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Multi-part Tariffs and Differentiated Commodity Taxation

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Abstract
We study commodity taxation in markets where firms, such as Internet Service Providers, energy suppliers and payment card platforms, adopt multi-part tariffs. We show that ad valorem taxes can correct underprovision and hence increase welfare, provided the government applies differentiated tax rates to the usage and access parts of the tariff. We obtain this result in different settings, including vertically interlinked markets, markets where firms adopt menus of tariffs to screen consumers and where they compete with multi-part tariffs. Our results suggest that exempting these markets from taxation may be inefficient.

JEL Classification: D42, D61, H21

Keywords: Commodity taxation, multi-part tariffs, price discrimination

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Table of contents

1 Introduction

Multi-part tariffs are common among telephone and Internet connection providers, energy distributors (electricity and gas), payment card platforms and parking operators. These firms often charge consumers a fee for access in addition to a payment that depends on the amount or duration of usage. These markets are generally characterized by significant concentration on the supply side, which is likely to result in underprovision.[1] Governments often apply indirect taxation (e.g., VAT and excise taxes) to the above industries and, given their importance in modern economies, the question arises of whether and how to design taxes without seriously distorting provision and reducing growth. As we argue shortly below, this question is part of an ongoing policy debate regarding the reform of indirect taxes applying to essential services. However, quite surprisingly, existing research has devoted little attention to the design of taxation in presence of multi-part tariffs.

Motivated by the above considerations, we study taxation of goods and services when providers charge multi-part tariffs. We explore a relevant dimension along which taxes can be designed to reduce their distortionary impact: the parts of the tariff to which they apply. This dimension has so far been ignored by the literature, although there are several examples of such differentiation in reality.[2] We show that by applying different ad valorem tax rates to each part of these tariffs, the government can correct underprovision, and hence increase welfare, with a *positive* ad valorem tax on usage or access.

Our findings hinge on a simple observation. In markets where suppliers adopt linear prices, the latter are the result of a trade-off between the revenue gain from inframarginal units and the net loss from marginal ones. In equilibrium, therefore, suppliers operate on the elastic part of the demand curve. Thus, if the government introduces a commodity tax (either unit or ad valorem), the suppliers' optimal response is to reduce provision. Consequently, if the good or service is underprovided (as it is usually the case with imperfect competition), taxation aggravates the distortion (Auerbach and Hines, 2002). However, when the suppliers adopt multi-part tariffs, the trade-off governing their choice of prices is different. Typically,

[1] For example, the main U.S. cable operators hold de facto monopolies for high-speed services in several local markets. The Federal Communications Commission (FCC) reported that about 20% of households have access to a single broadband provider for a service of up to 4Mbits/s. This share rises to 30% and 55% for speeds up to 10Mbit/s and 25Mbits/s, respectively (see https://www.fcc.gov/document/chairman-remarks-facts-and-future-broadband-competition).

[2] For instance, excise taxes on either access or usage exist in the telecom sector. In several U.S. states, subscribers to wireless telecommunication services pay a separate per-line tax on top of VAT and other state-level taxes. Other examples include taxes on SMS, calls, handsets and SIM cards, applied by countries such as Argentina, Brazil, Mexico, Greece, Turkey, Ukraine and Pakistan (Katz, 2015; Matheson and Petit, 2017).

suppliers design the usage fees to induce the level of usage that maximizes the net surplus from consumption, which they can capture via the access fees (Oi, 1971). As we show, this logic implies that taxation can have counterintuitive effects: ad valorem taxes can increase output, provided that different tax rates are set on the "usage" and "access" part of the tariff. Thus, differentiated ad valorem taxes can produce a double dividend, by correcting distortions due to market power while raising revenue for the government. In contrast, we do not find efficiency-enhancing effects for unit or uniform ad valorem taxes.

In Section 3, we introduce an ad-hoc model to convey the basic mechanism of our analysis. We then provide foundations to this model, exploring several settings where underprovision takes place in presence of multi-part tariffs. First, we consider a model with identical consumers and a monopolist providing access to a piece of infrastructure which is essential to consume some final goods (Section 4.1). Examples include Internet Service Providers enabling consumers to reach online content (e.g., movies, music, games, apps) and payment card systems that consumers use for purchases. Although the infrastructure supplier can recover consumer surplus via the access fee, consumers are charged a usage fee higher than marginal cost in equilibrium. As shown by Economides and Hermalin (2015), restricting consumption at the margin allows the infrastructure supplier to capture part of the surplus that would otherwise accrue to the sellers of final goods. The combination of this fee with the mark-up set by final good suppliers implies that there is underprovision. We show that the government can correct this distortion with a positive ad valorem tax on usage, as long as the marginal cost is not exceedingly large. If this condition holds, the equilibrium quantity lies on the inelastic part of consumer demand, implying that the supplier's optimal response to the tax is to decrease the usage fee and increase provision. We extend this model in Section 5 where we consider a duopoly of infrastructure providers, showing that our main results continue to hold.

Second, we consider taxation when the infrastructure provider offers a menu of tariffs to screen different consumer types (Section 4.2). We assume perfect competition in the final goods market, thereby abstracting from the distortion analyzed in the preceding setup. Therefore, this setup also applies to situations where the infrastructure is not used to reach final good providers, as in the case of parking at a garage or consuming energy for heating and operating domestic appliances. In this context, the distortion is due to the provider's seeking to reduce the information rent left to the heavy users, which implies underproviding the light users (Maskin and Riley, 1984). We find that an ad valorem tax on usage increases welfare as long as the supplier's gains from restricting the consumption of light users (to reduce the information rent) and its marginal cost are sufficiently low. When these conditions hold, the usage fees are such that consumers' demand is inelastic in equilibrium. Furthermore, taxing access can also increase efficiency in this context. Indeed, the access tax weakens the supplier's incentive to distort the consumption of light users, because the revenue collected with access fees is partly taxed away. As a result, when restricting consumption of light users has a strong effect on the information rent, the supplier responds to the access tax by reducing their usage fee and thus alleviating the underprovision.

Our findings provide useful insights for the design of fiscal instruments applied to network services. In most countries, these industries are subject to VAT, sales or excise taxes, but such levies typically apply uniformly to all parts of the tariffs. Our results suggest that such an implementation may worsen market distortions, so governments may consider adopting differentiated tax rates on separate tariff parts. More generally, our findings contribute to a lively policy debate on the restructuring of commodity taxation applying to essential services. A prime example is Internet access, which has attracted attention in light of the rapid digitization of economic activity (European Commission, 2014). The U.S. Congress recently passed the Permanent Internet Tax Freedom Act, which restricts taxation of Internet access services. One of the main arguments in favor of the ban was that taxes would discourage consumers from using Internet services, sapping their growth. In contrast, France has reportedly considered taxing Internet connections and downloaded data, to replace declining sources of revenue.[3] Our results show that, when appropriately designed, taxation need not result in reduced consumption.

2 Related literature

Our study relates to the longstanding literature that compares ad valorem to specific tax rates. These taxes are equivalent under perfect competition, but not under imperfect competition (Suits and Musgrave, 1955). Generally, the welfare dominance of either instrument depends on market conditions, such as the degree of concentration and whether one considers Cournot or Bertrand equilibrium (Delipalla and Keen, 1992; Skeath and Trandel, 1994; Anderson et al., 2001; Wang et al., 2018).[4] Recently, Peitz and Reisinger (2014) show that it is more

[3]The Permanent Internet Tax Freedom Act prohibits federal, state and local governments from taxing Internet access, although some states have retained the right to impose preexisting taxes. See https://www.congress.gov/bill/114th-congress/house-bill/235. The French government proposed in 2008 "an infinitesimal sales tax on Internet access and mobile telephony" (see http://content.time.com/time/world/article/0,8599,1702223,00.html) and, more recently, a "new tax on the use of bandwidth by large operators".

[4]See Auerbach and Hines (2002) for an overview of commodity taxation in imperfectly competitive markets.

efficient to levy an ad valorem tax in the downstream than in the upstream market.[5] Wang and Wright (2017) show that ad valorem taxes allow efficient price discrimination across goods with different costs and values, unlike unit taxes. Differently from our paper, this literature focuses on firms that are restricted to linear pricing. Furthermore, the comparison between tax instruments is generally about which instrument produces the smaller distortion. Instead, in our paper, (differentiated) ad valorem taxes dominate because they reduce the distortions linked to the structure of the multi-part tariff.

Few papers on commodity taxation have considered multi-part tariffs. To our knowledge, none allows for different tax rates on different parts of the tariff, which is our key contribution.[6] Laffont (1987) studies taxation of a monopolist that discriminates consumers using non-linear tariffs. He shows that, when the government uses specific taxes, the welfare-optimal policy is a subsidy to increase production. Cheung (1998) shows that the dominance of ad valorem taxes established by Skeath and Trandel (1994) remains intact under a nonlinear pricing monopolist. Jensen and Schjelderup (2011) show that these results remain valid even if some consumers are excluded. They also find that ad valorem or specific taxation increases the usage fee for all consumers, but most likely reduces the access fee.[7]

An important finding of our paper is that taxation can increase total surplus. Efficiency-enhancing effects of taxation have been shown in other papers, although the underlying mechanisms are different from ours. Hamilton (2009) considers multi-product transactions in retail markets, finding that excise ad valorem taxes decrease equilibrium output per product in the short-run, but increase it in the long-run (with endogenous entry). Carbonnier (2014) finds that nonlinear and price-dependent tax schedules can result in lower prices and thus higher welfare. Cremer and Thisse (1994) show, in a framework with endogenous vertical product differentiation, that a small uniform ad valorem tax lowers consumer prices and increases welfare.[8]

[5] A recent literature has pointed out an analogy between the effects of prices set by upstream firms and taxation on other firms in the supply chain (see Economides and Hermalin, 2015; Johnson, 2017; Gaudin and White, 2014). We contribute to this literature by studying the effect of fiscal policy on upstream firms.

[6] Some studies have explicitly considered differentiated taxation, though not in presence of nonlinear pricing. Cremer et al. (2001) prove that differentiated commodity tax rates are a relevant policy instrument besides income taxes for optimal tax policies, contrary to the well-known finding by Atkinson and Stiglitz (1976). Cremer and Thisse (1994) show that differentiating commodity tax rates according to the quality of the product possibly increases welfare.

[7] Some authors (e.g., De Borger, 2000) have also analyzed the design of two-part tariffs by public bodies, but not the effects of taxation on firms that adopt nonlinear pricing.

[8] The result that taxation can reduce the consumer price is referred to in the tax incidence literature as "undershifting". This phenomenon is what Edgeworth (1925) called the "Taxation Paradox". Hotelling (1932) illustrates that this result can hold with imperfect competition. Recently, Agrawal and Hoyt (2019) show in a multi-product setting that undershifting can occur when products are complements and with perfectly

The paper also relates to the literature on taxation in two-sided markets. As pointed out by Rochet and Tirole (2006), in such markets "the volume of transactions [...] depends on the structure and not only on the overall level of the fees charged by the platform". Taxing one or both sides of the market (at different rates) can thus have counterintuitive effects, by affecting the structure of fees. Kind et al. (2008), show that the supply of goods provided by a two-sided platform may increase under ad valorem taxation. More recently, Bourreau et al. (2018) find that an ad valorem tax on subscriptions or on advertising may raise welfare. Belleflamme and Toulemonde (2018) show that taxation may either increase or decrease the profits of competing two-sided platforms. Tremblay (2018) distinguishes taxation at the access and the transaction level, and finds that either tax may increase welfare because of network effects. Bloch and Demange (2018) find that taxing access and data revenues at different rates is the most effective way to reduce a platform's incentive to collect data. We connect to this literature because, in our model, differentiated taxes correct distortions by altering the structure of fees as well. However, our findings do not hinge on two-sided effects.

3 A bare-bones model

In this section, we provide a stylized model of taxation in a market with multi-part tariffs, to convey the main message of our analysis. We provide foundations for this ad-hoc model in Section 4.

Consider a monopolist firm, I, providing access to a piece of infrastructure. There are Θ different types of consumers, indexed by $i = \{1, ..., \Theta\}$ and each group is of unit size. We assume I can observe the quantity consumed by each individual and is therefore able to charge a two-part tariff $A_i + p_i q$, where A_i is the access fee, p_i the usage fee charged to a type-i consumer and q the quantity consumed. Firm I could be an Internet Service Provider (ISP), a distributor of natural gas or electricity, a payment card platform or a parking operator. Accordingly, the quantity q can represent Gigabytes of data, cubic meters of gas, kilowatts of electricity, card transactions or the duration of parking, respectively.[9] The monopolist's marginal cost is $c \geq 0$.

Let $u_i(q)$ be the utility a type-i consumer derives from consuming q units, which is

competitive suppliers. The mechanism driving the results is very different than in our model.

[9]Two-part tariffs are common among energy distributors (Ito, 2014), parking operators (Inci, 2015) and payment card platforms (Bedre-Defolie and Calvano, 2013). Telecom suppliers often adopt more complex tariffs (Economides and Hermalin, 2015). However, we show in Appendices C.8 and D.1, that there is no loss of generality in restricting attention to two-part tariffs.

increasing and concave. We denote by q_i the quantities chosen (i.e., demand) by type-i consumers, given the usage fee p_i. These quantities are such that:

$$\frac{du_i}{dq} = p_i, \qquad i = 1, ..., \Theta. \tag{3.1}$$

The government can levy unit or ad valorem commodity taxes. We assume the latter can be differentiated according to the part of the tariff applied by I. Let τ be the unit tax rate, t_A the ad valorem tax rate on the access fee and t_p the ad valorem tax rate on the usage charge. The standard ad valorem tax $t_A = t_p = t$ is a special case of this tax system.[10] Hence, firm I's net of tax profit (which we assume to be concave in p_i for $i = 1...\Theta$) is

$$\pi_I \quad = \textstyle\sum_{i=1...\Theta} A_i \left(1 - t_A\right) + \left[p_i \left(1 - t_p\right) - \tau - c\right] q_i. \tag{3.2}$$

In a standard monopoly model (Oi, 1971), firm I can capture each consumer's net surplus, $u_i(q_i) - p_i q_i$, with the access fee. However, in more realistic settings there may be constraints on the monopolist's ability to extract consumer surplus. To convey the essence of our results, at this stage we model these constraints in an ad-hoc way, assuming there is an exogenous part of surplus, F_i, that I cannot extract from consumers of type i. A possible foundation for F_i is that part of the surplus may be captured by providers of complementary goods. For example, connection to an ISP enables consumers to reach online content (e.g., music, video, games, apps) and part of consumer surplus accrues to content providers (Economides and Hermalin, 2015). A second possible foundation relates to the presence of multiple consumer types: the supplier must leave an information rent to some types in order to obtain the intended self-selection (Maskin and Riley, 1984). We model these foundations for F_i in Sections 4.1 and 4.2, respectively.

Given p_i and q_i for $i = 1...\Theta$, A_i satisfies the following equality

$$A_i = u_i(q_i) - p_i q_i - F_i, \qquad i = 1, ..., \Theta. \tag{3.3}$$

We assume F_j to be a function of the usage fees, p_i, for $i, j = 1...\Theta$. Indeed, as we show in Section 4.1, the price of goods complementary to the network infrastructure, and thus the surplus extracted by the providers of those goods, decrease in I's usage fee. Furthermore, as

[10] Our choice of the tax base for t_A and t_p is based on several considerations. First, as will become clear, distortions in the market are due to the *structure* of the two-part tariff. Second, these taxes should be easy to calculate and implement for the tax administration. Third, excise taxes on either access or usage already exist in some countries for markets such as telecom, as we have argued in the Introduction (footnote 2).

we show in Section 4.2, when I proposes a menu of tariffs to screen consumers, raising the price paid at the margin by certain types makes screening more effective, because information rents decrease. Accordingly, we assume that $\frac{\partial F_j}{\partial p_i} \leq 0,\ i,j = 1..\Theta$.[11]

We define welfare as the sum of profits, consumer surplus and tax revenue, which simplifies to $\sum_{i=1...\Theta} u_i(q_i) - cq_i$. It is straightforward that the allocation maximizing welfare is such that $\frac{du_i}{dq} = c$. We denote the equilibrium usage fees (conditional on the tax rates) as p_i. Using equations (3.2) and (3.3), these fees satisfy the following first-order conditions:

$$\begin{aligned} \frac{d\pi_I}{dp_i} &= (1-t_A)\left[\left(\frac{du_i}{dq_i} - p_i\right)\frac{dq_i}{dp_i} - q_i - \sum_{j=1,...,\Theta}\frac{\partial F_j}{\partial p_i}\right] + \\ &+ (1-t_p)\left(p_i\frac{dq_i}{dp_i} + q_i\right) - (c+\tau)\frac{dq_i}{dp_i} = 0, \qquad i = 1,...,\Theta. \end{aligned} \tag{3.4}$$

In the absence of taxation ($t_A = t_p = \tau = 0$), and given (3.1) and (3.4) we obtain

$$\frac{du_i}{dq} = p_i = c + \frac{\sum_{j=1...\Theta}\frac{\partial F_j}{\partial p_i}}{\frac{dq_i}{dp_i}}, \qquad i = 1,...,\Theta, \tag{3.5}$$

This expression implies that, in equilibrium, consumers' marginal utility from consumption is weakly higher than the marginal cost c (given $\frac{\partial F_j}{\partial p_i} \leq 0,\ i,j = 1,...,\Theta$ and $\frac{dq_i}{dp_i} < 0$). As long as $\frac{\partial F_j}{\partial p_i} < 0$ for some j, raising p_i above c produces a net gain. Hence, I sets the usage fees in a way that restricts consumption with respect to the welfare-optimal level, to relax the constraint (3.3).

This market inefficiency calls for government intervention. The focus of our subsequent analysis is whether taxation can alleviate this distortion. As a first step, we consider standard instruments that the literature has concentrated on so far (see Appendix B for the proofs of the statements that follow). Consider first the effect of unit taxes. Differentiating (3.4) delivers $\frac{dp_i}{d\tau} > 0$, and, hence $\frac{dq_i}{d\tau} < 0$. The intuition is that when τ rises, the effect on firm I is the same as that of an increase in the production cost. Thus, firm I's best response is to reduce total provision, which is achieved by increasing p_i. Consider now uniform ad valorem taxation, i.e. $t_A = t_p = t$. We obtain, by differentiating (3.4), that $\frac{dp_i}{dt} \geq 0$ and therefore $\frac{dq_i}{dt} \leq 0$. The intuition is that the tax reduces the total revenue collected by the monopolist and, hence, makes provision implicitly more costly. Thus, neither unit nor uniform ad valorem taxation can alleviate the restrictions imposed by firm I, and both actually worsen the distortion if $c > 0$.

[11] For example, suppose there are no complementary goods, that $\Theta = 2$ and type 1 potentially mimics type 2. Given these assumptions, we would have $\frac{\partial F_2}{\partial p_2} = \frac{\partial F_1}{\partial p_1} = \frac{\partial F_2}{\partial p_1} = 0$ and $\frac{\partial F_1}{\partial p_2} < 0$.

In other words, conditional on relying on these standard instruments, the welfare-maximizing policy is to subsidize the provider. This is a well-known result from the commodity taxation literature (Auerbach and Hines, 2002).

We now analyze the effects of differentiated ad valorem taxes. Differentiating (3.4) with respect to the usage tax, t_p, we get

$$\frac{dp_i}{dt_p} = \frac{\left(\frac{p_i}{q_i}\frac{dq_i}{dp_i} + 1\right) q_i}{\frac{d^2\pi_I}{dp_i^2}}, \qquad i = 1, ..., \Theta. \tag{3.6}$$

Because the denominator on the right hand side of (3.6) is negative by concavity of π_I, the tax t_p reduces p_i when the numerator is positive. This condition holds if and only if the equilibrium consumption level, q_i, is on the *inelastic* part of consumer demand. To understand, consider that t_p targets the revenue collected through the usage fee, $p_i q_i$. Therefore, firm I has an incentive to change p_i in a way that reduces such revenue. If q_i is on the inelastic part of demand, the ensuing change in p_i is *negative.* Consequently, consumption by type-i individuals increases. We characterize the conditions on parameters such that $dp_i/dt_p < 0$ holds in Sections 4.1 and 4.2. However, to get a sense of when $dp_i/dt_p < 0$ can hold, consider an initial equilibrium with zero taxes. Given q_i decreases with p_i, the condition $p_i < -q_i/\frac{dq_i}{dp_i}$ holds when neither term on the right hand side of (3.5) is exceedingly large.

Observe that $dp_i/dt_p < 0$ could not hold if firm I were restricted to charging linear tariffs (i.e., $A_i = 0$). Indeed, firms with market power that charge linear prices operate on the *elastic* part of demand and, thus, typically respond to ad valorem taxes by reducing output (Auerbach and Hines, 2002).

Finally, consider the effect of t_A on p_i. Differentiating (3.4) and using (3.1), we obtain

$$\frac{dp_i}{dt_A} = \frac{\left(\frac{du_i}{dq} - p_i\right)\frac{dq_i}{dp_i} - q_i - \sum_{j=1...\Theta}\frac{\partial F_j}{\partial p_i}}{\frac{d^2\pi_I}{dp_i^2}} = \frac{-q_i - \sum_{j=1...\Theta}\frac{\partial F_j}{\partial p_i}}{\frac{d^2\pi_I}{dp_i^2}}, \qquad i = 1, ..., \Theta. \tag{3.7}$$

To understand this expression, note that t_A gives firm I an incentive to reduce its access fees. Given (3.3) holds in equilibrium, reducing such fees involves changing the usage fees. The sign of the required change depends on p_i's effect on type-i consumers' net surplus, $u_i - p_i q_i$, and the revenue constraints, $\sum_{j=1...\Theta} F_j$. Again, we postpone characterizing the conditions on parameters such that $dp_i/dt_A < 0$ to Sections 4.1 and 4.2. However, we expect that $dp_i/dt_A < 0$ when the terms $\sum_{j=1...\Theta}\frac{\partial F_j}{\partial p_i}$ are large in magnitude. Notice from (3.5) that these terms increase p_i and thus reduce q_i, all else given.

To summarize, we have illustrated that differentiated ad valorem taxation can *increase* provision and welfare in presence of multi-part tariffs. To our knowledge, this is a previously unnoticed effect of commodity taxes.

4 Foundations of the bare-bones model

In this section, we provide foundations of the ad-hoc model presented above. In particular, we characterize the term F_i introduced in expression (3.3) and its relation to the tariffs set by the infrastructure provider. In Section 4.1, we consider a setting with identical consumers, where the infrastructure is essential to consume some final goods. In that setting, F_i is linked to the rent captured by the providers of final goods when the latter have market power. In Section 4.2, we consider taxation when the supplier offers a menu of tariffs, screening different consumer types (light and heavy users). In that setting, F_i originates from the information rent left to heavy users.

4.1 A representative consumer framework

This section builds upon the model of Economides and Hermalin (2015). We consider a unit mass of identical individuals who want to consume N goods, indexed by $j = 1, ..., N$. Each good is supplied by a different monopolist, also indexed by j. The utility function is

$$U = \sum_{j=1}^{N} u(q_j) + y, \tag{4.1}$$

where q_j are units of good j and y is a numeraire good. We assume $\frac{du}{dq_j} > 0 > \frac{d^2u}{dq_j^2}$.[12] To consume these goods, individuals need to access an essential infrastructure, provided by firm I.[13] For convenience, we refer to the good provided by I as the "infrastructure" good and to the N goods as "final" goods.

As in Economides and Hermalin (2015), firm I can be thought of as an ISP that connects consumers to providers of digital content (e.g. music and video). I could also be a payment

[12] In this setup, we treat the N goods as independent. In Appendix C.2, we provide an alternative version of the model where these goods are imperfect substitutes and show that our results are not affected qualitatively.

[13] In Appendix C.3, we show that our results are robust to allowing consumers to acquire goods without using firm I's infrastructure as long as bypassing the infrastructure is costly (for instance, carrying more cash instead of using a card may imply a cost for consumers). Moreover, we consider competition between infrastructure providers in Section 5.

card platform that enables consumers to purchase goods supplied by several retailers.[14]

Given all consumers acquire access to the infrastructure good in equilibrium (see below), the profit of final good provider j reads

$$\pi_j = (x_j - \phi)\, q_j, \quad j = 1, .., N, \tag{4.2}$$

where x_j denotes the price of a unit of good j and ϕ is the marginal cost, assumed symmetric for all final good providers. We assume for convenience that such providers charge linear prices.[15]

For ease of exposition, we assume that for each unit of final good a consumer needs a fixed quantity of the infrastructure good, which we normalize to one. For example, watching a movie online entails downloading a given quantity of data (say, 1 GB). Furthermore, individuals need to make one transaction with the payment card provider per purchase. Given this assumption, consumption of $\sum_1^N q_j$ units of the final goods entails using the same quantity of the infrastructure good.

Firm I charges a two-part tariff of the form

$$T_I = A + p \sum_1^N q_j, \tag{4.3}$$

where A is the access fee and p is a per unit (usage) fee.[16]

[14] We ignore payments from final goods providers to the infrastructure provider here. In some circumstances, these payments may be limited by regulation (e.g., net neutrality rules for ISPs). However, they are common in other markets, e.g. payment card platforms that charge merchants. In Appendix C.4.1, we let the usage fee p be charged to final good providers. However, the physical incidence of p is irrelevant for the analysis, because this fee is similar to a tax on consumption of final goods by the infrastructure provider (Weyl and Fabinger, 2013). Alternatively, the infrastructure provider could charge a lump-sum access fee to the providers of final goods. We show in Appendix C.4.2 that our main results are not affected as long as final good providers have some bargaining power, so that I is unable to extract all their profit.

[15] Linear prices are natural for such final good providers as brick-and-mortar retailers. Several digital content providers charge linear prices (e.g., the movie stores on iTunes and Google Play), though some adopt nonlinear tariffs (e.g., Netflix). We show in Appendix C.5 that our results are robust to this modification, as long as at least one final good provider charges linear prices. Note that the linear formulation can also capture the case where content providers are ad-financed (Economides and Hermalin, 2015). In this interpretation, x_j can be seen as the quantity of ads per unit of content. Normalizing the advertising rate to one, $x_j q_j$ is the per-consumer ad revenue. Given that ads decrease utility, the effect on consumers is the same as that of a monetary price.

[16] In Appendix C.8, we show that firm I makes at least as much profit with a two-part tariff than with a three-part tariff of the form $T_I = A + p \cdot max\left\{0, \sum_1^N q_j - L\right\}$, where L is a consumption limit (i.e., a certain quantity of service bundled with access) and p is a per unit fee applying to all units over the limit (an "overage charge"). Such a three-part tariff encompasses most of the tariff structures encountered in reality. First, if $L > 0$ and p is finite, this tariff has a "loose" limit: the consumer can exceed the limit, but has to pay an

The individual's budget constraint reads

$$M \geq A + p\sum_{1}^{N} q_j + \sum_{1}^{N} x_j q_j + y, \tag{4.4}$$

where M is the exogenously given income. Thus, firm I earns the before-tax profits

$$\pi_I = A + (p - c)\sum_{1}^{N} q_j, \tag{4.5}$$

where $c \geq 0$ is the marginal cost of the infrastructure good.

Social welfare is defined as the sum of consumer surplus, firm profits and tax revenues, which equals total surplus:

$$W = \sum_{j=1}^{N} u(q_j) - (c + \phi) q_j + M. \tag{4.6}$$

From this expression, one obtains the socially optimal consumption levels q^* (we drop the index j because these quantities are symmetric for all final goods), such that:

$$\frac{du}{dq_j} = c + \phi, \quad j = 1, .., N. \tag{4.7}$$

That is, the socially optimal quantities are such that marginal utility equals the sum of marginal costs of provision.

We have shown in Section 3 that standard tax instruments (i.e. a uniform ad valorem tax or a unit tax on the infrastructure good) cannot increase welfare.[17] Because this result naturally extends to the current setting as well, we focus here on differentiated ad valorem tax rates. That is, we allow for a "usage" tax rate t_p that applies to $p\sum_1^N q_j$, and an "access" tax rate t_A, which applies to the access payment A, where $t_k \in [-1, 1]$, $k = A, p$.

We assume the following timing of moves. First, the government sets t_A and t_p. Then, the infrastructure provider sets its tariff. Next, consumers decide whether to acquire access. Thereafter, the final good providers simultaneously decide on their prices, x_j. Finally, consumers choose q_j.[18]

overage charge per unit. Second, if $L > 0$ and $p = \infty$, the tariff has a "strict" limit, which cannot be exceeded. Obviously, when $L = 0$, the tariff has only two parts. See, e.g., arstechnica.com for examples of tariffs set by residential ISPs in the US that fit this description (http://tiny.cc/xp9h9y).

[17]In Appendix C.6, we also analyze taxes on final goods, and show that such taxes cannot increase welfare.

[18]We consider an alternative timing, with sellers of final goods moving before I, in Appendix C.1. Changing

We solve the model by backward induction, starting from the consumer's problem. Given the budget constraint binds and using (4.3), utility is

$$U = \begin{cases} \sum_1^N \left[u\left(q_j\right) - \left(p + x_j\right) q_j\right] - A + M & \text{if acquiring access} \\ M & \text{otherwise.} \end{cases} \tag{4.8}$$

To satisfy the consumer's participation constraint, the infrastructure provider can at most extract the consumer's net surplus from consumption, i.e. $A \leq \sum_1^N \left[u\left(q_j\right) - \left(p + x_j\right) q_j\right]$ in equilibrium. Under this condition, all consumers acquire access to the infrastructure. Maximizing (4.8) with respect to q_j yields

$$\frac{du}{dq_j} = p + x_j, \; j = 1, .., N, \tag{4.9}$$

which defines a consumer's demand for good j, denoted $q_j\left(p, x_j\right)$. Note that, to save notation, in the following we omit the arguments of this demand. Clearly, the usage fee, p, and the final goods price, x_j, have the same effect on this demand, that is $\frac{\partial q_j}{\partial p} = \frac{\partial q_j}{\partial x_j} < 0$.

Each final good provider, $j = 1, .., N$, maximizes (4.2) with respect to x_j. Assuming concavity, the equilibrium price of good j (given p and the tax rates), is determined by

$$\frac{\partial \pi_j}{\partial x_j} = q_j + \left(x_j - \phi\right) \frac{\partial q_j}{\partial x_j} = 0 \Rightarrow x_j = \phi - \frac{q_j}{\frac{\partial q_j}{\partial x_j}}, \qquad j = 1, .., N. \tag{4.10}$$

Thus, x_j follows from the standard monopoly markup rule. Because consumers' utility is symmetric and separable, and final good suppliers are symmetric as well, the price defined by (4.10) is identical for all j. Therefore, we denote the equilibrium price by x and the demand for each final good as q, dropping the index j.

Intuitively, because consumer demand for final goods depends on p as well as x, the infrastructure provider may influence the price set by final good providers through its usage fee. The effect of p on x is ambiguous a priori. However, it is reasonable to expect that in most circumstances x decreases with p, because p reduces the surplus consumers get from each additional unit of final goods. A sufficient condition to ensure this intuitive outcome is that consumers' demand is not exceedingly convex.[19] To streamline the exposition, in line with

the timing affects the pricing decisions by firm I, but not the main results regarding the effects of taxation.

[19]Starting from (4.10) and using $\partial q/\partial p = \partial q/\partial x$, we get $\partial x/\partial p < 0$ if and only if $\partial q/\partial x + (x - \phi)\left(\partial^2 q/\partial x^2\right) < 0$. This condition holds as long as $\partial^2 q/\partial x^2$ is either non-positive or relatively small in magnitude.

Economides and Hermalin (2015), we assume that this condition holds, so that

$$\partial x/\partial p < 0. \tag{4.11}$$

However, as we show in Appendix A.1, the combined price $p + x$ increases in p, i.e.

$$\frac{d\,(p+x)}{dp} = 1 + \frac{\partial x}{\partial p} > 0. \tag{4.12}$$

Thus, the overall effect of an increase in the usage fee on consumption is negative

$$\frac{dq}{dp} = \frac{\partial q}{\partial p} + \frac{\partial q}{\partial x}\frac{\partial x}{\partial p} = \frac{\partial q}{\partial p}\left(1 + \frac{\partial x}{\partial p}\right) < 0. \tag{4.13}$$

We now turn to the infrastructure provider's problem:

$$\max_{A,p} \pi_I = (1 - t_A)\,A + (p\,(1 - t_p) - c)\,Nq, \quad \text{s.t.} \quad A \leq N\,[u\,(q) - (p + x)\,q]\,. \tag{4.14}$$

The participation constraint binds in equilibrium, because the infrastructure provider could otherwise increase A without changing consumer behavior and make strictly higher profits. Hence, we have

$$A = N\,[u\,(q) - (p + x)\,q]\,. \tag{4.15}$$

Thus, the infrastructure provider cannot extract the whole consumer surplus, because part of it, Nxq, accrues to the suppliers of final goods. This term provides a foundation to the term F_i in expression (3.3) of Section 3 (given a single consumer type).

Using the access fee characterized above, the maximization problem of firm I simplifies to

$$\max_{p} \pi_I = N\,[(1 - t_A)\,(u\,(q) - pq - xq) + ((1 - t_p)\,p - c)\,q]\,. \tag{4.16}$$

The equilibrium usage fee (conditional on t_A and t_p), denoted p, satisfies the following first-order condition

$$N\,(1 - t_A)\left[\left(\frac{du}{dq} - p\right)\frac{dq}{dp} - q - \left(x\frac{dq}{dp} + q\frac{\partial x}{\partial p}\right)\right] + N\,(1 - t_p)\left(p\frac{dq}{dp} + q\right) - Nc\frac{dq}{dp} = 0. \tag{4.17}$$

The last term in square parentheses captures the effect of the usage fee on the part of consumer surplus, Nxq, that accrues to final goods providers. Using the equilibrium condition (4.9) in the above expression, we get

$$p=\frac{1}{1-t_p}\left(\left(t_p-t_A\right)\frac{q}{\frac{dq}{dp}}+\left(1-t_A\right)\frac{\frac{\partial x}{\partial p}q}{\frac{dq}{dp}}+c\right). \tag{4.18}$$

To analyze the effects of taxation, it is useful to first consider as a benchmark the case of zero taxes. Setting $t_A=t_p=0$ in (4.17) yields

$$\frac{du}{dq}=\frac{x\frac{dq}{dp}+q\frac{\partial x}{\partial p}}{\frac{dq}{dp}}+c. \tag{4.19}$$

The numerator on the right hand side of this expression corresponds to $\sum_{j=1...\Theta}\frac{dF_j}{dp_i}$ in expression (3.5). It captures the distortion that the infrastructure provider induces to extract part of the surplus otherwise accruing to the suppliers of final goods (Economides and Hermalin, 2015). Indeed, using (4.18), equation (4.19) simplifies to

$$p=\frac{\frac{\partial x}{\partial p}q}{\frac{dq}{dp}}+c, \tag{4.20}$$

which shows that, despite its ability to extract surplus via the access fee, the infrastructure provider charges a usage fee above the marginal cost. Doing so induces the suppliers of final goods to reduce their own prices, so that the additional revenue from usage that firm I receives from increasing p exceeds the reduction in the access fee needed to maintain consumer participation.

Summing up, because $p>c$ and $x>\phi$ with no taxes, consumption falls short of the socially optimal level, i.e., $q<q^*$. We now analyze how the government can design its instruments to alleviate this distortion. Focus first on the effects of the tax on access. Totally differentiating (4.17) with respect to p and t_A and using (4.9) yields

$$\frac{\partial p}{\partial t_A}=-\frac{dA/dp}{{\partial^2\pi_I}/{\partial p^2}}=-\frac{\left(1+\frac{\partial x}{\partial p}\right)qN}{{\partial^2\pi_I}/{\partial p^2}}>0. \tag{4.21}$$

Firm I responds to t_A by reducing A. In this setting, the implication is that p increases. To see why, consider that, in equilibrium, A captures the consumer's net surplus, satisfying (4.15). This surplus decreases with p, given that the combined price of final goods, $p+x$, increases with the usage fee (see (4.12)). In other words, I reacts to the access tax by relying less on the access fee and more on usage fee to extract surplus from consumers.

Consider now the effects of the usage tax. Totally differentiating (4.17) with respect to p

and t_p delivers

$$\frac{\partial p}{\partial t_p} = \frac{\left(1 + \frac{p}{q}\frac{dq}{dp}\right) qN}{\partial^2 \pi_I / \partial p^2} = \frac{\left(1 + \frac{\partial x}{\partial p}\right)\left(\left(1 - t_A\right) q + c\frac{\partial q}{\partial p}\right) N}{\left(1 - t_p\right) \partial^2 \pi_I / \partial p^2}. \tag{4.22}$$

The first equality in (4.22) indicates that p decreases with t_p if and only if q lies on the *inelastic* part of consumer demand, i.e. $\frac{p}{q}\frac{dq}{dp} > -1$ holds. Under this condition, the tax base, Npq, decreases with p. Thus, *reducing* p is the provider's optimal response as t_p increases. The last equality in (4.22) follows from the equilibrium usage fee in (4.18) and equation (4.13). We obtain

$$\frac{\partial p}{\partial t_p} < 0 \Longleftrightarrow c < -\frac{\left(1 - t_A\right) q}{\frac{\partial q}{\partial p}}. \tag{4.23}$$

Hence, if the marginal cost c is sufficiently low, an increase in the usage tax reduces the usage fee.[20] The reason is that higher marginal costs imply a higher usage fee, p, and a lower consumption level, q. Hence, a higher marginal cost makes it less likely that q lies on the inelastic part of demand. Similarly, the usage fee is less likely to decrease with t_p the larger is t_A, because the access tax increases p (as pointed out above). Note also that the condition (4.23) tends to be more stringent when the cost of final goods, ϕ, increases, because this cost results in a higher price x and reduces q, all else equal.

In sum, because q decreases with the usage fee, the previous results imply that consumption can be stimulated either by reducing the tax on access or, provided (4.23) holds, *increasing* the tax on usage.[21] Therefore, taxation can reduce the distortion stemming from the providers' market power.

The effects of a change in either tax on welfare are given by

$$\frac{\partial W}{\partial t_k} = N\left(\frac{du}{dq} - c - \phi\right)\frac{dq}{dt_k} = N\left(p + x - c - \phi\right)\frac{dq}{dt_k}, \quad k = A, p. \tag{4.24}$$

Thus, because at equilibrium $p + x > c + \phi$ holds, the government can increase welfare by *raising* t_p, provided that the marginal cost c is small enough (as specified in (4.23)). This finding readily brings us to the optimal tax rates. Ideally, the government should implement

[20] The right hand side of (4.23) also depends on c, but remains strictly positive when c approaches zero. By continuity, the inequality holds when c is small enough.

[21] Although taxation affects the access fee as well, there is no effect on market participation because all consumers connect in equilibrium. In Appendix E, we consider an extension where only consumers with a sufficient valuation for access connect. While the effects of taxing usage on the consumption of final goods do not change, there may be a reduction in the number of consumers connecting. However, a tax on access expands the number of connections. As a result, taxing usage (resp. access) is optimal when the demand for access tends to be inelastic (resp. elastic).

the socially optimal allocation, which is characterized by $p + x = c + \phi$. However, because $x > \phi$ in equilibrium, the optimum can only be achieved if p is below the marginal cost c. For the sake of exposition, we restrict our attention to equilibria where $p \geq c$.[22] Given $p + x$ increases with p (see (4.12)), the constrained optimum is such that $p = c$. Hence, using (4.18), we conclude

$$p = c \Longleftrightarrow t_p = \frac{t_A \left(1 + \frac{\partial x}{\partial p}\right) - \frac{\partial x}{\partial p}}{1 + \frac{c}{q}\frac{dq}{dp}}, \tag{4.25}$$

which suggests that an infinite set of pairs (t_p, t_A) implements the constrained optimum. Many of these pairs involve positive tax rates on access and usage. However, if (4.23) holds, it is sufficient to tax usage only, setting $t_p^* = -\frac{\frac{\partial x}{\partial p}}{1+\frac{c}{q}\frac{dq}{dp}} > 0$ and $t_A^* = 0$.[23]

Proposition 1. *Consider a setting with homogeneous consumers, imperfect competition in the markets for final goods and a monopolist infrastructure provider adopting a multi-part tariff. If the marginal cost of infrastructure usage is sufficiently low (see* (4.23)*), an ad valorem usage tax increases consumption and social welfare.*

Note that a small marginal cost is fairly realistic for the main applications of this model. For instance, the cost for ISPs of delivering an additional Gigabyte of data is close to zero. Similarly, there is virtually no cost of handling an additional transaction for payment card platforms.[24]

Our analysis focuses on efficiency, but we can also shed some light on the distributional consequences of taxation in this setting. It is straightforward to show that the tax on usage reduces the profit of the infrastructure supplier. The effect on final good providers' profits is instead positive as long as $\frac{\partial p}{\partial t_p} < 0$, because final good providers can charge higher prices and sell higher quantities if the usage fee decreases with t_p. To continue, the infrastructure provider captures the whole net consumer surplus. Therefore, the effect of taxation on consumers is zero. Note also that we have assumed that tax revenue has the same weight as the other components of social welfare. That is, the cost of public funds equals one. Assuming a larger weight would of course increase the welfare benefit of the taxes we consider. For example, the government could use their revenue to reduce other distortionary taxes in the economy. Thus,

[22] Usage fees below marginal cost are possibly not feasible in applications such as telecom and payment cards, where marginal costs are most likely very small. However, our conclusions about the optimal tax rates do not change if we allow for $p < c$, as we show in Appendix C.7.

[23] Note that when $t_A = 0$, (4.23) is sufficient to ensure the denominator of (4.25) is positive, given $0 > \frac{dq}{dp} = \frac{\partial q}{\partial p}\left(1 + \frac{\partial x}{\partial p}\right) > \frac{\partial q}{\partial p}$.

[24] Telecom firms may face issues of network congestion, which could affect the optimal usage fees. We discuss these issues in Section 6.

if provision increases with t_p, this tax would produce a double dividend.

4.2 Menus of tariffs and screening

In the industries where multi-part tariffs are common, such as telecom and energy distribution, firms often propose menus of tariffs with the goal of screening consumers. We now explore the effects of differentiated taxation in this context, characterizing the conditions such that imposing differentiated taxes on access and usage increases welfare. In so doing, we provide an additional foundation for the ad-hoc model in Section 3.

We again consider a monopolist infrastructure provider I that consumers use to access final goods. However, we relax the assumption of homogeneous consumers and allow them to differ in the utility they get from final goods. To keep the setup as simple as possible, we assume there is only one such good ($N = 1$). We consider two types of individuals, indexed by $i = h, \ell$, where h stands for "heavy user" and ℓ for "light user." We normalize the total number of consumers to one, denoting the share of type h by $\sigma \in (0, 1)$. The utility function is

$$u(q, \alpha_i) + y, \quad i = h, \ell, \quad \text{with} \quad \frac{\partial^2 u}{\partial q \partial \alpha} > 0, \tag{4.26}$$

The preference parameter α_i, assumed private information, determines a consumer's utility from the final good, with $\alpha_h > \alpha_\ell > 0$. This parameter determines also the intensity of consumers' infrastructure network usage. The infrastructure provider engages in second-degree price discrimination, by proposing a menu of tariffs. We retain the same timing as in the previous sections and restrict again attention to equilibria with two-part tariffs.[25]

To concentrate on the effects of taxation when the infrastructure provider screens consumers, we assume there is perfect competition in the final good market. Therefore, the price of a unit of final good equals marginal cost, ϕ. Hence, unlike in Section 4.1, influencing the final good's price is not a motive driving the choice of tariffs by the infrastructure provider. In fact, in this setting we could ignore final goods altogether (dropping ϕ from the expressions that follow). Consequently, this model can also apply to situations where connecting to providers of final goods is not the main purpose of using the infrastructure, as in, e.g., the case of energy distribution or parking. Hence, one may also interpret q as the length of stay in a parking garage or as the amount of energy consumed for heating and operating domestic appliances.

The net utility of a type-i consumer, conditional on choosing the tariff intended for type

[25] Analogously to Section 4.1, the focus on two-part tariffs is without loss of generality (see Appendix D.1).

$\tilde{i} = h, \ell$ and given the quantity q, is

$$u(q, \alpha_i) + M - A_{\tilde{i}} - (p_{\tilde{i}} + \phi)\, q, \quad i, \tilde{i} = h, \ell. \tag{4.27}$$

We denote the quantity chosen by such consumer as $q_{i\tilde{i}}$ and the ensuing gross utility, $u(q_{i\tilde{i}}, \alpha_i)$, as $u_{i\tilde{i}}$. We drop the double index when $i = \tilde{i}$ (i.e., when consumers choose the intended tariff). In equilibrium, participation and incentive compatibility constraints are satisfied and all consumers self-select into the tariff intended for their type. Hence, the quantities q_i satisfy the following first-order conditions

$$\frac{\partial u(q, \alpha_i)}{\partial q} = p_i + \phi, \ \forall i, \tag{4.28}$$

that is, marginal utility equals the sum of unit prices, $p_i + \phi$.

Tax revenue amounts to $R = t_p (p_h q_h \sigma + p_\ell q_\ell (1 - \sigma)) + t_A (A_h \sigma + A_\ell (1 - \sigma))$ and welfare, defined ad the sum of this revenue plus consumer surplus and firms' profits, is:

$$W = \sigma (u_h - (c + \phi) q_h) + (1 - \sigma)(u_\ell - (c + \phi) q_\ell) + M. \tag{4.29}$$

Hence, the socially optimal consumption levels, q_i^*, are such that the marginal utility equals the sum of marginal cost, that is $\frac{\partial u(q,\alpha_i)}{\partial q} = c + \phi$.

The infrastructure provider's problem can be written as

$$\begin{aligned}
\max_{A_h, p_h, A_\ell, p_\ell} \quad & \sigma [(1 - t_A) A_h + (1 - t_p) p_h q_h - c q_h] + (1 - \sigma) [(1 - t_A) A_\ell + (1 - t_p) p_\ell q_\ell - c q_\ell] \\
s.t. \quad & V_i \equiv u(q_i, \alpha_i) + M - A_i - (p_i + \phi) q_i \geq M, \quad i = h, \ell, \quad \text{and} \\
& V_i \geq u(q_{i\tilde{i}}, \alpha_i) + M - A_{\tilde{i}} - (p_{\tilde{i}} + \phi) q_{i\tilde{i}}, \quad i, \tilde{i} = h, \ell, \ i \neq \tilde{i}.
\end{aligned}$$

The first set of constraints are the participation constraints and the second set represent the incentive compatibility constraints. We relegate the standard steps to solve this problem to Appendix A.2 (see Laffont and Martimort, 2001). We find that in equilibrium the participation constraint is binding for $i = \ell$, whereas the incentive compatibility constraint is binding for $i = h$. The other constraints are slack. The equilibrium access fees are thus given by

$$A_\ell = u_\ell - (p_\ell + \phi) q_\ell, \quad A_h = u_h - (p_h + \phi) q_h - (u_{h\ell} - u_\ell), \tag{4.30}$$

where $u_i \equiv u(q_i, \alpha_i)$ and $u_{h\ell} \equiv u(q_\ell, \alpha_h)$. The fee charged to the light users captures all their net surplus from consumption. By contrast, heavy users pay an access fee which does not capture their whole surplus: they receive some information rent to ensure they do not mimic the other type. This rent, captured by the last term in parentheses in (4.30), provides another foundation to the term F_i introduced in Section 3 (see (3.3)).

The equilibrium usage fees are

$$\begin{aligned} p_h &= \frac{1}{1-t_p}\left((t_p - t_A)\frac{q_h}{\frac{\partial q_h}{\partial p_h}} + c\right), \\ p_\ell &= \frac{1}{1-t_p}\left((t_p - t_A)\frac{q_\ell}{\frac{\partial q_\ell}{\partial p_\ell}} + (1-t_A)\frac{\sigma}{1-\sigma}\left(\frac{\partial u_{h\ell}}{\partial q_\ell} - \frac{\partial u_\ell}{\partial q_\ell}\right) + c\right). \end{aligned} \tag{4.31}$$

To analyze the effects of taxation, it is useful to first consider as a benchmark the equilibrium with zero taxes. Setting $t_A = t_p = 0$, we get

$$p_h = c, \quad \text{and} \quad p_\ell = \frac{\sigma}{1-\sigma}\left(\frac{\partial u_{h\ell}}{\partial q_\ell} - \frac{\partial u_\ell}{\partial q_\ell}\right) + c. \tag{4.32}$$

In the absence of taxation, firm I does not distort heavy users' consumption as compared to the socially optimal level. By contrast, the firm does impose a restriction to the light users, who pay a usage fee above marginal cost, as captured by the right hand side of the expression for p_ℓ in (4.32). This result stems from the trade-off between rent-extraction and efficiency (Maskin and Riley, 1984). By restricting light users' consumption, the infrastructure provider makes mimicking less appealing to heavy users, that value the marginal unit of consumption more than light users, given $\frac{\partial^2 u}{\partial q \partial \alpha} > 0$. In so doing, firm I reduces heavy users' information rent and can thus extract more revenue from them through the access fee, A_h (see (4.30)). As a result, there is underprovision (only) to the light users because $p_\ell > c$ and, in turn, $q_\ell < q_\ell^*$.

We now analyze whether taxation can correct this underprovision. The effects of the access tax, t_A, on the usage fees are given by

$$\frac{\partial p_h}{\partial t_A} = \sigma\frac{\partial A_h/\partial p_h}{\partial^2 \pi_I/\partial p_h^2} = -\frac{\sigma q_h}{\partial^2 \pi_I/\partial p_h^2} > 0, \tag{4.33}$$

$$\frac{\partial p_\ell}{\partial t_A} = \frac{(1-\sigma)\,\partial A_\ell/\partial p_\ell + \sigma \partial A_h/\partial p_\ell}{\partial^2 \pi_I/\partial p_\ell^2} = -(1-\sigma)\frac{q_\ell + \frac{\sigma}{1-\sigma}\left(\frac{\partial u_{h\ell}}{\partial q_\ell} - \frac{\partial u_\ell}{\partial q_\ell}\right)\frac{\partial q_\ell}{\partial p_\ell}}{\partial^2 \pi_I/\partial p_\ell^2}. \tag{4.34}$$

Intuitively, firm I responds to this tax by reducing the access fees. These fees are pinned down, respectively, by the binding participation (for $i = \ell$) and incentive compatibility (for $i = h$) constraints, resulting in (4.30). Hence, to reduce A_h and A_ℓ, firm I changes the usage fees. Specifically, a higher p_h reduces A_h, because p_h reduces the net surplus of heavy users,

$u_h - (p_h + \phi)\, q_h$. However, p_h has no effect on A_ℓ. Therefore, we find that ${}^{\partial p_h}/_{\partial t_A} > 0$. Taxing access gives the infrastructure supplier an incentive to collect more revenue from heavy users through the usage fee.

By contrast, collecting less revenue through access fees does not necessarily entail an increase in the usage fee for light users. On the one hand, a higher p_ℓ reduces the net surplus of light users, which in turn reduces A_ℓ. On the other hand, consumption of heavy users who mimic is reduced as well. Therefore, the information rent decreases, which raises A_h. These opposing effects are captured, respectively, by the first and second terms at the numerator of the rightmost fraction in (4.34). The sign of ${}^{\partial p_\ell}/_{\partial t_A}$ depends on which effect dominates. Specifically, when distorting the consumption of light users strongly affects the heavy users' information rent, firm I responds to the tax on access by reducing p_ℓ. In other words, the supplier's incentive to restrict the consumption of light users is weakened, because the gain from reducing heavy users' information rent is partly taxed away.

Consider now the effects of the tax on usage, which are given by

$$\frac{\partial p_h}{\partial t_p} = \sigma \frac{q_h + p_h \frac{\partial q_h}{\partial p_h}}{{}^{\partial^2 \pi_I}/_{\partial p_h^2}} = \sigma \frac{(1 - t_A)\, q_h + c \frac{\partial q_h}{\partial p_h}}{{}^{\partial^2 \pi_I}/_{\partial p_h^2}\, (1 - t_p)}, \tag{4.35}$$

$$\frac{\partial p_\ell}{\partial t_p} = (1 - \sigma) \frac{q_\ell + p_\ell \frac{\partial q_\ell}{\partial p_\ell}}{{}^{\partial^2 \pi_I}/_{\partial p_\ell^2}} = -(1 - \sigma) \frac{(1 - t_A) \left[q_\ell + \frac{\sigma}{1-\sigma} \left(\frac{\partial u_{h\ell}}{\partial q_\ell} - \frac{\partial u_\ell}{\partial q_\ell} \right) \frac{\partial q_\ell}{\partial p_\ell} \right] + c \frac{\partial q_\ell}{\partial p_\ell}}{{}^{\partial^2 \pi_I}/_{\partial p_\ell^2}\, (1 - t_p)}. \tag{4.36}$$

As in our previous settings, the effect of t_p on each usage fee, p_i, is negative if and only if the revenues generated by such fee, $p_i q_i$, increase with p_i. That is, p_i decreases with t_p if and only if q_i lies on the inelastic part of type-i consumers' demand. In the case of heavy users, this condition holds under a very similar condition to (4.23). Specifically, ${}^{\partial p_h}/_{\partial t_p} < 0$ if the marginal cost and the tax on access (which tends to increase p_h, as pointed out above) are sufficiently small. As in the previous section, this condition is more likely to be satisfied the smaller is ϕ. The condition determining the sign of ${}^{\partial p_\ell}/_{\partial t_p}$ is more interesting. Expression (4.36) suggests that the above-mentioned conditions are not sufficient for p_ℓ to decrease with t_p: even if c and t_A are zero, p_ℓ can increase with t_p when the effect of this fee on the information rent left to heavy users, captured by $\frac{\sigma}{1-\sigma} \left(\frac{\partial u_{h\ell}}{\partial q_\ell} - \frac{\partial u_\ell}{\partial q_\ell} \right) \frac{\partial q_\ell}{\partial p_\ell}$, is large. Furthermore, a higher t_A makes it more likely that ${}^{\partial p_\ell}/_{\partial t_p} > 0$ if and only if the term in square parentheses in (4.36) is positive, i.e. $\frac{\sigma}{1-\sigma} \left(\frac{\partial u_{h\ell}}{\partial q_\ell} - \frac{\partial u_\ell}{\partial q_\ell} \right) \frac{\partial q_\ell}{\partial p_\ell}$ is small.

We now analyze whether taxation can increase welfare. Differentiating (4.29) with respect to t_k and using (4.28), we obtain

$$\frac{\partial W}{\partial t_k} = \sigma \left(p_h - c\right) \frac{dq_h}{dt_k} + (1-\sigma) \left(p_\ell - c\right) \frac{dq_\ell}{dt_k}, \quad k = A, p, \tag{4.37}$$

where $\frac{dq_i}{dt_k} = \frac{\partial q_i}{\partial p_i} \frac{\partial p_i}{\partial t_k}$, with $i = \ell, h$. Furthermore, setting $t_p = t_A = 0$ and using (4.32), we get

$$\left.\frac{\partial W}{\partial t_k}\right|_{t_A=0, t_p=0} = \sigma \left(\frac{\partial u_{h\ell}}{\partial q_\ell} - \frac{\partial u_\ell}{\partial q_\ell} \right) \frac{dq_\ell}{dt_k} > 0 \Longleftrightarrow \frac{\partial p_\ell}{\partial t_k} < 0, \quad k = A, p. \tag{4.38}$$

Because there is underprovision to the light users, the government can increase welfare by introducing either a tax on access or one on usage, provided p_ℓ decreases with such taxes. That is, the government can alleviate the restriction firm I imposes on light users by taxing access if the impact of a marginal increase in p_ℓ on the information rent of heavy users is large. Instead, if the impact of p_ℓ on the information rent and the marginal cost c are both small, a similar effect can be obtained by taxing usage. Based on our previous results, we can state:

Proposition 2. *Consider a setting where a monopolist infrastructure provider implements a menu of multi-part tariffs to screen consumers. If the marginal cost of infrastructure usage and the effect of restricting the consumption of light users on heavy users' information rent are small, introducing an ad valorem usage tax increases welfare. Instead, if restricting the consumption of light users has a large effect on the information rent, welfare increases with an ad valorem access tax.*

A small marginal cost is a reasonable assumption for ISPs, credit card platforms and parking providers. In line with Proposition 1, Proposition 2 suggests that this condition makes it more likely that a tax on usage increases output and welfare. However, this condition is not sufficient: the impact of restricting usage by light users on the information rent should also be relatively small. We can expect the latter condition to apply when either the share of heavy users in the population, σ, is small and/or when their marginal utility from consumption is similar to that of light users (i.e., the difference between α_h and α_ℓ is small). By contrast, a tax on access is more likely to increase welfare when the impact of restricting usage by light users on the information rent is large.

We can again shed some light on the distributional implications of taxation. It is straightforward to show that taxes reduce the profit of the infrastructure provider and leave the profit of final good providers unchanged. Moreover, because the surplus of light users is entirely captured by the provider, the net effect of taxation on light users is also zero. However, when taxes induce an increase in consumption by the light users, the information rent left to heavy users increases and they are better off.[26] Note also that, as in the previous section, we ignore the potential welfare benefits from the tax revenue that government could eventually use to reduce other distortionary taxes in the economy.

We conclude this part by discussing the implications of relaxing some of our assumptions. An infrastructure provider may also screen consumers on the quality of service (e.g. download speed in the case of ISPs). Adding this dimension to the model should not change the main conclusions. We expect the heavy users to have a higher willingness to pay for high-quality service. Hence, light users would be offered plans with lower quality as well as restricted volumes. The effect of the usage fee on information rents could be either magnified or reduced, depending on whether quality and volume are complements or substitutes.

Finally, we assumed I serves both consumer types. However, in some circumstances, the firm may prefer to exclude the light users, because serving them entails an exceedingly high rent to the heavy ones. This outcome can occur, for example, if the light users' willingness to pay is substantially smaller than that of the heavy users. Exclusion may also become more profitable with taxation. Nevertheless, provided both types are served in laissez faire, there should always exist positive (possibly small) tax rates such that welfare increases and exclusion does not take place.

5 Competition among infrastructure providers

We now extend the model of Section 4.1 to allow for competition among infrastructure providers. For brevity, we relegate much of the analysis to Appendix F, and focus here on describing the model setup and the results.

We consider a horizontally differentiated duopoly, with infrastructure providers located at the extremes of a Hotelling line, i.e. firm m is located at point 0 and firm n at point 1. The unit mass of consumers is uniformly distributed on the $[0, 1]$ interval.[27] The utility a consumer located at point $z \in [0, 1]$ gets from connecting via supplier $s = m, n$ and consuming q units of final good is

[26]To understand, replace A_h from (4.30) in (4.27), obtaining $U_h = u_{h\ell} - u_\ell$, and note that $\frac{\partial u_{h\ell}}{\partial q_\ell} - \frac{\partial u_\ell}{\partial q_\ell} > 0$.

[27]Horizontal differentiation may arise from differences in technologies (e.g. Cable vs DSL for ISPs, fossil vs. renewable sources for energy distributors) and other services bundled with access (e.g. TV channels bundled with Internet access). Horizontal differentiation may also capture heterogeneous coverage, which is quite common for network industries such as telecom. Depending on their physical location, some consumers may prefer using one provider rather than the other, because better coverage implies higher quality and reliability of service. See, e.g., Chen and Savage (2011), Dessein (2003), and Granier and Podesta (2010) for previous models of competition in the telecom and energy sectors that assume horizontally differentiated providers.

$$U_s = \int_0^q (\alpha - r)\, dr + V - \beta\,|z - l_s| + M - T_s - T_{f,s}, \quad s = m, n. \tag{5.1}$$

The first term of (5.1) is the utility from consuming the final good. To ease notation, let there be only one such good ($N = 1$). To simplify the analysis, we adopt a specific form for such utility (as in Economides and Hermalin, 2015). We assume $\alpha > \phi + c$. The second term in (5.1), $V - \beta\,|z - l_s|$, is the utility of a consumer at location z acquiring access from s, where V is the gross intrinsic surplus from acquiring access, β is the transportation cost, and l_s represents the firm's position on the Hotelling line, with $l_m = 0$ and $l_n = 1$. We assume consumers acquire access from exclusively one provider and focus on the case where their surplus is large enough that all acquire access in equilibrium. The last two terms in (5.1) are the tariffs paid to infrastructure provider s and to the supplier of the final good, respectively. We again focus on two-part tariffs set by infrastructure providers, $T_s = A_s + p_s q$. Instead, the provider of the final good sets a linear price, so $T_{f,s} = x_s q$. Note that we allow the latter firm to charge a different price to consumers according to which of the providers they subscribe to. This assumption is not essential, but simplifies the analysis (see below).[28]

Social welfare is the sum of consumer surplus, profits and government revenue. This sum simplifies to

$$\begin{aligned} W \;=\; & \int_0^{D_m} \left(\int_0^{q_m} (\alpha - r)\, dr - z\beta - (c + \phi)\, q_m \right) dz + \\ & + \int_{D_m}^1 \left(\int_0^{q_n} (\alpha - r)\, dr - (1 - z)\,\beta - (c + \phi)\, q_n \right) dz + M. \end{aligned} \tag{5.2}$$

where D_s is the market share of infrastructure provider s and q_s the quantity consumed by an individual acquiring access from provider s.

To derive the socially optimal allocation, we maximize (5.2) with respect to q_m, q_n, and D_m, obtaining $q_s^* = \alpha - \phi - c$ and $D_m^* = D_n^* = \frac{1}{2}$. Consumers choose the infrastructure provider that is closest to their location, and their consumption of the final good is such that marginal utility equals the combined marginal cost of provision by the infrastructure and final good providers.

Setting $t_A = t_p = 0$, the equilibrium prices and quantities are such that: $p_s = \frac{\alpha - \phi + 2c}{3}$, $x_s = \frac{\alpha + 2\phi - c}{3}$, $q_s = \frac{\alpha - \phi - c}{3}$ and $D_s = \frac{1}{2}$, for $s = m, n$. In the absence of taxation, consumers

[28] There are examples of such price discrimination. Streaming music services Spotify and Deezer offer special rates to subscribers of specific ISPs (e.g. Orange in France), whereas Netflix allows Comcast's Xfinity subscribers free access to its content for limited time periods (such as the "Watchathon" week).

pay a usage fee above the marginal cost of infrastructure usage, c. The intuition is again that raising the usage fee allows to extract more surplus from consumers, because it induces the final good provider to reduce its own markup. Given this fee and the mark-up imposed by the final good supplier, there is underprovision in equilibrium ($q_s < q_s^*$).

We find that taxation produces similar effects as in the model of Section 4.1. In particular, introducing a tax on usage t_p increases consumption as long as the marginal costs are not exceedingly large. Although the infrastructure providers raise the access fee, this increase has only distributional but not welfare consequences, because the market is fully covered.

Proposition 3. *With competing infrastructure providers that adopt multi-part tariffs, an ad valorem usage tax increases consumption and social welfare as long as the marginal cost of infrastructure usage is sufficiently small.*

Before concluding, we briefly comment on relaxing the assumption of price discrimination by the final good supplier. When an infrastructure provider, say m, raises its usage fee above marginal cost, it induces a reduction in the price of the final good. Suppose the final good supplier does not discriminate consumers according to which infrastructure provider they connect to. Then, m does not gain from such price reduction, because consumers get to benefit even if they connect to the other provider, n. Thus, both providers would set $p_s = c$ in equilibrium. Nevertheless, because the final good supplier charges a monopoly markup, there is still underprovision without taxes. Consequently, provided it induces a reduction in p_s, the tax t_p still increases welfare. The analysis of this case is available upon request.

6 Concluding remarks

We studied commodity taxation with multi-part tariffs, allowing for differentiated ad valorem taxes on the various parts of the tariff. We have modeled different market situations where multi-part tariffs generate distortions, showing that consumption and welfare can increase with differentiated taxes. Our results imply that tax exemptions for goods or services that are priced according to multi-part tariffs may be inefficient.

Although we have shown our results hold in several specifications, we briefly discuss some issues that were left out of the analysis. We have ignored consumption externalities such as pollution and congestion. The former is relevant for some applications of the model, such as energy. If the supplier does not internalize the externality, the welfare gains of increased provision must be weighed against the increased external costs. Congestion often takes place on digital and transport networks. Unlike pollution, this external cost should be internalized by the supplier (particularly in the case of a monopoly) and therefore reflected in the usage fee. This effect implies a larger usage fee, which in turn means that the conditions ensuring that the usage fee decreases with taxation become stricter (see (3.6)). Whether these conditions hold depends of course on how large congestion is, which is an empirical question. We note that, as shown by Economides and Hermalin (2015), underprovision may occur even if the supplier internalizes congestion. Hence, the taxes we study may still increase welfare.

Our analysis also abstracted from long-run issues. By reducing the net revenue collected from each consumer, taxation may reduce an infrastructure supplier's incentives to invest in capacity and service improvements. On the other hand, if taxation does increase usage, incentives to invest may be strengthened. Moreover, final good suppliers' investments may increase. In addition, tax revenue can be used to fund public initiatives to enhance infrastructure investment (e.g., universal service funds; see OECD, 2015 for an overview of these initiatives). We have also ignored how the market structure may be affected by taxes in the long run. Because taxation reduces profits, it may discourage entry in the infrastructure market. However, if taxes stimulate usage, they indirectly benefit the providers of the final goods, possibly leading to additional entry. A complete analysis of the effects of taxation in a dynamic context is left for future work.

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Appendix

A Proofs

A.1 Proof of (4.12)

We differentiate $p + x$ with respect to p and get $d\left(p+x(p)\right)/dp = 1 + \partial x/\partial p = \left(\partial q/\partial p\right) / \left(2\frac{\partial q}{\partial p} + (x-\phi)\frac{\partial^2 q}{\partial p^2}\right) > 0$. The last equality follows from totally differentiating (4.10) and the fact that $\partial q/\partial p = \partial q/\partial x < 0$. The denominator in the last expression is negative by second order conditions of the final good providers' problem.

A.2 Proof of (4.31)

The infrastructure provider's problem is

$$\begin{aligned} \max_{A_h, p_h, A_\ell, p_\ell} \quad & \sigma\left(\left(1-t_A\right)A_h + \left(1-t_p\right)p_h q_h - c q_h\right) + (1-\sigma)\left(\left(1-t_A\right)A_\ell + \left(1-t_p\right)p_\ell q_\ell - c q_\ell\right) \\ s.t. \quad & V_i \geq M \quad i = h, \ell \quad \text{and} \quad V_i \geq V_{i\tilde{i}} \quad i, \tilde{i} = h, \ell \end{aligned}$$

where the first set of constraints are the participation constraints (PCs) and the second set of constraints are the incentive compatibility constraints (ICCs). Moreover,

$$V_i = u(q_i, \alpha_i) + M - A_i - \left(p_i + \phi\right) q_i \quad i = h, \ell,$$

$$V_{i\tilde{i}} = u(q_{i\tilde{i}}, \alpha_i) + M - A_{\tilde{i}} - \left(p_{\tilde{i}} + \phi\right) q_{i\tilde{i}} \quad i, \tilde{i} = h, \ell, \ i \neq \tilde{i},$$

are, respectively, the indirect utility levels of a type-i consumer adopting the intended tariff, and that of a mimicker. Following standard steps (Laffont and Martimort, 2001), it can be shown that the solution has to be such that the PCs are slack for $i = h$ and the ICCs are slack for $i = \ell$ and $\tilde{i} = h$, whereas the other constraints are binding. Furthermore, in order to relax the ICC for $i = h$, it is optimal to have $q_{h\ell}$ arbitrarily close to q_ℓ (this can be implemented by imposing an extra fee for usage immediately beyond q_ℓ, which would however not be paid in equilibrium). Hence, we rewrite the problem as

$$\begin{aligned} \max_{A_h, p_h, A_\ell, p_\ell} \quad & \sigma\left(\left(1-t_A\right)A_h + \left(1-t_p\right)p_h q_h - c q_h\right) + (1-\sigma)\left(\left(1-t_A\right)A_\ell + \left(1-t_p\right)p_\ell q_\ell - c q_\ell\right) \\ s.t. \quad & V_\ell = M \quad \text{and} \quad V_h = V_{h\ell}. \end{aligned}$$

From the equality constraints, we get

$$A_\ell = u(q_\ell, \alpha_\ell) - (p_\ell + \phi)\, q_\ell, \tag{A.1}$$

$$A_h = u(q_h, \alpha_h) - (p_h + \phi)\, q_h - (u(q_\ell, \alpha_h) - (p_\ell + \phi)\, q_\ell - A_\ell)\,. \tag{A.2}$$

We can therefore rewrite the profit maximization problem as

$$\max_{p_h, p_\ell} \quad \sigma\left[(1 - t_A)\left(u(q_\ell, \alpha_\ell) - u(q_\ell, \alpha_h) + u(q_h, \alpha_h) - (p_h + \phi)\, q_h\right) + (1 - t_p)\, p_h q_h - c q_h\right] + \\ + (1 - \sigma)\left[(1 - t_A)\left(u(q_\ell, \alpha_\ell) - (p_\ell + \phi)\, q_\ell\right) + (1 - t_p)\, p_\ell q_\ell - c q_\ell\right] \tag{A.3}$$

Using the equilibrium conditions $\frac{\partial u(q, \alpha_i)}{\partial q} = p_i + \phi$, for $i = h, \ell$, we can write the first-order conditions (FOCs):

$$\frac{d\pi}{dp_h} = (t_A - t_p)\, q_h + \frac{\partial q_h}{\partial p_h}\left((1 - t_p)\, p_h - c\right) = 0, \tag{A.4}$$

$$\begin{aligned} \frac{d\pi}{dp_\ell} \quad &= \quad -\sigma\,(1 - t_A)\left(\frac{\partial u_h}{\partial q_\ell} - \frac{\partial u_\ell}{\partial q_\ell}\right)\frac{\partial q_\ell}{\partial p_\ell} + \\ &\quad + (1 - \sigma)\left(q_\ell\,(t_A - t_p) + \frac{\partial q_\ell}{\partial p_\ell}\left((1 - t_p)\, p_\ell - c\right)\right) = 0. \end{aligned} \tag{A.5}$$

We obtain (4.31) by rearranging the above FOCs. Furthermore, differentiating these FOCs, we obtain the comparative statics in (4.33) - (4.36).

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B Alternative tax instruments

Unit taxes. Differentiating (3.4) and given $\frac{dq_i}{dp_i} < 0$, we get

$$\frac{dp_i}{d\tau} = \frac{\frac{dq_i}{dp_i}}{\frac{d^2\pi_I}{dp_i^2}} > 0, \qquad i = 1, ..., \Theta. \tag{B.1}$$

The inequality follows from the fact that the numerator is negative and the denominator is also negative by concavity of π_I.

Uniform ad valorem taxes. Setting $t_A = t_p = t$ and differentiating (3.4) we get

$$\frac{dp_i}{dt} = \frac{\left(\frac{du_i}{dq} - p_i\right)\frac{dq_i}{dp_i} - q_i - \sum_{j=1...\Theta}\frac{dF_j}{dp_i} + p_i\frac{dq_i}{dp_i} + q_i}{\frac{d^2\pi_I}{dp_i^2}}, \qquad i = 1, ..., \Theta. \tag{B.2}$$

Given (3.4), the numerator of the above expression equals $\frac{c}{1-t}\frac{dq_i}{dp_i} \leq 0$. Therefore, given the denominator is also negative by concavity of π_I, we have $\frac{dp_i}{dt} > 0$ as long as $c > 0$ and $\frac{dp_i}{dt} = 0$ if $c = 0$.

C Extensions to Section 4.1

C.1 Alternative timing for the representative consumer model (section 4.1)

We consider the following timing. First, the government sets t_A and t_p. Thereafter, the sellers of final goods simultaneously set their prices x_j. Then, the infrastructure provider sets its own tariff. Next, consumers choose consumption levels q_j. We show that the results of Section 4.1 are robust to this change in timing. In a nutshell, we shall show that given this timing there is no response of final good prices to the usage fee p. Hence, firm I has no incentive to raise this fee above the marginal cost c. Nevertheless, because prices exceed marginal costs in the no-tax equilibrium, there is underprovision and taxes have the same implications as in the baseline model.

We solve the model backwards. Stage 4 is as in the main model, and the equilibrium consumption is characterized by (4.9). Then, at stage 3 the infrastructure provider solves the problem in (4.14), with the difference that x_j is given. Again, the infrastructure provider sets A to extract all consumer surplus, and solves (4.16). Differentiating π_I with respect to p, using the equilibrium condition $\frac{du}{dq} = p + x$ (we omit the index j because the final good sellers are symmetric), we obtain

$$p = \frac{c}{1 - t_p} + \frac{q\,(t_p - t_A)}{\frac{dq}{dp}\,(1 - t_p)}. \tag{C.1}$$

Hence, if $t_p = t_A = 0$, then $p = c$. The main difference with respect to (4.18) is that there is no effect of p on x, because x is given when firm I decides its tariff. However, as we will show, the effect of taxes on p is similar to the main text. Furthermore, given the generalized price of final goods is higher than marginal cost, taxes have a similar effect on social welfare.

Let us now solve the problem of the sellers $j = 1, .., N$. Each firm j maximizes its profit $\pi_j = (x_j - \phi)\, q_j$ with respect to x_j. Assuming this profit is concave, the equilibrium price of good j is determined by

$$\frac{\partial \pi_j}{\partial x_j} = q_j + (x_j - \phi)\left(\frac{\partial q_j}{\partial x_j} + q_j \frac{\partial p}{\partial x_j}\right) = 0, \qquad j = 1, .., N. \tag{C.2}$$

Hence, given the prices are symmetric across final good providers, we find

$$x = \phi - \frac{q}{\frac{\partial q}{\partial x} + p\frac{\partial p}{\partial x}}.$$

We assume the denominator is negative, implying that $x > \phi$, which is intuitive. Consider now the effect of taxes on p:

$$\frac{\partial p}{\partial t_A} = -\frac{-\frac{q}{\frac{dq}{dp}(1-t_p)}}{\partial^2 \pi_I / \partial p^2} > 0, \qquad \frac{\partial p}{\partial t_p} = \frac{q + p\frac{dq}{dp}}{\partial^2 \pi_I / \partial p^2} = \frac{(1 - t_A)\, q + c\frac{\partial q}{\partial p}}{(1 - t_p)\, \partial^2 \pi_I / \partial p^2}.$$

Hence the condition for p to increase in t_p is isomorphic to (4.23).

Finally, we argue that introducing a tax t_p increases welfare, as long as $\frac{\partial p}{\partial t_p} < 0$. Let us start from the laissez faire equilibrium. Given that $x + p > c + \phi$, we have $q < q^*$, where the latter is such that $u'(q^*) = c + \phi$. As argued above, there is no effect of p on x. Hence, $p + x$ decreases with t_p and, thus, $\frac{\partial q}{\partial t_p} > 0$, as long as the condition (4.23) holds. Introducing a tax on usage therefore alleviates underprovision and increases welfare.

C.2 Sellers provide imperfect substitutes

We now show that the main results of Section 4.1 continue to hold when final good suppliers provide imperfect substitutes, and hence compete with each other. In a nutshell, we show that, as long as final good providers have market power (even though they compete), their prices respond to changes in the usage fee p. Hence, firm I restricts consumption at the marginal and taxation has similar effects as in the baseline model.

For tractability, we focus on the case where $N = 2$ and we assume the final good providers are symmetric. Consumer utility is

$$U(q_1, q_2) - x_1 q_1 - x_2 q_2 - p(q_1 + q_2) - A. \tag{C.3}$$

We assume that $\frac{\partial U}{\partial q_i} > 0$ and $\frac{\partial^2 U}{\partial q_i^2} < \frac{\partial^2 U}{\partial q_1 \partial q_2} < 0$, which implies that the two goods are imperfect substitutes. Let us consider the consumer's utility maximization problem. The FOCs are

$$f_i \equiv \frac{\partial U}{\partial q_i} - x_i - p = 0, \quad i = 1, 2. \tag{C.4}$$

Hence, we get

$$\frac{\partial q_1}{\partial x_1} = -\frac{det\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial q_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial q_2} \end{bmatrix}}{det\begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} \end{bmatrix}}, \qquad \frac{\partial q_1}{\partial p} = -\frac{det\begin{bmatrix} \frac{\partial f_1}{\partial p} & \frac{\partial f_1}{\partial q_2} \\ \frac{\partial f_2}{\partial p} & \frac{\partial f_2}{\partial q_2} \end{bmatrix}}{det\begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \frac{\partial f_1}{\partial q_2} \\ \frac{\partial f_2}{\partial q_1} & \frac{\partial f_2}{\partial q_2} \end{bmatrix}}.$$

We proceed assuming that the denominator of these expressions is positive, as required by second-order conditions of the consumer's problem. The numerator of the first expression is $\frac{\partial f_1}{\partial x_1}\frac{\partial f_2}{\partial q_2} > 0$, because $\frac{\partial f_2}{\partial x_1} = 0$, $\frac{\partial f_1}{\partial x_1} < 0$ and $\frac{\partial f_1}{\partial q_2} < 0$. Hence, $\frac{\partial q_1}{\partial x_1} < 0$. Similarly, we can show that $\frac{\partial q_1}{\partial x_2} > 0$, because $-\frac{\partial f_2}{\partial x_2}\frac{\partial f_1}{\partial q_2} < 0$. Furthermore, we have $\frac{\partial q_1}{\partial p} < 0$. To see this, consider that $\frac{\partial f_1}{\partial p}\frac{\partial f_2}{\partial q_2} - \frac{\partial f_2}{\partial p}\frac{\partial f_1}{\partial q_2} = \frac{\partial f_1}{\partial p}\left(\frac{\partial f_2}{\partial q_2} - \frac{\partial f_1}{\partial q_2}\right) > 0$, because $\frac{\partial f_1}{\partial p} = \frac{\partial f_2}{\partial p} < 0$ and $\frac{\partial^2 U}{\partial q_i^2} < \frac{\partial^2 U}{\partial q_1 \partial q_2}$.

Consider now the effect of p on the prices set by sellers. Each seller's profit is $\pi_j = (x_j - \phi) q_j$. The FOCs are

$$g_j \equiv q_j + (x_j - \phi)\frac{\partial q_j}{\partial x_j} = 0, \quad j = 1, 2. \tag{C.5}$$

Therefore, we have

$$\frac{\partial x_1}{\partial p} = -\frac{det\begin{bmatrix} \frac{\partial g_1}{\partial p} & \frac{\partial g_1}{\partial x_2} \\ \frac{\partial g_2}{\partial p} & \frac{\partial g_2}{\partial x_2} \end{bmatrix}}{det\begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} \end{bmatrix}}.$$

We proceed again assuming that the denominator is positive. Hence, we have $\frac{\partial x_1}{\partial p} < 0$ if and only if $\frac{\partial g_1}{\partial p}\frac{\partial g_2}{\partial x_2} - \frac{\partial g_2}{\partial p}\frac{\partial g_1}{\partial x_2} > 0$. Because $\frac{\partial g_1}{\partial x_2} > 0$ (which follows from the assumption that the goods are imperfect substitutes) and $\frac{\partial g_2}{\partial x_2} < 0$ by the second-order conditions of the sellers' problem, we have that $\frac{\partial x_1}{\partial p} < 0$ if and only if $\frac{\partial g_i}{\partial p} < 0$. This inequality holds as long as consumer demands is not exceedingly convex, similarly to what we have assumed in the main text. Given this condition, we have $\frac{\partial x_i}{\partial p} < 0$.

Let us now consider the problem of firm I. Its profit is

$$\pi_I = (1 - t_A)\left[U(q_1, q_2) - x_1 q_1 - x_2 q_2\right] + (t_A - t_p)\, p\,(q_1 + q_2) - c\,(q_1 + q_2)\,, \tag{C.6}$$

and the FOC for the optimal usage fee is given by

$$\begin{aligned} \frac{\partial \pi_I}{\partial p} &= \sum_{i=1,2}\left((1 - t_A)\left(\frac{\partial U}{\partial q_i} - x_i\right) + (t_A - t_p)\, p - c\right)\left(\frac{\partial q_i}{\partial p} + \sum_{j=1,2}\frac{\partial q_i}{\partial x_j}\frac{\partial x_i}{\partial p}\right) \\ &\quad - (1 - t_A)\sum_{i=1,2} q_i \frac{\partial x_i}{\partial p} + (t_A - t_p)\,(q_1 + q_2) = 0. \end{aligned} \tag{C.7}$$

Denoting $\frac{dq_i}{dp} \equiv \frac{\partial q_i}{\partial p} + \sum_{j=1,2}\frac{\partial q_i}{\partial x_j}\frac{\partial x_i}{\partial p}$, using the FOCs of the consumer problem and rearranging, we get

$$p = \frac{1}{1 - t_p}\left((t_p - t_A)\frac{\sum_{i=1,2} q_i}{\sum_{i=1,2}\frac{dq_i}{dp}} + (1 - t_A)\frac{\sum_{i=1,2}\frac{\partial x_i}{\partial p} q_i}{\sum_{i=1,2}\frac{dq_i}{dp}} + c\right). \tag{C.8}$$

When there are no taxes, the above expression becomes

$$p = \frac{\sum_{i=1,2}\frac{\partial x_i}{\partial p} q_i}{\sum_{i=1,2}\frac{dq_i}{dp}} + c. \tag{C.9}$$

Given that $\frac{dq_i}{dp} < 0$, it is optimal to price usage above marginal costs, i.e. $p > c$.

Finally, consider the effect of t_p on p. Starting from (C.7), the effect of a change in t_p on

p is given by

$$\frac{\partial p}{\partial t_p} = \frac{\sum_{i=1,2}\left(q_i + p\frac{dq_i}{dp}\right)}{\frac{\partial \pi_I^2}{\partial p^2}} = \frac{\sum_{i=1,2}\left(\left(1-t_A\right)q_i\left(1+\frac{\partial x_i}{\partial p}\right)+c\frac{dq_i}{dp}\right)}{\left(1-t_p\right)\frac{\partial \pi_I^2}{\partial p^2}}. \tag{C.10}$$

Given that the denominator is negative by second-order conditions of firm I's problem, and given $\left(1+\frac{\partial x_i}{\partial p}\right) > 0$, then $\frac{\partial p}{\partial t_p} < 0$ as long as c is sufficiently small.

C.3 Alternative way to access final goods

We now suppose that consumers can access the final goods without the services of the infrastructure provider. In a nutshell, we show that, assuming that using the alternative way to access final goods is costly to consumers, raising p still has the effect of reducing the demand faced by final good providers and, hence, induces them to reduce their price. As a result, firm I still has an incentive to set $p > c$, there is underprovision and taxes on infrastructure usage have similar effects as in the baseline model.

Let $q = q^I + q^W$ be the units of final good consumed, where q^I is the quantity of units accessed using the infrastructure provided by firm I and q^W the quantity accessed without using the infrastructure. For example, in the case of a payment card platform, q^W can be interpreted as the quantity of retail goods purchased using cash rather than the card. We assume that the consumer sustains a cost $m\left(q^W\right)$ when consuming q^W units of final good without the infrastructure, with $m\left(0\right)$, $m'\left(0\right) = 0$, and $m'\left(.\right)$ and $m''\left(.\right)$ strictly positive when q^W is positive. The idea is that not using the infrastructure provider to acquire final goods implies an extra cost or inconvenience to the user (e.g., carrying more cash instead of using a card), which increases with the number of goods that users acquire not using the infrastructure. We also assume that the function $m(.)$ is steep enough that using the infrastructure provider for at least some units of q is optimal, i.e. $m'\left(.\right) \geq p$. We assume without loss that there is a single final good ($N = 1$) and that the producer of this good charges the same price, x, to consumers irrespectively of how they acquire this good.

Given the tariff set by firm I and writing $q^W = q - q^I$, the utility of a user is

$$U = u\left(q\right) - pq^I - m\left(q - q^I\right) - xq - A. \tag{C.11}$$

Maximizing the above with respect to q^I and q, we get the following FOCs

$$\begin{aligned} -p + m' &= 0, \\ \frac{du}{dq} - m' - x &= 0, \end{aligned} \tag{C.12}$$

from which we obtain that $m' = p$, so that we can rewrite the last equation, describing the choice of q, as

$$\frac{du}{dq} = p + x. \tag{C.13}$$

This expression implies that, for any set of prices, consumption of the final good is chosen in the same way as in the baseline model (see (4.9)). Note that the above expression implies that $\frac{\partial q}{\partial p} = \frac{\partial q}{\partial x}$ and that $\frac{dq^I}{dp} < 0$.

Consider now the price setting decision by the final good provider. Its profit is $\pi = (x - \phi)\, q$, hence x satisfies the following FOC:

$$q + (x - \phi) \frac{\partial q}{\partial x} = 0, \tag{C.14}$$

which is isomorphic to (4.10). Hence, the same conditions as in the baseline model are sufficient to ensure that $\frac{\partial x}{\partial p} < 0$. As in the baseline model, we have $1 + \frac{\partial x}{\partial p} > 0$.

Consider now the choice of tariff by firm I. We have

$$\pi_I = A + (p - c)\, q^I, \quad \text{s.t.} \quad A \leq u(q) - pq^I - m\left(q - q^I\right) - xq.$$

In equilibrium, the participation constraint is satisfied at equality. Hence, we can writ

$$\pi_I = \left(u(q) - pq^I - m\left(q - q^I\right) - xq\right)(1 - t_A) + (p(1 - t_p) - c)\, q^I. \tag{C.15}$$

Using (C.13) and (C.12), we can write the FOC of this problem as

$$-(1 - t_A)\left(q^I\left(1 + \frac{\partial x}{\partial p}\right) + \frac{\partial x}{\partial p}\left(q - q^I\right)\right) + (1 - t_p)\left(q^I + p\frac{dq^I}{dp}\right) - c\frac{dq^I}{dp} = 0. \tag{C.16}$$

Therefore, we obtain

$$p = \frac{c}{1 - t_p} + \frac{(t_p - t_A)\, q^I + (1 - t_A) \frac{\partial x}{\partial p} q}{(1 - t_p) \frac{dq^I}{dp}}. \tag{C.17}$$

Without taxation, we get $p = c + \frac{q\frac{\partial x}{\partial p}}{\frac{dq^I}{dp}}$. The terms of this expression are essentially the same as (4.20). Note that the last one is positive. This expression therefore suggests that $p > c$. As in the baseline model, $p + x > \phi + c$ in the no tax equilibrium, so consumption of the final good is below the socially optimal level.

Consider now the effect of taxing usage of the infrastructure. Using (C.16) and (C.17), we get

$$\frac{dp}{dt_p} = \frac{q^I + p\frac{dq^I}{dp}}{\frac{d^2\pi_I}{dp^2}} = \frac{(1 - t_A)\left(q^I + \frac{\partial x}{\partial p}q\right) + c\frac{dq^I}{dp}}{\frac{d^2\pi_I}{dp^2}(1 - t_p)},$$

which given concavity of π_I and $q^w = q - q^I$, implies that

$$\frac{dp}{dt_p} < 0 \Longleftrightarrow c < -\frac{(1 - t_A)\left(q\left(1 + \frac{\partial x}{\partial p}\right) - q^w\right)}{\frac{dq^I}{dp}}.$$

Similarly to (4.23), this expression suggests that it is possible to reduce the combined price of final goods and, hence, increase consumption (given $\frac{dq}{dp} = \frac{\partial q}{\partial p}\left(1 + \frac{\partial x}{\partial p}\right) < 0$) with a tax on usage, if the marginal cost c is below a threshold. Given that $1 + \frac{\partial x}{\partial p} > 0$, this threshold is strictly positive as long as q^w is not exceedingly large.

C.4 Infrastructure supplier charging providers of final goods

C.4.1 Unit fee to final good providers

We argue that there is no loss of generality when assuming that firm I charges a usage fee to consumers and not to providers of final goods. Our reasoning follows standard tax incidence arguments (Weyl and Fabinger, 2013).

Consider first the case where I charges p to consumers, as in section 4.1. Note that, to simplify notation, we use that prices and quantities of providers of final goods are symmetric, dropping the index j. Let x be the price charged by a final good provider. Denote x_c the price of the good as perceived by consumers, that is, including p paid to firm I. Therefore, $x + p = x_c$. Using this formulation, we can rewrite consumer utility (4.8) as

$$U = N\left(u\left(q\right) - x_c q\right) - A + M, \tag{C.18}$$

whereas the profit of the provider of final good j is

$$\pi_j = (x - \phi)\, q. \tag{C.19}$$

Therefore, the price x is defined by the following FOC:

$$\frac{\partial q}{\partial x}(x-\phi)+q=\frac{\partial q}{\partial x}(x_c-\phi-p)+q=0. \tag{C.20}$$

In equilibrium, firm I sets $A=N\left(u\left(q\right)-\left(x+p\right)q\right)=N\left(u\left(q\right)-x_cq\right)$, therefore its profit is

$$\pi_I=N\left[\left(1-t_A\right)\left(u\left(q\right)-x_cq\right)+\left(p\left(1-t_p\right)-c\right)q\right]. \tag{C.21}$$

Suppose now that p is paid by providers of final goods, rather than consumers. For consistency, we assume p is subject to the same tax rate as above, t_p. Let x' be the price of final goods and x_c' the price perceived by consumers. Differently from the previous case, because p is paid by final good suppliers, $x'=x_c'$. We can therefore write consumer utility as

$$U=N\left(u\left(q\right)-x_c'q\right)-A+M, \tag{C.22}$$

which is isomorphic to (C.18). The profit of a provider of final goods, j, is

$$\pi_j=\left(x'-\phi-p\right)q, \tag{C.23}$$

given it has to pay p for each unit produced. Therefore, the price x' is defined by the following FOC:

$$\frac{\partial q}{\partial x'}(x'-\phi-p)+q=\frac{\partial q}{\partial x'}(x_c'-\phi-p)+q=0. \tag{C.24}$$

This condition is therefore isomorphic to (C.20). Finally, in equilibrium, firm I sets $A=N\left(u\left(q\right)-x_c'q\right)$, therefore its profit is

$$\pi_I=N\left[\left(1-t_A\right)\left(u\left(q\right)-x_c'q\right)+\left(p\left(1-t_p\right)-c\right)q\right]. \tag{C.25}$$

Comparing the above expressions, it is clear that the expressions that characterize the equilibrium where p is physically paid by consumers are isomorphic to the expressions characterizing the equilibrium where p is paid by providers of final goods. Hence, there is no loss of generality in assuming that p is physically paid by consumers.

C.4.2 Access fee to final good providers

Assume that firm I charges a lump-sum access fee, A_j, to each final good provider j. At stage 3, provider j maximizes profits $\left(x_j-\phi\right)q_j-A_j$, so its first-order conditions are as in (4.10).

Firm j acquires access if and only if$A_j \leq (x_j - \phi) q_j$. At stage 2, firm I sets the access fee to consumers as in the main model, see (4.15), to ensure all consumers connect. We again assume that final good providers are symmetric. To capture in a simple way the idea that firm I may have to negotiate an access payment with each provider j, and thus be unable to extract the latter's entire profit with such fee, let $\gamma \in [0, 1]$ represent the share of such profit that can be extracted (capturing I's bargaining power). Hence, I sets A_j such that $A_j = \gamma (x_j - \phi) q_j$. After replacing A_j, we rewrite profits of firm I as (we again omit subscripts given symmetry)

$$\pi_I = N \left[(1 - t_A) \left(u(q) - q \left(x (1 - \gamma) - \gamma \phi \right) \right) + (t_A - t_p) pq - cq \right].$$

Assuming $t_A = t_p = 0$ and using the consumers' equilibrium condition $\frac{du}{dq} = p + x$, the first-order condition for the choice of p is:

$$\frac{d\pi_I}{dp} = N \left[(p + x) \frac{dq}{dp} - \left((x (1 - \gamma) - \gamma \phi) \frac{dq}{dp} + (1 - \gamma) q \frac{\partial x}{\partial p} \right) - c \frac{dq}{dp} \right] = 0.$$

Suppose that $\gamma = 1$. The above expression boils down to $p + x = c + \phi$. Therefore, the combination of the usage fee and the price set by final good providers is such that there is no distortion with respect to the socially optimal allocation. Suppose now $\gamma < 1$. We can rearrange the above first-order condition to obtain

$$p = c - \gamma (x - \phi) + \frac{(1 - \gamma) q \frac{\partial x}{\partial p}}{\frac{dq}{dp}}.$$

Using the equilibrium price of final goods, $x = \phi - \frac{q}{\frac{\partial q}{\partial x}}$, the above expression can be rewritten as

$$p + x = c + \phi + (1 - \gamma) \left(\frac{q \frac{\partial x}{\partial p}}{\frac{dq}{dp}} - \frac{q}{\frac{\partial q}{\partial x}} \right)$$

Because $\frac{dq}{dp} < 0$ and $\frac{\partial q}{\partial x} < 0$, and $\frac{\partial x}{\partial p} < 0$ by assumption, we have $(1 - \gamma) \left(\frac{q \frac{\partial x}{\partial p}}{\frac{dq}{dp}} - \frac{q}{\frac{\partial q}{\partial x}} \right) > 0$. Hence, the equilibrium level of consumption is smaller than the socially optimal one. Therefore, there is scope for taxation to reduce the distortion, as in the main text. One can therefore follow similar steps to argue that, given a sufficiently small c, a tax on usage increases provision and welfare.

C.5 Final good providers charging nonlinear tariffs

We consider the presence of final good providers that charge nonlinear tariffs. We show that such providers tend to extract surplus from consumers using lump-sum payments. Nonetheless, the main results do not change. In particular, firm I still has an incentive to charge a usage fee above the marginal cost. As long as there is at least one final good provider charging linear prices, we still obtain that an ad valorem tax on usage increases consumption when the marginal cost is small enough.

Assume there are two groups of final goods producers. There are N_n symmetric suppliers charging a nonlinear tariff and we assume without loss of generality that they charge a two part tariff of the form $B_j + x_j q_j$. Their profit is therefore

$$B_j + (x_j - \phi)\, q_j, \quad j = 1, ..., N_n. \tag{C.26}$$

In the above expression, B_j is the access fee charged by supplier j, x_j is the usage fee and q_j the quantity of good j. Moreover, there are $L \equiv (N - N_n)$ symmetric final goods suppliers which charge linear prices, as in our main analysis, whose profit is

$$(x_j - \phi)\, q_j, \quad j = N_n + 1, ..., N. \tag{C.27}$$

The surplus a consumer obtains from consuming final good j is given by

$$\begin{aligned} u(q_j) - B_j - x_j q_j - p q_j, \quad j = 1, ..., N_n, \\ u(q_j) - x_j q_j - p q_j, \quad j = N_n + 1, ..., N. \end{aligned} \tag{C.28}$$

Consider the pricing strategy of a final good provider $j \in \{1, ..., N_n\}$. Given $B_j \leq u(q_j) - x_j q_j - p q_j$ is necessary and sufficient to ensure that all consumers who acquire access to the infrastructure good also acquire access to good j, the condition

$$B_j = u(q_j) - x_j q_j - p q_j, j = 1, ..., N_n \tag{C.29}$$

holds in equilibrium. Hence, the final goods providers earn the following profit

$$\pi_j = u(q_j) - (p + \phi)\, q_j, j = 1, ..., N_n, \quad \pi_j = (x_j - \phi)\, q_j, j = N_n + 1, ..., N. \tag{C.30}$$

It follows that the equilibrium price x_j satisfies (4.10) for $j = N_n + 1, ..., N$. As for the producers adopting nonlinear tariffs, we have

$$\frac{d\pi_j}{dx_j} = \left(\frac{du}{dq_j} - (p+\phi)\right)\frac{\partial q_j}{\partial x_j} = 0 \Rightarrow x_j = \phi, j = 1, ..., N_n, \tag{C.31}$$

where the last equality follows from the equilibrium condition $\frac{du}{dq_j} = p + x_j$. Henceforth, we assume $\phi = 0$. We make this assumption to simplify the analysis and to capture the fact that many content providers charge only access fees (e.g., Netflix). Therefore, the final goods providers who adopt nonlinear tariffs do not charge for usage. Notice that the solution to all final goods providers problem is symmetric. Hence, to simplify notation, we are now going to denote $q_j \equiv q_n$ for $j = 1, ..., N_n$ and $q_j \equiv q_r$ for $j = N_n + 1, ..., N$. Similarly, we denote $x_j \equiv x_n = 0$ and $B_j \equiv B_n$ for $j = 1, ..., N_n$ and $x_j \equiv x_r$ for $j = N_n + 1, ..., N$.

Let us now consider the problem of firm I, which maximizes

$$\pi_I \quad = \quad Q\left(A\left(1-t_A\right) + \left(p\left(1-t_p\right) - c\right)\sum_{1}^{N} q_j\right), \tag{C.32}$$

subject to

$$A \leq N_n u(q_n) + Lu(q_r) - p\left(N_n q_n + L q_r\right) - N_n B_n - L x_r q_r. \tag{C.33}$$

After replacing from (C.29) and (C.31) and noting that this constraint must also be satisfied with equality in equilibrium, we get

$$A = L\left(u(q_r) - p q_r - x_r q_r\right). \tag{C.34}$$

Therefore,

$$\pi_I \quad = \quad \left(1-t_A\right)\left(Lu(q_r) - Lpq_r - Lx_r q_r\right) + \left(p\left(1-t_p\right) - c\right)\left(N_n q_n + L q_r\right). \tag{C.35}$$

We now maximize the above expression with respect to p. Using the equilibrium condition $\frac{du}{dq_l} = p + x_r$, the first-order condition of this problem is

$$\begin{gathered}\tfrac{d\pi_I}{dp} = \left(1-t_A\right) L\left(-q_r - \tfrac{\partial x_r}{\partial p} q_r\right) + \\ \left(1-t_p\right)\left(N_n\left(\tfrac{dq_n}{dp}p + q_n\right) + L\left(\tfrac{dq_r}{dp}p + q_r\right)\right) - c\left(N_n\tfrac{dq_n}{dp} + L\tfrac{dq_r}{dp}\right) = 0.\end{gathered} \tag{C.36}$$

Rearranging this expression we obtain

$$p = \frac{1}{1-t_p}\left(c + \frac{(1-t_A)\,L\frac{\partial x_r}{\partial p}q_r - (1-t_p)\,N_n q_n + (t_p - t_A)\,Lq_r}{N_n\frac{dq_n}{dp} + L\frac{dq_r}{dp}}\right) \tag{C.37}$$

Evaluating (C.37) at zero tax rates, we get

$$p = c + \frac{L\frac{\partial x_r}{\partial p}q_r - N_n q_n}{L\frac{dq_r}{dp} + N_n\frac{dq_n}{dp}}. \tag{C.38}$$

Observe that, when $N_n = 0$, the expression boils down to (4.18). By contrast, when $L = 0$, we get a standard monopoly pricing formula. The intuition for the latter is that the nonlinear pricing final good providers capture the entire consumer surplus via their access fee, B_n, except for what consumers pay for usage of the infrastructure, pq_n. Hence, if there are no linear pricing providers, the infrastructure provider can only generate revenue through its usage fee, i.e. $A = 0$ (see (C.34)). That is, the infrastructure provider is effectively constrained to linear pricing.

Let us now consider the effect of the tax on usage. Differentiating (C.36) and replacing from (C.37) we obtain

$$\frac{\partial p}{\partial t_p} = \frac{p\left(N_n\frac{dq_n}{dp} + L\frac{dq_r}{dp}\right) + N_n q_n + Lq_r}{\frac{d^2\pi_I}{dp^2}} = \frac{(1-t_A)\,Lq_r\left(1 + \frac{\partial x_r}{\partial p}\right) + c\left(N_n\frac{dq_n}{dp} + L\frac{dq_r}{dp}\right)}{\frac{d^2\pi_I}{dp^2}\,(1-t_p)}. \tag{C.39}$$

Therefore,

$$\frac{\partial p}{\partial t_p} < 0 \iff c < -\frac{(1-t_A)\,Lq_r\left(1 + \frac{\partial x_r}{\partial p}\right)}{N_n\frac{dq_n}{dp} + L\frac{dq_r}{dp}}. \tag{C.40}$$

This expression shows that as long as $L \geq 1$, the condition for the tax on usage to decrease the usage fee (and hence, consumption) is very similar to (4.23), which is derived assuming $N_n = 0$. However, when $N_n > 0$ there is an additional term in the denominator, which implies that satisfying the condition for p to decrease with t_p becomes harder. Note also that, if $L = 0$, then $\frac{\partial p}{\partial t_p} > 0$ because $cN_n\frac{dq_n}{dp} < 0$. Thus, when $L = 0$ and $c > 0$, the effect of the tax on p is positive. When both c and L are zero, however, taxation has no effect.

C.6 Taxing final goods

Unit tax on final goods. Assume the government imposes a unit tax, τ_F, on the final goods. A final good provider's profit is

$$\pi_j = (x_j - \phi - \tau_F)\, q_j, \quad j = 1, .., N. \tag{C.41}$$

The first-order conditions from maximizing (C.41) are

$$\frac{\partial \pi_j}{\partial x_j} = q_j + (x_j - \phi - \tau_F)\, \frac{\partial q_j}{\partial x_j} = 0, \qquad j = 1, .., N. \tag{C.42}$$

By differentiating (C.42), we get

$$\frac{\partial x_j}{\partial \tau_F} = \frac{\frac{\partial q_j}{\partial x_j}}{\frac{d^2 \pi_j}{dx_j^2}} > 0, \qquad j = 1, .., N. \tag{C.43}$$

The inequality follows from the fact that the numerator is negative because $\frac{\partial q_j}{\partial x_j} > 0$, and the denominator is negative by concavity of π_j.

Ad valorem tax on final goods. Assume the government imposes an ad valorem tax, t_F, on final goods. The final goods' producers profit is

$$\pi_j = (1 - t_F)\, x_j q_j - \phi q_j, \quad j = 1, .., N, \tag{C.44}$$

The first-order conditions from maximizing (C.44) are

$$\frac{\partial \pi_j}{\partial x_j} = (1 - t_F) \left(q_j + x_j \frac{\partial q_j}{\partial x_j} \right) - \phi \frac{\partial q_j}{\partial x_j} = 0, \qquad j = 1, .., N. \tag{C.45}$$

By differentiating (C.45), we get

$$\frac{\partial x_j}{\partial t_F} = \frac{q_j + x_j \frac{\partial q_j}{\partial x_j}}{\frac{d^2 \pi_j}{dx_j^2}} \geq 0, \qquad j = 1, .., N. \tag{C.46}$$

Given (C.45), the numerator of the above expression equals $\frac{\phi}{1 - t_F} \frac{\partial q_j}{\partial x_j}$. As the denominator is also negative by concavity of π_j, we therefore get $\frac{\partial x_j}{\partial t_F} \geq 0$ for $\phi \geq 0$.

C.7 Allowing for $p < c$ when deriving optimal tax rates

We relax the assumption that $p \geq c$ when deriving the optimal tax rates and show that our main results continue to hold. Starting from the analysis in Section 4.1, the optimal usage fee described in expression (4.18) is

$$p = \frac{1}{1-t_p}\left((t_p - t_A)\frac{q}{\frac{dq}{dp}} + (1-t_A)\frac{\frac{\partial x}{\partial p}q}{\frac{dq}{dp}} + c\right). \tag{C.47}$$

To implement the first-best allocation, the government should set taxes such that $p+x-c-\phi = 0$. Plugging (C.47) into $p+x-c-\phi = 0$, using $x - \phi = -\frac{q}{\frac{\partial q}{\partial x}}$ from the final good providers' problem and $\frac{\partial q}{\partial x} = \frac{\partial q}{\partial p}$, we get

$$\frac{1}{1-t_p}\left[t_p\left(\frac{q}{\frac{dq}{dp}} + c\right) - t_A\left(1 + \frac{\partial x}{\partial p}\right)\frac{q}{\frac{dq}{dp}} + \frac{\frac{\partial x}{\partial p}q}{\frac{dq}{dp}}\right] = \frac{q}{\frac{\partial q}{\partial p}}.$$

Multiplying by $\frac{dq}{dp}$, dividing by q and solving for t_p yields

$$t_p = \frac{1 + t_A\left(1 + \frac{\partial x}{\partial p}\right)}{1 + \frac{c}{q}\frac{dq}{dp} + \left(1 + \frac{\partial x}{\partial p}\right)}. \tag{C.48}$$

The numerator of this expression is strictly positive, because $0 < \left(1 + \frac{\partial x}{\partial p}\right) < 1$ and $-1 \leq t_A \leq 1$. Furthermore, given $t_A = 0$ and under the condition (4.23) in the text, the denominator is positive as well. Hence, the government can implement the optimal allocation setting $t_A = 0$ and the positive t_p described in (C.48).

In our analysis above, we assumed that the government is unconstrained in implementing its tax rates, i.e. we allowed for $p < 0$ in the optimum. However, because a negative usage fee may not be feasible, we now derive the optimal tax rates under the constraint $p \geq 0$. In the analysis above, the optimum requires that $p = c + \frac{q}{\frac{\partial q}{\partial x}}$, with $\frac{q}{\frac{\partial q}{\partial x}} = -(x - \phi)$. Suppose now that $c + \frac{q}{\frac{\partial q}{\partial x}} < 0$ at the optimal allocation which implies that the constraint $p \geq 0$ is binding. Using (4.18), setting $p = 0$, and solving for t_p we obtain

$$t_p = -\frac{\partial x}{\partial p} + t_A\left(1 + \frac{\partial x}{\partial p}\right) - \frac{c}{q}\frac{dq}{dp}.$$

Given $-\frac{\partial x}{\partial p} > 0$ and $\frac{dq}{dp} < 0$, the above expression is strictly positive when $t_A = 0$. Therefore,

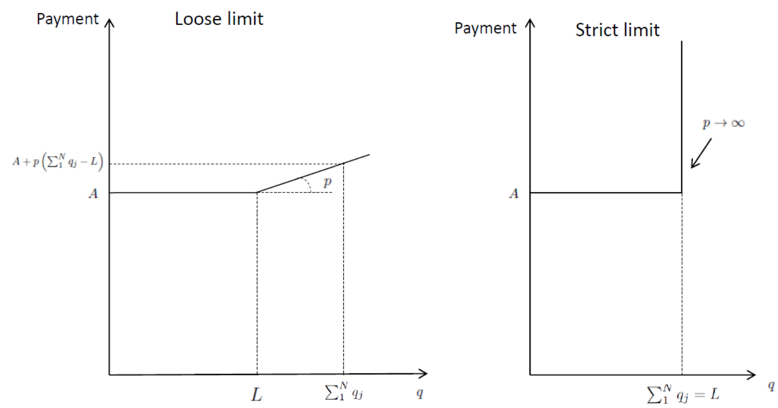

Figure 1: Illustration of the structure of three-part tariffs.

the government can again implement the constrained optimal allocation setting $t_A = 0$ and $t_p > 0$.

C.8 Introducing three-part tariffs

We show that there is no loss of generality in assuming that the firm adopts the two-part tariff as given in (4.3), instead of a three-part tariff of the form

$$T_I = A + p \cdot max\left\{0, \sum_1^N q_j - L\right\}, \tag{C.49}$$

where L is a consumption limit (i.e., a certain quantity of service bundled with access) and p is a per unit fee of this good applying to all units over the limit (an "overage charge"). Figure 1 provides an illustration of this tariff).

We generalize the tax structure assumed in the main text as follows: t_p applies to the payment a consumer makes for usage, the latter being valued at the marginal unit price. Formally, the tax base for t_p is $p\sum_1^N q_j$ if $\sum_1^N q_j \geq L$, and zero otherwise. The tax rate t_A applies instead to the access payment A. Letting Q denote the quantity of consumers that acquires access, firm I's tax burden is thus

$$b_I = Qt_A A \quad if \quad \sum_1^N q_j < L, \tag{C.50}$$

$$b_I = Q\left(t_A A + t_p p \sum_1^N q_j\right) \quad if \quad \sum_1^N q_j \geq L. \tag{C.51}$$

Observe that this tax structure coincides with the one we assume in the paper when the firm adopts a two-part tariff, i.e. $L = 0$.

Given the above assumptions, we claim

Lemma C.1. *Assume the infrastructure provider adopts a multi-part tariff as in* (C.49). *There always exists a two-part tariff that induces either the same equilibrium or one where I makes strictly higher after-tax profits.*

Lemma C.1 implies that, without loss of generality, we can focus on two-part tariffs such as (4.3). Before turning to the proof, consider the intuition, which runs as follows. Consider an equilibrium where the firm adopts a tariff as in (C.49) and $\sum_1^N q_j > L$ holds.[29] Because the access fee A is sunk at the last stage, only the usage fee p is relevant for the consumer's decision at the margin. Suppose the firm adopts (4.3) and sets p equal to the same overage price as in the original (multi-part) tariff. Then, the quantity $\sum_1^N q_j$ chosen by the consumer does not change, provided the prices of the final goods are also the same. This is indeed the case, because the demand faced by the final good sellers is unchanged. Furthermore, although the consumer pays an additional amount pL for usage with the two-part tariff, the firm can discount it from the access fee, as shown in the left panel of Figure 1, so that the total charge on the consumer does not change. Finally, the firm pays a usage tax equal to $t_p p \sum_1^N q_j$ in both equilibria, but the access tax payment is smaller.

Suppose now the initial equilibrium entails a binding consumption limit $\sum_1^N q_j = L$ and p is larger than the net surplus consumers get from the marginal additional unit. The firm can adopt a two-part tariff such that individuals still consume L units (see the right panel of Figure 1). However, the final good sellers charge lower prices in the new equilibrium. The reason is that they perceive consumers' demand as less elastic when the consumption limit is binding. Hence, the firm is able to extract a larger total payment from each consumer than with a three-part tariff, thus making more profit.[30]

Proof of Lemma C.1

We divide the proof into two parts. In Part 1 we consider the case where $\sum_1^N q_j > L$, while in Part 2 we look at the case where $\sum_1^N q_j = L$ and the consumption limit binds. In each part,

[29] The case where $\sum_1^N q_j < L$ is trivial, because p is irrelevant for the consumer.

[30] Economides and Hermalin (2015, Proposition 4) show that, with a finite number of final good sellers, a strict limit results in lower profits for the infrastructure provider than a two-part tariff. Note also that, when the usage tax is positive, the firm can always reduce the tax burden by avoiding a binding limit. The reason is that, if the limit is binding, the firm can always reduce p without changing the quantity of consumption.

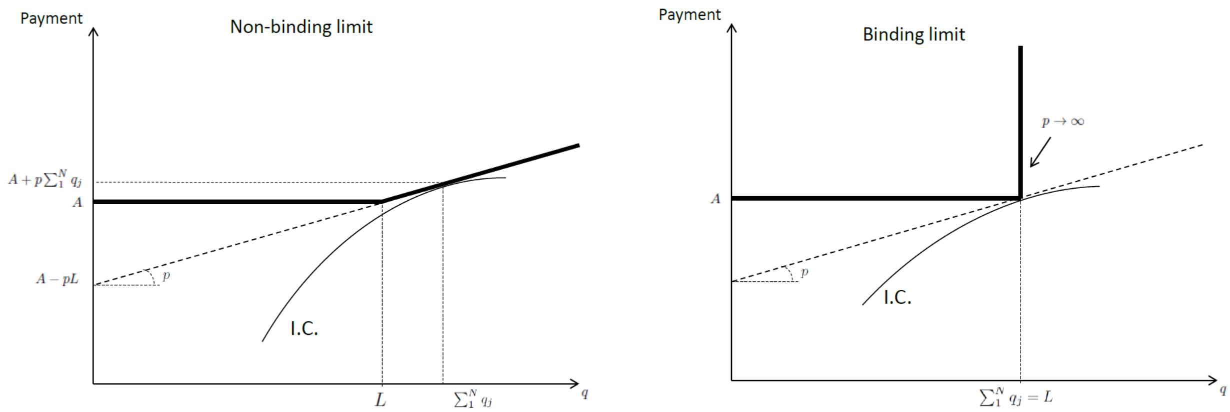

Figure 2: The bold line denotes the tariff, the solid curve is the indifference curve. Left panel: equilibrium with a non-binding consumption limit. The dashed line is the two-part tariff that induces the same consumption and total payment. Right panel: equilibrium with strict (binding) consumption limit. The dashed line is the two-part tariff that induces the same consumption.

we proceed by steps. First, we prove that, starting from an initial equilibrium when the tariff is as in (C.49), the infrastructure provider can implement the same equilibrium by adopting a two-part tariff, obtaining the same pre tax profit. Second, we show that the equilibrium with the two-part tariff is such that the after-tax profit cannot be lower than in the original equilibrium.

Part 1. *STEP* 1: Consider an equilibrium where the infrastructure provider sets a multi-part tariff, prices for the final goods are $\{x_j\}_{j=1,..,N}$, consumers buy q_j units from seller j and that $\sum_1^N q_j > L$ holds. Let Q be the number of consumers that connect to the infrastructure provider. We refer to this equilibrium as the "original" equilibrium, and denote the components of the firm's tariff with the subscript o. Hence

$$T_I = A_o + p_o\left(\sum_1^N q_j - L\right), \qquad b_I = Q\left(t_p\cdot p_o\cdot\sum_1^N q_j + t_A\cdot A_o\right), \qquad \pi_I = Q\left(T_I - b_I - c\sum_1^N q_j\right). \tag{C.52}$$

In the original equilibrium, the quantity q_j that maximizes

$$U = \sum_1^N u\left(q_j\right) + M - A_o - p_o\left(\sum_1^N q_j - L\right) - \sum_1^N x_j q_j, \tag{C.53}$$

is implicitly given by the following FOC

$$\frac{du}{dq_j} = p_o + x_j, \quad j = 1, .., N. \tag{C.54}$$

Suppose now the infrastructure provider adopts the two-part tariff $T_{2P} = A + p\sum_1^N q_j$, with $p = p_o$ and $A = A_o - p_oL$. Assume that the set of prices $\{x_j\}_{j=1,..,N}$ in the equilibrium with T_{2P} is identical to the original equilibrium (we prove that this condition holds below). Because $p = p_o$, the FOC for consumption is identical to (C.54), implying that q_js are identical in the two scenarios, for any j.

We now prove that the set $\{x_j\}_{j=1,..,N}$ under T_{2P} is indeed the same as in the original equilibrium. Recall that Q and the prices of the other sellers are given when seller j sets x_j. Hence, (C.54) locally defines the demand function $q_j(p_o, x_j)$ faced by seller j. The equilibrium price x_j satisfies

$$\frac{\partial \pi_j}{\partial x_j} = q_j(p_o, x_j) + (x_j - \phi)\frac{\partial q_j(p_o, x_j)}{\partial x_j} = 0.$$

The overage price p_o in the original equilibrium is given in a neighborhood of the equilibrium quantity q_j and the same applies to p in T_{2P}. Furthermore, because $p = p_o$ by assumption, then $q_j(p, x_j) = q_j(p_o, x_j)$. Hence, if x_j is the unique maximizer of π_j in the original equilibrium with a multi-part tariff, it must be also when T_{2P} is implemented. It follows that the set of prices $\{x_j\}_{j=1,..,N}$ does not change if the infrastructure provider adopts T_{2P} . Accordingly, also the quantities q_j and Q are unchanged.

We have therefore established that, by adopting T_{2P}, the infrastructure provider can induce the same equilibrium as with a multi-part tariff, obtaining the same pre-tax profit $A + (p - c)\sum_1^N q_j = A_o - p_oL + (p_o - c)\sum_1^N q_j$.

STEP 2: Next, we show that the tax burden b_I in the original equilibrium is weakly larger than in the equilibrium with T_{2P}. To see this, consider that the burden when T_{2P} is adopted is

$$b_{2P} = Q\left(t_p \cdot p \cdot \sum_1^N q_j + t_A \cdot A_{2P}\right) = Q\left(t_p \cdot p_o \cdot \sum_1^N q_j + t_A \cdot (A - p_oL)\right),$$

while the tax burden in the original equilibrium is

$$b_I = Q\left(t_p \cdot p_o \cdot \sum_1^N q_j + t_A \cdot A_o\right).$$

The latter is weakly higher than the former, given that quantities are the same and $p_oL \geq 0$.

Part 2. *STEP 1*: Suppose that the consumption limit is binding in the original equilibrium (that is, with the multi-part tariff as in (C.49)). Faced with the tariff T_I and the set of prices $\{x_j\}_{j=1,..,N}$, consumers get $\sum_1^N q_j = L$ units, with $p_o + x_j > u'(q_j)$. Consider the choice of consumption quantities q_j. Because the constraint $\sum_1^N q_j \le L$ is binding, the set of equilibrium quantities $\{q_j\}_{j=1,..,N}$ is the solution to the following problem

$$\max_{\{q_j\}_{j=1,..,N}} \quad \sum_1^N u(q_j) + M - A - \sum_1^N x_j q_j \quad \text{s.t.} \sum_1^N q_j = L.$$

Denoting by λ the Lagrange-multiplier associated with the constraint $\sum_1^N q_j = L$, the set of FOCs are given by

$$\frac{du}{dq_j} = \lambda + x_j, \quad j = 1, .., N. \tag{C.55}$$

Consider now the problem of final good provider j when I adopts a multi-part tariff. Equation (C.55) implicitly defines the demand $q_j(\lambda, x_j)$ for j's good. Because the infrastructure provider's tariff and the set $\{x_j\}_{-j}$ of other sellers' prices are given when j sets x_j, the equilibrium value of x_j satisfies the following FOC

$$\frac{\partial \pi_j}{\partial x_j} = q_j(\lambda, x_j) + (x_j - \phi)\frac{\partial q_j(\lambda, x_j)}{\partial x_j} = 0, \tag{C.56}$$

where

$$\frac{dq_j}{dx_j} = \frac{\partial q_j}{\partial x_j} + \frac{\partial q_j}{\partial \lambda}\frac{\partial \lambda}{\partial x_j} = \frac{\partial q_j}{\partial x_j}\left(1 + \frac{\partial \lambda}{\partial x_j}\right). \tag{C.57}$$

The last equality follows from the fact that $\frac{\partial q_j}{\partial x_j} = \frac{\partial q_j}{\partial \lambda} < 0$, as implied by (C.55) and concavity of $u(q_j)$. Furthermore, $\frac{\partial \lambda}{\partial x_j} < 0$, because the shadow price of additional consumption decreases when the final good becomes more expensive. Finally, the fact that sellers are symmetric implies that $x_j = x$ for any j. Therefore, given (C.55), we also have $q_j = \frac{L}{N}$ for any j.

Let us now assume that the infrastructure provider adopts a two-part tariff $T_{2P} = A + p\sum_1^N q_j$ designed such that (i) the equilibrium quantity is $q_j = \frac{L}{N}$ for each final good (although there is no binding limit) and (ii) the number of connected consumers Q are identical to the original equilibrium. We show that adopting this tariff yields a pre tax profit for the infrastructure provider which is weakly higher than in the original equilibrium. When the consumer faces T_{2P} and a set of prices $\{x_j\}_{j=1,..,N}$, the quantity q satisfies

$$\frac{du}{dq_j} = p + x_j, \quad j = 1, .., N. \tag{C.58}$$

By assumption, the usage fee p is chosen such that (C.58) yields $q = \frac{L}{N}$. This equality can hold only if there are no differences between the prices charged by the sellers, but this is indeed the case. Because Q, T_{2P} and the set $\{x_j\}_{-j}$ of other sellers' prices are given when j sets x_j, seller j's price for the final good is implicitly determined by

$$\frac{\partial \pi_j}{\partial x_j} = q_j\left(p, x_j\right) + \left(x_j - \phi\right) \frac{\partial q_j\left(p, x_j\right)}{\partial x_j} = 0. \tag{C.59}$$

Because p is constant and the same for all j, x_j is identical for all sellers and we denote it by x_{2P}.

Let us now evaluate the two FOCs (C.56) and (C.59) in the neighborhood of the same value for q. They differ in their second terms. Because $\frac{\partial q_j}{\partial x_j} = \frac{1}{u''(q_j)} < 0$, we get that $\frac{dq_j}{dx_j} = \frac{\partial q_j}{\partial x_j}\left(1 + \frac{\partial x_j}{\partial \lambda}\right) > \frac{\partial q_j}{\partial x_j}$, which means that the level of x_j that satisfies (C.56) must be larger than that which satisfies (C.59). Therefore, the equilibrium quantities q are identical in the two equilibria, but the price for the final good is smaller when the two-part tariff is adopted, i.e., $x_{2P} < x$. Accordingly, the surplus a connecting consumer gets from consumption, $\sum_1^N u\left(q\right) - Nxq$, is smaller in the original equilibrium than under T_{2P}. Assume the infrastructure provider intends to serve the same number of consumers, Q. Because the consumer gets more net surplus from consumption, the infrastructure provider can extract from each consumer a total payment that is at least as large as in the original equilibrium. Recall that, in this equilibrium, each consumer pays just A_o to the infrastructure provider. With T_{2P}, a consumer's total payment is $A + p\sum_1^N q_j = A + pL$, which exceeds A_o by our previous arguments. Thus, by adopting T_{2P}, the infrastructure provider's pre-tax profit cannot be smaller.

Note that the tariff T_{2P} described above must be such that $p < p_o$. To see why, recall that, by assumption, $p_o + x > \left.\frac{du}{dq_j}\right|_{\frac{L}{N}}$ in the original equilibrium. If $p \geq p_o$ when T_{2P} is adopted, because (as we have shown) $p + x$ increases with p, then $p + x > \left.\frac{du}{dq_j}\right|_{\frac{L}{N}}$, implying that the consumer would choose a quantity q smaller than $\frac{L}{N}$. This outcome contradicts our assumption that T_{2P} induces $q = \frac{L}{N}$.

STEP 2: The infrastructure provider's after-tax profit is $\pi_I = A_o - \left(t_p p_o L + t_A A_o\right) - cL$ in the original equilibrium with a multi-part tariff and $\pi_{I,2P} = A - \left(t_p pL + t_A A\right) - cL$ under the two-part tariff. The difference between the after-tax profits is thus given by

$$\pi_{I,2P} - \pi_I = Q\left[\left(A + pL - A_o\right) + t_A\left(A_o - A\right) + t_p L\left(p_o - p\right)\right].$$

With non-negative tax rates, the above difference cannot be negative, because $A + pL \geq A_o$ and $p_o > p$. The term in squared brackets is bounded from below by $pL + t_p L (p_o - p) \geq 0$.

D Extensions to Section 4.2

D.1 Three-part tariffs with menus of tariffs

In this section, we argue that there is no loss of generality in assuming the firm adopts a menu of two-part tariffs, as done in Section 4.2, compared to the case where the firm is allowed to charge three-part tariffs.

Lemma D.1. *For any equilibrium where the infrastructure provider adopts a menu of multi-part tariffs as in* (C.49)*, there exists a menu of two-part tariffs* $A_i + p_i q, \quad i = h, \ell$ *which induces the same equilibrium quantities, but yields weakly higher after-tax profits for the downstream firm.*

The intuition for Lemma D.1 is similar to that for Lemma C.1. A menu of multi-part tariffs can be replaced by two-part tariffs inducing the same consumption by the type they are intended for, and a total payment to the downstream firm that is at least arbitrarily close to the initial one, while tax payments cannot be higher. Thus, the downstream firm cannot be worse off.

Proof of Lemma D.1

We divide the proof in two steps. First, we show that the pre tax profit of the downstream firm when adopting a multi-part tariff is arbitrarily close to the profit when the two-part tariff menu is adopted. Second, we show that the downstream firm's tax burden when adopting the menu of two-part tariffs cannot be higher than with a menu of multi-part tariffs. Therefore, the infrastructure provider's post-tax profit is at least arbitrarily close to that in the original equilibrium.

STEP 1: Consider a menu of tariffs $T_M = \{T_\ell, T_h\}$, with the structure described in (C.49), and the associated equilibrium. Let b_I be the associated tax payment by the infrastructure provider. Let also q_h (resp. q_ℓ) be the quantity chosen by type-h (resp. type-ℓ) when choosing the tariff intended for its type. Further, let $T_{h\ell}$ (resp. $T_{\ell h}$) be the out-of-equilibrium payment that type h (resp. ℓ) makes when choosing the tariff intended for the other type. Similarly, $q_{h\ell}$ and $q_{\ell h}$ are the out-of-equilibrium quantities. For convenience, we refer to this equilibrium as the “original” equilibrium, and denote the components of the infrastructure provider’s tariff with the subscript o. Let $U_i(T_{i\tilde{i}}, q_{i\tilde{i}})$ be the utility of a consumer of type i given the payment $T_{i\tilde{i}}$ and the quantity $q_{i\tilde{i}}$, with $i, \tilde{i} = h, \ell$ and recall we drop the double index when $i = \tilde{i}$. By assumption, the following conditions must be satisfied in the original equilibrium

$$U_h(T_h, q_h) \geq U_h\left(T_{h\ell}, q_{h\ell}\right) \tag{D.1}$$

$$U_\ell(T_\ell, q_\ell) \geq U_\ell(T_{\ell h}, q_{\ell h}) \tag{D.2}$$

$$U_i(T_i, q_i) \geq M \quad i = h, \ell \tag{D.3}$$

Conditions (D.1) and (D.2) are incentive compatibility constraints. Conditions (D.3) are participation constraints.

Using standard arguments (see, e.g., Laffont and Martimort, 2001), one can show that (D.1) and (D.3) for $i = \ell$ are binding in the original equilibrium, whereas (D.3) for $i = h$ and (D.2) are slack. Furthermore, as we now argue, the condition $q_{h\ell} = q_\ell$ must hold in the original equilibrium. To see this, consider that when an h-type mimics an ℓ-type, the former’s utility is $U_h\left(T_{h\ell}, q_{h\ell}\right) = u\left(q_{h\ell}, \alpha_h\right) - A_{o\ell} - \phi q_{h\ell} - p_{o\ell} \cdot max\left\{q_{h\ell} - L_\ell; 0\right\}$. Because $\frac{\partial^2 u}{\partial q \partial \alpha} > 0$, whenever $q_{h\ell} > q_\ell \geq L_\ell$ the condition $\frac{\partial u(q_\ell, \alpha_h)}{\partial q} - \phi > p_{o\ell} \geq \frac{\partial u(q_\ell, \alpha_\ell)}{\partial q} - \phi$ holds. Similarly, whenever $q_{h\ell} > q_\ell$ and $q_\ell < L_\ell$, the condition $\frac{\partial u(q_\ell, \alpha_h)}{\partial q} - \phi > \frac{\partial u(q_\ell, \alpha_\ell)}{\partial q} - \phi = 0$ holds. Therefore, the marginal surplus a mimicker gets from consuming units in excess of q_ℓ is strictly positive. Hence, the infrastructure provider can always reduce $q_{h\ell}$ up to $q_{h\ell} = q_\ell$, and thereby reduce the mimicker’s utility $U_h\left(T_{h\ell}, q_{h\ell}\right)$. This move is profitable because it does not produce any change in the equilibrium quantities, but reduces the information rent left to the h-types and, hence, raises the total payment that the infrastructure provider can extract. Note that the optimality of $q_{h\ell} = q_\ell$ implies that $L_\ell = q_\ell$ in the original equilibrium: because $\frac{\partial^2 u}{\partial q \partial \alpha} > 0$, $q_{h\ell} = q_\ell$ can hold only if $p_{o\ell} \geq \frac{\partial u(q_\ell, \alpha_h)}{\partial q} - \phi > \frac{\partial u(q_\ell, \alpha_\ell)}{\partial q} - \phi$.

Keeping the original tariff T_ℓ fixed, the infrastructure provider can replace T_h by a two-part tariff $T_{2P,h} \equiv A_h + p_h q$ such that $p_h = u'\left(q_h, \alpha_h\right)$ and $A_h = T_h - p_h q_h$, obtaining the same payment from h-types, and inducing the same quantity q_h. Observe that, because (D.2) is slack in the original equilibrium, it is also when $T_{2P,h}$ is adopted, given that q_h and the total payment $A_h + p_h q_h = T_h$ are unchanged. The same applies to (D.3) for $i = h$.

Assume now that $T_{2P,h}$ is the tariff intended for h-types. The infrastructure provider can replace the original tariff T_ℓ with a “quasi” two-part tariff $T_{2P,\ell} \equiv A_\ell + p_\ell q + z \cdot max\left\{0, q - L_{2P}\right\}$, with $p_\ell = u'\left(L_\ell, \alpha_\ell\right) - \phi$, such that the following conditions are satisfied: (i) q_ℓ does not change, (ii) constraints (D.1) and (D.3) for $i = \ell$ remain binding and (iii) the total payment collected from h- and ℓ-types is arbitrarily close to their payment in the original equilibrium. To see how these conditions can be satisfied, note first that $p_\ell = u'(L_\ell, \alpha_\ell) - \phi$ implies that $q_\ell = L_\ell$ when $T_{2P,\ell}$ is implemented. Second, by setting $A_{2P,\ell} = T_\ell - p_\ell q_\ell$, the infrastructure provider ensures that the ℓ-type's total payment is the same as in the original equilibrium. Third, by setting z large enough and $L_{2P} = L_\ell + \epsilon$, where $\epsilon > 0$ and arbitrarily small, the infrastructure provider ensures that an h-type mimicker consumes a quantity arbitrarily close to q_ℓ. Hence, the mimicker's payoff $U_h(T_{h\ell}, q_{h\ell})$ is arbitrarily close to her payoff in the original equilibrium. Thus, adopting $T_{2P,h}$ and $T_{2P,\ell}$ implies that (D.2) is slack and (D.3) for $i = \ell$ is binding. It follows that the h-type's payment $T_{2P,h} = A_{2P,h} + p_h q_h$ can also be made arbitrarily close to that in the original equilibrium. Thus, the firm's pre-tax profit is arbitrarily close to the original one, $T_\ell(1-\sigma) + T_h\sigma$. Note that $T_{2P,\ell}$ is such that $z \cdot max\{0, q - L_{2P}\} = 0$ in equilibrium.

STEP 2: Given positive tax rates, the firm's tax burden when adopting $\{T_{2P,\ell}, T_{2P,h}\}$ is bounded from above by the burden in the original equilibrium, which is given by

$$b_I = t_p\left(p_{o\ell} \cdot L_\ell \cdot (1-\sigma) + p_{oh} \cdot q_h \cdot \sigma\right) + t_A\left((1-\sigma) A_{o\ell} + \sigma A_{oh}\right),$$

with $p_{o\ell} > \frac{\partial u(L_\ell, \alpha_\ell)}{\partial q} - \phi$ and $p_{oh} = \frac{\partial u(q_h, \alpha_h)}{\partial q} - \phi$. By contrast, when $\{T_{2P,\ell}, T_{2P,h}\}$, the tax burden is

$$b_{I,2P} = t_p\left(p_\ell \cdot L_\ell \cdot (1-\sigma) + p_h \cdot q_h \cdot \sigma\right) + t_A\left((1-\sigma) A_\ell + \sigma A_h\right),$$

Because $A_{2P,\ell} = T_\ell - p_\ell q_\ell = A_\ell - p_\ell L_\ell$ and $A_{2P,h} = T_h - p_h q_h = A_h - p_h L_h$, we get

$$b_{I,2P} = t_p\left(p_\ell \cdot L_\ell \cdot (1-\sigma) + p_h \cdot q_h \cdot \sigma\right) + t_A\left((1-\sigma)(A_\ell - p_\ell L_\ell) + \sigma(A_h - p_h L_h)\right).$$

with $p_\ell = \frac{\partial u(L_\ell, \alpha_\ell)}{\partial q} - \phi$ and $p_h = \frac{\partial u(q_h, \alpha_h)}{\partial q} - \phi$. Hence, $p_h = p_{oh}$. Furthermore, $p_{o\ell} > \frac{\partial u(q_\ell, \alpha_\ell)}{\partial q} - \phi = p_\ell$. Therefore, $b_{I,2P} \leq b_I$ holds.

E Consumer heterogeneity in the value of access

In the model of Section 4.2, consumers differ in the utility they derive from final goods. In this Section, we consider a different type of heterogeneity, namely in the intrinsic valuation for accessing the infrastructure, independently of final goods. For example, in the case of Internet connections, some consumers may value non-data-intensive services, such as email or instant messaging, in addition to data-intensive ones such as video and gaming. Furthermore, depending on their location, consumers may face different levels of quality due to, e.g., uneven network coverage.

To focus on the implications of this heterogeneity, we extend the model of Section 4.1, where consumers have identical valuations for final goods as well as access. We assume consumer utility is

$$\sum_{j=1...N} \int_0^{q_j} (\alpha - r)dr + y + (V - \beta z) \quad \text{with} \quad z \sim U(0, \bar{z}), \tag{E.1}$$

where $V > 0$ is the common valuation for access, independently of consumption of final goods. The term $z \in [0, \bar{z}]$, multiplied by the positive parameter β, captures an idiosyncratic cost of access. Hence, the last term in (E.1) can be interpreted as the idiosyncratic utility from access. In the case of Internet connections, V may represent the utility from accessing services, such as email or instant messaging. The parameter z may capture instead the cost of using the Internet for these activities, compared to other networks (e.g., traditional mail or the telephone). In the case of a highway, V may represent the value of accessing other destinations, while z may capture the transportation cost of accessing the highway.

Note that, for ease of exposition, we adopt a specific form of utility from final goods (identical to that in Economides and Hermalin, 2015). We also assume without loss that $N = 1$, to reduce notation clutter. Apart from these modifications, we retain the same assumptions as in Section 4.1.

Given that consumers' budget constraints bind, we can write utility as

$$U = \begin{cases} \int_0^q (\alpha - r)\, dr - (p + x)\, q + V - \beta z - A + M & \text{if acquire access,} \\ M & \text{otherwise.} \end{cases} \tag{E.2}$$

Note that we dropped the index j for final goods as $N = 1$. A consumer acquires access if and only if she receives a positive utility from doing so. Hence, the total number of individuals connecting is given by $Q\,(p, x, A) = \widetilde{z}$, where

$$\tilde{z} = \frac{\int_0^q (\alpha - r)\, dr - (p + x)\, q + V - A}{\beta}, \tag{E.3}$$

denotes the marginal consumer who is indifferent between acquiring access and not. It is straightforward to show that higher prices reduce $\tilde{z}$ and, thus, the number of consumers that connect (the "extensive margin" of consumption).

We begin by characterizing the equilibrium level of consumption of final goods by those who get access (the "intensive margin"). Maximizing (E.2), we obtain that the equilibrium q is such that $u'(q) = p + x$. Moreover, because the profit of the final good provider is $\pi = Q(x - \phi)q$, and Q is given when x is chosen, the optimal price is determined by the same expression as (4.10). Furthermore, as in Section 4.1, x and q are decreasing in p as long as consumer demand is not exceedingly convex (which we assume).

The infrastructure provider maximizes its profit

$$\pi_I = Q\left[(1 - t_A) A + ((1 - t_p) p - c) q\right] \tag{E.4}$$

with respect to the usage and access fee. The solution is

$$p = \frac{1}{1 - t_p}\left((t_p - t_A)\frac{q}{\frac{dq}{dp}} + (1 - t_A)\frac{\frac{\partial x}{\partial p}q}{\frac{dq}{dp}} + c\right), \tag{E.5}$$

$$A = -\frac{Q}{\frac{\partial Q}{\partial A}} - ((1 - t_p)p - c)q = -\frac{Q}{\frac{\partial Q}{\partial A}} - \frac{q^2}{\frac{dq}{dp}}\left((t_p - t_A) + (1 - t_A)\frac{\partial x}{\partial p} + c\right). \tag{E.6}$$

See Appendix E.1 for the derivation. We note that the equilibrium usage fee in (E.5) is identical to that in (4.18). It is therefore not surprising that the effects of taxes on p and q are unchanged with respect to Section 4.1. These are given by (see Appendix E.2 for the derivation):

$$\frac{\partial p}{\partial t_A} > 0, \qquad \frac{\partial p}{\partial t_p} < 0 \Longleftrightarrow c < -\frac{(1 - t_A)q}{\frac{\partial q}{\partial p}}, \tag{E.7}$$

$$\frac{dq}{dt_A} = \frac{dq}{dp}\frac{\partial p}{\partial t_A} < 0, \qquad \frac{dq}{dt_p} = \frac{dq}{dp}\frac{\partial p}{\partial t_p} > 0 \Longleftrightarrow c < -\frac{(1 - t_A)q}{\frac{\partial q}{\partial p}}. \tag{E.8}$$

Concerning the access fee in (E.6), the infrastructure provider now faces a trade-off between extracting more surplus from inframarginal customers and losing the marginal ones. The effects of the tax rates on A are given by

$$\frac{\partial A}{\partial t_A} = \left(Q + A\frac{\partial Q}{\partial A}\right)\frac{\partial^2 \pi_I}{\partial p^2} - A\frac{\partial Q}{\partial p}\frac{\partial^2 \pi_I}{\partial p \partial A}, \tag{E.9}$$

$$\frac{\partial A}{\partial t_p} = pq\frac{\partial Q}{\partial A}\frac{\partial^2 \pi_I}{\partial p^2} - \left[qQ + p\left(Q\frac{dq}{dp} + q\frac{\partial Q}{\partial p}\right)\right]\frac{\partial^2 \pi_I}{\partial p \partial A}. \tag{E.10}$$

These expressions are quite involved and hard to sign. Therefore, the effect of the tax rates on the extensive margin

$$\frac{dQ}{dt_k} = \frac{\partial Q}{\partial A}\frac{\partial A}{\partial t_k} + \frac{\partial Q}{\partial p}\frac{\partial p}{\partial t_k} \quad k = A, p, \tag{E.11}$$

is also ambiguous. To illustrate the ambiguity, consider an increase in the tax on usage and assume $\frac{\partial p}{\partial t_p} < 0$ holds (see (E.7)). The tax results in a higher consumption by those who acquire access. Although this effect increases the consumer's willingness to pay for access, the net effect on the extensive margin depends also on the change in the access fee, which is ambiguous. Nonetheless, we can show that, when $t_A = t_p = 0$ and the marginal cost c is small, the following holds

$$\left.\frac{dQ}{dt_p}\right|_{t_A=0,t_p=0} \gtrless 0 \Longleftrightarrow \left.\frac{dQ}{dt_A}\right|_{t_A=0,t_p=0} \lessgtr 0. \tag{E.12}$$

Which implies that the government can expand the extensive margin of consumption by taxing either access or usage. See Appendix E.3 for the proof.

We now examine the implications for welfare. To simplify the analytics, henceforth we assume that $c = \phi = 0$. These assumptions apply particularly to the case of ISPs and digital final goods. Given these assumptions, we get $\left.\frac{dQ}{dt_p}\right|_{t_A=0,t_p=0} < 0$ and $\left.\frac{dQ}{dt_A}\right|_{t_A=0,t_p=0} > 0$. Thus, taxing usage (resp. access) expands consumption on the intensive (resp. extensive) margin, but restricts the other one. The effect of a marginal change in tax rates is given by

$$\frac{\partial W}{\partial t_k} = Qu'(q^e)\frac{dq}{dt_k} + (u(q) + V - \beta Q)\frac{dQ}{dt_k}, \quad k = A, p. \tag{E.13}$$

The first term in this expression represents the effect of changes in the intensive margin of consumption, which is given by the marginal utility of the final goods, multiplied by the change in consumption induced by the tax. By (E.8), the latter is positive (resp. negative) when t_p (resp. t_A) increases. The second term in (E.13) captures the change in welfare due to changes in the extensive margin of consumption. The factor in brackets is the gross surplus generated by the marginal consumer that acquires access to the infrastructure.

Because each tax rate expands consumption on either the intensive or the extensive margin, but reduces consumption on the other, the effect on welfare is also ambiguous. However, we can show (see Appendix E.4) that

$$\left.\frac{\partial W}{\partial t_p}\right|_{t_A=0,t_p=0} > 0 \Longleftrightarrow V > \frac{\alpha^2}{2}, \tag{E.14}$$

$$\left.\frac{\partial W}{\partial t_A}\right|_{t_A=0,t_p=0} > 0 \Longleftrightarrow V < \frac{\alpha^2}{2}. \tag{E.15}$$

Welfare increases with the tax on usage when the common valuation for access, V, is large compared to the marginal utility from the final goods α. By contrast, the tax on access increases welfare when α is relatively large compared to V. To understand this finding, note from (E.8) that q increases with t_p, and so does the total surplus of consumers that acquire access. On the other hand, A increases, with a negative effect on the quantity of consumers that acquires access. The first effect has greater impact on aggregate surplus the larger is the initial consumer base, Q, which increases with V. Although Q increases with α as well, this parameter also increases the welfare loss from the marginal consumer that does not acquire access. Hence, when V is large enough, welfare increases with a tax on usage. By the same token, when α is large compared to V, welfare increases with a tax on access.

In other words, a large intrinsic valuation implies the initial base of consumers that acquire access is relatively large. That is, a larger V makes the demand for access less elastic. Hence, the increase in consumption of final goods (intensive margin) that results from taxing usage has a positive impact on a large number of consumers, dominating the lost surplus from those who stop using the infrastructure and consuming the final goods (extensive margin). By contrast, with a relatively small initial base of consumers, total surplus increases with a tax on access.

Although the analytical complexity of the problem prevents a full characterization of the optimal tax rates, the conditions provided above are also sufficient for taxes on either usage or access being globally optimal (conditional on the other being zero). Indeed, we show in Appendix E.5 that

$$V > \frac{\alpha^2}{2} \Longrightarrow t_p^* > 0, \tag{E.16}$$

$$\frac{\alpha^2}{2} > V \Longrightarrow t_A^* > 0. \tag{E.17}$$

We summarize in the following

Proposition E.1. *Consider a setting where consumers differ in the intrinsic valuation for access to the infrastructure, imperfect competition in the markets for final goods and a monopolist infrastructure provider adopting a multi-part tariff. Assume also a small marginal cost of infrastructure usage. A tax on usage (respectively, access) increases total surplus if consumers' valuation for access (respectively, for consumption of the final goods) is relatively large.*

E.1 Proof of (E.5) and (E.6)

The first-order conditions of the infrastructure provider are

$$\begin{aligned}\frac{\partial \pi_I}{\partial p} &= \frac{\partial Q}{\partial p} A\left(1-t_A\right)+\left[qQ\left(1-t_p\right)+\left(p\left(1-t_p\right)-c\right)\left(q\frac{\partial Q}{\partial p}+Q\frac{dq}{dp}\right)\right]=0,\\ \frac{\partial \pi_I}{\partial A} &= \frac{\partial Q}{\partial A} A\left(1-t_A\right)+Q\left(1-t_A\right)+\left(\left(1-t_p\right)p-c\right)q\frac{\partial Q}{\partial A}=0.\end{aligned} \tag{E.18}$$

Note that only the direct effect of p on Q matters. Indeed, $\frac{dQ}{dp}=\frac{\partial Q}{\partial p}+\frac{\partial Q}{\partial q}\frac{dq}{dp}$, but the last term is equal to zero because (E.3) and $u'(q)=p+x$ hold in equilibrium. Combining these expressions and solving for p and A, we get (E.5) and (E.6).

E.2 Proof of (E.7), (E.9), and (E.10)

Total differentiation of (E.18) yields

$$\frac{\partial p}{\partial t_p}=-\frac{det\begin{bmatrix}\frac{\partial^2\pi_I}{\partial p\partial t_p} & \frac{\partial^2\pi_I}{\partial p\partial A}\\ \frac{\partial^2\pi_I}{\partial A\partial t_p} & \frac{\partial^2\pi_I}{\partial A^2}\end{bmatrix}}{det\begin{bmatrix}\frac{\partial^2\pi_I}{\partial p^2} & \frac{\partial^2\pi_I}{\partial p\partial A}\\ \frac{\partial^2\pi_I}{\partial p\partial A} & \frac{\partial^2\pi_I}{\partial A^2}\end{bmatrix}},\qquad \frac{\partial p}{\partial t_A}=-\frac{det\begin{bmatrix}\frac{\partial^2\pi_I}{\partial p\partial t_A} & \frac{\partial^2\pi_I}{\partial p\partial A}\\ \frac{\partial^2\pi_I}{\partial A\partial t_A} & \frac{\partial^2\pi_I}{\partial A^2}\end{bmatrix}}{det\begin{bmatrix}\frac{\partial^2\pi_I}{\partial p^2} & \frac{\partial^2\pi_I}{\partial p\partial A}\\ \frac{\partial^2\pi_I}{\partial p\partial A} & \frac{\partial^2\pi_I}{\partial A^2}\end{bmatrix}}. \tag{E.19}$$

We proceed assuming that the determinants at the denominator of these expressions (Hessian matrix) are positive, as required by the second-order conditions of the profit maximization problem. Therefore,

$$\frac{\partial p}{\partial t_p}<0 \Leftrightarrow \frac{\partial^2\pi_I}{\partial p\partial t_p}\frac{\partial^2\pi_I}{\partial A^2}-\frac{\partial^2\pi_I}{\partial A\partial t_p}\frac{\partial^2\pi_I}{\partial p\partial A}>0, \tag{E.20}$$

$$\frac{\partial p}{\partial t_A}>0 \Leftrightarrow \frac{\partial^2\pi_I}{\partial p\partial t_A}\frac{\partial^2\pi_I}{\partial A^2}-\frac{\partial^2\pi_I}{\partial A\partial t_A}\frac{\partial^2\pi_I}{\partial p\partial A}<0. \tag{E.21}$$

It is useful to note the following equalities that follow from (E.18), $\frac{\partial Q}{\partial A}=-\frac{F'(\tilde{z})}{\beta}$ and the fact that $\frac{\partial Q}{\partial p}=-\frac{F'(\tilde{z})}{\beta}N\left(q+p\frac{dq}{dp}\right)$:

$$A\left(1-t_A\right)+N\left(\left(1-t_p\right)p-c\right)q=-\frac{Q\left(1-t_A\right)}{\frac{\partial Q}{\partial A}}, \tag{E.22}$$

$$\left(\left(1-t_p\right)\left(q+p\frac{dq}{dp}\right)-c\frac{dq}{dp}\right)\frac{\partial Q}{\partial A}=\left(1-t_A\right)\frac{\partial Q}{\partial p}. \tag{E.23}$$

$$\frac{\partial Q}{\partial p} = q\left(1+\frac{\partial x}{\partial p}\right)\frac{\partial Q}{\partial A}. \tag{E.24}$$

We now establish the sign of $\frac{\partial p}{\partial t_p}$. Using the FOCs in (E.18), we get $\frac{\partial^2 \pi_I}{\partial A \partial t_p} = -pq\frac{\partial Q}{\partial A}$, $\frac{\partial^2 \pi_I}{\partial p \partial t_p} = -\left(qQ + pQ\frac{dq}{dp} + pq\frac{\partial Q}{\partial p}\right) = -\frac{Q}{1-t_p}\left(q\left(1-t_A\right)\left(1+\frac{\partial x}{\partial p}\right)+c\frac{dq}{dp}\right) - pq\frac{\partial Q}{\partial p}$. The last equality follows from replacing p from (E.5). Furthermore, using (E.22) and (E.23), we get $\frac{\partial^2 \pi_I}{\partial A^2} = \left(1-t_A\right)\left(-\frac{\partial^2 Q}{\partial A^2}\frac{Q}{\frac{\partial Q}{\partial A}} + 2\frac{\partial Q}{\partial A}\right)$. Note that, because π_I is concave by assumption, this expression has to be negative. Finally, using (E.22) and (E.23), we have $\frac{\partial^2 \pi_I}{\partial p \partial A} = \left(1-t_A\right)\left(2\frac{\partial Q}{\partial p} - \frac{\partial^2 Q}{\partial p \partial A}\frac{Q}{\frac{\partial Q}{\partial A}}\right)$. Combining these equalities, and using (E.24), we get

$$\frac{\partial p}{\partial t_p} = \frac{\partial^2 \pi_I}{\partial p \partial t_p}\frac{\partial^2 \pi_I}{\partial A^2} - \frac{\partial^2 \pi_I}{\partial A \partial t_p}\frac{\partial^2 \pi_I}{\partial p \partial A} = -Q\frac{1+\frac{\partial x}{\partial p}}{1-t_p}\left(\left(1-t_A\right)q + c\frac{\partial q}{\partial p}\right)\left(-\frac{\partial^2 Q}{\partial A^2}\frac{Q}{\frac{\partial Q}{\partial A}} + 2\frac{\partial Q}{\partial A}\right). \tag{E.25}$$

Because $Q\frac{1+\frac{\partial x}{\partial p}}{1-t_p} > 0$ and the last term in brackets is negative, we get $\frac{\partial p}{\partial t_p} < 0$ if and only if $\left(1-t_A\right)q + c\frac{\partial q}{\partial p} > 0$.

We turn now to $\frac{\partial p}{\partial t_A}$. Using the previous results, and the fact that $\frac{\partial^2 \pi_I}{\partial A \partial t_A} = -\left(A\frac{\partial Q}{\partial A} + Q\right)$ and $\frac{\partial^2 \pi_I}{\partial p \partial t_A} = -A\frac{\partial Q}{\partial p}$, rearranging the terms, we obtain

$$\frac{\partial p}{\partial t_A} = \frac{\partial^2 \pi_I}{\partial p \partial t_A}\frac{\partial^2 \pi_I}{\partial A^2} - \frac{\partial^2 \pi_I}{\partial A \partial t_A}\frac{\partial^2 \pi_I}{\partial p \partial A} = Qq\left(1+\frac{\partial x}{\partial p}\right)\left(-\frac{\partial^2 Q}{\partial A^2}\frac{Q}{\frac{\partial Q}{\partial A}} + 2\frac{\partial Q}{\partial A}\right). \tag{E.26}$$

Again, because the last term in brackets is negative and $Qq\left(1+\frac{\partial x}{\partial p}\right) > 0$, we have $\frac{\partial p}{\partial t_A} > 0$. Observe also that, when $c \to 0$, (E.19), (E.25) and (E.26) imply that

$$\frac{dp}{dt_A} = -\frac{1-t_p}{1-t_A}\frac{dp}{dt_p}. \tag{E.27}$$

Let us now consider the effect of tax rates on A. We get

$$\frac{\partial A}{\partial t_p} = -\frac{det\begin{bmatrix}\frac{\partial^2 \pi_I}{\partial p^2} & \frac{\partial^2 \pi_I}{\partial p \partial t_p}\\ \frac{\partial^2 \pi_I}{\partial p \partial A} & \frac{\partial^2 \pi_I}{\partial A \partial t_p}\end{bmatrix}}{det\begin{bmatrix}\frac{\partial^2 \pi_I}{\partial p^2} & \frac{\partial^2 \pi_I}{\partial p \partial A}\\ \frac{\partial^2 \pi_I}{\partial p \partial A} & \frac{\partial^2 \pi_I}{\partial A^2}\end{bmatrix}}, \qquad \frac{\partial A}{\partial t_A} = -\frac{det\begin{bmatrix}\frac{\partial^2 \pi_I}{\partial p^2} & \frac{\partial^2 \pi_I}{\partial p \partial t_A}\\ \frac{\partial^2 \pi_I}{\partial p \partial A} & \frac{\partial^2 \pi_I}{\partial A \partial t_A}\end{bmatrix}}{det\begin{bmatrix}\frac{\partial^2 \pi_I}{\partial p^2} & \frac{\partial^2 \pi_I}{\partial p \partial A}\\ \frac{\partial^2 \pi_I}{\partial p \partial A} & \frac{\partial^2 \pi_I}{\partial A^2}\end{bmatrix}}.$$

As before, we assume that the denominator of these expressions is positive. Therefore, we have

$$\frac{\partial A}{\partial t_p} > 0 \Leftrightarrow \frac{\partial^2 \pi_I}{\partial p^2}\frac{\partial^2 \pi_I}{\partial A \partial t_p} - \frac{\partial^2 \pi_I}{\partial p \partial A}\frac{\partial^2 \pi_I}{\partial p \partial t_p} < 0,$$

$$\frac{\partial A}{\partial t_A} > 0 \Leftrightarrow \frac{\partial^2 \pi_I}{\partial p^2}\frac{\partial^2 \pi_I}{\partial A \partial t_A} - \frac{\partial^2 \pi_I}{\partial p \partial A}\frac{\partial^2 \pi_I}{\partial p \partial t_A} < 0.$$

Because $\frac{\partial^2 \pi_I}{\partial A \partial t_p} = -pq\frac{\partial Q}{\partial A}$, $\frac{\partial^2 \pi_I}{\partial p \partial t_p} = -\left[qQ + p\left(Q\frac{dq}{dp} + q\frac{\partial Q}{\partial p}\right)\right]$, $\frac{\partial^2 \pi_I}{\partial A \partial t_A} = -\left(A\frac{\partial Q}{\partial A} + Q\right)$, and $\frac{\partial^2 \pi_I}{\partial p \partial t_A} = -A\frac{\partial Q}{\partial p}$, we obtain (E.9) and (E.10).

E.3 Effects of tax rates on the extensive margin (Proof of (E.12))

We provide the proof for the case where $c = 0$. If the result holds under this condition, it must hold also for positive values of c that are sufficiently close to zero. To show this result we solve the infrastructure provider's problem sequentially. First, we maximize π_I with respect to p, treating A as a parameter. The solution, denoted $p(A)$, is implicitly defined by the following

$$\frac{\partial \pi_I}{\partial p} = \frac{\partial Q}{\partial p} A(1 - t_A) + (1 - t_p)\left[qQ + p\left(q\frac{\partial Q}{\partial p} + Q\frac{dq}{dp}\right)\right] = 0. \tag{E.28}$$

Next, we maximize π_I with respect to A, recalling that $p(A)$ is defined by (E.28). We have $\frac{d\pi_0}{dA} = \frac{\partial \pi_I}{\partial A} + \frac{\partial p}{\partial A}\frac{\partial \pi_I}{\partial p} = \frac{\partial \pi_I}{\partial A}$. The last equality follows from the fact that $\frac{\partial \pi_I}{\partial p} = 0$. Hence, the FOC of the problem is

$$\frac{d\pi_I}{dA} = \frac{\partial \pi_I}{\partial A} = -Q(1 - t_p)\left(q + p(A)\cdot\frac{dq}{dp}\right)\frac{\frac{\partial Q}{\partial A}}{\frac{\partial Q}{\partial p}} + Q(1 - t_A) = 0, \tag{E.29}$$

where we have made use of (E.28) to replace $A(1 - t_A) + (1 - t_p)pq = \frac{-Q(1-t_p)\left(q+p\frac{dq}{dp}\right)}{\frac{\partial Q}{\partial p}}$. Differentiation of (E.29) with respect to t_A (recalling that p is a function of A) results in $\frac{\partial^2 \pi_I}{\partial A \partial t_A} + \frac{\partial^2 \pi_I}{\partial A^2}\frac{\partial A}{\partial t_A} = 0$, which, after some rearrangements, brings to

$$\frac{\partial A}{\partial t_A} = \frac{Q + K\frac{dp}{dt_A}}{\frac{\partial^2 \pi_I}{\partial A^2}},$$

where $\frac{dp}{dt_A} = \frac{\partial p}{\partial A}\frac{dA}{dt_A}$ and

$$K \equiv Q(1 - t_p)\left(\left(q + p(A)\cdot\frac{dq}{dp}\right)\cdot\frac{d\left(\frac{\partial Q}{\partial A}/\frac{\partial Q}{\partial p}\right)}{dp} + \left(2\frac{dq}{dp} + p(A)\cdot\frac{d^2 q}{dp^2}\right)\frac{\partial Q}{\partial A}/\frac{\partial Q}{\partial p}\right).$$

Following similar steps, we get

$$\frac{\partial A}{\partial t_p} = \frac{-Q\left(q + p\frac{dq}{dp}\right)\frac{\partial Q}{\partial A}/\frac{\partial Q}{\partial p} + K\frac{\partial p}{\partial t_p}}{\frac{\partial^2 \pi_I}{\partial A^2}} = \frac{-Q + K\frac{dp}{dt_p}}{\frac{\partial^2 \pi_I}{\partial A^2}},$$

where the last equality follows from (E.24).

Let us now consider the effect of changes in t_A and t_p on Q. We have

$$\frac{dQ}{dt_k} = \frac{\partial Q}{\partial A}\frac{\partial A}{\partial t_k} + \frac{\partial Q}{\partial p}\frac{dp}{dt_k} \quad k = A, p.$$

Replacing $\frac{\partial Q}{\partial A} = -\frac{F'(\tilde{z})}{\beta}$ and $\frac{\partial Q}{\partial p} = -\frac{F'(\tilde{z})}{\beta}\left(q + p\frac{dq}{dp}\right)$, and using the above results, we get

$$\frac{dQ}{dt_A} = -\frac{F'(\tilde{z})}{\beta}\left(\frac{Q + K\frac{dp}{dt_A}}{\frac{\partial^2 \pi_0}{\partial A^2}} + \left(q + p\frac{dq}{dp}\right)\cdot\frac{dp}{dt_A}\right),$$

$$\frac{dQ}{dt_p} = -\frac{F'(\tilde{z})}{\beta}\left(\frac{-Q + K\frac{dp}{dt_p}}{\frac{\partial^2 \pi_0}{\partial A^2}} + \left(q + p\frac{dq}{dp}\right)\cdot\frac{dp}{dt_p}\right).$$

Given (E.27), these two expressions must have opposite sign when $t_A = t_p$.

E.4 Proof of (E.14) and (E.15)

Given the assumptions made in the text, by solving (E.18) we obtain the following equilibrium prices and quantities

$$p = \frac{\alpha(1 - 2t_p + t_A)}{3 + t_A - 4t_p}, \quad A = \frac{V}{2} - \frac{(\alpha(1 - t_p))^2(1 - 4t_p + 3t_A)}{4(3 - 4t_p + t_A)^2(1 - t_A)}, \tag{E.30}$$

$$x = q = \frac{\alpha(1 - t_p)}{3 + t_A - 4t_p}, \quad Q = \frac{1}{\beta}\left(\frac{V}{2} + \frac{(\alpha(1 - t_p))^2}{4(3 - 4t_p + t_A)^2(1 - t_A)}\right). \tag{E.31}$$

Note that we restrict attention to equilibria such that both p and A are non-negative. This restriction implies that we focus on combinations of tax rates such that $1 - 2t_p + t_A \geq 0$ holds. Using the above expressions, we get

$$\frac{dQ}{dt_p} = -\frac{\alpha^2(1-t_p)(1-2t_p+t_A)}{2\beta(3-4t_p+t_A)^2(1-t_A)} < 0, \tag{E.32}$$

$$\frac{dQ}{dt_A} = \frac{(\alpha(1-t_p))^2(1-2t_p+t_A)}{2\beta(3-4t_p+t_A)^2(1-t_A)^2} > 0. \tag{E.33}$$

Using (E.30) and (E.31), we have $p + x = u'(q) = \left(\frac{2\alpha}{3}\right)$ and $u(q) + V - \beta Q = \frac{7\alpha^2}{36} + \frac{V}{2}$ when $t_A = t_p = 0$. Using these equalities, we can rearrange (E.32) and (E.33) to get (E.14) and (E.15).

E.5 Proof of Proposition E.1

The proof is organized in two parts, and each part is divided in two steps. Part 1 focuses on the optimal t_p given $t_A = 0$. In step 1, we establish that, conditional on $t_A = 0$, the only value of t_p that is potentially a global maximizer of W within the $[-1, 0]$ interval is -1. In step 2, we establish that, when $V \geq \frac{\alpha^2}{2}$, there exists a value $t_p > 0$ such that W is strictly larger than when $t_p = -1$. Taken together, these findings imply that the optimal t_p must be positive. Part 2 focuses on the optimal t_A given $t_p = 0$. In step 1 we establish that, conditional on $t_p = 0$, the only potential global maximizer of W on the $t_A \in [-1, 0]$ interval is -1. In step 2, we establish that, when $\frac{N\alpha^2}{2} > V$, there exists a value $t_A > 0$ such that W is strictly larger than when $t_A = -1$. Taken together, these findings imply that the optimal t_A must be positive.

Part 1. *STEP 1*: The first order derivative $\left.\frac{\partial W}{\partial t_p}\right|_{t_A=0}$ can be written as $-\alpha^2 \frac{X+Y}{8(3-4t_p)^4\beta}$, where

$$X \equiv -2(3-4t_p)(1+t_p(7+2t_p(-9+4t_p)))V,$$

$$Y \equiv \alpha^2(1-t_p)^2(1-4t_p)(3-t_p(3+2t_p)).$$

X and Y are monotonically decreasing in t_p when $t_p \in [-1, 0]$. Furthermore, as long as $V > \frac{\alpha^2}{2}$ holds, we have $X + Y > 0$ when $t_p = -1$, and $X + Y < 0$ when $t_p = 0$. Consequently, the only local maximizers on the $t_p \in [-1, 0]$ interval are its extremes. However, because we have already established that $\left.\frac{\partial W}{\partial t_p}\right|_{t_A=0,t_p=0} > 0$ when $V > \frac{\alpha^2}{2}$, $t_p = 0$ cannot be a global maximizer of W. Therefore, the only potential maximizer of W on the $t_p \in [-1, 0]$ interval is $t_p = -1$.

STEP 2: If we can establish that W is larger when $t_p = \frac{1}{2}$ than when $t_p = -1$, we can conclude that the global maximizer of W must be such that $t_p > 0$. We have

$$W|_{t_p=1/2,t_A=0} = \frac{1}{\beta}\left(\frac{3}{8}V^2 + \frac{7}{32}\left(\alpha^2 V\right) + \frac{11}{512}\alpha^4\right)$$

$$W|_{t_p=-1,t_A=0} = \frac{1}{\beta}\left(\frac{3}{8}V^2 + \frac{19}{98}\left(\alpha^2 V\right) + \frac{17}{686}\alpha^4\right)$$

Because the difference between these terms is increasing in V, and $W|_{t_p=1/2,t_A=0} > W|_{t_p=-1,t_A=0}$ holds when $V = \frac{\alpha^2}{2}$, the claim is proved.

Part 2. *STEP 1*: The first order derivative $\left.\frac{\partial W}{\partial t_A}\right|_{t_p=0}$ can be written as $-\alpha^2 \frac{X+Y}{8\beta(1-t_A)^3(3+t_A)^4}$, where

$$X \equiv (3+t_A)\left(2(1-t_A)(1+t_A(2+t_A)(-5+2t_A))V\right),$$

$$Y \equiv \alpha^2(1+3t_A)(-3+t_A(3+2t_A)).$$

Under the assumption that $\frac{\alpha^2}{2} > V$, these expressions are monotonically decreasing with t_A on the interval $[-1,0]$. Furthermore, they are all positive when $t_A = -1$, but their sum is negative when $t_A = 0$, because, $\left.\frac{\partial W}{\partial t_A}\right|_{t_p=0,t_A=0} > 0$. It follows that the only local maximizers of W on the interval $t_A \in [-1,0]$ are $t_A = -1$ and $t_A = 0$. However, the latter cannot be a global maximizer of W, because $\left.\frac{\partial W}{\partial t_A}\right|_{t_p=0,t_A=0} > 0$.

STEP 2: If we can show that $\exists t_A > 0$ such that W is larger than when $t_A = -1$, we can conclude that the global maximizer of W must be such that $t_A > 0$. We have

$$W|_{t_p=0,t_A=2/3} = \frac{1}{\beta}\left(0.375V^2 + 0.22V\alpha^2 + 0.027\alpha^4\right)$$

$$W|_{t_p=0,t_A=-1} = \frac{1}{\beta}\left(0.375V^2 + 0.218V\alpha^2 + 0.021\alpha^4\right)$$

Because the difference between these terms is increasing in α^2, and $W|_{t_p=0,t_A=2/3} > W|_{t_p=0,t_A=-1}$ when $\frac{\alpha^2}{2} = V$, the claim is proved.

F Competition between infrastructure providers: proofs

We solve the model by proceeding backwards. At stage 3, consumers subscribing to $s = m, n$ choose the level of consumption q_s maximizing (5.1) given the prices set by the infrastructure and final goods providers. We obtain that $q_s = \alpha - p_s - x_s$. Given this outcome and taking the number of subscribers to each infrastructure provider as given, at stage 2 the final good provider maximizes its profit, $\pi_f = q_m (x_m - \phi) D_m + q_n (x_n - \phi) D_n$, with respect to x_m and x_n. Solving the system of first-order conditions $\frac{\partial \pi_f}{\partial x_s} = 0,\ s = m, n$, we obtain $x_s = \frac{1}{2}(\alpha - p_s + \phi)$.

Finally, at stage 1, consumers decide which infrastructure provider they choose, taking as given its tariffs $(p_s,\ A_s)$. We first determine the marginal consumer $\overline{z}$ who is indifferent between the two infrastructure providers. Equating $U_n(z) = U_m(z)$ and solving for z, we find

$$\overline{z} = \frac{1}{2} + \frac{8(A_n - A_m) + 2\alpha (p_n - p_m) + p_m^2 - p_n^2}{16\beta}. \tag{F.1}$$

Demand for firm m is given by all consumers to the left of $\overline{z}$ on the Hotelling line, while that for firm n is given by all consumers to its right. That is, $D_m = \overline{z}$ and $D_n = 1 - \overline{z}$.

The infrastructure providers simultaneously maximize their profits π_s with respect to $(p_s,\ A_s)$, given the choices of the other players. Let t_p and t_A be the tax rates. The profit of firm s is $\pi_s = D_s [(1 - t_A) A_s + (1 - t_p) p_s q_s - c q_s]$. We assume this function is concave in p_s, A_s. From the system of first-order conditions $\frac{\partial \pi_s}{\partial p_s} = 0,\ s = m, n$ and $\frac{\partial \pi_s}{\partial A_s} = 0,\ s = m, n$, we find

$$A_s = \beta - \frac{(\alpha - \phi)^2 (1 + t_A - 2t_p)(1 - t_p)^2}{(1 - t_A)(3 + t_A - 4t_p)^2}, \qquad p_s = \frac{(\alpha - \phi)(1 + t_A - 2t_p) + 2c}{3 + t_A - 4t_p}. \tag{F.2}$$

Substituting p_s and x_s in q_s characterized above, we find

$$q_s = \frac{(\alpha - \phi)(1 - t_p) - c}{3 + t_A - 4t_p}. \tag{F.3}$$

Setting $t_A = t_p = 0$, we obtain that $p_s = \frac{\alpha - \phi + 2c}{3}$, $x_s = \frac{\alpha + 2\phi - c}{3}$, $q_s = \frac{\alpha - \phi - c}{3}$ and $D_m = D_n = \frac{1}{2}$. In the absence of taxation, both infrastructure providers charge individuals for consumption. To derive the socially optimal allocation, we maximize (5.2) with respect to $(q_m,\ q_n,\ D_m)$, which yields $q_s^* = \alpha - \phi - c$ and $D_m^* = D_n^* = \frac{1}{2}$. Hence, as in the baseline model, there is underprovision of the final good $(q_s < q_s^*)$ in the no-tax equilibrium.

From (F.2), we find that that

$$\frac{\partial p_s}{\partial t_A} = \frac{2((\alpha - \phi)(1 - t_p) - c)}{(3 + t_A - 4t_p)^2}, \qquad \frac{\partial p_s}{\partial t_p} = -\frac{2(\alpha - \phi)(1 - t_A) - 8c}{(3 + t_A - 4t_p)^2}.$$

Hence, the infrastructure suppliers respond to an increase in t_p by reducing p_s, as long as the marginal cost c is not exceedingly large. By contrast, raising t_A increases p_s if c is small. As a result, we find

$$\frac{\partial q_s}{\partial t_A} = -\frac{(\alpha - \phi)(1 - t_p) - c}{(3 + t_A - 4t_p)^2}, \qquad \frac{\partial q_s}{\partial t_p} = \frac{(\alpha - \phi)(1 - t_A) - 4c}{(3 + t_A - 4t_p)^2}.$$

The effects of taxes on welfare are given by

$$\frac{\partial W}{\partial t_A} = -\frac{(\alpha - \phi)^2 (1 - t_p)(2 + t_A - 3t_p) + c}{(3 + t_A - 4t_p)^3}, \qquad \frac{\partial W}{\partial t_p} = \frac{(\alpha - \phi)^2 (1 - t_A)(2 + t_A - 3t_p) - 4c}{(3 + t_A - 4t_p)^3}.$$

Hence, starting from $t_A = t_p = 0$, a tax on usage increases welfare as long as c is sufficiently small.