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Patent Protection and Threat of Litigation in Oligopoly

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Patent Protection and Threat of Litigation in Oligopoly

Carlo Capuano*, **Iacopo Grassi**** **and Riccardo Martina*****

Abstract
In a context of imperfect patent protection, this paper analyses the strategic use of patents from a novel perspective; patents are seen as a means available to the incumbent firm to control entry and, more importantly, to influence the post-entry market interaction process effectively, by creating the conditions that favour collusion. The level of patent protection chosen by the incumbent affects the likelihood that a potential entrant will be found guilty of patent infringement. This mechanism can operate as a punishment device that eases the conditions for collusion sustainability. Therefore, in a sense, patent protection can be regarded as an instrument allowing replication of the monopoly outcome in the context of a contestable market.

Keywords: patents, patent portfolio, litigation, collusion, foreclosing, entry game

JEL Classification: D43, K21, L13

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1 Introduction

In 2007, Steve Jobs, CEO of Apple Inc., contacted Ed Colligan, CEO of the now defunct Palm Inc., and threatened Palm with patent litigation unless Colligan agreed not to make unsolicited job offers to Apple employees. This "no-poaching" request was an explicit attempt to suppress competition. In an e-mail to Jobs, Colligan said that the agreement was "not only wrong, but likely illegal." In response, Jobs told Colligan "I'm sure you realize the asymmetry in the financial resources of our respective companies" and to "take a look at our patent portfolio before you make a final decision here".[1] This anecdotal evidence highlights use of the threat of patent litigation as an instrument to enforce a collusive outcome.

The traditional economic wisdom considers patents to be an anti-collusive instrument since, in a Schumpeterian perspective (Schumpeter 1942), patents should grant firms temporary monopoly; however, if this statement were the case, in industries characterized by relevant R&D expenditures and huge patent portfolios (e.g., knowledge-based sectors), we would rarely see instances of firm collusion. However, the empirical evidence shows that competitors do enter in markets protected by patents, and collusion does emerge among patenting firms. Accusations of collusion have involved companies in highly innovative markets all over the world. For instance, in 2005, in the USA Samsung pleaded guilty to conspiring with Infineon and Hynix Semiconductor, to fix Dynamic Random Access Memory (DRAM) price;[2] in 2006, the French government fined 13 perfume brands including L'Oréal, Chanel, LVMH's Sephora and Hutchison Whampoa's Marionnaud for price collusion between 1997 and 2000;[3] in 2008, in the USA, LG Display Co., Chunghwa Picture Tubes and Sharp Corp., agreed to plead guilty to conspiring to fix the prices of Liquid Crystal Display (LCD) apnels;[4] a similar fine was imposed in 2010 in Europe on LG, Chimei Innolux, AU Optronics, Chunghwa Picture Tubes Ltd., and HannStar Display Corp.;[5] in 2012, South Korea's antitrust regulator fined Samsung Electronics and LG Electronics for conspiring to fix the prices of some appliances (washing

[1] https://www.macworld.com/article/2026075/steve-jobs-threatened-palm-with-patent-suit-if-it-objected-to-nopoaching-pact.html; Documents available at https://www.documentcloud.org/documents/560664-applepalm.html

[2] https://www.nytimes.com/2006/03/23/technology/3-to-plead-guilty-in-samsung-pricefixing-case.html

[3] https://www.wsj.com/articles/SB114238994783198532

[4] https://www.reuters.com/article/us-lcd-doj/lg-display-sharp-chunghwa-say-guilty-in-lcd-case-idUSTRE4AB7TA20081112

[5] https://www.ft.com/content/671be466-02f3-11e0-bb1e-00144feabdc0

machines, flat-panel TVs, laptop computers).[6] Finally, some of Silicon Valley's giants (Apple, Adobe, Google, Intel among others) were accused of collusion over an agreement not to hire each other's staff, in order to keep wages low.[7] Overall, it is clear that these markets have a common fundamental characteristic: the existence of a huge number of patents protecting their innovations.

The proliferation of patents is a recognized phenomenon that has relevant economic and legal implications. Its scale is impressive:[8] in the last 30 years, the number of patents has more than quadrupled, leading to an increase in both patent intensity and the proportion of patents that remain unexploited (the so called new patent paradox).[9] Not surprisingly, the proliferation in the number of patents has determined an increase in litigation rates in many industries.[10] However, only a small fraction of these eventually are contested, with an even smaller fraction of disputes over patent infringement resulting in a verdict.[11] Firms' use of patents as a complementary instrument in the definition of their competitive strategies, has been assessed in the Industrial Organization literature from several perspectives. Patents have been considered a quality signal for markets and investors,[12] a defensive tool in patent infringement lawsuits,[13] a means to defend a dominant position,[14] or as

[6]https://www.reuters.com/article/us-lcd-settlement/lcd-makers-settle-price-fixing-case-for-553-million-idUSTRE7BQ0KK20111227

[7]https://www.theguardian.com/technology/2014/apr/24/apple-google-settle-antitrust-lawsuit-hiring-collusion.

[8]Brink Lindsey, of the Niskanen Centre, in an interview with The Economist, stated: "Over the past 30-40 years, there has been a big rise in patent protection. Today the balance is out of whack. The Patent and Trademark Office grants about five times as many patents as it did in the 1980s. Standards for patentability have declined. And patents have expanded in scope, to include things like software and business methods. For instance, Amazon's 1-Click button was patented. So what we have seen is a dramatic expansion in the number of monopolies that have been created". The Economist, 20th July 2018, available on line at https://www.economist.com/open-future/2018/07/20/why-is-vigorous-economic-competition-a-good-thing.

[9]See Hall and Ziedonis (2001), Blind et al. (2006), Shrestha (2010), and Pénin (2012).

[10]The Price Waterhouse Cooper 2014 Litigation Study, available on line at http://www.pwc.com/us/en/forensic-services/publications/patent-litigation-study.html based on US Patent and Trademark Office Data, reports an annual 8% growth in patent actions filed from 1991 through 2013.

[11]See Lemley (2001), Kesan and Ball (2010), Allison et al. (2014).

[12]See, inter alia, Long (2002), Gambardella (2013), Comino and Graziano (2015).

[13]See Hall and Ziedonis (2001), Ziedonis (2004), Choi and Gerlach (2017).

[14]An analysis of entry deterrence strategies based on the refusal to license or on the threat of litigations, can be found in Lerner and Tirole (2004), Robledo (2005), Agarwal et al. (2009), Gavin and Toh (2010). See Somaya (2012) for a survey of the

affecting R&D decisions.[15]

This paper analyses the strategic use of patents from a novel perspective; patents are seen as a means available to the incumbent firm to control entry and, more importantly, as influencing the post-entry market interaction process effectively, by creating the conditions that might favour collusion.[16] In our context, the strategic value of a patent lies in the option offered to the owner to start a litigation process which, in principle, could weaken the aggressive stance of rival firms. More specifically, an incumbent firm might choose to protect its status with patents. However, whatever the amount of resources invested in this process, there is no patent system that provides complete protection since, in principle, a court could invalidate the relevant patent, thus granting to a rival the possibility of entry.[17] Therefore, the monopoly position can be challenged and an incumbent might find it profitable to explore another opportunity offered by patents, that is, to set up a pro-collusive industry environment. More specifically, since in the context of an infinitely repeated game, the sustainability of tacit collusion is inversely related to the level of the profit in the punishment stage the threat of denunciation for patent infringement facilitates collusion, lowering the profits in the punishment stage and, thus, reducing the level of the critical discount factor.

This paper contributes to the literature which examines how the sustainability of collusion is affected by firms' asymmetries. According to the Industrial Organization literature, the conventional wisdom holds that collusion is most likely to occur in the case of symmetric firms and that the coordination problem becomes harder when firms are different.[18] Papers on collusion in the case of asymmetry, focus on product differentiation,[19] asymmetric capacity,[20] different discount factors,[21] cost

literature on the strategic use of patents.

[15]See Jeon (2016).

[16]In an entry game, a firm that possess a patent might find it optimal not to sue the entrant: see, e.g., Choi (1998), Aoki and Hu (1999), Yiannaka and Fulton (2006).

[17]See Lemley and Shapiro (2005) for an introduction to the probabilistic nature of patent protection.

[18]See Ivaldi et al. (2003) and Motta (2004).

[19]See Deneckere (1983), Häckner (1994); more recently Grassi (2014).

[20]See Lambson (1994); Compte et al. (2002); Vasconcelos (2005)

[21]See Harrington (1989).

asymmetry,[22] or the presence of a ringleader.[23] This literature recognizes that firms operating in asymmetric industries find it difficult to sustain a collusive agreement as a non-cooperative Nash equilibrium in a repeated game, since the incentive constraint is more severe. Differently from the previous literature, which focuses on structural differences (such as cost), we focus on the asymmetry arising from the different structure of the patent portfolio, selected strategically by the firm. This allows the incumbent to control market access: by fixing the level of patent protection, the incumbent raises the entrant's expected costs, thereby reducing the incentive to defect from the collusive path. This asymmetry turns out to facilitate collusion. Moreover, this mechanism may sustain collusion even if the firms' discount factors are different.

The paper is organized as follows: Section 1 introduces the model; Section 2 describes the equilibrium outcomes for three different game scenarios: (i) full deterrence; (ii) non-collusive entry; (iii) accommodation and collusion. Section 3 presents the main results. Section 4 concludes. Appendix A provides the proofs; Appendices B and C extend the analysis to product differentiation, and lawsuit cost asymmetries; Appendix D describes symbols and notations used in the paper.

2 The Model

Consider a monopolistic industry where the incumbent I chooses to protect its market dominance by accumulating a portfolio of patents, which, in principle, could annihilate all threats of entry by severely reducing the opportunities to rival firms to enter the market. It is assumed that firms have perfect information on the characteristics of their (actual or potential) rivals. Firm I can oppose entry by suing the potential entrant for patent infringement. It is assumed that, at the litigation stage, the incumbent's probability of success in court is positively related to the size (breadth) of its patent portfolio, which can be thought of as an indirect measure of the degree of protection acquired by the incumbent. Patent protection is costly: to guarantee a probability of success $\beta \in [0, 1]$ at

[22] Collusion with cost asymmetries would require the inefficient firm to shut down, and side payments between the firms. Schmalensee (1987) states that: "When side payments are not possible, total industry profits may have to be reduced in order to attain an equitable division of the gains from collusion. Colluding firms must solve non-trivial bargaining problems." Therefore, when side payments are ruled out, joint-payoff maximization is entirely implausible. On the other hand, if all the firms produce positive amounts, the outcome is inefficient. Patinkin (1947), and Bain (1948) are seminal contributions; recently, Miklos-Thal (2011) focuses on retaliation schemes that might make collusion sustainable under cost asymmetry.

[23] See Ganslandt et al. (2012) and Davies and De (2013).

the litigation stage, the incumbent must incur a cost $x(\beta) > 0$, with $x'(\beta) > 0$ and $x''(\beta) > 0$. To rule out the less interesting case of complete patent protection, it is assumed that the relative monetary costs involved to achieve this would be too high.

A priori, as the entrant starts production in the market, both the incumbent and the entrant can appeal to the court for protection of their right either to act as the monopolist (incumbent), or to invalidate the patents (potential entrant). The probabilities of success β for the incumbent and $1-\beta$ for the entrant are assumed not to be affected by the type of player appealing to the court. In the case of a lawsuit, both players incur a cost $L > 0$.[24]In the event of a successful case for firm I, it will receive the fine $F > 0$, imposed on the entrant which then exits the market. However, if the court's decision favours the entrant, no fine is imposed and both firms continue to compete in the market. Following the court decision, the firms play an infinitely repeated non-cooperative game. Notice that, under any legal patent protection system, no player can be tried twice for the same violation.

Firms play a complete information game, that unfolds as follows:

- at the pre-entry stage $t = 0$, the incumbent chooses the degree of protection β;
- at stage $t = 1$, the entrant observes β and decides whether to enter the market;
- at stage $t = 2$, according to the entry decision at $t = 1$, the firms compete in the market;
- at stage $t = 3$, in the case of entry at $t = 1$, either of the two firms can choose to start a lawsuit;
- at stage $t = 4$, the court announces its verdict (conditional on either of the two firms having appealed to the court);
- at stage $t = 5$, the market structure is defined by the court's decision (either monopoly or duopoly) and the firms play an infinitely repeated non-cooperative game.

Firms discount the profit from stage $t = 5$ at the discount factors δ_E and δ_I.[25] It is important to notice that, given the structure of the

[24]Asymmetry in lawsuit costs is discussed in Appendix C.

[25]Different firms may discount future profits differently for at least two reasons. First, the smallest firms may be subject to financial constraints based on some kind

game, if, at the pre-entry stage $t = 0$, the firms are sufficiently patient to sustain the collusion, the incumbent will choose not to seek protection ($\beta = 0$). Thus, the firms will never collude if any of them has sued its rival. This is because the critical value of the discount factor necessary to sustain collusion along the equilibrium path, at stage $t = 3$, is the same as the one required to sustain collusion at the beginning of the game (when the incumbent chooses $\beta = 0$). In this scenario, to avoid legal costs, no firm would choose to protect itself or to sue its rival.

Let C, N, and M define the respective per-period profits, for the cases of collusion, non-cooperative duopoly, and monopoly, such that $M > C > N$. If entry occurs, at $t = 2$ the firms can either play non-cooperatively(obtaining N), or collusively (obtaining C). At $t = 3$, the firms can sue each other if any of them deviates from collusion, This leads to a non-collusive scenario. In these cases, without litigation, the entrant would earn a non-cooperative profit N in each period, starting from $t = 5$. In the case of successful litigation for the entrant (with probability $1-\beta$, it retains its non-cooperative profits and its expected earning will be null; in the case of unsuccessful litigation (with probability β), the entrant will exit the market, forego any future non-cooperative profits, and be obliged to pay a fine F to the incumbent. In both the cases, the entrant is liable for the costs of lawsuit (L) As a consequence, the expected value of the trial for the entrant, is given by:

$$\beta \left(-\frac{\delta_E}{1-\delta_E} N - F \right) - L \tag{1}$$

Equation (1) allows us to state the following Lemma.

Lemma 1 *The entrant's expected profit from litigation is decreasing in* β, F, *and* L; *it is always negative.*

By enforcing a positive level of patent protection, the incumbent effectively reduces the entrant's expected profit in the case of trial. As a consequence, suing the rival is a dominated strategy for the entrant; i.e., the entrant never sues the incumbent.

Analogously, the incumbent's expected value of the trial, is given by:

$$\beta \left(\frac{\delta_I}{1-\delta_I} (M - N) + F \right) - L \tag{2}$$

of credit market imperfection (e.g., less favourable interest rates). Second, the time preferences of the managers of those firms might be different. Some managers may discount future heavily (e.g., those that are due to retire or expect to be sacked in the near future). Some managers' preference may be more in line with the preference of the firm (e.g., those with firm's stock options). For a seminal contribution on collusion with asymmetric discount factors, see Harrington (1989).

Equation (2) allows us to state the following Lemma.

Lemma 2 *The incumbent's expected profit from litigation is increasing in β and F, and decreasing in L; it is positive if L is sufficient low, i.e. $L \leq \beta\left(\frac{\delta_I}{1-\delta_I}(M-N)+F\right)$.*

When the expected value of the trial is positive, the incumbent always has an incentive to sue its rival, that is, its litigation threat is credible and is part of the Subgame Perfect Nash Equilibrium (SPNE) of the game. Alternatively, if the costs of a lawsuit are too high, the incumbent will have no incentive to sue its rival and, in this case, will not invest in patent protection (i.e.,$\beta = 0$). In this case entry is never deterred, and the oligopolistic competition follows Friedman (1971).[26] To avoid the trivial case where no patent protection is implemented in equilibrium, hereafter, we consider lawsuit costs to be low enough to make any threat of suing credible in any scenario. Thus, we expect the incumbent will sue its rival in the case of full deterrence, the case of non-cooperative entry accommodation, and following any deviation from the collusive outcome.

The level of patent protection β, chosen by the incumbent at stage $t = 0$, defines the structure of the game to be played in the successive stages. More precisely, the incumbent can choose to prevent entry or to adopt a non-cooperative strategy or to accommodate entry in order to favour collusion. Accordingly, in the following we characterize the equilibrium outcome for three different scenarios that can emerge in the game: (i) full deterrence; (ii) non-collusive entry; (iii) accommodation and collusion.

2.1 Full deterrence (fd)

At stage $t = 0$, the incumbent sets $\beta > 0$. If entry occurs and the firms do not collude, both obtain Nash profits. The incumbent reacts to entry by suing the rival, a lawsuit starts, and the firms incur a cost $L > 0$. If the court finds infringement (with probability β), the entrant pays a fine $F > 0$ and exits the market. If the court finds no infringement (with probability $1-\beta$), both firms remain in the market and play an infinite horizon repeated non-cooperative game, with simultaneous decisions at each stage.

[26] When $\beta = 0$ the model collapses to the one described in Friedam (1971), where collusion is sustainable if and only if both firms have a discount factor that is not lower than the critical value $\sigma = (D-C)/(D-N)$, where D is the one-shot deviation profit.

The first non-cooperative strategy is full deterrence: the incumbent sets a level of β such that the entrant's non-cooperative profit, Π_E^{nc}, is equal to zero.

$$\Pi_E^{nc} = N - L + \beta(-F) + (1-\beta)\left(\frac{\delta_E N}{1-\delta_E}\right). \tag{3}$$

where:

E denotes the entrant;

nc denotes the non-cooperative case;

Π_E^{nc} is the entrant's non-cooperative expected profit;

$\beta \in [0,1]$ is the incumbent's probability of a successful lawsuit (i.e., the level of demanded patent protection);

$L \geq 0$ is the fixed litigation cost;

$F \geq 0$ is the fine imposed on the entrant and transferred to the incumbent if the court decrees an infringement;

$N \geq 0$ is the one-period Nash equilibrium profit for a duopolistic firm;

$\delta_E \in [0,1)$ is the entrant's discount factor.

Given the full deterrence strategy pursued by the incumbent, the competitor never enters the market and the incumbent preserves its monopoly. On the contrary, if $\beta = 0$ entry will always occur, since the entrant's expected profit $\Pi_E^{nc}(\beta = 0)$ will be strictly positive. The threshold value of β that makes the potential entrant indifferent between staying out of the market or entering is given by:

$$\beta^{fd} = \frac{N - L(1-\delta_E)}{\delta_E N + F(1-\delta_E)}. \tag{4}$$

where fd denotes the full deterrence case.

Notice that, β^{fd} is unaffected by the cost of implementing patent protection $x(\beta)$ and is decreasing with respect to the litigation cost L. Hence, if $\beta \geq \beta^{fd}$, the incumbent prevents entry and obtains the following profit:[27]

$$\Pi_I^{fd} = \frac{M}{1-\delta_I} - x\left(\beta^{fd}\right). \tag{5}$$

where:

I denotes the incumbent;

Π_I^{fd} is the incumbent's full deterrence profit;

$x(\beta^{fd})$ is the cost of implementing a level of patent protection equal to β^{fd};

[27] This conduct represents a strategic barrier to entry.

M is the one-period monopolistic profit, such that $M > N$;
$\delta_I \in [0, 1)$is the incumbent's discount factor.

2.2 Non-collusive entry (nc)

If deterrence is too costly, the incumbent can set a positive level of $\beta \in \left[0, \beta^{fd}\right)$, allowing it to sue the rival in the case of entry. The optimal β maximizes the incumbent's expected profit Π_I^{nc}:

$$\Pi_I^{nc} = N - L + \beta\left(\frac{\delta_I M}{1-\delta_I} + F\right) + (1-\beta)\left(\frac{\delta_I N}{1-\delta_I}\right) - x(\beta) \tag{6}$$

The first order condition of the maximization problem can be written as:

$$\frac{\delta_I\left(M-N\right)}{1-\delta_I} + F = x'(\beta^{nc}) \tag{7}$$

Notice that, given the strict convexity of the function $x(\beta)$, β^{nc} is increasing with respect to the monopoly profit, to the incumbent's discount factor and to the fine (possibly) imposed on the entrant, and is decreasing with respect to the size of the duopoly non-cooperative profit. Moreover, β^{nc} is unaffected by the litigation cost L.

Moreover, if $\beta^{nc} \geq \beta^{fd}$, the full deterrence strategy dominates the non-collusive strategy and the incumbent sets $\beta = \beta^{fd}$.[28]

2.3 Accommodation and collusion (ac)

If the incumbent chooses to collude, it will set $\beta = \beta^{ac}$, to make collusion sustainable. This means that neither firm has an incentive to sue its rival and both will move along the collusive path. In this case, firms' expected profits will be given by:

$$\Pi_E^{ac} = \frac{C}{1-\delta_E} \tag{8}$$

$$\Pi_I^{ac} = \frac{C}{1-\delta_I} - x(\beta^{ac}) \tag{9}$$

where:
ac denotes the collusive case;
C is the firm's per period collusive profit, such that $M > C > N$.

In our framework, collusion can emerge at equilibrium under two different scenarios. The first occurs when the firms are sufficiently patient

[28]This conduct represents a innocent barrier to entry.

so that collusion is achieved even in the absence of any patent protection at stage $t = 0$. According to Friedman (1971), this happens when:

$$\delta_I \geq \sigma_I(0) = \frac{D - C}{D - N} \tag{10}$$

$$\delta_E \geq \sigma_E(0) = \frac{D - C}{D - N} \tag{11}$$

where:

D is the per-period deviation profit, such that $D > C$;

$\sigma_I(0)$ and $\sigma_E(0)$ are the critical values of the discount factors when $\beta = 0$;

The second case occurs when condition (10) or (11) (or both) are not satisfied at $\beta = 0$, and the incumbent chooses a positive level of patent protection $\beta^{ac} > 0$. In this case, the firms will adopt the following modified trigger strategy:

- when the new competitor enters, the firms collude in the first period;

- in successive periods, the firms will follows the collusive path, unless one of them has defected; if a deviation occurs, the incumbent sues the rival and will play the Nash equilibrium strategy forever;[29]

- if the verdict is favourable to the entrant, the firms will continue to play the Nash equilibrium strategy; if the verdict is favourable to the incumbent, the entrant will exit the market and the incumbent will become the monopolist.

These strategies will constitute an equilibrium if, both: (i) no firm has an incentive to deviate from the collusive path, and (ii) neither firm goes to court.

To investigate collusion sustainability, we start by considering the entrant's incentive to deviate. Its expected profit from defection is:

$$\Pi_E^{dev} = (D - L) + \beta^{ac}(-F) + (1 - \beta^{ac})\frac{\delta_E N}{1 - \delta_E} \tag{12}$$

where:

dev denotes the deviation case;

Π_E^{dev} is the entrant's deviation profit.

The entrant chooses to collude if and only if the two following constraints are satisfied: (i) the participation constraint, $\Pi_E^{ac} \geq 0$, and (ii) the incentive compatibility constraint, $\Pi_E^{ac} \geq \Pi_E^{dev}$. Since for any $\beta > 0$,

[29] Notice that, according to Lemma 1, the entrant never has an incentive to sue the incumbent (regardless of which deviates).

$\Pi_E^{dev} > 0$, for the sustainability of collusion is sufficient that the latter constraint is satisfied. That is:

$$\frac{C}{1-\delta_E} \geq (D-L) + \beta^{ac}(-F) + (1-\beta^{ac})\frac{\delta_E N}{1-\delta_E} \tag{13}$$

which can be rewritten as:

$$\delta_E \geq \sigma_E(\beta^{ac}) = \frac{(D-L) - F\beta^{ac} - C}{(D-L) - N + \beta^{ac}(N-F)} \tag{14}$$

where $\sigma_E(\beta^{ac})$ is the entrant's critical discount factor when $\beta = \beta^{ac}$.

Condition (14) shows that a threshold value exists for the entrant's discount factor, $\sigma_E(\beta^{ac})$, which satisfies the constraint (13) as an equality. As a consequence, the collusive strategy is part of a SPNE only if the entrant's discount factor, δ_E, is not smaller than $\sigma_E(\beta^{ac})$.

It is easy to check that an increase in the level of patent protection β, in the legal cost of a trial L, or in the fine for violation F, reduce the entrant's critical discount factor, $\sigma_E(\beta^{ac})$, thus facilitating collusion.

Consider now the incumbent's incentive to deviate from the collusive path. Having fixed β^{ac}, deviation from the collusive path by the incumbent requires both the choice of the market variable as a best reply to the rival's collusive action, and the choice to sue the entrant for patent right infringement. In this scenario, the profit from deviation is given by:

$$\Pi_I^{dev}(\beta^{ac}) = D + \beta^{ac}\left(F + \frac{\delta_I M}{1-\delta_I}\right) + (1-\beta^{ac})\frac{\delta_I N}{1-\delta_I} - x(\beta^{ac}) - L \tag{15}$$

Analogously, the incumbent colludes if and only if the following two constraints are satisfied: (i) the participation constraint, $\Pi_I^{ac} \geq 0$, and (ii) the incentive compatibility constraint, $\Pi_I^{ac} \geq \Pi_I^{dev}$. Since for any $\beta > 0$, $\Pi_I^{dev} > 0$, for collusion sustainability it is sufficient that the latter constraint is satisfied. That is:

$$\frac{C}{1-\delta_I} \geq D - L + \beta^{ac}\left(F + \frac{\delta_I M}{1-\delta_I}\right) + (1-\beta^{ac})\frac{\delta_I N}{1-\delta_I} \tag{16}$$

which can be rewritten as:

$$\delta_I \geq \sigma_I(\beta^{ac}) = \frac{(D-L) - C + F\beta^{ac}}{(D-L) - N - \beta^{ac}(M-N-F)} \tag{17}$$

Where $\sigma_I(\beta^{ac})$ is the incumbent's critical discount factor when $\beta = \beta^{ac}$.

Thus, there exists a critical value of the incumbent's discount factor, $\sigma_I(\beta^{ac})$, such that the constraint (16) is satisfied as an equality. Consequently, the collusive strategy is part of a SPNE only if the entrant's discount factor, δ_I, is not smaller than $\sigma_I(\beta^{ac})$.

Notice that an increase in the level of patent protection (β^{ac}), or in the level of the fine (F), increases the value of the incumbent's critical discount factor ($\sigma_I(\beta^{ac})$), making collusion harder to sustain. Conversely, an increase in the litigation costs (L) decreases the incumbent's critical discount factor making collusion easier to sustain.

Figure (1) shows the incumbent's and the entrant's critical discount factors as a functions of the level of patent protection β^{ac} chosen by the incumbent. The horizontal dotted line represents the critical discount factor when $\beta = 0$ (no patent protection): in this case, the incumbent's and the entrant's critical discount factors turn out to be equal. Moving from $\beta^{ac} = 0$ to $\beta^{ac} = 0^+$ (that is a positive level of patent protection close to zero) we observe a downward jump for both the critical discount factors; when $\beta^{ac} > 0$, the decreasing continuous line represents $\sigma_E(\beta^{ac})$ and the increasing dotted line describes $\sigma_I(\beta^{ac})$.

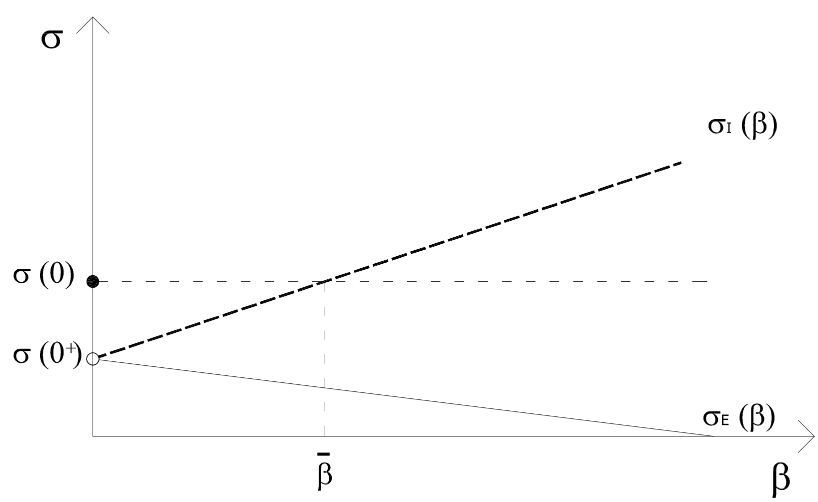

Figure 1: The critical discount factors for sustaining collusion.

The following proposition shows how the implemented β affects the critical discount factors:

Proposition 1. Denoting by $\overline{\beta}$ the positive level of patent protection

such that $\sigma_I(\overline{\beta}) = \sigma(0)$, we obtain that:
- if $\beta \leq \overline{\beta}$, then $\sigma_E(\beta) \leq \sigma(0)$ and $\sigma_I(\beta) \leq \sigma(0)$;
- if $\beta > \overline{\beta}$, then $\sigma_E(\beta) < \sigma(0)$ but $\sigma_I(\beta) > \sigma(0)$.

Proof. See Appendix A. ■

From the previous proposition we obtain the following result:

Result 1. If $\beta < \overline{\beta}$, increasing patent protection facilitates collusion; if $\beta \geq \overline{\beta}$, increasing patent protection facilitates collusion only if the incumbent is sufficiently patient.

If $\beta < \overline{\beta}$, both critical discount factors are smaller than in the no patent protection case ($\beta = 0$), hence collusion is easier to sustain. If $\beta \geq \overline{\beta}$, the critical discount factor of the entrant continues to be smaller while the incumbent's critical discount factor is higher than the one obtained for $\beta = 0$; hence, a patient incumbent may increase patent protection in order to induce an impatient entrant to collude.

In both cases, the level of patent protection becomes a strategic tool for the incumbent, since it can have a crucial effect on the sustainability of collusion. This result can be explained easily: moving from $\beta = 0$, a marginal increase in β has a negative and discrete impact equal to L on the expected gain from deviation for both firms. This creates a discontinuity and a downward jump in the discount factors. However, an additional increase in the level of patent protection has a different impact on both the entrant's and the incumbent's deviation profits and, hence, on their critical discount factors. More specifically, an increase in β raises the expected fine that must be paid by the entrant, thus reducing its deviation profits and, consequently, the critical value of its discount factor. For the incumbent, it has the opposite effect: an increase in β raises the expected value of the fine it will receive, thereby increasing the profitability of a deviation and, consequently, the critical value of the discount factor. Notice that, for values of β in the interval $\left[0, \overline{\beta}\right]$, an increase in β implements lower levels of the critical discount factors than those computed in the case of no patent protection. Figure (1)shows that, for levels of β higher than $\overline{\beta}$, $\sigma_E(\beta)$ continues to decrease, while $\sigma_I(\beta)$ is higher than $\sigma(0)$.

3 The game equilibria

Depending on the values of the parameters, the model can generate different equilibrium outcomes. In particular, collusion emerges as the

SPNE of the game if (i) it is sustainable and (ii) it is more profitable than alternative strategies. We first analyse the sustainability of collusion.

As already shown, the critical discount factors $\sigma_I(\beta)$ and $\sigma_E(\beta)$ depend crucially on the level of β implemented by the incumbent (see Figure 1). Proposition 2 defines the sufficient conditions for collusion sustainability:

Proposition 2. Define $\widetilde{\beta}_i$ as the value of β such that $\sigma_i(\beta) = \delta_i$ (where $i = I, E$). Collusion is sustainable in the accommodating subgame if:
(a) $\delta_I \geq \sigma(0^+)$;
or
(b) $\delta_I \geq \sigma_I(\widetilde{\beta}_E)$ and $\delta_E < \sigma(0^+)$.

Proof. See Appendix A. ■

According to the previous proposition, collusion is sustainable if and only if, for each firm, the individual discount factor is not smaller than its critical discount factor. Since $\sigma_I(\beta)$ is increasing in β and $\sigma(0^+)$ is the minimum value of the function $\sigma_I(\beta)$, if $\delta_I < \sigma(0^+)$, by definition collusion cannot be sustainable; hence, we must have $\delta_I \geq \sigma(0^+)$. Now compare the two firms' discount factors, δ_I and δ_E. Two possible cases have to be considered: (i) suppose that the incumbent is the less patient player (i.e., $\delta_E > \delta_I$): if $\delta_I < \sigma(0^+)$ collusion cannot be sustainable; if $\delta_I \geq \sigma(0)$, we have two patient firms and collusion is sustainable at $\beta = 0$; if $\sigma(0^+) \leq \delta_I < \sigma(0)$ collusion is sustainable at $\beta = 0^+$; (ii) suppose that the entrant is the less patient player (i.e. $\delta_I \geq \delta_E$). As in the previous case, if $\delta_E \geq \sigma(0)$, collusion will be sustainable at $\beta = 0$; if $\sigma(0^+) \leq \delta_E < \sigma(0)$ collusion may be sustainable at $\beta = 0^+$. Finally, if $\widetilde{\beta}_I \geq \widetilde{\beta}_E$ collusion is sustainable if $\beta = \widetilde{\beta}_E$.

Figure 2 depicts the relevant case where $\delta_E < \sigma(0^+)$ and $\delta_I > \sigma(0^+)$: in this case collusion is sustainable with $\beta > 0$.

Considering the complete game, we obtain the following result:

Result 2. Collusion is sustainable if and only if the incumbent implements levels of patent protection equal to:

1. $\beta^{ac} = 0$ when $\delta_E \geq \sigma(0)$ and $\delta_I \geq \sigma(0)$
2. $\beta^{ac} = 0^+$ when $\delta_E \geq \sigma(0^+)$, $\delta_I \geq \sigma(0^+)$ and $\min[\delta_E, \delta_I] < \sigma(0)$

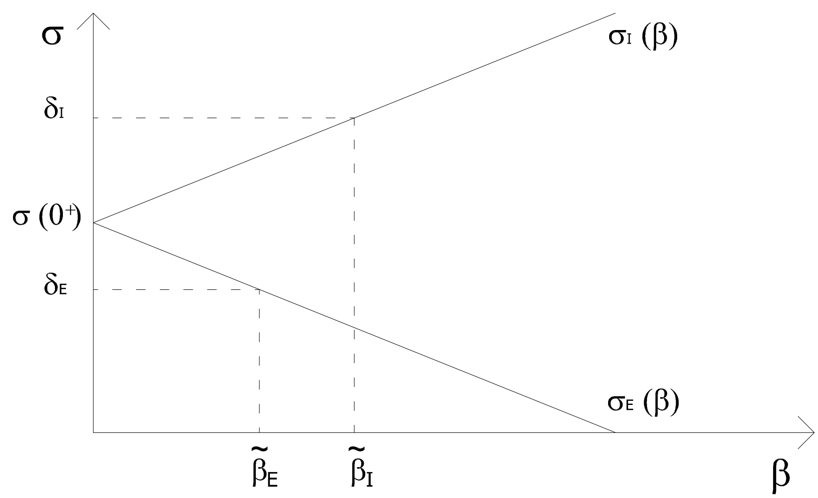

Figure 2: The case of collusion sustainable with $\beta > 0$

3. $\beta^{ac} = \widetilde{\beta}_E$ when $\delta_E < \sigma\left(0^+\right), \delta_I \geq \sigma\left(0^+\right)$ and $\widetilde{\beta}_I \geq \widetilde{\beta}_E$

Result (2) characterizes the degree of patent protection the incumbent should choose to facilitate collusion with the entrant. This amount may tend to zero; however, in some cases the game equilibrium is given by $\beta > 0$. If both firms are patient (i.e. $\delta_E, \delta_i \geq \sigma(0)$), collusion is sustainable even though the incumbent does not invest in patent protection. Since such investment is costly, the optimal level of patent protection turns out to be $\beta^{ac} = 0$. If both firms are moderately patient (i.e., their discount factors are not smaller than $\sigma\left(0^+\right)$ and at least one of the two values is smaller than $\sigma(0)$), even an infinitesimal level of patent protection $\beta^{ac} = 0^+$ suffices to make collusion sustainable. This level is sufficient to give the incumbent the possibility of suing the entrant in the case of deviation: both firms bear the lawsuit cost L reducing their critical discount factors to $\sigma\left(0^+\right) < \sigma(0)$. If the incumbent is at least moderately patient, but the entrant is not patient (i.e., $\delta_E \geq \sigma\left(0^+\right)$ and $\delta_I < \sigma\left(0^+\right)$), the optimal level of patent protection is given by $\beta^{ac} = \widetilde{\beta}_E$. Starting from $\beta = 0^+$, increasing β reduces the entrant's discount factor and increases the incumbent's discount factor. At $\beta = \widetilde{\beta}_E$, the entrant has no unilateral incentives to deviate from collusion (i.e., $\sigma_E\left(\widetilde{\beta}_E\right) = \delta_E$); if conditions are such that even the incumbent has no incentives to deviate (i.e., if $\sigma_I\left(\widetilde{\beta}_E\right) \leq \delta_I$) collusion turns out to be sustainable in equilibrium. The latter condition is satisfied if and only if $\widetilde{\beta}_I \geq \widetilde{\beta}_E$.

Sustainability does not imply that collusion emerges in equilibrium, since the incumbent can find more profitable alternative strategies. The following result summarizes the conditions characterizing all the game equilibria.

Result 3. The game equilibria are as follows:

1. (the deterrence case) When $\Pi_I^{fd}\left(\beta^{fd}\right) \geq \max\left[\Pi_I^{nc}\left(\beta^{nc}\right), \Pi_I^{ac}\left(\beta^{ac}\right), 0\right]$ the incumbent maintain her monopolistic position fixing $\beta = \beta^{fd}$.

2. (the non-cooperative case) When $\Pi_I^{nc}\left(\beta^{nc}\right) \geq \max\left[\Pi_I^{fd}\left(\beta^{fd}\right), \Pi_I^{ac}\left(\beta^{ac}\right), 0\right]$ the incumbent fixes $\beta = \beta^{nc}$, entry occurs and firms face the trial.

3. (the collusive case) When collusion is sustainable and $\Pi_I^{ac}\left(\beta^{ac}\right) \geq \max\left[\Pi_I^{fd}\left(\beta^{fd}\right), \Pi_I^{nc}\left(\beta^{nc}\right), 0\right]$ the incumbent fixes $\beta = \beta^{ac}$, entry occurs and firms collude.

Result (3) characterizes the game equilibria. The incumbent chooses the strategy leading to the maximum expected profit; in the case of collusion, the latter must be sustainable. The ranking among the expected profits and the emerging equilibrium, depend on the parameters. For example, if the lawsuit costs are sufficiently high, full deterrence may emerge as the equilibrium, since a low level of patent protection is sufficient to reduce the entrant's expected profits to zero; in other words, low investment in patents is sufficient to allow the monopolist to annihilate the threat of entry. In contrast, if the lawsuits costs are low, full deterrence requires a larger investment; this may not be profitable and the incumbent may prefer to pursue an alternative strategy. Analogously, if the cost of implementing β is sufficiently low, the incumbent will extend its patent portfolio to increase the probability of success at litigation; if the cost of implementing β is sufficiently high, , the firm will determining a minimum size for its patent portfolio, necessary to favour collusive behaviour in the repeated market game. To sum up, the incumbent may find it optimal to collude with the entrant if the litigation costs are low enough and the patent protection costs are sufficiently high.

3.1 Numerical simulations

The general framework proposed in this paper can be extended to any kind of oligopolistic interaction. Here we present two numerical simulations. The first describes the case of full deterrence emerging as the game equilibrium; the second corresponds to the case depicted in Figure (2), where the positive level of patent protection implemented by the incumbent leads to collusion in the market. Any hypothesis about product differentiation (horizontal or vertical), cost asymmetry, geographical distance between markets, and so on, will affect only the level of the one-shot profits (N, C, D, and M), but will not alter the constraints characterizing the model, which may determine a collusive equilibrium.[30]

We assume that the inverse market demand function is linear and is given by $P(q_I, q_E) = 1-(q_I+q_E)$, where P, q_I and q_E are respectively the market price, the incumbent's and the entrant's output levels. Firms, whose costs are normalized to zero, compete à la Cournot. The profits under monopoly, Cournot-Nash competition, collusion and deviation, are given by $M = 1/4$, $N = 1/9$, $C = 1/8$, and $D = 9/64$. Also, we set $F = N$, $L = N/10$, and assume that the firms' respective discount factors are equal to $\delta_I = 0.50$ and $\delta_E = 0.20$. In this configuration, were a system of patent protection not available, collusion would not be sustainable: it is easy to check that $\sigma(0) = 0.529421 > \delta_I = 0.50 > \delta_E = 0.20$. On the contrary, when patent protection is possible, we have $\widetilde{\beta}_I = 0.0375 > \widetilde{\beta}_E = 0.0075$; thus, collusion is sustainable fixing $\beta^{ac} = \widetilde{\beta}_E$. Collusion emerges as the equilibrium if it provides the highest profit.

Consider, first, the case where $x(\beta) = \frac{\beta^2}{4}$. The incumbent has three strategies: it can foreclose on the entry, setting $\beta^{fd} = 0.92$, it can choose the non-collusive strategy, setting $\beta^{nc} = 0.39$, or it can accommodate and collude, setting $\beta^{ac} = \widetilde{\beta}_E = 0.0075$. It is easy to obtain that $\Pi_I^{fd} = 0.28 > \Pi_I^{nc} = 0.27 > \Pi_I^{ac} = 0.25$: therefore, in this case, even though collusion is sustainable, full deterrence emerges as the equilibrium outcome of the game.

Consider, now, the case of high costs of patent protection; that is, $x(\beta) = \beta^2$. This will have a negative effect on the incumbent's expected profit. Now, the values of β for the three strategies are given by: $\beta^{fd} = 0.92$, $\beta^{nc} = 0.18$, and $\beta^{ac} = \widetilde{\beta}_E = 0.0075$. It is easy to verify that $\Pi_I^{ac} = 0.25 > \Pi_I^{nc} = 0.22 > \Pi_I^{fd} < 0$: therefore, collusion is sustainable and more profitable with respect to the alternative strategies and it will emerge as the equilibrium outcome of the game.

[30] Appendix B provides a simple model of differentiation; Appendix C analyses the case where the firms' litigation costs differ.

4 Conclusion

From a Schumpeterian perspective, the idea of patents as pro-collusive instruments, seems contradictory, since patents are seen as granting (at least) temporary monopolistic conditions and, therefore, exclude entry per se. However, this idea relies crucially on the implicit assumption that the patent system guarantees complete protection. If this assumption is removed, the scenario changes dramatically since, if the probability of conviction of an imitator is different from 1, entry may occur. In this context, the accumulation of a patent portfolio and the threat of patent litigation may mitigate the pro-competitive effects induced by entry, since they may operate as an anti-competitive non-price device: a patient incumbent might induce an impatient entrant to collude and, thus, prevent aggressive entry.

Collusion may then emerge as the equilibrium strategy adopted by the incumbent in the context of a game, characterized strongly by the uncertainty inherent in the judicial system decision process. Strategic investment in "protection" through the patent system is the insurance policy available to the incumbent. It is by the very nature of the non-cooperative repeated game, that the "insurance" investment takes the form of a means to promote collusion. This paper shows how this intriguing process may successfully unfold.

Appendix A

Proof of Proposition 1.

The proof is developed through a sequence of steps.

(i) Since $\forall L > 0$, $\Pi_E^{ac}(\beta = 0^+) = \Pi_E^{ac}(\beta = 0)$ and $\Pi_E^{dev}(\beta = 0^+) = (D - L) + \delta_E \left(\frac{N)}{1-\delta_E}\right) < D + \delta_E \left(\frac{N)}{1-\delta_E}\right) = \Pi_E^{dev}(\beta = 0)$, then $\forall L > 0$, $\sigma_E(0^+) < \sigma_E(0)$.

(ii) Since $\forall L > 0, \frac{\partial \Pi_E^{ac}(\beta)}{\partial \beta} = 0$ and $\frac{\partial \Pi_E^{dev}(\beta)}{\partial \beta} = -F - \delta_E \frac{N}{1-\delta_E} < 0$, then $\frac{\partial \sigma_E(\beta)}{\partial \beta} < 0$.

(iii) from (i) and (ii) we have that $\forall \beta > 0$ and $L > 0$, $\sigma_E(\beta) < \sigma_E(0)$.

(iv) Since $\forall L > 0$, $\Pi_I^{ac}(\beta = 0^+) = \Pi_I^{ac}(\beta = 0)$ and $\Pi_I^{dev}(\beta = 0^+) = D - L + \left(\frac{\delta_I}{1-\delta_I} N\right) < D + \left(\frac{\delta_I}{1-\delta_I} N\right) = \Pi_I^{dev}(\beta = 0)$, then $\forall L > 0$, $\sigma_I(0^+) < \sigma_I(0)$.

(v) Since $\sigma_I(\beta)$ is continuous in β, $\forall L > 0, \frac{\partial \sigma_I(\beta)}{\partial \beta} > 0$, $\sigma_I(0^+) < \sigma_I(0)$ and $\lim_{\beta \to \infty} \ \sigma_I(\beta) = \infty$; then $\exists! \overline{\beta} : \sigma_I\left(\overline{\beta}\right) = \sigma_I(0)$.

(iv) from (iv) and (vi) we have $\forall \beta \leq \overline{\beta}$, $\sigma_I(\beta) \leq \sigma_I(0)$, while $\forall \beta > \overline{\beta}$, $\sigma_I(\beta) > \sigma_I(0)$.

(v) from (iii) and (iv) we have $\forall \beta \leq \overline{\beta}$, $\sigma_I(\beta) \leq \sigma_I(0)$ and $\sigma_E(\beta) < \sigma_E(0)$, while $\forall \beta > \overline{\beta}$, $\sigma_I(\beta) > \sigma_I(0)$ and $\sigma_E(\beta) < \sigma_E(0)$.

Proof of Proposition 2.

The proof is developed through a sequence of steps. Collusion sustainability requires that $\delta_I \geq \sigma_I(\beta)$ and $\delta_E \geq \sigma_E(\beta)$.

(i) Since $\frac{\partial \sigma_I(\beta)}{\partial \beta} > 0$ and $0^+ = \arg\min_\beta \sigma_I(\beta)$, collusion is sustainable only if $\delta_I \geq \sigma(0^+)$.

Assume that $\delta_I \geq \sigma(0^+)$.

(ii) When $\delta_E > \delta_I$:

- if $\delta_I \geq \sigma(0)$, then $\delta_E \geq \sigma(0)$ and collusion is sustainable at $\beta = 0$;

- if $\delta_I \in [\sigma(0^+); \sigma(0))$ then $\delta_E \geq \sigma(0^+)$ and collusion is sustainable at $\beta = 0^+$;

(iii) When $\delta_I \geq \delta_E$:

- if $\delta_E \geq \sigma(0)$, then $\delta_I \geq \sigma(0)$ and collusion is sustainable at $\beta = 0$;

- if $\delta_E \in [\sigma(0^+); \sigma(0))$ then $\delta_I \geq \sigma(0^+)$ and collusion is sustainable at $\beta = 0^+$;

-if $\delta_E < \sigma(0^+)$, $\exists! \widetilde{\beta}_E : \delta_E = \sigma_E(\widetilde{\beta}_E)$, then collusion is sustainable only if $\delta_I \geq \sigma_I(\widetilde{\beta}_E)$; otherwise collusion is not sustainable.

Appendix B

In this Appendix we derive our results in a simple duopoly model with product differentiation. Firms face the following demand functions:

$$p_i = 1 - q_i - hq_j \qquad i,j = [1,2] \quad i \neq j$$

where $h \in [0,1]$ measures the degree of differentiation: when $h = 0$ we have two separate markets, and firms play as monopolist; when $h = 1$, goods are perfect substitutes and firms play a duopolistic Nash competition. Marginal costs of production are assumed to be zero. The one-shot profits under Cournot-Nash, collusion, deviation, and monopoly, are respectively:

$$N = \frac{1}{(h+2)^2}; \qquad C = \frac{1}{4(h+1)}; \qquad D = \frac{1}{16}\frac{(h+2)^2}{(h+1)^2} \qquad M = \frac{1}{4}$$

The cost function to implement patent protection is $x(\beta) = \beta^2$. In order to make the model treatable, we set the following (credible) configuration, with: $F = \frac{1}{8}, L = \frac{1}{128}, \delta_I = \frac{7}{12}$, and $\delta_E = \frac{1}{5}$. According to Equations(4) and (7), and Result (2), the levels of β in the different scenarios are:

$$\beta^{fd} = \begin{cases} \nexists & if\ h < 0.74398 \\ \frac{1}{16}\frac{156-4h-h^2}{4h+h^2+6} & if\ h \geq 0.74398 \end{cases}$$

$$\beta^{nc} = \frac{1}{80}\frac{76h + 19h^2 + 20}{(h+2)^2}$$

$$\beta^{ac} = \begin{cases} 0^{+} & if\ h < 0.67773 \\ \frac{1}{32}\frac{-12h+11h^2+18h^3+7h^4-4}{(4h+h^2+5)(h+1)^2} & if\ h \geq 0.67773 \end{cases}$$

Thus, according to Equations(5), (6), and(9), we obtain the following expected profits:

$$\Pi_I^{fd} = \begin{cases} \nexists & if\ h < 0.74398 \\ \frac{1}{1280}\frac{43\,104h+22\,984h^2+6104h^3+763h^4-94\,032}{(4h+h^2+6)^2} & if\ h \geq 0.74398 \end{cases}$$

$$\Pi_I^{nc} = \frac{1}{6400}\frac{62\,880h + 20\,696h^2 + 2488h^3 + 311h^4 + 61\,040}{(h+2)^4}$$

$$\Pi_I^{ac} = \begin{cases} \frac{12}{5(4h+4)} & if\ h < 0.67773 \\ \frac{12}{5(4h+4)} - \frac{\left(7h^4+18h^3+11h^2-12h-4\right)^2}{256(h+1)^4(h^2+4h+6)^2} & if\ h \geq 0.67773 \end{cases}$$

Full deterrence is possible only for low levels of differentiation ($h < 0.74398$). When goods are weak substitutes, Nash profits are so high that foreclosing is not possible; when goods are strong substitutes ($h \geq 0.74398$) there exists a β^{fd} decreasing in h that allows to implement deterrence. As the substitutability between the goods increases, the duopolistic profit obtained by the entrant decreases and the level of investment in patent protection necessary to deter entry decreases.

The non-cooperative equilibrium is implemented by fixing β^{nc} . In this scenario, the marginal profit by increasing β is negatively related to the Nash profit; as the substitutability between the goods increases, the duopolistic profit obtained by the entrant at the Nash equilibrium decreases; as a consequence, the level of patent protection increases.

Finally, β^{ac} is defined for any value of h: for low levels of differentiation a minimal level β is sufficient to sustain collusion (i.e., $\beta = 0^+$); for high levels of differentiation, collusion sustainability requires higher levels of β. When $h \geq 0.67773$, β^{ac} is increasing with respect to h: as the substitutability between the goods increases, the duopolistic profit obtained by the entrant in the Nash reversion decreases; in order to increase the punishment, the investment in patent protection increases.[31] Figure (3) illustrates the critical values of the β as a function of the differentiation parameter h in the three scenarios.

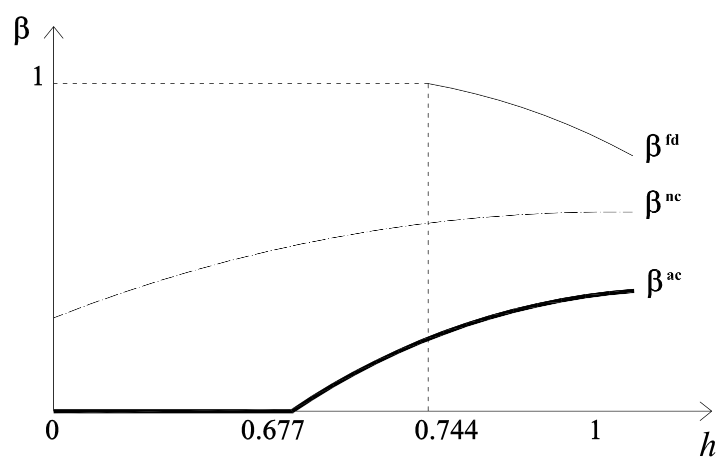

Figure 3: The β as functions of the differentiation parameter h.

[31] Part of the punishment consists in depriving the entrant of the Nash profit, i.e. the higher the Nash profit, the harder the punishment.

It is worthy to notice that the deterrence strategy is always dominated; collusion emerges as an equilibrium outcome when $h \geq 0.83748$ since $\Pi^{ac} \geq \Pi^{nc}$, with $\beta > 0$.

Appendix C

So far, we have assumed that firms pay the same lawsuit cost L. Hereafter, we remove this assumption, considering the case where the suing firm (the incumbent) pays a higher lawsuit cost; i.e., $L_I > L_E$. As shown in Figure (4), this asymmetry modifies the incumbent's critical discount factor, described in Equation (17), making bigger the downward jump, when patent protection passes from 0 to 0^+. On the contrary, the entrant's critical discount factors, described in Equation (14), does not change.

Figure (4) describes the new scenario. The curve $\sigma_I(\beta, L_I)$ (continuous line) describes the incumbent's critical discount factors for $L_I > L_E$. The curve $\sigma_I(\beta, L)$(dotted line) illustrates the case where$L_I = L_E = L$, as assumed in the paper. According to point(3) of Result (1), we note that increasing the lawsuit cost (from L to L_I) increases the set of the parameters that make collusion sustainable at the equilibrium, for $\beta^{ac} > 0$. Consider the firms' discount factors δ_I and δ_E as shown in Figure (4). If both firms face the same lawsuit cost the collusive equilibria is not sustainable with a positive amount of β, since $\tilde{\beta}_E(L) > \tilde{\beta}_I(L)$, and this is in contrast with point 3 of Result 1. On the contrary, if $L_I > L_E$, we may have $\tilde{\beta}_I(L_I) > \tilde{\beta}_E(L)$, that is the necessary condition for the sustainability of collusion in equilibrium, with $\beta > 0$.

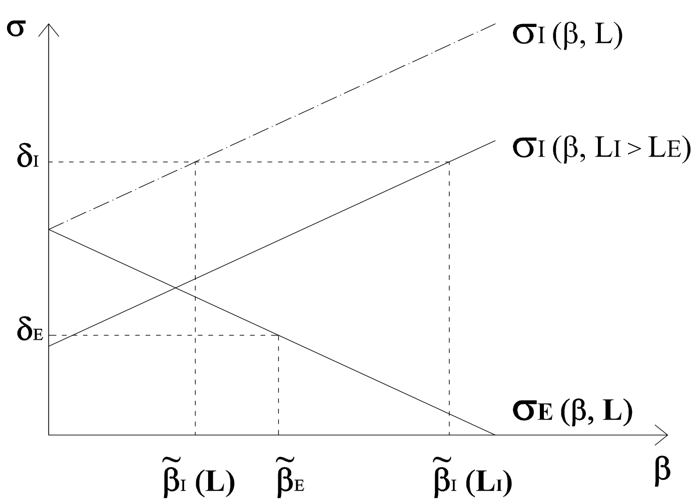

Figure 4: The case with different lawsuit costs ($L_I > L_E$)

Appendix D

Table 1: Symbols and notations.

I	Incumbent.
E	Entrant.
N	one-shot Nash profit.
C	one-shot collusion profit.
D	one-shot deviation profit.
M	one-shot monopoly profit.
Π_i	firm i's expected profits.
δ_i	firm i's discount factor.
σ	critical discount factor.
β	probability to win the patent litigation trial.
L	litigation cost of the trial.
F	fine charged to the entrant, in case of infringement.
nc	non-cooperative case.
fd	full deterrence case.
ac	accommodation and collusion case.
dev	deviation case.

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