of a simultaneous game of incomplete information with two side private information on the signal the Cbs receive at the time when they decide and implement monetary policy reply. We concentrate on pure strategy equilibria. Without any loss in generality[12] all the results in this section are reported for the normalization $Var(k_i) = \sigma_k^2 = 1$ and for the case when the initial public signal to the governments about the marginal benefit of surprise inflation is the same in both countries, i.e. $\bar{k}_i = \bar{k}_j = \bar{k}$. The type space is given by $K \subseteq R^2$ the pure strategy set is given by $M = \{(m_i(k_i),\, m_j(k_j)\}$. The equilibrium concept is the Bayes-Nash equilibrium and, since the model is linear quadratic, attention will be restricted to linear pure strategies of the form $m_i(k_i) = A_i + B_i k_i$.

In a Bayes-Nash equilibrium, money supplies have to satisfy the following condition, for $i \neq j$:

$$m_i^{BN}(k_i) = Arg \max_{m_i(k)} E_{k_j}(W_i) \tag{15}$$

$$= \int_{k_j} W_i \left\{p_i[(m_i(k_i), m_j(k_j)], x_i[(m_i(k_i)]\right\} h(k_j \mid k_i) dk_j \tag{16}$$

which, for country i, can be written as follows:

$$m_i(k) + a m_i(k) = a m_i^e + a[E_k(m_j(k) \mid k_i) - m_j^e] \frac{\psi_i}{1+a}(1 - k_i \varepsilon_i) - b(\varepsilon_i - \varepsilon_j) \tag{17}$$

where $E_k[m_j(k) \mid k_i] = A_j + B_j E(k_i \mid k_j) = A_j + B_j[\bar{k} + \rho_k(k_i - \bar{k})]$ represents the conditional expectation that the Cb of country i has about the money supply played by the Cb in country j, given the joint normality assumption about the k_i, k_j parameters. Bayes- Nash equilibrium strategies are computed by solving for the undetermined coefficients A_i and B_i and for the expected money supplies, m_i^e, m_j^e, given the private agents information set,

$$A_i = \frac{\gamma}{1+a}\psi_i + \frac{(1-\rho_k)a\bar{k}}{1+2a}[aB_i + (1+a)B_j] - \frac{b(\varepsilon_i - \varepsilon_j)}{1+2a} \tag{18}$$

$$B_i = -\frac{\gamma}{\Lambda}[\frac{a}{1+a}\rho_k \psi_j \varepsilon_j + \psi_i \varepsilon_i] \tag{19}$$

[12] The comparative statics result with respect to σ_k^2 can be shown to be the same as in the model under complete information.

where $\Lambda = (1+a)^2 - a^2\rho_k^2 > 0$.

Under rational expectations, private agents can anticipate the predictable component of money supply (inflationary bias), given their information set. This is given by $m_i^e = \frac{\gamma}{1+a}\psi_i$ and equilibrium money supply are given by

$$m_i^{BN} = A_i + B_i k_i \tag{20}$$

where A_i and B_i are given by (18) and (19)[13].

Proposition 1 ***Lemma 3 Proposition 2*** *There exists a unique pure strategy Bayes-Nash equilibrium in the monetary stabilization game, where money supplies are given by (20).*

Proof. See Appendix.

Following the simultaneous stabilization game, the macroeconomic outcomes are realized and can be obtained by substituting the Bayes-Nash equilibrium money supply processes in country i and j into (3), and (6). The reduced form for the inflation and output processes are given by

$$p_i^{BN} = \frac{\gamma}{1+a}\psi_i + (1+a)B_i k_i + aB_j[\overline{k}(1-\rho_k) - k_j] \tag{21}$$

and

$$x_i^{BN} = \frac{a^2 B_j + a(1+a)B_i}{1+2a}\gamma\overline{k}(1-\rho_k) + B_i k_i - \frac{b}{1+2a}\gamma(\varepsilon_i - \varepsilon_j) + \varepsilon_i \tag{22}$$

Macroeconomic outcomes depend on the size of the shocks hitting the two economies, on the preferences assigned to the Cbs and on the (equilibrium) beliefs held by each Cb about the preferences of the other policymaker. In particular, differently from the case of complete information, domestic inflation process in the two countries cannot be insulated from elements affecting the foreign economy. The intuition for the result is that private information does not allow the Cb, willing to pursue this aim, to perfectly insulate the inflation process at home from the inflation process abroad. This feature of the macroeconomic outcomes from the stabilization game will be crucial in reintroducing strategic interdependence between governments decisions at the delegation stage and will shape the relationship between commitment and degree of openness.

[13] As expected, for $\rho_k \to 1$ the equilibrium outcome in (20) is the same as in (10) under the hypothesis of symmetric types.

4.2 The delegation game under private information

By substituting equations (21) and (22) into the payoff function of the government we get the reduced form of the welfare function of the government in the first stage. The perfect Nash equilibrium strategies ψ_i^{**} for the game under imperfect commitment will be given by

$$\begin{aligned} \psi_i^{**} &= \underset{\psi_i}{ArgMax}\, E_{\varepsilon_i,\varepsilon_j,k_i,k_j}(W_i^G) && (23) \\ &= \int_\Omega \{-\frac{1}{2}\left[p_i^{BN}(\psi_i)\right]^2 + \psi_i^G(1-\overline{k}\epsilon_i) \\ &\quad \times [x_i^{BN}(\psi_i)]\} dG(k_i,k_j,\varepsilon_i,\varepsilon_j) && (24) \end{aligned}$$

Therefore governments' strategies will satisfy the following reaction functions

$$c_1\psi_i^{**} = -c_2(1-\rho_k^2)\rho_k\rho_\epsilon\sigma_\varepsilon^2\psi_j^{**} + c_3\sigma_\varepsilon^2\psi_i^G \tag{25}$$

where c_1, c_2 and c_3 are positive constants defined in the appendix, ρ_k is the coefficient of correlation across the private signals received by the bankers, ρ_ϵ the coefficient of correlation across the economic shocks.

Setting $a = 0$ we get the optimal Cb preferences in the case of a closed economy, i.e. $\psi_i^{**} = \frac{\overline{k}^2\sigma_\varepsilon^2}{1+(1+\overline{k}^2)\sigma_\varepsilon^2}\psi_i^G$.

By evaluating (25) in a symmetric equilibrium ($\psi_i^G = \psi_j^G$) we get the following solution:

$$\tilde{\psi}_i = c_4\psi_i^G \tag{26}$$

where $c_4 > 0$ is a constant defined in the appendix and characterized in the next section.

The results above can be synthesized as follows

Proposition 3 *There is a unique perfect equilibrium for the delegation game under private information of bankers about k_i. Governments' equilibrium strategies will exhibit strategic substitutability (complementarity) if and only if the sign of $\rho_k\rho_\varepsilon$ is positive (negative).* ***Proof****: see Appendix.*

The proposition above states that the sign of the correlation across shocks to the level of economic activity and the correlation across the private signals received by the Cbs in each country play a major role in the strategic design of monetary institution in the case of open economies. Without further restrictions on the sign of ρ_k and ρ_ε the model would allow us to construct a

taxonomy for the strategic interrelationship at the institutional design stage. This taxonomy would predict that the sign and the magnitude of the incentive to precommit in open economies would depend on the correlation structure across the economic shocks hitting the two economies as well as on the correlation structure of the private signals.

However, as argued before, we take the view that, given the forces we mean to capture in our interpretation of the private signal technology, the correlation between the external pressures arising from interests and influence of different agents in the society and the correlation between the scientific view among academic scholars of the benefits and viability of stabilization policy affecting the Cb's perceived marginal benefits has been positive in the main industrialized countries. Therefore the results of the model will be analyzed for the case of $\rho_k > 0$.

Under this restriction the results will depend upon the sign of ρ_ε. We defined ϵ as a reduced form shock hitting the level of economic activity in the two countries, so that the correlation structure between these shocks across countries is not restricted within the model. If the bulk of the variance in the level of output in the two countries is mainly due to shocks in the terms of trade (or to factor supplies or factor movements across countries), then the correlation is likely to be negative. If, on the other hand, the bulk of the variance is due to common demand or technological shocks, then the correlation is likely to be positive. What is the prevailing source of common shocks for given two countries is an empirical question that is not addressed in this paper and would provide additional restrictions on the sign of ρ_ε[14].

The interpretation of the results in the case of strategic substitutability ($\rho_\epsilon > 0$) versus strategic complementarity ($\rho_\epsilon < 0$) can be easily disentangled. In the following we provide intuition for the former case, though similar arguments apply to the latter case. Strategic substitutability is equivalent to governments' marginal benefit from increasing ψ_i be decreasing in ψ_j.

For $\rho_\epsilon > 0$, both governments rationally expect the two economies to be in the same state of the world in the future (booms or slumps)[15]. Moreover,

[14]The reduced form shock considered in the model will make the discussion euristic and depending on the main source of variability in the output process across countries. However once a prevailing structural source of variability in the output shock is identified, the model allows for well defined predictions on the features of the equilibrium outcome of the game. Notice also that the model only allows for contemporaneuos correlations in the business cycles across countries, i.e. the economic shocks hitting the two economies are assumed to be perfectly synchronized. Different issues would of course arise in a dynamic specification without changing the nature of the basic results, provided that the hypothesis that shocks to the economic activity in both countries would affect to some extent the real exchange rate as well as the balance of trade.

[15]Since $\rho_k > 0$, it is also the case that a higher (lower) than average signal about the

because of strategic complementarity at the stabilization stage, they will expect a matching between money supply in the stabilization game. This expected matching between money supplies affects the expected variability of inflation (remember that in each country the equilibrium inflation outcome is not insulated from the foreign component) and the amount of stabilization that is expected to be imported from abroad. Consider the argument from the point of view of the domestic government: if the foreign government (marginally) increases ψ_j, the foreign Cb will have a more flexible stance in the stabilization stage. Due to $\rho_\epsilon > 0$ and by strategic complementarity at the stabilization stage, the home government expects the domestic Cb to match the behavior of the foreign government *in the states of the world when it is needed for stabilizing the domestic economy.* Therefore, a marginal increase in the foreign banker preferences for flexibility ψ_j, by increasing the variance of inflation process and by reducing the marginal cost of inflation for any given domestic banker, will coeteris paribus, reshape the incentive of the domestic government so to make it more willing to buy domestic credibility in change for a reduction in the preference for flexibility. As a result, the marginal benefit of a given type of Cb, ψ_i, to a government ψ_i^G will be reduced and, in equilibrium, the domestic government will rationally reduce the level of precommitment ψ_i after an increase in ψ_j, leading to negatively sloped reaction function at the institutional design level[16].

Before exploring the comparative statics properties of the present model let us discuss some of its welfare implications that will also be useful for a more thorough description of the forces at work behind the results.

Simple intuition would suggest that the expected welfare to the domestic government is increasing (decreasing) in the level of stabilization provided

marginal benefit from money surprise in country i is rationally expected to be matched by a higher (lower) than average marginal benefit in country j.

[16] Under the hypothesis that $\rho_\epsilon < 0$ both economies will be in different states of the world (booms or slumps) most of the times. Therefore, because of the incentive to the Cbs to match money supply at the stabilization stage, a more activistic monetary policy abroad will trigger the wrong stabilization policy at home. Coeteris paribus the home government will be willing to crowd out this effect by reducing the level of commitment leading to strategic complementarity at the institutional stage of the game.

Moreover, given that $\rho_k > 0$ it is also the case that a high marginal benefit from money surprise in country i is matched by an high marginal benefit in country j. Given strategic complementarity in the money supplies at the stabilization stage, this positive correlation, will increase the ex-ante expected variance of CPI inflation. This can not be traded off against an increase in expected benefit in terms of surprise inflation and hence, in order to reduce the future variance of inflation each country will appoint a more conservative CB in a subgame perfect Nash equilibrium. Under strategic substitutability, the increase in the variance following an increase in ψ_F is larger then the increase in the expected benefits, leading to an increase in the degree of conservativeness in the domestic country's Cb.

by the foreign Cb when the shocks are positively (negatively) correlated. In other words if $\rho_\varepsilon > 0$ we would expect that, at the institutional design stage, the foreign government, by increasing the preferences for stabilization of its own banker (paying it in terms of a larger inflationary bias), is providing a public good in terms of a higher level of stabilization policy at the world level exactly in the states of the world when it is needed [17]. Based on the nature of the externality at the institutional design level it seems reasonable to argue that the expected welfare to government i is increasing in ψ_j.

This argument, however, does not hold in general, that is for any of the possible government's types. To see the reason, consider, for example, a government i with $\psi_i^G = 0$ and a world where positively correlated shocks represent the bulk of the variability in output ($\rho_\epsilon > 0$). Since this type of government does not care about stabilization, but only about the increased variance of inflation imported from abroad, it will not take an increase in ψ_j as a public good[18]. Because of these arguments we further limit our analysis to the case of a symmetric equilibrium, $\psi_i^G = \psi_j^G$, warning that the conclusions do not necessarily carry over in the presence of important differences in the governments preferences.

Proposition 4 *In a symmetric equilibrium $\psi_i^G = \psi_j^G$, a sufficient condition for the expected welfare to government i to be increasing in the government j's strategy under strategic substitutability ($\rho_\epsilon > 0$) is $\rho_\epsilon > \bar{\rho}_\epsilon = \frac{a(1+a)}{1+2a}$. The expected welfare to government i is decreasing in the government j's strategy under strategic complementarity ($\rho_\epsilon < 0$).* ***Proof****: see Appendix.*

Proposition 3 states that, when the variability in output processes are due to shocks that lead to negative correlations the expected welfare to the

[17] This feature would give the institutional design game in the first stage between governments welfare properties rather different from the standard duopoly analysis with quantity or pricing strategies. In particular, consider the case of the standard Cournot duopoly with quantity competition, in that case strategic complementarity is equivalent to a firm 1 decreasing marginal profit in the quantity played by firm 2 moreover, firm 1 expected profits also decreasing in firm 2 strategy.

[18] It is immediate to show that for this extreme government the reaction function is given by $\psi_i = 0$ (dominant strategy) and the bliss point is given by the origin of the axes in the governments strategy space. A similar feature is present in a world where the bulk of the output variability is due to shocks that are negatively correlated, ($\rho_\epsilon < 0$), in this case a larger preference for stabilization abroad will trigger a larger variance of inflation at home. Therefore strategic complementarities at the monetary policy stage would imply a procyclical stabilization policy at home but since the type of government we are considering does not care about stabilization this will not be taken into account at the delegation stage.

domestic government will be reduced after the increase in the preferences for stabilization abroad since it will induce a reply by the domestic Cb that is procyclical and also a larger expected variance of price inflation increases.

On the other hand in the cases when the fundamental shocks lead to positive correlations across outputs in the two countries, the expected welfare in each country is increasing in the preferences for stabilization in the other country if the level of correlation is high enough. In other words, a marginal increase in the amount stabilization imported from abroad must be (in expected terms) worth enough to the domestic government to counteract the expected increase in the welfare cost due to a larger variance in the inflation process, this happens to be true if the probability to end up in similar states of the world is high enough[19].

Notice also that the fact that the contract curve in the $\psi_i - \psi_j$ plane lies outward with respect to the Nash equilibrium in strategic substitutes does not mean that cooperation leading to a lower degree of commitment in both countries does necessarily improve welfare. (See fig.1) The reason is that in a cooperative regime at the institutional design level, the expectations of private agents have to be taken into account and credibility would be lost in both countries. This is not a welfare property peculiar to the present model and is nothing more than the same effect analyzed in Rogoff (1985b), relocated at the institutional design stage.

In synthesis, the equilibrium outcome of the game analyzed in the present section depends mainly on the correlation structure of the shocks to the level of activity in the two countries. The feature of the equilibrium outcome of the game will depend on the nature and the sign of the prevailing sources of variability in the output shocks. If the main source of variability in the level of economic activity as summarized in the (reduced form) ϵ shocks is due to the structural shocks to the terms of trade between countries the structure of the correlation will be likely to be negative leading to the strategic complementarity results. On the other hand when the main source of variability in the output levels is related to structural shocks in different components of demand these are likely to be positively correlated[20] leading to the strategic substitutability result. This outcome of the game shows how the institutional

[19] For $\rho_\varepsilon > 0$, an increase in ψ_j will trigger two effects on the expected welfare in country i: on the one hand it will make monetary policy more flexible (stabilization policy at the world level as public good) and, at the same time will rise the variance of the inflation process. If the first effect prevails welfare in country i is increasing in ψ_j. However, both effects raise the marginal cost of flexibility in country i and therefore will induce strategic substitutability between the governments strategies.

[20] Consider for example the shocks in money demand and wealth effects due to stock markets fluctuation.

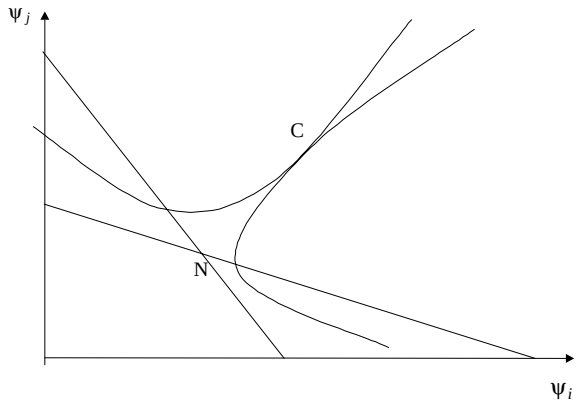


Figure 1: $\rho_\varepsilon > 0$

design and the degree of commitment in a given country may interact with institutional design in other countries. The main driving forces shaping the incentives to commitment as predicted by the model are found in the nature of the main sources of common shocks to the level economic activity across countries, and on the degree of similarity in the social and economic environment surrounding the action of the Cb as captured by the k parameter.

Therefore the conclusion is that, provided that the academic views and other factors influencing the perceived marginal benefits from stabilization policies are positively correlated and the (reduced form) shocks to the level of economic activity are also positively correlated across countries, the spillover effect from one country to the other is negative i.e. a larger degree of inflation aversion in the government in a given country leads to a larger degree of inflation aversion for the Cb in that country and is matched by a lower degree of inflation aversion in the foreign country Cb leading to a lower degree of commitment at the institutional design stage. The opposite occurs under strategic complementarity.

4.3 Open economies, correlated shocks and commitment

We move now to further characterize the equilibrium outcome and to report comparative statics results to analyze the forces shaping the incentive to commitment of monetary policy in the presence of open economies. In particular we will analyze under which conditions the presence of two side private information at the monetary policy stage increases the incentive to precommitment, how this is affected by the degree of openness, by the degree

of correlation between the economic shocks and by the degree of homogeneity in the socio- political environment in which the Cb's action is cast.

A general result about the incentive to appoint a "conservative" Cb ($\psi_i^{**} < \psi_i^G$) for any possible structure of correlation in the economic shocks cannot be obtained in this model for the same reasons outlined above in the discussion of the public good aspect related to the provision stabilization policies. For example in the case of strategic complementarity ($\rho_\epsilon < 0$) a Cb with high preferences for flexibility abroad will induce the domestic government to relax the commitment of monetary policy since the existence of a negative correlation across output shocks will imply that a release of precommitment abroad will induce more activistic stance by any domestic Cb in the states of the world where it is not required. Therefore in this case there will always exist a value of ψ_j^G high enough to induce the domestic government to delegate monetary policy to a Cb with "non conservative" objectives.

In the case of strategic substitutability at the delegation stage ($\rho_\epsilon > 0$), on the other hand, the incentive to precommit is always present as shown in the next proposition.

Proposition 5 *For any ψ_i^G and ψ_j^G, $\rho_\varepsilon > 0$ (strategic substitutability between the government's strategies) is a sufficient condition for the existence of incentives to precommit monetary policy ($\psi_i^{**} < \psi_i^G$).* ***Proof****: See Appendix*

The intuition for the result is that the incentive to precommit monetary policy to a conservative Cb exists under strategic substitutability between the governments' strategies since, as explained above, the strategic complementarity in the monetary stabilization stage and the nature of correlation among the economic shocks are such that both economies will be in the same state most of the time and therefore any government is willing to free ride on the stabilization policy provided by the other government. This result cannot be obtained, in general, under strategic complementarity since in this latter case more stabilization expected to be pursued abroad is a bad news to the domestic government because it is expected to trigger the "wrong" stabilization reply by the domestic Cb, so that it has to be countervailed by reinforcing the domestic Cb's preferences for stabilization. This effect may counteract the direct effect to precommitment that obtains in a closed economy.

This result suggests that symmetry in the preferences across countries is an important feature to understand why in the presence of open economies countries *may* or *may not* have incentive to build on the institutional ap-

proach the solution of the credibility problem[21]. By restricting attention to the symmetric equilibrium it is possible to prove that a conservative Cb results obtains in the present model. The results about the characterization of the symmetric equilibrium are reported in the following proposition.

Proposition 6 *In a symmetric equilibrium ($\psi_i^G = \psi_j^G$) the following results hold:*

1) $\psi_i^{**} < \psi_i^G$, i.e. in open economies there always exist incentive to commit monetary policy;
2) If $\rho_\varepsilon > 0$ then $\psi_i^* - \psi_i^{**} > 0$;
3) If $\rho_\varepsilon < 0$ and $(1+2a) \mid \rho_\varepsilon \mid < a\rho_k < 1$ then $\psi_i^* - \psi_i^{**} > 0$.
***Proof**: See Appendix.*

Result 1) states that when the differences in preferences between governments are small enough, there is always incentive to precommit monetary policy, due to the fact that the direct incentive to precommit countervails any possible incentive to relax the commitment due to openness.

Result 2) states that in the case of private signals the optimal degree of commitment increases compared to the case of complete information, ($\psi_i^{**} < \psi_i^*$) when the correlation across economic shocks is positive (strategic substitutability) because of the public good effect.

Result 3) states that this may also be the case for negatively correlated economic shocks provided that this correlation is sufficiently small compared to the correlation between private signals. The reason is that a large (negative) correlation across economic shocks, entails a large negative effect of stabilization provision abroad, and then a large incentive to reduce the commitment due to openness.

Finally we study the comparative statics properties of the symmetric equilibrium for the case when $\rho_\varepsilon > 0$. This will allow us to address the analysis of the relations between commitment and the degree of openness,

[21] In a more general model this may lead to an explanation about why some countries may *not* find the adoption of the institutional solution to the credibility problem as a convenient one. Further analysis of this point is left for future work. Here we limit ourselves to remark that, by enlarging the model to more than two countries and by allowing for etherogeneous correlation structure between inputs, the final outcome of the game is much richer than in the present setting. In particular the optimal degree of commitment may depend on the sign and degree of homogeneity of the socio political environment sorrounding the central bank institution and on the nature of the shocks hitting the economies.

A possible candidate for these implications Russia. Evidence on the asymmetric business cycles compared to Europe or Us and differences in social structure and then preferences for stabilization policies may explain difficulties in the lack of incentives the society has to precommit monetary policy in this country.

the correlations across the economic shocks and across the signal on the marginal benefit of surprise inflation. The results are summarized in the following proposition

Proposition 7 *In a symmetric equilibrium the following results hold:*

1') if $\rho_\varepsilon > 0$ *then* $\frac{d\psi^{**}}{d\rho_\varepsilon} < 0$ *, i.e. the degree of commitment is increasing in the correlation across economic shocks when the correlation is positive;*

2') if $\rho_\varepsilon > 0$ *then* $\frac{d\psi^{**}}{d\rho_k} < 0$*, for* $0 < \rho_k < \bar{\rho}_k < 1$*, and* $\frac{d\psi^{**}}{d\rho_k} > 0$*, for* $\bar{\rho}_k < \rho_k < 1$*, i.e. the degree of commitment is initially increasing and then decreasing in the level of* ρ_k*;*

3') if $\rho_\varepsilon > 0$ *then* $\frac{d(\psi^* - \psi^{**})}{da} > 0$ *i.e. the larger the degree of openness the larger the incentive to commit monetary policy in the case of private signals compared to the case of complete information.*

Proof*: see Appendix*

Result 1') states that the international component of stabilization policy (the public good aspects of stabilization provision and the imported component of the inflation process) becomes more important the higher the degree of correlation across economic shocks, leading to a larger incentive to precommitment as this correlation increases. If $\rho_\varepsilon > 0$, by appointing a banker caring about stabilization, each government is providing a public good. The value of this externality to each of the governments is higher, the higher the likelihood that the two economies will be in the same state most of the times. Therefore the higher the value of the externality, the stronger will be the incentive to precommit monetary policy at home by relying on the stabilization policy provided abroad.

The intuition for result 2') is less straightforward and deserves some more detailed discussion. For $\rho_\varepsilon > 0$ a given increase in the value of $\rho_k > 0$ entails two effects. On the one hand by increasing ρ_k the signal k_i received by the domestic banker is expected to contain more information on the attitude the foreign banker has about stabilization k_j. Therefore, from the point of view of the government, an increase in ρ_k entails less variance in the equilibrium inflation process, that in turn would induce a reduction in the credibility cost of a less conservative Cb. This effect would call for a relaxation in the degree of commitment[22]. On the other hand a larger value of ρ_k increases the possibility to exploit the public good provided by the foreign banker in term of stabilization policy since it makes the domestic banker more sensitive to

[22]This effect has the same nature as in the models on information sharing in the industrial organization literature.

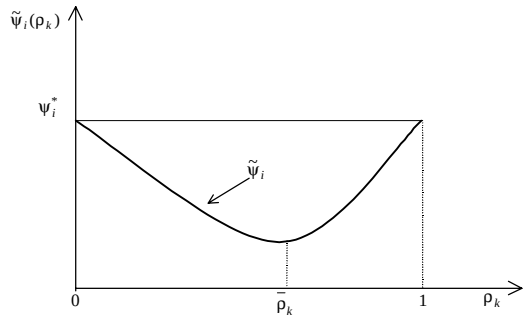


Figure 2:

the foreign monetary stance. In other words for $\rho_k > 0$, the larger the value of ρ_k the more likely are the domestic and foreign Cbs to observe similar signals about the value of the marginal benefit from surprise inflation. The closer will be the match of money supplies in the stabilization stage, the larger will be the attempt of each country to rely on the stabilization provided by the foreign Cb. The increased possibility to free ride on the stabilization policy provided abroad gives incentive to restrict the level of commitment. Result 2' in the proposition shows that for low (high) levels of ρ_k the second (first) effect prevails. See figure 2 and appendix for details.

Compared to the case of complete information, where, as we have shown in section 3, it is the case that the degree of openness is a substitute for commitment, these results may yield different conclusions in the relationship between the optimal degree of commitment and degree of openness. In particular, under the hypothesis of $\rho_\epsilon > 0$, countries may have incentive to increase the degree of commitment because each country would like to free ride on the world stabilization activity provided abroad saving on credibility cost. This effect is particularly strong when the correlation among the economic shocks across countries is large. Moreover the degree of commitment is initially increasing and then decreasing in the degree of similarity in the socio-institutional environment in which the action of the banker is set and in the factors affecting the view that Cbs have about the nature and the convenience of stabilization in different states of the world. The prediction of the model developed here is that, coeteris paribus, we should observe more attention to the institutional solution in countries with similar social and institutional environments and where the conflict over the provision of stabilization at the world level arises from shocks that are positively correlated

(money demand or correlated tecnological shocks rather than shocks in the terms of trade). It is also worth noticing that after, an initial increase the conflict over the provision of worldwide stabilization tends to disappear as soon as the degree of homogeneity in the two countries becomes large enough ($\rho_k \to 1$).

Therefore the model provides some rationale for the spreading of the institutional solution to the credibility problem in the most developed countries. Despite the presence of an increasing trade, and possibly increasing variance in both nominal and real macroeconomic variables, all effects that would induce a rational government to relax the commitment of monetary policy to independent and conservative Cb, the last decades have also witnessed an increased degree of similarity in the economic shocks hitting the economies and an increased social and institutional homogeneity in the markets and the institutions defining the environment in which the action of the Cb is cast. A larger similarity in the economic shocks across unambiguously provides a rational government with a larger incentive to increase the degree of commitment. At the initial stages of increasing similarity in the social and institutional environment that provides the Cb with signals and pressures about the viability of the stabilization policy the incentive to precommitment gets further reinforced. However, as societies become more and more similar this second effect tends to disappear.

5 Conclusions

The last decades have been characterized by strong interest and debate on the institutional reforms of agencies in charge of monetary policy. The conventional wisdom has interpreted this reforms as the institutional solution to the credibility problem in monetary policymaking. However despite a credibility model, for example like the one provided in Romer (1993), predicts that the commitment of monetary policy is decreasing in the degree of openness, these reforms have taken place during a period where the degree of openness across most developed western country has increased. Moreover comparative statics results show that the degree of commitment should decrease after an increase in the variance of the output shock (like the oil shocks). As a conclusion the standard commitment model for open economies relies mainly on shifts in the preferences of the median voter in these countries as a force to explain the increasing attention to the institutional solution. In this paper we analyzed a simple two country model with monetary spillover incorporating the hypothesis of imperfect commitment and private information. Our findings can be summarized as follows: when the economy is open to trade,

the viability of the institutional solution to the credibility problem depends on the nature of the correlation structure across the economic shocks and the degree of similarity in the social and institutional environment surrounding the central bank. When the preferences in the two countries are similar enough (symmetric equilibrium) and the correlation between shocks to output is positive a conservative Cb results obtains in the case of open economies and the optimal degree of commitment is increasing in the degree of correlation across shocks to output. This result is due to the fact that, under the hypothesis on the correlation across the output shocks, stabilization policy provided in each single country is a public good and therefore, at the institutional design level each country tend to rely on the other to provide it by saving on credibility costs.

As suggested by Romer (1993) in his empirical and theoretical study on this issue, most developed countries have found some means of overcoming the problem of dynamic inconsistency of optimal monetary policy. These means mainly rely on institutional arrangements warranting Cb's independence and commitment to monetary targets, despite the increasing degree of openness among these countries would have suggested a relaxation of the optimal level of commitment (the more effective the "built in check", the lower the need for an institutional solution). The results derived in this paper show that in the presence of positive correlation between shocks to the level of economic activity and in the presence of similar social and institutional environment in which the action of the Cb is cast, strategic considerations at the institutional design stage will countervail the reduced incentive to precommit monetary policy coming from an increased degree of openness in open economies.

Most of the analysis was pointed to the cases that could provide an explanation based on the commitment approach for the conditions under which some countries even in the presence of increased trade may find in their interest to undertake reforms of monetary institutions pointing to more independence and to commitment of their Cbs to monetary targets. The model predicts that when the shocks to the level of output are negatively correlated (as in the case when the bulk of the relative variance in the output process is due to shocks in the terms of trade) the incentive to strategic precommitment may be reduced because of interdependence. Therefore the model predicts that it is also possible that open economies may *not* find convenient to undertake such reforms. A more proper analysis of this aspect and the empirical testing of the predictions of the current model is left for future work.

6 References

Benigno P. (1998), "A Simple Approach to International Monetary Policy Coordination", *Mimeo*, Princeton University.

Canzoneri M. and D. Henderson (1988), "Is Sovereign Policy Making Bad?", *Carnegie Rochester Conference Series*, **28**, pp. 93-140.

_ and _(1991) *Monetary Policy in Interdependent Economies: A Game Theoretic Approach*, The Mit Press, Cambridge, Mass.

Corsetti G. and P. Pesenti (1999), "Welfare and Macroeconomic Interdependence", *Mimeo*, Revised Draft of Nber W.P. No 6307, December 1997.

Cukierman A. (1993), *Central Bank Strategy, Credibility, and Independence. Theory and Evidence*, The Mit Press, Cambridge, Mass.

Dolado J., M. Griffith and J. Padilla, (1994), "Delegation in International Monetary Policy Games", *European Economic Review*, **38**, pp. 1057-1069.

Eijfinger S., Hoeberichts M. and E. Shaling, (1997), "Optimal Central Bank Conservativeness in an Open Economy", *mimeo*, Bank of England, London.

Eijfinger S.and E. Shaling (1995), "Optimal Commitment in an Open Economy: Credibility vs Flexibility", Bank of England Discussion Papers.

Fisher S. (1995), "Central Bank Independence Revisited", *American Economic Review Papers and Proceedings*, May, **85**, n.2, pp. 201-206.

Lohmann S. (1993), "Electoral Cycles and International Policy Cooperation", *European Economic Review*, **37**, pp.1373-1391.

McCallum B. (1995), "Two Fallacies Concerning Central-Bank Independence", *American Economic Review Papers and Proceedings*, May, **85**, n.2, pp. 207-211.

Muscatelli A. (1998), "Optimal Inflation Contracts and Inflation Targets with Uncertain Central Bank Preferences: Accountability through Independence?", *The Economic Journal,* **108**, pp.529-542.

Obstfeld and Rogoff (1995) "Exchange Rate Dynamics Redux", *Journal of Political Economy,* **103**, pp. 624-660.

Persson T. and G. Tabellini, (1993) "Designing Institutions for Monetary Stability": Carnegie-Rochester-Conference-Series-on-Public-Policy; 39, pp. 53-84.

_ and _, (1995), "*Double Edged Incentives: Institutions and Policy Coordination*", CEPR Discussion Paper, No. **1141**, London.

_ and _ (1996), "*Monetary Cohabitation in Europe*", CEPR Discussion Paper, No. **1380**, London.

Rogoff K. (1985a), "The Optimal Degree of Commitment to an Intermediate Monetary Target", *The Quarterly Journal of Economics*", **100**, pp.1169-1190.

_ (1985b) "Can International Monetary Policy Cooperation be Counterproductive?", *Journal of International Economics*, **18**, pp. 199-217.

Romer D. (1993) "Openness and Inflation: Theory and Evidence", *The Quarterly Journal of Economics,* **108** pp.869-903.

Romer D. (1998) "A New Assessment of Openness and Inflation: Reply", *The Quarterly Journal of Economics,* **113,** pp.649-652.

Schelling T. (1984), *Choice and Consequence*, Harvard University Press, Cambridge, Mass.

Svensson L. (1995), "Optimal Inflation Targets. 'Conservative' Central Banks, and Linear Inflation Contracts", *Cepr Discussion Paper, n.***1149**.

Terra C. (1998), "Openness and Inflation: a New Assessment", *The Quarterly Journal of Economics,* **113,** pp.641-648.

Walsh C. (1995), "Optimal Contracts for Independent Central Bankers", *American Economic Review,* **85**, n.1, pp. 150-167.

A Appendix

Proof of proposition 1.

We solve for linear type contingent strategies of the form $m_i(k_i) = A_i + B_i k_i$. In a Bayes-Nash equilibrium, money supplies have to satisfy the following condition, for $i \neq j$:

$$\begin{aligned} m_i^{BN}(k_i) &= \underset{m_i(k)}{Arg\max}\, E_{k_j}(W_i^G) \\ &= \int_{k_j} W_i^G \left\{p_i[(m_i(k_i), m_j(k_j)], x_i[(m_i(k_i)]\right\} f(k_j \mid k_i) dk_j \end{aligned} \tag{A1}$$

which, for both countries, can be written as follows:

$$\begin{aligned} (1+a)m_i - am_i^e - E_i(m_j(k) \quad &| \quad k_i) + am_j^e \\ &= \frac{\gamma\psi_i}{1+a}(1 - k_i\varepsilon_i) - b(\varepsilon_i - \varepsilon_j) \end{aligned} \tag{A2}$$

$$\begin{aligned} (1+a)m_j - am_j^e - E_j(m_i(k) \quad &| \quad k_j) + am_i^e \\ &= \frac{\gamma\psi_j}{1+a}(1 - k_j\varepsilon_j) + b(\varepsilon_i - \varepsilon_j) \end{aligned} \tag{A2'}$$

where, $E_k[m_j(k) \mid k_i] = A_j + B_j(1-\rho_k)(k_i - \hat{k})$ represents the conditional expectation that the CB of country i has about the money supply played by the CB in country j, given the normality assumption. Eqs.(A2) and (A2') can be rewritten as

$$\begin{aligned} m_i &= \underbrace{\frac{1}{1+a}\left[\frac{\gamma\psi_i}{1+a} + a(m_i^e - m_j^e) + a\alpha_j + a\beta_j(1-\rho_k)(k_i - \hat{k})\right]}_{A_i} \\ &\quad + \underbrace{\frac{1}{1+a}(a\beta_j\rho_k - \frac{\gamma\psi_i\varepsilon_i}{1+a})}_{B_i} k_i \end{aligned} \tag{A3}$$

a symmetric expression holds for country j.

It is then possible to solve jointly the following systems of equations:

$$\begin{aligned} B_i &= \frac{1}{1+a}(aB_j\rho_k - \frac{\gamma\varepsilon_i}{1+a}\psi_i) \\ B_j &= \frac{1}{1+a}(aB_i\rho_k - \frac{\gamma\varepsilon_j}{1+a}\psi_j) \end{aligned} \tag{A4}$$

and

$$\begin{aligned} A_i &= \frac{\gamma\psi_i}{(1+a)^2} + a(m_i^e - m_j^e) + \frac{a}{1+a}[A_j + B_j(1-\rho_k)(k_i - \hat{k})] \\ A_j &= \frac{\gamma\psi_j}{(1+a)^2} - a(m_i^e - m_j^e) + \frac{a}{1+a}[A_i + B_i(1-\rho_k)(k_j - \hat{k})] \end{aligned} \tag{A5}$$

Substituting A4 and A5 into A3 we can compute private agents expectations, given their information set ad the policy regime as follows

$$\begin{aligned} m_i^e &= \int_\Omega (A_i + B_i k_i) f(\varepsilon_i, \varepsilon_j) g(k_i, k_j) d\varepsilon_i d\varepsilon_j dk_i dk_j \\ m_j^e &= \int_\Omega (A_j + B_j k_j) f(\varepsilon_i, \varepsilon_j) g(k_i, k_j) d\varepsilon_i d\varepsilon_j dk_i dk_j \end{aligned} \tag{A6}$$

Solving systems (A4), (A5) and (A6) we get the following expressions for the undetermined coefficients

$$\begin{aligned} A_i &= \frac{\gamma}{1+a}\psi_i + \frac{a^2 B_i + a(1+a)B_j(1-\rho_k)}{1+2a}\overline{k} - \frac{b(\varepsilon_i - \varepsilon_j)}{1+2a} \\ A_j &= \frac{\gamma}{1+a}\psi_j + \frac{a^2 B_j + a(1+a)B_i(1-\rho_k)}{1+2a}\overline{k} + \frac{b(\varepsilon_i - \varepsilon_j)}{1+2a} \end{aligned} \tag{A7}$$

and

$$\begin{aligned} B_i &= -\frac{1}{\Lambda}(\frac{a}{1+a}\gamma\rho_k\psi_j\varepsilon_j - \gamma\psi_i\varepsilon_i) \\ B_j &= -\frac{1}{\Lambda}(\frac{a}{1+a}\gamma\rho_k\psi_i\varepsilon_i - \gamma\psi_j\varepsilon_j) \end{aligned} \tag{A8}$$

where $\Lambda = (1+a)^2 - a^2\rho_k^2 > 0$. Moreover, the solution for the rational expectations of the agents about the equilibrium money inflation rate, given their information set will be given by taking expectations of (A6), given (A7) and (A8), will be the following:

$$
\begin{aligned}
m_i^e &= \frac{\gamma\psi_i}{1+a} \qquad \text{(A9)}\\
m_j^e &= \frac{\gamma\psi_j}{1+a}
\end{aligned}
$$

■

Proof of proposition 2

The first order conditions for government i's problem is the following

$$
\begin{aligned}
\psi_i^{**} &= \underset{\psi_i}{ArgMax}\, E_{\varepsilon_i,\varepsilon_j,k_i,k_j}(W_i^G) = \qquad \text{(A10)}\\
&\int_\Omega \{-\frac{1}{2}\left[p_i^{BN}(\psi_i,\psi_j,\varepsilon_i,\varepsilon_j,k_i,k_j)\right]^2 + \psi_i^G(1-\overline{k}_i\epsilon_i)\times\\
&\times[x_i^{BN}(\psi_i,\psi_j,\varepsilon_i,\varepsilon_j,k_i,k_j)]\}f(\varepsilon_i,\varepsilon_j)g(k_i,k_j)d\varepsilon_i d\varepsilon_j dk_i dk_j
\end{aligned}
$$

by substituting the Bayes Nash equilibrium money supplies in (A10), by taking expectations and maximizing with respect to ψ_i, we get:

$$c_1\psi_i^{**} = -c_2(1-\rho_k^2)\rho_k\rho_\epsilon\sigma_\varepsilon^2\psi_j^{**} + c_3\sigma_\varepsilon^2\psi_i^G$$

that is (25) in the text, where, $c_1 = \frac{\gamma^2}{(1+a)^2} + \frac{\gamma^2(1+\bar{k}^2)[(1+a)^2-2a^2\rho_k^2]\sigma_\varepsilon^2}{\Lambda^2} + \frac{a^4\gamma^2(1+\rho_k^2\bar{k}^2)\rho_k^2\sigma_\varepsilon^2}{(1+a)^2\Lambda^2} > 0$, $c_2 = \frac{\gamma^2a^3}{\Lambda^2(1+a)} > 0$, $c_3 = \frac{\gamma^2\bar{k}^2}{1+2a} > 0$.

To get the solution in the symmetric equilibrium set $\psi_i = \psi_j$ and get $\tilde{\psi}_i = c_4\psi_i^G$, that is (26) in the text, where $c_4 = \frac{(1+a)^2}{(1+2a)}\frac{\Lambda^2 d_0}{D}$, $D = \Lambda^2 + (1+a)^2[\Lambda^2 - a^2\rho_k^2]d_1 + a^4d_2 + a^3(1+a)d_3$ and $\Lambda = [(1+a)^2 - a^2\rho_k^2]$, $d_0 = \overline{k}^2\sigma_\varepsilon^2$, $d_1 = (1+\overline{k}^2)\sigma_\varepsilon^2$, $d_2 = (1+\rho_k^2\overline{k}^2)\rho_k^2\sigma_\varepsilon^2$, $d_3 = (1-\rho_k^2)\rho_k\rho_\varepsilon\sigma_\varepsilon^2$ ■

Proof of Proposition 3.

Compute the expected welfare in country i and differentiate it with respect to ψ_j and get

$$
\begin{aligned}
\frac{dEW_G^i}{d\psi_j} &= \frac{\gamma^2\bar{k}^2[a^3(1-\rho_k^2)+a(1+2a)]}{\Lambda(1+2a)(1+a)}\rho_\varepsilon\sigma_\varepsilon^2\psi_i^G \qquad \text{(A11)}\\
&-\frac{\gamma^2a^2(1-\rho_k^2)}{\Lambda^2}[\frac{a}{1+a}\rho_k\rho_\varepsilon\sigma_\varepsilon^2\psi_i + \sigma_\varepsilon^2\psi_j]
\end{aligned}
$$

Let us discuss two cases by evaluating it at the symmetric equilibrium $\psi_i = \psi_j$:

a) $\rho_\varepsilon > 0$; in this case the first term in (A11) is positive and measures the marginal benefit from increased externality from stabilization, the second one is the marginal welfare cost due to the increased variance of inflation. After some algebra it is possible to show that a sufficient condition for the first term to dominate is $\rho_\varepsilon > \frac{a(1+a)}{1+2a}$.

b) $\rho_\varepsilon < 0$, in this case the first term of (A11) is negative. The second term is also negative since $(\frac{a}{1+a}\rho_k\rho_\varepsilon + 1)\psi_i$ is positive ■

Proof of Proposition 4.

If $\rho_\varepsilon > 0$ a sufficient condition for the result is $c_1 > c_3$ in (25) that is easily verified after some algebra, given that, $0 < a < 1$ and $\bar{k}^2\sigma_\varepsilon^2 < 1$ ■

Proof of Proposition 5.

To prove result 1 is immediate since, under the assumptions $0 < a < 1$ and $\bar{k}^2\sigma_\varepsilon^2 < 1$, the following expression holds true:

$$\frac{(1+a)^2}{(1+2a)}\frac{\Lambda^2 d_0}{\{\Lambda^2 + (1+a)^2[\Lambda^2 - a^2\rho_k^2]d_1 + a^4 d_2 + a^3(1+a)d_3\}} < 1$$

To prove result 2 and 3, consider that $\psi_i^* - \psi_i^{**} > 0$ can be rewritten as

$$(1-\rho_k^2)a^3[a\rho_k^2\sigma_\varepsilon^2 + (1+2a)\rho_k\rho_\varepsilon\sigma_\varepsilon^2] > 0$$

that holds true for $\rho_\varepsilon > 0$. For $\rho_\varepsilon < 0$ the inequality holds for $a\rho_k > (1+2a) \mid \rho_\varepsilon \mid$ ■

Proof of Proposition 6.

To prove result 1', notice that in a symmetric equilibrium

$$\frac{d\tilde{\psi}_i}{d\rho_\varepsilon} = -\frac{\bar{k}^2\Lambda^2(1+a)^3(1+2a)a^3(1-\rho_k^2)\rho_k}{D^2}$$

is negative for $\rho_\varepsilon > 0$ and positive otherwise.

To prove result 2' requires different steps.

Step 1. Consider $\tilde{\psi}_i$ as a function of ρ_k. It is immediate to show, since $D > 0$, that $\tilde{\psi}_i$ is continuous for $\rho_k \in [0,1]$. We also know from proposition 5 that $\forall\rho_k \in [0,1]$, $\tilde{\psi}_i \leq \psi_i^*$.

Step 2.Compute the limit of $\tilde{\psi}_i$ at the boundary of $\rho_k \in [0,1]$ and get the result $\lim_{\rho_k\to 0}\tilde{\psi}_i = \lim_{\rho_k\to 1}\tilde{\psi}_i = \psi_i^*$.

Step 3. Compute the derivative $\frac{d\tilde{\psi}_i}{d\rho_k} = \frac{(1+a)^2 d_0\psi_i^G}{1+2a}\Lambda\frac{2\frac{d\Lambda}{d\rho_k}D - \Lambda\frac{dD}{d\rho_k}}{D^2}$ and notice that: $\lim_{\rho_k\to 0}\frac{d\tilde{\psi}_i}{d\rho_k} < 0$ and $\lim_{\rho_k\to 1}\frac{d\tilde{\psi}_i}{d\rho_k} > 0$, so that $\tilde{\psi}_i$ as a function of ρ_k is decreasing for $\rho_k \to 0$ and increasing for $\rho_k \to 1$ and always lies below ψ_i^*.

Therefore to get result 2', it remains to be shown that $\frac{d\tilde{\psi}_i}{d\rho_k}$ changes sign only once in the relevant range. This is addressed in the next step.

Step 4. To show that $\rho_\varepsilon > 0$ is a sufficient condition for the existence of $0 < \bar{\rho}_k < 1$ such that $\frac{d\tilde{\psi}_i}{d\rho_k} < 0$, $\forall \rho_k \in [0,\ \bar{\rho}_k]$, and $\frac{d\tilde{\psi}_i}{d\rho_k} > 0$ $\forall \rho_k \in [\ \bar{\rho}_k, 1]$, notice that $\frac{d\tilde{\psi}_i}{d\rho_k} = \frac{\Gamma}{D^2} P(\rho_k)$, where $\Gamma = \frac{(1+a)^2 d_0 \psi_i^G}{1+2a}\Lambda > 0$ and

$$\begin{aligned} P(\rho_k) &= a^5(1+a)\rho_\varepsilon \rho_k^4 + 2a^4[2(1+a)^2 - 2a^2]\rho_k^3 \\ &\quad +3a^3[(1+a)^3 - a^2(1+a)]\rho_\varepsilon \rho_k^2 - \\ &\quad 2a^4(1+a)^2\rho_k - a^3(1+a)^3\rho_\varepsilon \end{aligned}$$

is a fourth degree polynomial in ρ_k. By applying Descartes rule, we notice that for $\rho_\varepsilon > 0$ the coefficient of the polynomial change sign only once, therefore there will exist one single positive real root $\bar{\rho}_k$ such that $P(\rho_k) = 0$. By definition of $P(\rho_k)$, any root of $\frac{d\tilde{\psi}_i}{d\rho_k} = 0$ must be a root of $P(\rho_k) = 0$ and viceversa. Therefore, by continuity, $\frac{d\tilde{\psi}_i}{d\rho_k}$ can change sign only once. But since we know, frome step 3 that it is decreasing for $\rho_k \to 0$ and increasing for $\rho_k \to 1$, therefore it must be the case that $0 < \bar{\rho}_k < 1$ and $\frac{d\tilde{\psi}_i}{d\rho_k} < 0$, $\forall \rho_k \in [0,$ $\bar{\rho}_k]$, and $\frac{d\tilde{\psi}_i}{d\rho_k} > 0$ $\forall \rho_k \in [\ \bar{\rho}_k, 1]$.

Finally we prove result 3'. We notice that $\psi_i^* - \psi_i^{**}$ as a function of a can be rewritten as $(1-\rho_k^2)a^3[a\rho_k^2\sigma_\varepsilon^2 + (1+2a)\rho_k\rho_\varepsilon\sigma_\varepsilon^2]$, that is continuous, increasing and positive for $a \in (0, 1]$ ■