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Criminal Networks, Market Externalities and Optimal Leniency

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Criminal Networks, Market Externalities and Optimal Leniency

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Abstract
We analyze the relationship between competition and self-reporting incentives within a criminal network formed by a supplier of an illegal good and two dealers distributing the good to final consumers. The Legislator designs a leniency program to deter crime. We show that the comparison between the optimal amnesty with competition and monopoly in the dealership market depends on the strength of the externalities between dealers at the reporting stage. While in monopoly a leniency program is al- ways feasible, the opposite may happen with competition. This impossibility result is more relevant when the demand for the illegal product is large, when the market is neither too competitive nor too concentrated and when dealers know too much about each other. Moreover, in contrast to monopoly, the policy does not necessarily increase welfare in a competitive environment.

Keywords: Accomplice-witnesses, Criminal Organizations, Leniency, Whistle-Blower

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1 Introduction

Organized crime has existed in various parts of the world for centuries. However, only recently, pressured by the growing influence of criminal organizations into the polity — see, e.g., Acemoglu et al. (2013), Alesina et al. (2018) and Barone and Narciso (2015) among many others — governments all over the world have been forced to reform their legal and judicial systems to strengthen deterrence. A notable example is the introduction in many countries of accomplice-witness regulations (also known as leniency programs) whose aim is to encourage criminals to blow the whistle and cooperate with prosecutors in exchange for legal benefits.

The existing literature has examined various aspects of the optimal design of leniency programs intended to fight organized crime; for example, by considering the effects of these programs on: the criminals' propensity to subvert the law; the timing of their self-reporting decision; the link with their organization structure; the incentives of the whistleblowers to disclose their insider information; etc.[1] But these models are silent about the effects of competition between local dealers in the markets for illegal goods on their incentive to blow the whistle. Are competitive criminal networks more or less fragile than monopolistic ones? What are the externalities that dealers exert on their rivals and suppliers of illegal goods? How can a Legislator take advantage of these externalities to deter crime?

To answer these questions we analyze the relationship between competition and the incentives to blow the whistle within a criminal network which we model as a vertical supply-chain. We develop a simple model involving a benevolent Legislator and a criminal network formed by an upstream supplier of an illegal good — e.g., illegal drugs, guns, ammunition, counterfeit products etc. — and two local dealers who distribute the supplier's product in the final market. The Legislator designs a leniency program to minimize the total quantity of illegal goods.

Following Segal (1999) and the most recent IO literature (see, e.g., Rey and Tirole, 2007 and Segal and Whinston, 2003, among others) we assume that the supplier contracts with all dealers simultaneously and that a dealer does not observe the contract offered to his rival (secret offers). Every dealer is offered a quantity to be distributed in the final market and a tariff which must be paid to the supplier. The key assumption of our model is that as dealers sell more, they attract more attention from law enforcers — i.e., the probability of each dealer being prosecuted is increasing with the quantity of illegal goods that he smuggles. If a dealer is prosecuted, he decides whether or not to blow the whistle in exchange for an amnesty that depends on whether the rival also cooperates with justice. To capture the idea that prosecutors can play dealers one against the other in order to increase the conviction's risk for the whole network — including the supplier who is usually less exposed to prosecution

[1] See, e.g., Acconcia et al. (2014), Landeo and Spier (2018a, 2018b), Piccolo and Immordino (2017), Gamba et al. (2018) and Immordino et al. (2018).

— we assume that each dealer owns information about the other members of the network.

Under the assumption of quantity forcing contracts and 'passive beliefs' off equilibrium path, we characterize a symmetric Perfect Bayesian Nash Equilibrium in which every dealer distributes the same amount of illegal product. This equilibrium features a standard opportunism problem: the supplier cannot monopolize the downstream market because committing to deliver the monopoly outcome (to both dealers) would generate the temptation to secretly increase the sales of one dealer at the other's expense. Within this context, our objective is to study whether competition facilitates or hardens the introduction of a leniency program compared to monopoly.

First, we show that the comparison between the optimal amnesty with a competitive dealership market and with monopoly is ambiguous and relies on the model's underlying parameters. Specifically, when the informational externality between the dealers is strong enough, it is easier for the Legislator to induce cooperation in a competitive setting rather than in monopoly. The reason being that it is relatively more convenient for each dealer to engage in a rush to the courthouse and 'share' the amnesty with his competitor rather than facing the risk of being betrayed by him. Hence, the equilibrium amnesty is lower with competition than with monopoly. The opposite is true if the informational externality is weak.

Second, we show that while a leniency program is always feasible with monopoly, it might not be in a competitive environment: an impossibility result. This is because when more dealers simultaneously decide to cooperate with justice, the first in line at the prosecutor's office is randomly determined (see, e.g., Harrington Jr, 2008 and Chen and Rey, 2013). Hence, even when full amnesty is granted, a dealer's expected sanction is always positive when he blows the whistle and, depending on the underlying parameters, could be higher than the expected sanction obtained when the dealer decides to remain silent and face the trial. In this latter case the Legislator is unable to induce both dealers to blow the whistle. Such impossibility result is more relevant when the information externality is not too large since, in this case, the risk of being convicted for a dealer who decides not to blow the whistle is (relatively) small. An interesting implication is that the introduction of leniency programs should be complemented by policies targeting final users (consumers) of illegal products in addition to their suppliers — e.g., harsher punishments for drug consumption. Our analysis suggests that these policies should be seen as complements rather than substitutes.

Third, while in monopoly the policy is always welfare enhancing — i.e., it always reduces crime — in competition it may not do so when criminals are poorly informed on the other members of the network and when the law enforcement system is sufficiently effective. In this case, by reducing the dealers' marginal sanctions, the introduction of a leniency program reduces the supplier's marginal cost of producing the illegal good, thus increasing production and harming welfare.

Finally, we show that monopoly is always preferable to competition although in competition dealers can be played one against the other. Indeed, the supply of illegal goods in competition is always larger than in monopoly suggesting that criminal supply chains with monopoly both upstream and downstream are easier to fight although the amnesty required in a monopolistic setting may be lower than that required in competitive environments. This result may explain why although groups with a strong hierarchical structure, continue to be involved in the drug trade, there is evidence that looser, horizontal networks are becoming increasingly significant. In 2017, Europol estimated that such networks accounted for 30-40 per cent of organized crime groups operating in the European Union (United Nations Office on Drugs and Crime: World Drug Report 2017).

We then extend the baseline analysis by considering: (*i*) how our results change when instead of quantity forcing contracts the supplier uses simpler linear tariffs; (*ii*) how the supplier's ability to overcome the opportunism problem (through public contracts) affects the Legislator's behavior; and (*iii*) the effect of the number of dealers in the retail market on the feasibility of the program. We show that welfare is higher with linear pricing than with more general quantity forcing contracts, because double marginalization reduces the equilibrium level of illegal goods and makes it easier to induce cooperation. The same effect is at play when comparing secret versus public offers since overcoming the opportunism problem leads to soften competition in the dealership market, which triggers a drop in total quantity. Hence, surprisingly, social welfare is higher when suppliers feature strong commitment power — i.e., more trust among the members of the criminal network increases social welfare. Finally, we argue that the size of the network has a non-monotone effect on the Legislator's ability to introduce a leniency program and that this is hardest when the downstream market is neither too concentrated nor too competitive.

Background and motivating evidence. Our model fits many illegal markets in which products that are forbidden by law are introduced in the economy by upstream suppliers using local dealers for delivery to final consumers.[2] Clearly, one of the most relevant among these markets is drug trafficking. Indeed, organized crime groups have been involved in international drug trades for almost 100 years and drugs continue to play an important role in organized crime groups. The 2010 estimate in the European Union was equivalent to roughly 25 per cent of overall criminal proceeds, making drugs the third largest source of income from organized crime after tax fraud and counterfeiting (United Nations Office on Drugs and Crime, World Drug Report 2017). Compared with organized crime groups operating in other areas, such as financial and economic crime, property crime and counterfeiting, organized crime groups involved in drug trafficking tend to be larger (Europol, 2013). In terms of criminal trafficking networks, Colombia and Mexico provide two important examples. In

[2]More generally, our model can be applied not only to the offenses enacted by traditional criminal organizations like the mafias, but also to crimes committed by groups of wrongdoers rather than by individuals acting in isolation, with the key requirement of these organizations being organized vertically — e.g., terrorism, financial frauds, corporate crimes, collective tax evasion, etc.

Colombia, the rise and fall of the Medellin and Cali cartels illustrate the vulnerabilities of large, hierarchical criminal trafficking organizations. Both major cartels in Colombia were hierarchically structured and proved to be vulnerable targets for Colombian and international law enforcement agencies (Bagley, 2013). Indeed, hierarchical structures have a major weakness: they can be dismantled when detected by the authorities and this motivates our information structure.

The evolution of trafficking routes, the gradual shift towards a global perspective may explain why in drug markets traffickers of different groups are distributed along the supply chain. The trafficking in illicit drugs from producer countries (e.g., Afghanistan, Colombia, etc.) to consumer markets requires a certain level of organization and the participation of several actors with different roles along the supply chain. Although the drugs market produces a large amount of money, the presence of numerous competitors entails that many drug suppliers are unable to apply high profit margins and do not have the power to charge more than their marginal costs. Indeed, several studies (see, e.g., Reuter et al., 1990 and Levitt and Venkatesh, 2000) have shown that many of the criminals involved in drug trafficking, especially those at the bottom of the supply chain, make modest incomes. It is a well known fact that profits connected to the trafficking, low entry barriers, and repression by law enforcement agencies make the illicit drugs market a competitive business with numerous competitors (see, e.g., Pearson et al., 2001 and Kenney, 2007). Although some criminal organizations (for instance Colombians for cocaine) have a dominant position in the import and wholesale distribution of drugs (Paoli and Reuter, 2008) there are numerous local groups which compete with several others in order to smuggle drugs (Savona and Riccardi, 2015). This important feature of the criminal market motivates our choice of modeling criminal networks as a vertical supply chaing in which the upstream supplier has full bargaining power vis-à-vis local dealers who (re)sell the upstream product to final consumers.

Related literature. Our analysis is related to the strand of literature on organized crime. Jennings (1984), Polo (1995), Konrad and Skaperdas(1997, 1998) and Garoupa (2000) started to model criminal organizations as vertical structures, whose heads need to discipline their fellows with implicit rewards and credible threats (see, e.g., also Baccara and Bar-Isaac (2008) who consider both vertical and horizontal organizations). More related to our work Kugler et al. (2005) analyze an oligopoly model in which criminal organizations compete on criminal activities and engage in corruption.[3] However all these models are silent on the role of leniency programs as a tool to generate conflicts within criminal organizations, which is instead the building block of our analysis.

The idea of applying leniency programs to fight organized crime builds upon the antitrust law enforcement literature, which studies the effects of reduced sanctions on cartel formation in oligopolistic markets — see, e.g., Motta and Polo (2003) and Spagnolo (2003), Rey (2003),

[3]See also Mansour et al. (2006).

Spagnolo (2008), Aubert et al. (2006), Chen and Harrington Jr (2007), Chen and Rey (2013) and Harrington Jr (2008) among many others.[4] The main difference between this literature and papers that deal with organized crime is that while cartels are horizontal institutions, criminal organizations are typically hierarchical. The optimal design of leniency programs meant to fight organized crime has recently been discussed in Acconcia et al. (2014), who also provide an empirical analysis of the phenomenon, Piccolo and Immordino (2017), who emphasize the benefits and the costs of these programs when whistle-blowers can hide their insider information, Gamba et al. (2018), who study the costs and benefits of leniency programs when corruption is a potential danger and Immordino et al. (2018), in which the organizational structure of criminal organizations is endogenous and determined jointly with the leniency program. More recently, Landeo and Spier (2018a, 2018b), have examined, both theoretically and experimentally, the concept of 'order leniency' mechanisms where the degree of leniency granted to each criminal is determined by his position in the self-reporting queue. They show that granting leniency to the first informant generates a race to the court house which is also at the heart of our analysis, even though we do no explicitly allow for order leniency. Differently than us, however, all these models are silent about the relationship between competition in the dealership markets and the effectiveness of leniency programs.

The rest of the paper is organized as follows. In Section 2 we set up the baseline model, provide the equilibrium analysis and determine the optimal policy. In Section 3 we extend the model by considering linear tariffs, public contracts and a more general market structure in the retail industry. Section 4 concludes. Proofs are in the Appendix.

2 The baseline model

The game involves a benevolent Legislator and a criminal 'network' formed by an upstream supplier (hereafter U) of an illegal good — e.g., illegal drugs, guns, ammunition, counterfeit products etc. — and 2 dealers (each denoted by D_i) converting each unit of the supplier's product into one unit of the final good (in Section 3 we consider the general case of $N \geq 2$ dealers). The Legislator, having forbidden welfare reducing acts, designs a leniency program with the goal of deterring crime — i.e., to minimize the total quantity of illegal goods smuggled in the economy.

We denote by x_i the quantity smuggled by dealer i. The downstream demand function is $P(x_1 + x_2)$, with $P'(\cdot) < 0$. Following the previous literature, the upstream supplier contracts with all dealers simultaneously — see, e.g., Segal (1999). Contracts are secret: a dealer does not observe the contracts that the supplier offers to the rival (we relax this assumption in Section 3). The supplier offers quantity-forcing contracts (x_i, t_i): dealer D_i is

[4] Related and more recent models are offered by Harrington (2013), Sauvagnat (2014, 2015) and Silbye (2010).

offered a quantity x_i to be distributed to the final consumers and a tariff t_i that has to be paid to the supplier (in Section 3 we also consider linear pricing).

D_i is prosecuted with probability $\rho(x_i) \in [0,1]$, with $\rho'(\cdot) > 0$ and $\rho''(\cdot) \geq 0$ — i.e., a dealer's prosecution risk is increasing in the quantity of illegal goods he smuggles.

Without loss of generality, sanctions are normalized to 1 for all criminals. Yet, the conviction technology depends on the dealers' reporting behavior. For simplicity we rule out the possibility that U self reports and that a dealer decides to cooperate before an investigation opens. The possibility of spontaneous cooperation seems realistic for cartels and corruption cases, but less so in our organized crime framework.[5] As a matter of fact, the anecdotal evidence suggests that criminals accept to cooperate with justice only after being captured. This may reflect, for example, psychological costs (not modeled for simplicity) resulting from apprehension and the fear of imprisonment, which materialize only when a criminal is about to face the trial.[6] Alternatively, one could imagine that while a whistleblower may see self-reporting as a 'risky bet', he could learn to trust the prosecutors ruling his case once he is captured, or even be persuaded to cooperate with them.

If D_i does not blow the whistle, he is convicted with probability $q + k\theta$, where $k \in \{0, 1\}$ is an indicator function which is equal to 1 if the other dealer blows the whistle, and 0 otherwise. The parameter q is the baseline probability of conviction when prosecutors cannot rely on whistleblowers' testimonies. The parameter θ measures the accuracy of the dealers' testimony and captures the *informational externality* that a whistleblower imposes on his rival (when the latter does not cooperate with justice). The idea being that each dealer owns information on the other members of the network, and that those pieces of information complement each other. For the conviction probability to be smaller than 1 we assume that $\theta \leq 1 - q$.

By contrast, if D_i blows the whistle, he enjoys an amnesty $\phi(k)$ on the original sanction that depends on whether the other dealer cooperates ($k = 1$) or not ($k = 0$). Following Harrington Jr (2008) and Chen and Rey (2013) it is assumed that if more dealers simultaneously decide to apply for leniency, they each obtain an amnesty[7]

$$\phi(k) \triangleq \tfrac{1}{1+k}\phi.$$

Conditional on $n \in \{0, 1, 2\}$ dealers blowing the whistle, the upstream supplier is convicted with probability $n\theta$, with $\theta \leq \frac{1}{2}$. For simplicity, we do not model ordered leniency

[5] In the antitrust law enforcement literature Spagnolo (2003), and the recent experimental studies by Hinloopen and Soetevent (2008) and Bigoni et al. (2012) among others have clarified that there may be a marked difference between ex-post and ex-ante leniency.

[6] In criminal proceedings the imprisonment of defendants is mandatory even before the definitive verdict. In addition, the trial can be very long depending on the importance of the charges and the number of defendants. — see, e.g., Acconcia et al. (2014).

[7] One interpretation of this simple rule is that who is first in line at the prosecutor's office is randomly determined with equal probability.

which is extensively analyzed in Landeo and Spier (2018a, 2018b).[8]

Players are risk neutral. All sanctions will be interpreted as the monetary equivalent of the imprisonment terms, fines, damages, and so forth, to which the criminals expose themselves.[9] For simplicity, we assume that whistleblowers are not exposed to retaliation by their former partners (see Remark 1 for a discussion on retribution costs).

The timing of the game is as follows.

t=0 The Legislator commits to an amnesty $\phi \in [0, 1]$.

t=1 U simultaneously offers contracts to the dealers. Every dealer D_i who accepts U's contract obtains the quantity x_i and pays the tariff t_i accordingly.

t=2 Dealers sell their quantities in the final market and profits realize.

t=3 With probability $\rho(x_i)$ dealer D_i is prosecuted.

t=4 If D_i is prosecuted, he decides whether to blow the whistle or to remain loyal to the criminal network (without knowing the rival's behavior). The trial uncertainty resolves and sanctions are imposed.

The equilibrium that we characterize is equivalent to the one of a game in which dealers set prices but are capacity constrained in the downstream market, because the supplier produces to order before prices are set and final demand realizes (Rey and Tirole, 2007).[10] Essentially, price competition with capacity constraints as in Kreps and Scheinkman (1983) leads to a Cournot outcome.

We consider a Perfect Bayesian Nash Equilibrium with the standard 'passive beliefs' refinement (Hart et al., 1990; McAfee and Schwartz, 1994; Rey and Tirole, 2007). With passive beliefs and multiple dealers, a dealer's conjecture about the contracts offered to other dealers is not influenced by an out-of-equilibrium offer he receives. This is a natural refinement for games with secret contracting and production to order because, from the perspective of the upstream supplier, each dealer forms a separate market (Rey and Tirole, 2007). In addition, as shown by McAfee and Schwartz (1994), passive beliefs correspond to wary beliefs in our game.[11]

The analysis will be conducted under the following conditions:

[8]However, our qualitatively insights would not change even with ordered leniency as long as the introduction of the policy creates a race to the court-house (more below).

[9]This assumption is made only for the sake of simplicity. Our insights readily extend to non-monetary sanctions as long as their cost is not excessively large, in which case granting a positive discount is still optimal (see Garoupa, 1997 for an overview on optimal law enforcement with non-monetary sanctions).

[10]Alternatively, one could imagine that a downstream firm cannot instantaneously reorder the supplier's product and satisfy customers when their demand is larger than expected, or reduce it when demand is unexpectedly low.

[11]With quantity competition, equilibria with passive beliefs are equivalent to contract equilibria (Cremer and Riordan, 1987)—i.e., each equilibrium with passive beliefs is also a contract equilibrium and vice versa. By contrast, with price competition the two equilibrium concepts are not equivalent.

Assumption 1. $P(0) > \rho'(0)$.

This assumption guarantees that production is always positive since the (maximum) market price $P(0)$ is larger than the maximal marginal sanction.

Assumption 2. The inverse demand function satisfies the following conditions:

(i) $2P'(x) + P''(x)x < -k$, with $k > 0$ and sufficiently large;

(ii) $\lim_{x\to+\infty} P(x) = 0$ and $|P'(x)| < +\infty$ for every x.

Part (i) of Assumption 2 implies that all profit functions are strictly concave and that quantities are strategic substitutes.[12] Part (ii) ensures that the equilibrium market price is zero as the quantity gets unbounded and that the equilibrium quantity is positive.

Assumption 3. Let $\overline{x} \triangleq \rho^{-1}(1)$, then

$$P(\overline{x}) + \overline{x}P'(\overline{x}) \leq \rho'(\overline{x}) \min\{q, \theta\}.$$

This assumption implies that it is never optimal to choose a relatively high production because (for large sales) the change in revenue generated by an extra unit of illegal product smuggled in the market is lower than the corresponding change in conviction risk (i.e., the cost associated with an extra unit of production).

Every player's outside option is normalized to zero. We will focus on a symmetric equilibrium of the game in which: every dealer distributes the same amount of illegal product, and (conditional on being under investigation) they either all blow the whistle or remain loyal to the organization.

2.1 Monopoly benchmark

We first solve the case of a monopolistic criminal network in which the supplier uses one dealer only. Clearly, when there is no leniency program, the expected sanction faced by the monopolistic dealer is $\rho(x)q$ — i.e., the probability of being investigated times the probability of being convicted. Hence, it can be easily shown that U requires a transfer $t^M_{\varnothing}(x) \triangleq P(x)x - \rho(x)q$, and solves the following maximization problem

$$\max_{x\geq 0} P(x)x - \rho(x)q,$$

whose first order condition

$$P(x) + P'(x)x = \rho'(x)q,$$

yields the monopoly quantity $x^M_{\varnothing}$ without leniency.

[12] This is a standard assumption in games with quantity competition (see e.g.,Vives, 2001).

Clearly, the higher the conviction risk for the dealer — i.e., the larger q — the lower the surplus that the supplier can extract since the dealer needs to be compensated with a lower transfer. Therefore, since the probability of investigation depends on the quantity of illegal goods smuggled by the dealer, a higher conviction risk also increases the marginal sanction, which is equivalent to the supplier facing a higher marginal cost, whereby decreasing the optimal quantity.

Suppose now that a leniency program is in place. Given an amnesty ϕ announced by the Legislator, the monopolistic dealer blows the whistle if and only if (conditional on being investigated) the expected utility of remaining loyal to the organization is lower than the expected utility of cooperating with justice — i.e.,

$$-q \leq -(1-\phi) \quad \Leftrightarrow \quad \phi \geq 1-q.$$

Essentially, the monopolistic dealer blows the whistle only when the probability of being acquitted $1-q$ is lower than the fraction of the sanction ϕ waived by the Legislator in case of cooperation.

Suppose that condition $\phi \geq 1-q$, since the dealer's outside option has been normalized to zero, the optimal transfer required by the supplier is

$$t^M(x) \triangleq \underbrace{P(x)\,x}_{\text{Revenue}} - \underbrace{\rho(x)(1-\phi)}_{\text{Exp. sanction}},$$

which, as intuition suggests, is increasing in the downstream revenue and decreasing in the expected sanction.

Moreover, since the dealer blows the whistle in the candidate equilibrium under consideration, U's expected sanction is $\rho(x)\,\theta$, which is increasing both in the amount x of illegal product smuggled into the market and in the accuracy of the dealer's testimony θ.

Hence, U solves the following maximization problem

$$\max_{x \geq 0} P(x)\,x - \rho(x)(\theta + 1 - \phi).$$

Differentiating with respect to x, the monopoly quantity when a leniency program is in place, $x^M(\phi)$ solves the following first order condition

$$\underbrace{P(x) + P'(x)\,x}_{\text{Marginal revenue}} = \underbrace{\rho'(x)(\theta + 1 - \phi)}_{\text{Marginal sanctions}}. \tag{1}$$

This expression has a simple interpretation: the supplier chooses the amount of illegal goods that equalize marginal benefits and marginal costs. Recall that in our context marginal costs reflect the impact of a larger level of production on the supplier's expected sanction and on the dealer's break-even transfer.

We can show the following result.

Lemma 1 $x^M(\phi)$ *is increasing in* ϕ.

The intuition is straightforward. When the amnesty ϕ grows large, the dealer's expected sanction decreases, so the supplier will charge him a higher transfer anticipating the lower sanction. This reduces the marginal cost of producing the illegal good, whereby increasing production: a *dark side of leniency* that echoes the findings of Acconcia et al. (2014), Piccolo and Immordino (2017), Gamba et al. (2018) and Immordino et al. (2018).

Moving backward to the first stage of the game, the Legislator's problem is to choose the amnesty ϕ that minimizes $x^M(\phi)$ subject to $\phi \geq 1-q$. The next result describes the solution of this problem and the optimal policy under monopoly.

Proposition 1 *The Legislator always finds optimal to introduce a leniency program. The optimal amnesty is* $\phi^M = 1-q$ *and the amount of illegal products introduced in the economy is* $x^M \triangleq x^M\left(\phi^M\right)$, *with*

$$P\left(x^M\right) + P'\left(x^M\right)x^M = \rho'\left(x^M\right)(\theta + q). \tag{2}$$

Intuitively, since the quantity of the illegal product is increasing with the amnesty, the Legislator chooses the lowest possible amnesty that induces the dealer to blow the whistle. By increasing the conviction risk for the whole organization, this strategy rises U's marginal cost of production and is therefore always welfare enhancing since it reduces the amount of illegal products introduced in the economy.

2.2 Competition in the downstream market

We now turn to analyze the case of competition in the downstream market. As before, we first briefly describe what would be the outcome of the game in the absence of a leniency program. Consider a symmetric equilibrium in which each dealer smuggles $x^*_\varnothing$ units of the illegal product. The transfer required by U to D_i is then

$$t_i\left(x_i, x^*_\varnothing\right) \triangleq P\left(x_i + x^*_\varnothing\right)x_i - \rho\left(x_i\right)q.$$

Hence, since contracts are secret, the bilateral problem between U and R_i (see, e.g., Rey and Tirole, 2007) is

$$\max_{x_i \geq 0} P\left(x_i + x^*_\varnothing\right)x_i - \rho\left(x_i\right)q,$$

whose first-order condition, after imposing symmetry is

$$P'\left(2x^*_\varnothing\right)x^*_\varnothing + P\left(2x^*_\varnothing\right) = \rho'\left(x^*_\varnothing\right)q,$$

which immediately yields $x^*_\varnothing < x^M_\varnothing$ since the supplier faces the opportunism problem when the downstream market is competitive. Notice that, under the hypothesis of secret contracts, the supplier maximizes the bilateral profit with each dealer, which implies that he does not internalize the effect of selling an additional unit to a dealer on the rival's profit. Hence, the dealers only accept contracts with the 'Cournot' quantity (since otherwise each of them would expect the supplier to secretly sell a larger quantity to the rival). This prevents the supplier from achieving the monopoly profit: the opportunism problem.

Next, suppose that a leniency program is in place. Consider again a symmetric equilibrium in which both dealers produce x^* units of the illegal product and blow the whistle if they are investigated. An equilibrium with these features exists if and only if the discounted sanction that each dealer bears by blowing the whistle ($y_i = 1$) is lower than the expected sanction he bears remaining loyal to the network ($y_i = 0$). That is:

$$\underbrace{\mathrm{E}\left[1 - \phi\left(k\right)\right]}_{y_i=1} \leq \underbrace{\mathrm{E}\left[q + k\theta\right]}_{y_i=0}, \tag{3}$$

where expectations are taken over the event that the other dealer is investigated.[13] We have

$$\mathrm{E}\left[1 - \phi\left(k\right)\right] \triangleq \underbrace{(1 - \rho\left(x^*\right))(1 - \phi)}_{k=0} + \underbrace{\rho\left(x^*\right)\left(1 - \tfrac{\phi}{2}\right)}_{k=1}. \tag{4}$$

Notice that the amnesty granted by the Legislator decreases with the number of whistleblowers — i.e., it drops from from ϕ when $k = 0$ to $\frac{\phi}{2}$ when $k = 1$.

Similarly, we have

$$\mathrm{E}\left[q + k\theta\right] \triangleq \underbrace{(1 - \rho\left(x^*\right))q}_{k=0} + \underbrace{\rho\left(x^*\right)\left(q + \theta\right)}_{k=1}. \tag{5}$$

These equations have an interesting interpretation. Condition (4)) reflects the so-called '*rush-to-the-courthouse*' effect, which (other things being equal) tends to reduce the individual prize for cooperation: with more than one dealer, the Legislator gains the ability to induce the network members to self-report and betray their partners for fear of being otherwise betrayed by them.[14] Condition (5)), instead, reflects a *domino effect* à la Baccara and Bar-Isaac (2008): since each whistleblower provides evidence not only against the boss but also against his rival dealer, when deciding to remain loyal to the network each dealer must take into account the risk of being betrayed by his rival.

Comparing (4) with (5), we state the following key result.

[13] Recall that, by assumption, when a dealer is prosecuted he does not know whether his rival has been prosecuted or not.

[14] See, e.g., Bigoni et al. (2012), among others.

Proposition 2 *In a symmetric equilibrium where U sells x^* units of the illegal product to each dealer, cooperation occurs if and only if*

$$\phi \geq \Phi(x^*) \triangleq \phi^M - \frac{\rho(x^*)(2\theta - (1-q))}{2 - \rho(x^*)}, \tag{6}$$

with $\Phi(x^*) \leq \phi^M$ if and only if $\theta \geq \theta^* \triangleq \frac{1-q}{2} \in \left[0, \frac{1}{2}\right]$. Moreover, $\Phi'(x^*) \geq 0$ if and only if $\theta \leq \theta^*$.

The comparison between the minimal enforceable amnesty with competition and with monopoly is ambiguous and depends on the model's underlying parameters. When the informational externality between the dealers is strong enough — i.e., $\theta \in [\theta^*, 1/2]$ — it is easier for the Legislator to induce cooperation with competition than with monopoly. The reason is that when θ is sufficiently large, it is relatively more convenient for each dealer to take the risk of a rush to the courthouse and obtain the amnesty with probability $\frac{1}{2}$, rather than facing the risk of being betrayed by the rival. The opposite holds true when the informational externality is weak — i.e., for $\theta < \theta^*$. In this region of parameters the minimal amnesty that the Legislator has to grant in order to induce cooperation is higher with competition than with monopoly because as θ drops, the value of not cooperating with justice is higher for each dealer — i.e., the domino effect weakens.

Notice that as x^* grows large the impact on the minimal enforceable amnesty is ambiguous and depends only on θ:

Since at equilibrium x^* will depend on the amnesty announced at the outset of the game, in order to determine whether a leniency program is viable or not, we must first characterize the equilibrium production level as a function of ϕ, then solve the Legislator's problem and finally impose conditions such that the solution of this problem — i.e., the optimal amnesty — is lower than 1 (recall that we are ruling out rewards).

In a symmetric equilibrium in which all dealers blow the whistle, D_i's expected utility is

$$\begin{aligned} u_i(x_i, x^*) \triangleq\ & P(x_i + x^*)x_i - t_i + \\ & - \rho(x_i)\mathrm{E}[1 - \phi(k)] - (1 - \rho(x_i))\mathrm{E}[q + \theta k]. \end{aligned}$$

As a result, each dealer's break-even transfer is

$$t_i(x_i, x^*) \triangleq P(x_i + x^*)x_i - \rho(x_i)\mathrm{E}[1 - \phi(k)] - (1 - \rho(x_i))\mathrm{E}[q + \theta k].$$

It then follows that the supplier's bilateral contracting problem when choosing x_i is

$$\max_{x_i \geq 0} P(x_i + x^*)x_i - \rho(x_i)\mathrm{E}[1 - \phi(k)] - (1 - \rho(x_i))\mathrm{E}[q + \theta k] - \mathrm{E}[S(x^*, x_i|y_i)],$$

where $S(x^*|y_i)$ is U's expected sanction conditional on D_i's reporting behavior $y_i \in \{0, 1\}$.

It then follows that $\mathrm{E}[S(x^*, x_i|y_i)] =$

$$2\theta\rho(x_i)\rho(x^*) + \theta(1-\rho(x_i))\rho(x^*) + \theta(1-\rho(x^*))\rho(x_i) = \theta\rho(x_i) + \theta\rho(x^*),$$

where the contribution x_i to the expected sanction of U is made explicit.

Differentiating with respect to x_i and imposing symmetry — i.e., $x_i = x^*$ — we have

$$\underbrace{P(2x^*) + P'(2x^*)x^*}_{D_i\text{'s marginal revenue}} = \underbrace{\rho'(x^*)\theta}_{U\text{'s marginal sanction}} + \underbrace{\rho'(x^*)\left[1-\phi(1-\tfrac{1}{2}\rho(x^*)) - (q+\theta\rho(x^*))\right]}_{D_i\text{'s marginal sanction}}. \quad (7)$$

This condition shows that, in the equilibrium, each dealer smuggles a quantity of illegal good that equalizes the marginal revenue with the marginal cost, which reflects how an extra unit of product smuggled by the dealer affects his own (expected) sanction and that of the supplier. However, since contracts are secret, U's choice of x_i cannot internalize the positive impact that a larger quantity produced by D_i has on the expected sanction of the rival (we will discuss this point more in depth in Section 3.2.

Let $x^*(\phi)$ be the solution of equation (7). We can show the following

Lemma 2 *The quantity $x^*(\phi)$ is increasing in ϕ.*

As in the monopoly benchmark, a higher amnesty reduces the marginal sanction of each dealer, which in turn increases individual quantities via a lower marginal cost of production. The Legislator then chooses the optimal amnesty to minimize each dealer's individual quantity subject to the 'incentive compatibility constraint' (6) — i.e.,

$$\phi^* \triangleq \arg\min_{\phi\in[0,1]} \{x^*(\phi) : \phi \geq \Phi(x^*(\phi))\}.$$

Hence, Lemma 2 immediately implies that (since $x^*(\phi)$ is increasing in ϕ) the optimal amnesty must be such that each dealer is indifferent between blowing the whistle and remaining loyal to his partners — i.e., if it exists, ϕ^* must solve $\phi = \Phi(x^*(\phi))$. As explained before, the optimal policy is shaped by the tension between the domino and the race-to-the-court-house effects. Clearly, with general demand and cost functions understanding which of these forces prevails can be a rather complex task. Nevertheless, in the following proposition we are able to characterize a condition that guarantees the existence of a solution with $\phi^* \leq 1$.

Proposition 3 *With competition, a leniency program may not be viable. When $\theta \leq \theta^*$ the program is viable if and only if $x^*(1) \leq \underline{x}$, with*

$$\underline{x} \triangleq \rho^{-1}\left(\frac{2q}{1-2\theta}\right).$$

By contrast, when $\theta \geq \theta^*$, the program is viable if $x^*(1) \leq \underline{x}$.

When the program is viable, the optimal amnesty $\phi^* \in [0,1]$ solves $\phi = \Phi(x^*(\phi))$. The region of parameters in which $x^*(1) \leq \underline{x}$ expands as θ grows large.

This proposition shows that, in contrast to monopoly, a leniency program may not be viable with competition. The Legislator could, indeed, be unable to induce the dealers to blow the whistle even when granting maximal amnesty — i.e., $\phi = 1$. The reason for this hinges on the negative externality that they exert on one another through the race-to-the-court-house effect: a dealer who decides to cooperate with justice may have to share the amnesty with his rival, so that even with $\phi = 1$ his expected sanction is positive and increasing in the market size. Indeed, the larger the market, the larger the quantity smuggled by each dealer $x^*(1)$ will be and so higher the probability of the rival being investigated, which exacerbates the race to the court house (relative to the domino effect). When $x^*(1)$ is small enough, this effect is negligible and the leniency programs becomes always viable. The reason why the region of parameters in which this happens expands as θ grows large is as follows: when the information externality between the dealers become more severe production drops since the marginal sanction increases and, in addition, also the domino effect becomes more severe since the probability of being succesfully accused by the rival increases with θ; therefore, remaining loyal to the network becomes very risky for the dealers, whereby making a leniency program easier to implement. Indeed, when θ is not too large — i.e., $\theta \leq \theta^*$ — then $x^*(1) \leq \underline{x}$ is necessary and sufficient.

An interesting implication of this result is that in order to limit demand, the introduction of leniency programs should be complemented by policies targeting final users (consumers) of illegal products in addition to their suppliers — e.g., policies harshening the sanctions for drug consumption. These policies should be seen as complements rather than substitutes with leniency programs.

Assuming that the program is viable, we can now turn to study the question of whether the Legislator actually wants to introduce a leniency program or not.

Proposition 4 *When a leniency program is feasible the individual quantity of each dealer solves*

$$P(2x^*) + P'(2x^*)\, x^* = \rho'(x^*)\, \theta.$$

Moreover, introducing a leniency program reduces crime if $\theta \geq q$.

Hence, in contrast to monopoly, with competition the Legislator introduces the leniency program if and only if the accuracy of the dealer's testimony is higher than the baseline probability of conviction — i.e., $\theta \geq q$. An interesting point underlined in the following result is that monopoly is always preferable to competition although in competition dealers can be played one against the other.

Corollary 1 *The supply of illegal goods in competition is always larger than in monopoly — i.e.,* $2x^* > x^M$.

This suggests that criminal networks with strong monopoly power are easier to fight although this may require a lower amnesty than with competition. This result is particularly worrisome because, although groups with a strong hierarchical structure continue to be involved in the drug trade, there is evidence that looser horizontal networks are becoming increasingly significant. In 2017, Europol estimated that such networks accounted for 30-40 per cent of organized crime groups operating in countries in the European Union (United Nations Office on Drugs and Crime, World Drug Report 2017).

Example. In order to better understand the results we now develop a linear quadratic example that allows us to obtain closed-form solution and to obtain a full characterization of the optimal policy. Suppose that $\rho(x_i) = x_i$ and that $P(\cdot) = \max\{0, a - (x_1 + x_2)\}$. Assume that $a \geq \theta - q + 1$ and $a \leq \frac{7}{2} - q$ so to have $0 \leq \rho(x_i) \leq 1$ in the equilibrium. First, notice that solving the first-order condition (7) we have

$$x^*(\phi) = 2\frac{a + q - \theta - (1 - \phi)}{6 - 2\theta + \phi},$$

and thus

$$\Phi(x^*(\phi)) = -\frac{q + 2\theta - 1}{7 - a - q - \theta}\phi + \frac{2\theta(2\theta - a) + 6(1 - q)}{7 - a - q - \theta}.$$

Notice that $\Phi(x^*(\phi))$ is linear in ϕ. Hence, $x^*(1) < \underline{x}$ is necessary and sufficient for the leniency program to be viable. This requires the demand intercept to be small — i.e.,

$$x^*(1) = 2\frac{a + q - \theta}{7 - 2\theta} < \underline{x} \triangleq \frac{2q}{1 - 2\theta} \quad \Leftrightarrow \quad a \leq \frac{6q + \theta(1 - 2\theta)}{1 - 2\theta}.$$

The optimal amnesty, which solves $\phi = \Phi(x^*(\phi))$, is

$$\phi^* = \phi^M - \frac{(a - \theta)(q + 2\theta - 1)}{6 + \theta - a},$$

and the equilibrium quantity is $x^* = \frac{a-\theta}{3} \geq x^*_{\varnothing} = \frac{a-q}{3}$ if and only if $\theta \geq q$.

Remark 1. So far, we have assumed that dealers can only blow the whistle when they are prosecuted — i.e., criminals never cooperate spontaneously before being investigated. This hypothesis seems reasonable in our organized crime framework and for good reasons as explained before. Nevertheless, one may still wonder what would happen with spontaneous reporting — i.e., with a policy that rewards informants only if the decision of cooperating with justice is taken before investigation. The answer is straightforward. A criminal considering whether cooperating with justice or remaining loyal to the organization still faces the trade off between taking the risk of a race to the court-house and the risk of facing a trial in which he is accused by his former partners. Yet, since cooperation takes place before prosecution, the interesting interaction between the market structure and the incentive to blow the whistle vanishes.

Moreover, we also assumed that whistleblowers do not face retribution by their former partners. Introducing this additional ingredient is straightforward, and it would make the introduction of a leniency program even more difficult, whereby reinforcing the impossibility result highlighted above.

3 Extensions

In this section we develop a few natural extensions of the baseline model.

3.1 Linear (wholesale) pricing

So far, we have assumed that the supplier offers each dealer a quantity forcing contract, which allows the former to fully extract the downstream surplus. An interesting question is whether our main results hinge on the specific nature of this contract space. Many papers in the industrial organization literature compare contracts involving fixed payments (non linear contracts) with simpler contracts that only specify wholesale prices (linear contracts).[15] In contrast to non-linear contracts, with linear pricing, dealers make positive profits and the equilibrium features the so called 'double marginalization' phenomenon, which occurs when dealers can exert market power on the final market. Essentially, with linear wholesale prices, the two dealers exercise monopoly power vis-à-vis final consumers by pushing price above marginal costs. But, to do this, they must contract the amount of output they sell and the supplier does so too in order to extract profits from them.

We show that the qualitative insights of the baseline model do not change with linear contracts, although we argue that sustaining a leniency program is easier with linear than with non-linear contracts. Accordingly, suppose that U can only set the wholesale price (w_i hereafter) that each dealer has to pay him in order to buy every unit of illegal product. As before, focus on secret contracts — i.e., D_i cannot observe the wholesale price w_j that U offers to his rival. Again, we assume passive beliefs off the equilibrium path. For simplicity, in order to deal with (strictly) concave maximization problems both upstream and downstream, we assume that the inverse demand function is sufficiently concave relative to the detection probability — i.e., $P''(\cdot) < 0$ and $|P''(\cdot)|/\rho''(\cdot)$ sufficiently large — and that third order derivatives are negligible.[16]

[15] Linear prices are usually justified in the literature as a shortcut to model information frictions — e.g., dealers' private information or moral hazard — that allow the downstream units to obtain positive rents at the supplier's expense.

[16] The analysis remains unaltered as long as $\rho'''(x) \geq 0 \geq P'''(2x)$ for every x.

The first point to notice is that, focusing on a symmetric equilibrium in which each dealer smuggles x^{**} units of the illegal product, the incentive compatibility constraint that secures cooperation has the same shape as the one with quantity forcing contracts — i.e., $\phi \geq \Phi(x^{**})$. Hence, the main difference between linear and quantity forcing contracts hinges on the rule according to which U sets the wholesale prices — i.e., on the extent of the double marginalization effect.

Consider D_i's maximization problem. For a given wholesale price w_i, he solves

$$\max_{x_i \geq 0} P(x_i + x^{**})x_i - w_i x_i - \rho(x_i) \mathrm{E}[1 - \phi(k)] - (1 - \rho(x_i)) \mathrm{E}[q + \theta k],$$

whose first order condition implies

$$P(x_i + x^{**}) + P'(x_i + x^{**})x_i = w_i + \rho'(x_i)[1 - \phi(1 - \tfrac{1}{2}\rho(x^{**})) - (q + \theta\rho(x^{**}))]. \quad (8)$$

Intuitively, when D_i chooses the amount of illegal product to smuggle he trades off the market value of an extra unit of output with its cost, which now reflects the impact of said extra unit on the expected sanction, as well as the price w_i of this extra unit.

Let $x(w_i)$ be the solution with respect to x_i of (8). It follows that

Lemma 3 *$x(w_i)$ is decreasing in w_i — i.e., $x'(w_i) < 0$.*

The intuition is straightforward. As the illegal product becomes more expensive, the dealers have an incentive to sell less.

The bilateral contracting problem between U and D_i is

$$\max_{w_i \geq 0} x_i(w_i) w_i - \theta\rho(x_i(w_i)) - \theta\rho(x^{**}),$$

Notice that with linear pricing, U does not internalize the impact of his choice on the dealers' expected sanction. Differentiating with respect to w_i, we obtain the following first-order condition

$$x_i'(w_i) w_i + x_i(w_i) - \theta\rho'(x_i(w_i)) x_i'(w_i) = 0.$$

Hence, other things being equal, a higher wholesale price w_i affects U's profit only through D_i's choice of how much illegal product to smuggle, which implicitly reflects his expected sanction.

In a symmetric equilibrium in which every dealer pays a wholesale price w^{**} and smuggles x^{**} units of the illegal product, we have

$$x'(w^{**}) w^{**} + x^{**} - \theta\rho'(x^{**}) x'(w^{**}) = 0 \quad \Longrightarrow \quad w^{**} = \theta\rho'(x^{**}) - \frac{x^{**}}{x'(w^{**})} > 0. \quad (9)$$

Inserting (9) into (8) and imposing symmetry, the equilibrium output x^{**} solves

$$P(2x^{**}) + P'(2x^{**})x^{**} =$$
$$\theta\rho'(x^{**}) + \rho'(x^{**})\left[1 - \phi(1 - \tfrac{1}{2}\rho(x^{**})) - (q + \theta\rho(x^{**}))\right] - \underbrace{\frac{x^{**}}{x'(w^{**})}}_{\text{Double marginalization}}.$$

The last term in the right-hand side of this condition reflects the double marginalization effect, which boosts U's marginal cost of production.

Let $x^{**}(\phi)$ be the solution of the above equation. We can prove the following Lemma.

Lemma 4 *The function $x^{**}(\phi)$ is increasing in ϕ. Moreover, $x^{**}(\phi) < x^{*}(\phi)$ for every ϕ.*

As intuition suggests, other things being equal, if the leniency program is feasible with two part tariffs it is also feasible with linear pricing, but the opposite is not true. Indeed, since $x^{**}(1) < x^{*}(1)$ then it may well happen that $x^{**}(1) < \overline{x} < x^{*}(1)$. In words, since with linear pricing U cuts back his supply of illegal goods due to the double marginalization effect, the probability with which each dealer is prosecuted drops. This weakens the race-to-the-court-house effect and makes it easier for the Legislator to induce cooperation compared to the case of quantity forcing contracts. Except for this feature the qualitative insights of our analysis do not change: a leniency program is either unfeasible if both θ and $x^{**}(1)$ are large enough, or it features an optimal amnesty that solves $\phi = \Phi(x^{**}(\phi))$.

In line with standard vertical contracting models, with linear pricing the equilibrium amount of illegal goods smuggled in the economy is lower than that obtained with more general quantity forcing contracts. Specifically, in an interior solution $x^{**} \triangleq x^{**}(\phi^{**})$ solves

$$P'(2x^{**})x^{**} + P(2x^{**}) = \theta\rho'(x^{**}) - \frac{x^{**}}{x'(w^{**})}.$$

As a result:

Proposition 5 *The optimal amnesty with linear pricing ϕ^{**} is lower than the optimal amnesty with two part tariffs. Moreover, $x^{**} < x^{*}$.*

Hence, in contrast to legal markets, in our context welfare is higher with linear pricing than with more general quantity forcing contracts.

3.2 Public contracts

Following the idea that contracts can always be (secretly) renegotiated ex post, up to this point we have assumed that contracts are secret. Suppose, instead, that U has enough credibility to commit to a contract that is the same for both dealers. Assume, as in the baseline model, quantity forcing contracts. Notice that, once more, the incentive compatibility constraint that secures cooperation has the same shape as in the baseline. Hence, as before, the key difference between secret and public contracts hinges on the way quantity is determined in each regime.

Following the same logic as before, it can be shown that U's maximization problem is

$$\max_{x\geq 0} P(2x)\, 2x - 2\left[\rho(x)\left(1-\phi(1-\tfrac{1}{2}\rho(x))\right) + (1-\rho(x))\left(q+\theta\rho(x)\right)\right] - 2\theta\rho(x)\,.$$

One key difference between private and public contracts is that with the latter ones U does not need to solve two independent bilateral contracting problems. Now commitment allows U to internalize the negative externalities between the dealers both on the market price, by preventing the opportunism problem, and the expected sanctions, by accounting for the effect of a dealer's higher sales level on the prosecution risk of his rival (and viceversa).

Differentiating with respect to x, the first-order condition for an interior optimum yields immediately

$$\underbrace{2\left[P'(2x)\, 2x + P(2x)\right]}_{\text{Aggregate marg. revenue}} = \underbrace{2\theta\rho'(x)}_{U\text{'s marg. sanction}} + \underbrace{2\rho'(x)\left[1-\phi(1-\tfrac{1}{2}\rho(x)) - (q+\theta\rho(x))\right]}_{\text{Aggregate marg. sanction}} + $$
$$+ \underbrace{\rho'(x)\left[\rho(x)\,\phi + (1-\rho(x))\,\theta\right]}_{\text{Internalization of externalities}}.$$

Compared to the case of secret contracts, there are two new effects. First, the supplier's ability to commit to a given quantity wipes out the opportunism problem. Second, when choosing the optimal quantity, U takes into account the fact that higher sales also imply a higher prosecution risk, which hardens the race to the court-house. We can then show the following.

Lemma 5 *The function $\hat{x}(\phi)$ is increasing in ϕ. Moreover, $\hat{x}(\phi) < x^*(\phi)$ for every ϕ.*

Likewise the case of linear pricing, if the leniency program is feasible with two part tariffs it is also feasible with linear pricing, but the opposite is not true. In an interior solution the optimal amount of illegal good solves

$$P'(2\hat{x})\,\hat{x} + P(2\hat{x}) = \theta\rho'(\hat{x}) + \rho'(\hat{x})\left(\rho(\hat{x})\tfrac{\phi}{2} + (1-\rho(x))\,\theta\right) - \hat{x}P'(2\hat{x})\,,$$

which immediately implies that $\hat{x} < x^*$. In terms of welfare, the Legislator prefers fighting suppliers with strong commitment power, since this delivers quantities closer to the monopoly benchmark. In other words, more trust among the members of the network is, interestingly enough, beneficial to social welfare.

3.3 Network size

We now extend the baseline model by considering the case of $N \geq 3$ dealers. As before, we focus on a symmetric equilibrium in which all retailers produce x_N^* units of illegal goods and blow the whistle once they are prosecuted. For simplicity, we normalize $q + (N-1)\theta = 1$ so that when only one dealer remains loyal to the network (while his $N-1$ rivals blow the whistle) he is convicted with probability 1. As in the baseline model, we also posit that $\theta \leq \frac{1}{N}$ so that U's expected sanction never exceeds 1.

With N rivals in dealership market, D_i blows the whistle if and only if

$$\underbrace{\mathrm{E}\left[1-\phi\left(k\right)\right]}_{y_i=1} \leq \underbrace{\mathrm{E}\left[1-\left(N-1\right)\theta+k\theta\right]}_{y_i=0}, \tag{10}$$

where, from D_i's perspective, $k = 0, ..., N-1$ represents now the number of rivals who blow the whistle.

By the binomial theorem (see the Appendix) it can be shown that

$$\mathrm{E}\left[1-\phi\left(k\right)\right] \triangleq \textstyle\sum_{k=0}^{N-1}\left(1-\phi\left(k\right)\right)\binom{N-1}{k}\rho\left(x_N^*\right)^k\left(1-\rho\left(x_N^*\right)\right)^{N-1-k} = 1-\phi\frac{1-\left(1-\rho\left(x_N^*\right)\right)^N}{N\rho\left(x_N^*\right)},$$

where $1-\left(1-\rho\left(x_N^*\right)\right)^N$ is the probability that at least one dealer is prosecuted. Similarly, it can be shown (see the Appendix) that

$$\mathrm{E}\left[q+k\theta\right] \triangleq \textstyle\sum_{k=0}^{N-1}\left(q+k\theta\right)\binom{N-1}{k}\rho\left(x_N^*\right)^k\left(1-\rho\left(x_N^*\right)\right)^{N-1-k} = 1-\left(N-1\right)\theta\left(1-\rho\left(x_N^*\right)\right).$$

Solving (10) with respect to ϕ we have

$$\phi \geq \Phi_N\left(x_N^*\right) \triangleq \frac{\theta N\left(N-1\right)\rho\left(x_N^*\right)\left(1-\rho\left(x_N^*\right)\right)}{1-\left(1-\rho\left(x_N^*\right)\right)^N},$$

with $\Phi_N\left(x\right) \geq 0$ for every $x \geq 0$.

Recalling that, by definition, $\rho\left(\overline{x}\right) = 1$, we can thus show the following.

Lemma 6 *Holding N constant, the function $\Phi_N\left(x\right)$ is single peaked with respect to x, with*

$$1 > \Phi_N\left(0\right) = \theta\left(N-1\right) > \Phi\left(\overline{x}\right) = 0.$$

Moreover, holding $x \in \left[0, \overline{x}\right]$ constant, $\Phi_N\left(x\right) < \Phi_{N+1}\left(x\right)$ for every $N \geq 3$.

Increasing the level of sales has (for given size of the network) two main effects on the dealers' propensity to blow the whistle. On one hand, ceteris paribus, the minimal feasible amnesty drops because the domino effect becomes stronger. Indeed, when D_i does not blow the whistle, the probability of being betrayed by his rivals is increasing with the equilibrium sales level — i.e., a dealer that does not cooperate with the justice is less likely to be accused by his rivals (since each dealer is prosecuted with a relatively low probability). On the other hand, the minimal feasible amnesty increases because (in expected terms) D_i shares more often the prize for cooperation with his rivals. Hence, for intermediate values of x the two effects tend to balance out, and it is harder for the Legislator to induce cooperation.

Similarly, holding production constant, the effect of the network size (N) is also ambiguous. On the one hand, a larger number of dealers reduces the expected prize for cooperation, whereby reducing the willingness to blow the whistle of each dealer. On the other hand, a larger network size also increases probability of conviction for each dealer that decides to remain loyal to the organization. The first effects tend to increase $\Phi_N(\cdot)$ while the second tends to reduce it; yet, on balance, the former always dominates. Hence, holding individual sales constant, it is harder to induce dealers to cooperate with the justice as the size of the network increases.

Notice that U's optimal level of sales will also endogenously depend on N. Hence, the forces just described above will be simultaneously at play in equilibrium.

In order to solve the game, recall that D_i's break-even transfer is

$$t_i(x_i, x^*) = P(x_i + (N-1)x_N^*)\,x_i + \\ - \rho(x_i)\,\mathrm{E}[1-\phi(k)] - (1-\rho(x_i))\,\mathrm{E}[q+\theta k].$$

The bilateral contracting problem between U and D_i is therefore

$$\max_{x_i \geq 0} t_i(x_i, x_N^*) - \mathrm{E}[S(x_N^*|y_i)],$$

and it can be shown (see the Appendix) that

$$\mathrm{E}[S(x_N^*|y_i)] = \theta(\rho(x_i) + (N-1)\rho(x_N^*)).$$

Differentiating the objective of U with respect to x_i and imposing symmetry, in an interior solution we have

$$P(Nx_N^*) + P'(Nx_N^*)\,x_N^* = \rho'(x_N^*)\left[(N-1)\theta(1-\rho(x_N^*)) - \phi\frac{1-\left(1-\rho\left(x_N^*\right)\right)^N}{N\rho\left(x_N^*\right)}\right] + \rho'(x_N^*)\,\theta,$$

whose solution $x_N^*(\phi)$ is again increasing in ϕ.[17] The intuition is as before. A more intense amnesty translates into a lower marginal sanction (marginal cost), which tends to increase the amount of goods that each dealer smuggles in the equilibrium. As before a sufficient condition for the program to be viable is $x_N^*(1) \leq \Phi_N^{-1}(1)$.

Notice that the impact of N on this condition is ambiguous. The reason is that when

[17] As before, this requires the sale revenue to be sufficiently concave. The proof follows the same logic as the proof of Lemma 2.

N is very large the probability of each dealer being investigated is low because each sells an arbitrarily small quantity of illegal goods. Hence, the externalities imposed both through the domino and the courthouse effects are negligible, which is therefore similar to the monopoly problem (where the program is always feasible). The same is true when N is small because in this case the probability of each dealer being convicted is not negligible and it is more likely that all members of the organization blow the whistle. Hence, when the dealership market is either very concentrated or very competitive, inducing cooperation is relatively easy. By contrast, when N takes intermediate values it becomes relatively harder owing to the interaction between the direct effects of N on the domino and the race-to-court-house effects and the indirect effect on the probability of prosecution via market competition.

Of course, if the program is feasible, the optimal quantity then solves

$$P'\left(Nx_N^*\right)x_N^* + P\left(Nx_N^*\right) = \rho'\left(x_N^*\right)\theta.$$

As before, the Legislator prefers to grant an amnesty to whistleblowers if and only if $\theta \geq q$, but only if this is feasible — i.e., if $1 \geq \Phi_N\left(x_N^*\left(1\right)\right)$.

4 Concluding remarks

Motivated by the overwhelming evidence on the network nature of many criminal activities, we have analyzed the relationship between competition in illegal markets and incentives to blow the whistle within a criminal network modeled as a vertical supply-chain. We developed a simple model involving a benevolent Legislator and a criminal network formed by an upstream supplier of an illegal good — e.g., drugs, guns, ammunition, counterfeit products etc. — and local dealers who distribute the supplier's product on the final market. We have shown that the comparison between the optimal amnesty with competition and monopoly in the dealership market is ambiguous and depends on the strength of the (negative) externality that dealers impose one another through their reporting behavior. Surprisingly, while in monopoly the introduction of a leniency program is always feasible and beneficial to welfare, with competition the policy might not be feasible even though welfare enhancing. This impossibility result arises when the demand for the illegal product is large, when the downstream market is neither too competitive nor too concentrated and when the dealers have accurate enough information on each other. With linear and/or public contracts the conditions under which a leniency program is viable are milder than with secret contracts.

These results suggest that the use of leniency programs in criminal proceedings is a delicate issue and that their efficacy depends not only on features inherently linked to the supply side of the market — i.e., the internal structure of criminal organizations, their size, efficiency and strength etc. — but also on the characteristics of the demand for the products provided by these organizations. Specifically, when the number of people who consume these products is sufficiently large — as it is the case for opioids and counterfeit products — even granting large enough amnesties may not suffice to induce criminals to cooperate with justice. In these cases, the introduction of leniency programs should be complemented by other forms of policies targeting final users of illegal products in addition to their suppliers — e.g., harsher punishments for drug consumption. Our analysis suggests that these policies should be seen as complements rather than substitutes. We hope to examine this link more in depth in future research.

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A Appendix

Proof of Lemma (1). Using the definition of $x^M(\phi)$, the implicit function theorem yields

$$\frac{\partial x^M}{\partial \phi} = -\frac{\rho'(\cdot)}{2P'(\cdot) + P''(\cdot)\,x^M(\phi) - \rho''(\cdot)(\theta + 1 - \phi)},$$

which is positive because of Assumption 2 and the convexity of $\rho(\cdot)$. ■

Proof of Proposition (1). If the Legislator implements a leniency program, she chooses the lowest value that induces the dealer to blow the whistle: $\phi = 1 - q$, because of Lemma (1). Comparing the equations identifying x^M (with $\phi = 1 - q$) and $x^M_{\varnothing}$, it is immediate to notice that $x^M \le x^M_{\varnothing}$ for any positive θ, because x^M has a larger marginal cost. Hence the Legislator always chooses to implement leniency $\phi^M = 1 - q$. Substituting $\phi = \phi^M$ in equation (1) we obtain equation (2). ■

Proof of Proposition (2). Finding ϕ that solves inequality (3) we obtain:

$$(1-\rho(x^*))(1-\phi)+\rho(x^*)\left(1-\tfrac{\phi}{2}\right) \le (1-\rho(x^*))q+\rho(x^*)(q+\theta) \quad \Leftrightarrow \quad \phi \ge \phi^M - \frac{\rho(x^*)(2\theta-(1-q))}{2-\rho(x^*)}.$$

The relationship between $\Phi(x^*)$ and ϕ^M follows immediately. Finally

$$\frac{\partial \Phi(\cdot)}{\partial x^*} = -\rho'(\cdot)\frac{[2-\rho(\cdot)](2\theta-1+q)+\rho(\cdot)(2\theta-1+q)}{[2-\rho(\cdot)]^2} = -\rho'(\cdot)\frac{2(2\theta-1+q)}{[2-\rho(\cdot)]^2}.$$

Hence, $\Phi'(x^*) \ge 0$ if and only if $\theta \le \theta^*$. ■

Proof of Lemma (2). Using the definition of $x^*(\phi)$, the implicit function theorem implies immediately

$$\frac{\partial x^*(\cdot)}{\partial \phi} = -\frac{\rho'(x^*)\left(1-\frac{\rho(x^*)}{2}\right)}{P''(\cdot)+2P'(\cdot)x^*(\cdot)-\rho''(\cdot)\left[\theta+1-\phi(1-\frac{1}{2}\rho(\cdot))-q-\theta\rho(\cdot)\right]-\rho'(\cdot)^2\left(\frac{\phi}{2}-\theta\right)}.$$

This expression is positive because of Assumption A2 and the convexity of $\rho(\cdot)$. ■

Proof of Proposition (3). To begin with, observe that $\Phi(x^*(0)) \ge 0$. Indeed, since $1 \ge q + \theta$ it follows that

$$\Phi(x^*(0)) \ge 0 \quad \Leftrightarrow \quad 2\frac{1-q-\rho(x^*)\,\theta}{2-\rho(x^*)} \ge 0.$$

Then, a sufficient condition for the existence of ϕ^* is $\Phi(x^*(1)) \le 1$ which implies $x^*(1) < \underline{x}$.

Notice that when $x^*(1) > \underline{x}$ the viability of the program depends on θ. When $\theta \ge \theta^*$ then $\Phi(\cdot)$ is decreasing in ϕ, and so the program is not viable. When $\theta \le$and that $\Phi(\cdot)$ falls always zbovd 1.—,and the program si not viable. ■

Proof of Lemma (3). Using the definition of $x^*(w_i)$, the implicit function theorem implies immediately

$$\frac{\partial x\left(\cdot\right)}{\partial w_i} = \frac{1}{P'(\cdot) + 2P''(\cdot)x(\cdot) - \rho''\left(x(\cdot)\right)\left[1 - \phi\left(1 - \frac{1}{2}\rho(\cdot)\right) - q - \theta\rho(\cdot)\right]}.$$

Recall that in an equilibrium with leniency

$$1 - \phi(1 - \tfrac{1}{2}\rho\left(x^{**}\right)) \geq q + \theta\rho\left(x^{**}\right).$$

Hence, Assumption A2 and the convexity of $\rho(\cdot)$ imply that $x'\left(w_i\right) < 0$. ■

Proof of Lemma (4). To begin with, recall that $x^{**} \triangleq x\left(w^{**}\right)$ by definition. Hence,

$$\frac{\partial x'\left(w^{**}\right)}{\partial x^{**}} = -\frac{3P''(\cdot)+2x^{**}P'''(\cdot)-\rho'''(\cdot)[1-\phi(1-\frac{1}{2}\rho(\cdot))-q-\theta\rho(\cdot)]-\rho''(\cdot)\rho'(\cdot)(\frac{\phi}{2}-\theta)}{\left[P'(\cdot)+2P''(\cdot)x^{**}-\rho''(\cdot)\left(1-\phi\left(1-\frac{1}{2}\rho(\cdot)\right)\right)-q-\theta\rho(\cdot))\right]^2},$$

which is positive if $\frac{|P''(\cdot)|}{\rho''(\cdot)} \geq \frac{\rho'(\cdot)}{3}$ since we assumed that $\rho'''\left(\cdot\right)$ and $P'''\left(\cdot\right)$ are negligible.

Then, using the definition of $x^{**}\left(\phi\right)$ the implicit function theorem yields immediately

$$\frac{\partial x^{**}\left(\cdot\right)}{\partial\phi} = -\frac{1-\frac{1}{2}\rho(\cdot)}{3P'(\cdot)+2P''(\cdot)x^{**}(\cdot)-\rho''(\cdot)\left[\theta(1-\rho(\cdot))+1-\phi\left(1-\frac{1}{2}\rho(\cdot)\right)-q\right]-\rho'(\cdot)^2\left[\frac{\phi}{2}-\theta\right]+\underbrace{\frac{x'\left(\cdot\right) - x^{**}\frac{\partial x'(\cdot)}{\partial x^{**}}}{x'\left(\cdot\right)^2}}_{-}},$$

which is positive under Assumption A2. ■

Proof of Proposition (5)**.** The proof follows immediately from the fact that $x'\left(\cdot\right) < 0$. ■

Proof of Lemma (5). First, showing that $\hat{x}\left(\phi\right) < x^*\left(\phi\right)$ for every $\phi \in [0,1]$ is immediate since $\rho'\left(\cdot\right) > 0$. Second, using the definition of $\hat{x}\left(\phi\right)$, the implicit function theorem implies immediately

$$\frac{\partial\hat{x}\left(\cdot\right)}{\partial\phi} = -\frac{\rho'(\cdot)}{3P'(\cdot)+2P''(\cdot)\hat{x}(\phi)-\rho''(\cdot)\left[\theta(1-\rho(\cdot))+1-\phi\left(1-\frac{1}{2}\rho(\cdot)\right)-q\right]-\rho'(\cdot)^2\left[\frac{\phi}{2}-\theta\right]-\rho'(x)^2\frac{\phi-\theta}{2}-\rho''(\cdot)\frac{\rho(\cdot)\phi+(1-\rho(\cdot))\theta}{2}},$$

which is positive under Assumption A2.

Finally, showing that $\hat{x}\left(\phi\right) < x^*\left(\phi\right)$ is immediate since $\rho'\left(x\right) > 0$. ■

Expected utilities with N dealers. Given an equilibrium candidate in which all dealers sell x^* units of illegal good, the expected utility of a dealer who blows the whistle is

$$\mathrm{E}\left[1-\phi\left(k\right)\right] = \textstyle\sum_{k=0}^{N-1}\left(1-\phi\left(k\right)\right)\binom{N-1}{k}\rho\left(x_N^*\right)^k\left(1-\rho\left(x_N^*\right)\right)^{N-1-k} =$$

$$1 - \sum_{k=0}^{N-1}\frac{\phi}{k+1}\frac{(N-1)!}{k!(N-1-k)!}\rho\left(x_N^*\right)^k\left(1-\rho\left(x_N^*\right)\right)^{N-1-k} =$$

$$1 - \frac{\phi}{N}\sum_{k=0}^{N-1}\frac{N!}{(k+1)!(N-(k+1))!}\rho\left(x_N^*\right)^k\left(1-\rho\left(x_N^*\right)\right)^{N-1-k}.$$

By a simple change of variables — i.e., $l = k + 1$ — we have

$$\frac{\phi}{N}\sum_{k=0}^{N-1}\frac{N!}{(k+1)!(N-(k+1))!}\rho\left(x_N^*\right)^k\left(1-\rho\left(x_N^*\right)\right)^{N-1-k} =$$

$$\frac{\phi}{N}\sum_{l=1}^{N}\frac{N!}{l!(N-l)!}\rho\left(x_N^*\right)^{l-1}\left(1-\rho\left(x_N^*\right)\right)^{N-l} =$$

$$\frac{\phi}{\rho N}\sum_{l=1}^{N}\frac{N!}{l!(N-l)!}\rho\left(x_N^*\right)^{l}\left(1-\rho\left(x_N^*\right)\right)^{N-l} =$$

$$\frac{\phi}{\rho N}\sum_{l=0}^{N}\frac{N!}{l!(N-l)!}\rho\left(x_N^*\right)^{l}\left(1-\rho\left(x_N^*\right)\right)^{N-l} - \frac{\phi}{\rho\left(x_N^*\right)N}\left(1-\rho\left(x_N^*\right)\right)^{N} =$$

$$= \frac{\phi}{\rho\left(x_N^*\right)N}\left(1-\left(1-\rho\left(x_N^*\right)\right)^{N}\right).$$

Hence,

$$\mathrm{E}\left[1-\phi\left(k\right)\right] = 1 - \frac{\phi}{\rho\left(x_N^*\right)N}\left(1-\left(1-\rho\left(x_N^*\right)\right)^{N}\right). \quad \blacksquare$$

By a similar logic one immediately obtains

$$\mathrm{E}\left[q+kq\right] = q + \left(N-1\right)\theta\left(1-\rho\left(x_N^*\right)\right) = 1-\left(N-1\right)\theta\left(1-\rho\left(x_N^*\right)\right),$$

and

$$\mathrm{E}\left[S\left(x_N^*|y_i\right)\right] = \theta\left(\rho\left(x_i\right)+\left(N-1\right)\rho\left(x_N^*\right)\right).$$

Proof of Lemma (6). To begin with recall that

$$\Phi_N\left(x\right) \triangleq \frac{\theta N\left(N-1\right)\rho\left(x\right)\left(1-\rho\left(x\right)\right)}{1-\left(1-\rho\left(x\right)\right)^N}$$

Hence, $\Phi_N\left(1\right) = 0$ since $\rho\left(\overline{x}\right) = 1$ by definition and $\Phi_N\left(0\right) = \theta\left(N-1\right)$ since $\rho\left(0\right) = 0$. Moreover, notice that

$$\frac{\partial\Phi_N\left(x\right)}{\partial x} = \frac{\partial\Phi_N\left(\cdot\right)}{\partial\rho\left(\cdot\right)}\frac{\partial\rho\left(\cdot\right)}{\partial x} \quad \Rightarrow \quad \mathrm{sign}\frac{\partial\Phi_N\left(x\right)}{\partial x} = \mathrm{sign}\frac{\partial\Phi_N\left(\cdot\right)}{\partial\rho\left(\cdot\right)},$$

since $\rho'\left(\cdot\right) > 0$. Next, abusing slightly notation, let

$$\eta\left(\rho\right) \triangleq \frac{\rho\left(1-\rho\right)}{1-\left(1-\rho\right)^N},$$

we then have

$$\eta'\left(\rho\right) = \frac{\left(1-\left(1-\rho\right)^N\right)\left(1-2\rho\right)-\rho N\left(1-\rho\right)^N}{\left(1-\left(1-\rho\right)^N\right)^2},$$

which is strictly negative for $\rho \geq \frac{1}{2}$ and (strictly) positive for ρ close to 0 — i.e., by using de l'Hoptial rule

$$\lim_{\rho\to 0}\frac{\partial\eta\left(\rho\right)}{\partial\rho}=\frac{N-3}{2N}>0.$$

Hence, $\eta\left(\rho\right)$ features a global maximum and so does $\Phi_N\left(x\right)$. We now show that this maximum is unique, implying that $\Phi_N\left(x\right)$ is single peaked with respect to x. Let

$$\varphi\left(\rho\right)\triangleq\left(1-\left(1-\rho\right)^N\right)\left(1-2\rho\right)-\rho N\left(1-\rho\right)^N,$$

we show that $\rho^{\max}$ is the unique solution of $\varphi\left(\rho\right)=0$. To this purpose, notice that

$$\varphi'\left(\rho\right)=\left(1-\rho\right)^{N-1}\left(2\left(1-\rho\right)+N\rho\left(N-1\right)\right)-2,$$

with

$$\varphi'\left(0.5\right)=\frac{N\left(N-1\right)}{2^N}+\frac{1}{2^{N-1}}-2<0,$$

and $\varphi'\left(0\right)=0$. Next, notice that

$$\varphi''\left(\rho\right)=N\left(1-\rho\right)^{N-2}\left(N-3-\rho\left(N\left(N-1\right)-2\right)\right),$$

which, for every $N\geq 3$, is positive if and only if

$$\rho\leq\frac{N-3}{N\left(N-1\right)-2}<\frac{1}{2}.$$

Hence, the curvature of $\varphi\left(\cdot\right)$ changes sign only once, implying that $\varphi\left(\cdot\right)$ has a unique positive zero $\rho^{\max}<\frac{1}{2}$. This directly implies that $\eta\left(\rho\right)$ is single peaked and that it is maximized by some $\rho^{\max}\in\left(0,1\right)$. As a result, also $\Phi_N\left(x\right)$ is single peaked with respect to x — i.e., $x^{\max}=\rho^{-1}\left(\rho^{\max}\right)$.

Finally, showing that, for given x, $\Phi_N\left(x\right)$ is increasing in N is straightforward. ■