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Marcello D'Amato[*], Christian Di Pietro[**] and Marco M. Sorge[***]

Abstract
Historical examinations of credit markets provide ample evidence on the coexistence of a variety of banking models, some of which specialize in information-intensive business practices. This paper studies the operation of markets in which asymmetrically informed lenders compete for investment projects with stochastic returns. We explore how the business model underlying informed lending — profit maximization (e.g. for profit relational lenders) vs. inter-member surplus redistribution (e.g. credit cooperatives) — shapes relative comparative advantages and affects market efficiency. Three findings stand out. First, consistent with real world evidence, a variety of market configurations — in terms of e.g. credit volumes and market shares — may obtain in equilibrium. Second, market failures (overlending) always prove mitigated when both types of lenders are operative, relative to a world in which equally uninformed lenders only populate the banking landscape. Third, market interaction between asymmetrically informed lenders can generate multiple equilibria. Hence, small changes in the business conditions or other fundamentals can cause large shifts in the allocation of credit leading to either highly selective markets or ones which rather endorse credit provision to undeserving entrepreneurs.

Keywords: Credit markets, Asymmetric information, Universal banks, Credit cooperatives

JEL Classification: D2, D4, G2

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Table of contents

1 Introduction

Modern credit markets have experienced rapidly increasing competition, as a result of e.g. regulatory change and financial innovation, which have loosen barriers to entry and expanded the scope of activities of financial intermediaries. Along with conventional credit institutions (e.g. commercial banks), other types of financial intermediaries – e.g. community and cooperative banks – have emerged and flourished in industrialized economies, often proving able to withstand critical situations under sensible competitive pressure. This process has fostered dramatic changes in the competitive structure of local credit markets and the allocation of bank credit to small and medium enterprises (SMEs) as well as larger businesses relying on short-term financing (e.g. Ayadi *et al.*, 2010).

The U.S. banking system, where large commercial banks coexist with a number of relatively small community lenders, is a case in point. Stemming from a long-standing concern about the concentration of banking power, this dual structure of the U.S. banking industry has historically featured a clear-cut distinction between the business models of the two lending institutions: while universal banks typically rely on centralized decision-making based on hard financial information and on a form of risk diversification in the presence of market imperfections (e.g. adverse selection) to meet their profit targets, small banks rather exploit more direct information about the local communities they are closely tied to and profitably lever on relational banking practices (e.g. Petersen and Rajan, 1994; Berger and Udell, 1995, 2002).

In a similar fashion, credit cooperatives have rapidly established themselves as significant players in the European financial services industry. Founded in local areas for the immediate community in the late 19th century, these institutions were originally focused on overcoming financial exclusion of the rural poor and working-class people in industrializing economies – e.g. the Raiffeisen movement in Germany and Holland (Guinnanne, 2001, 2002). While experiencing pronounced growth in the subsequent decades, cooperative banks have also engaged in several forms of consolidation and successfully expanded the scope of their operations across financial services and beyond local markets, as a response to intensified competition within retail banking. Nowadays, credit cooperatives comprise some of the largest financial institutions in Europe, with a few of them ranking persistently in the world-wide top quarter (e.g. Ayadi *et al.*, 2010).

Carrying unique features inherent to member ownership, the cooperative banking model does not rest on an exclusive notion of efficiency as measured by the ability to create value for its shareholders

– i.e. maximizing profits. Credit cooperatives typically pursue distinctive member-related objectives – e.g. surplus redistribution – and grant preferential access to credit to their members, that are also the cooperatives' clients, depositors, and borrowers (e.g. Fonteyne, 2007). Member ownership, originally conceived as a device to overcome credit market failures due to information asymmetries, therefore lies at the core of the surplus redistribution mechanism within the supported community. In fact, whereas for universal banks the redistribution occurs because of information asymmetries and through the market, in the case of cooperative banks redistribution concerns are explicit and exogenous with respect to market forces.

In any market where organizations with distinctive features and business models coexist, natural questions about competitiveness and efficiency arise. The goal of the present paper is to explore the operation of credit markets in which two kinds of lenders compete on (fixed size) investment projects of heterogeneous quality: *informed lenders*, for which quality is perfectly observable, and *uninformed lenders*, that are only aware of the distribution of characteristics of the population of entrepreneurs. Consistent with the diversity of business models observed in modern credit markets, two scenarios are explicitly considered: in the first one, informed lenders act as pure profit-seeking organizations and therefore maximize profits from lending activities (e.g. for-profit relational banks); in the second one, by contrast, informed lending is driven by a set of redistribution concerns, according to which the expected surplus from project financing is shared – according to some exogenously set rule – within the pool of their clients/members (e.g. credit cooperatives).

Our purpose is to emphasize two propositions. First, irrespective of the actual business model underlying informed lending, the inefficiency of the equilibrium resulting in credit markets in which both types of lenders are operative always proves ameliorated, relative to a world in which uninformed ones populate the banking landscape. Second, under some circumstances, the combination of market competition between asymmetrically informed lenders – AILs henceforth (e.g. Stroebel, 2016) – and either of these institutional patterns (profit-maximization vs. full redistribution with equal treatment) lead to multiple equilibria in the credit market. Hence, small changes in the business conditions or other fundamentals – e.g. the distribution of quality – can cause large shifts in the allocation of credit leading to either highly selective markets (with reduced inefficiency) or ones which rather endorse credit provision to undeserved entrepreneurs (hence exacerbating market failures).

The basic framework of analysis is a simple partial equilibrium model in which, given their own objectives and relative comparative (dis)advantages, all lenders engage in price (interest rate) competition in the presence of free entry. The characteristics of loan contracts will naturally reflect the lenders' asymmetric informational footing in the absence of collateral provisions: while uninformed lenders are bound to offer uniform price contracts, a project-contingent pricing rule is enforced by informed ones. We posit, by contrast, that uninformed lenders face lower costs of intermediation relative to informed ones – a comparative disadvantage in the market for funds. This is exogenously imposed and yet consistent with a simple theory of information acquisition with fixed costs, in which any lender must be indifferent between acquiring the information (e.g. screening) technology or not, whose outcome would have specialized lenders on the supply side of the credit market. A partition of the fund market might also be due to e.g. an unbalanced access to different sources of funding (e.g. wholesale debt, bond issued on international financial market), or to the peculiar specialized lenders' feature of gathering information on opaque projects, vis-à-vis universal banks' specialization on providing ancillary services to fund owners.

We first consider a simple framework in which all lenders are equally (un)informed, and overlending occurs in equilibrium. Our focus on a benchmark case featuring "two much investment" as the the essential market failure has a twofold rationale. First, information asymmetries in credit markets may well result in low-quality entrepreneurs benefiting from an excess of credit (e.g. De Meza and Webb, 1987; De Meza, 2002). Intuitively, whenever the utility in the credit relationship is increasing in the entrepreneur's type, good entrepreneurs are attracted and advantageous selection occurs. The resulting high average quality of the served pool in turn generates a positive externality on entrepreneurs endowed with lower quality projects, by allowing lenders to reduce the price charged on loan contracts. Most remarkably, and unlike adverse selection models (e.g. Akerlof, 1970), the overlending equilibrium is unique under reasonable assumptions, even when all lenders act as price takers. Second, recent attempts in measuring the relative importance of overlending vis-à-vis credit rationing show that the former has been an empirically relevant phenomenon for opaque enterprises in some advanced industries (Bonnet *et al.*, 2016).

When open to competition between AILs, by contrast, credit markets will generically feature equilibrium multiplicity, leading to highly diversified allocations of the quality of projects to either lender's type and therefore to different market shares. Remarkably, in any competitive equilibrium in which both types of lenders are active, a simple vertical segmentation of the credit market is supported, in which informed lenders exploit their informational advantage to attract the upper tail of the quality distribution (cream skimming). While the uniform price contract offered by the uninformed will still reflect the average quality of the pool of served entrepreneurs, the strength of the (positive) externality imposed by high-quality entrepreneurs on low-quality ones might be dramatically reduced, as long as the informed lenders provide an inside financing option for top quality entrepreneurs and therefore constrain the process of cross-subsidization among borrowers. This in turn creates room for multiple competitive equilibria – prices and volumes of intermediated credit – to arise.

The consequences of competition between conventional profit-maximizing lenders and informed ones which rather exhibit redistribution concerns are less straightforward. One important implication of redistribution is that it increases the cost of lending to good entrepreneurs. To set ideas, let the goal of informed lenders be that of extending the gains from credit provision to the largest feasible client base which does not violate their budget requirements, provided the whole aggregate surplus is redistributed over the pool of signing entrepreneurs. In order to attract high-quality projects, which helps meet their redistribution task, informed lenders could then be forced to provide borrowers with an overly large share of the projects' surplus and/or implement a more favorable redistribution scheme, with adverse effects on their overall budget. In sharp contrast to the profit-maximization case, a fundamental disconnection in the allocation of quality to either lending institutions can then be supported in equilibrium, according to which uninformed lenders attract 'peaches' and 'lemons', with the medial quality segment being covered by informed ones.

When redistribution concerns shape the bottom line objective of informed lenders, a further source of equilibrium multiplicity, other than the market-related one, is introduced. Remarkably, a varying amount of redistribution from high- to low quality projects induced by uniform pricing from their uninformed competitors alters the amount of redistribution that is feasible for the informed and yet does not violate their balanced budget requirement. As a consequence, different surplus sharing rules consistent with equal treatment are in principle enforceable in equilibrium.

Whatever the behavioral pattern underlying informed lending, we show that informed lending in credit markets plagued with opaque borrowers is likely to be viable occur in the presence of depressed investment perspectives (as summarized in the distribution of entrepreneurs' types). This result is in line with recent scholarly work documenting that relationship (informed) lending is more likely to relax credit constraints in bad times (cyclical downturns) rather than in good ones (e.g. Beck *et* *al.*, 2014). However, if the prior distribution of quality supports the existence of multiple equilibria featuring both types of intermediaries, the effect of a small change in the business conditions (boom versus recession) on market coverage is ambiguous: depending on the prevailing equilibrium, such a change can either ameliorate or exacerbate overlending, by causing uninformed lenders to serve highly inefficient (underserving) entrepreneurs. Consequently, even the prediction on the relationship between credit volumes intermediated by the uninformed and the business environment is not unambiguous.

According to our analysis, successful entry (and operation) of informed lenders along with informed ones is found unable to overcome market failures, though the overlending issue always proves ameliorated. When the market exhibits multiple equilibria with both types of lenders featuring positive market shares, overlending proves lower in high-price equilibria relative to low-price ones. Thus, informed lending acts as a *discipline device* for uninformed lending, by endogenously causing a relocation of quality of investment projects – and the associated failure risk – across the two types of financial intermediaries.

Equilibrium multiplicity, whether related to standard profit-maximizing behavior in competitive markets or rather induced by the coexistence of two different redistribution structures – one implicit and endogenous to market forces (e.g. universal banks), the other explicit and exogenously given (e.g. credit cooperatives) – may help explain why economies sharing the same fundamentals (e.g. distribution of entrepreneurs) need not exhibit the same aggregate volume of intermediated financial resources or similar market shares for the operating financial institutions. This finding is consistent with the empirically documented varying success and persistence of alternative banking models, such as credit cooperatives, along with universal banks in advanced economies (e.g. Fonteyne, 2007; Ayadi *et al.*, 2010).

The paper is structured as follows. In Section 2 we review the relevant literature. Section 3 lays down the basic framework of analysis, whereas Section 4 is devoted to the benchmark case, in which uninformed, profit-maximizing lenders only populate the credit market. In Sections 5 and 6, we turn to study lending competition between AILs, contrasting the equilibrium outcomes in terms of both market segmentations and efficiency under the specified behavioral dichotomy (profit maximization vs. surplus redistribution). Section 7 discusses some policy implications. Finally, Section 8 offers concluding remarks. All the proofs are reported in the Appendix.

2 Related literature

A standard tenet of the early literature on credit markets imperfections holds that information asymmetries among market participants (borrowers and lenders) play a key role in determining the efficiency of credit allocation (e.g. Akerlof, 1970; Jaffee and Russel, 1976; Stiglitz and Weiss, 1981; De Meza and Webb, 1987). The ability to collect and process borrower-specific information has always been deemed crucial to effective lending decisions. While conventional credit institutions (e.g. commercial banks) might indeed face imperfect information about borrowers' creditworthiness, others financial intermediaries (e.g. relational lenders) have historically proved able to build up sound informational expertise, mostly benefiting from peer monitoring effects (e.g. Banerjee *et al.*, 1994) and/or long-lasting credit relationships (e.g. Rajan, 1992; Von Thadden, 1995). In this respect, a fairly recent strand of scholarly work has emphasized the emergence of information-induced competitive advantage of such lending institutions (e.g. Sharpe, 1990), as well as the enhancing effects of *soft information* availability on their screening power over new loan applications (e.g. Agarwal and Hauswald, 2010).

The market effects of competition between AILs, on the other hand, has only recently drawn interest. While the empirical literature on the topic is still limited (e.g. Petersen and Rajan, 1994; Karlan and Zinman, 2009; Stroebel, 2016), a relatively large number of theoretical studies have dealt with the operation of credit markets as well as the organization of the banking industry in the presence of lenders which compete on an asymmetric informational footing. The objectives of these studies are rather mixed and include, yet are not limited to, the analysis of the information monopoly generated by long-term lending relationship (e.g. Sharpe, 1990), of the impact of competition on banks' strategic investment in screening technology (e.g. Hauswald and Marquez, 2006), and of firms' optimal mix of financing resources in the presence of AILs offering terms of trade which vary over the business cycle (e.g. Bolton *et al.*, 2016). We contribute to this literature by studying the impact on market outcomes of an asymmetric allocation of borrower-specific information across competing lenders, conditional on their bottom line objectives. Our framework of analysis close in spirit to that of De Meza and Webb (1987): in equilibrium, bad entrepreneurs benefit from banks overlending relative to the efficiency level. Whereas the role of informed lending in a model leading to underinvestment (e.g. Stiglitz and Weiss, 1981) would presumably be that of filling the market, our first contribution is to show that De Meza and Webb (1987)'s ovelending result survives the (threat of) entry by informed lenders.

A relevant strand of literature has been concerned with the design of firms' constitution and the resulting assignment of property rights, in order to study efficiency of ownership structures (e.g. Hart and Moore, 1996, 1998; Bontems and Fulton, 2009). Further studies have rather investigated the links between organizational structures and information-dependent allocation of capital to investment projects (e.g. Stein, 2002), optimal design of credit cooperatives as information machines (Banerjee *et al.*, 1994) as well as economic consequences of different objective and behavior patterns in production cooperative (e.g. Bonin *et al.*, 1993). Our analysis shows that the business objective of informed lenders crucially alters the allocation of investment projects to either financial intermediary and hence the emergence of diverse market configurations compatible with a competitive setting (vertical segmentation vs. disconnection).

For the purposes of the analysis, a number of otherwise relevant issues are assumed away from our framework. First, informed lenders are not assumed to hold any market power due to their informational monopoly (e.g. Sharpe, 1990; Dell'Ariccia and Marquez, 2004). That is, we model the two lending entities – informed vs. uninformed – as competitive ones in which the assumed informational asymmetry does not prevent group-specific free entry[1].

Second, informational asymmetries between different lending entities are exogenously taken, i.e. they do not result from strategic information gathering considerations in the presence of e.g. fierce competition on credit market (e.g. Hauswald and Marquez, 2006) nor they stem from lenders' learning ability under e.g. continuation lending terms (e.g. Rajan, 1992; Von Thadden, 1995; Bolton *et al.*, 2016). As said, this assumption could be easily relaxed to allow for ex ante costly information acquisition.

Third, we do not account for the possibility of multiple lending, though of course our borrowers face a choice between different contracts offered by informed and uninformed lenders. Whether and under which conditions borrowers opt for a mix of funding sources in a world of credit market imperfections is a different issue from the ones discussed here and deserves some attention on its own right (e.g. Detragiache *et al.*, 2000; Farinha and Santos, 2002; Houston and James, 1996). In fact, while not fully complying with observed features of modern banking, this simplification allows us to shed light on the equilibrium segmentation of the credit market stemming from the competitive interplay between different redistribution mechanisms: the one induced by adverse selection in the uninformed sector, and the other by the behavior pattern established within the informed one.

[1]See Dell'Ariccia *et al.* (1999) for an equilibrium model of the effects of adverse selection on the market structure of the banking industry.

3 The model

We consider a standard fixed-investment model with a continuum of heterogeneous entrepreneurs. Each entrepreneur is endowed with a risky project yielding a return $\Pi > 1$ if successful; if unsuccessful the returns are normalized to zero. Projects differ with respect to q, defined to be the probability of success; q is distributed over the unit interval, $q \in [0, 1] \equiv Q$, following a twice continuously differentiable distribution function $F(q)$, with density $f(q) > 0$ for all q.

Entrepreneurs are risk neutral and have no private wealth, so external finance lending one unit of funds is needed to implement the project. The financing of the project can only be raised through a standard debt contract which specifies the price $\rho \in (0, \Pi)$ for the unit loan, possibly type-contingent, depending on whether information about q is available to the lender or not. Under limited liability the expected utility of the entrepreneur q is given by

$$u(\rho; q) = (\Pi - \rho) \cdot q \tag{1}$$

If not involved in the implementation of the project, the entrepreneur can raise income in an alternative occupation yielding an expected return of $u_0 > 0$.[2] Hence, for any given debt contract $\{\rho, 1\}$, the entrepreneur will choose to implement the project according to which of the two occupations provides the best expected result, i.e. $\max\{u(\rho; q), u_0\}$.

Two types of risk-neutral outside financiers can enter the market and compete on projects by offering loan contracts. The first type of credit institution is a coalition of *uninformed lenders* who do not have precise information about the quality of the project, and only know the distribution $F(q)$.

We denote this institution with the subscript n (for not informed) along with all the terms of the contract they offer. Notice that so far the model is a simplified version of the model analyzed in De Meza and Webb (1987). We extend the analysis by considering the presence of another type of credit institution made of a coalition of *informed lenders* who perfectly know about the quality type of the project, i.e. about q. We denote this institution by subscript i, along with all the terms of the contract they fix. As a consequence of complete information, these credit institutions compete for each single project in the market.

The two types of credit institutions face two different prices for the funds they intermediate,

[2] Notice that the outside option $u_0 > 0$ is taken to be exogenous and not to be a function of q, or on any other element of the credit market. We discuss below the implications of such an assumption.

denoted R_n and R_i for the coalition of uninformed and informed lenders respectively. To make the problem interesting we posit the following parametric restriction:

$$R_n < R_i < \Pi - u_0 \tag{2}$$

The first (strict) inequality in (2) states that informed lenders have access to funds for finance at a higher cost than the uninformed. The second (strict) inequality by contrast ensures that there exists a non-degenerate subset of entrepreneurs who strictly prefer the (minimum price) contract that informed lenders may offer (i..e. the one inducing zero profits on each financed project) over the outside option u_0. Accordingly, the set of efficient projects

$$Q^* := \{q : q \geq q^*\} \subset Q, \tag{3}$$

is non empty, where $q^* := \frac{R_n + u_0}{\Pi} < 1$.[3] For future reference let $\Gamma = \{F, \Pi, R_n, R_i, u_0\}$ denote the model's parameter set, restricted to be consistent with (2).

Since a competitive credit market with free entry by both types of credit institutions is considered, all contracts offered must satisfy a zero profit condition, whenever a credit institution of either type is present in the market at equilibrium. As a consequence, in any competitive equilibrium where both types of credit institutions are present, entrepreneurs, in choosing whether to implement the project, will confront the outside option with the best offer in the credit market, given their type q. In other words entrepreneurs will choose whether or not to implement the project and, in the affirmative, which of the offers by one of the lending institutions to accept. More formally, entrepreneurs will make their choice according to

$$Max\,\{u(\rho_i(q); q), u(\rho_n; q), u_0\} \tag{4}$$

where the first argument denotes the utility obtained if a contract with an informed lender is subscribed by the entrepreneur, the second argument denotes the utility obtained if a contract with an informed lender is chosen, the third term denotes the utility obtained in the alternative occupation in case investment is not undertaken.

[3] Let $q \in Q$ be publicly observable. Then the aggregate surplus in the credit market is given by $\int_Q [(\Pi q - R_n)\, I(q) + u_0\,(1 - I(q))]\, d\mathcal{F}(q)$, where $I(q) = 1$ for entrepreneurs $q \in Q$ entering the market and $I(q) = 0$ otherwise. Maximizing this function over $I(q) = \{0, 1\}$ delivers q^*. Notice that this notion of efficiency pertains to the *state* q and not to the measure of the pool of served entrepreneurs in the neighborhood of the least quality project having access to the credit market.

Notice that this is a specific feature of the model in that the alternative options of the credit contract with a specific type of lender is endogenously defined in the credit market as the equilibrium policy chosen by the competitors.

As for the members of the coalition of uninformed lenders, in a symmetric equilibrium, contracts offered by identical members of n stipulate a uniform price ρ_n in exchange for a credit of unit size, since no collateral is available. Expected profits for credit institutions of type n are given by

$$\mathrm{E}_F[\pi_n(\rho_n, Q_n)] = \int_{Q_n} [\rho_n \cdot q - R_n] \, dF(q) \tag{5}$$

where Q_n is the set of entrepreneurs for which it is preferred to sign a credit contract with institutions of type n over other alternatives and hence

$$Q_n = \{q : \; u(\rho_n; q) \geq \max\{u_0, u(\rho_i; q)\}\} . \tag{6}$$

In any competitive equilibrium with free entry, it must hold (5) equal to zero.

Contracts offered by identical members of i can be made contingent on the observed q so that $\rho_i(q)$ is offered. Indeed, as an immediate consequence of competition within that group, members of the i type of credit institution must offer contracts such that their profit is driven to zero on every project of a given quality, i.e. for any $q \in Q$. Expected profits for credit institutions of type i are given by

$$\mathrm{E}_F[\pi_i(\rho(q), q)] = \rho_i(q) \cdot q - R_i, \quad q \in Q_i \tag{7}$$

where Q_i is the subset of projects for which it is preferred to sign a credit contract with institutions of type i over other alternatives, that is

$$Q_i = \{q : \; u(\rho_i(q); q) \geq \max\{u_0, u(\rho_n; q)\}\} \tag{8}$$

In any competitive equilibrium with free entry, (expected) profits (7) are equal to zero. Notice that, while uninformed lenders break even on average, pricing of informed lenders makes them earn zero profit on each served project.

To summarize, we will study the equilibrium of a credit market where the distribution $F(q)$ of the quality of the projects is revealed to both types of lenders and yet an entrepreneur's type q is perfectly observable only by lenders of type i. Then, all lenders decide whether or not to enter the market and offer contracts to the entrepreneurs (i.e. no offer corresponds to no entry).[4]

Entrepreneurs then choose whether to implement the project and select the contract that yields the higher expected utility, agreeing to make repayment subject to their terms. Once contracts are signed, financed projects are undertaken – each succeeding with own probability q – and payoffs y are realized, with $y = \Pi$ in the good state and zero otherwise.[5]

As mentioned in the introduction, the analysis will proceed by considering first the benchmark case where only uninformed lenders are present in the market; then we study the model described above where the two types of lenders compete for projects, both maximizing their expected profits. Finally, we will explore some consequences of a different behavioral assumption whence the credit institution made by informed lenders has a redistributive concern. For the ease of exposition, we postpone to section (6) the description of such behavioral assumption, since it also requires additional conditions in the definition of the competitive equilibrium with free-entry. Hence, for the moment, each type of credit institution is assumed to maximize expected profits. In this case, an equilibrium is a collection $\{\rho_i(q), \rho_n, Q_i, Q_n\}$ such that equations (5) and (7) take value at zero and such that Q_i and Q_n in (6) and (8) are satisfied and mutually consistent. In case of indifference among different credit contract, an entrepreneur will be assumed to choose a contract of type n[6].

4 Benchmark: Uninformed lenders only

In this section, following a standard model of overinvestment (De Meza and Webb, 1987), a credit market is considered where *all* competing lenders are *uninformed* about the actual quality of projects to be financed. Since quality is unobservable and entrepreneurs provide no collateral, the interest rate ρ_n must be independent of the entrepreneur's type, and hence entrepreneurs endowed with a project of quality q will sign the debt contract $\{\rho_n, 1\}$ if and only if $u(\rho_n; q) \geq u_0$. The supply of debt contracts will reflect lenders' beliefs about the average quality of entrepreneurs who accept it. In any

[4]In the absence of entry or other setup costs, the model indeed resembles a one-stage entry game, in which the actions of entry and price choices are simultaneous. We adopt the convention that a lender chooses to enter when she is indifferent. As we are not interested in deriving the number of operating lenders in equilibrium, our characterization of free-entry equilibria through (expected) zero-profit conditions will neglect the potential "non-integer problem". See Dos Santos Ferreira and Dufourt (2007) for a general analysis of symmetric price competition games played by actual and potential entrants.

[5]Notice that, under this timing assumption, uninformed lenders are unable to observe offers made by informed ones, which might have allowed the former to infer payoff-relevant information.

[6]The tie break rule is immaterial for the expected profits since the indifferent project will always be of measure zero

competitive equilibrium these beliefs correctly reflect the actual average quality of the projects being funded, so we state the following:

Definition 1. *For a given* Γ*, a competitive equilibrium with free entry* $\mathcal{N}(\Gamma)$ *is a price* $\rho_n^{\mathcal{N}}$ *and a set of projects* $Q_n^{\mathcal{N}} \subseteq Q$ *such that:*

$$\int_{Q_n^{\mathcal{N}}} \left[\rho_n^{\mathcal{N}} \cdot q - R_n\right] dF(q) = 0$$

$$Q_n^{\mathcal{N}} := \left\{q : u(\rho_n^{\mathcal{N}}) \geq u_0\right\} \tag{9}$$

that is, a zero-profit condition settles the market price for credit $\rho_N^{\mathcal{N}}$ consistent with entrepreneurs' incentives to prefer the debt contract over their outside option, here the superscript $\mathcal{N}$ denotes equilibrium choices. The following then holds:

Proposition 1. *For a given* Γ*:*

(i) An equilibrium always exists, and it is unique;

(ii) Overlending occurs in equilibrium, i.e. $Q^* \subset Q_n^{\mathcal{N}} = [\underline{q}^{\mathcal{N}}, 1]$.

Proof. See the Appendix. □

Proposition 1 states that, for any given parameter set Γ, a unique equilibrium market allocation of projects exists, and it features excess investment relative to the efficient level. Intuitively, in the presence of a uniform price contract, the borrowers' utility from signing the debt contract increases monotonically with their own (privately known) type, for a given (type-independent) outside option u_0. Hence, at any feasible price entrepreneurs with high-quality projects are attracted. The resulting higher average quality of the pool in turn generates a positive externality on entrepreneurs endowed with lower quality projects, by allowing uninformed lenders to reduce the price charged on debt contracts. As in De Meza and Webb (1987), overlending thus arises in equilibrium.

That the credit market never unravels follows immediately from the fact that the efficient competitive outcome – the one resulting from perfect observability of the quality of projects – calls for a non-zero measure subset of the entrepreneurs' population to be served (i.e. Q^*). Since the market mechanism results in good projects drawing bad ones in, a competitive equilibrium under asymmetric information will always exist.

The intuition for the uniqueness result is as follows. Recall that, in any equilibrium, competition across lenders in the presence of free-entry drives (expected) profits to zero. So, fix a competitive equilibrium $(\rho_n^{\mathcal{N}}, Q^{\mathcal{N}})$, and suppose the price (interest rate) is lowered, then borrowing becomes more attractive and low-quality entrepreneurs are therefore attracted. The market would therefore expand at the bottom and the average quality of the pool of traded projects would shrink, forcing competing lenders to require a higher price on debt contracts in order to break even, i.e. a contradiction. Hence, there cannot exist another equilibrium.

Notice that uniqueness does not necessarily rely on the assumption of type independent outside option u_0. It can be easily proven that uniqueness obtains under reasonable assumptions incorporating the idea that entrepreneurs who have better investment opportunities are also more productive in the alternative occupation.[7]

A direct implication of Proposition 1 is that, in equilibrium, the interest rate will be negatively related to the average quality of served entrepreneurs, as measured by the conditional mean of the underlying distribution on the equilibrium support $Q_n^{\mathcal{N}}$. Hence, by simple comparative statics, when the distribution of quality in the population of entrepreneurs features a relatively large measure of high-quality projects, the (uniform) market price for credit will be relatively low. Formally:

Corollary 1. *Consider a distribution $H(q)$ such that $H(q) \leq F(q)$ and $h(q) > 0$ for all $q \in Q$. Then, all else equal, $\rho_n^{\mathcal{N}(H)} < \rho_n^{\mathcal{N}(F)}$.*

Proof. See the Appendix. □

In other words the interest rate in this model is pro-cyclical in the sense that better investment perspectives (summarized in F) entail a lower interest rate and a larger degree of overlending in this model. This finding is consistent with the well-documented evidence on the positive response of credit provision to business cycle expansions, and its contraction during subsequent downturns (e.g. Berger and Udell, 2002). Similarly it can be easily shown that an increase in the cost of funds will make the credit market more selective so that a reduction in the volumes of trade will be obtained.[8]

[7] Specifically, denote with $v(q)$ a type dependent outside option, then it is easy to show that a set of sufficient conditions for uniqueness is the following: $v'(q) > 0$ and $v''(q) < 0$.

[8] This follows immediately from the zero profit condition in (9).

5 Market equilibrium with informed and uninformed profit-maximizing lenders

We now study the credit market in the presence of informed lenders. We proceed by solving for a competitive equilibrium in which (i) all lenders hold Nash-type conjectures about their rivals' entry and pricing decisions and play symmetric best-response strategies, (ii) free entry results in each operating lender earning (expected) zero profits and (iii) self-selection of entrepreneurs obtains in a way that is consistent with the lenders' equilibrium strategies[9].

The analysis is less straightforward than in the previous section. Notice that informed lenders compete among themselves in a Bertrand fashion for all projects they are aware of, since they have perfect information about their prospective clients (profits are competed away state by state i.e. for any q by competition among lenders in this group). Moreover, since competitive contracts offered by the informed lenders must be best reply to offers made by the uninformed ones, both existence and uniqueness of the equilibrium of the credit market can be, in principle, affected.

To facilitate intuition, consider the case where only informed lenders are present in the credit market. It is easy to see that a Bertrand-type argument would yield the following outcome: $\rho_i(q) = R_i/q$, so that the whole (state by state) surplus of the credit relationship would be left to the entrepreneurs and $u(q) = \max\{u_0, \Pi q - R_i\}$, where Q_i is given by the interval $\left[\frac{R_i+u_0}{\Pi}, 1\right]$. So if the two types of lenders would operate in two separated markets the equilibrium in these segmented markets would exist and it would be unique.

Consider now the case where informed and uninformed lenders coexist in the same credit market. Before stating our main results, let us introduce a formal definition of the equilibrium relevant for this case :

Definition 2. *For a given* Γ*, a competitive equilibrium with fee entry – denoted* $\mathcal{B}(\Gamma)$ *– is a strategy profile* $\left\{\rho_i^{\mathcal{B}}, \rho_n^{\mathcal{B}}, Q_n^{\mathcal{B}}\right\}$ *and a set* $Q_n^{\mathcal{B}}$ *such that:*

(i) $\int_{Q_n^{\mathcal{B}}} \left[\rho_n^{\mathcal{B}} \cdot q - R_n\right] dF(q) = 0$;

(ii) $\rho_i^{\mathcal{B}} \cdot q = R_i, \quad \forall q \in Q_i^{\mathcal{B}}$;

(iii) $Q_j^{\mathcal{B}} = \left\{q : u(\rho_j^{\mathcal{B}}; q) \geq \max\left\{u_0, u(\rho_k^{\mathcal{B}}; q)\right\}\right\}, \quad j \neq k \in \{i, n\}$.

[9] Asymmetric equilibria may in principle exist in this model. For the purposes of our analysis, we intentionally disregard this potential source of multiplicity.

Part (i) requires zero profit among uninformed lenders, part (ii) affirms the state by state pricing rule consistent with the zero profit condition by informed lenders, part (iii) defines the set of entrepreneurs who choose (in an incentive compatible fashion) to subscribe the debt contract with either of the two types of lenders. All lenders who enter the market hold correct expectations about both their rival's pricing choices and the pool of entrepreneurs who will accept the contract. As a consequence, the allocation of quality across lenders obtains consistent with the lenders' equilibrium strategies.

In the following we present results about existence and (non) uniqueness of the equilibrium, and its characterization in terms of conditions under which both types of lenders will enter the credit market or not.

We first establish that, though finitely many equilibria can emerge from the competitive interplay between informed and uninformed lenders, in any equilibrium a unique form of *vertical segmentation* emerges, in which informed lenders attract projects in the top segment of the quality spectrum, while uninformed ones attract and redistribute over medial and lower quality ones under uniform pricing.

To better illustrate this point, we start noticing that, in any competitive equilibrium (if existing), the type-contingent price $\rho_i^{\mathcal{B}}(q)$ offered by informed lenders is strictly decreasing in $q \in Q_i^{\mathcal{B}}$. As a consequence, if an equilibrium market configuration exists which supports both types of lenders, a unique marginal type $\tilde{q}^{\mathcal{B}}$ entrepreneur will find himself indifferent between the two contracts ($\tilde{q}^{\mathcal{B}}$ is assumed to accept the one offered by the informed), whereas each entrepreneur with project of quality higher than the marginal one's will enter the credit relationship with the informed lenders.

Formally:

Lemma 1. *In any equilibrium $\mathcal{B}(\Gamma)$:*

(i) There exists a unique constant $\beta(\Gamma) > 1$ such that $\rho_n^{\mathcal{B}} \in (R_i, \beta R_i)$, and $\rho_i^{\mathcal{B}}(q) \in [R_i, \beta R_i]$, $\forall q \in Q_i^{\mathcal{B}}$;

(ii) There exists a unique $\tilde{q}^{\mathcal{B}} \in (\frac{u_0}{\Pi}, 1)$ such that $\rho_n^{\mathcal{B}} = \rho_i^{\mathcal{B}}(\tilde{q}^{\mathcal{B}})$ and $\rho_n^{\mathcal{B}} > \rho_i^{\mathcal{B}}(q)$ iff $q > \tilde{q}^{\mathcal{B}}$.

Proof. See the Appendix. □

Part *(i)* states that there exists a unique range – as a function of the relative cost of funding – where equilibrium prices that support entry by both types of lenders can be set. Part *(ii)* states that for any given pricing strategies (that satisfy equilibrium conditions), there must exist a unique project such that the entrepreneur is indifferent between the credit contracts offered by the informed and the uninformed, whereby informed lenders serve top quality projects and the whole surplus is left to the entrepreneur.

The above Lemma does not restrict the equilibrium configuration in terms of entry so that, depending on the parameters, three possible equilibrium configurations are – in principle – allowed: i) one where the low-cost uninformed lenders may not enter due to adverse selection; ii) a second where both types of lenders enter the credit market; iii) a third where the high-cost informed lenders do not enter due to their larger costs of funding. The next proposition addresses existence, characterization in terms of entry by different types of lenders and equilibrium multiplicity.

Proposition 2. *For any given* Γ*:*

(i) There exists (generically) an odd number of equilibria;

(ii) in all equilibria uninformed lenders enter the market and there is no equilibrium such that only informed lenders enter the market;

(iii) a sufficient condition for the existence of an equilibrium such that both types of lenders enter the market is

$$E_F\left[q \middle| q \in \left[\frac{u_0}{\Pi - R_i}, 1\right)\right] < \frac{R_n}{R_i} \tag{10}$$

Proof. See the Appendix. □

Part (i) of the above proposition states that competition among informed and uninformed lenders can indeed produce equilibrium multiplicity. The intuition for this result is as follows. Suppose that ρ_n and $\rho_i(q)$ are two equilibrium (pricing) strategies by informed and uninformed lenders, respectively. Notice that $\rho_i(q)$ by informed lenders is a type-contingent schedule that does not depend upon the value of ρ_n. Suppose next that ρ_n is increased, then a re-composition effect arises at the two tails of the pool of projects served by uninformed lenders. At the bottom of the (conditional) distribution on the support Q_n, marginal projects are driven out of the credit market, as these entrepreneurs find the (exogenous) outside option more appealing after the increase in ρ_n. On the other hand, marginal entrepreneurs at the top of the (conditional) distribution also find the offer made by the informed more attractive. As a consequence, the average quality of the pool can raise or fall, depending on the values the specific F distribution takes at the extremes of Q_n. If the average quality decreases as a consequence of this re-composition, the increase in ρ_n could be validated, zero profits satisfied and a new self-enforcing equilibrium interest rate can emerge.

Most remarkably, in any competitive equilibrium in which both types of lenders are active, the uniform price contract offered by the uninformed will still reflect the average quality of the pool of served entrepreneurs. However, the strength of the (positive) externality imposed by high-quality entrepreneurs on low-quality ones might be dramatically reduced, as long as the informed lenders attract the top end of the market and therefore constrain cross-subsidization among borrowers.

According to part (*ii*) of Proposition 2, pooling contracts – and as we will see, the associated overlending in the credit market – are a necessary feature of any equilibrium configuration of the credit market in its lower segment. The possibility that informed lenders do not enter the market obviously relies on the extent of the cost disadvantage faced by the informed: if this cost is large enough, even the largest surplus that the informed may offer to the entrepreneurs falls short of that provided by the uninformed. By this argument, it naturally follows that the cost gap R_i/R_n can force the informed lenders not to engage competitive behavior.

On the relative cost of funding as well as on the properties of the (unconditional) quality distribution also hinges part (*iii*) of Proposition 2. It indeed clarifies that the value of information for type i lenders is crucially linked to the general business environment, as summarized in the initial distribution of the population of entrepreneurs $F(q)$. Intuitively, if uninformed lenders are bound to make (expected) negative profit by undercutting – and hence by attracting the whole of the upper tail of the quality distribution – they are unable to deter entry by informed lenders, and a (possibly non-unique) market equilibrium emerges where both types of intermediaries serve non-zero measure sets of entrepreneurs. All else equal, entry deterrence of the informed by the uninformed lenders is likely when the cost gap is sufficiently wide, and/or business perspectives as summarized in F are enough optimistic, i.e. the distribution of entrepreneurs features a sufficiently large number (here, the measure) of high-quality projects. A good enough business environment – e.g. one characterized by better investment perspectives – will allow uninformed lenders to price less aggressively and hence drive high-cost informed lenders out of the market.[10] In terms of potentially testable implications of the model, this result suggests that the volume of credit intermediated by uninformed lenders tends

[10]Notice that, for any given distribution $F(q)$ the condition at equation (10) is likely to be violated when R_i is large, as the left-hand side (the conditional expectation) strictly increases in the latter, whereas the cost gap goes down. Also, if the benchmark equilibrium price $\rho_n^{\mathcal{N}}$ lies strictly below the type i lenders' cost of funds R_i, then the sufficient condition stated in Proposition 2, part (*iii*) fails to hold for $R_n/R_i < R_n/\rho_n^{\mathcal{N}} = E[q|q \in [\underline{q}^{\mathcal{N}}, 1]] < E[q|q \in [\frac{u_0}{\Pi - R_i}, 1]]$.

to be larger in economies with better profit perspectives.

It is worth emphasizing that, when no competitive equilibrium with both types of intermediaries exists, the credit market never breaks down, insofar as uninformed lenders will always find themselves serving a non-zero measure set of (inefficient) projects. Put differently, the viability of informed lending does not impose any constraint on the set of distributions F for which the credit market does not unravel, relative to the benchmark scenario. By contrast, non-unique market equilibria can occur as a consequence of competition among asymmetrically informed lenders, as stated in the following

Corollary 2. *For a given* Γ*, let* $M := \int_Q q dF(q)$ *denote the population mean. Then, if* $M \geq \frac{R_n}{R_i}$*, the competitive equilibrium (when it exists) is generically non-unique.*

Proof. See the Appendix. □

When multiple market equilibria exist, they can be ranked according to the associated degree of overlending. Suppose $\rho_n^{\mathcal{B}}$ and $\rho_n^{\mathcal{B}'}$ are part of two competitive equilibria, with $\rho_n^{\mathcal{B}} < \rho_n^{\mathcal{B}'}$. Then the set of projects financed by uninformed lenders under the higher price $\rho_n^{\mathcal{B}'}$ will contract and feature an inferior average quality, whereas the trade volume of informed lenders expands at the bottom. However, overlending proves lower in the high-price equilibrium relative to the low-price one, since the pricing rule of uninformed lenders screens out the lower extreme of the conditional distribution of entrepreneurs entering the credit relationship, while causing a relocation of quality across the two types of intermediaries within the same (vertical) market configuration.

Summarizing the results in this section, we have shown that, when informed lenders find entry profitable, they position themselves at the top segment of the market, reducing the (average) surplus that uninformed lenders redistribute across served entrepreneurs. As a main consequence, any equilibrium outcome featuring both types of lenders improves upon the inefficient allocation generated by a credit market in which only uninformed lenders are operative, though (aggregate) overlending never disappears. Formally, let $\mu(X) = \int_X dF$ denote the measure of some set $X \subseteq Q$ according to distribution $F(q)$. Then

Corollary 3. *For a given* Γ*:*

(i) In any equilibrium $\mathcal{B}(\Gamma)$*, the least quality project served* $\underline{q}^{\mathcal{B}}$ *lies strictly below* q^**;*

(ii) In any equilibrium $\mathcal{B}(\Gamma)$*, it holds* $\underline{q}^{\mathcal{B}} \geq \underline{q}^{\mathcal{N}}$*, where the inequality is strict if* $\mu(Q_j) > 0$ *for* $j \in \{i, n\}$*.*

Proof. See the Appendix. □

6 Market equilibrium when informed lenders have redistribution concerns

As stated in the introduction, our analysis of the coexistence of both uninformed and informed lenders is motivated by the aim of understanding the consequences of competition between conventional (universal) profit-maximizing banks and specialized lenders which may rather exhibit redistribution concerns.

We capture these concerns by letting the contract offered by the informed stipulate two interdependent provisions: a *sharing rule* α, which governs the allocation of the surplus generated by each single project, if successful, between the entrepreneur and the lender; and an *inter-member redistribution* scheme D, which governs the redistribution of (possibly only a part of) the aggregate surplus flowing to lenders, and then re-assigned to the pool of subscribing entrepreneurs. As the aggregate amount of surplus available for redistribution crucially depends both on the number (here, the measure) of entrepreneurs participating in the contract and on the surplus that lenders retain on each individual project, the two contractual provisions crucially interact in providing the entrepreneurs with the appropriate incentives to join the pool.

To set ideas, consider first the case where all lenders in the economy have perfect information about the projects' quality. Let $\mu(X) = \int_X dF$ denote the measure of some set $X \subseteq Q$ according to the distribution F. The expected utility of entrepreneur q from entering the credit relationship reads as:

$$u(\alpha, D; q) = \alpha(q)\left[\Pi q - R_i\right] + D(q, \alpha(q), \mu(Q_i)), \quad \alpha \in [0,1], \quad D \geq 0$$

where the first addend in the right-hand side is the share of the (expected) surplus associated with project q, which is left to the entrepreneur by the terms of contract, while the second one is the (possibly type-dependent) fraction of the aggregate surplus – as a function of the retained surplus on each project $q \in Q_i$ and of the measure of the set Q_i of entrepreneurs accepting the contract –

redistributed to the borrowers. The optimal contract $\{\alpha, D, 1\}$ will then solve the following program:

$$\max_{\alpha, D, Q_i} \int_{Q_i} V(\alpha, D)\, dF(q) \tag{11}$$

$$s.t. \quad \int_{Q_i} (1-\alpha)\left[\Pi q - R_i\right] - D dF(q) \geq 0 \tag{12}$$

$$u(\alpha, D; q) \geq u_0 \tag{13}$$

where $V(\cdot)$ is exogenously specified to represent the preferences of the informed for lending to – and redistributing over – a defined pool of entrepreneurs Q_i, while any feasible choice for (α, D) is restricted not to generate losses on the part of lenders (relation (12)) and is compatible with the incentives of entrepreneurs to accept the contract (relation 13). In the presence of free entry, the relation (12) will hold as an equality, requiring the optimal contract offered by the informed to enforce full redistribution.

Even this highly simplified formulation of the contracting problem faced by informed lenders highlights the critical tension between *sharing* and *redistributing*. Assume the objective of informed lenders is that of maximizing the amount of surplus to be redistributed to the entrepreneurs. All else equal, granting a sufficiently large share of the expected surplus on single projects q to enlarge the clients base might reduce the aggregate surplus which stands available for redistribution, as the latter depends on the measure of the pool of the entrepreneurs willing to participate in the relationship. By contrast, setting the lowest share compatible with the incentives of the entrepreneurs to join the pool might increase the aggregate surplus, depending on whether a more favorable redistribution scheme proves able to counterbalance the repulsive effect generated by a weaker surplus sharing contractual provision. Also, depending on the the properties of the distribution of quality $F(q)$, multiple combinations of sharing $\alpha(q)$ and redistribution $D(q)$ might allow the lenders to achieve their objective; and even when a unique optimal pair $(\alpha(q), D(q))$ exists, it might be associated with a multiplicity of pools Q_i over which the surplus can be equivalently redistributed. In general, the specific form of the objective $V(\cdot, \cdot)$ will dictate how this tension gets resolved.[11]

The occurrence of market competition with uninformed lenders further complicates the picture. Recall that these lenders are bound to offer uniform price contracts, which engender a positive relation-

[11] Notice that, because of the presence of redistributive concerns, competition across informed lenders cannot operate up to the point that the whole project-specific surplus is left to the entrepreneur.This is indeed the case when informed lenders behave as profit maximizers and have no redistribution concerns. In this case, $D = 0$ for all $q \in Q_i$, and the share of the project-specific surplus retained bu the lenders is zero, i.e. $\alpha(q) = 1$ for all $q \in Q$.

ship between the quality of the financed project and the (expected) utility of the relative entrepreneur (i.e. the higher the quality, the larger the utility). To attract the top tail of the quality distribution, which helps meet the redistribution purpose (the higher the quality of a given project, the higher the associated expected surplus), informed lenders could then be forced to provide the entrepreneurs with an overly large share of the projects' surplus and/or implement a more favorable redistribution scheme, with adverse effects on their budget constraint.

The essence of the main implications of redistributive concerns can however be obtained by exogenously imposing some discipline on the various forces involved. To this end, we make three simplifying assumptions about the contract design problem faced by informed lenders. First, their objective is that of maximizing the size (here, the measure) of their client base over which to redistribute the aggregate surplus, and not the surplus itself. Second, the sharing rule α is taken to be exogenous, as e.g. enacted by statutory provisions. Third, the redistribution scheme offered by the informed is restricted to enforce *(almost) equal treatment* of signing entrepreneurs: in this case, the aggregate surplus flowing to the lenders is redistributed across the served entrepreneurs of sufficiently high quality (where the quality threshold is endogenously determined, see below), so as to provide them with the same level of utility (ex-ante equality).

Formally, let $\rho_i(q)$ denote the price of the debt contract offered by informed lenders. Then (almost) equal treatment involves granting a constant expected utility $u(\rho_i(q); q) = (\Pi - \rho_i(q)) \cdot q = \Pi c$ to served entrepreneurs, for some budget-balancing sharing parameter $c \in (0, 1)$, provided $\mu(Q_i)$ is maximized.[12] Since $\rho_i(q)$ is restricted to be non-negative, we have $\rho_i(q) = \max\left\{0, \Pi\left(1 - \frac{c}{q}\right)\right\}$, or equivalently

$$\rho_i(q) = \begin{cases} \Pi\left(1 - \frac{c}{q}\right), & q \in Q_i : q > c \\ 0, & q \in Q_i : q \leq c \end{cases}$$

Notice the share c defines a quality threshold within the set Q_i: a utility level of Πc is granted to all projects above this threshold, whereas all the expected return Πq is left to the entrepreneurs on projects below it.

Most importantly, the share c depends on the total amount of surplus that informed lenders manage to raise, provided the balanced budget requirement is met; and the contract $\{\rho_i(q), 1\}$ – or equivalently

[12] The restriction $c < 1$ is necessary for redistribution to occur. In terms of the above-mentioned contractual program, the price $\rho_i(q)$ satisfies $u(\rho_i(q); q) = (\Pi - \rho_i(q))q = \alpha(\Pi q - R_i) + D$ from which $\rho_i(q) = (1 - \alpha)\Pi - (D - \alpha R_i)/q$. Equal treatment is obtained when e.g. $\alpha = 0$ and $D = \Pi c$.

the surplus share c – is offered to all the entrepreneurs who have an incentive to join the pool. Hence, the pricing strategy $\rho_i(q)$ must be a schedule, in contrast to the type-by-type pricing obtained in the absence of redistribution concerns.

We adopt the following notion of a competitive equilibrium in the presence of redistribution concerns:

Definition 3.

For a given Γ, a competitive equilibrium with free entry – denoted $\mathcal{B}(\Gamma)$ – is a strategy profile $\left\{c^{\mathcal{B}}, \rho_n^{\mathcal{B}}, Q_i^{\mathcal{B}}\right\}$ and a set $Q_n^{\mathcal{B}}$ such that:

(i) $\int_{Q_n^{\mathcal{B}}} \left[\rho_n^{\mathcal{B}} \cdot q - R_n\right] dF(q)$;

(i) $c^{\mathcal{B}} \in \arg\max \mu(Q_i^{\mathcal{B}}) \quad s.t. \quad \int_{Q_i^{\mathcal{B}}} \left[\Pi(q - c^{\mathcal{B}}) - R_i\right] d\mathcal{F}(q) = 0$;

(iii) $Q_j^{\mathcal{B}} = \left\{q \,\middle|\, u(\rho_j^{\mathcal{B}}; q) \geq \max\left\{u_0, u(\rho_k^{\mathcal{B}}; q)\right\}\right\}, \quad j \neq k \in \{i, n\}.$

Part (i) is the expected zero profit condition for the uninformed, part (ii) states that surplus redistribution enforced by informed lenders must be consistent with their maximal measure objective, and cannot violate their balanced budget requirement. Part (iii) finally states that the allocation of projects to the two types of intermediaries must be consistent with the incentives faced by the entrepreneurs.

Notice that the critical trade-off highlighted above survives the adoption of this set of assumptions. On the one hand, attracting higher quality projects requires distorting upwards the surplus redistribution scheme to be granted to the served entrepreneurs; on the other, this also generates larger project-specific revenues in the top segment of the quality spectrum. The value of each choice, in turn, crucially depends on the entrepreneurs' (endogenously determined) borrowing option represented by the contract offered by the uninformed.

The pricing strategy resulting from the interplay of asymmetric information and redistributive concerns is therefore quite involved. For any expected pricing ρ_n of their uninformed competitors, informed lenders set the highest share $c(\tilde{q})$ compatible with the entrepreneurs' incentives to accept the associated contract, where $\tilde{q}$ identifies the marginal entrepreneur which is exactly indifferent between the two borrowing options. Among all couples (c, Q_i) which are consistent with full redistribution, they then select the one(s) yielding Q_i with maximal measure. Since both $c(\tilde{q})$ and the revenues on the highest quality project served are increasing in $\tilde{q}$, a "go-for-the-top" strategy may either facilitate or hinder the achievement of the maximum measure objective, provided it fulfills the balanced budget requirement.[13] In contrast to the profit-maximization scenario, the value to informed lenders of attracting top quality entrepreneurs is therefore reduced, and feasible redistribution (if any) might have to involve medial or low quality ones. In either case, when the price charged by uninformed lenders is sufficiently low (i.e. $\rho_n < R_i$), the informed still find entry in any segment of the market unprofitable[14].

Consistent with the self-selection of entrepreneurs in the two borrowing opportunities, the informed lenders can only make profits over the interval $(c(\tilde{q}), \tilde{q}]$, whereas for $q \in Q_i$ with $q \leq c(\tilde{q})$ the whole expected surplus is left by the terms of contract to the entrepreneurs. Hence, while any project involves a constant cost of funding R_i, expected revenues per project are nondecreasing in $q \in Q_i$ for any given share $c(\tilde{q})$. As a major consequence, if equilibrium pricing strategies of the informed lenders induce $Q_i \subseteq (c(\tilde{q}), \tilde{q}]$, then Q_i is strictly convex and uniquely determined by its supremum $\tilde{q}$[15]. The following indeed holds true:

Lemma 2. *In any equilibrium $\mathcal{B}(\Gamma)$, if $Q_i^{\mathcal{B}} \subseteq (c(\tilde{q}^{\mathcal{B}}), \tilde{q}^{\mathcal{B}}]$, where $\tilde{q}^{\mathcal{B}} : u(\rho_n^{\mathcal{B}}; \tilde{q}^{\mathcal{B}}) = u(c(\tilde{q}^{\mathcal{B}}); \tilde{q}^{\mathcal{B}})$, then $Q_i^{\mathcal{B}}$ is a connected set.*

Proof. See the Appendix. □

Notice that, from the injectiveness of $c(\tilde{q})$, it follows that whenever informed lenders aim at serving higher quality projects, a larger fraction of the return Π must be granted to $q \in Q_i$. Again, different forms of market segmentations and/or different market shares within the same segmentation can in principle be supported at equilibrium.[16]

Given the complexity of the players' interactions in place, we do not attempt to characterize existence and/or uniqueness of the model's equilibrium under full (surplus) redistribution. Rather, we next provide a number of key insights into the qualitative properties of market outcomes, by stating the following

[13] For given $q \in Q_i$, these profits amount to $(q - c(\tilde{q}))\Pi - R_i$. When evaluated at $q = \tilde{q}$, they reduce to $\rho_n \cdot \tilde{q} - R_I$, and hence increase with $\tilde{q}$.

[14] Let $I_{\rho_n} = \{(c, \tilde{q}) | (\tilde{q} - c)\Pi > R_i\}$ denote the set of contracts that entrust informed lenders with strictly positive profits on project $\tilde{q} \leq 1$, when their uninformed competitors price at ρ_n. When $\rho_n < R_I$, it holds $\tilde{q} - c = \frac{\rho_n}{\Pi}\tilde{q} < \frac{R_i}{\Pi}\tilde{q} \leq \frac{R_i}{\Pi}$, and I_{ρ_n} is empty.

[15] This follows readily from the equivalence between maximizing the measure $\mu(Q_i)$ under full redistribution, and maximizing expected revenues over Q_i.

[16] By contrast, if $Q_i \supset (c(\tilde{q}), \tilde{q}]$, then this set is no further restricted, and different pools of projects may exist over which a given level of expected surplus can be redistributed. A form of indeterminacy then arises: for any budget-balancing $c(\tilde{q})$, there may exist more than one Q_i compatible with the maximal measure objective.

Proposition 3. *For any given* Γ*:*

(i) There exists no equilibrium in which $Q_n^{\mathcal{B}} = [\tilde{q}^{\mathcal{B}}, 1]$*;*

(ii) if an equilibrium exists with $\tilde{q}^{\mathcal{B}} < 1$*, then* $Q_n^{\mathcal{B}}$ *is disconnected, i.e.*

$$Q_n^{\mathcal{B}} = \left[\underline{q}^{\mathcal{B}}, q(c^{\mathcal{B}})\right) \cup \left(\tilde{q}^{\mathcal{B}}, 1\right]$$

where $\underline{q}^{\mathcal{B}}$ *is the marginal entrepreneur for whom* $u(\rho_n^{\mathcal{B}}; \underline{q}^{\mathcal{B}}) = u_0$ *and* $q(c^{\mathcal{B}}) := \min\left(Q_i^{\mathcal{B}}\right)$*;*

(iii) let $\mathcal{B}(\Gamma)$ *and* $\hat{\mathcal{B}}(\Gamma)$ *denote any two distinct equilibria. Then* $c^{\mathcal{B}} \neq \hat{c}^{\mathcal{B}}$ *only if* $\rho_n^{\mathcal{B}} \neq \hat{\rho}_n^{\mathcal{B}}$*.*

Proof. See the Appendix. □

Part (i) states that, in the presence of redistributive concerns, no vertical segmentation of the credit market can emerge in which the uninformed lenders attract high-quality projects with informed ones covering the residual. Put simply, redistribution across entrepreneurs endowed with low-quality projects is too costly for the informed, for any cost gap R_i/R_n. Conversely, if a competitive equilibrium exists where the best entrepreneurs – the 'peaches' – accept the pooling contract offered by the uninformed, then low-quality entrepreneurs – 'lemons' – will follow suit (part (ii)). A fundamental *disconnection* in the market configuration then emerges, where informed lenders serve and redistribute over projects of medial quality.

The potential for multiple equilibria in the presence of redistributive concerns lies on the fact that a varying amount of redistribution from high- to low quality projects induced by uniform pricing from their competitors alters the amount of redistribution that is feasible for the informed and yet does not violate their balanced budget requirement. As a consequence, different surplus sharing rules consistent with equal treatment are in principle enforceable in equilibrium. In this respect, the last part of Proposition 3 crucially points out that, for any price charged by the uninformed, there exists at most one sharing rule which is measure-maximizing when full redistribution is enforced.

Whatever the market segmentation induced at equilibrium, any equilibrium with redistribution still fails to restore market efficiency. However, the performance of the credit market is improved as aggregate overlending shrinks relative to the benchmark outcome $\mathcal{N}(\Gamma)$. Again, the higher price of the uninformed lenders' contract – that is required to break even when informed lenders capture the top tail of the quality redistribution – repulse marginal borrowers, inducing them to exploit their outside option. Formally:

Corollary 4. *For a given* Γ*:*

(i) *In any equilibrium* $\mathcal{B}(\Gamma)$*, the least quality project served* $\underline{q}^{\mathcal{B}}$ *lies strictly below* q^**;*

(ii) *In any equilibrium* $\mathcal{B}(\Gamma)$*, it holds* $\underline{q}^{\mathcal{B}} \geq \underline{q}^{\mathcal{N}}$*, where the inequality is strict if* $\mu(Q_j) > 0$ *for* $j \in \{i, n\}$*.*

Proof. See the Appendix. □

7 Policy considerations

7.1 Usury laws

As in early papers on asymmetric information in credit markets (e.g. Stiglitz and Weiss, 1981; Mankiw, 1986; De Meza and Webb, 1987), changes in the interest rate (of uninformed lenders) alter the average quality (and/or the riskiness) of the pool of served borrowers. Our analysis shows that, for a given distribution of quality F, the price charged by uninformed lenders (i) determines the lower bound of the aggregate pool of entrepreneurs receiving credit in equilibrium, and (ii) acts as screening device by driving lower quality projects out of the market. As a consequence, a usury law (interest rate ceiling) either is ineffective or can have highly disruptive effects.

Consider first the benchmark equilibrium $\mathcal{N}(\Gamma)$. The equilibrium price $\rho_n^{\mathcal{N}}$ is uniquely determined by the cost R_n and the properties of F. For any interest rate ceiling lying below $\rho_n^{\mathcal{N}}$, uninformed lenders can never break even, and the market collapses.

By the same token, when both types of lenders coexist in equilbrium, a usury law would either cause the market to disappear or prevent existence of high-price equilibria, i.e. those in which $\rho_n^{\mathcal{B}}$ is relatively high and overlending is relatively limited. If anything, such a policy intervention proves efficiency-reducing.

7.2 Refinancing rate

In this subsection we investigate the effects of altering the interest rate on banks refinancing operations. Set by the monetary authority (e.g. the Governing Council of the ECB) to provide liquidity to the banking system, the refinancing rate can be broadly interpreted as the cost of funding for the banks.

Let us denote with R such cost of funding for each bank in the system, and with α the (indivisible) information cost. When α is borne, lenders gain perfect information about projects. Consistent with our previous notation, we can write $R = R_n$ and $R_i = R + \alpha$.

Consider first the benchmark equilibrium. We know that, for a given Γ, the equilibrium price $\rho_n^{\mathcal{N}}$ always exists and is an interior fixed point of the following

$$\frac{R}{\rho_n^{\mathcal{N}}} = E_F\left[q|q \geq \frac{u_0}{\Pi - \rho_n^{\mathcal{N}}}\right] \tag{14}$$

An increase in R shifts upward the graph of the LHS of (17), hence the fixed point $\rho_n^{\mathcal{N}}$ moves to the right. To prove it analytically, we use the implicit function theorem to define the function $\rho_n^{\mathcal{N}}(R)$ and obtain

$$\frac{d\rho_n^{\mathcal{N}}}{dR} = -\frac{-\frac{1}{\rho_n^{\mathcal{N}}}}{\frac{dE_F\left[q|q\geq\frac{u_0}{\Pi-\rho_n^{\mathcal{N}}(R)}\right]}{d\rho_n^{\mathcal{N}}} + \frac{R}{(\rho_n^{\mathcal{N}})^2}} = \frac{\rho_n^{\mathcal{N}}}{\frac{dE_F\left[q|q\geq\frac{u_0}{\Pi-\rho_n^{\mathcal{N}}(R)}\right]}{d\rho_n^{\mathcal{N}}}(\rho_n^{\mathcal{N}})^2 + R} > 0$$

As a consequence, the lower bound of $Q_n^{\mathcal{N}}$ – i.e. $\underline{q}^{\mathcal{N}}$ – is increasing in R. For any given distribution of quality F, increasing the refinancing rate tightens the measure of served entrepreneurs yet improves on the allocation of credit in terms of market efficiency.

Consider now a credit market in which both uninformed and informed lenders act as profit-maximizers. A sufficient condition to have both types of lenders active in the credit market is (Proposition 2)

$$E_F\left[q|q \geq \frac{u_0}{\Pi - \rho_n^{\mathcal{N}}(R)}\right] \leq \frac{R}{R+\alpha} \tag{15}$$

The LHS is a strictly increasing function of R, whose curvature depends on the properties of F, whereas the RHS is strictly increasing and concave in R. For $R \to 0$ we have that

$$\frac{0}{0+\alpha} = 0 < E_F[0,1] = E_F\left[q|q \geq \frac{u_0}{\Pi - \rho_n^{\mathcal{N}}(0)}\right] \tag{16}$$

For R close to zero this inequality holds by continuity for given Γ. Hence the inequality (15) is violated for very low costs of funding, reducing the incentive for lenders to acquire information at cost α prior to entering the market. Notice also that for $R > \Pi - u_0 - \alpha$, the lenders who would choose to become informed are bound to make negative profits and hence never opt for information acquisition. Thus, for sufficiently low costs of funding, there exists a trade-off between enlarging the measure of served projects and favoring the biodiversity of lending institutions in the credit market.

8 Concluding remarks

Historical examinations of financial markets in advanced economies have provided ample evidence on the coexistence of different banking models, some of which specialize in information-intensive business practices. This paper studies the operation of credit markets in which informed banks (e.g. credit cooperatives) compete vis-à-vis uninformed ones (e.g. universal banks) for heterogeneous quality projects. We explore how the business model underlying informed lending – profit-maximization vs. inter-member surplus redistribution – shapes relative comparative advantages and affects market efficiency. Against a benchmark where unobservable quality always results in a unique equilibrium with overlending, we show that a variety of (symmetric) equilibria and market segmentations may emerge with competing AILs under either of the two behavioral patterns. When informed lenders act as profit maximizers, multiple equilibria with positive (non-zero measure) market shares for both types of lenders can arise. However, a unique form of vertical segmentation obtains in equilibrium, in which informed lenders cover the upper tail of the quality distribution. By contrast, when a redistribution concern shapes informed lenders' decision-making, a fundamental disconnection in the allocation of credit emerges: 'peaches' and 'lemons' are attracted by uninformed lenders, with the medial quality segment being served by informed ones. Remarkably, whatever the underlying behavioral model, the operation of informed lenders is found unable to overcome market failures, though overlending never proves exacerbated.

When the distribution of characteristics in the population of entrepreneurs support the existence of multiple equilibria, credit markets reduce inefficient credit provision relative to a world in which all lenders are equally uninformed. This discipline effect of informed lending however comes with the risk of inducing fragility in the credit market, as modest changes in the business environment or other fundamentals (e.g. lenders' cost of capital) can in fact produce large shifts in the allocation of credit leading to either highly selective markets (high-price equilibria), or ones which over-fund ventures of the lowest quality (low-price equilibria). Remarkably, in the presence of AILs, severe market failures may result from small shocks — such as a shift in the distribution of investment prospects — which would otherwise prove efficiency-enhancing.

The model also delivers some policy implications. Since Stiglitz and Weiss (1981), usury laws – i.e. interest rate ceilings – have been proposed to correct credit market inefficiencies. Our analysis corroborates the view that such a policy intervention is likely harmful in the presence of overlending.

In markets with AILs, by contrast, the adverse effect of usury laws is distribution-specific: conditional on the underlying distribution of projects it can either cause the collapse of the whole market, or prevent less inefficient (high-price) outcomes from arising if equilibrium multiplicity occurs.

From a regulatory perspective, our results suggest that lending practices relying on the extensive use of soft information – as mostly occur in credit cooperatives because of their peculiar governance structure and mission – should be carefully contemplated by the sector as well as prudential regulation authorities, given the efficiency gains possibly stemming from their operation. Given the resulting lower risk exposure, relative to the benchmark scenario, the rationale for "one size fits all" regulation of the banking industry – such as minimum capital requirements – is called into question.

Appendix

Proof of Proposition 1

(i) Let E_F and $\mu(X) = \int_X dF(q)$ denote the conditional expectation operator and the measure associated with F for some set $X \subseteq Q$, respectively. We start noticing that the first line of equation (9) is equivalent to requiring the couple $(\rho_n^{\mathcal{N}}, Q_n^{\mathcal{N}})$ to fulfill $\rho_n^{\mathcal{N}} \cdot E_F\left[q | q \in Q_n^{\mathcal{N}}\right] = R_n$. Hence, for a non-empty $Q_n^{\mathcal{N}}$ to be part of a competitive equilibrium, it must be that $\rho_n^{\mathcal{N}} \in [R_n, \Pi - u_0]$ and $Q_n^{\mathcal{N}} = [\underline{q}^{\mathcal{N}}, 1]$, where $\underline{q}^{\mathcal{N}} = \frac{u_0}{\Pi - \rho_n^{\mathcal{N}}}$ satisfies the entrepreneurs' incentive-compatibility constraint with equality. For a given Γ, in any competitive equilibrium with $\mu(Q_n^{\mathcal{N}}) > 0$, $\rho_n^{\mathcal{N}}$ must be an interior fixed point of the following

$$\underbrace{\frac{R_n}{\rho_n}}_{g_1(\rho_n)} = \underbrace{E_F[q | q \geq \frac{u_0}{\Pi - \rho_n}]}_{g_2(\rho_n)} \tag{17}$$

with $g_1'(\cdot) < 0 < g_2'(\cdot)$ on $[R_n, \Pi - u_0]$. Since

$$1 = g_1(R_n) > g_2(R_n) = E_F[q | q \geq \frac{u_0}{\Pi - R_n}]$$

and

$$g_1(\Pi - u_0) < g_2(\Pi - u_0) = 1$$

by continuity the assertion follows.

(ii) Since $u_0 = \Pi q^* - R_n$, from (17) one obtains

$$\underline{q}^N - q^* = \frac{R_n}{\Pi}\left(\frac{\underline{q}^N}{E_F[q | q \geq \underline{q}^N]} - 1\right) < 0$$

that is $\underline{q}^N \notin Q^*$.

Proof of Corollary 1

We consider a first-order stochastic dominance change in the distribution of quality, i.e. $H(q) \leq F(q)$ for all $q \in Q$. Since

$$E_D[Q_n] = \int_{\frac{u_0}{\Pi - \rho_n}}^{1} (1 - D)\, dq - \frac{u_0}{\Pi - \rho_n} \cdot D\left(\frac{u_0}{\Pi - \rho_n}\right), \quad D \in \{F, H\}$$

we have $E_H[Q_n] > E_F[Q_n]$ for all $\rho_n \in [R_n, \Pi - u_0)$, whence the assertion.

Proof of Lemma 1

(i) From definition 2, existence of a competitive equilibrium with both types of lenders is equivalent to the following system admitting a (possibly non-unique) solution $\rho_n^{\mathcal{B}}$

$$\mathbb{S}(\rho_n^{\mathcal{B}}) = \begin{cases} \underline{q}^{\mathcal{B}} = \frac{u_0}{\Pi - \rho_n^{\mathcal{B}}} \\ \tilde{q}^{\mathcal{B}} = \frac{R_i}{\rho_n^{\mathcal{B}}} \\ \underline{q}^{\mathcal{B}} < \tilde{q}^{\mathcal{B}} < 1 \\ E_F[\, q \,|q \in [\underline{q}^{\mathcal{B}}, \tilde{q}^{\mathcal{B}})\,] = \frac{R_n}{\rho_n^{\mathcal{B}}} \end{cases}$$

Notice that $\underline{q}^{\mathcal{B}} < \tilde{q}^{\mathcal{B}} < 1$ obtains if and only if $R_i < \rho_n^{\mathcal{B}} < \beta R_i$, where $\beta := \frac{R_i + u_0}{\Pi}$. Moreover, since $\min(Q_i^{\mathcal{B}}) \geq \beta^{-1}$ and $\rho_i^{\mathcal{B}}(\beta^{-1}) = \beta R_i$, we have $R_i \leq \rho_i^{\mathcal{B}}(q) \leq \beta R_i$ for all $q \in Q_i^{\mathcal{B}}$.

(ii) Given the monotonicity properties of equilibrium pricing rules, if an $\tilde{q}$ exists such that $\rho_i^{\mathcal{B}}(\tilde{q}) = \rho_n^{\mathcal{B}}$, then it is unique. Suppose there is no such a marginal project. That is, suppose the intersection between the closures of $Q_i^{\mathcal{B}}$ and $Q_n^{\mathcal{B}}$ is empty. Then at least part of these projects will find it convenient to accept the contract offered by uninformed lenders, which cannot prevent them from doing so because of imperfect information. This apparently causes a contraction in the price $\rho_n^{\mathcal{B}}$ defined above, contradicting the fact that the latter was part of a competitive equilibrium.

Proof of Proposition 2

(i) Consider a uniform price offered by uninformed lenders such that $\rho_n = \beta R_i$, then $Q_n = \left\{\beta^{-1}\right\}$, and type n lenders earn strictly positive expected profits. Hence, if $E[\pi_n(\rho_n, Q_n)] < 0$ when $\rho_n = R_i$, by continuity of the (expected) profit function (5) there will exist at least one couple (ρ'_n, Q'_n) – or an odd number thereof, by virtue of the sign permanence theorem – satisfying $\rho'_n < \beta R_i$ and $Q'_n = [\frac{u_0}{\Pi - \rho'_n}, \frac{R_i}{\rho'_n}]$, such that both $\mu(Q_n)$ and $\mu(Q_i)$ are strictly positive. If, by contrast, $E[\pi_n(\rho_n, Q_n)] > 0$ when $\rho_n = R_i$, by the same continuity argument either multiple (even-numbered) or no competitive equilibria with $\mu(Q_j) > 0$, $j \in \{i, n\}$, exist for $\rho_n \in (R_i, \beta R_i)$. Also, since $E_F[q|q \in Q_n]$ is monotonically increasing in $\underline{q}$ when $\rho_n \leq R_I$ – as this price implies no entry by informed lenders – whereas $\underline{q}$ is monotonically increasing in $\rho_n \in (0, R_i)$, there certainly exists a price-set couple (ρ''_n, Q''_n) with $\rho''_n < R_i$ and non-degenerate $Q''_n = [\frac{u_0}{\Pi - \rho''_n}, 1]$ satisfying the (expected) zero-profit condition and hence replicating the benchmark outcome, where $\mu(Q_n) > 0$.

(ii) Follows readily from the proof of part (i);

(iii) Building on the proof of Lemma 1, $\rho_n^{\mathcal{B}} \in (R_i, \beta R_i)$ must be a fixed-point of the mapping

$$\underbrace{E_F\left[q \middle| q \in \left[\frac{u_0}{\Pi - \rho_n}, \frac{R_i}{\rho_n}\right)\right]}_{k_1(\rho_n)} = \underbrace{\frac{R_n}{\rho_n}}_{k_2(\rho_n)} \tag{18}$$

While the mapping $k_2(\cdot)$ is ever decreasing, the mapping $k_1(\cdot)$ depends on the conditional average quality of entrepreneurs who would accept the type n lenders' contract at the prevailing price ρ_n, and is in principle non-monotonic. Notice that $k_1(\cdot) \to E_F\left[q|q \in [u_0/(\Pi - R_i), 1]\right]$ and $k_2(\cdot) \to R_n/R_i$ when $\rho_n \to R_i$, while $k_1(\cdot) \to \beta^{-1}$ and $k_2(\cdot) \to \beta^{-1}R_n/R_i$ when $\rho_n \to \beta R_i$. Hence a $\rho_n^{\mathcal{B}} \in (R_i, \beta R_i)$ solving (18) exists if $k_1(R_i) < k_2(R_i)$, which is condition (10).

Proof of Corollary 2

Follows readily from the proof of Proposition 2, part i).

Proof of Corollary 3

(i) Notice that in any competitive equilibrium it holds $\tilde{q}^{\mathcal{B}} \in (\beta^{-1}, 1) \subset Q^*$. Let $\underline{q}^{\mathcal{B}}$ be part of $\mathcal{B}(\Gamma)$. Since[17]

$$\underline{q}^{\mathcal{B}} < q^* \quad \Leftrightarrow \quad E_F[q|q \in [\underline{q}^{\mathcal{B}}, \tilde{q}^{\mathcal{B}})\,] \in \operatorname{int} Q^*$$

it follows that $\underline{q}^{\mathcal{B}} > q^*$ – from which $E_F[q|q \in [\underline{q}^{\mathcal{B}}, \tilde{q}^{\mathcal{B}})\,] > q^*$ – leads to a contradiction;

(ii) The assertion is trivially true when $\rho_n^{\mathcal{B}} \leq R_i$. Consider now the case $\rho_n^{\mathcal{B}} > R_i$, and assume $\rho_n^{\mathcal{B}} \leq \rho_n^{\mathcal{N}}$, which is equivalent to assuming $\underline{q}^{\mathcal{B}} \leq \underline{q}^{\mathcal{N}}$. By the equilibrium conditions, we have

$$\rho_n^{\mathcal{B}} \cdot E_F[q\big|q \in [\underline{q}^{\mathcal{B}}, \tilde{q}^{\mathcal{B}}]] = R_n = \rho_n^{\mathcal{N}} \cdot E_F[q\big|q \in [\underline{q}^{\mathcal{N}}, 1]]$$

from which $E_F[q\big|q \in [\underline{q}^{\mathcal{B}}, \tilde{q}^{\mathcal{B}}]] \geq E_F[q\big|q \in [\underline{q}^{\mathcal{N}}, 1]]$. But since

$$E_F[q\big|q \in [\underline{q}^{\mathcal{B}}, \tilde{q}^{\mathcal{B}}]] < E_F[q\big|q \in [\underline{q}^{\mathcal{B}}, 1]] \leq E_F[q\big|q \in [\underline{q}^{\mathcal{N}}, 1]]$$

[17] This equivalence readily follows from the definition of q^* and the equilibrium value of $\underline{q}^{\mathcal{B}}$ as defined in $\mathbb{S}(\rho_n)$.

we obtain a contradiction. Hence, it must be $\underline{q}^{\mathcal{B}} > \underline{q}^{\mathcal{N}}$.

Proof of Lemma 2

Here we show that the equilibrium set $Q_i^{\mathcal{B}}$ is a convex set. Suppose not, then it must be the case that $Q_i^{\mathcal{B}} = Q_i' \cup Q_i''$ where:

$$cl(Q_i') \cap cl(Q_i'') = \emptyset \tag{19}$$

$$\int_{Q_i'} [\rho_i(q)q - R_i]\, dF(q) > 0 \tag{20}$$

$$\int_{Q_i''} [\rho_i(q)q - R_i]\, dF(q) < 0 \tag{21}$$

Then there exists an Q_i''' such that $\mu\left(Q_i'''\right) = \mu\left(Q_i''\right)$ and:

$$cl(Q_i''') \cap cl(Q_i') = \{q\} \tag{22}$$

$$\int_{Q_i' \cup Q_I'''} [\rho_i(q)q - R_i]\, dF(q) > 0 \tag{23}$$

This in turn implies that there exists a nonzero measure set Q_I° such that

$$\int_{Q_i' \cup Q_i''' \cup Q_i^{\circ}} [\rho_i(q)q - R_I]\, dF(q) = 0 \tag{24}$$

$$\mu\left(Q_i''' \cup Q_i^{\circ}\right) > \mu\left(Q_i''\right) \tag{25}$$

which contradicts the fact that $Q_i^{\mathcal{B}}$ is measure-maximizing. Hence, $Q_i^{\mathcal{B}}$ must be convex.

Proof of Proposition 3

(i) Assume there exists such an equilibrium. To meet the full redistribution constraint, it must be the case that $\tilde{q}^{\mathcal{B}}$ satisfies

$$\Pi\left(\tilde{q}^{\mathcal{B}} - c^{\mathcal{B}}\right) > R_i \tag{26}$$

which is equivalent to having

$$\Pi\tilde{q}^{\mathcal{B}} - (\Pi - \rho_n^{\mathcal{B}})\tilde{q}^{\mathcal{B}} = \rho_n^{\mathcal{B}} \cdot \tilde{q}^{\mathcal{B}} > R_i \tag{27}$$

Given the (expected) zero-profit condition on the side of uninformed lenders – i,e. $\rho_n^{\mathcal{B}} \cdot E_F[q|q \geq \tilde{q}^{\mathcal{B}}] = R_n$ – one has

$$R_n > \rho_n^{\mathcal{B}} \cdot \tilde{q}^{\mathcal{B}} > R_i \tag{28}$$

i.e. a contradiction.

(ii) When $\tilde{q}^{\mathcal{B}} < 1$, then $(\tilde{q}^{\mathcal{B}}, 1] \subset Q_n^{\mathcal{B}}$. Given uniform pricing by uninformed lenders, we have

$$\rho_n^{\mathcal{B}} \cdot E_F[Q_n^{\mathcal{B}}] < \Pi(\tilde{q}^{\mathcal{B}} - c^{\mathcal{B}}) \Leftrightarrow E_F[Q_n^{\mathcal{B}}] < \tilde{q}^{\mathcal{B}}$$

hence there exists a non-zero measure subset of projects $P(q)$ such that $P(q) \subset Q_n^{\mathcal{B}}$ and $q < E_F[Q_n^{\mathcal{B}}]$ for all $q \in P(q)$.

(iii) We show that $\rho_n^{\mathcal{B}} = \hat{\rho}_n^{\mathcal{B}}$ implies $c^{\mathcal{B}} = \hat{c}^{\mathcal{B}}$. This is trivial for any equilibrium with vertical segmentation in which $Q_i^{\mathcal{B}} = [\tilde{q}^{\mathcal{B}}, 1]$ and $Q_n^{\mathcal{B}} = [\underline{q}^{\mathcal{B}}, \tilde{q}^{\mathcal{B}})$. Consider now equilibrium configurations in which $Q_i^{\mathcal{B}} = [q(c^{\mathcal{B}}), \tilde{q}^{\mathcal{B}}]$ and $Q_n^{\mathcal{B}} = \left[\underline{q}^{\mathcal{B}}, (q(c^{\mathcal{B}}, \tilde{q}^{\mathcal{B}})\right) \cup \left(\tilde{q}^{\mathcal{B}}, 1\right]$, where the pair $\left(q(c^{\mathcal{B}}), \tilde{q}^{\mathcal{B}}\right)$ is budget-balancing for the informed, i.e. it solves

$$\int_{q(c^{\mathcal{B}})}^{\tilde{q}^{\mathcal{B}}} \left[\Pi(q - c^{\mathcal{B}}) - R_i\right] dF(q) = 0$$

By contradiction, assume that $\rho_n^{\mathcal{B}} = \hat{\rho}_n^{\mathcal{B}}$ when $\hat{c}^{\mathcal{B}} \neq c^{\mathcal{B}}$ – with no loss of generality, let $\hat{c}^{\mathcal{B}} < c^{\mathcal{B}}$. Then it must be the case that $E_F[q|q \in Q_n^{\mathcal{B}}] = E_F[q|q \in \hat{Q}_n^{\mathcal{B}}]$ and $\mu(Q_i^{\mathcal{B}}) = \mu(\hat{Q}_i^{\mathcal{B}})$. Since $\tilde{q}^{\mathcal{B}}(\hat{c}^{\mathcal{B}}) < \tilde{q}^{\mathcal{B}}(c^{\mathcal{B}})$, to preserve the average quality of the pool accepting the contract offered by the uninformed it needs $E_F[q|q \in [\underline{q}^{\mathcal{B}}, q(\hat{c}^{\mathcal{B}})] > E_F[q|q \in [\underline{q}^{\mathcal{B}}, q(c^{\mathcal{B}})]$. This requires $q(\hat{c}^{\mathcal{B}}) > q(c^{\mathcal{B}})$ – as $\underline{q}^{\mathcal{B}}$ is unaltered insofar as $\rho_n^{\mathcal{B}} = \hat{\rho}_N^{\mathcal{B}}$ – yet this contradicts $\mu(Q_i^{\mathcal{B}}) = \mu(\hat{Q}_i^{\mathcal{B}})$. Hence, $\rho_n^{\mathcal{B}} = \hat{\rho}_n^{\mathcal{B}}$ implies $c^{\mathcal{B}} = \hat{c}^{\mathcal{B}}$. From the constraints $\Pi c^{\mathcal{B}} = (\Pi - \rho_n^{\mathcal{B}})\tilde{q}^{\mathcal{B}}$ and $(\Pi - \rho_N^{\mathcal{B}})\underline{q}^{\mathcal{B}} = u_0$ it readily follows that $\rho_n^{\mathcal{B}} = \hat{\rho}_n^{\mathcal{B}}$ also implies $\tilde{q}^{\mathcal{B}} = \hat{\tilde{q}}^{\mathcal{B}}$. This in turn results in a unique budget-balancing $q(c^{\mathcal{B}})$ associated to $c^{\mathcal{B}}$.

Proof of Corollary 3

Recall that in any competitive equilibrium, and irrespective of the associated market segmentation, it holds $\underline{q}^{\mathcal{B}} = \min\left(Q_n^{\mathcal{B}} \cup Q_i^{\mathcal{B}}\right)$.

(i) Assume $\underline{q}^{\mathcal{B}} > q^*$. Then:

$$\frac{u_0}{\Pi - \rho_n^{\mathcal{B}}} > \frac{u_0 + R_n}{\Pi} \quad \Leftrightarrow \quad \rho_n^{\mathcal{B}} > \frac{\Pi R_n}{u_0 + R_n}$$

Using the (expected) zero-profit condition of the informed delivers

$$\frac{\Pi R_n}{u_0 + R_n} \cdot E_F[Q_n^{\mathcal{B}}] < R_n \quad \Leftrightarrow \quad E_F[Q_n^{\mathcal{B}}] < q^*$$

i.e. a contradiction.

(ii) For any competitive equilibrium with vertical segmentation – informed at the top, uninformed in the middle – the proof is analogous to that of Corollary 2. To show the assertion for equilibria with disconnection, we prove by contradiction that ρ_n cannot be part of a such equilibrium if $\rho_n \leq \rho_n^{\mathcal{N}}$, i.e. if $E_F[q|q \in [\underline{q}^{\mathcal{N}}, 1]] \leq \frac{R_n}{\rho_n}$. Assume the opposite, i.e. assume there exists $\rho_n^{\mathcal{B}} \leq \rho_n^{\mathcal{N}}$. Then there must exist $\tilde{q}^{\mathcal{B}} \in [\frac{R_i}{\rho_n^{\mathcal{B}}}, 1)$ and $\underline{q}^{\mathcal{B}} \leq \underline{q}^{\mathcal{N}}$ such that

$$E_F[q|q \in [\underline{q}^{\mathcal{B}}, q(c^{\mathcal{B}})) \cup (\tilde{q}^{\mathcal{B}}, 1]] = \frac{R_n}{\rho_n^{\mathcal{B}}}$$

and

$$E_F[q|q \in [q(c^{\mathcal{B}}), \tilde{q}^{\mathcal{B}}]] = \frac{\Pi - \rho_n^{\mathcal{B}}}{\Pi} \tilde{q}^{\mathcal{B}} + \frac{R_i}{\Pi}$$

with the left-hand side conditional average in the latter equation being increasing in $\tilde{q}^{\mathcal{B}}$ and satisfying:

$$E_F[q|q \in [q(c^{\mathcal{B}}), \tilde{q}^{\mathcal{B}}]] < E_F[q|q \in [\underline{q}^{\mathcal{B}}, 1]] < E_F[q|q \in [\underline{q}^{\mathcal{N}}, 1]]$$

Since for $\tilde{q}^{\mathcal{B}} = \frac{R_i}{\rho_n^{\mathcal{B}}}$ it holds

$$E_F[q|q \in [\underline{q}^{\mathcal{B}}, 1] > E_F[q|q \in [\underline{q}^{\mathcal{B}}, \tilde{q}^{\mathcal{B}}]] = \frac{R_i}{\rho_n^{\mathcal{B}}} > \frac{R_n}{\rho_n^{\mathcal{B}}} \geq E_F[q|q \in [\underline{q}^{\mathcal{N}}, 1]]$$

we have a contradiction.

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