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Does the Master's Eye Fatten the Cattle? Maintenance and Care of Collateral under Purchase and Leasing Contracts

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Does the Master's Eye Fatten the Cattle? Maintenance and Care of Collateral under Purchase and Leasing Contracts

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Abstract
The paper presents a theory of leasing in which asset *use and maintenance* shape the firm's decision between purchasing or leasing productive assets. When the asset purchase is financed through a secured debt contract and the value of the asset is sensitive to the user's uncontractible maintenance decision, maintenance may be privately unprofitable for the user and cause asset depletion. This jeopardises the return to the financiers and erodes the benefit of collateral pledging, particularly relevant for financially constrained firms. Such a shortcoming can be overcome with a leasing contract that delegates the maintenance to the lessor. However, delegation generates a novel agency problem on the lessee, who, by not paying for maintenance, may practice inefficiently low levels of care and asset abuse that increase the expected cost of maintenance for the lessor. The paper characterises circumstances in which it may be optimal to lease rather than buy, finding that the reliance on leasing may be non-monotone in financing constraints.
JEL Classification: D82, G32.

Keywords: Collateral, Financial constraints, Leasing, Maintenance.

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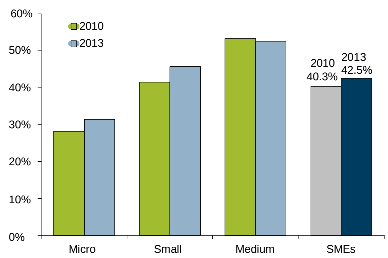
1 Introduction

Over the past decade or more, there has been a clear trend among many capital intensive industries, such as the construction and distribution sectors (e.g. agriculture, manufacturing, mining & utilities and construction), to finance larger sums of their machinery and industrial equipment through leasing.[1] This trend is common across all firm sizes, but is especially relevant for SME's. According to the 2019 ECB and European Commission Survey on the Access to Finance (SAFE), leasing is a reliable and robust form of finance for 45% of SMEs in the EU. This is corroborated by a survey on the use of leasing amongst European SME's conducted by Oxford Economics (2015), which finds that 42.5% of the SME's use leasing in 2013, up from 40.3% in 2010.[2] If we decompose the reliance on leasing across firm sizes, we see that this is mainly due to small and medium firms, while micro firms still lag behind.



Proportion of SMEs using leasing by firm size, 2010 and 2013 (source: Oxford Economics/EFG)

Given that leasing can finance up to 100% of the purchase price of an asset, and that small firms are more prone to facing financing constraints, why is the reliance on leasing for such firms not higher?

[1] We will throughout the paper refer to renting or operating leasing as synonyms, although there are differences between them related, for example, to the duration of the contract, the accounting treatment, the redemption option. We abstract however from these features in the paper.

[2] We deliberately neglect the 2020 figures, heavily hit by the pandemic and the recession that has followed.

In this paper we provide one possible rationale by identifying in the incentive problems related to asset use and maintenance one possible determinant of such differential reliance on leasing across firms' size.

By buying a productive asset, a firm not only obtains the right to its use in production, but can also use it as (inside) collateral. However, when the second hand asset value is uncertain due to agency problems, pledging the asset to financiers may fail to increase the firm's debt capacity. One of the factors that may affect the asset residual value is the maintenance performed on it (Igawa and Kanatas, 1990). When the degree of maintenance cannot be carefully specified as part of the loan agreement, it may be privately unprofitable for the user/owner to carry it out, because costly. This may jeopardize the return to the financiers in case of default, thus eroding the benefit of collateral pledging. Leasing (renting) overcomes this shortcoming, as the maintenance is delegated to the lessor, who, by performing preventative maintenance, preserves the asset value.

However, a closer look shows that leasing does not fully solve the incentive problems related to the maintenance of an asset. Indeed, a novel moral hazard problem arises on the lessee, who, by not paying for maintenance, may practice inefficiently low levels of care, where by care we denote all the unverifiable activities or actions that the *user* of an asset exerts in managing it that may affect its value. Thus, while leasing preserves the maintenance incentives, it cannot prevent the asset depletion due to carelessness in its use. The paper aims to identify whether and how the incentive problems related to asset use and maintenance, interacted with limited financial resources, shape the firm's decision between purchasing and/or leasing/renting productive assets.

We propose a one-period model in which a firm with an investment project but insufficient resources relies on bank lending to carry it out. The project uses one capital input that can be purchased or leased/rented. The capital input is redeployable, but depreciates in production. The degree of depreciation depends negatively on maintenance. When the user of the asset also owns it, maintenance is carried out in house, and is non-contractible. When the asset is leased, maintenance is delegated to the lessor.

Maintenance is costly, and its cost varies with the intensity with which capital is used. In periods of low demand, the intensity of usage of the capital good is limited (soft usage) and maintenance involves only a non-pecuniary cost. In periods of high demand, the intensity of usage of the capital good is high (hard usage) and maintenance involves also a monetary cost. Such cost can nevertheless be reduced by the (good) care with which the capital good is managed by the entrepreneur in the course of use. Care cannot be delegated, it is unobservable to third parties and has only a fixed non-monetary cost.

Maintenance is always valuable, i.e., relative to a situation with no maintenance, the extra value that the capital good has with maintenance is larger than the maintenance cost. However, when owners of the capital good, entrepreneurs are opportunistic in the sense that they may give up maintenance (and care) when is not privately optimal to carry it out, thereby reducing the residual value of the capital good.

Entrepreneur's opportunism may result in credit rationing and underinvestment. In particular, when the entrepreneur has got sufficient resources to entirely finance the project, she can purchase the capital good and keep its residual value upon production. The full right to the asset residual value allows her to internalize the maintenance (and care) incentives. When her wealth is lower, she has to borrow from an external financier to carry out the desired investment. The debt obligations are repaid out of cash flows and, if insufficient, by pledging (part of) the ex-post asset value in case of default. Having to part with the asset, the entrepreneur's maintenance benefits *when default occurs* may fall short of the cost, making maintenance suboptimal and jeopardizing the lender's returns. To preserve the maintenance incentive, investment has to be downsized, more so the lower the internal wealth. The reduction in profits following the scaling down of production may be so pronounced to induce the entrepreneur to stop carrying out the maintenance on the capital good in case of default, with a subsequent efficiency loss.

One way to restore maintenance incentives also in the low state and limit underinvestment and capital depletion is to rely on a leasing contract that delegates maintenance to the lessor. The latter, being the unconditional owner of the capital good, has always the incentive to carry it out. However, this only partly solves the problem as, despite not having the ownership, the entrepreneur/lessee still keeps daily control of the asset, whose care has a non-monetary cost, but no benefit for her. Exerting care is therefore not privately optimal for the lessee. This increases the expected cost of maintenance faced by the lessor, the rental fee charged to the lessee and, ultimately, reduces the level of investment.

Whether *credit rationed* firms prefer to lease rather than purchase the capital good depends on how the above described agency problems interact with the firm's financial constraints *and with market conditions.* Suppose the market conditions are favourable, i.e., the probability of success of the project is high, and suppose the expected benefit from care is high, i.e., the reduction in expected maintenance cost is high. When the input is purchased, full repossession by the entrepreneur in the success state implies that care is always exerted, and thus a benefit in terms of lower expected maintenance cost. Conversely, when the input is leased, maintenance is carried out by the lessor, but the maintenance cost he faces, because care cannot be enticed, is higher, with a subsequent increase in the leasing fee. This may be so high to make it worthwhile for the entrepreneur to buy the capital good and carry out the maintenance in house. Thus, leasing does not arise when the expected benefit of care is high.

When the expected benefit from care is low, there is little benefit from it, as the expected cost of maintenance for the entrepreneur is not much reduced from high care. Thus, while wealthy firms still prefer to purchase the capital input, less wealthy ones substitute buying with leasing, as the latter, by restoring the maintenance benefits, allows to relax financing constraints and increase the firm's borrowing capacity. Such substitution is full for sufficiently credit rationed firms who lease 100% of their capital, and delegate the maintenance to the lessor.

There is nevertheless a hybrid scenario occurring for intermediate values of the expected benefit from high care. In this case, as the severity of the financing constraints increases, it is still initially optimal to substitute buying with leasing. However, since such substitution implies a raising leasing fee, for highly credit rationed firms it may be cheaper to buy rather than lease the capital goods, giving up the maintenance in the case of failure. The reliance on leasing is therefore non-monotone in financing constraints.

The rationales for purchasing/leasing highlighted in our paper are related to some of the reasons firms generally invoke to motivate their reliance on leasing. According to a survey by Oxford Economics (2015), one important reason to use leasing is the ability to use assets without bearing the risks of ownership, like the risks on second hand value. This is precisely one of the predictions we get from our model. When an asset value is sensitive to the maintenance decision, the risk on its second hand value is high and purchasing it with a collateralised credit contract may not be feasible. This problem may be especially severe when the firm is financially constrained and unable to provide alternative collateral. In such circumstances, the inability to provide credible inside (and outside) collateral makes it more likely that the asset will be leased, and the maintenance delegated to the lessor. Such considerations may in turn contribute to explain some other commonly observed features of leasing, namely the possibility of financing up to 100% of the purchase price of an assets, as well as the bundling of finance with optional services, like installation, maintenance and repair of the leased asset (Leaseurope, 2015).[3]

The remainder of the paper is organized as follows. In Section 2 we present a brief sketch of the literature. In Section 3, we introduce the model. In Section 4, we analyze the contract problem when the capital goods can only be purchased, describing the benchmark case and the first-best contract in Section 4.1, the effects of financial constraints on the firm's maintenance and care decisions in Section 4.2 and the equilibrium outcome in Section 4.3. In Section 5, we introduce the possibility for firms to lease rather than buy capital inputs. In Section 6 we consider the case in which the firm can both purchase and lease them and derive three possible financing regimes. In Section 7 we conclude. All the proofs, unless otherwise specified, are in the Appendix.

2 Related literature

The paper is related to two strands of the literature. On one side there is the literature on credit rationing and collateral pledging, and the costs related to it. On the other side the

[3]Other reasons provided by firms for relying on leasing include the lower price of financing the asset relative to other forms of financing, the better cash flow management, the ability to adapt the contract terms to the company's needs, the predictability and transparency of lease payments or the ability to upgrade and renew assets more frequently than purchasing allows.

literature on moral hazard problems in leasing contracts.

As regards the first, since the seminal work of Bester (1985) showing the possibility for lenders in asymmetrically informed environments to eliminate credit rationing relying on collateral requirements, a large literature has flourished highlighting the potential costs of using collateral as a sorting device. Although this literature has mainly emphasized the lower value that assets may have for lenders than for the borrower (Bester (1985, 1987), Besanko and Thakor (1987), Chan and Kanatas (1985), among others), there are various other reasons for the existence of a deadweight loss attached to collateralization. Igawa and Kanatas (1990), for example, have focused on some possible incentive effects induced by the collateral requirements. In particular, when the maintenance of the pledged assets cannot be specified as part of the loan agreement, it can be privately unprofitable for the borrower to carry it out and collateral may fail to play the typical sorting role highlighted by the literature on credit rationing.[4]

In our work, we assume away differences between lenders and borrower in the valuation of the assets and, in line with Igawa and Kanatas (1990), view the secured contract's (transactions) costs as resulting from a moral hazard problem in maintaining the value of pledged assets. Despite this modelling analogy (stemming from the incentive effects of collateral requirements), there are many differences relative to our work. First, while in our paper the firm needs funding to buy the productive asset and uses both cash flows and the asset residual value to repay the loan, in Igawa and Kanatas (1990) the firm already owns the productive asset and pledges it to the lenders to mitigate the asymmetric information

[4] In Igawa and Kanatas (1990), firms with privately known success probability *own* a productive asset and need a fixed size loan to finance a project. They can apply for a secured loan by pledging the asset, for an unsecured loan, or they can self-finance by selling the asset to subsequently rent it. The authors show that high quality firms choose secured contracts, low quality firms choose unsecured contracts and intermediate quality firms choose to self-finance with rental contracts.

problem (to signal its quality). Thus, in our setting the firm pledges inside collateral rather than outside collateral.

Another difference between the two papers concerns the sources of asymmetric information. In Igawa and Kanatas, the firm profitability is private information, and the moral hazard in the maintenance of the asset prevents sorting of types. A similar maintenance incentive is present in our model, but there is no adverse selection. The relevance of the moral hazard problem in maintenance in our setting stems from the cost of capital being large relative to the firm's expected cash flows, which makes maintenance privately unprofitable for the owner. This problem can be overcome by leasing the asset and delegating its maintenance to the lessor, who, being the owner, has the incentive to preserve it to its highest value. However, delegation of maintenance to the lessor can generate a novel agency problem on the lessee related to the asset use. Indeed, having no right to the asset's residual value, the lessee may practice inefficiently low levels of care in the management of the asset. This causes asset depletion and affects the maintenance cost borne by the lessor.

This distinction between use and maintenance allows us to relate our work also to the literature on moral hazard problems in leasing contracts. This has emphasized the agency problems that arise in the use of an asset when the owner does not coincide with the user. The latter, not having the right to the asset's residual value, does not bear the full cost of abuse (Smith and Wakeman, 1985). This problem has been modelled by Eisfeldt and Rampini (2009), who construct a model in which leasing emerges from the trade-off between the lessor's better ability relative to a secured lender to repossess the asset and an agency problem with regard to the care with which the asset is used, which increases the rate at which the asset depreciates. This allows the lessor to extend more credit to a financially constrained firm relative to the case where he makes a loan to the firm , increasing the debt capacity of leasing relative to secured lending.

A trade-off also emerges in our paper, driven however from the incentive problem in maintenance faced by the owner of a capital good when this is purchased with a secured loan. Such problem is overcome with leasing, as maintenance is delegated to the lessor. However, similar to Eisfeldt and Rampini (2009), separating ownership and control introduces an incentive problem for the entrepreneur in the form of a lack of care in the management of the capital good that, rather than increasing the depreciation rate, increases the expected cost of maintenance faced by the lessor, and thus, through the leasing fee paid by the lessee, the cost of leasing. The actual mix of secured lending and leasing depends on how the maintenance and use incentives interact with the firm's financial constraints. As Eisfeldt and Rampini (2009), we also find that leasing may relax financing constraints, financing 100% of the asset purchase price even, a result that is in line with one of the reasons firms often invoke to motivate their reliance on leasing (Leaseurope, 2015), but that, to the best of our knowledge, has been so far absent in the theoretical literature. And this is precisely one of the novel findings of our paper, i.e., that even a penniless entrepreneur can access the capital to carry out production. However, we also find that the reliance on leasing may be non-monotone in financing costraints, thus rationalizing the evidence provided by Beck, Demirguc-Kunt and Maksimovic (2008) who show that small firms do not use disproportionately more leasing compared with larger firms.

The actual existence of a moral hazard problem in leasing contracts has been empirically documented by Schneider (2010) who examines the driving outcomes of long-term lessees and owner-operators of taxis in New York, finding that moral hazard explains a consistent fraction of lessees' accidents, driving violations, and vehicle inspection failures.

The paper is also related to the literature studying the impact of financial constraints on leasing choices (Krishnan and Moyer, 1994; Sharpe and Nguyen, 1995; Eisfeldt and Rampini, 2009). This literature has shown that more financially constrained firms lease more of their capital, which is consistent with the prediction we get from our model. do (for similar results, see also Krishnan

Besides this literature, many other contributions have suggested alternative explanations for leasing. In addition to the traditional tax-related incentives to lease or buy (Miller and Upton, 1976; Myers, Dill and Bautista, 1976; Franks and Hodges, 1987), several other factors affect the leasing versus buying decision. Asset characteristics, for example, are important determinants of the leasing versus buy decisions. In particular, leasing is more attractive for more liquid and less specific assets, which are more easily redeployable (Klein, Crawford, and Alchian, 1978). Empirical evidence consistent with this is found by Gavazza (2010, 2011). Hendel and Lizzeri (2002) and Johnson and Waldman (2003) develop a theoretical analysis of leasing contract in which leasing in the new-car market emerges as a response to the adverse-selection problem in the used-car market.

3 The model

Players and Environment: A risk-neutral entrepreneur has an investment project that uses a capital input (K). The invested input is converted into a verifiable state-contingent output, $Y \in \{0, y\}$. Uncertainty affects production through demand (i.e., production is demand-driven). Demand can be high, with probability p, or low, with probability $1 - p$. Following a period of high demand, the invested input generates output $Y = y$ according to a strictly concave production function, $y = Af(K)$, with $A > 0$ and $f'(K) > 0$ for all $K > 0$. Following a period of low demand, the invested input generates zero output $Y = 0$. The characteristics of the technology are common knowledge. The entrepreneur is a price-taker both in the input and in the output markets. The output price is normalized to one, and so is the price of the input.[5]

To buy the capital inputs K necessary for production, the entrepreneur has an initial wealth W and has access to external funding $L \geq 0$ from competitive investors/banks. Lending is exclusive, that is, the entrepreneur cannot borrow from multiple investors. In alternative to buying, the entrepreneur may lease the capital good from leasing firms.

Banks and leasing firms play different roles. Banks lend cash that is used by the entrepreneur to buy the capital input. In exchange for the loan L, investors receive a repayment R in case of success. In case of failure, because output is zero, by limited liability they receive zero. In case in which capital inputs are purchased, they can be entirely or partly pledged as collateral to creditors in case of default. Denote with λ the fraction of the capital good that goes to the bank in case of default. Unlike Eisfeldt and Rampini (2009), we assume that there is no loss in the scrap value of capital due to the transfer from the entrepreneur to the creditors.

The leasing firm buys the capital input and leases it to entrepreneurs in exchange for a rental fee F. Upon expiration of the leasing contract, assets are costlessly repossessed by the lessor. Thus, the lessor finances the purchase of the capital good with the rental fee and with the asset residual value upon expiration of the leasing contract. We assume that the scrap value of capital when repossessed by the lessor cannot be lower than when repossessed by the

[5]This normalization is without loss of generality because we use a partial equilibrium setting.

creditors.

Banks and lessors have a cost of raising funds on the market equal to r and r^R, respectively, with $r \leq r^R$. This assumption is consistent with the investors playing the role of specialized financial intermediaries. Each party is protected by limited liability.

Maintenance and Care: Capital inputs are redeployable. The degree of redeployability depends on the depreciation rate and is affected by their liquidity. The input liquidity, that we denote with $\gamma \in (0, 1]$, depends on the input exogenous characteristics. More specific inputs, tailored to the user's needs, are less liquid and cannot be easily resold in the secondary market. Less specific inputs, whose purpose is more general, are more liquid and can be resold at near the purchase price. A regards the input depreciation rate, this is partly exogenous and partly endogenous. The exogenous part is denoted by $\overline{\delta}$, with $0 < \overline{\delta} < 1$.[6] The endogenous part depends on the maintenance carried out by the owner of the capital good and slows down the exogenous depreciation rate. Maintenance consists in the periodical work needed to keep an equipment in good working condition and mitigate its wear-and-tear. It is unobservable by third parties (it is non-contractible) and is carried out by the owner of the asset. It is denoted by $\mu \in \{0, \overline{\mu}\}$, with $\overline{\mu} < \overline{\delta}$ and $\mu = 0$ meaning no maintenance. Thus, $\left(1 - \overline{\delta} + \mu\right) \gamma K$ is the scrap value of capital and a fraction of such value, $\mu\gamma K$, can be ascribed to the maintenance activity. Maintenance has both a pecuniary and a non-pecuniary cost. The non-pecuniary cost is constant and equal to $\eta > 0$. It can be justified with the hassle that the owner of the capital good has to incur to have it serviced, like finding the garage, taking an appointment or taking it there. The pecuniary cost is affected by the intensity with which the capital good has been used in the production process, which depends both on the level of demand and on

[6] Unlike Rampini, we assume that the rate at which the capital depreciates is the same whether the good is purchased or leased.

the level of care. By care, denoted with $a \in \{a_0, a_1\}$, where a_1 stands for high care and a_0 for low care, we identify the unverifiable activities carried out by the entrepreneur in the use of the asset that affect its value. It has a non-pecuniary cost ϕ that is always borne by the entrepreneur, but its benefits are enjoyed by the owner of the capital good in the form of lower expected maintenance costs. In particular, when the level of demand is high (which occurs with probability p), inputs are intensively used in the production process and the expected pecuniary cost of maintenance is $(1-q)\,mK$, if high care is exerted $(a = a_1)$, and mK, with $m > 0$, if low care is exerted $(a = a_0)$.[7] When the level of demand is low, maintenance has zero pecuniary cost and no care decision is taken.

To make the problem interesting, we assume that the benefit of maintenance per unit of capital, $\overline{\mu}\gamma$, is higher than its cost, m. Formally:

Assumption 1 *$\overline{\mu}\gamma > m$.*

Assumption 1 implies that, under zero non-pecuniary cost of maintenance and care, $\phi = \eta = 0$, maintenance is valuable even under low care. Indeed, when $\overline{\mu}\gamma > m$, the extra value of the capital good that can be ascribed to the maintenance activity is larger than the pecuniary maintenance cost for any level of capital input invested, also in the case of low care.

However, for positive non-pecuniary costs ϕ and η, the optimal care and maintenance decisions could depend on the investment level, K. To simplify exposition, in the following we restrict the attention to investment projects with ϕ and η sufficiently low so that high care and maintenance are welfare improving for all relevant K. This translates in the following assumptions:

[7] In particular, under high care the pecuniary cost of maintenance is equal to zero with probability q, and to mK with probability $1-q$. It turns out that upon observing a maintenance cost mK, it is not possible to say with certainty whether the entrepreneur has exerted high care or low care.

Assumption 2 *(i)* $K \geq \frac{\phi+\eta}{\overline{\mu}\gamma-(1-q)m}$*, and (ii)* $K \geq \frac{\phi}{qm}$*, for all relevant* K*.*

The first assumption ensures that the maintenance value given high care is positive, i.e., $(\overline{\mu}\gamma - (1-q)\,m)\,K \geq \eta + \phi$. The second assumption ensures that the maintenance value given high care exceeds the maintenance value given low care, i.e., $(\overline{\mu}\gamma - (1-q)\,m)\,K - \eta - \phi \geq (\overline{\mu}\gamma - m)\,K - \eta$. This reduces to $qmK \geq \phi$, that can be interpreted as the benefit of care in terms of reduced cost of maintenance exceeding its non-pecuniary cost. By this last assumption, the care incentive is internalised in the maintenance incentive, i.e., whenever it is optimal to carry out maintenance, it is optimal to exert care.[8]

Timing: The sequence of events is as follows.

At $t = 0$, competitive banks and rental firms make contract offers to the entrepreneur. The bank contract offer specifies the size of the loan, L, the repayment obligation, R, the amount of capital input to be purchased, K, and the fraction λ of the capital good that goes to the bank in case of default. The leasing firm contract offer specifies the leasing fee, F, and the amount of capital input to be leased, K. At $t = 1$, the entrepreneur chooses the contract, and thus buys or leases the capital input K. At $t = 2$ uncertainty resolves, production takes place and the unobservable care decision is taken, if any. At $t = 3$, the party who owns the good decides the level of maintenance. At $t = 4$, repayments are made.

As a consequence, in the case of leasing, the separation between ownership and control introduces an agency problem as the entrepreneur, not being the owner of the capital good, may choose a suboptimal level of care in its management. This in turn, by affecting the expected cost of maintenance faced by the lessor, affects the rental fee charged to the lessee.

[8]**Notice that, satisfaction of the first assumption also ensures that the maintenance value under low demand is positive, i.e.,** $\overline{\mu}\gamma K > \eta$.

4 Buying

4.1 The benchmark

In this section, we establish the benchmark outcome to evaluate the efficiency of the various equilibria that we will characterize in the following sections. We assume that the capital goods can only be purchased and define the first-best as the situation where there is symmetric information and maintenance and care are both observable and verifiable by a third party and can be included in an enforceable contract.[9] In this setting, the entrepreneur can finance the investment that maximizes the firm's value. This depends not only on the value of production, but also on the residual value of the capital input used in production. The latter is affected by the intensity of usage in the production process, i.e., the level of demand, and by the degree of maintenance and care. In each state of the world, maintenance and care are set to the level which produces the largest residual asset value net of their costs.

By assuming that both maintenance and care are enforceable, the entrepreneur can implement the level of investment that maximizes her expected payoff, conditional on high care and maintenance in the case of high demand, and maintenance in the case of low demand.[10] In such circumstances, the entrepreneur's expected payoff is

$$\Pi(K) = p(Af(K) - (1-q)\, mK) - rK + (1 - \overline{\delta} + \overline{\mu})\gamma K - \eta - p\phi.$$

Denote by K^{FB} the level of capital input that maximizes $\Pi(K)$. It solves the following first-order condition:

$$pAf'(K^{FB}) = r + p\,(1-q)\, m - (1 - \overline{\delta} + \overline{\mu})\gamma. \tag{1}$$

From the above discussion, it follows that provided that ϕ and η are not too high, the first-best

[9] We introduce the possibility to lease the capital good for the entrepreneur in Section 5.

[10] Because the capital usage is soft, no care decision is required in case of low demand.

outcome involves an investment in capital good equal to K^{FB}, with the entrepreneur exerting high care and carrying out maintenance equal to $\overline{\mu}$.

4.2 The choice of a financially constrained entrepreneur

In this section we assume that care and maintenance are not contractible and analyze the setting in which the entrepreneur uses internal wealth (W) and bank loans (L) to purchase the capital input necessary for production.

From the previous section, we know that the first-best outcome involves high care and maintenance, given that η and ϕ are not too high. However, when maintenance is not enforceable, there may be circumstances in which an entrepreneur prefers not to carry it out. This may happen under low demand if too large a fraction of the capital good is pledged as collateral.[11] The limited maintenance benefit introduces an agency problem on the entrepreneur, who, anticipating that she might not repossess (all) the capital input, might give up maintenance, thereby jeopardising the return the bank obtains in case of default. This may lead to credit rationing and underinvestment.

Under the assumptions of the model, the entrepreneur's optimization problem is defined by programme $\mathcal{P}_{\mathcal{B}}$:

$$\max_{K,L,R,\lambda} p(Af(K)-R)+p((1-\overline{\delta}+\overline{\mu})\gamma-(1-q)\,m)K+(1-p)(1-\lambda)(1-\overline{\delta}+\overline{\mu})\gamma K-\eta-p\phi$$

[11] **Under high demand, the capital good is never repossessed by the investors, which implies that by Assumption 2 both maintenance and care are optimal.**

$$\text{st } pR + (1-p)\lambda(1-\overline{\delta}+\overline{\mu})\gamma K \geq rL \quad (2)$$

$$(1-\lambda)\overline{\mu}\gamma K \geq \eta \quad (3)$$

$$Af(K) - (1-q)\, mK \geq R \quad (4)$$

$$L + W \geq K \quad (5)$$

$$\lambda \in [0,1]. \quad (6)$$

Condition (2) is the participation constraint requiring that investors get non-negative returns. Competition in the banking sector implies that it is binding. If not, it would be possible to lower R, and increase the entrepreneur's profits. Constraint (3) is the incentive compatibility condition guaranteeing that the entrepreneur carries out the maintenance in the bad state. Condition (4) is the limited liability constraint stating that the cash flows in the good state are sufficient to repay the investors and the expected maintenance cost (thus, investors in the good state are paid out of cash flows and not of assets) the repayment to the investors R cannot exceed the net cash flows available (i.e., net of pecuniary maintenance costs), while condition (5) is the resource constraint ensuring that the investment does not exceed available funds. Last, constraint (6) states that the fraction of the capital good that goes to the bank in case of default is in the unit interval.

To see where the incentive constraint (3) comes from, consider that the maintenance decision takes place after the uncertainty realizes. While carrying out maintenance (and care) is always optimal for the entrepreneur if the good state realizes, given that she keeps the capital good, it might be privately unprofitable if the bad state realizes. Indeed, if the fraction of the capital good λ pledged as collateral is sufficiently high, the benefit of maintenance in terms of increased value of the capital good $(1-\lambda)\overline{\mu}\gamma K$ may fall short of the cost η. It turns out that the maximum pledgeable capital can never exceed the one solving constraint (3).

Using $L = K - W$ from (5) in (2) gives $pR = (K-W)r - (1-p)\lambda(1-\overline{\delta}+\overline{\mu})\gamma K$. By combining (2) and (3), and substituting out in (4) gives $p(Af(K) - (1-q)\,mK) \geq (K-W)r - (1-p)\frac{\overline{\mu}\gamma K-\eta}{\overline{\mu}}(1-\overline{\delta}+\overline{\mu})$. Moreover, (3) and (6) lead to $\lambda \in [0, 1 - \frac{\eta}{\overline{\mu}\gamma K}]$ since $\frac{\eta}{\overline{\mu}\gamma K} < 1$. Indeed, we have restricted the analysis to investment projects where maintenance is welfare improving for all relevant K, which implies $\overline{\mu}\gamma K > \eta$.

The optimal value of λ depends on whether constraint (3) is binding at the optimum. If it is binding, then $\lambda = 1 - \frac{\eta}{\overline{\mu}\gamma K}$. For all the cases in which it is slack, since pledging collateral involves no cost, the optimal sharing rule is indeterminate and multiple solutions arise. To rule this out, we assume that in default all incentive feasible assets are used to repay investors. This is without loss of generality and in line with a vast theoretical literature showing that pledging collateral to creditors, by mitigating agency problems, increases debt capacity.

Substituting out R from the participation constraint in the entrepreneur's profits, program $\mathcal{P}_{\mathcal{B}}$ can be written as

$$\max_{K}\, p(Af(K) - (1-q)\,mK) + (1-\overline{\delta}+\overline{\mu})\gamma K - Kr - \eta - p\phi$$

subject to the financing constraint:

$$p(Af(K) - (1-q)\,mK) + (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\overline{\mu}\gamma K - \eta}{\overline{\mu}} - (K-W)\,r \geq 0. \tag{7}$$

The level of the capital input that maximizes the objective function is the first-best investment, K^{FB}, solving (1). If it also satisfies constraint (7), then K^{FB} is the level of capital solving program $\mathcal{P}_{\mathcal{B}}$.

Proposition 1 underlines the condition for the first-best investment to be achieved even when care and maintenance are not contractible, and characterizes the first-best secured debt contract.

Proposition 1 *The first-best investment, K^{FB}, solves program $\mathcal{P}_{\mathcal{B}}$ if and only if it satisfies the financing constraint (7). At the first-best, the loan size is $L^{FB} = K^{FB} - W$, the fraction of residual capital pledged to investors is $\lambda^{FB} = \frac{r(K^{FB}-W)}{(1-p)(1-\overline{\delta}+\overline{\mu})\gamma K^{FB}}$ and the repayment to investors is $R^{FB} = 0$ if $r(K^{FB} - W) \leq (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\overline{\mu}\gamma K^{FB}-\eta}{\overline{\mu}}$, and $\lambda^{FB} = \frac{\overline{\mu}\gamma K^{FB}-\eta}{\overline{\mu}\gamma K^{FB}}$ and $R^{FB} = \frac{r(K^{FB}-W)}{p} - \frac{(1-p)(1-\overline{\delta}+\overline{\mu})}{p}\frac{\overline{\mu}\gamma K^{FB}-\eta}{\overline{\mu}}$, otherwise.*

Proposition 1 states that to be implementable the first-best investment in the capital good has to satisfy the financing condition (7). It requires that the expected value of the highest pledgeable capital asset, $(1-p)(1-\overline{\delta}+\overline{\mu})\frac{\overline{\mu}\gamma K^{FB}-\eta}{\overline{\mu}}$, covers the part of the loan not paid for by the available net expected cash flows (i.e., exceeds the difference between the loan value, $(K^{FB} - W)r$, and the expected cash flows net of monetary maintenance costs, $p(Af(K^{FB}) - (1-q)\, mK)$). If K^{FB} does not satisfy constraint (7), then the debt obligation cannot be covered by the available resources, and an agency problem emerges: the entrepreneur has no incentive to do maintenance in the event of failure since the benefit from doing so is too small with respect to its cost.

The severity of the financing constraint is decreasing both in the cash flows and in the residual value of the capital input. Moreover, it depends not only on the magnitude of the expected value of the firm, but also on its composition. Indeed, as highlighted in the next corollary, for any fixed firm's profits, the agency problem is more relevant for projects whose expected cash flows are tiny relative to the residual value of capital. Let $\Pi^{FB} \equiv \Pi(K^{FB})$ the firm's profits at the first-best.

Corollary 1 *For any Π^{FB}, the financing condition (7) becomes more severe for firms with investment projects characterized by lower cash flows and higher residual value of capital assets.*

The intuition is as follows. The expected profits of a firm under both high care and maintenance consists of two parts: the expected net cash flows generated by the production process, $pAf(K) - rK$, and the residual capital value net of the expected maintenance cost, $(1 - \overline{\delta} + \overline{\mu})\gamma K - p(1-q)mK$. The higher the weight of the cash flows compared to that of the residual capital value, the more liquid the firm. Let us consider two firms, firm 1 and firm 2, with the same first-best profits but a different composition of the same. More precisely, suppose that firm 1 is characterized by greater capital productivity, A, and greater capital depreciation rate, $\overline{\delta}$, than firm 2. Under our assumptions, it is more difficult for firm 2 to implement the first-best investment than for firm 1. Indeed, to make up for the lower cash flows relative to firm 1, firm 2 has to pledge a larger fraction of the capital residual value as collateral to investors. This tightens the incentive constraint and makes it harder to satisfy it. Thus, the cash flows have to be large enough relative to the residual value of capital for the financing condition (7) to be satisfied.

In the following, we will derive the optimal investment as a function of wealth and we will show under which circumstances it is optimal for the entrepreneur to carry out maintenance even under low demand.

4.3 The equilibrium outcomes

In the previous section we have shown that an agency problem on the entrepreneur emerges if the first-best level of capital does not satisfy the financing condition (7). Indeed, to undertake the first-best production, the entrepreneur invests $K = K^{FB}$. When $W < K^{FB}$, the loan necessary to carry out the project is L^{FB}. If the cash flows in the case of high demand are too low to repay the debt, then the entrepreneur has to pledge collateral in the event of failure. The first-best residual capital value under low demand is $(1-\overline{\delta}+\overline{\mu})\gamma K^{FB}$. However, to preserve the maintenance incentives, from constraint (3), the fraction λ of the asset residual value pledged as collateral in case of failure cannot exceed $\frac{\overline{\mu}\gamma K^{FB}-\eta}{\overline{\mu}\gamma K^{FB}}$. Indeed, if a larger fraction is pledged, then the fraction retained by the entrepreneur becomes too small to make it worthwhile to carry out the maintenance, namely the benefit from maintenance falls short of its non-pecuniary cost.

Proposition 2 sets the minimum level of wealth at which the first-best outcome can be implemented.

Proposition 2 *The entrepreneur finances the first-best investment, K^{FB} if and only if her initial wealth W is greater than a critical level $W_2 \geq 0$, with W_2 increasing in η.*

Proposition 2 shows that if the entrepreneur is wealthy enough ($W \geq W_2$), the first-best outcome is implemented at equilibrium. The optimal contract has the features described by Proposition 1, with a loan $L \leq L^{FB} = K^{FB} - W_2$.

For wealth levels below W_2, the entrepreneur needs a larger loan to implement the first-best investment. However, pledging a larger fraction of the asset residual value as collateral to investors is not a viable route to obtain the loan as this destroys the entrepreneur's incentives to carry out the maintenance and jeopardizes the return to investors in the failure state. This reduces the entrepreneur's borrowing capacity and give rise to two possible scenarios. In the first, the entrepreneur reduces the need for external funds by downsizing the investment to a level that makes it always worthwhile to do the maintenance. In the second, the entrepreneur neglects the incentive constraint and chooses the level of investment that maximizes the firm value giving up maintenance in case of failure. The optimality of inducing or not maintenance in the bad state of the world depends on the firm's profits resulting in the two scenarios, which in turn depends on the entrepreneur's initial wealth.

When wealth is sufficiently close to W_2, it is possible to restore the entrepreneur's maintenance incentive by reducing the reliance on external finance, i.e., through a reduction in investment. Let $K^{FC}(W)$ be the maximum pledgeable capital when the entrepreneur's wealth is W, i.e., the level of capital inputs such that constraint (7) is binding. As W decreases, the investment level keeps decreasing. However, for sufficiently high non-pecuniary cost of maintenance, there is a level of wealth, $W_1 < W_2$, at which maintenance is given up. This can occur either because the reduction in investment is so pronounced that it is preferable to stop enticing maintenance from the entrepreneur in case of low demand, or because the financing condition can no longer be satisfied by reducing the investment level. Denote by $K^{NM} < K^{FB}$ such investment level, i.e., the one that maximizes the entrepreneur's expected payoff under high care and maintenance in the case of high demand and zero maintenance in the case of low demand.[12]

Proposition 3 characterizes the equilibrium outcomes for a financially constrained entrepreneur (and for sufficiently high non pecuniary cost of maintenance).

Proposition 3 *Suppose that the capital good can only be purchased. Assume η greater than a threshold $\hat{\eta}$ and suppose $W < W_2$, with $W_2 > 0$. There exists a critical level of the entrepreneur's initial wealth $W_1 < W_2$ such that (i) for $W_1 \leq W < W_2$, the entrepreneur invests $K^{FC}(W) < K^{FB}$, carries out maintenance both in the event of success and in the event*

[12]For the analysis of this programme with the formal derivation of K^{NM}, see Appendix A.

of default; (ii) for $W < W_1$, the entrepreneur invests $K^{NM} < K^{FB}$, carries out maintenance only in the event of success. Moreover, W_1 is increasing in η, and $K^{NM} > K^{FC}(W_1)$ if and only if $p > p_0$, with $p_0 \equiv \frac{\overline{\mu}}{(1-\overline{\delta}+2\overline{\mu})}$.

When η is large and $W \geq W_2$, the entrepreneur carries out maintenance and exerts care both in case of success and failure. When $W < W_2$, to make it worthwhile for the entrepreneur to do maintenance, the investment is reduced sufficiently so that the financing condition (7) is satisfied. This is equivalent to choosing the level of capital inputs, $K^{FC}(W)$, such that constraint (7) is binding. In this case, the loan size is $L^{FC}(W) \equiv K^{FC}(W) - W$, the repayment is $R^{FC}(W) \equiv f(K^{FC}(W)) - (1-q)\, mK^{FC}(W)$, and the fraction of residual capital pledged to investors $\lambda^{FC}(W) \equiv \frac{\eta}{\overline{\mu}\gamma K^{FB} - \overline{\mu}\gamma K^{FC}(W)} < 1$.

If $W < W_1$, maintenance is given up in the event of failure and the investment level is $K^{NM} < K^{FB}$. Moreover, the optimal loan size is $L^{NM}(W) \equiv K^{NM} - W$, the repayment and the fraction of residual capital pledged to investors are $R^{NM}(W) = \frac{r(K^{NM}-W)}{p} - \frac{(1-p)(1-\overline{\delta})\gamma K^{NM}}{p}$ and $\lambda^{NM}(W) = 1$.

Proposition 3 also states that if the likelihood of high demand overcomes a threshold p_0, then $K^{NM} > K^{FC}(W_1)$. Thus, if the probability of success is sufficiently high, there is a U-shaped relationship between investment and internal wealth.

To see where this result comes from, notice that the financing constraint (7) is concave in K and reaches its maximum value at a level of capital input, $\hat{K}^{FC} < K^{FB}$. Thus, there exist a level of wealth below which the financing condition cannot be satisfied even reducing the investment level. Let be W^{FC} the level of wealth such that $K^{FC}(W^{FC}) = \hat{K}^{FC}$. Such level W^{FC} is no higher than the threshold W_1 introduced in the proposition. In particular, denote by $\Pi(K^{FC}(W))$ the expected value of a financially constrained firm with initial wealth $W \in$

$[W^{FC}, W_2)$ under positive maintenance in both states, and by $\Pi^{NM^\star}$ the expected value of a firm under zero maintenance in case of low demand. If $\Pi(K^{FC}(W^{FC})) \equiv \Pi(W^{FC}) \geq \Pi^{NM^\star}$, then $W_1 = W^{FC}$, and if $\Pi(W^{FC}) < \Pi^{NM^\star}$, then $W_1 > W^{FC}$.

For any level of capital, K, the expected firm value is greater when maintenance is carried out in both states of the world rather than in the good state only. Hence, $\Pi(W^{FC}) > \Pi^{NM^\star}$ whenever $K^{NM} \leq K^{FC}(W^{FC})$. At $p = p_0$, K^{NM} is exactly equal to $K^{FC}(W^{FC})$, and $\Pi(W^{FC}) > \Pi^{NM^\star}$. However, higher values of p reduce the expected losses due to the lower maintenance performed in the case of failure and positively affect K^{NM} by increasing the marginal productivity of capital inputs. This implies that K^{NM} is greater than $K^{FC}(W^{FC})$ for all $p > p_0$.

There are therefore two patterns of investment at $W = W_1$ according to whether $p \lesseqgtr p_0$. Fig. 2 depicts the case in which $p \leq p_0$. The middle panel depicts the investment levels across the wealth areas, while the bottom one the profit levels. The top panel instead describes the relevance of the incentive problem across the wealth areas. Since $K^{NM} \leq K^{FC}(W^{FC})$, the profits under no maintenance $\Pi^{NM^\star}$ are lower than those in which it is still possible to entice maintenance, $\Pi(W^{FC})$, and then W_1 is equal to W^{FC}.

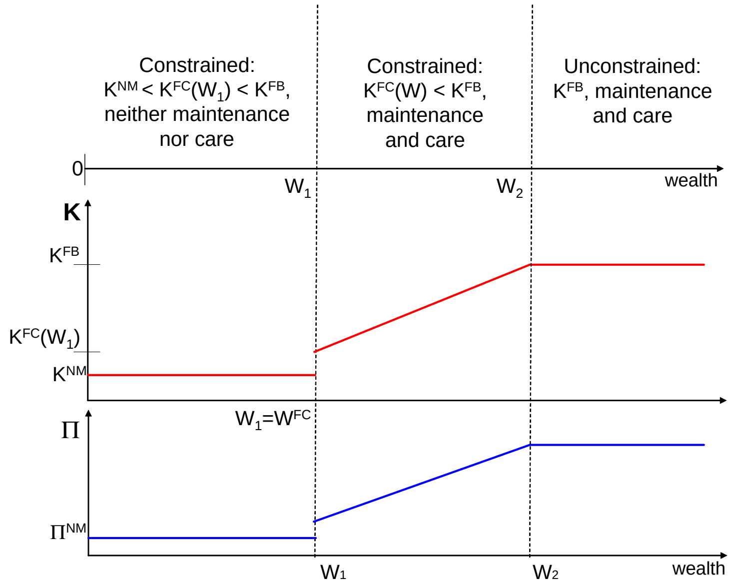


Fig. 2: Wealth areas, investment and profits when the capital inputs can only be purchased and $p < p_0$.

In the scenario with $p > p_0$, depicted in Fig.3, K^{NM} is greater than $K^{FC}(W_1)$, but either $\Pi(W^{FC}) \geq \Pi^{NM\star}$ or $\Pi(W^{FC}) < \Pi^{NM\star}$, depending on the parameters of the model. When $\Pi(W^{FC}) \geq \Pi^{NM\star}$, W_1 is equal to W^{FC}, as in the previous scenario. When $\Pi(W^{FC}) < \Pi^{NM\star}$, W_1 such that $\Pi(W_1) = \Pi^{NM\star}$ is larger than W^{FC}.

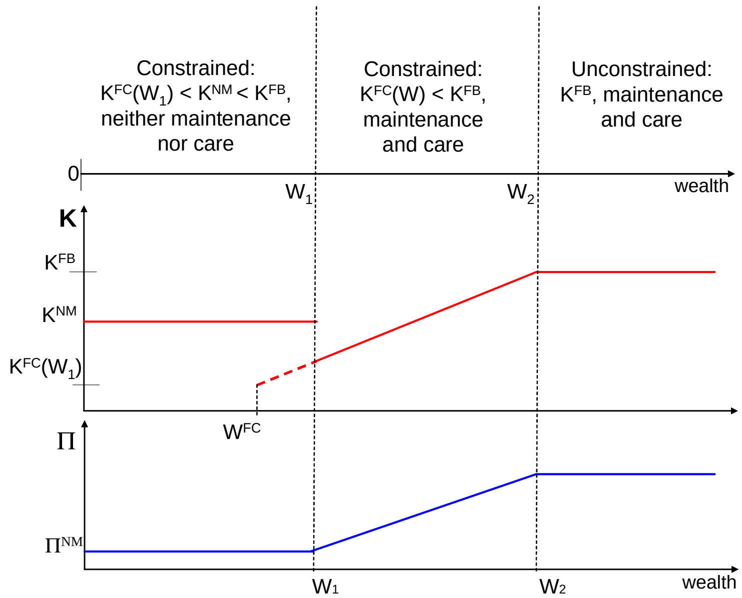


Fig. 3: Wealth areas, investment and profits when the capital inputs can only be purchased and $p > p_0$.

To gain an intuition for the above results, consider that under buying, the entrepreneur has always an incentive to carry out the maintenance (and the care) in the good state as she owns the capital good. In the bad state, in order for the entrepreneur to have an incentive to carry out the maintenance it must be the case that the expected benefit of maintenance on the fraction of the capital good she has a right to in case of default exceeds its non-pecuniary cost. For sufficiently high wealth, the fraction of the capital input pledged as collateral is small and the entrepreneur is enticed to do the maintenance.

As wealth decreases, to keep satisfying the incentive constraint, the entrepreneur has to downsize the investment with a subsequent reduction in profits. Two scenarios may then arise. In the one depicted in Fig. 2, at $W < W^{FC}$, the financing condition cannot be satisfied even reducing the investment level, and maintenance is not carried out. When this occurs, the profits under maintenance fall short of those under no maintenance. In the scenario depicted in Fig. 3, instead, the reduction in profits implied by the reduction in output is so pronounced to induce the entrepreneur with wealth no higher than W_1 to give up maintenance even if further reductions in the investment level could still satisfy the financing condition. When this last scenario arises, the investment level under no maintenance overshoots the one under maintenance and involves an increase in production so high to compensate for the loss due to the higher expected cost of no maintenance (in terms of reduced residual value of the capital good). When this occurs, the profits under maintenance equal those under no maintenance. Any reduction in wealth involves a further reduction in investment, and thus in profits, that makes the no maintenance regime optimal.

A last remark is in order. Proposition 3 has described the equilibrium outcomes for sufficiently high non pecuniary cost of maintenance and shown that investment may be non-monotone in initial wealth and that maintenance may be given up for sufficiently low levels of wealth. If, instead, η is small (i.e., $\eta \leq \hat{\eta}$), the entrepreneur prefers to reduce the investment in order to satisfy the financing condition (7), by choosing the level of capital inputs $K^{FC}(W)$, and carry out maintenance both in case of success and failure for all levels of initial wealth.[13]

5 Leasing

In the previous section, we have seen that it may be costly (or too costly) to induce the entrepreneur to do the maintenance. In the present section we want to investigate whether it is possible to overcome this incentive problem by relying on leasing contracts. In particular, we give the entrepreneur the possibility to lease the capital inputs rather than purchasing them. This allows the contractor to get the right to use the asset, leaving its servicing to the lessor, thereby saving the asset maintenance costs (and the related agency costs). However, as highlighted by Alchian and Demsetz (1972) and studied by Eisfeldt and Rampini (2009), the separation between ownership and control introduces a novel agency problem as, not being the owner of the capital good, the contractor may behave opportunistically and choose a suboptimal level of attention in its management. This affects the liquidation value of the capital good in case of high demand and jeopardizes the return to the lessor.

To model the leasing decision, we assume that in the market there are leasing firms that buy capital goods incurring a financing cost r^R and rent them to firms upon the payment of a leasing fee F. For sake of clarity, we assume $r^R = r$.[14] The entrepreneur has to choose the

[13] A formal analysis of this statement is in Lemma 4 in Appendix B.

[14] All our results remain true for all $r^R = r + \varepsilon$, with $\varepsilon > 0$ and small enough.

level of attention to exert in the management of the leased good when production is high. We assume that this choice is not observable by the lessor, who carries out maintenance.

Assumptions 1 guarantees that maintenance is Pareto improving regardless of care for sufficiently low values of η and ϕ. This implies that the lessor chooses to carry out maintenance both under soft capital usage (low demand) and under strong capital usage (high demand), even if the entrepreneur performs low care in managing the capital good. We assume that, unlike the case in which maintenance is carried out by the entrepreneur, the leasing company does not face the non-pecuniary cost of maintenance, i.e., $\eta = 0$. This can be justified with the fact that, along with leasing, maintenance is one of the lessor's main activities, carried out within the company's premises. As such, it does not involve the kind of costs faced by someone who owns the good, having purchased it as a production input, but cannot service it directly.[15]

The financial contract sets the level of investment in the capital good K and the leasing fee F to solve the following problem, $\mathcal{P}_{\mathcal{R}}$:

$$\max_{K,F} p\left[Af\left(K\right) - F\right]$$

subject to the lessor's participation constraint given that he carries out maintenance in both states of the world:

$$pF + [(1 - \overline{\delta} + \overline{\mu})\gamma - pm]K \geq rK. \tag{8}$$

The participation constraint (8) has to be binding at the optimum. If not, it would be possible to lower F and increase the entrepreneur's profits. Substituting out F from (8) in

[15] All our qualitative results continue to hold if we relax this assumption.

the entrepreneur's profits, the optimisation problem $\mathcal{P}_{\mathcal{R}}$ can be written as

$$\max_{K} \Pi^R(K) = pAf(K) + \left[(1 - \overline{\delta} + \overline{\mu})\gamma - pm\right] K - rK.$$

Denote by K^R the level of capital input that maximizes $\Pi^R(K)$ and solves $\mathcal{P}_{\mathcal{R}}$, and define $\Pi^{R\star} \equiv \Pi^R\left(K^R\right)$. It satisfies the following first-order condition:

$$pAf'(K^R) = r + pm - (1 - \overline{\delta} + \overline{\mu})\gamma. \tag{9}$$

Proposition 4 describes the optimal investment level and leasing fee under the leasing contract.

Proposition 4 *The equilibrium outcome solving program $\mathcal{P}_{\mathcal{R}}$ involves an investment in capital good equal to $K^R < K^{FB}$ defined by condition (9), with the entrepreneur exerting low care and the lessor carrying out maintenance in both states of the world. Moreover, the rental fee is $F^R = [r - (1 - \overline{\delta} + \overline{\mu})\gamma + pm]K^R$.*

The above proposition shows that the maintenance incentive that may break down under a purchase contract can be restored by relying on a leasing contract. However, delegating maintenance to the lessor does not allow the entrepreneur/lessee to fully solve her moral hazard problem. Indeed, despite not having the ownership, the lessee still keeps the control of the asset, and can exert a suboptimal level of care in managing it. This increases the cost of maintenance for the lessor and thus the rental fee, determining a reduction in the level of investment relative to the first-best. It turns out that, depending on the extent of the underinvestment problem, a leasing contract may be Pareto improving relative to a purchase contract. To rule out uninteresting scenarios, we assume that an unconstrained entrepreneur always prefers to buy the capital rather than lease it, despite the lessor's zero non-pecuniary cost of maintenance. This is equivalent to assuming that the higher maintenance cost due to lack of care exceeds its non pecuniary cost, that is, $pqmK^R \geq \eta$.

6 Buying and Leasing

We have so far considered two alternative ways for the firm to get hold of the capital inputs necessary for production. However, it is often the case that the capital inputs deployed by the firm are divisible and can be partly purchased and partly leased. Thus, the firm can use internal and external resources to purchase a fraction of them and lease the rest. To account for this possibility, we denote with K_b the capital purchased and with K_r the capital leased and we assume they are perfect substitutes in production, i.e., $K = K_b + K_r$. This assumption implies that to the two scenarios described in Section 4.2, a third one is added, where the entrepreneur can lease part of the capital inputs and buy the rest. In this scenario, the entrepreneur's optimization problem is defined by programme $\mathcal{P}_{\mathcal{B}-\mathcal{R}}$:

$$\max_{K,K_b,K_r,L,R,\lambda,F} p(Af(K)-R-F)+((p+(1-p)(1-\lambda))(1-\overline{\delta}+\overline{\mu})\gamma-p\,(1-q)\,m)K_b-\eta-p\phi-\mathbf{Wr}$$

$$\text{st } pF + [(1-\overline{\delta}+\overline{\mu})\gamma - pm]K_r \geq rK_r \tag{10}$$

$$pR + (1-p)\lambda(1-\overline{\delta}+\overline{\mu})\gamma K_b \geq rL \tag{11}$$

$$(1-\lambda)\overline{\mu}\gamma K_b \geq \eta \tag{12}$$

$$Af(K) - (1-q)\,mK_b \geq F + R \tag{13}$$

$$W + L \geq K_b \tag{14}$$

$$\lambda \in [0,1] \tag{15}$$

$$K_b + K_r = K \tag{16}$$

Conditions (10) and (11) are the lessor and investors' participation constraints, respectively. Competition in the leasing and in the credit market implies that they are both binding at the optimum. Constraint (12) is the incentive compatibility condition guaranteeing that the entrepreneur performs the maintenance on the purchased capital also in the bad state. Since we have assumed that in default all incentive feasible owned assets are used to repay investors, it is binding at the optimum. Condition (13) is the limited liability constraint stating that the sum of repayments due to the lessor and investors in the case of success, $F + R$, does not exceed the net cash flows available, while condition (14) is the resource constraint ensuring that the investment in owned capital, K_b, does not exceed available funds, and constraint (15) guarantees that the fraction of the purchased capital good that goes to the bank in case of default is in the unit interval. Last, condition (16) states that the total investment cannot exceed the sum of the leased and bought capital inputs.

Using $L = K_b - W$ from (14) and $\lambda = \frac{K_b\overline{\mu}\gamma - \eta}{K_b\overline{\mu}\gamma}$ from (12) in (11) gives $pR = r(K_b - W) - (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\overline{\mu}\gamma K_b - \eta}{\overline{\mu}}$. Using $pF = [r - (1-\overline{\delta}+\overline{\mu})\gamma + pm]K_r$ from (10), and $K = K_b + K_r$ from (16) and substituting out in (13) gives

$$p[Af(K_b + K_r) - m(1-q)K_b - mK_r] + (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\overline{\mu}\gamma K_b - \eta}{\overline{\mu}} + \\ + \left[((1-\overline{\delta}+\overline{\mu})\gamma)\right]K_r - r(K_b + K_r - W) \geq 0. \tag{17}$$

Finally, by combining (15) and (12) one obtains

$$K_b \geq \frac{\eta}{\overline{\mu}\gamma}. \tag{18}$$

Hence, for maintenance to be convenient for the entrepreneur, the purchased capital has to exceed a minimum threshold equal to $\frac{\eta}{\overline{\mu}\gamma}$.

Substituting out F and R from the participation constraints and K from (16) in the entrepreneur's profits, program $\mathcal{P}_{\mathcal{B}-\mathcal{R}}$ can be written as:

$$\max_{K_b,K_r} \Pi^{BR}(K_b,K_r) \equiv pAf(K_b+K_r) - [r-((1-\overline{\delta}+\overline{\mu})\gamma - p(1-q)m)]K_b -$$

$$+[r-((1-\overline{\delta}+\overline{\mu})\gamma - pm)]K_r - Wr - \eta - p\phi$$

subject to the financing constraint (17) and to the maintenance incentive constraint (18).

The combination of purchased and rented capital inputs that maximizes the objective function is $K_b = K^{FB}$ and $K_r = 0$, where K^{FB} is the first-best investment solving (1). Indeed, involving higher pecuniary maintenance costs, renting is costly and an unconstrained entrepreneur always prefers to buy all the capital inputs. Thus, since $K_b = K^{FB}$ satisfies (18) by assumption, if the combination $K_b = K^{FB}$ and $K_r = 0$ also meets the financing constraint (17), then it solves program $\mathcal{P}_{\mathcal{B}}$. From Proposition 2, we know that the first-best can be implemented for all levels of initial wealth above W_2, and then the maximum expected profit that the entrepreneur can obtain for all $W \geq W_2$ is $\Pi^{BR}(W) = \Pi^{FB}$, where $\Pi^{BR}(W) \equiv \max_{K_b,K_r} \Pi^{BR}(K_b,K_r)$, given constraints (17) and (18).

If $W < W_2$, the combination $K_b = K^{FB}$ and $K_r = 0$ does not satisfy constraint (17). In this case, the entrepreneur can reduce the purchased capital and, eventually, choose to rent part of the invested capital. Lemma 1 establishes that leasing relaxes financial constraints only for investment in capital inputs below a threshold level.

Lemma 1 *Leasing capital inputs relaxes the financial constraint (17) if and only if the total investment is* $K \leq K^R$.

Lemma 1 implies that entrepreneur always buys all the capital inputs if her initial wealth W is such that $K^{FC}(W) \geq K^R$. Thus, for all $W \geq W_1^{BR}$, with $W_1^{BR} < W_2$ and such that

$K^{FC}(W_1^{BR}) = K^R$, her expected profit is $\Pi^{BR}(W) = \Pi(K^{FC}(W)) \geq \Pi(K^R)$.[16] However, if $W < W_1^{BR}$, then the entrepreneur may optimally choose to lease a fraction of the capital to slacken the financial constraint. This decision depends on whether the benefit of the higher investment level offsets the higher maintenance costs due to the low care exerted by the lessee.

Next lemmas compare the leasing contract with the purchase contract with and without maintenance under low demand. In particular, Lemma 2 states that the investment in the capital input financed with a leasing contract is below the investment under no maintenance if the probability of default is sufficiently low $(1-p < 1-p_1)$ and the expected benefit from high care is high enough $(q > q_1(p))$. Lemma 3 focuses on comparing profits in three alternative scenarios: the leasing scenario, where the investment is K^R and the expected profit Π^{R^*}, the buying scenario with the entrepreneur carrying out maintenance only in the event of success, where the investment is K^{NM} and the expected profit Π^{NM^*}, and the buying scenario with the entrepreneur carrying out maintenance both in the event of success and failure, when the investment is $K^{FC}(W_1^{BR}) \equiv K^R$ and the expected profit $\Pi^{BR}(W_1^{BR})$. Interestingly, it points out conditions on p and q for buying the capital input to be preferred to leasing it, regardless of the financial constraints.

Lemma 2 *The optimal investment in capital good under leasing (K^R) is below the one under buying with no maintenance in the event of failure (K^{NM}) if and only if $p \geq p_1$ and $q > q_1(p)$, with $p_1 \equiv 1 + \frac{m}{\overline{\mu}\gamma} > p_0$, and $q_1(p) \equiv \frac{(1-p)\overline{\mu}\gamma}{pm}$.*

Lemma 3 *There exist $\underline{p}$ and $\overline{p}$, and $\underline{q}(p)$ and $\overline{q}(p)$, with $\overline{p} > \underline{p} > p_1$ and $\overline{q}(p) > \underline{q}(p) > q_1(p)$ such that (i) $\Pi^{NM\star} > \Pi^{BR}(W_1^{BR}) > \Pi^{R\star}$ for all $p \geq \overline{p}$ and $q > \overline{q}(p)$, (ii) $\Pi^{BR}(W_1^{BR}) >$*

[16] A level of wealth $W_1^{BR} \in (0, W_2)$ such that $K^{FC}(W_1^{BR}) = K^R$ always exists since $K^R > \hat{K^{FC}}$.

$\Pi^{NM\star} > \Pi^{R\star}$ *for all* $p \geq \overline{p}$ *and* $\underline{q}(p) \leq q < \overline{q}(p)$ *or* $\underline{p} \leq p < \overline{p}$ *and* $q > \underline{q}(p)$, *(iii)* $\Pi^{BR}(W_1^{BR}) > \Pi^{R\star} > \Pi^{NM\star}$ *otherwise.*

Proposition 5 *Assume* $p < \underline{p}$ *or* $p \geq \underline{p}$ *and* $q \leq \underline{q}(p)$. *There exist two critical levels of the entrepreneur's initial wealth* W_0^{BR} *and* W_1^{BR}, *with* $W_0^{BR} < W_1^{BR} < W_2$ *such that*

(i) *for* $W > W_2$, *the entrepreneur invests* K^{FB}, *buying all the capital inputs;*

(ii) *for* $W_1^{BR} < W \leq W_2$, *the entrepreneur invests* $K^{FC}(W) \leq K^{FB}$, *buying all the capital inputs;*

(iii) *for* $W_0^{BR} < W \leq W_1^{BR}$, *the entrepreneur invests* $K^{FC}(W_1^{BR}) = K^R$, *leasing a fraction* $K_r^{BR}(W) \equiv \frac{r(W_1^{BR}-W)}{p[(1-\overline{\delta}+\overline{\mu})\gamma - qm]}$ *and buying the rest* $K_b^{BR}(W) \equiv K^R - \frac{r(W_1^{BR}-W)}{p[(1-\overline{\delta}+\overline{\mu})\gamma - qm]} \geq \frac{\eta + p\phi}{pmq}$;

(iv) *for* $W \leq W_0^{BR}$, *the entrepreneur invests* K^R, *leasing all the capital inputs. Moreover,* $W_1^{BR} > W_1$.

Proposition 5 is illustrated in Figure 4, which depicts the wealth areas, the investment levels and the profits when both secured lending and leasing are available. The continuous lines in the middle panel show the investment in the capital input when both leasing and secured lending (purchase) are available. The population of entrepreneurs is distributed into three wealth areas with different degrees of credit rationing. For each area, the figure shows whether there is credit rationing as well as the inputs are purchased or leased. Sufficiently rich entrepreneurs ($W \geq W_2$), finance the first-best investment K^{FB} by purchasing the capital input with internal wealth and a secured loan (constant red line). Because the loan size is not too high, the entrepreneur has the incentive to carry out both maintenance and care on the capital goods. As wealth comes down toward W_2, the loan size increases to compensate for the lack of internal wealth. When $W_1^{BR} < W < W_2$, the loan needed to finance the first-best investment implies a large repayment obligation and the need to pledge a large fraction of the capital input to investors that leaves the entrepreneur with a return from carrying out the maintenance lower than the return from giving it pu. Banks must therefore ration the entrepreneur to prevent opportunistic behavior, whence credit rationing. Thus, to entice maintenance (i.e., to satisfy the financing condition (17)), the investment has to be reduced sufficiently. This is equivalent to choosing the level of the capital inputs, $K^{FC}(W)$, such that constraint (17) is binding (upward sloping red line). When $W \leq W_1^{BR}$, as well as secured lending, the entrepreneur starts relying on leasing and the investment level is equal to K^R. In particular, as wealth decreases below W_1^{BR}, the entrepreneur compensates the progressively lower secured loan received (dotted red line) with leasing (dotted blue line) and keeps the investment constant at K^R (green line). For $W \leq W_0^{BR}$, the entrepreneur leases 100% of the capital goods (blue line). **The leasing/purchase decision is therefore monotone in wealth.**

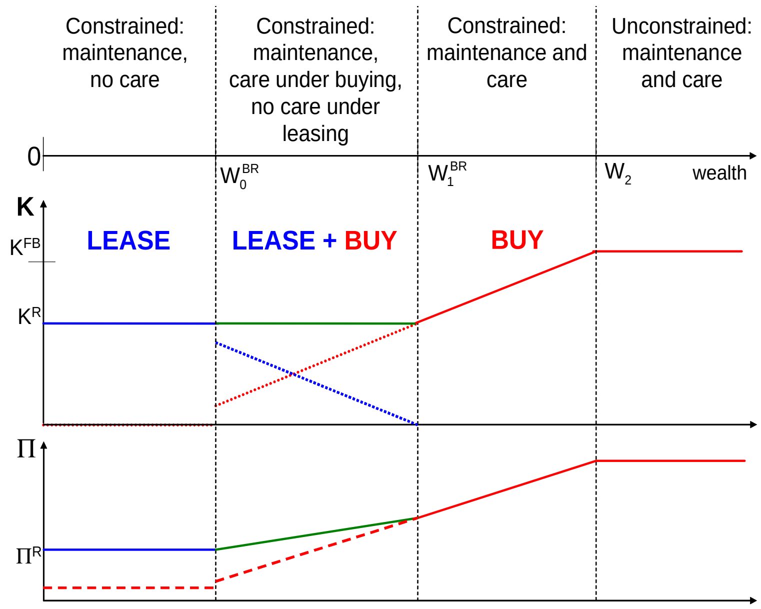


Fig. 4: Wealth areas, investment and profits when the capital input can be purchased and

leased.

The effect of leasing on profits can be seen in the continuous line in the bottom diagram of Figure 4, showing that they are monotone in wealth. To see why, consider that the possibility to lease part of the capital good allows the entrepreneur to slacken the financial constraint and keep the investment constant at $K^{\mathbf{R}}$. For the fraction of capital that is purchased, maintenance and care are carried out by the entrepreneur, while, for the fraction leased, maintenance is delegated to the lessor and care cannot be enticed. The lack of care translates in a higher expected cost of maintenance for the lessor, with a subsequent increase in the leasing fee F. When wealth is not too low (close to W_1^{BR}), the reliance on leasing is negligible and leasing is beneficial as, by relaxing financing constraints, allows to keep investment constant. As wealth decreases, the reliance on leasing increases and the subsequent increase in the leasing fee determines a reduction in profits, as shown by the green line in the bottom panel. When all the capital goods are leased, at $W \leq W_0^{BR}$, the profits are constant (blue line in the bottom panel).

The above scenario arises when either $p < \underline{p}$ or $p \geq \underline{p}$ and $q \leq \underline{q}(p)$. For the complementary parameter space, i.e., $p \geq \overline{p}$ and $\underline{q}(p) \leq q < \overline{q}(p)$ or $\underline{p} \leq p < \overline{p}$ and $q > \underline{q}(p)$, the investment, the profits and the leasing/purchase decision are non-monotone in wealth. Such scenario is described in Proposition 6 and depicted in Figure 5.

Proposition 6 *Assume $p \geq \overline{p}$ and $\underline{q}(p) \leq q < \overline{q}(p)$ or $\underline{p} \leq p < \overline{p}$ and $q > \underline{q}(p)$. There exist two critical levels of the entrepreneur's initial wealth $W_0^{BR'} > W_0^{BR}$ and W_1^{BR}, with $W_0^{BR'} < W_1^{BR} < W_2$ such that (i) for $W > W_2$, the entrepreneur invests K^{FB}, buying all the capital inputs; (ii) for $W_1^{BR} < W \leq W_2$, the entrepreneur invests $K^{FC}(W) \leq K^{FB}$, buying all the capital inputs; (iii) for $W_0^{BR'} < W \leq W_1^{BR}$, the entrepreneur* *invests* $K^{FC}(W_1^{BR}) = K^R$*, leasing a fraction* $K_r^{BR}(W) \equiv \frac{r(W_1^{BR}-W)}{p[(1-\bar{\delta}+\bar{\mu})\gamma-qm]}$ *and buying the rest* $K_b^{BR}(W) \equiv K^R - \frac{r(W_1^{BR}-W)}{p[(1-\bar{\delta}+\bar{\mu})\gamma-qm]}$*; (iv) for* $W \leq W_0^{BR'}$*, the entrepreneur invests* K^{NM}*, buying all the capital inputs and giving up maintenance altogether in the event of failure. Moreover,* $W_0^{BR'} < W_1$*.*

The results in Proposition 6 do not differ from those in Proposition 5 when $W \geq W_1^{BR}$. For $W < W_1^{BR}$, the capital inputs are partly purchased and partly leased and the investment is kept constant at K^R. As in the previous case, the reliance on leasing increases as wealth decreases and the subsequent increase in the leasing fee determines a reduction in profits, as shown by the green line in the bottom panel of Figure 5. However, unlike the case described in Proposition 5, there is a level of wealth, $W_0^{BR'}$, at which the increase in the leasing fee F due to the higher maintenance cost borne by the lessor for the entrepreneur's lack of care is so high to lower profits below the level obtainable when the capital input is purchased but no maintenance may be enticed. This is described in Figure 5. Again, the continuous lines in the top panel show the investment in the capital input when both leasing and secured lending (purchase) are available. There are no differences with the findings of the complementary parameter space depicted in Figure 4 if the entrepreneur has initial wealth $W \geq W_1^{BR}(W_0^{BR'})$. In this case the entrepreneur buys the capital input downsizing the investment below the first-best for $W < W_2$. When $W \leq W_1^{BR}$, as well as on secured lending, the entrepreneur compensates the progressively lower secured loan received (dotted red line) with leasing (dotted blue line) and keeps the investment constant at K^R (green line). However, at $W = W_1^{BR'} > W_1^{BR}$, the entrepreneur stops relying on leasing and buys an amount of the capital input $K^{NM} > K^R$ (red line). Thus, the reliance on leasing is non-monotone in wealth.

As regards the effect of leasing on profits, this can be seen in the bottom diagram of

Figure 5. Again, for $W \leq W_1^{BR}$, they are decreasing in wealth, as shown by the green line. At $W = W_0^{BR\prime} > W_0^{BR}$, they are constant and equal to Π^{NM}, those under no maintenance, and higher than those that would be obtained by leasing up to 100% of the capital input, Π^R.

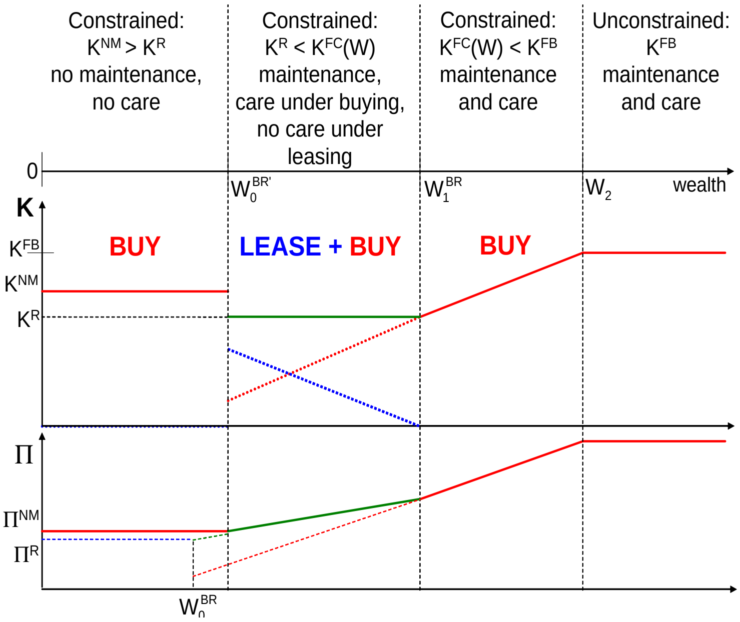


Fig. 5: Wealth areas, investment and profits when the capital input can be purchased and leased

The scenarios described in Propositions 3, 5 and 6 are depicted in Fig. 6. For sufficiently high probability of success, $p > \bar{p}$, the emergence of each scenario depends on the expected benefit of care, q, i.e., the expected reduction in the cost of maintenance due to high care. When it is high (green area), there is a large benefit from care and it is worthwhile for the entrepreneur to buy the capital good and carry out the maintenance in house. This is true also for credit rationed firms who may give up maintenance altogether in the case of failure but still prefer buying to leasing.

When the expected benefit from high care is low (area between purple and blue line), there is little benefit from it, as the expected cost of maintenance is not much reduced from high care. Thus, while cash rich firms still prefer to purchase the capital input, less cash rich ones start substituting buying with leasing, as the latter allows to relax financing constraints and increase the firm's borrowing capacity. Such substitution is full for sufficiently credit rationed firms who lease 100% of their capital, and delegate the maintenance to the lessor.

There is nevertheless a hybrid scenario occurring for intermediate values of the expected benefit from high care (area between blue and green line). In this case, as the severity of the financing constraints increases, it is still initially optimal to substitute buying with leasing. However, since such substitution implies a raising leasing fee, for highly credit rationed firms it may be cheaper to buy rather than lease the capital goods, giving up the maintenance in the case of failure. The reliance on leasing is therefore non-monotone in financing constraints.

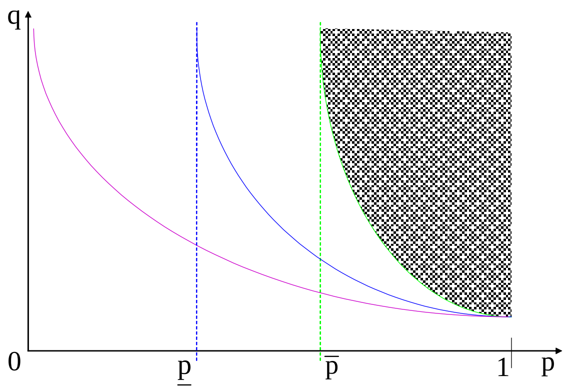


Fig. 6: Financing regimes

7 Theoretical predictions

From the above analysis, we can derive testable predictions on the relation between the contract choice and the characteristics of the assets invested in the project.

The key mechanism that makes the leasing contract emerge in equilibrium in our setting has to do with the incentive problems arising from the maintenance of the asset value. When such problems exist, pledging the asset as collateral may fail to secure lending to credit rationed firms. In such cases, leasing may be the most efficient way to get hold of these assets and overcome the credit rationing problem, despite the suboptimal level of care accompanying leasing. Assets with such characteristics are typically those whose physical life exceeds the firm's economic life. This may explain why precisely these types of assets are more predisposed to being leased rather than being purchased.

These considerations allow us to formulate the following theoretical predictions.

Prediction 1. *Assets whose value is sensitive to maintenance are more likely leased than purchased.*

Prediction 2. *Firms using the same type of assets are more likely to lease the more financially constrained they are.*

Prediction 3. *Firms relying on leasing can finance up to 100% of the purchase price of the assets.*

Prediction 4. *Firms with investment projects characterized by less liquid assets (lower cash flows and higher residual value of capital) are more likely to lease.*

Prediction 5. *Firms are more likely to lease capital goods in periods of recession than during expansions.*

8 Conclusion

The paper has presented a theory of leasing in which asset use and maintenance shape the firm's decision between purchasing and/or leasing productive assets. When the maintenance of the asset cannot be carefully specified as part of the loan agreement, a collateralized loan contract is time-inconsistent as the entrepreneur cannot be trusted that she will carry out maintenance, jeopardizing the lender's returns. As a result, the lender will only offer unsecured loan contracts, with a subsequent efficiency loss. One way out to restore maintenance incentives and avoid capital depletion is to rely on a leasing contract. With such a contract, the maintenance is delegated to the lessor. However, despite not having the ownership, the lessee still keeps control of the asset, and can exert a suboptimal level of (unobservable) care in managing it. The paper characterizes circumstances in which it may be optimal to rent rather than buy. We thus provide a new theory of leasing that not only rationalizes some observed features of renting/leasing contracts, but also offers some novel testable predictions. Our static analysis predicts that entrepreneurs using assets whose value is sensitive to maintenance are more likely to lease than purchase their assets. Moreover, within the same sector, they are more likely to lease the more financially constrained they are and the less liquid their assets are. We leave the empirical verification of these predictions to future research.

Appendix A

Buying for a financially constrained entrepreneur: the scenario with no maintenance upon low demand

Under the assumption that the entrepreneur is financially constrained and no maintenance is induced upon low demand, the optimal capital K, loan size L, repayment R, and fraction of the capital residual value that goes to the lender in the event of default λ, solve the following maximization problem $\mathcal{P}_{\mathcal{NM}}$:

$$\max_{K,L,R,\lambda} p(Af(K)-R)+p((1-\overline{\delta}+\overline{\mu})\gamma-(1-q)\,m)K+(1-p)(1-\lambda)(1-\overline{\delta})\gamma K-p(\eta+\phi)-Wr$$

under the constraint that investors get non-negative returns

$$pR+(1-p)\lambda(1-\overline{\delta})\gamma K-Lr\geq 0, \tag{19}$$

the limited liability constraints (4) and (6), and the resource constraint (5).

Participation constraint (19) has to be binding at the optimum. Substituting out $L=K-W$ from the resource constraint gives $pR=(K-W)r-(1-p)\lambda(1-\overline{\delta})\gamma K$. By combining the participation constraint and the limited liability constraints gives $p(Af(K)-(1-q)mK)+(1-p)(1-\overline{\delta})\gamma K\geq(K-W)r$. Substituting out pR in the entrepreneur's profits, the optimisation problem $\mathcal{P}_{\mathcal{NM}}$ can be written as:

$$\max_{K}\Pi^{NM}(K)\equiv pAf(K)+[(1-\overline{\delta})\gamma+p(\overline{\mu}\gamma-(1-q)\,m)]K-rK-p(\eta+\phi) \tag{20}$$

subject to

$$p(Af(K)-(1-q)mK)+(1-p)(1-\overline{\delta})\gamma K\geq(K-W)r. \tag{21}$$

The investment level maximizing the entrepreneur's expected profit under high care and high maintenance in the event of success and zero maintenance in the event of failure,

$\Pi^{NM}(K)$, is K^{NM} solving the following first-order condition:

$$pAf'(K^{NM}) = r + p\,(1-q)\,m - (1-\overline{\delta} + p\overline{\mu})\gamma. \tag{22}$$

By assuming that assume that K^{NM} is always implementable with a secured debt contract, regardless of the entrepreneur's initial wealth, it solves the optimisation problem $\mathcal{P}_{\mathcal{NM}}$ and the entrepreneur's expected profit in this scenario is $\Pi^{NM\star} \equiv \Pi^{NM}\left(K^{NM}\right)$ for any W.

Appendix B

Proof of Proposition 1. In the text. ■

Proof of Corollary 1. Define by $\Pi^{FB}(A, \overline{\delta})$ the first best profit for given A and $\overline{\delta}$. Let be $\overline{\delta}(A)$ the function implicitly defined by $\Pi^{FB}(A, \overline{\delta}(A)) - \Pi^{FB} \equiv 0$, with $\frac{\partial\overline{\delta}(A)}{\partial A} = \frac{pf(K^{FB})}{\gamma K^{FB}}$. Define $IC_1(A) \equiv p(Af(K^{FB}) - (1-q)\,mK^{FB}) + (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\overline{\mu}\gamma K^{FB}-\eta}{\overline{\mu}} - \left(K^{FB} - W\right)r$. By the chain rule:

$$\frac{\mathrm{d}IC_1}{\mathrm{d}A} = \frac{\partial IC_1}{\partial K^{FB}}\left(\frac{\partial K^{FB}}{\partial A} + \frac{\partial K^{FB}}{\partial\overline{\delta}(A)}\frac{\partial\overline{\delta}(A)}{\partial A}\right) + \frac{\partial IC_1}{\partial A} + \frac{\partial IC_1}{\partial\overline{\delta}(A)}\frac{\partial\overline{\delta}(A)}{\partial A}.$$

From (1), $\frac{\partial IC_1}{\partial K^{FB}} = p(Af'(K^{FB}) - \left[r + p(1-q)m - (1-p)(1-\overline{\delta}+\overline{\mu})\gamma\right] = -p\left(1-\overline{\delta}+\overline{\mu}\right)\gamma < 0$. From the implicit function theorem and the concavity of $f(K)$, $\frac{\partial K^{FB}}{\partial A} = -\frac{f'(K^{FB})}{Af''(K^{FB})} > 0$. Finally, $\frac{\partial IC_1}{\partial A} = pf(K^{FB}) > 0$. Moreover, $\frac{\partial K^{FB}}{\partial\overline{\delta}(A)} = \frac{\gamma}{pAf''(K^{FB})} < 0$ and $\frac{\partial IC_1}{\partial\overline{\delta}(A)} = -(1-p)\frac{\overline{\mu}\gamma K^{FB}-\eta}{\overline{\mu}} < 0$. Hence

$$\frac{\mathrm{d}IC_1}{\mathrm{d}A} =$$

$$-p(1-\overline{\delta}+\overline{\mu})\gamma\left(-\frac{f'(K^{FB})}{Af''(K^{FB})} + \frac{f(K^{FB})}{Af''(K^{FB})K^{FB}}\right) + pf(K^{FB}) - (1-p)\frac{\overline{\mu}\gamma K^{FB}-\eta}{\overline{\mu}}\frac{pf(K^{FB})}{\gamma K^{FB}} =$$

$$\frac{p(1-\overline{\delta}+\overline{\mu})\gamma}{-Af''(K^{FB})K^{FB}}\left(f(K^{FB}) - f'(K^{FB})K^{FB}\right) + pf(K^{FB})\left(1-(1-p)+\frac{(1-p)\eta}{\overline{\mu}\gamma K^{FB}}\right) > 0.$$

Indeed, from the assumption of positive expected profits and from condition (1):

$$pf(K^{FB}) > \frac{\left[r + p(1-q)m - (1-\overline{\delta}+\overline{\mu})\gamma\right]}{A}K^{FB} = pf'(K^{FB})K^{FB}.$$

■

Proof of Proposition 2. Let $\alpha(K) \equiv p(Af(K) - (1-q)\,mK) + (1-p)(1-\overline{\delta}+\overline{\mu})\gamma K - Kr$, and $z(\eta) = (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\eta}{\overline{\mu}}$. The financial constraint (7) can be written as $\alpha(K) \geq z(\eta) - Wr$. If $\alpha(K^{FB}) \geq z(\eta)$, (7) is satisfied for any $W \geq 0$ and, then, $W_2 = 0$. Now assume $\alpha(K^{FB}) < z(\eta)$. In this case, if $W = 0$ (7) is not satisfied and, then, $W_2 \neq 0$. If $W = K^{FB}$, (7) is satisfied since $p(Af(K^{FB}) - (1-q)\,mK^{FB}) + (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\overline{\mu}\gamma K^{FB}-\eta}{\overline{\mu}} > 0$. The Bolzano-Weierstrass theorem implies that there exists $W_2 \in (0,\, K^{FB})$ such that $\alpha(K^{FB}) = z(\eta) - W_2 r$ and $\alpha(K^{FB}) \lesseqgtr z(\eta) - Wr$ if and only if $W \lesseqgtr W_2$. Finally, W_2 is increasing in η since $z'(\eta) = (1-p)\frac{(1-\overline{\delta}+\overline{\mu})}{\overline{\mu}} > 0$. ■

Lemma 4 *There exists $\hat{\eta}$ such if $\eta \leq \hat{\eta}$ the entrepreneur invests $K^{FC}(W) < K^{FB}$, carries out maintenance both in the event of success and in the event of default and performs high care for all $W < W_2$, with $W_2 > 0$.*

Proof Let $\hat{K}^{FC} = \arg\max \alpha(K)$, with $\alpha(K)$ defined in the proof of Proposition 2. From the concavity of $f(K)$, $\hat{K}^{FC} < K^{FB}$. The firm's expected value, given high care and maintenance, when $K = K^{FC}(W)$ is

$$\Pi(K^{FC}(W)) \equiv \alpha(K^{FC}(W)) + p(1-\overline{\delta}+\overline{\mu})\gamma K^{FC}(W) - \eta - p\phi =$$

$$= p(1-\overline{\delta}+\overline{\mu})\gamma K^{FC}(W) - Wr - p(\phi+\eta) + (1-p)(1-\overline{\delta})\frac{\eta}{\overline{\mu}},$$

and the firm's expected value, given no maintenance in the event of failure, when $K = K^{NM}$ is

$$\Pi^{NM\star} \equiv \alpha(K^{NM}) + p(1-\overline{\delta}+\overline{\mu})\gamma K^{NM} - p(\phi+\eta) - (1-p)\overline{\mu}\gamma K^{NM}.$$

We shall next prove the result in four steps.

Step 1: *There exists $p_0 \in (0, 1)$ such that $\hat{K}^{FC} < K^{NM}$ if and only if $p > p_0$.*

$\hat{K}^{FC}$ is such that $\alpha'(\hat{K}^{FC}) = 0$, that is, $p(Af'(\hat{K}^{FC}) - (1-q)\,m) = r - (1-p)(1-\overline{\delta}+\overline{\mu})\gamma$. K^{NM} is such that $p(Af'(K^{NM}) - (1-q)\,m) = r - (1-\overline{\delta}+p\overline{\mu})\gamma$. Since $f'(K)$ is decreasing in K and $(r-(1-p)(1-\overline{\delta}+\overline{\mu})\gamma) - (r-(1-\overline{\delta}+p\overline{\mu})\gamma) = p(1-\overline{\delta}+2\overline{\mu})\gamma - \overline{\mu}\gamma > 0$ for all $p > \frac{\overline{\mu}}{(1-\overline{\delta}+2\overline{\mu})}$, then $\hat{K}^{FC} < K^{NM}$ if and only if $p > p_0 \equiv \frac{\overline{\mu}}{(1-\overline{\delta}+2\overline{\mu})}$.

Step 2: *Let be η^{FC} such that $z(\eta^{FC}) \equiv \alpha(\hat{K}^{FC})$. $\alpha(K) = z(\eta)$ for some $K \leq K^{FB}$ if and only if $\eta \leq \eta^{FC}$.*

By definition of $\hat{K}^{FC}$, $\alpha(K) \leq \alpha(\hat{K}^{FC}) = z(\eta^{FC}) < z(\eta)$ for all $K \leq K^{FB}$. Moreover, $z(0) = 0 < \alpha(K^{FB})$. The continuity of $z(\eta)$ implies that for any $K \in (\hat{K}^{FC}, K^{FB})$ there is η such that $z(\eta) = \alpha(K)$. Moreover, $\eta \leq \eta^{FC}$. Indeed, for all $\eta > \eta^{FC}$, $\alpha(K) \leq \alpha(\hat{K}^{FC}) \leq z(\eta^{FC}) < z(\eta)$ for any $K \in (\hat{K}^{FC}, K^{FB})$.

Step 3: *If $p \leq p_0$, then $\hat{\eta} = \eta^{FC}$.*

For any $\eta \leq \eta^{FC}$ denote with $K^{FC}(0,\eta)$ the investment in capital inputs which satisfies constraint (7) given $W = 0$. By Step 2 $K^{FC}(0,\eta)$ esists, is into $[\hat{K}^{FC}, K^{FB})$, and $K^{FC}(0,\eta^{FC}) = \hat{K}^{FC}$. Since $\Pi'(K) > 0$ for all $K < K^{FB}$ and since $K^{NM} \leq K^{FC} \leq K^{FC}(0,\eta) < K^{FB}$ by Step 1, then $\Pi(K^{FC}(0,\eta)) \geq \Pi(\hat{K}^{FC}) \geq \Pi(K^{NM})$. Moreover, $\Pi(K^{NM}) > \Pi^{NM\star}$ by assumption. Hence, $\Pi(^{FC}(0,\eta)) > \Pi^{NM\star}$ for all $\eta \leq \eta^{FC}$ and $\hat{\eta} = \eta^{FC}$.

Step 4: *If $p > p_0$, then $\hat{\eta} \leq \eta^{FC}$.*

Since $\Pi'(K) > 0$ for all $K < K^{FB}$ and since $\hat{K}^{FC} < K^{NM} < K^{FB}$ by Step 1, then $\Pi(\hat{K}^{FC}) < \Pi(K^{NM})$. However, $\Pi(K^{NM}) > \Pi^{NM\star}$ by Assumption 2. Hence, depending on the parameters of the model, either $\Pi(\hat{K}^{FC}) \geq \Pi^{NM*}$ or $\Pi(\hat{K}^{FC}) < \Pi^{NM\star}$. In the first case, $\Pi(K^{FC}(0,\eta)) \geq \Pi(\hat{K}^{FC}) > \Pi^{NM\star}$ for all $\eta \leq \eta^{FC}$ and $\hat{\eta} = \eta^{FC}$. In the second case, there exists $\eta' \leq \eta^{FC}$ such that $\Pi(K^{FC}(0,\eta')) = \Pi^{NM*}$ (Bolzano's theorem) and $\hat{\eta} = \eta'$. ■

Proof of Proposition 3 Assume $\eta > \hat{\eta}$. First consider the case $\hat{\eta} = \eta^{FC}$. By definition of η^{FC}, $\alpha(\hat{K}^{FC}) < z(\eta)$ for all $\eta > \eta^{FC}$. Hence $K^{FC}(W)$ which satisfies constraint (7) exists only if $W \geq W_1$, with $W_1 = \frac{1}{r}(z(\eta) - \alpha(K^{FC}))$, and $K^{FC}(W_1) = \hat{K}^{FC} \geq K^{NM}$ (Step 3 of the proof of Lemma 4). This concludes the proof for the case $\hat{\eta} = \eta^{FC}$ since, from the proof of Lemma 4, we know that $\Pi(\hat{K}^{FC}) \geq \Pi^{NM\star}$.

Now consider the case $\hat{\eta} < \eta^{FC}$. By definition of $\hat{\eta}$, $\alpha(K^{FC}(0,\hat{\eta})) < z(\eta)$ for all $\eta > \hat{\eta}$, with $K^{FC}(0,\eta)$ defined in Step 4 of the proof of Lemma 4. Hence, $K^{FC}(W)$ which satisfies constraint (7) exists only if $W \geq W_1$, with $W_1 = \frac{1}{r}(z(\eta) - \alpha(K^{FC}(0,\hat{\eta}))$, and $K^{FC}(W_1) = K^{FC}(0,\hat{\eta}) < K^{NM}$. This concludes the proof for the case $\hat{\eta} < \eta^{FC}$ since, from the proof of Lemma 4, we know that $\Pi(\hat{K}^{FC}) < \Pi^{NM\star} = \Pi(K^{FC}(0,\hat{\eta}))$.

To prove the last part of Proposition, notice that by Step 4 of the proof of Lemma 4 if $p > p_0$, $\eta \leq \hat{\eta}$ and $K^{FC}(W_1) < K^{NM}$. ■

Proof of Proposition 4. The level of investment that solves $\mathcal{P}_{\mathcal{R}}$, K^R, is lower than K^{FB} by the concavity of $f(\cdot)$. By substituting K^R into (8), one gets the rental fee, F^R. ■

Proof of Lemma 1.

Let $\alpha_{br}(K_b, K_r) \equiv pAf(K_b + K_r) - \left[r - (1-p)(1-\overline{\delta}+\overline{\mu})\gamma + p\,(1-q)\,m\right] K_b - \left[r - ((1-\overline{\delta}+\overline{\mu})\gamma + pm)\right] K_r$. The financial constraint (17) can be written as $\alpha_{br}(K_b, K_r) \geq$

$z(\eta) - Wr$, with $z(\eta)$ defined in the proof of Proposition 2. For any given level of purchased capital, K_b, leasing capital relaxes constraint (17) if

$$\frac{\partial \alpha_{br}(K_b, K_r)}{\partial K_r} = p\frac{\partial f(K_b + K_r)}{\partial K_r} - r + [(1 - \overline{\delta} + \overline{\mu})\gamma - pm] > 0 \tag{23}$$

which is true if and only if $K_b + K_r < K^R$, with K^R solving (23) with equality, by the concavity of the production function. ■

Proof of Lemma 2. K^R and K^{NM} solve the first-order conditions (9) and (22), respectively. By comparing (9) and (22), the strict concavity of the production function implies that $K^R \geq K^{NM}$ if and only if $q \leq q_1(p) \equiv \frac{(1-p)\overline{\mu}\gamma}{pm}$. Since $q \leq 1$ and $q_1(p) \geq 1$ for all $p \leq p_1$, it follows that if $p \leq p_1$ then $K^R \geq K^{NM}$ for all q, if $p > p_1$ then $K^R \geq K^{NM}$ if and only if $q \leq q_1(p)$. ■

Proof of Lemma 3 .

Step 1: $\Pi^{BR}(W_1^{BR}) \geq \Pi^{R\star}$ *for all possible* p *and* q.

$\Pi^{BR}(W_1^{BR}) - \Pi^{R\star} = \Pi(K^R) - \Pi^{R\star} = pqmK^R - \eta > 0$ for all possible p and q by assumption.

Step 2: *There exist* $\underline{p} > p_1$ *and* $\underline{q}(p) > q_1(p)$ *such that* $\Pi^{NM\star} > \Pi^{R\star}$ *if and only if* $p \geq \underline{p}$ *and* $q > \underline{q}(p)$.

First assume $p < p_1$. By combining $\Pi^{R\star} > \Pi^R\left(K^{NM}\right)$, true by definition of $\Pi^{R\star}$, and $\Pi^R(K) - \Pi^{NM}(K) = ((1-p)\overline{\mu}\gamma - pqm)K + p(\eta + \phi) > 0$ for any K, true for all $p < p_1$, one gets $\Pi^{R\star} > \Pi^{NM\star}$ for all q. A similar argument can be used to show that $\Pi^{R\star} > \Pi^{NM\star}$ if $p \geq p_1$ and $q < q_1(p)$.

Assume now $p \geq p_1$ and $q \geq q_1(p)$. Define the function $\Delta_{NM}^R : [p_1, 1] \times [q_1(p_1), 1]$, with $\Delta_{NM}^R(p, q) = \Pi^{R\star} - \Pi^{NM\star}$, and notice that 1) $\frac{\partial \Delta_{NM}^R(p,q)}{\partial p} = Af\left(K^R\right) - Af\left(K^{NM}\right) - mK^R - (\overline{\mu}\gamma - (1-q)\,m)K^{NM} + (\eta + \phi) < 0$ since $K^R \leq K^{NM}$ and $(\overline{\mu}\gamma - (1-q)\,m)K^{NM} > (\eta + \phi)$

by Assumption 2, 2) $\frac{\partial \Delta^R_{NM}(p,q)}{\partial q} = -\frac{\partial \Pi^{NM\star}}{\partial q} = -pmK^{NM} < 0$. If $p = p_1$, $q_1(p) = 1$ and $\Delta^R_{NM}(p_1, 1) > 0$. On the other hand, $\lim_{p\to 1} \Pi^{NM\star} = \lim_{p\to 1} \Pi^{FB} > \lim_{p\to 1} \Pi^{R\star}$, and $\lim_{p\to 1} \Delta^R_{NM}(p, q) < 0$ for all q. By the intermediate value theorem there exists $p^0 \in (p_1, 1)$ and $q^0 \in (q_1(p_1), 1)$ so that $\Delta^R_{NM}(p^0, q^0) = 0$. By the implicit function theorem, there is a neighborhood I_p of p^0, a neighborhood I_q of q^0, and an implicitly defined continuous function $\underline{q}(p)$, with $\underline{q} : I_p \to I_q$, so that for all $p \in I_p$, $\Delta^R_{NM}(p, \underline{q}(p)) = 0$ and $\frac{\partial \underline{q}(p)}{\partial p} = -\frac{\frac{\partial \Delta^R_{NM}(p,q)}{\partial q}}{\frac{\partial \Delta^R_{NM}(p,q)}{\partial p}} < 0$.

Let $\underline{p} \equiv \inf\{p : \Delta^R_{NM}(p, \underline{q}(p)) = 0\}$. Clearly, $\underline{p} \geq p_1$, and $\underline{q}(p) \geq q_1(p)$ for all $p \geq \underline{p}$, since $\frac{\partial \Delta^R_{NM}(p,q)}{\partial q} < 0$ and $\Delta^R_{NM}(p^0, q_1(p^0)) = p^0(\eta + \phi) > 0$, by definition of $q_1(p)$. By combining $\frac{\partial \Delta^R_{NM}(p,q)}{\partial p} < 0$ and $\frac{\partial \underline{q}(p)}{\partial p} < 0$ one gets $\Delta^R_{NM}(p, \underline{q}(\underline{p})) < 0$ if and only if $p > \underline{p}$. For all $p > \underline{p}$, $\frac{\partial \Delta^R_{NM}(p,q)}{\partial q} < 0$ implies $\Delta^R_{NM}(p, q) < 0$ if and only if $q > \underline{q}(p)$.

Step 3: *If $p < \underline{p}$, $\Pi^{BR}(W_1^{BR}) > \Pi^{R\star} > \Pi^{NM\star}$ for all possible q. If $p \geq \underline{p}$, $\Pi^{BR}(W_1^{BR}) > \Pi^{R\star} > \Pi^{NM\star}$ for all $q < \underline{q}(p)$.*

The result follows immediately by combining Steps 1 and 2.

Step 4: *There exist $\overline{p} > \underline{p}$ and $\overline{q}(p) > \underline{q}(p)$ such that $\Pi^{BR}(W_1^{BR}) < \Pi^{NM\star}$ if and only if $p \geq \overline{p}$ and $q > \overline{q}(p)$.*

From Step 3 we know that $\Pi^{BR}(W_1^{BR}) > \Pi^{NM\star}$ if $p \leq \underline{p}$ and if $p \geq \underline{p}$ and $q < \underline{q}(p)$.

Assume $p \geq \underline{p}$ and $q \geq \underline{q}(p)$ and define the function $\Delta^{BR}_{NM} : [\underline{p}, 1] \times [\underline{q}(\underline{p}), 1]$, with $\Delta^{BR}_{NM}(p, q) = \Pi(K^R) - \Pi^{NM\star}$ and $\Pi(K^R) = \Pi^{BR}(W_1^{BR})$ by definition of W_1^{BR}. Moreover, 1) $\frac{\partial \Delta^{BR}_{NM}(p,q)}{\partial p} = Af\left(K^R\right) - Af\left(K^{NM}\right) - (1-q)\, mK^R - \left(\overline{\mu}\gamma - \left((1-q)\, m\right)K^{NM} - \eta\right) < 0$ since $K^R \leq K^{NM}$ and $(\overline{\mu}\gamma - (1-q)\, m)K^{NM} > \eta$ by Assumption 2, 2) $\frac{\partial \Delta^{BR}_{NM}(p,q)}{\partial q} = -\frac{\partial \Pi^{NM\star}}{\partial q} = -pm(K^{NM} - K^R) < 0$.

From the definition of $\underline{q}(p)$, if follows that $\lim_{q\to \underline{q}(p)} \Pi^{NM\star} = \Pi^{R\star} < \lim_{q\to \underline{q}(p)} \Pi(K^R)$ for all p. Moreover, $\lim_{p\to 1} \Pi^{NM\star} = \lim_{p\to 1} \Pi^{FB} > \lim_{p\to 1} \Pi^{R\star}$ for all q. Since $\lim_{q\to \underline{q}(p)} \Delta^{BR}_{NM}(p, q) < 0$ for all p and $\lim_{p \to 1} \Delta_{NM}^{BR}(p,\, q) > 0$ for all q, from the intermediate value theorem it follows that there exist $p^0 \in [\underline{p}, 1]$ and $q^0 \in [\underline{q}(\underline{p}), 1]$ such that $\Pi(K^R) - \Pi^{NM\star} = 0$. By the implicit function theorem, there is a neighborhood I_p of p^0, a neighborhood I_q of q^0, and an implicitly defined continuous function $\overline{q}(p)$, with $\overline{q} : I_p \to I_q$, so that for all $p \in I_p$, $\Delta_{NM}^{BR}(p,\, \overline{q}(p)) = 0$, and $\frac{\partial \overline{q}(p)}{\partial p} = -\frac{\frac{\partial \Delta_{NM}^{BR}(p,\,q)}{\partial q}}{\frac{\partial \Delta_{NM}^{BR}(p,\,q)}{\partial p}} = -\frac{pm(K^R - K^{NM})}{Af(K^R) - Af(K^{NM}) - m(1-q)K^R - (\overline{\mu}\gamma - (1-q)m)K^{NM} + \eta + pqm\frac{\partial K^R}{\partial p}}$. Moreover, $\frac{\partial \overline{q}(p)}{\partial p} < 0$ since $K^R \leq K^{NM}$ for all $p \geq p_1$ and $q > q_1(p)$, $(\overline{\mu}\gamma - (1-q)\, m)K^{NM} > (\eta + \phi)$ by Assumption 2, and $\frac{\partial K^R}{\partial p} > 0$. Let $\overline{p} \equiv \inf\{p : \Delta_{NM}^{BR}(p,\, \overline{q}(p)) = 0\}$. Clearly, $\overline{p} \geq \underline{p}$, and $\overline{q}(p) \geq \underline{q}(p)$ for all $p \geq \overline{p}$, since $\frac{\partial \Delta_{NM}^{BR}(p,\,q)}{\partial q} < 0$ and $\Delta_{NM}^{BR}(p^0,\, \underline{q}(p^0)) = \Pi(K^R) - \Pi^{R\star} > 0$, by definition of $\underline{q}(p)$. By combining $\frac{\partial \Delta_{NM}^{BR}(p,\,q)}{\partial p} < 0$ and $\frac{\partial \overline{q}(p)}{\partial p} < 0$ one gets $\Delta_{NM}^{BR}(p,\, \overline{q}(\overline{p})) < 0$ if and only if $p > \overline{p}$. For all $p > \overline{p}$, $\frac{\partial \Delta_{NM}^{BR}(p,\,q)}{\partial q} < 0$ implies $\Delta_{NM}^{BR}(p,\, q) < 0$ if and only if $q > \overline{q}(p)$.

Step 5: *If $p \in [\underline{p}, \overline{p})$, $\Pi^{BR}(W_1^{BR}) > \Pi^{NM\star} \geq \Pi^{R\star}$ for all $q \geq \underline{q}(p)$. If $p \geq \overline{p}$, $\Pi^{BR}(W_1^{BR}) > \Pi^{NM\star} \geq \Pi^{R\star}$ for all $q \in [\underline{q}(p),\, \overline{q}(p))$. If $p \geq \overline{p}$, $\Pi^{NM\star} \geq \Pi^{BR}(W_1^{BR}) > \Pi^{R\star}$ for all $q \geq \overline{q}(p)$.*

The results follow immediately by combining Steps 1, 2 and 4. ■

Proof of Proposition 5. From Proposition 2 we know that $K = K^{FB}$ solves programme $\mathcal{P}_{\mathcal{B}-\mathcal{R}}$ for all $W > W_2$. Moreover, since leasing is costly, $K_b = K^{FB}$ and $K_r = 0$.

From Lemma 1 we know that $\alpha_{br}(K_b, K_r)$ is increasing in K_r if and only if $K_b + K_r \leq K^R < K^{FB}$. This implies that for all $W \in (W_1^{BR},\, W_2]$ $K = K_b = K^{FC}(W) < K^{FB}$ and $K_r = 0$, with $K^{FC}(W)$ defined in Section 4.3.

Now consider the case where $W \leq W_1^{BR}$. Financing constraint (17) reaches its maximum value at $K_b = 0$ and $K_r = K^R$. Indeed

$$\frac{\partial \alpha_{br}(K_b, K_r)}{\partial K_b} = p\frac{\partial f(K_b + K_r)}{\partial K_b} - r + [(1-p)(1-\overline{\delta} + \overline{\mu})\gamma + p\,(1-q)\, m] \tag{24}$$

and $\frac{\partial \alpha_{br}(K_b,K_r)}{\partial K_r} - \frac{\partial \alpha_{br}(K_b,K_r)}{\partial K_b} = p\left((1-\overline{\delta}+\overline{\mu})\gamma - qm\right) > 0$ for all (K_b, K_r). Moreover, $\hat{K}^{FC}$ solving (24) is lower than K^R from the concavity of the production function. Since $K_b \geq \frac{\eta}{\overline{\mu}\gamma}$ by constraint (**??**), the maximum feasible value of $\alpha_{br}(K_b, K_r)$ is $\hat{\alpha}_{br} \equiv \alpha_{br}(\frac{\eta}{\overline{\mu}\gamma}, K^R - \frac{\eta}{\overline{\mu}\gamma})$. Let $\hat{W}^{BR} \equiv \max\{z(\eta) - \hat{\alpha}_{br}, 0\}$ and $\Pi_0^{BR} \equiv \Pi^{BR}(\frac{\eta}{\overline{\mu}\gamma}, K^R - \frac{\eta}{\overline{\mu}\gamma})$. For all $W \in (\hat{W}^{BR}, W_1^{BR}]$ the investment level solving programme $\mathcal{P}_{\mathcal{B}-\mathcal{R}}$ is $K = K^R$. Substituting $K_r = K^R - K_b$ in (**??**) and remembering that $W_1^{BR} = K^R - \frac{1}{r}[p(Af(K^R) - (1-q)mK^R) + (1-p)(1-\overline{\delta}+\overline{\mu})\gamma K^R - (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\eta}{\overline{\mu}}]$ by definition, one gets:

$$\begin{aligned}
p[Af(K^R) - m\,(1-q)\,K_b - m(K^R - K_b)] + (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\overline{\mu}\gamma K_b - \eta}{\overline{\mu}} + \\
+ \left[((1-\overline{\delta}+\overline{\mu})\gamma)\right](K^R - K_b) - r\left(K_b + (K^R - K_b) - W\right) = \\
= p(Af(K^R) - mK^R) + (1-\overline{\delta}+\overline{\mu})\gamma K^R - (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\eta}{\overline{\mu}} - r\left(K^R - W\right) + \\
-p[(1-\overline{\delta}+\overline{\mu})\gamma - qm]K_b \pm pqmK^R \pm p(1-\overline{\delta}+\overline{\mu})\gamma K^R = \\
= p(Af(K^R) - (1-q)mK^R) + (1-p)(1-\overline{\delta}+\overline{\mu})\gamma K^R - (1-p)(1-\overline{\delta}+\overline{\mu})\frac{\eta}{\overline{\mu}} - rK^R + rW + \\
+p[(1-\overline{\delta}+\overline{\mu})\gamma - qm](K^R - K_b) = p[(1-\overline{\delta}+\overline{\mu})\gamma - qm](K^R - K_b) - r(W_1^{BR} - W) = 0.
\end{aligned}$$

Thus

$$K_b = K^R - \frac{r(W_1^{BR} - W)}{p[(1-\overline{\delta}+\overline{\mu})\gamma - qm]} \equiv K_b^{BR}(W).$$

and

$$K_r = K^R - (K^R - \frac{r(W_1^{BR} - W)}{p[(1-\overline{\delta}+\overline{\mu})\gamma - qm]}) = \frac{r(W_1^{BR} - W)}{p[(1-\overline{\delta}+\overline{\mu})\gamma - qm]} \equiv K_r^{BR}(W).$$

The entrepreneur expected profit, given $K_b = K_b^{BR}(W)$ and $K_r = K_r^{BR}(W)$, is

$$\begin{aligned}
\Pi^{BR}(W) \equiv & pAf(K^R) - [r - ((1-\overline{\delta}+\overline{\mu})\gamma - pm)]K^R + pqmK_b^{BR}(W) - Wr - \eta - p\phi = \\
= & \Pi^{R\star} + pqmK_b^{BR}(W) - \eta - p\phi \geq \Pi^{R\star} \iff K_b^{BR}(W) \geq \frac{\eta + p\phi}{pqm} > \frac{\eta}{\overline{\mu}\gamma}.
\end{aligned}$$

By substituting out $K_b^{BR}(W)$ one gets

$$\Pi^{BR}(W) \geq \Pi^{R\star} \iff K^R - \frac{r(W_1^{BR} - W)}{p[(1-\overline{\delta}+\overline{\mu})\gamma - qm]} \geq \frac{\eta + p\phi}{pqm} \iff W \geq W_0^{BR},$$

with $W_0^{BR} \equiv W_1^{BR} - \frac{p[(1-\overline{\delta}+\overline{\mu})\gamma - qm]}{r} K^R + \frac{[(1-\overline{\delta}+\overline{\mu})\gamma - qm](\eta+p\phi)}{rqm}$. Since $\Pi^{R\star} \geq \Pi^{NM\star}$ by assumption, this implies that at the optimum $K_b = 0$ and $K_r = K^R$ for all $W \leq W_0^{BR}$, with $W_0^{BR} > \hat{W}^{BR}$.

To conclude the proof, we have to show that $W_1^{BR} > W_1$. To this aims we first show that $\frac{\partial \Pi^{BR}(W)}{\partial W} \geq 0$. Suppose, by way of obtaining a contradiction, that $\frac{\partial \Pi^{BR}(W)}{\partial W} < 0$ for some W. This means that there exist W' and $W'' > W'$ such that $\Pi^{BR}(W'^{BR}(W'')$. But, this is not possible by definition of $\Pi^{BR}(W)$. Indeed, an entrepreneur with initial wealth equal to W'' may invest a fraction $W = W' < W''$ of her wealth and enjoy higher expected profit $\Pi^{BR}(W')$. Thus, $\frac{\partial \Pi^{BR}(W)}{\partial W} \geq 0$ for all W. This, combined with $\Pi^{BR}(W) \geq \Pi(K^{FC}(W))$ for any $W \leq W_1^{BR}$, implies that W_1 cannot to be higher than or equal to W_1^{BR}. ■

Proof of Proposition 6. The proof is analogous to that of Proposition 5. Indeed, $p \geq \overline{p}$ and $\underline{q}(p) \leq q < \overline{q}(p)$ or $\underline{p} \leq p < \overline{p}$ and $q > \underline{q}(p)$ imply that $\Pi^{R^\star} < \Pi^{NM^\star} < \Pi^{BR}(W_1^{BR})$ by Propositions 3 and 3. Thus, there exists $W_0^{BR'} > W_0^{BR}$ such that $\Pi^{R^\star} = \Pi^{BR}(W_0^{BR}) < \Pi^{NM^\star} = \Pi^{BR}(W_0^{BR'}) < \Pi^{BR}(W_1^{BR})$. To prove that $W_0^{BR'} < W_1$ notice that since $\Pi^{BR}(W) \geq \Pi(K^{FC}(W))$ for any $W \leq W_1^{BR}$, $\Pi^{BR}(W_1) > \Pi^{NM\star}$. This, combined with $\frac{\partial \Pi^{BR}(W)}{\partial W} < 0$, implies $W_0^{BR'} < W_1$. ■

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