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When Prohibiting Platform Parity Agreements Harms Consumers

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Abstract
We consider a three-level supply chain where a monopolistic seller distributes its product both directly through its own distribution channel and indirectly through platforms accessed by intermediaries competing for final consumers. In this setting, we examine the welfare effects of platform parity agreements, namely contractual provisions according to which the seller cannot charge different prices for the same product distributed through different platforms. We find that these agreements mitigate the marginalization problem both in a wholesale and an agency model. However, only in the former model platform parity unambiguously increases consumer surplus; in the latter, it also increases the commissions paid by the monopolist to the platforms, whereby exacerbating the marginalization problem. On the net, platform parity benefits consumers in the agency model when competition between direct and indirect distribution is sufficiently intense. Interestingly, in both models consumers' preferences are always aligned with the platforms' but not with the seller's.

JEL classification: L42, L50, L81.

Keywords: Agency Model, Distribution Channels, Platform Parity Agreements, Wholesale Model.

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1 Introduction

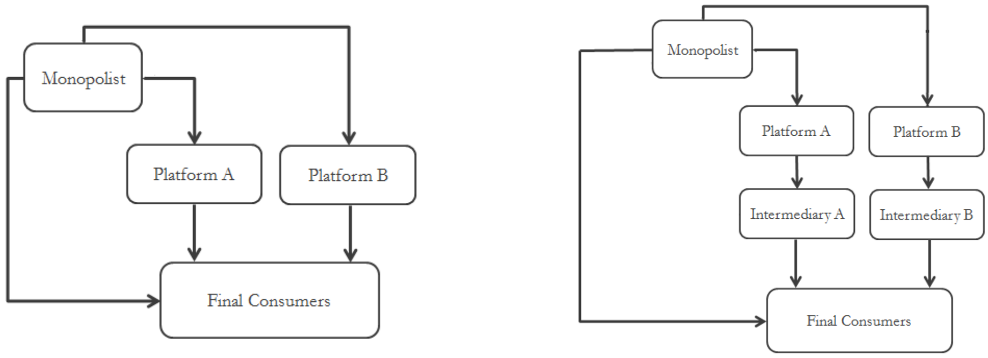
Motivated by the recent antitrust scrutiny of price parity provisions, a number of contributions have examined the effects of such contractual agreements on firms' profits and consumer welfare (e.g., Edelman and Wright, 2015, Johansen and Vergé, 2016, among others). These models consider one or more competing sellers supplying products both through their own direct distribution channels and competing platforms, which are accessed by final consumers (see Figure 1 panel a).

Within this framework, two different types of contractual arrangements between a seller and a platform are considered, namely *narrow* and *wide price parity agreements*. Under a wide parity agreement, the price charged to final consumers in the direct distribution channel must not be lower than the price charged to final consumers through either of the indirect booking channels; not exceed the price charged in the direct distribution channel and, in addition, the prices charged through the two platforms must be identical. Instead, under a narrow parity agreement, the prices charged for products distributed through a certain platform may be different from the prices charged to consumers booking through another platform. However, as with the wide parity agreement, the price charged to final consumers in the direct distribution channel must not be lower than the price charged to final consumers through either of the indirect booking channels.

The literature concludes that typically wide parity agreements are anti-competitive, absent efficiencies, unless upstream competition (i.e., platform competition) is relatively fierce and sellers can delist from a platform.[1] The basic intuition behind this result is rather simple and is aligned with the theory of harm developed by several antitrust authorities (see, e.g., Akman, 2016). The considered agreements soften platform competition because a platform setting high commissions (or fees) will not lose market share since sellers cannot offer more favorable prices through alternative distribution channels, including the direct distribution channel, which may involve lower costs. Instead, the platform can charge high fees knowing that those fees will be spread across all transactions and that consumers will not be able to find lower-cost alternatives elsewhere.[2] Similar results apply to narrow parity agreements: when platforms are *must have*, platforms may not undercut each other even when possible. The reason is that any reduction in fees would not be compensated by an increase in sales,

[1] If a seller can exit one of the platforms and faces competition from other sellers in that platform, then it can leverage the possibility of delisting credibly and effectively, in which case wide parity may lead to lower commissions and final prices. Intuitively, this is because by exiting one platform a seller de facto reduces its marginal cost and hence can steal business from its competitors, whereas the sales lost are not particularly valuable if sellers are close competitors.

[2] In addition to this, wide parity agreements may also limit the entry and expansion of new platforms, and thus have a negative impact on investment and innovation (see Boik and Corts, 2016). Platforms that are not yet established will not be able to compete effectively in the supply of indirect distribution services by offering lower fees (in return for which they might negotiate lower prices with final consumers). Incumbent platforms will thus be able to capture an increasingly large share of consumer traffic, as network effects draw consumers and sellers to the most heavily used platforms.



(a) *Vertical structures considered in the available models.*

(b) *Vertical structure considered in our model.*

Figure 1: Industry vertical structure.

since sellers would have no incentive to reduce the price charged in the undercutting platform at the expense of their direct distribution channels (with a price tied to the price of the high commission platform) when direct and indirect distribution are close substitutes for a majority of final consumers.

These conclusions may well apply to industries characterized by the structure described above, such as the hotel booking industry, in which platforms compete with each other to attract final consumers. However, in several other cases (e.g., the airline ticket distribution industry) platforms are accessed by specialized intermediaries, which in turn are in competition with each other in the retail market (see Figure 1 panel b). In these industries, which involve vertical supply chains that are more complex than the hotel booking industry's, it arises a multiple marginalization problem which is different from the one studied in the available models: as in the literature, a seller will mark up the commissions charged by the platforms, which in turn will negotiate fees above their marginal costs. However, in addition to this, intermediaries will mark up the prices offered by the sellers, implying that final prices reflect two mark-ups. These differences in vertical structure are likely to have important economic implications as for the competitive effect of price parity clauses, also because the contractual provisions themselves are rather different with respect to the narrow and wide price parity agreements detailed above.

Notably, in these industries the so called *platform parity provisions* are typically negotiated between sellers and platforms. Such clauses require sellers to provide the same products and related content (e.g., ancillary services) to intermediaries using one platform on no worse terms and conditions than the seller itself would apply to users of other platforms. The cumulative effect of such provisions in agreements of several platforms could lead to a situation where users of all platforms have access to the same content. However, these provisions do not prevent sellers from distributing exclusive content through their *direct* distribution channels (e.g., their own websites). Furthermore, sellers are not required to ensure that final consumers pay the same prices in all *indirect* distribution channels (e.g., at intermediaries relying on different platforms). For these reasons, such agreements cannot be considered *price* parity agreements, as the wide and narrow parity agreements discussed above. The only operative constraint imposed by the parity agreements observed in the industries under consideration concerns the prices offered by a seller through the indirect distribution channels, which constitute the content that intermediaries distribute. Since these prices are forced to be identical, we may refer to this clause as a *platform (or content) parity agreement.*

We argue that the effects of such agreements on consumer welfare cannot be inferred from the analyses conducted in the available literature. The reasons are as follows. First, since, unlike the wide and narrow parity agreements reviewed above, prices set on the direct sale channels are unconstrained, each platform's incentive to negotiate high commissions or fees is limited, given that sellers will be able to increase the prices offered to the intermediaries that distribute their products indirectly and, therefore, divert business from the platform demanding high fees to their own direct distribution channels. Second, while platform parity may allow platforms to negotiate higher commissions, those extra rents are bound to be competed away because platforms compete with each other to increase their share of consumer traffic by expanding their network and supporting the competitive position of their intermediaries. Finally, since a seller must be concerned about the impact of multiple mark-ups on sales, platform parity may reduce its incentives to set high prices when distributing through platforms, thus mitigating the marginalization problem and leading to lower final prices and higher sales. This is because any increase in the prices offered through intermediaries using a given platform will lead to a parallel increase in the prices offered through the intermediaries dealing with the other platforms. These price increases will then be passed on to consumers in the form of higher retail prices and will cause a reduction in the demand served through the indirect distribution channel, which will be offset only in part by the increase in the demand served through the seller's direct distribution channel. Therefore, while platform parity may have similar rent shifting effects upstream to wide or narrow parity, its impact on consumer welfare is bound to be different.

To illustrate these points we build a three-level supply chain model where a monopolistic seller distributes its products both directly through its own distribution channel and indirectly through two platforms accessed by intermediaries, which in turn rely on the IT infrastructure provided by specialized platforms to buy the seller's product on behalf of final consumers (Figure 1 panel b). We assume that platforms and intermediaries are in exclusive relationships (e.g., because of switching costs) and, following Boik and Corts (2016) and industry practice, that contracts are linear — i.e., input prices do not vary with sales volumes.

As a benchmark, we first consider a wholesale industry where the seller sets the price on its direct channel and bilaterally negotiates a commission (wholesale price) with each platform for every unit of product purchased through that platform. Every platform, in turn, charges its own intermediary an 'access' price for each unit of product sold to final consumers. Finally, intermediaries set retail prices in competition between themselves and with the monopolist's direct distribution channel. In this setting, we find that a platform parity agreement is always pro-competitive even in the absence of efficiencies: the constraint on wholesale prices imposed through such a provision is used by the monopolist as a commitment device to mitigate the multiple marginalization problem. Indeed, when the platforms cannot be price discriminated, any attempt of the monopolist to increase the wholesale price charged to one platform immediately translates into a parallel shift of the wholesale price charged to the other platform. As a result, both platforms symmetrically increase the access price charged to the intermediaries, whereby leading to higher retail prices (since both intermediaries will mark up the increased access prices). By contrast, without platform parity, when the monopolist increases the wholesale price charged to a platform, the platform takes as given the wholesale price charged to its rival because contracts are secret and our solution concept is contract equilibrium. Hence, the multiple marginalization problem is relatively less important for the monopolist compared to the regime with platform parity. In other words, the excessive pass on rate that occurs under the parity provision refrains the monopolist from charging a wholesale price that is too high in equilibrium. This mandates a lower access price, a lower final price and thus a higher consumer surplus in the indirect distribution channel. In addition, since prices are strategic complements, lower prices in the indirect channel induce lower prices also in the direct channel, which benefits consumers in that segment too. Finally, by reducing the multiple marginalization problem, the provision also increases profits — i.e., the monopolist, the platforms and the intermediaries are better off with than without platform parity. Hence, in the wholesale model all players have aligned preferences.

Building on these insights we then consider an agency model (see, e.g., Johnson, 2017) which seems to reflect more closely real life practices. In this model each platform negotiates a per-unit commission with the monopolist for each unit of product purchased through that platform. The monopolist charges intermediaries an access price[3] that they must pay for each unit of product purchased. Intermediaries set retail prices (paid by final consumers) in competition between themselves and with the monopolist's direct distribution channel. In industries with such a vertical structure, we find that platform parity provisions might be pro-competitive (absent efficiencies) depending on the degree of product differentiation across distribution channels. Specifically, we show that the constraint on access prices implied by a platform parity agreement generates a new trade-off shaped by the following effects. First, each platform anticipates that, being concerned with double marginalization, under the parity provision the monopolist has a lower incentive to pass on commissions to the intermediaries. Hence, as in Boik and Corts (2016), platforms' fees are higher under the parity provision, which clearly harms consumers because it creates marginalization. Second,

[3]We define the access price as the (unit) input price (net of commissions, surcharges and discounts) that an intermediary has to pay the monopolist in order to operate a transaction through a given platform.

as in the wholesale model, the provision mitigates the marginalization problem between the monopolist and the intermediaries, which benefits consumers. Third, since the price in the direct channel is lower with platform parity than without platform parity (because the monopolist has an incentive to divert business towards that channel when the provision is in place), consumers on that segment benefit from the provision.

The net effect points in the direction of increasing consumer surplus when the products or the services provided through different distribution channels are not too differentiated. Essentially, in equilibrium, competition in the product market — i.e., within and between the distribution channels — erodes the intermediaries' mark ups and magnifies the pro-competitive effect of the provision on the marginalization problem. Notably, when platforms benefit from the provision consumers do too — i.e., their preferences are aligned with respect to the choice of the contractual provision. By contrast, absent efficiencies, the seller and the intermediaries are always better off without the provision. However, we also find that platforms can persuade more easily the intermediaries to prefer the parity regime through appropriate side payments than the monopolist. In other words, whenever the joint profit of the platforms and the intermediaries is higher with than without the parity provision, the total industry profit may well be lower without the provision — i.e., the price that the monopolist would require to accept the parity regime is higher than the gain obtained jointly by the platforms and the intermediaries. As a result, in these cases, the only way to increase consumer surplus is to allocate more decision rights to the platforms than the monopolist.

Summing up, based on our analysis we can conclude that, even with an upstream monopoly, content (i.e., platform) parity provisions cannot be presumed anti-competitive absent efficiencies. Interestingly, consumers and platforms' preferences are always aligned: as long as platforms benefit from platform parity, consumers gain as well (which is not always the case for the seller and the intermediaries). Hence, in practice, the likelihood that the introduction of such a provision benefits consumers is higher when platforms are not against it.

Finally, we develop some interesting extensions and robustness checks of the baseline model. First, we show that the welfare results discussed above hold qualitatively when considering multiple (more than two) competing platforms. Second, if the monopolist can commit to the price set on the direct channel before contracting with platforms, the pro-competitive effect of the provision survives only in the wholesale model. The reason why with commitment the provision is anticompetitive in the agency model is that the monopolist cannot compensate the effect of increased platforms' commissions with a lower price in the direct channel. This is because being chosen at the outset of the game that price is set 'efficiently' regardless of whether the provision is in place or not. Third, we show that the monopolist may use the quality of the product distributed through the indirect channel strategically in order to break down platform parity at the consumers' expense.

The remainder of the paper is organized as follows. After reviewing the related literature, in Section 2 we set-up the wholesale model. In Section 3, we analyze the agency model. Section 4 concludes. Proofs are in the Appendix. Additional results are presented in an online Appendix.

Related literature. Our paper contributes to the literature on wholesale and platform most-favored nation (MFN) clauses.[4] Early contributions (DeGraba and Postlewaite, 1992, McAfee and Schwartz, 1994, DeGraba, 1996, Marx and Shaffer, 2004) consider sequential contracting between a manufacturer and a number of retailers, and investigate the role played by MFN clauses in mitigating the time inconsistency problem faced by the supplier of a durable input. Following a number of antitrust investigations against the use of such clauses in wholesale markets (see, e.g., Avilés-Lucero and Boik, 2018) the potential pro- and anti-competitive effects of these provisions have been informally discussed in the law and economics literature (see, e.g., Baker and Chevalier, 2012).

More recent contributions investigate the welfare effects of the adoption of MFN clauses in online markets. Boik and Corts (2016), for example, consider a monopolist facing two competing platforms, which first simultaneously choose whether to impose a price parity agreement, then set per-unit commissions. After observing these choices, the monopolist sets final prices on both platforms (the so called agency model). Within this framework, a price parity clause is unambiguously anti-competitive since it raises platforms' commissions and retail prices. Similar results are found by Johnson (2017), who models competition in the upstream market. The anti-competitive nature of price parity provisions is challenged by Johansen and Vergé (2016), who consider endogenous platform participation and the presence of direct sales channels in addition to upstream competition (two ingredients that are also present in our model).[5] In their setting, if upstream competition is fierce enough, consumers benefit from the introduction of a narrow or a wide price parity clause provided that sellers can delist from platforms charging excessively high commissions. The mechanism through which parity agreements benefit consumers in our model is different from the argument put forward by Johansen and Vergé (2016), since we consider a monopolistic seller who never finds it optimal to delist from a platform.

Other contributions (Ronayne and Taylor, 2018, Calzada et al., 2018, Wang and White, 2016, Shen and White, 2019) suggest that platforms use these clauses to avoid *shoowrooming* — i.e., that consumers use the platform to learn of products, but then buy through the firms' direct sales channel if they find a lower price.[6] Edelman and Wright (2015), instead, assume

[4]Other works examine MFN clauses that sellers offer to consumers: see, e.g., Cooper (1986), Butz (1990), Schnitzer (1994).

[5]Actually, because of preference for variety, in our model delisting never occurs in equilibrium.

[6]In a model with a monopolistic platform, Ronayne and Taylor (2018) show that price parity agreements are profitable for the platform and reduce consumer surplus, whereas Calzada et al. (2018) find that these clauses may induce single homing by sellers, thereby reducing the products offered on each platform. Similar results are found by Wang and White (2016), who assume that the platform reduces consumers' search costs. However, when considering competing vertically differentiated platforms, they show that, under some circumstances, a narrow price parity agreement can enhance consumer surplus. Finally, Shen and White

that, as a result of costly investments, platforms are able to provide benefits to buyers,[7] and they show that a price parity clause leads to inflated retail prices, excessive adoption of the platforms' services, over-investment in benefits to buyers, and ultimately a reduction in consumer surplus. The welfare effects of price parity clauses in the agency model are also discussed in the law and economics literature (e.g., Ezrachi, 2015). In all these models, the pro-competitive effect of price parity agreements is driven by the presence of efficiencies, which are instead absent in our model.

Finally, as for the comparison between the wholesale and the agency model, Foros et al. (2017) show that, even if platforms' commission rates remain the same across the two business models (for exogenous reasons), the use of price parity clauses may facilitate the adoption of the agency model which, in turn, may involve lower consumer prices. Our analysis shows that, like also in Johnson (2017), the agency model increases consumer surplus and platforms' profits when contracts and parity provisions are endogenously chosen within each business model.

2 The wholesale benchmark

In order to highlight the beneficial effects of platform parity agreements in the clearest possible way, it is useful to start with the standard wholesale framework, where these provisions unambiguously increase consumer welfare and firms' profits. We will then turn to the agency model and show that, in industries with such a business structure, platform parity agreements increase welfare as long as competition within and between the distribution channels is fierce enough — i.e., when the products or the services sold in these market segments are not too differentiated.

Markets and players. Consider a multi-channel and multi-tier industry in which a monopolist (M) sells its product through a direct channel and two competing platforms (each denoted by P_i, with $i = A, B$) accessed by intermediaries competing to attract final consumers. Suppose, for simplicity, that intermediaries and platforms are in exclusive relationships — e.g., because of switching costs.[8] Final consumers can buy the monopolist's product either through the direct sale channel or through the intermediaries.

The monopolist. The monopolist sets a price p_d for sales through its direct distribution

(2019) assume that the platform can make a recommendation to each consumer about which product to buy, finding that the main effect of price parity agreements is to shift surplus from sellers to the platform, and that these clauses can increase total welfare.

[7]These benefits can consist in offering complementary products, reducing transaction costs, and offering financial rebates. Moreover, consumers incur in a cost to join a platform and (like in Wang and White, 2016) the platform charges commissions to sellers as well as buyers.

[8]We assume exclusivity in order to understand the role of platforms in multi-channel and multi-layer markets. In fact, in the opposite scenario where platforms compete to attract intermediaries and are homogeneous, they would make zero profits in equilibrium and play no role in the analysis.

system. Moreover, it can also sign distribution contracts with the platforms. Following the literature (see, e.g., Boik and Corts, 2016, and Gaudin, 2019) we assume linear contracts: each contract specifies a unit (wholesale) price t_i for every purchase processed through platform i (we discuss two-part tariffs in Section 2.3). Production costs are assumed to be linear and normalized to zero without loss of generality. The monopolist's profit is

$$\pi^M(\cdot) \triangleq \sum_{i=A,B} t_i q_i + p_d q_d,$$

with q_i and q_d denoting the quantities sold through platform $i = A, B$ and the direct distribution system, respectively.

Platforms. Platforms also use linear contracts when dealing with intermediaries. Hence, each platform $i = A, B$ charges a unit (access) price w_i to its exclusive intermediary. Accordingly (normalizing to zero their costs) each platform's profit is

$$\pi_i^P(\cdot) \triangleq (w_i - t_i) q_i.$$

The platforms' outside option is normalized to zero without loss of generality. Hence, in this business model platforms act as wholesale distributors, aggregating many dispersed retailers. Their (implicit) role is to help sellers to reach out retailers and save on transaction costs.

Intermediaries. Each intermediary I_i charges a retail price p_i at which final consumers can buy M's product. Hence, I_i's profit is

$$\pi_i^I(\cdot) \triangleq (p_i - w_i) q_i,$$

where, for simplicity, the distribution cost is equal to zero. Again, we normalize to zero (without loss of generality) the intermediaries' outside option.

Demand functions. In line with the empirical evidence (see, e.g., Cazaubiel et al., 2018) we assume that consumers perceive the products sold through the direct and the indirect channel as imperfect substitutes. Some consumers may in fact prefer to purchase through the indirect channel because intermediaries offer (un-modelled) additional services that M is unable or unwilling to supply in the direct channel.

To this purpose, following the literature (e.g., Johansen and Vergé, 2016), for the sake of tractability, we assume that the demand system reflects the preferences of a representative consumer, whose utility function is

$$U(\cdot) \triangleq \sum_{j=A,B,d} q_j - \frac{1}{2} \sum_{j=A,B,d} q_j^2 - \gamma \sum_{i,j=A,B,d;,j \neq i} q_j q_i - \sum_{j=A,B,d} p_j q_j + m, \quad (1)$$

where m is the utility from income. Notice that since this utility function is strictly concave, it displays preference for variety. This means that the representative consumer prefers to have more consumption options.[9] As we will explain, this property will have important consequences on our analysis.

Standard techniques then yield the (direct) demand functions

$$q_A \triangleq D^A(p_A, p_B, p_d) = \frac{1-\gamma-(1+\gamma)\,p_A+\gamma\,(p_B+p_d)}{(1-\gamma)(1+2\gamma)},$$
$$q_B \triangleq D^B(p_B, p_A, p_d) = \frac{1-\gamma-(1+\gamma)\,p_B+\gamma\,(p_A+p_d)}{(1-\gamma)(1+2\gamma)},$$

and

$$q_d \triangleq D^d(p_d, p_A, p_B) = \frac{1-\gamma-(1+\gamma)\,p_d+\gamma\,(p_A+p_B)}{(1-\gamma)(1+2\gamma)}.$$

Hence, γ reflects the degree of substitutability between products within and across distribution channels. In the online Appendix we show that the analysis' results do not change qualitatively if we assume that the products distributed in the indirect channel are perceived by the consumer as closer substitutes compared to the product distributed in the direct channel. We assume that $\gamma \in [0, \overline{\gamma}]$, with $\overline{\gamma} \approx 0.9$ in order to guarantee that second-order conditions hold (see Appendix 2).

Timing. The timing of the game is as follows (see Figure 2):

$t = 1$ M simultaneously offers contracts — i.e., t_A and t_B.

$t = 2$ Upon observing its own offer t_i, platform P_i sets w_i.

$t = 3$ Upon observing its own offer w_i, intermediary I_i sets p_i and M sets p_d, simultaneously.

$t = 4$ After observing the vector of prices $\mathbf{p} \triangleq (p_d, p_A, p_B)$, final consumers allocate their demand.

Platform (content) parity. We consider two versions of the game, depending on the presence or not of a *platform (content) parity* agreement between the monopolist and the platforms. Specifically:

- When a *platform parity* agreement is in place, the monopolist commits to offer the same wholesale price to the competing platforms — i.e., $t_A = t_B = t$.
- Without such agreement, the monopolist is free to offer different prices $t_A \neq t_B$ to the platforms.

[9] For example, in the airline ticket industry, the consumer may want to have the ability to book in different ways because it may not always be able to reach directly the airline website.

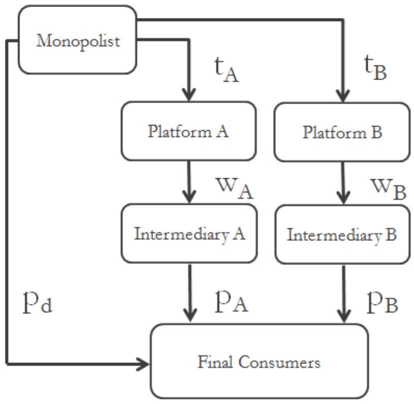


Figure 2: The wholesale model.

Equilibrium concept. For the sake of tractability, as in Johansen and Vergé (2016) and Rey and Vergé (2017), our solution concept will be *Contract Equilibrium* (see Crémer and Riordan, 1987, and Horn and Wolinsky, 1988). We focus on symmetric equilibria in which, depending on whether a platform parity agreement is in place ($k = 1$) or not ($k = 0$):[10]

- M sets $p^*_{d,k}$ and charges the same wholesale price t^*_k to both platforms.
- Both platforms charge the same access price w^*_k to the intermediaries.
- Each intermediary charges the same retail price p^*_k to final consumers.

As noted by Rey and Vergé (2017), this equilibrium concept has some of the features of a perfect Bayesian Nash equilibrium with passive beliefs: P_i chooses the access price w_i charged to I_i assuming that its rival remains under the equilibrium contract t^*_k, even if P_i has received an out-of-equilibrium contract $t_i \neq t^*_k$ from M. This is in line with the market-by-market bargaining restriction of Hart and Tirole (1990) and with the passive beliefs or pairwise-proofness assumption of McAfee and Schwartz (1994). Similarly, at the stage in which retail prices are set, I_i chooses p_i assuming that its rival remains under the equilibrium contract w^*_k, even if I_i has received an out-of-equilibrium contract $w_i \neq w^*_k$ from P_i. Hence, the equilibrium concept that we use discards the possibility of multilateral deviations by the monopolist, who is the only player making multiple offers in the game. We discuss alternative equilibrium concepts in Section 3.8.

Multi-product monopolist. To begin with, it is useful to characterize the solution of the benchmark in which M sells directly to all consumers — i.e., the outcome of the game in which M is vertically integrated with the platforms and the intermediaries. In this hypo-

[10]It can be easily proved that, in our linear demand setting, there is a unique Contract Equilibrium and that this equilibrium is symmetric. However, to simplify exposition, we directly focus on a symmetric equilibrium.

thetical scenario, M's maximization problem is

$$\max_{p_A,p_B,p_d} \sum_{i=A,B} p_i D^i(p_i, p_{-i}, p_d) + p_d D^d(p_d, p_i, p_{-i}).$$

It can be shown (see the Appendix) that M charges the same price for all products

$$p^M = \frac{1}{2},$$

and that efficient quantities are symmetric since the utility function (1) displays preferences for variety — i.e.,

$$q_A^M = q_B^M = q_d^M = \frac{1}{2+4\gamma}.$$

In short, when M behaves as a multi-product monopolist, it fully internalizes the effects of intra- and inter-channel competition. In the rest of the analysis we will often refer to this benchmark in order to identify the extent of multiple marginalization.

2.1 Equilibrium analysis

In what follows we characterize the equilibrium of the game with and without platform parity. We will then study its impact on consumer surplus and firms' profit.

First we characterize the intermediaries' pricing behavior. Consider a symmetric equilibrium in which intermediaries charge p_k^*, platforms charge w_k^* and the monopolist charges t_k^* and $p_{d,k}^*$ on the direct channel. Hence, for any $k = 0, 1$ and for every offer w_i (received from P_i), I_i solves the following maximization problem

$$\max_{p_i} D^i \left(p_i, p_k^*, p_{d,k}^*\right) (p_i - w_i).$$

whose standard first-order condition is

$$\frac{1-\gamma-(1+\gamma)\,p_i+\gamma\left(p_k^*+p_{d,k}^*\right)}{(1-\gamma)(1+2\gamma)} - (p_i - w_i)\,\frac{1+\gamma}{(1-\gamma)(1+2\gamma)} = 0. \tag{2}$$

Given the equilibrium candidate under consideration, this condition defines I_i's 'best reply' to w_i — i.e.,

$$p_k\left(w_i\right) \triangleq \frac{w_i}{2} + \frac{1-\gamma+\gamma\left(p_k^*+p_{d,k}^*\right)}{2\left(1+\gamma\right)}, \quad \forall k = 1, 0. \tag{3}$$

As intuition suggests, this expression is increasing in the access price w_i and (since prices are strategic complements) in p_k^* and $p_{d,k}^*$ — i.e., the rivals' equilibrium prices in the indirect and direct channels, respectively.

2.1.1 Equilibrium with platform parity

When a platform parity agreement is in place, M charges the same wholesale price (t) to both platforms. Hence, P_i solves the following maximization problem

$$\max_{w_i} D^i\left(p_1\left(w_i\right), p_1^*, p_{d,1}^*\right)\left(w_i - t\right),$$

whose (standard) first-order condition yields

$$\frac{1-\gamma-(1+\gamma)\, p_1\left(w_i\right)+\gamma\left(p_1^*+p_{d,1}^*\right)}{(1-\gamma)(1+2\gamma)} - \frac{1+\gamma}{(1-\gamma)(1+2\gamma)} \frac{\partial p_1\left(w_i\right)}{\partial w_i}\left(w_i-t\right)=0.$$

Using (2) yields

$$-\left(p_1\left(w_i\right)-w_i\right)+\frac{\partial p_1\left(w_i\right)}{\partial w_i}\left(w_i-t\right)=0,$$

whose solution is

$$w_1^*(t) \triangleq \frac{t}{2}+\frac{1-\gamma+\gamma\left(p_1^*+p_{d,1}^*\right)}{2\,(1+\gamma)}. \tag{4}$$

This function is symmetric — i.e., it is the same for both platforms — and, as intuition suggests, it is increasing in the (retail) prices charged for the rival products (p_1^* and $p_{d,1}^*$).

Next, substituting (4) into (3) and solving for p_1^*, we obtain:

$$p_1^*(t) \triangleq \frac{1+\gamma}{4+\gamma} t+\frac{3(1-\gamma+\gamma p_{d,1}^*)}{4+\gamma}.$$

Notice that $p_1^*(t)$ is increasing in t: when M charges a higher wholesale price to the platforms, they increase the access prices charged to the intermediaries, which increases the retail price charged for their product: a multiple marginalization effect.

Finally, we can examine M's maximization problem(s). At the final (pricing) stage M solves

$$\max_{p_d} 2tD^i\left(p_1^*\left(t\right), p_1^*\left(t\right), p_d\right)+p_d D^d\left(p_d, p_1^*\left(t\right), p_1^*\left(t\right)\right),$$

whose first-order condition is

$$\underbrace{\frac{1-\gamma-(1+\gamma)\, p_d+2\gamma p_1^*\left(t\right)}{(1-\gamma)(1+2\gamma)} - p_d \frac{1+\gamma}{(1-\gamma)(1+2\gamma)}}_{\text{Monopoly rule}} + \underbrace{\frac{2\gamma t}{(1-\gamma)(1+2\gamma)}}_{\text{Horizontal (or channel) externality}} = 0. \tag{5}$$

The first term in this expression reflects the standard monopoly trade-off: increasing p_d lowers demand in the direct market but it increases M's profit on each unit of sale. The second term, instead, captures the externality that a higher price in the direct channel creates on the indirect one: a higher p_d increases the intermediaries' demand and thus (for given wholesale prices) the revenue that M collects from the platforms.

Letting $p^*_{d,1}(t)$ denote the solution of (5), M's maximization problem at stage 1 is

$$\max_t 2tD^i\left(p^*_1(t), p^*_1(t), p^*_{d,1}(t)\right) + p^*_{d,1}(t)\, D^d\left(p^*_{d,1}(t), p^*_1(t), p^*_1(t)\right).$$

Differentiating with respect to t, by the Envelope Theorem we obtain the following first-order condition

$$\underbrace{\frac{2\gamma t}{(1-\gamma)(1+2\gamma)}\frac{\partial p^*_1(t)}{\partial t} + \frac{1-\gamma-p^*_1(t)+\gamma p^*_{d,1}(t)}{(1-\gamma)(1+2\gamma)}}_{\text{Vertical externality}} + \underbrace{\frac{\gamma p^*_{d,1}(t)}{(1-\gamma)(1+2\gamma)}\frac{\partial p^*_1(t)}{\partial t}}_{\text{Horizontal (or channel) externality}} = 0.$$

Once again, this condition features two terms reflecting the impact of a higher t on M's total profit. First, other things being equal, a higher t increases the revenues from the indirect channel; but, since this also increases the (equilibrium) retail price in that channel, demand drops and so does M's revenue. Second, by increasing $p^*_1(t)$, a higher t also increases demand on the direct channel.

Imposing symmetry and solving the first-order conditions derived above, we can state the following.

Proposition 1 *With platform parity, the symmetric equilibrium of the wholesale model has the following features:*

(i) The monopolist sets

$$p^*_{d,1} = p^M + \underbrace{\frac{\gamma}{(1+2\gamma)(4+\gamma)}}_{\textit{Channel externality}},$$

and charges

$$t^*_1 = p^M + \frac{3\gamma^2}{2(1+2\gamma)(4+\gamma)}.$$

(ii) The platforms charge

$$w^*_1 = t^*_1 + \underbrace{\frac{1-\gamma}{4+\gamma}}_{P_i\textit{'s mark-up}}.$$

(iii) The intermediaries set

$$p^*_1 = w^*_1 + \underbrace{\frac{1-\gamma}{2(4+\gamma)}}_{I_i\textit{'s mark-up}},$$

with $p^*_1 \geq p^*_{d,1}$.

In the equilibrium every player makes a positive profit — i.e., $p^*_1 \geq w^*_1 \geq t^*_1 > 0$. The reason is rather intuitive: under linear contracts, both the platforms and the intermediaries pass on their 'input prices' (the wholesale and the access price respectively) in order to secure positive margins. As a result, retail prices are higher than the price that would be charged by a multi-product monopolist: a multiple marginalization effect.

2.1.2 Equilibrium without platform parity

Next, we analyze the regime without platform parity, where M can charge different wholesale prices to the platforms. P_i solves the following maximization problem

$$\max_{w_i} D^i\left(p_0\left(w_i\right), p_0^*, p_{d,0}^*\right)\left(w_i - t_i\right),$$

whose first-order condition, using (2), is

$$-\left(p_0\left(w_i\right) - w_i\right) + \frac{\partial p_0\left(w_i\right)}{\partial w_i}\left(w_i - t_i\right) = 0,$$

yielding P_i's best reply to every t_i chosen by M — i.e.,

$$w_0^*(t_i) \triangleq \frac{t_i}{2} + \frac{1-\gamma+\gamma\left(p_0^* + p_{d,0}^*\right)}{2\left(1+\gamma\right)}.$$

Substituting $w_0(t_i)$ into $p_0(w_i)$ we obtain I_i's retail price as a function of t_i — i.e.,

$$p_0^*(t_i) \triangleq \frac{t_i}{4} + \frac{3\left(1-\gamma+\gamma(p_0^* + p_{d,0}^*)\right)}{4\left(1+\gamma\right)},$$

which, as expected, is increasing in t_i (the marginal cost) and the equilibrium prices charged in the final market — i.e., p_0^* and $p_{d,0}^*$.

Consider M's behavior. For every pair of contracts $\mathbf{t} \triangleq (t_i, t_{-i})$ offered to the platforms, M chooses p_d in order to solve

$$\max_{p_d} \textstyle\sum_{i=A,B} t_i D^i(p_0^*\left(t_i\right), p_0^*(t_{-i}), p_d) + p_d D^d(p_d, p_0^*(t_A), p_0^*(t_B)),$$

whose first-order condition is

$$\underbrace{\frac{1-\gamma-\left(1+\gamma\right)p_d + \gamma\sum_{i=A,B} p_0^*\left(t_i\right)}{(1-\gamma)(1+2\gamma)} - p_d\frac{1+\gamma}{(1-\gamma)(1+2\gamma)}}_{\text{Monopoly rule}} + \underbrace{\frac{\gamma}{(1-\gamma)(1+2\gamma)}\textstyle\sum_{i=A,B} t_i}_{\text{Horizontal (or channel) externality}} = 0. \qquad (6)$$

As before, at the (final) pricing stage M must take into account the positive effect of increasing p_d on the intermediaries' demand, and thus on its revenue from the indirect channel.

Let $p_{d,0}^*\left(\mathbf{t}\right)$ be the solution of (6). Moving backward at the contracting stage, M chooses the wholesale price charged to P_i in order to solve

$$\max_{t_i} p^*_{d,0}(\mathbf{t})\, D^d(p^*_{d,0}(\mathbf{t}), p^*_0(t_i), p^*) + t^*_0 D^{-i}(p^*_0, p^*_0(t_i), p^*_{d,0}(\mathbf{t})) + t_i D^i(p^*_0(t_i), p^*_0, p^*_{d,0}(\mathbf{t})),$$

whose first-order condition (by using the Envelope Theorem) is

$$\underbrace{\frac{1-\gamma-(1+\gamma)\,p^*_0(t_i)+\gamma\left(p^*_0+p^*_{d,0}(\mathbf{t})\right)}{(1-\gamma)(1+2\gamma)} - t_i\frac{1+\gamma}{(1-\gamma)(1+2\gamma)}\frac{\partial p^*_0(t_i)}{\partial t_i}}_{\text{Monopoly rule}} +$$
$$\underbrace{+\frac{\gamma}{(1-\gamma)(1+2\gamma)}\frac{\partial p^*_0(t_i)}{\partial t_i}\left(p^*_{d,0}(\mathbf{t})+t^*_0\right)}_{\text{Channel externality}} = 0.$$

Since we are considering a contract equilibrium, in the absence of a platform parity agreement, the monopolist itself, when contracting with P_i, takes as given the equilibrium contract offered to the other platform (and the resulting equilibrium price p^*_0). Hence, increasing t_i has three main effects on M's profit. First, it increases M's demand on the direct channel because $p^*_0(t_i)$ is increasing. Second, a higher t_i also increases the revenue that M obtains from P_{-i} because (ceteris paribus) it increases I_{-i}'s demand. Both effects create a channel externality which adds to the third more standard effect: a higher t_i increases the revenue that M collects from P_i, but it also lowers the demand for the product distributed through P_i since $p^*_0(t_i)$ is increasing.

Imposing symmetry and solving the first-order conditions derived above, we can state the following.

Proposition 2 *Without platform parity, the symmetric equilibrium of the wholesale model has the following features:*

(*i*) *The monopolist sets*

$$p^*_{d,0} = p^M + \underbrace{\frac{3\gamma(1+\gamma)}{(1+2\gamma)(8+5\gamma)}}_{\textit{Channel externality}},$$

and charges

$$t^*_0 = p^M + \frac{3\gamma(1+3\gamma)}{2(1+2\gamma)(8+5\gamma)}.$$

(*ii*) *The platforms charge*

$$w^*_0 = t^*_0 + \underbrace{\frac{2(1-\gamma)}{8+5\gamma}}_{P_i\textit{'s mark-up}}.$$

(*iii*) *The intermediaries set*

$$p^*_0 = w^*_0 + \underbrace{\frac{1-\gamma}{8+5\gamma}}_{I_i\textit{'s mark-up}},$$

with $p_0^* > p_{d,0}^*$.

The equilibrium features multiple marginalization also in the regime without parity — i.e., $p_0^* \geq w_0^* \geq t_0^* > 0$. The reason is as before: each level of the supply chain creates a mark up, the sum is ultimately passed on to final consumers.

2.2 Welfare

We can now examine the welfare effects of platform parity. Before stating the main result of the section, it is useful to observe that, when the parity rule is in place, the final price in the indirect channel is more sensitive to the wholesale price charged by the monopolist to the platforms — i.e.,

$$\frac{\partial p_1^*(t)}{\partial t} - \frac{\partial p_0^*(t_i)}{\partial t_i} = \frac{3\gamma}{4(\gamma+4)} > 0.$$

In other words, the rate at which intermediaries pass on wholesale prices to final consumers is higher when the provision is in place. In this regime, a platform (say P_i) that is charged a higher wholesale price anticipates that the other platform (say P_{-i}) faces the same price increase. Hence, when P_i observes a higher t it expects P_{-i} to pass on this higher wholesale price to its intermediary via a higher access price w_{-i}, which will in turn induce I_{-i} to increase the final price p_{-i}. As a result, when t increases, P_i has two reasons for charging I_i a higher access price w_i: first, because it faces a higher wholesale price and needs to mark up more its own intermediary: a standard vertical externality; second, because it expects a higher final price by I_{-i} and, as a result, a higher demand for I_i's product: a horizontal externality introduced by platform parity. By contrast, when the provision is not in place, P_i only observes its own wholesale price and, in the contract equilibrium, it assumes that the rival remains under the equilibrium contract.

Interestingly, the gap between the rate at which intermediaries pass on with and without parity is increasing in γ (see Figure 3). Essentially, with parity an increase in t is equivalent to a common cost shock, which is passed on to a greater extent the more competition there is (the greater γ is). By contrast, without parity, a higher t_i is equivalent to an idiosyncratic cost shock, and (with linear demand) the pass on rate does not depend on γ.

Hence, when products become closer substitutes, the intermediaries are more responsive to the prices of their rivals because competition is more intense. Thus, the horizontal externality introduced by the parity provision is relatively more pronounced when γ is large. By contrast, when γ is small, intermediaries care less about the price of their rivals. In this case, the dominating force is the vertical externality — e.g., in the limit of $\gamma \to 0$ the pass on rate is nearly the same regardless of whether there is parity or not.

We can thus state the following.

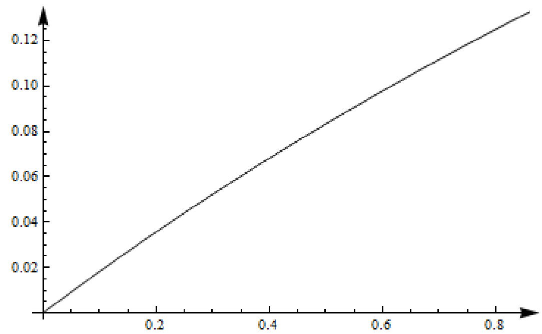


Figure 3: Difference $\frac{\partial p_1^*(t)}{\partial t} - \frac{\partial p_0^*(t_i)}{\partial t_i}$ as a function of γ.

Proposition 3 *In the wholesale model the introduction of a platform parity agreement lowers prices at every level of the supply chain — i.e., $t_1^* < t_0^*$, $w_1^* < w_0^*$, $p_{d,1}^* < p_{d,0}^*$ and $p_1^* < p_0^*$ — and thus it always benefits consumers. Moreover, it also increases firms' individual profits.*

Hence, the provision can be interpreted as a commitment device that forces the monopolist to mitigate the multiple marginalization effect by choosing a lower wholesale price. The intuition hinges on the effect discussed above. Platform parity is equivalent to introducing a common component into the platforms and the intermediaries' cost function: an increase of such component has a symmetric impact on the pass on rates charged at both levels of the supply chain. In other words, when platforms cannot be price discriminated, any attempt of the monopolist to increase the (wholesale) price charged to one platform immediately translates into a parallel shift of the (wholesale) price charged to the other platform. As a result, both platforms increase the access price charged to their intermediaries, whereby leading to higher retail prices. By contrast, without platform parity, it is as if the cost function of the competing platforms and intermediaries are affected by idiosyncratic components only. In fact, contract equilibrium implies that when the monopolist increases the (wholesale) price charged to a platform (say P_i), this platform takes as given the access price charged by its rival since (as for the logic of passive beliefs) it remains under the equilibrium contract t_0^*. Therefore, a higher t_i only translates into a higher retail price charged by I_i, which makes the multiple marginalization problem less problematic for M.

Precisely the increased pass on rate that occurs under the parity provision refrains M from charging a wholesale price that is too high in equilibrium. This mandates a lower access price, a lower final price and thus a higher consumer surplus in the indirect distribution channel. In addition, since prices are strategic complements, this effect propagates to the direct channel, leading to lower final prices in that segment also. As a result, also consumers in the direct channel benefit from the parity provision. Figure 4 shows the difference between consumer surplus with and without the parity provision. It can be seen that, as γ increases, the positive impact of the provision on consumer surplus grows larger: as competition in the final market becomes more intense, the difference between the intermediaries' pass on rate with and without platform parity increases (recall Figure 3), which strengthens the beneficial commitment effect of the provision.

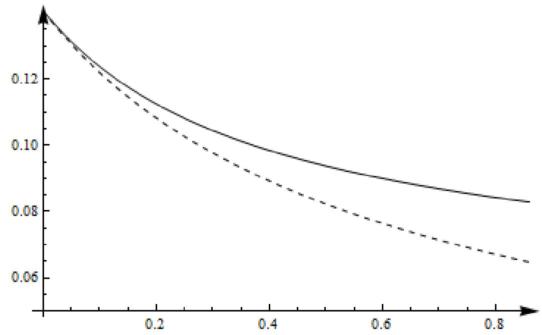


Figure 4: Consumer surplus with platform parity (continuous line) and without platform parity (dashed line) as functions of γ.

Interestingly, platform parity echoes the effect of assuming symmetric beliefs off equilibrium path when using PBE as solution concept (see, e.g., Rey and Tirole, 2007, and Pagnozzi and Piccolo, 2011). Essentially, as in our analysis, with symmetric beliefs a platform receiving from the monopolist an offer different from what it expects in equilibrium, believes that the competing platform has received the same offer, which generates welfare effects equivalent to those discussed above. Yet, in contrast to the previous literature, in our three layer model where the monopolist can also sell directly to consumers, the effect on consumer surplus is pro-competitive rather than anti-competitive.

Finally, it should also be noted that, by reducing multiple marginalization, the provision increases profits at every level of the supply chain. This result is in line with Hart and Tirole (1990) and McAfee and Schwartz (1994) who consider symmetric beliefs in a model with a single (monopolistic) manufacturer and two independent and competing retailers. They show that, with private contracts, the manufacturer's profit is higher with symmetric than with passive beliefs.

Hence, in the wholesale model platform parity is also total welfare enhancing.

2.3 Discussion

Before turning to the agency model, a few remarks are in order.

Two-layer parity. So far, we have considered price parity only at the platform level. One may wonder what would happen when such a provision is imposed also at the intermediaries' level — i.e., when $w_i = w$ in addition to $t_i = t$ for every $i = A, B$. The effect of introducing this additional constraint amplifies the pro-competitive effect discussed above because, in addition to M, also platforms will now have a lower incentive to increase their access prices being afraid of creating excessive marginalization.

Two-part tariffs. The effects highlighted above hinge on the multiple marginalization problem that is created by the assumption of linear contracts. One may wonder what would happen with two-part tariffs. In the wholesale model the answer is simple. When the monopolist can charge a fixed fee to the platforms, who can in turn charge a fixed fee to the intermediaries, the double marginalization problem wipes out. The reason is that the monopolist internalizes the profits of the entire indirect channel via the fixed fee charged to the platforms. This is because, by doing so the monopolist is able to internalize the fixed fee charged by the platforms to the intermediaries, and thus the intermediaries' profit. However, the monopolist cannot reach the efficient solution p^M because intermediaries compete downstream: conditional on wholesale prices being equal to marginal costs, prices are too low compared to what M would like to choose. In this case, platform parity can help the monopolist in restoring its monopoly power by increasing symmetrically the platforms' wholesale prices so to relax competition in the indirect channel. As a result, with two-part tariffs, platform parity is likely to be anti-competitive. Yet, when extracting profits by means of a fixed fee is costly (e.g., because of frictions akin moral hazard or adverse selection) our conclusions still hold as long as the cost of extracting profits up-front is sufficiently high (a similar point is discussed in Rey and Vergé, 2017).[11]

RPM. Up until now we have assumed that M cannot control the retail prices charged by the intermediaries. When M can dictate the retail prices in the indirect channel — i.e., when Resale Price Maintenance (henceforth, RPM) is allowed — the multiple marginalization problem wipes out (see, Motta, 2004, Ch. 6, for a survey). Hence, consumer surplus and M's profit increase, whereas the intermediaries and the platforms make zero profit. Obviously, in this case, platform parity is always welfare neutral. Notice also that while imposing a price-cap is equivalent to forcing prices (because intermediaries would like to set prices higher than M), imposing a price-floor is welfare neutral since it does not help M to solve the multiple marginalization problem.

RPM is welfare improving also when it is imposed by the platforms, and not by M. This is because platforms have an incentive to squeeze the intermediaries' mark-up, exactly as M would do. In this case platforms would still make positive profits because of double marginalization (see, e.g., also Gaudin, 2019). Therefore, following the same logic of our baseline model, platform parity would still be pro-competitive.[12]

In sum, in the wholesale model platform parity is likely to increase welfare also when retail price restrictions are imposed along the supply chain. We will see in the next section that this conclusion changes in the agency model, where the distribution of the bargaining power is different.

Price commitment. Up until now, we have assumed that M sets the price on the direct channel at the last stage of the game (i.e., simultaneously with the intermediaries). Would

[11]This occurs, for example, when by charging a fixed fee $T_i > 0$ the monopolist gains T_i but the platform loses $(1 + \mu)T_i$, and μ is sufficiently large (see, e.g., Calzolari et al., 2018). See the online Appendix for a complete analysis of this case.

[12]The formal argument is standard and omitted for brevity. Proofs are available upon request.

the results change when M can commit to p_d before the contracting takes place? In the online Appendix we show that, when M can credibly commit to the price charged in the direct channel, it acts as a Stackelberg leader and sets that price efficiently regardless of whether platform parity is in place or not. Therefore, the beneficial effect of platform parity is the same as in the baseline model, except that it must be diluted from the channel externality which wipes out because of commitment.

Multiple Platforms. Finally, it should be clear that the logic of the pro-competitive mechanism described above does not change when M deals with $N > 2$ symmetric platforms. In fact, as seen for γ, it can be shown (see the online Appendix) that the difference between the pass on rate with and without parity is increasing in N: as the market becomes more competitive, intermediaries are more sensitive to cost variations. As a result, the beneficial effect of platform parity is more pronounced as competition in the industry intensifies.

3 The agency model

We can now turn to analyze the *agency model*. In this business model a contract between M and P_i specifies a commission (fee) f_i paid by M to P_i for each unit distributed by I_i. In addition, M sets the access price τ_i that it charges I_i for every unit sold through P_i. Following the literature (e.g., Boik and Corts, 2016, and Johansen and Vergé, 2016, among others) the timing of the game is as follows (see Figure 5):

$t = 1$ Platforms secretly offer commissions to M;

$t = 2$ M accepts or refuses these offers, and sets the access prices;

$t = 3$ The monopolist and the intermediaries set final prices and demand is allocated across and within the two channels.

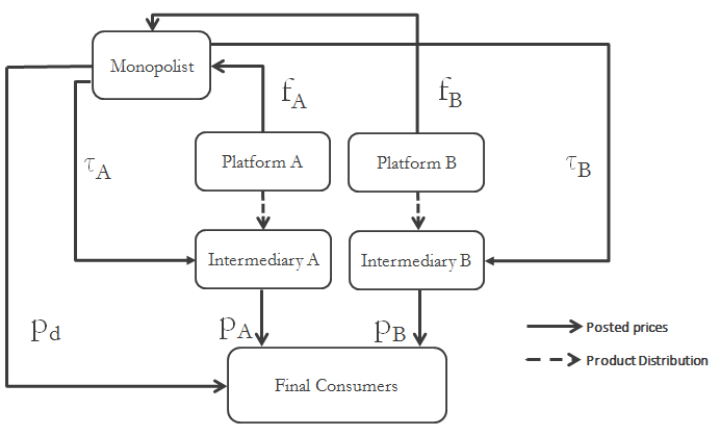


Figure 5: The agency model.

We consider again two different versions of the game, depending on the possibility of signing a *platform parity agreement* between the monopolist and the platforms. Specifically:

- When a platform parity agreement is in place ($k = 1$), M commits to post the same access price on both platforms — i.e., $\tau_i = \tau$ for every $i = A, B$.
- Without the agreement ($k = 0$), M is free to charge intermediaries different access prices — i.e., it may happen that $\tau_A \neq \tau_B$.

As before, the equilibrium concept is *contract equilibrium* and we restrict our attention to symmetric equilibria in which, for every $k = 1, 0$:[13]

- Both platforms offer the same commission f_k^* to M;
- M accepts both offers and sets the same access price τ_k^* on each platform;
- Intermediaries charge the same retail price p_k^* to final consumers, while M charges $p_{d,k}^*$ on the direct channel.

It should be noted that, compared to the previous section, we have now changed both the direction of payment flows and the distribution of bargaining power — i.e., while in the wholesale model M had full bargaining power *vis-à-vis* platforms, now it is the opposite. The reason is intuitive: if M could set the commissions f_A and f_B, platforms would clearly make zero profit. The analysis would then be equivalent to the wholesale model: since platforms play no role, the parity provision would again unambiguously increase consumer surplus because it reduces the intermediaries' mark-ups. Hence, in giving full contractual power to the platforms against the monopolist, we are amplifying the anticompetitive effect of platform parity.

3.1 Equilibrium with platform parity

With platform parity each intermediary I_i solves

$$\max_{p_i} D^i\left(p_i, p_1^*, p_{d,1}^*\right)(p_i - \tau),$$

whose (standard) first-order condition is

$$\frac{1-\gamma-(1+\gamma)\,p_i+\gamma\left(p_1^*+p_{d,1}^*\right)}{(1-\gamma)(1+2\gamma)} - \frac{1+\gamma}{(1-\gamma)(1+2\gamma)}(p_i-\tau) = 0. \tag{7}$$

[13] As in the wholesale model, we focus directly on a symmetric equilibrium for simplicity of exposition and without loss of generality.

The solution of (7) yields the (symmetric) equilibrium of the (pricing) game between the intermediaries

$$p_1^*(\tau) \triangleq \frac{1+\gamma}{2+\gamma}\tau + \frac{1+\gamma(p_{d,1}^*-1)}{2+\gamma},$$

which, as expected, is increasing in the (common) wholesale price τ.

For any pair of commissions (f_A, f_B) negotiated with the platforms, M solves

$$\max_{p_d} \sum_{i=A,B} (\tau - f_i) D^i(p_1^*(\tau), p_1^*(\tau), p_d) + p_d D^d(p_d, p_1^*(\tau), p_1^*(\tau)),$$

whose first-order condition is

$$\underbrace{\frac{1-\gamma-(1+\gamma)\,p_d+2\gamma p_1^*\,(t)}{(1-\gamma)(1+2\gamma)} - p_d\frac{1+\gamma}{(1-\gamma)(1+2\gamma)}}_{\text{Monopoly rule}} + \underbrace{\left(2\tau - \sum_{i=A,B} f_i\right)\frac{\gamma}{(1-\gamma)(1+2\gamma)}}_{\text{Channel externality}} = 0. \tag{8}$$

As in the wholesale model, this condition reflects both the standard monopoly trade-off and the channel externality due to the impact of p_d on the profit that M obtains through the indirect channel.

For any pair (f_A, f_B), let $p_{d,1}^*(\tau, f_A, f_B)$ denote the solution of equation (8). Following a backward induction logic, M sets τ by solving

$$\max_{\tau} \sum_{i=A,B} (\tau - f_i) D^i(p_1^*(\tau), p_1^*(\tau), p_{d,1}^*(\tau, f_A, f_B)) + p_{d,1}^*(\tau, f_A, f_B) D^d(p_{d,1}^*(\tau, f_A, f_B), 2p_1^*(\tau)).$$

whose first-order condition (by the Envelope Theorem) is

$$\underbrace{2\frac{1-\gamma-p_1^*(\tau)+\gamma p_{d,1}^*(\tau, f_A, f_B)}{(1-\gamma)(1+2\gamma)}}_{\text{Revenue Enhancing}} - \underbrace{\sum_{i=A,B} (\tau - f_i)\,\frac{1}{(1-\gamma)(1+2\gamma)}\frac{\partial p_1^*(\tau)}{\partial \tau}}_{\text{Demand Reduction}} +$$

$$\underbrace{+2p_{d,1}^*(\tau, f_A, f_B)\frac{\partial p_1^*(\tau)}{\partial \tau}\frac{\gamma}{1+\gamma\,(1-2\gamma)}}_{\text{Channel Externality}} = 0. \tag{9}$$

Hence, M's choice of τ is shaped by the following effects. First, a higher τ increases the retail prices on the indirect channel, which leads to a higher demand on the direct channel: a positive channel externality. Second, higher retail prices in the indirect channel reduce the demand faced by each intermediary: a demand reduction effect. Third, for a given demand, a higher τ increases the revenue that M collects from the intermediaries: a revenue enhancing effect.

Solving equation (9) yields M's optimal wholesale price $\tau_1^*(f_A, f_B)$. Thus, going back to the first stage of the game, we can solve each platform's maximization problem. Because the consumer displays preference for variety, we neglect for the moment M's participation constraint, which is verified in the Appendix. Hence, P_i solves

$$\max_{f_i} f_i D^i(p_1^*(f_i, f_1^*), p_1^*(f_i, f_1^*), p_{d,1}^*(f_i, f_1^*)),$$

where, to save on notation, we defined

$$p_{d,1}^*(\tau_1^*(f_i, f_1^*), f_i, f_1^*) \triangleq p_{d,1}^*(f_i, f_1^*),$$

and

$$p_1^*(\tau_1^*(f_i, f_1^*)) \triangleq p_1^*(f_i, f_1^*).$$

Differentiating with respect to f_i, we obtain

$$\underbrace{\frac{1-\gamma-p_1^*(f_i, f_1^*)+\gamma p_{d,1}^*(f_i, f_1^*)}{(1-\gamma)(1+2\gamma)}}_{\text{Revenue Enhancing}} - \underbrace{\frac{f_i}{(1-\gamma)(1+2\gamma)}\frac{\partial p_1^*(f_i, f_1^*)}{\partial f_i}}_{\text{Multiple Marginalization}} +$$

$$+\underbrace{\frac{\gamma f_i}{(1-\gamma)(1+2\gamma)}\frac{\partial p_{d,1}^*(f_i, f_1^*)}{\partial f_i}}_{\text{Business Stealing}} = 0.$$

The interpretation of this condition is as follows. First, holding demand constant, when P_i charges M a higher fee, it earns higher revenues. Second, ceteris paribus, a higher f_i also induces M to charge a higher access price. Hence, retail prices in the indirect channel increase, whereby reducing the volume of sales on that platform: a multiple mark-ups problem. Third, since a higher f_i exacerbates the marginalization problem, M has an incentive to reduce the price on the direct channel in order to increase demand on the (relatively) more efficient direct channel, whereby reducing P_i's demand.

Imposing symmetry, we can state the following.

Proposition 4 *With platform parity, the symmetric equilibrium of the agency model has the following features:*

(i) *Each platform charges M*

$$f_1^* = \underbrace{(1-\gamma)\frac{4+10\gamma-\gamma^2-10\gamma^3-4\gamma^4}{6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4}}_{(>0)\ P_i\text{'s mark-up}}.$$

(ii) *The monopolist sets*

$$p_{d,1}^* = p^M + \underbrace{\frac{\gamma(1-\gamma^2)}{2(6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4)}}_{\text{Channel externality}},$$

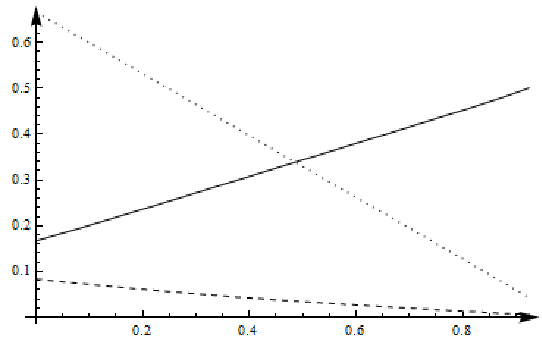


Figure 6: Intermediaries' (dashed line), platforms' (dotted line) and M's (continuous line) markups as functions of γ in the agency model with a platform parity agreement.

and charges each intermediary

$$\tau_1^* = f_1^* + \underbrace{\frac{2 + 9\gamma + 11\gamma^2 - 6\gamma^3 - 13\gamma^4 - 4\gamma^5}{2\left(6 + 15\gamma - \gamma^2 - 15\gamma^3 - 6\gamma^4\right)}}_{(>0)\ M\text{'s mark-up}},$$

with $\tau_1^* > f_1^*$.

(*iii*) *The intermediaries set*

$$p_1^* = \tau_1^* + \underbrace{\frac{(1-\gamma)^2(1+\gamma)(1+2\gamma)}{2\left(6 + 15\gamma - \gamma^2 - 15\gamma^3 - 6\gamma^4\right)}}_{(>0)\ I_i\text{'s mark-up}},$$

with $p_1^* \geq p_{d,1}^*$ *and* $p_1^* \geq \tau_1^*$.

The equilibrium of the game features multiple mark-ups even in the agency model. The reason is straightforward: in order to earn profits platforms must charge positive commissions to M, who is thus forced to pass on these higher commissions to the intermediaries. As a result, in the equilibrium, intermediaries set a retail price that is higher than the price charged by the multi-product monopolist. Moreover, M will also charge a higher price on the direct channel because demand in that segment increases in response to higher prices in the indirect channel.

Figure 6 shows that while the intermediaries and the platforms' mark-ups are decreasing in γ, as implied by tougher competition between and within the two distribution channels, M's mark-up is increasing in γ because more competition reduces the multiple mark-ups problem, thus M behaves more efficiently.

3.2 Equilibrium without platform parity

Next, assume that there is no platform parity agreement — i.e., M can charge different access prices to the intermediaries. Recall that contracts are secret: I_i observes only τ_i but not the access price charged to its rival. Hence, taking as given the equilibrium price of its competitors — i.e., p_0^* and $p_{d,0}^*$ — I_i solves

$$\max_{p_i} D^i\left(p_i, p_0^*, p_{d,0}^*\right)\left(p_i - \tau_i\right),$$

whose first-order condition immediately yields

$$p_0^*(\tau_i) \triangleq \frac{\tau_i}{2} + \frac{1 + \gamma p_0^* - \gamma(1 - p_{d,0}^*)}{2(1+\gamma)}.$$

As intuition suggests, $p_0^*(\tau_i)$ is increasing in τ_i and (due to strategic complementarity) in p_0^* and $p_{d,0}^*$.

Moving to stage 3, after observing f_A and f_B negotiated with the platforms, M sets the price p_d in order to solve

$$\max_{p_d} \sum_{i=A,B} (\tau_i - f_i) D^i(p_0^*(\tau_i), p_0^*(\tau_{-i}), p_d) + p_d D^d(p_d, p_0^*(\tau_A), p_0^*(\tau_B)).$$

The first-order condition is

$$\underbrace{\frac{1-\gamma-(1+\gamma)\,p_d+\gamma\left(p_0^*(\tau_A)+p_0^*(\tau_B)\right)}{(1-\gamma)(1+2\gamma)} - p_d\frac{1+\gamma}{(1-\gamma)(1+2\gamma)}}_{\text{Monopoly rule}}+$$

$$\underbrace{+\frac{\gamma}{(1-\gamma)(1+2\gamma)}\sum_{i=A,B}(\tau_i - f_i)}_{\text{Channel externality}} = 0.$$

Once again, this condition reflects the standard monopoly trade-off and the channel externality.

Let $p_{d,0}^*(\tau_A, \tau_B, f_A, f_B)$ denote the solution to the considered first-order condition. At stage 2, M chooses τ_A and τ_B in order to solve

$$\max_{\tau_A,\tau_B} \sum_{i=A,B} (\tau_i - f_i) D^i(p_0^*(\tau_i), p_0^*(\tau_{-i}), p_{d,0}^*(\cdot)) + p_{d,0}^*(\cdot) D^d(p_{d,0}^*(\cdot), p_0^*(\tau_A), p_0^*(\tau_B)).$$

By the Envelope Theorem, the first-order condition with respect to τ_i ($i = A, B$) is

$$\underbrace{\frac{1-\gamma-(1+\gamma)\,p_0^*(\tau_i)+\gamma\left(p_0^*(\tau_{-i})+p_{d,0}^*(\cdot)\right)}{(1-\gamma)(1+2\gamma)} - \frac{1+\gamma}{(1-\gamma)(1+2\gamma)}(\tau_i - f_i)\frac{\partial p_0^*(\tau_i)}{\partial \tau_i}}_{\text{Monopoly rule}}+$$

$$\underbrace{+\frac{\gamma}{(1-\gamma)(1+2\gamma)}\left[\tau_{-i} - f_{-i} + p_{d,0}^*(\cdot)\right]\frac{\partial p_0^*(\tau_i)}{\partial \tau_i}}_{\text{Channel externality}} = 0.$$

Clearly, M's profit increases as τ_i grows large because (for a given demand) M collects higher revenues from I_i. However, since contracts are secret, a higher τ_i only increases the retail price charged by I_i. Hence, other things being equal, I_i's demand drops, whereas I_{-i}'s demand on the indirect channel and M's demand on the direct channel increase.

By solving the system of first-order conditions, we obtain the monopolist's price on platform P_i as a function $\tau_0^*(f_i, f_{-i})$. To save on notation, let

$$p_{d,0}^*(\tau_0^*(f_i, f_{-i}), \tau_0^*(f_{-i}, f_i), f_i, f_{-i}) \triangleq p_{d,0}^*(f_i, f_{-i}),$$

and

$$p_0^*(\tau_0^*(f_i, f_{-i})) \triangleq p_0^*(f_i, f_{-i}).$$

Then, P_i solves the following maximization problem at stage 1:

$$\max_{f_i} f_i D^i(p_0^*(f_i, f_0^*), p_0^*(f_0^*, f_i), p_{d,0}^*(f_i, f_0^*)),$$

whose first-order condition is

$$\underbrace{\frac{1-\gamma-(1+\gamma)\,p_0^*(f_i, f_0^*)+\gamma\left(p_0^*(f_0^*, f_i)+p_{d,0}^*(f_i, f_0^*)\right)}{(1-\gamma)(1+2\gamma)} - f_i\frac{1+\gamma}{(1-\gamma)(1+2\gamma)}\frac{\partial p_0^*(f_i, f_0^*)}{\partial f_i}}_{\text{Monopoly rule}}+$$

$$\underbrace{+f_i\frac{\gamma}{(1-\gamma)(1+2\gamma)}\left[\frac{\partial p_0^*(f_0^*, f_i)}{\partial f_i}+\frac{\partial p_{d,0}^*(f_i, f_0^*)}{\partial f_i}\right]}_{\text{Channel externality}} = 0.$$

There are three effects that shape P_i's optimal commission. First, for a given demand, by increasing the commission charged to M, platform P_i earns a higher profit. Second, since (ceteris paribus) M reacts to a higher f_i by increasing the access price charged to I_i, fewer consumers demand P_i's product. Third, since the price charged by I_i increases, the demand on platform P_{-i} and on the direct channel increase. Intuitively, as I_i becomes less competitive, M has an incentive to reduce τ_{-i} and p_d.

Imposing symmetry we can state the following.

Proposition 5 *Without platform parity, the symmetric equilibrium of the agency model has the following features:*

(*i*) *Each platform charges M*

$$f_0^* = \underbrace{\frac{8\,(1-\gamma^2)\,(4+4\gamma-9\gamma^2)}{64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4}}_{(>0)\ P_i\text{'s mark-up}}.$$

(ii) The monopolist sets

$$p_{d,0}^* = p^M + \underbrace{\frac{\gamma(1+\gamma)(8-9\gamma^2)}{64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4}}_{(>0) \text{ Channel externality}},$$

and charges each intermediary

$$\tau_0^* = f_0^* + \underbrace{\frac{32+128\gamma+100\gamma^2-148\gamma^3-153\gamma^4}{2(64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4)}}_{(>0)\ M_i\text{'s mark-up}},$$

with $\tau_0^* > f_0^*$.

(iii) Each intermediary sets

$$p_0^* = \tau_0^* + \underbrace{\frac{(1-\gamma)(1+2\gamma)(8-9\gamma^2)}{64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4}}_{(>0)\ I_i\text{'s mark-up}},$$

with $p_0^* \geq p_{d,0}^*$ *and* $p_0^* \geq \tau_0^*$..

Once again, the equilibrium features multiple marginalization. Platforms and intermediaries' mark-ups are decreasing in γ whereas M's mark-up is increasing in γ (see Figure 7).

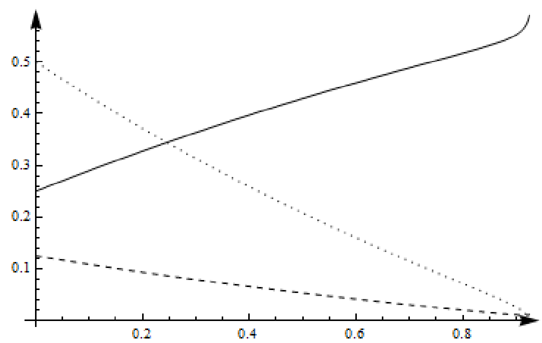


Figure 7: Intermediaries' (dashed line), platforms' (dotted line) and M's (continuous line) markups as functions of γ in the model without platform parity agreements.

3.3 Welfare

We can now study how the introduction of a platform parity agreement affects consumer surplus and industry profits. To begin with, it is useful to examine the effect of such an agreement on commissions, access prices and final (retail) prices.

Proposition 6 *With platform parity:*

(i) P_i $(i = A, B)$ *charges* M *a higher commission — i.e.,* $f_1^* > f_0^*$;

(ii) M *always charges a lower price on the direct channel, but it charges a higher access price if and only if* γ *is not too large — i.e.,* $p_{d,1}^* < p_{d,0}^*$ *for every* $\gamma \in [0, \overline{\gamma}]$, *while there is* $\check{\gamma} \in (0, \overline{\gamma})$ *such that* $\tau_1^* < \tau_0^*$ *if and only if* $\gamma \geq \check{\gamma}$.

(iii) I_i $(i = A, B)$ *sets a higher (retail) price if and only if* γ *is not too large — i.e., there is* $\gamma^* \in (0, \check{\gamma})$ *such that* $p_1^* < p_0^*$ *if and only if* $\gamma \geq \gamma^*$. *Notice that* $\gamma^* < \check{\gamma}$, *so that* $\tau_1^* < \tau_0^*$ *implies* $p_1^* < p_0^*$.

The reason why commissions are higher under platform parity is as in Boik and Corts (2016): each platform anticipates that, being concerned with double marginalization, under the parity provision M has a lower incentive to pass on commissions to the intermediaries, because the increase in a platform's commission must be passed on to both intermediaries and not just to that platform's intermediary. Hence, they charge M more. Interestingly, the difference between the commissions with and without parity is decreasing in γ (see Figure 8). The reason is that, as products become more differentiated (lower γ), competition (within and across channels) intensifies and, as a result, mark-ups and profits in both regimes decrease, whereby aligning the equilibrium commissions with and without parity.

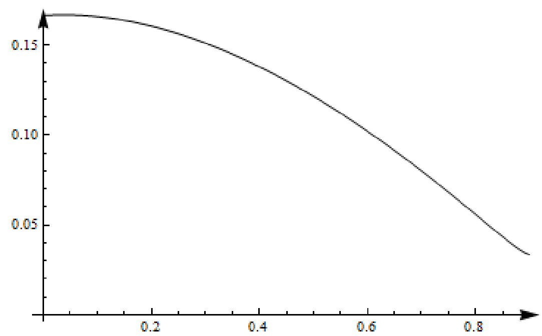


Figure 8: Difference $f_1^* - f_0^*$ as a function of γ.

The effect of the provision on the price charged on the direct channel is straightforward. Since platform parity increases commissions, M charges a lower p_d in order to divert business towards the (relatively cheaper) direct channel. Notice, however, that diverting business towards direct distribution at the expense of direct distribution is not necessarily efficient for M because of consumers' taste for variety.

The effect of the provision on the access price is ambiguous. Since the parity provision induces platforms to charge higher commissions, M marks up more the intermediaries. But, by doing so, it also reduces their demand (since, in turn, intermediaries will charge higher retail prices) and thus the volume of indirect sales. Clearly, when γ is large enough, the second effect prevails, because with parity pass on rates are higher when γ is large, whereas the opposite is true without parity.

Finally, the effect of platform parity on the retail prices is ambiguous too since it reflects the non-monotone impact of γ on the access price. Of course, whenever the provision lowers the equilibrium access price, it also lowers the retail price in the indirect channel (since it implies lower marginal costs for the intermediaries).

Summing up, in the agency model there are three main welfare effects associated with platform parity. First, other things being equal, by increasing commissions, the parity provision harms consumers because it creates more marginalization: the dark side of platform parity. Second, ceteris paribus, the provision mitigates the marginalization problem between the monopolist and the intermediaries, which benefits consumers: the bright side of platform parity. Third, by lowering prices on the direct channel, the provision increases consumer surplus in that segment and creates a competitive pressure on the indirect channel, which (ceteris paribus) benefits the consumers purchasing from the intermediaries.

Building on these insights, we can state the following.

Proposition 7 *There are two thresholds $\tilde{\gamma} \in (0,1)$ and $\hat{\gamma} \in (0,1)$, with $\hat{\gamma} > \tilde{\gamma}$, such that the introduction of platform parity:*

(i) benefits consumers if and only if $\gamma \geq \tilde{\gamma}$;

(ii) always damages the monopolist and the intermediaries;

(iii) benefits platforms if and only if $\gamma \geq \hat{\gamma}$.

Hence, platform parity benefits consumers for γ large. The intuition is the following. Intensified competition increases the difference between τ_0^* and τ_1^* and, at the same time, it reduces the difference between f_1^* and f_0^*. In other words, as in the wholesale model, the wedge between the rate at which intermediaries pass on with and without parity is increasing in γ — i.e.,

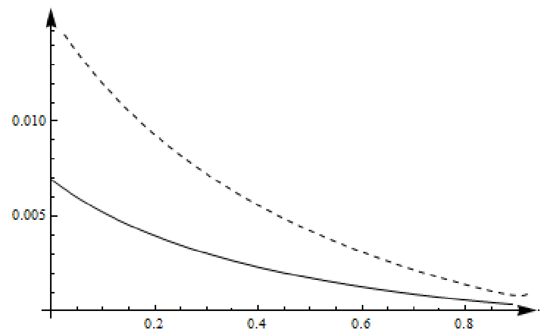
$$\frac{\partial p_1^*(\tau)}{\partial \tau} - \frac{\partial p_0^*(\tau_i)}{\partial \tau_i} = \frac{\gamma}{2(2+\gamma)},$$

whereas the wedge between the commissions decreases (as shown in Figure 8). Moreover, intensified competition also lowers prices in the direct channel, which benefits consumers on both channels.

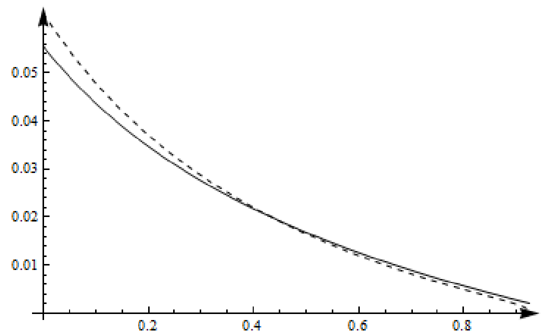
Interestingly, in contrast with the wholesale benchmark, in the agency model platforms benefit from the parity provision only if γ is sufficiently large. The reason is that, when γ is large enough, platform parity exerts a strong downward pressure on the mark-ups, whereby allowing platforms to increase commissions without producing an effect that is too negative on demand. The same argument explains why the monopolist and the intermediaries are hurt by the provision. The monopolist gains lower margins and pays higher commissions. The intermediaries earn lower margins because prices on the direct channel are lower with rather than without the provision and (for γ large) prices on the indirect channel are lower with the provision.

Hence, a novel policy implication of our analysis is that platforms' incentives are aligned with consumers' interests since — i.e., whenever the parity provision benefits platforms, it also benefits consumers ($\hat{\gamma} > \tilde{\gamma}$), but not the other way around. Hence, from a practical point of view, the likelihood that these provisions benefit consumers is higher when their introduction is not opposed by platforms. By contrast, the monopolist and the intermediaries's incentives are always misaligned with consumer surplus.

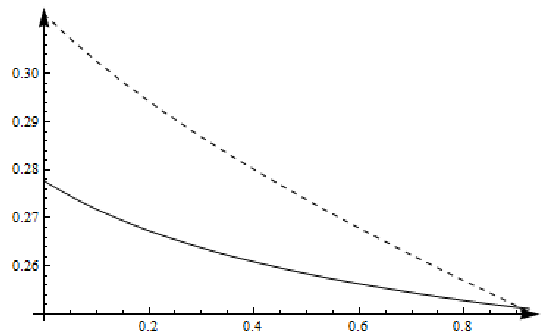
Figure 9 describes the impact of platform parity on profits and consumer surplus.



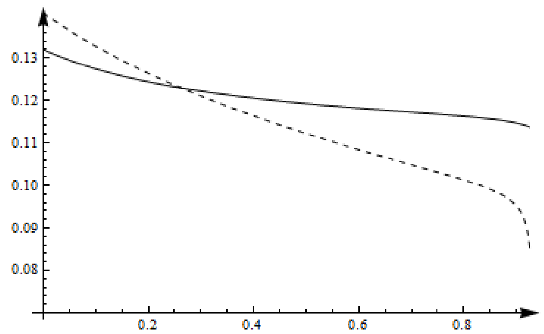
(a) *Intermediaries' profit in the presence of a platform parity clause (continuous line) and without parity agreements (dashed line).*



(b) *Platforms' profit in the presence of a platform parity clause (continuous line) and without parity agreements (dashed line).*



(c) *Monopolist's profit in the presence of a platform parity clause (continuous line) and without parity agreements (dashed line).*



(d) *Consumer surplus in the presence of a platform parity clause (continuous line) and without parity agreements (dashed line).*

Figure 9: Equilibrium profits and consumer surplus in the agency model as functions of γ.

Finally, in the next proposition we examine to what extent platforms can use side payments to align their incentives concerning the parity regime with the other players in the industry.

Proposition 8 *There exist thresholds $\gamma_T > \gamma_M > \gamma_I > \hat{\gamma}$ such that the introduction of platform parity:*

- *increases the joint profit of platforms and intermediaries if and only if $\gamma > \gamma_I$;*
- *increases the joint profit of platforms and the monopolist if and only if $\gamma > \gamma_M$;*

- *increases the total industry profit if and only if* $\gamma > \gamma_T$.

The intuition is straightforward. When competition in the product market is sufficiently fierce, the beneficial effect of the provision on the profit of the platforms outweighs the negative effect on the profit of the intermediaries. In fact, for γ large, the provision not only allows platforms to increase commissions without producing a too negative effect on demand, but it also reduces the access price charged by M to the intermediaries — i.e., $\tau_1^* < \tau_0^*$. Hence, platforms can persuade intermediaries to accept the parity regime by making appropriate side payments that align their incentives. However, persuading the monopolist is harder: whenever the joint profit of the platforms and the intermediaries is higher with than without the parity provision, the total industry profit may well be lower without the provision — i.e., the price that M would require to accept the parity regime is higher than the gain obtained jointly by the platforms and the intermediaries. As a result, in these cases, the only way to increase consumer surplus is to allocate more decision rights to the platforms than the monopolist, who is the most resilient player in the industry to the introduction of a consumer welfare enhancing parity provision.

3.4 Wholesale *vs* agency model

In this section we compare firms' profits and consumer surplus in the two business models examined before. We consider two alternative scenarios. First, we suppose that, in both business models, platforms (cooperatively) choose whether or not requiring platform parity. In this case, from the foregoing analysis, it follows that platforms would always choose to require the provision if the wholesale model is adopted, whereas in the agency model they would do so only for $\gamma \geq \hat{\gamma}$. Hence, to obtain a meaningful comparison between the two business models, in the following, we compare

- firms' profits and consumer surplus with platform parity in the wholesale model and without platform parity in the agency model in the region of parameters where $\gamma < \hat{\gamma}$;
- firms' profits and consumer surplus under platform parity in both business models in the region of parameters where $\gamma \geq \hat{\gamma}$.

Alternatively, we suppose that the monopolist, instead of platforms, decides whether to introduce platform parity, within each business model. In this case, from the foregoing analysis, we know that, for every $\gamma \in [0, \overline{\gamma}]$, M would introduce platform parity in the wholesale model and would not do so in the agency model.

The comparison between the two business models is summarized by the following Proposition.

Proposition 9 *Suppose that, within each business model, the presence of platform parity is dictated either by M or cooperatively by the platforms. In both cases, the two business models compare as follows:*

- *M is better off in the wholesale model, unless γ is sufficiently large;*
- *platforms are better off in the agency model;*
- *intermediaries are better off in the wholesale model;*
- *consumers are better off in the agency model.*

The result concerning consumer surplus reflects the fact that in the wholesale model the multiple marginalization problem is more pronounced than in the agency model. Intuitively, in the wholesale model platforms can influence the intermediaries' mark-ups through the access price, and when setting this price they only internalize the provision's effect on their own demand, and not the externality on the direct channel. By contrast, in the agency model M directly controls the intermediaries' mark-ups (by choosing the access price) and has a clear incentive to reduce multiple marginalization to minimize the (negative) channel externality. As a consequence, as in Johnson (2017), regardless of the parity regime, final prices are lower in the agency model than in the wholesale model.

Not surprisingly, and in line with the results of Johnson (2017), platforms gain more in the agency model, since in this case they exploit a first-mover advantage vis-à-vis the monopolist. Interestingly, however, M itself can be better off in the agency model. The trade off is as follows. As pointed out above, regardless of the parity regime, shifting from the wholesale to the agency model reduces the multiple marginalization problem, which, ceteris paribus, benefits M; however, in the agency model M loses bargaining power vis-à-vis platforms. Since in the agency model (in both parity regimes), platforms' commissions are decreasing in γ, it follows that, overall, M is worse off in that business model when γ is relatively small, and better off otherwise. Moreover, since, in the agency model, platforms' commissions are higher with platform parity, the range of γ where the adoption of the agency model benefits M enlarges when the provision is not in place — i.e., when M can choose whether to introduce platform parity. Clearly, intermediaries prefer the wholesale model because final prices in the agency model are lower: with an agency structure (in equilibrium) there is more competition both within and across distribution channels.

Interestingly, platforms' incentives over the choice of the business model are aligned with those of consumers. Specifically, if platforms can (cooperatively) choose the business model, then, regardless of which player is then allowed to dictate the parity regime, platforms would always choose the agency model, which maximizes consumer surplus.

3.5 Commitment

In this section we examine the agency model under the assumption that M can credibly commit to the price on the direct channel before contracting with platforms. Clearly, the intermediaries' maximization problem is the same as in the no commitment scenario analyzed above, except for the fact that they now observe p_d before setting their price. The analysis follows the same backward logic as in the wholesale model. Hence, for brevity, we relegate the details to the Appendix and state here only the main result.

Proposition 10 *If M can credibly commit to the price on the direct channel before contracting with platforms, it chooses p^M regardless of whether platform parity is in place or not. Platforms charge higher commissions under platform parity and final prices on the indirect channel are still upward distorted with respect to p^M — i.e.,*

$$p_1^* = p^M + \underbrace{\frac{(1-\gamma)(5+2\gamma)}{6(2+\gamma)}}_{\textit{Multiple marginalization}} > p_0^* = p^M + \underbrace{\frac{(1-\gamma)(6+\gamma)}{2(8+3\gamma)}}_{\textit{Multiple marginalization}}.$$

Hence, with commitment, platform parity always harms consumers in the agency model.

When M commits to the efficient price p^M the beneficial effect of the parity provision is completely dissipated. The reason is the following. The introduction of a price parity provision induces the platforms to increase the commissions charged to the monopolist, who is thus forced to increase the access price charged to the intermediaries in order to mark up these higher commissions. However, under commitment, M cannot compensate the effect of increased marginalization with a lower price in the direct channel because that price is set efficiently regardless of whether the provision is in place or not. Hence, with price commitment, platform parity always harms consumers.

3.6 $N > 2$ competing platforms

We now investigate the effect of increased competition in the indirect channel in the agency model. To do so, we assume that M deals with $N \geq 2$ symmetric platforms, each being in an exclusive relationship with one intermediary. Following the previous approach, consider a representative consumer whose preferences are described by the following linear-quadratic utility function

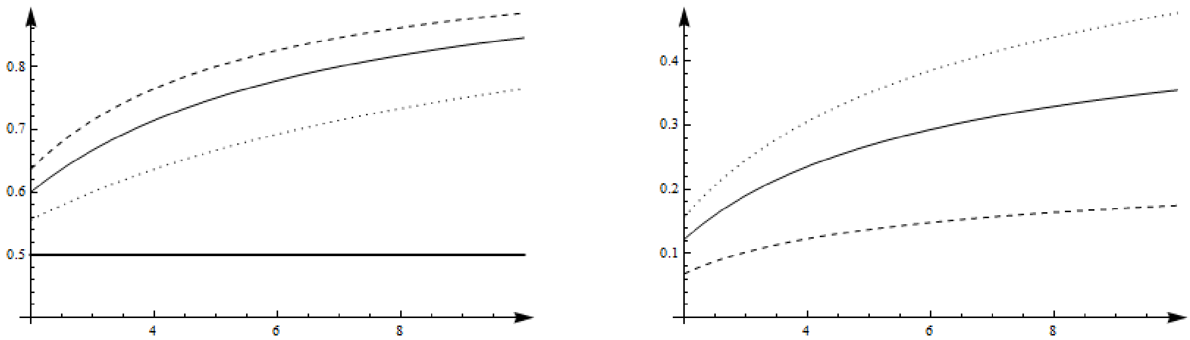
$$U(\cdot) \triangleq \sum_{i \in \mathcal{I}} q_i - \frac{1}{2} \sum_{i \in \mathcal{I}} q_i^2 - \gamma \sum_{i,j \in \mathcal{I}, j \neq i} q_i q_j - \sum_{i \in \mathcal{I}} p_j q_j + m,$$

where $\mathcal{I} \triangleq \{1, \ldots, N, d\}$. Standard techniques then yield the (direct) demand functions

$$q_i(\cdot) = \frac{1-\gamma}{1+(N-1)\gamma - N\gamma^2}\left(1 - \frac{1+(N-1)\gamma}{1-\gamma}p_i + \frac{\gamma}{1-\gamma}\sum_{j\in\mathcal{I}\setminus\{i\}} p_j\right).$$

Finally, as before, we assume that platform parity either applies to all platforms or none.

For brevity we only provide a graphical analysis of the results that are detailed in the Appendix. To begin with, it is useful to understand how N affects the bright side of platform parity — i.e., how the wedge between the rate at which intermediaries pass on with and without parity varies with N. Figure 10 panel a shows that, for a given γ, as the number of platforms increases the bright side of platform parity amplifies. The intuition is as before: increased competition (as reflected by a larger N) leads intermediaries to be more responsive to 'marginal costs', which refrains M from charging higher access prices.



(a) *Derivatives* $\frac{\partial p_0^*(\tau_i)}{\partial \tau_i}$ *(thick line) and* $\frac{\partial p_1^*(\tau)}{\partial \tau}$ *as functions of* $N \geq 2$.

(b) *Difference* $f_1^* - f_0^*$ *as function of* $N \geq 2$.

Figure 10: Parameter's value: $\gamma = .25$ (dotted lines), $\gamma = .5$ (continuous lines), $\gamma = .75$ (dashed lines).

However, Figure 10 panel b shows that also the dark side of the provision increases with N — i.e., the difference between $f_1^* - f_0^*$ is positive and increasing in N. The reason is again related to the impact of the parity provision on the multiple marginalization effect: other things being equal, as N grows large the rate at which intermediaries pass on costs is higher with than without platform parity. This greater responsiveness implies that commissions are passed on by the monopolist to the intermediaries at a rate that is (ceteris paribus) lower with platform parity and, even more so, when N is large. As a result, the difference between the commissions charged by the platforms to the monopolist with and without the provision is increasing with N.

Figure 11 illustrates the net effect of an increase of N on consumer surplus. It can be seen that for sufficiently large values of γ, the positive effect of N on the bright side of the provision dominates the effect on the dark side. By contrast, for values of γ below $\tilde{\gamma}$ — i.e., when the parity provision damages consumers in duopoly — a higher N tends to make the negative effect of the provision on consumer surplus even worst.

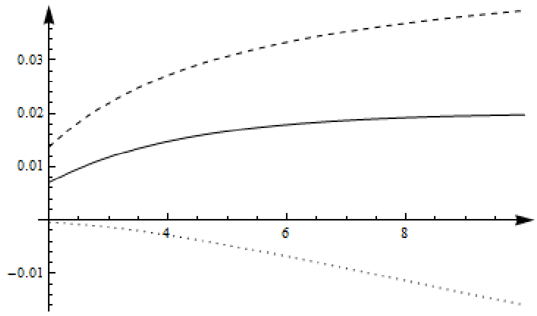


Figure 11: Difference between the consumer surplus in the presence and in the absence of a platform parity agreement as a function of $N \geq 2$. Parameter's value: $\gamma = .25$ (dotted lines), $\gamma = .5$ (continuous lines), $\gamma = .75$ (dashed lines).

3.7 Quality deterioration as a threat to break down parity

In this section we consider the case in which the products distributed in the indirect channel are of lower quality than the product sold on the direct channel — i.e., we assume that the two channels are vertically differentiated. The goal is to understand whether M has an incentive to distort quality in the indirect channel in order to induce platforms to drop the parity regime.

To model this feature in the simplest possible way, consider the following utility function of the representative consumer:

$$U(\cdot) \triangleq q_d + (1-c)\sum_{j=A,B} q_j - \frac{1}{2}\sum_{j=A,B,d} q_j^2 - \gamma \sum_{i,j=A,B,d;,j\neq i} q_j q_i - \sum_{j=A,B,d} p_j q_j + m,$$

where the parameter $c \in [0, 1-\gamma]$ can be interpreted as a measure of *quality deterioration*: with $c = 0$ there is no vertical differentiation between the two channels, but as c increases (ceteris paribus) consumers' utility from purchasing on the indirect channel drops.[14] For simplicity, we assume that the value of c is common knowledge. Standard techniques then yield the (direct) demand functions

$$q_i = \frac{1-c-\gamma-(1+\gamma)\,p_i+\gamma\,(p_{-i}+p_d)}{(1-\gamma)(1+2\gamma)}, \quad \forall i = A, B$$
$$q_d = \frac{1-\gamma+2c\gamma-(1+\gamma)\,p_d+\gamma\,(p_A+p_B)}{(1-\gamma)\,(1+2\gamma)}.$$

Hence, a lower quality (as reflected by a higher c) induces a parallel downward shift of the demand functions for the products distributed in the indirect channel, and an upward shift of the demand for the product directly distributed by M. Therefore, if we consider c as an

[14]The restriction $c \leq 1-\gamma$ guarantees that equilibrium quantities are positive.

exogenous parameter of the model, the analysis is the same as in the baseline model — i.e., the first-order conditions of the players' maximization problems have the same structure as before.

Let $\pi_k^M(c)$ and $\pi_k^P(c)$ denote M and the platforms' equilibrium profits for a given parity regime $k = 0, 1$, respectively. We can state the following.

Proposition 11 *$\pi_k^M(c)$ and $\pi_k^P(c)$ are decreasing in c for every $k \in \{0, 1\}$. Moreover,*

(i) $\pi_0^M(c) > \pi_1^M(c)$ for every $c \leq 1 - \gamma$;

(ii) $\pi_0^P(c) \geq \pi_1^P(c)$ if and only $\gamma \leq \hat{\gamma}$.

The intuition of this result is straightforward. When the quality of the product distributed in the indirect channel decreases, consumers are less willing to pay for that product: demand in the indirect channel drops. As a result, the monopolist and the platforms are worse off. Moreover, since a lower quality only shifts demand down but does not affect its slope, it does not influence the rate at which firms pass on costs and thus the extent of multiple marginalization. Hence, the results of the baseline model apply: M always prefers the regime without parity, whereas platforms prefer the provision as long as competition in the product market is sufficiently fierce.

This result suggest that, while M would always be better off by dropping parity, platforms may still want it for γ sufficiently large. However, when M can choose quality — i.e., when c is endogenous — it may use this choice as a strategic variable to align its incentives with the platforms'. Specifically, imagine that at the outset of the game M can commit to a menu of qualities depending on the parity regime which is then chosen cooperatively by the platforms — i.e., a menu (c_0, c_1) that specifies the quality of the product supplied on the indirect channel in every regime $k = 0, 1$.[15] After this preliminary stage, the game evolves as in the baseline model. We can state the following:

Proposition 12 *For every $\gamma > \hat{\gamma}$, there is a threshold $\bar{c} \in (0, 1 - \gamma)$ such that, by choosing $c_0 = 0$ and any $c_1 = c > \bar{c}$, M induces platforms to drop parity — i.e., $\pi_0^P(0) > \pi_1^P(c)$.*

Therefore, if M can commit to a menu of qualities as a function of the parity regime, it can induce the worse choice for consumers.[16] Clearly, this result holds only if M has enough commitment power *vis-à-vis* the platforms. The reason is that, if M lacks such commitment power, choosing $c_1 = c > \bar{c}$ is not subgame perfect: if platforms choose to introduce parity, M would then have an incentive to set $c_1 = 0$ if this choice can be reneged. In practice, however, even if the monopolist lacks commitment power, repeated interactions can allow it to build a reputation that makes the threat of reducing quality in the parity regime credible. We plan to examine the interesting dynamic aspects of platform parity in future works.

[15] Clearly, as explained before, if M could choose both quality and parity regime, it would choose $c = 0$ and $k = 0$.

[16] Clearly, in the wholesale model quality is not an issue because players' incentives are aligned — i.e., the monopolist, the platforms and consumers are better off under the parity provision.

3.8 Further remarks

Alternative bargaining. In the previous model we assumed that platforms make offers to the monopolist. The analysis considerably simplifies in the opposite scenario where M has full bargaining power and proposes contracts to the platforms. As intuition suggests, in this case M sets $f_i = 0$ in the equilibrium and platforms make zero profit. Hence, the main source of inefficiency hinges on the intermediaries' mark-ups. The logic is the same as in the wholesale model: M must charge a positive access price in order to make positive profits, which in turn induces the intermediaries to set excessively high retail prices. Platform parity is then a commitment device to mitigate double marginalization. Noteworthy, the same outcome realizes if M bilaterally bargains with each platform P_i over the commission f_i that maximizes their joint profit, and the bilateral surplus is then split by means of a fixed payment between M and P_i. Details are in the online Appendix.

RPM and Full Content Agreements. What would happen if M can fix retail prices in the agency model? As argued in the wholesale model, RPM (or even a price-cap) also in the agency model allows M to eliminate the intermediaries' mark-ups (e.g., by setting $\tau_{i,k} = p_{i,k}$ for every $i = A, B$ and $k = 0, 1$). In such a model, however, as in Johansen and Vergé (2016), wide and narrow price parity agreements are anti-competitive. By a similar logic, platform parity on the top of RPM cannot increase consumer welfare (in contrast to the wholesale model where platform parity is always welfare neutral).[17]

By contrast, the logic of our model suggests that full content agreements — i.e., agreements according to which the monopolist's negotiates with the platforms a retail price to be charged in the direct distribution channel in exchange of lower fees — are likely to be efficient. In fact, while those agreements may limit the moderating effect on platform's fees of competition from the direct distribution channel, the monopolist will not give up the leverage provided by its direct distribution channels unless platforms reduce their fees by as much as they would do in the absence of a full content agreement. Hence, access and retail prices need not be higher under a full content agreement. Furthermore, consumer welfare will tend to be higher if consumers can find the same content in their preferred distribution channel, which is precisely what a full content agreement ensures.

Interlocking relationships. As for the implications of our model in terms of antitrust policy, there is a further important reason why the *balance of harms* is likely to point in the direction of no intervention against platform parity provisions: unlike the model analyzed in this paper, platforms usually compete for the business of single-homing intermediaries as well as to increase their share of the business of multi-homing intermediaries. As a result, the presence of a platform parity agreement would incentivize platforms to lower their charges to the monopolist in order to gain market shares downstream. Hence, the likelihood that

[17]The formal argument is standard and omitted for brevity. Proofs are available upon request.

the net competitive effect of content parity is positive will be increased.

Competing sellers. With competing sellers in the upstream market, the effect of platform parity produces different results depending on the business model under consideration. Specifically, while upstream competition unambiguously magnifies the welfare effects of the parity provision in the wholesale model (since it reduces wholesale prices), in the agency model the result depends on the degree of competition between direct and indirect distribution, as well as on the possibility of the sellers to develop their own direct distribution channels (in competition not only between them but also with the indirect distribution channels). In this respect, it is useful to recall that Johnson (2017) shows that when there is no direct channel, the same logic of Boik and Corts (2016) applies with competing sellers; actually, competition in the upstream market makes the anticompetitive effect of parity provisions even worst because platforms charge higher commissions to the sellers, who pass on these higher commissions to the intermediary, whereby reducing consumer surplus. By contrast, Johansen and Vergé (2016) show that parity provisions may actually improve consumer surplus when competition in the upstream market is fierce enough and sellers, distributing their products also through their own direct channel, can delist from platforms charging commissions that are too high, and intensify competition through their direct channels.

Specific investments and the hold-up problem. We have also not considered the platforms' incentives to invest in activities that improve the quality of their services. The reason is again rather simple. Clearly, platforms have a greater incentive to increase quality as their profit grows large. Hence, in the wholesale model a parity provision increases welfare not only because it mitigates multiple marginalization, but also because it stimulates platforms' investments in quality. However, in the agency model the result depends on whether the provision increases or reduces platforms' profits, which as seen before depends on the extent of competition. The most interesting case is that in which $\gamma \geq \hat{\gamma}$, where the provision benefits both platforms and consumers. In this case, if the quality enhancing investment is a lump sum, M has a clear incentive to promise platform parity, wait for the investment to be undertaken, and then renege on the initial promise (of course, provided that the parity decision is in M's domain). Clearly, as long as M's renegotiation is anticipated by the platforms, the latter will not invest, whereby reducing consumer surplus.

Alternative solution concept. Following a recent literature, we have used Contract Equilibrium rather than PBE as a solution concept. The reason behind this choice is purely for tractability. As we have discussed before, Contract Equilibrium shares common features with passive beliefs, except for multilateral deviations which are often problematic for the existence of an equilibrium even with linear demand (see, e.g., Rey and Vergé, 2004). In addition, the three-layer structure of our model considerably complicates the analysis of PBE: in our game off equilibrium beliefs should be specified not only in the contracting problem between the monopolist and the platforms (depending on the business model under consideration) but also for the intermediaries.

This makes our setting rather peculiar. To see why consider (for example) the wholesale model (the same logic applies mutatis mutandis to the agency model). When an intermediary (say I_i) observes an off equilibrium contract, it faces the problem of conjecturing who has deviated along the supply chain. Specifically: does the unexpected contract reflect a deviation by M (and if so has I_{-i}'s contract been changed as well or not) or it simply reflects a deviation by P_i? The answer to this 'dilemma' has significantly different implications for the outcome of the game.

For example, if the deviation is attributed to P_i only, the 'no signal what you do not know principle' (Fudenberg and Tirole, 1991) implies that I_i has no reason to believe that I_{-i} has been offered a contract different than the equilibrium one: a logic coherent with passive beliefs. However, if the deviation is attributed to M, then I_i must believe that also I_{-i}'s contract has changed: a logic closer to the concept of wary beliefs (McAfee and Schwartz, 1994). Depending on which option one chooses, the equilibrium of the game may be rather complex to characterize either because wary beliefs are imposed twice in the game, or because they must coexist and be coherent with passive beliefs on the intermediaries' side. Nevertheless, the effect of platform parity seems robust to these issues: regardless of whether an equilibrium exists or not, with wary beliefs intermediaries are more responsive to unexpected offers than with passive beliefs because they are more suspicious about off equilibrium contracts. This greater responsiveness tends to magnify the commitment power that M can gain under platform parity, and hence to preserve the qualitative insights of our analysis. We plan to address these interesting issues in the future (perhaps in a simpler environment).

General demand function. Although, for the sake of tractability, throughout the analysis we considered a linear demand function, the basic trade off driving our results is robust to alternative specifications of the demand function. On the one hand, for every downward sloping demand function, platform parity yields higher platforms' commissions (see Boik and Corts, 2016) which, in turn, makes it more attractive for the monopolist to divert business towards the direct channel. On the other hand, also the bright side of platform parity is not specific to the assumption of linear demand. In fact, for every downward sloping demand function, with platform parity, an increase in the access price is equivalent to a common cost shock for the intermediaries, which is passed on to a greater extent as competition intensifies, whereas, without parity, a higher access price is equivalent to an idiosyncratic cost shock, which is passed on to a lesser extent when competition is fiercer.

4 Conclusion

In this paper, we have shown that in complex vertical industries — like, e.g., manufacturing industries with multi-tier supply chains (such as automotive, consumer appliances, electronic equipment, and apparel) or the airline ticket distribution industry — platform parity cannot be presumed anti-competitive in the absence of efficiencies.

We have seen that, if the wholesale model is adopted, as it is usually the case in traditional *brick-and-mortar* businesses, then these agreements are always pro-competitive, since they work as a commitment device to mitigate the multiple marginalization problem. Accordingly, not only consumers but also all firms in the supply chain benefit from these provisions. By contrast, when the agency model is employed, as it happens in many *two-sided markets*, even when platforms have full bargaining power in their relationship with the seller, being able to raise the commissions' level when a platform parity agreement is introduced, such a contractual provision might still be pro-competitive. In this case, consumers' preferences are always aligned with the platforms' but not with the seller's. Namely, as long as platforms benefit from platform parity agreements consumers gain as well. Hence, from a practical point of view, the likelihood that these provisions benefit consumers is higher when their introduction is not opposed by platforms.

In fact, we have also shown that while the platforms may persuade the intermediaries to accept the provision through appropriate side payments, persuading the monopolist is harder — i.e., even if the joint profit of the platforms and the intermediaries is higher with than without the provision, the total industry profit and the joint profit of the monopolist and the platforms may well drop when the provision is introduced. This implies that in the industry the incentives of the monopolist are the most likely not to be aligned with consumer surplus. Hence, in these cases, the only way to increase the welfare of the consumers is to allocate more decision rights on the industry structure to the platforms — i.e., to award them a stronger influence than the other players on whether the provisions under consideration should be banned or not.

Finally, we do acknowledge that in order to keep the analysis tractable we have sometimes neglected a few salient aspects of real life — e.g., upstream competition, merger incentives, quality provision with the related moral hazard problems, etc. We plan to explore these interesting issues in future research.

References

[1] Akman, P. (2016). A competition law assessment of platform most-favored-customer clauses. *Journal of Competition Law & Economics*, 12(4), 781-833.

[2] Avilés-Lucero, F., & Boik, A. (2018). Wholesale most-favored-nation clauses and price discrimination with negative consumption externalities: equivalence results. *Journal of Regulatory Economics*, 54(3), 266-291.

[3] Baker, J. B., & Chevalier, J. A. (2012). The competitive consequences of most-favored-nation provisions. *Antitrust*, 27, 20-26.

[4] Boik, A., & Corts, K. S. (2016). The effects of platform most-favored-nation clauses on competition and entry. *The Journal of Law and Economics*, 59(1), 105-134.

[5] Butz, D. A. (1990). Durable-Good Monopoly and Best-Price Provisions. *The American Economic Review*, 80(5), 1062-1076.

[6] Calzada, J., Manna, E., & Mantovani, A. (2019). Platform Price Parity Clauses and Segmentation. UB Economics-Working Papers, 2018, E19/387.

[7] Calzolari, G., Denicolò, V., & Zanchettin, P. (2017). Exclusive dealing with distortionary pricing. Working paper.

[8] Cazaubiel, A., Cure, M., Johansen, B. O., & Vergé, T. (2018). Substitution Between Online Distribution Channels: Evidence from the Oslo Hotel Market. University of Bergen, Department of Economics (No. 8/18).

[9] Cooper, T. E. (1986). Most-favored-customer pricing and tacit collusion. *The RAND Journal of Economics*, 17(3), 377-388.

[10] Cremer, J., & Riordan, M. H. (1987). On governing multilateral transactions with bilateral contracts. *The RAND Journal of Economics*, 436-451.

[11] DeGraba, P., & Postlewaite, A. (1992). Exclusivity clauses and best price policies in input markets. *Journal of Economics & Management Strategy*, 1(3), 423-454.

[12] DeGraba, P. (1996). Most-Favored-Customer Clauses and Multilateral Contracting: When Nondiscrimination Implies Uniformity. *Journal of Economics & Management Strategy*, 5(4), 565-579.

[13] Edelman, B., & Wright, J. (2015). Price coherence and excessive intermediation. *The Quarterly Journal of Economics*, 130(3), 1283-1328.

[14] Ezrachi, A. (2015). The competitive effects of parity clauses on online commerce. *European Competition Journal*, 11(2-3), 488-519.

[15] Foros, O., Kind, H. J., & Shaffer, G. (2017). Apple' s agency model and the role of most-favored-nation clauses. *The RAND Journal of Economics*, 48(3), 673-703.

[16] Fudenberg, D., & Tirole, J. (1991). *Game Theory*. MIT Press.

[17] Gaudin, G. (2019). Vertical relations, opportunism, and welfare. *The RAND Journal of Economics*, 50(2), 342-358.

[18] Hart, O., & Tirole, J. (1990). Vertical integration and market foreclosure. *Brookings papers on economic activity. Microeconomics*, 205-286.

[19] Horn, H., & Wolinsky, A. (1988). Bilateral monopolies and incentives for merger. *The RAND Journal of Economics*, 408-419.

[20] Johansen, B. O., & Vergé, T. (2016). Platform price parity clauses with direct sales. IDEI working.

[21] Johnson, J. P. (2017). The agency model and MFN clauses. *The Review of Economic Studies*, 84(3), 1151-1185.

[22] Marx, L.M., & Shaffer, G. (2004). Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity: Comment. *The American Economic Review*, 94(3), 796-801.

[23] McAfee, R. P., & Schwartz, M. (1994). Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity. *The American Economic Review*, 210-230.

[24] Motta, M. (2004). *Competition policy: theory and practice*. Cambridge University Press.

[25] Pagnozzi, M., & Piccolo, S. (2011). Vertical separation with private contracts. *The Economic Journal*, 122(559), 173-207.

[26] Rey, P., & Tirole, J. (2007). A primer on foreclosure. *Handbook of industrial organization*, 3, 2145-2220.

[27] Rey, P., & Vergé, T. (2004). Bilateral control with vertical contracts. *The RAND Journal of Economics*, 35(4), 728-746.

[28] Rey, P., & Vergé, T. (2017). Secret contracting in multilateral relations. Working paper.

[29] Ronayne, D., & Taylor, G. (2018). Competing Sales Channels. University of Oxford, Department of Economics (No. 843).

[30] Shen, B., & Wright, J. (2019). Why (don' t) firms free ride on an intermediary' s advice?. *International Journal of Industrial Organization*, 64, 27-54.

[31] Schnitzer, M. (1994). Dynamic duopoly with best-price clauses. *The RAND Journal of Economics*, 25(1), 186-196.

[32] Wang, C., & Wright, J. (2016). Search platforms: Showrooming and price parity clauses. Working Paper, National University of Singapore.

Appendix 1: Proofs

Multi-product monopolist. The first-order conditions of the multi-product monopolist's problem with respect to p_i, $i = A, B$, and p_d are

$$\frac{1-\gamma-(1+\gamma)\,p_i+\gamma\,(p_{-i}+p_d)}{(1-\gamma)(1+2\gamma)} - \frac{1+\gamma}{(1-\gamma)(1+2\gamma)}p_i + \frac{\gamma}{(1-\gamma)(1+2\gamma)}(p_{-i}+p_d) = 0,$$

and

$$\frac{\gamma}{(1-\gamma)(1+2\gamma)}\sum_{i=A,B} p_i + \frac{1-\gamma-(1+\gamma)\,p_d+\gamma\sum_{i=A,B} p_i}{(1-\gamma)(1+2\gamma)} - \frac{1+\gamma}{(1-\gamma)(1+2\gamma)}p_d = 0,$$

respectively. Solving the system of these three first-order conditions, we obtain the price charged by the multi-product monopolist on the three channels. ■

Proof of Proposition 1. The result can be directly obtained by solving the first-order conditions derived in Section 2.1.1. ■

Proof of Proposition 2. The result can be directly obtained by solving the first-order conditions derived in Section 2.1.2. ■

Proof of Proposition 3. The comparison between the equilibrium prices in the two regimes is as follows:

$$p_1^* - p_0^* = -\frac{3\gamma(1+2\gamma\,(1+2\gamma))}{2(4+\gamma)(1+2\gamma)(8+5\gamma)} < 0,$$

$$p_{d,1}^* - p_{d,0}^* = -\frac{3\gamma^2(5+2\gamma)}{2(4+\gamma)(1+2\gamma)(8+5\gamma)} < 0.$$

In the presence of platform parity, equilibrium profits are

$$\pi_{i,1}^I = \frac{1-\gamma^2}{4(4+\gamma)^2(1+2\gamma)},$$

$$\pi_{i,1}^P = \frac{1-\gamma^2}{2(4+\gamma)^2(1+2\gamma)},$$

and

$$\pi_1^M = \frac{3(1+\gamma)(4+11\gamma+3\gamma^2)}{2(4+\gamma)^2(1+2\gamma)^2}.$$

In the game without platform parity, we have

$$\pi_{i,0}^I = \frac{1-\gamma^2}{(1+2\gamma)(8+5\gamma)^2},$$

$$\pi_{i,0}^P = \frac{2(1-\gamma^2)}{(1+2\gamma)(8+5\gamma)^2},$$

and

$$\pi_0^M = \frac{96 + 432\gamma + 597\gamma^2 + 252\gamma^3}{4(1 + 2\gamma)^2(8 + 5\gamma)^2}.$$

Comparing these expression yields

$$\pi_{i,1}^I - \pi_{i,0}^I = \frac{3\gamma(16 + 7\gamma)(1 - \gamma^2)}{4(4 + \gamma)^2(1 + 2\gamma)(8 + 5\gamma)^2} > 0,$$

$$\pi_{i,1}^P - \pi_{i,0}^P = \frac{3\gamma(16 + 7\gamma)(1 - \gamma^2)}{2(4 + \gamma)^2(1 + 2\gamma)(8 + 5\gamma)^2} > 0,$$

$$\pi_1^M - \pi_0^M = \frac{9\gamma^2(8 + 98\gamma + 103\gamma^2 + 22\gamma^3)}{4(4 + \gamma)^2(1 + 2\gamma)^2(8 + 5\gamma)^2},$$

which concludes the proof. ■

Proof of Proposition 4. The equilibrium values are obtained by solving the first-order conditions derived in Section 3.1. However, to establish Propositions 4, it remains to be shown that the commissions values obtained by solving the first-order conditions of platforms' problem are such that M's participation constraint is satisfied. Specifically, since we assumed that M is allowed to delist from a platform, as in Johansen and Vergé (2016), the considered equilibrium exists if and only if, when the other platform P_{-i} offered the candidate equilibrium contract f_1^*, M does not find it profitable to decline P_i's offer f_1^*, thus being free to set prices on the two remaining channels — i.e., platform P_{-i} and the direct sale channel. If such a profitable deviation exists, then, in the symmetric contract equilibrium, platforms' commissions must be lower than f_1^*, and such that M's participation constraint is binding. Thus, in order to check whether a candidate equilibrium value f_1^* satisfies M's participation constraint, we should determine its profit when it declines P_i's offer, conditional on accepting the same offer f_1^* made by P_{-i}.

If M sells its product only through one platform, say P_i, then the linear-quadratic specification of the representative consumer's utility function becomes

$$U(q_i, q_d) \triangleq \sum_{j=i,d} q_j - \frac{1}{2}\sum_{j=i,d} q_j^2 - \gamma q_i q_d - \sum_{j=i,d} p_j q_j + m,$$

yielding the following demand functions

$$D(p_i, p_d)^i = \frac{1 - \gamma(1 - p_d) - p_i}{1 - \gamma^2}, \quad D^d(p_d, p_i) = \frac{1 - \gamma(1 - p_i) - p_d}{1 - \gamma^2}.$$

Assuming, for simplicity, that M's acceptance decisions are common knowledge, if M declined P_{-i}'s offer, then I_i solves[18]

$$\max_p D\left(p, \tilde{p}_d\right)(p - \tau),$$

where $\tilde{p}_d$ denotes the price that M is expected to set on the direct channel. Optimization

[18]Given that only one indirect channel is active, we omit subscripts, to ease notation.

yields a best-reply function

$$\tilde{p}(\tau) \triangleq \frac{\tau}{2} + \frac{1-\gamma(1-\tilde{p}_d)}{2}.$$

Accordingly, M sets τ and p_d by solving

$$\max_{\tau, p_d} p_d D^d(p_d, \tilde{p}(\tau)) + (\tau - f)D(\tilde{p}(\tau), p_d),$$

whose first-order conditions yield[19]

$$\tilde{\tau}(f) = \frac{4(1+f) - \gamma(4p_d^* + 3\gamma(1+\gamma+2f-\gamma\tilde{p}_d) - 2)}{8-9\gamma^2},$$

and

$$\tilde{p}_d(f) = \frac{4-\gamma(f+\gamma(5+\tilde{p}_d)-1)}{8-9\gamma^2}.$$

Thus, to check whether M's participation constraint is satisfied when platforms set f_1^*, we must substitute this value into the above expressions (and impose $\tilde{p}_d(f_1^*) = \tilde{p}_d$). Let us first consider the case in which platform parity is in place. We obtain

$$\tilde{\tau}_1 = \frac{40+84\gamma-46\gamma^2-93\gamma^3-3\gamma^4+14\gamma^5}{8\left(6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4\right)},$$

and

$$\tilde{p}_d = p^M + \frac{\gamma(1-\gamma)(2+\gamma)(1+2\gamma)}{8(6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4)},$$

from which it is easy to obtain M's *deviation profit*, which can finally be compared with the *equilibrium profit* — that is, the profit that M gets by accepting both platforms' offers. In Figure 12, we show that the equilibrium profit is, for every γ, higher than the deviation profit. ■

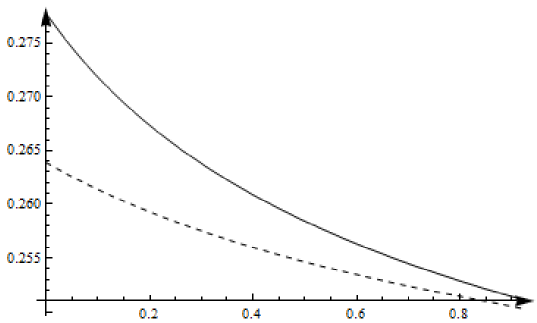


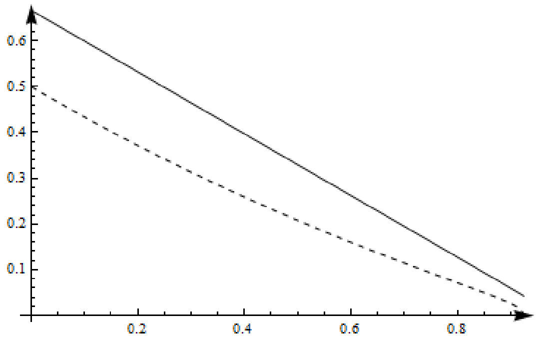
Figure 12: M's equilibrium profit (continuous line) and deviation profit (dashed line) in the game with platform parity agreement as a function of γ.

Proof of Proposition 5. The equilibrium values are obtained by solving the first-order conditions derived in Section 3.2. It can be easily seen that M's participation constraint is

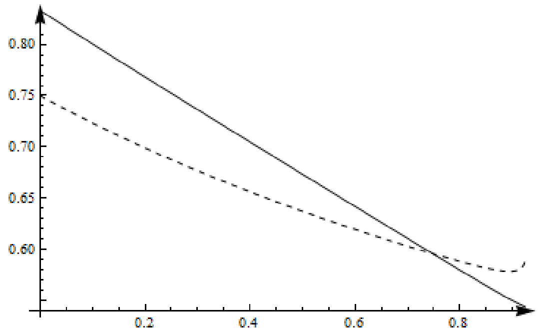
[19]The corresponding second-order condition is satisfied for every $\gamma < \frac{2\sqrt{2}}{3}$.

not binding, in the game without platform parity, when platforms offer f_0^*.[20] ■

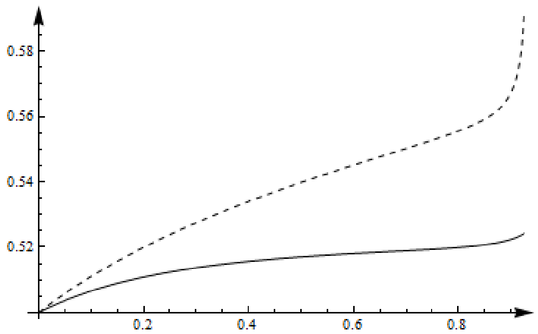
Proof of Proposition 6. The equilibrium values of commissions and prices in the two regimes, given in Propositions 4 and 5, are compared in Figure 13, which thus establishes Proposition 6. ■



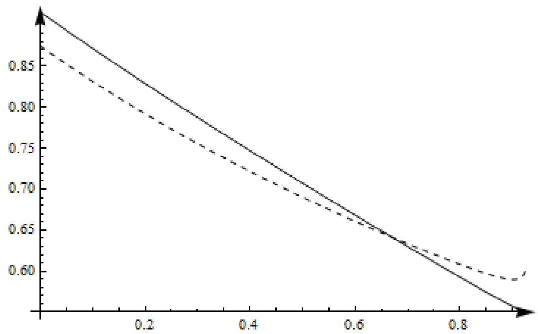
(a) *Equilibrium commissions* f_1^* *(continuous line) and* f_0^* *(dashed line) as functions of* γ.



(b) *Equilibrium wholesale prices* τ_1^* *(continuous line) and* τ_0^* *(dashed line) as functions of* γ.



(c) *Equilibrium price* $p_{d,1}^*$ *(continuous line) and* $p_{d,0}^*$ *(dashed line) as functions of* γ.



(d) *Equilibrium price* p_1^* *(continuous line) and* p_0^* *(dashed line) as functions of* γ.

Figure 13: Equilibrium values as functions of γ.

Proof of Proposition 7. As for equilibrium profits, if a platform parity agreement is in place, we find

$$\pi_{i,1}^I = \frac{(1+2\gamma)\left(1-\gamma^2\right)^3}{4\left(6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4\right)^2},$$

$$\pi_{i,1}^P = \frac{\left(1-\gamma^2\right)^2\left(4+10\gamma-\gamma^2-10\gamma^3-4\gamma^4\right)}{2\left(6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4\right)^2},$$

$$\pi_{i,1}^M = \frac{40+190\gamma+204\gamma^2-240\gamma^3-519\gamma^4-120\gamma^5+244\gamma^6+170\gamma^7+32\gamma^8}{4\left(6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4\right)^2},$$

[20] As argued by Johansen and Vergé (2016), the reason is as follows: if a platform charges a higher commission, in the absence of platform parity, M finds it optimal to set a higher price on that platform, rather than shut down that channel. Therefore, in the regime without platform parity, the equilibrium value for the platforms' commission is the *unconstrained* one stated in Proposition 5.

whereas, without platform parity

$$\pi_{i,0}^{I} = \frac{(1-\gamma^2)(1+2\gamma)\left(8-9\gamma^2\right)^2}{(64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4)^2},$$

$$\pi_{i,0}^{P} = \frac{8(1-\gamma^2)(1+\gamma)\left(8-9\gamma^2\right)\left(4+4\gamma-9\gamma^2\right)}{(64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4)^2},$$

$$\pi_{i,0}^{M} = \frac{1}{4} + \frac{(1+\gamma)^2\left(8-9\gamma^2\right)^2\left(4+4\gamma-9\gamma^2\right)}{(64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4)^2}.$$

These expressions are plotted in Figure 9, which establishes Proposition 7. ■

Proof of Proposition 8. The results follow from a direct comparison among the relevant joint profits in the two regimes: see Figure 14. ■

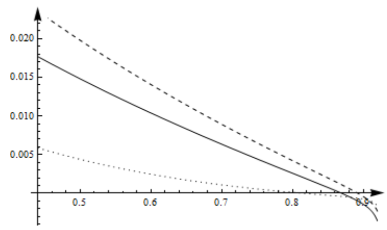
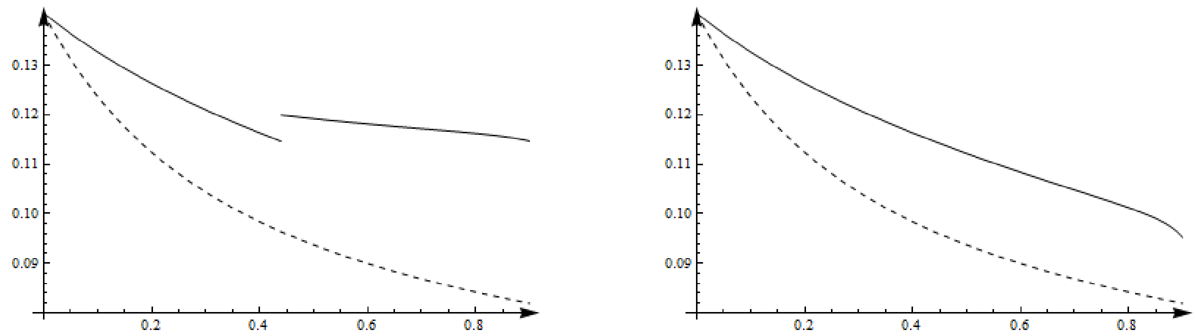


Figure 14: Differences of joint profits $2[\pi_{i,0}^{P} + \pi_{i,0}^{I} - (\pi_{i,1}^{P} + \pi_{i,1}^{I})]$ (dotted line), $\pi_0^M + 2\pi_{i,0}^{P} - (\pi_1^M + 2\pi_{i,1}^{P})$ (continuous line), $\pi_0^M + 2\pi_{i,0}^{P} + 2\pi_{i,0}^{I} - (\pi_1^M + 2\pi_{i,1}^{P} + 2\pi_{i,1}^{I})$ (dashed line), as functions of $\gamma \in (\hat{\gamma}, \overline{\gamma})$.

Proof of Proposition 9. The results easily follow from a direct comparison between the relevant profits. Details are available upon request. The comparison of consumer surplus is illustrated in Figure 15. ■



(a) *Parity regime chosen (cooperatively) by platforms. The discontinuity point in the agency model is at $\gamma = \hat{\gamma}$, in which the parity regime changes.*

(b) *Parity regime chosen by M.*

Figure 15: Consumer surplus in the agency model (continuous line) and in the wholesale model (dashed line) as function of γ.

Proof of Proposition 10. In the game with commitment, the intermediaries observe p_d before setting their price. Therefore, in the presence of platform parity, their equilibrium price is a function of the (common) access price and of the price on M's direct channel, denoted by $p_1^*(\tau, p_d)$. Similarly, in the absence of platform parity, the best-reply function of I_i is denoted by $p_0^*(\tau_i, p_d)$.
Equilibrium with platform parity. For any pair of commissions (f_A, f_B) negotiated with the platforms, and any announced value of p_d, M's optimization problem at the final pricing stage is as follows:

$$\max_{\tau} \sum_{i=A,B} (\tau - f_i) D^i(p_1^*(\tau, p_d), p_1^*(\tau, p_d), p_d) + p_d D^d(p_d, p_1^*(\tau, p_d), p_1^*(\tau, p_d)),$$

whose first-order condition is as in the baseline agency model of Section 3.1 and yields

$$\tau_1^*(f_i, f_{-i}, p_d) \triangleq \frac{2 + f_i + f_{-i} + 2\gamma(2p_d - 1)}{4}.$$

Moving backward at the previous stage of the game, P_i's problem is

$$\max_{f_i} f_i D^i(p_1^*(f_i, f_1^*, p_d), p_1^*(f_i, f_1^*, p_d), p_d),$$

where, to ease notation, $p_1^*(f_i, f_1^*, p_d) \triangleq p_1^*(\tau_1^*(f_i, f_1^*, p_d), p_d)$. The first-order condition for this optimization problem (which, again, is as in the corresponding problem without commitment), imposing symmetry, yields the equilibrium value

$$f_1^* = \frac{2}{3}(1 - \gamma),$$

which does not depend on p_d. Substituting this value into $\tau_1^*(\cdot)$ and $p_1^*(\cdot)$, we find

$$\tau_1^*(p_d) \triangleq \tau_1^*(f_1^*, f_1^*, p_d) = \frac{5(1-\gamma)}{6} + \gamma p_d$$

and

$$p_1^*(p_d) \triangleq p_1^*(f_1^*, f_1^*, p_d) = \frac{1}{6}\left(4 - 5\gamma + \frac{3}{2+\gamma}\right) + \gamma p_d.$$

Therefore, at the first stage of the game, M commits to the value of p_d which solves

$$\max_{p_d} 2(\tau_1^*(p_d) - f_1^*) D^i(p_1^*(p_d), p_1^*(p_d), p_d) + p_d D^d(p_d, p_1^*(p_d), p_1^*(p_d)),$$

whose first-order condition is

$$2\frac{\partial \tau_1^*(p_d)}{\partial p_d}\left(\frac{1 - \gamma - (1+\gamma)\, p_1^*(p_d) + \gamma\,(p_1^*(p_d) + p_d)}{(1-\gamma)(1+2\gamma)}\right) +$$

$$+2(\tau_1^*(p_d) - f_1^*)\left(\frac{\partial p_1^*(p_d)}{\partial p_d}\left(-\frac{1+\gamma}{(1-\gamma)(1+2\gamma)} + \frac{\gamma}{(1-\gamma)(1+2\gamma)}\right) + \frac{\gamma}{(1-\gamma)(1+2\gamma)}\right) +$$

$$+\left(\frac{1-\gamma-(1+\gamma)\,p_d+\gamma 2p_1^*(p_d)}{(1-\gamma)(1+2\gamma)}\right)+p_d\left(-\frac{1+\gamma}{(1-\gamma)(1+2\gamma)}+2\frac{\partial p_1^*(p_d)}{\partial p_d}\frac{\gamma}{(1-\gamma)(1+2\gamma)}\right)=0. \tag{10}$$

Solving this equation, we find

$$p_{d,1}^*=p^M,$$

which can be finally substituted into $\tau_1^*(p_d)$ and $p_1^*(p_d)$ to obtain

$$\tau_1^*=f_1^*+\underbrace{\frac{1+2\gamma}{6}}_{(>0)\ M\text{'s mark-up}}$$

and

$$p_1^*=\tau_1^*+\underbrace{\frac{1-\gamma}{6(2+\gamma)}}_{(>0)\ I_i\text{'s mark-up}},$$

with $p_1^*\geq\tau_1^*\geq p^M$.

Equilibrium without platform parity. For any pair of commissions (f_A,f_B) negotiated with the platforms, and any announced value of p_d, M's optimization problem at the final pricing stage is as follows:

$$\max_{\tau_A,\tau_B}\sum_{i=A,B}(\tau_i-f_i)D^i(p_0^*(\tau_i,p_d),p_0^*(\tau_{-i},p_d),p_d)+p_dD^d(p_d,p_0^*(\tau_A,p_d),p_0^*(\tau_B,p_d)),$$

whose first-order conditions are as in the baseline agency model of Section 3.2 and yield

$$\tau_0^*(f_i,p_d)\triangleq\frac{f_i}{2}+\frac{1+\gamma(1-p_0^*+2p_d+\gamma(3p_d-2))}{2(1+\gamma)},\quad\forall i=A,B.$$

Notice that, differently from the model without commitment, here the access price charged to I_i only depends on the commission negotiated with P_i.

Moving backward at the previous stage of the game, P_i's problem is

$$\max_{f_i}f_iD^i(p_0^*(f_i,p_d),p_0^*(f_0^*,p_d),p_d),$$

where, to ease notation, $p_0^*(f_i,p_d)\triangleq p_0^*(\tau_0^*(f_i,p_d),p_d)$. The first-order condition for this optimization problem, which is analogous to the corresponding one shown in Section 3.2, imposing symmetry (i.e., $f_i=f_0^*$), yields the equilibrium value

$$f_0^*=\frac{4(1-\gamma)^2}{8+5\gamma+3\gamma^2},$$

which, also in this case, does not depend on p_d. We can then obtain

$$p_0^*(p_d)\triangleq p_0^*(f_0^*,p_d)=\frac{(1-\gamma)(7+2\gamma)}{8+3\gamma}+\gamma p_d.$$

Therefore, at the first stage of the game, M commits to the value of p_d which solves

$$\max_{p_d} 2(\tau_0^*(f_0^*, p_d) - f_0^*)D^i(p_0^*(p_d), p_0^*(p_d), p_d) + p_d D^d(p_d, p_0^*(p_d), p_0^*(p_d)),$$

whose first-order condition is analogous to equation (10), and, also in this case, yields

$$p_{d,0}^* = p^M,$$

which can be finally substituted into $\tau_0^*(f_0^*, p_d)$ and $p_0^*(p_d)$, to obtain

$$\tau_0^* = f_0^* + \underbrace{\frac{4 + 8\gamma - \gamma^2}{16 + 6\gamma}}_{M\text{'s mark-up}},$$

and

$$p_0^* = \tau_0^* + \underbrace{\frac{(1-\gamma)^2}{8 - 5\gamma - 3\gamma^2}}_{I_i\text{'s mark-up}},$$

with $p_0^* \geq \tau_0^* \geq p^M$.
Consumer Welfare. Since, in both parity regimes, M commits to set the same price on the direct channel, to evaluate the effects of the parity provision on consumer welfare we only need to compare final prices in the indirect channel. It is easy to see that

$$p_1^* - p_0^* = \frac{(1-\gamma^2)(4+3\gamma)}{6(2+\gamma)(8+3\gamma)} > 0,$$

which concludes the proof. ■

$N \geq 2$ **competing platforms.** We begin by considering I_i's problem in regime $k = 1, 0$:

$$\max_{p_i} D^i\left(p_i, \sum_{j \in \mathcal{I}\setminus\{i\}} p_{j,k}^*\right)(p_i - \tau_i),$$

where, since we are restricting our attention to a symmetric equilibrium,

$$\sum_{j \in \mathcal{I}\setminus\{i\}} p_{j,k}^* = (N-1)p_k^* + p_{d,k}^*.$$

The (standard) first-order condition for the considered problem is

$$\frac{1-\gamma}{1+(N-1)\gamma - N\gamma^2} - \frac{1+(N-1)\gamma}{1+(N-1)\gamma - N\gamma^2}(p_i - \tau_i) +$$

$$+\frac{1-\gamma}{1+(N-1)\gamma - N\gamma^2}\left(\frac{\gamma}{1-\gamma}((N-1)p_k^* + p_{d,k}^*) - \frac{1+(N-1)\gamma}{1-\gamma}p_i\right) = 0. \quad (11)$$

Equilibrium with platform parity. When platform parity is in place (i.e., $\tau_i = \tau$ for all

$i = 1, \ldots, N$), by solving the system of first-order conditions (11), we obtain the equilibrium retail price

$$p_1^*(\tau) \triangleq \left(1 - \frac{1}{2+(N-1)\gamma}\right)\tau + \frac{1-\gamma(1-p_{d,1}^*)}{2+(N-1)\gamma}.$$

At the final pricing stage, M solves

$$\max_{p_d} \sum_{i=1,\ldots,N} (\tau - f_i) D^i\left(p_1^*(\tau), (N-1)p_1^*(\tau) + p_d\right) + p_d D^d(p_d, Np_1^*(\tau)),$$

whose first-order condition,

$$\frac{\gamma}{1+(N-1)\gamma - N\gamma^2} \sum_{i=1,\ldots,N} (\tau - f_i) + \frac{1-\gamma}{1+(N-1)\gamma - N\gamma^2} - \frac{1+(N-1)\gamma}{1+(N-1)\gamma - N\gamma^2} p_d +$$

$$+ \frac{\gamma}{1+(N-1)\gamma - N\gamma^2} N p_1^*(\tau) - \frac{1+(N-1)\gamma}{1+(N-1)\gamma - N\gamma^2} p_d = 0,$$

gives a function $p_{d,1}^*(\boldsymbol{f})$, where $\boldsymbol{f} \triangleq (f_1, \ldots, f_N) \in \mathbb{R}^N$. In the previous stage of the game, M sets the (common) access price by solving

$$\max_{\tau} \sum_{i=1,\ldots,N} (\tau - f_i) D^i\left(p_1^*(\tau), (N-1)p_1^*(\tau) + p_{d,1}^*(\boldsymbol{f})\right) + p_{d,1}^*(\boldsymbol{f}) D^d(p_{d,1}^*(\boldsymbol{f}), Np_1^*(\tau)),$$

whose first-order condition, by the Envelope Theorem, is

$$\frac{1-\gamma}{1+(N-1)\gamma - N\gamma^2} \left(1 - \frac{1+(N-1)\gamma}{1-\gamma} p_1^*(\tau) + \frac{\gamma}{1-\gamma}((N-1)p_1^*(\tau) + p_{d,1}^*(\boldsymbol{f}))\right) +$$

$$+ \frac{\partial p_1^*(\tau)}{\partial \tau} \left(-\frac{1+(N-1)\gamma}{1+(N-1)\gamma - N\gamma^2} + (N-1)\frac{\gamma}{1+(N-1)\gamma - N\gamma^2}\right)\left(\tau - \frac{1}{N}\sum_{i=1,\ldots,N} f_i\right) +$$

$$+ \frac{\partial p_1^*(\tau)}{\partial \tau} p_{d,1}^*(\boldsymbol{f}) \frac{\gamma}{1+(N-1)\gamma - N\gamma^2} = 0,$$

and it yields a best-reply function $\tau_1^*(\boldsymbol{f})$.

Finally, we consider P_i's problem. Notice that, when P_i offers a contract f_i to M, it believes that every other platform is offering the equilibrium commission f_1^*. Hence, for every offer f_i, M is expected to set prices $\tau_1^*(f_i, \boldsymbol{f^*_{-i,1}})$ and $p_{d,1}^*(f_i, \boldsymbol{f^*_{-i,1}})$, where we denote $\boldsymbol{f^*_{-i,1}} \triangleq (f_1^*, \ldots, f_1^*) \in \mathbb{R}^{N-1}$. Accordingly, intermediaries are expected to charge a retail price $p_1^*(f_i, \boldsymbol{f^*_{-i,1}}) \triangleq p_1^*(\tau_1^*(f_i, \boldsymbol{f^*_{-i,1}}))$. Thus, P_i's problem is as follows:

$$\max_{f_i} f_i D^i\left(p_1^*(f_i, \boldsymbol{f^*_{-i,1}}), (N-1)p_1^*(f_i, \boldsymbol{f^*_{-i,1}}) + p_{d,1}^*(f_i, \boldsymbol{f^*_{-i,1}})\right).$$

By solving the first-order condition,

$$\frac{1-\gamma}{1+(N-1)\gamma-N\gamma^2}-\frac{1+(N-1)\gamma}{1+(N-1)\gamma-N\gamma^2}p_1^*(f_i,\boldsymbol{f^*_{-i,1}})+$$

$$+\frac{\gamma((N-1)p_1^*(f_i,\boldsymbol{f^*_{-i,1}})+p_{d,1}^*(f_i,\boldsymbol{f^*_{-i,1}}))}{1+(N-1)\gamma-N\gamma^2}+f_i\left[-\frac{1+(N-1)\gamma}{1+(N-1)\gamma-N\gamma^2}\frac{\partial p_1^*(f_i,\boldsymbol{f^*_{-i,1}})}{\partial f_i}\right]+$$

$$+\frac{\gamma}{1+(N-1)\gamma-N\gamma^2}\left((N-1)\frac{\partial p_1^*(f_i,\boldsymbol{f^*_{-i,1}})}{\partial f_i}+\frac{\partial p_{d,1}^*(f_i,\boldsymbol{f^*_{-i,1}})}{\partial f_i}\right)\Bigg]=0.$$

and imposing symmetry — i.e., $f_i=f_1^*$ and $p_{d,1}^*(f_1^*,\boldsymbol{f_1^*})=p_{d,1}^*$ — we obtain the equilibrium values

$$f_1^*=\frac{1}{\Psi_1}N(1-\gamma)(8+\gamma(20(N-1)+\gamma(16-41N+16N^2-4N\gamma^2(N-1)^2+4\gamma(N-1)(1-5N+N^2))))$$

and

$$p_{d,1}^*=p^M+\frac{1}{\Psi_1}N\gamma(1-\gamma)(1+\gamma(N-1)),$$

where

$$\Psi_1\triangleq 8(1+N)-\gamma(20(1-N^2)-\gamma(16-24N-25N^2+16N^3)+$$
$$+4\gamma^2(1-5N+5N^3-N^4)+4N(N+1)(N-1)^2\gamma^3).$$

Finally, substituting these values into $\tau_1^*(\cdot)$ and $p_1^*(\cdot)$, we obtain

$$\tau_1^*=f_1^*+\frac{1}{2\Psi_1}(8-\gamma(20-28N+\gamma(-16+58N-36N^2)+\gamma^2(4-36N+63N^2-20N^3))+$$

$$-2\gamma^3N(N-1)(3-12N+2N^2)+4\gamma^4N^2(N-1)^2)$$

and

$$p_1^*=\tau_1^*+\frac{2}{\Psi_1}(1-\gamma)^2(1+\gamma(N-1))(1+N\gamma).$$

Equilibrium without platform parity. In the absence of platform parity, the first-order condition (11) yields I_i's best-reply function

$$p_0^*(\tau_i)\triangleq\frac{\tau_i}{2}+\frac{1+\gamma((N-1)p_0^*+p_{d,0}^*-1)}{2(1+\gamma(N-1))}.$$

Hence, at the final pricing stage, M's optimization problem is

$$\max_{p_d}\sum_{i=1,\dots,N}(\tau_i-f_i)D^i\left(p_0^*(\tau_i),(N-1)p_0^*(\tau_i)+p_d\right)+p_dD^d(p_d,Np_0^*(\tau_i)),$$

whose first-order condition is

$$\frac{\gamma}{1+(N-1)\gamma-N\gamma^2}\sum_{i=1,\dots,N}(\tau_i-f_i)-\frac{1+(N-1)\gamma}{1+(N-1)\gamma-N\gamma^2}p_d+$$

$$+\frac{1-\gamma}{1+(N-1)\gamma-N\gamma^2}\left(1-\frac{1+(N-1)\gamma}{1-\gamma}p_d+\frac{\gamma}{1-\gamma}Np_0^*(\tau_i)\right)=0.$$

Letting $p_{d,0}^*(\boldsymbol{f})$ denote the solution to the above equation, M's problem in the previous stage of the game is as follows:

$$\max_{\tau_1,\dots,\tau_N}\sum_{i=1,\dots,N}(\tau_i-f_i)D^i\left(p_0^*(\tau_i),\sum_{j\in\mathcal{I}\setminus\{i,d\}}p_0^*(\tau_j)+p_{d,0}^*(\boldsymbol{f})\right)+p_{d,0}^*(\boldsymbol{f})D^d\left(p_{d,0}^*(\boldsymbol{f}),\sum_{i\in\mathcal{I}\setminus\{d\}}p_0^*(\tau_i)\right).$$

By the Envelope Theorem, the first-order condition with respect to τ_i is

$$\frac{1-\gamma}{1+(N-1)\gamma-N\gamma^2}\left(1-\frac{1+(N-1)\gamma}{1-\gamma}p_0^*(\tau_i)+\frac{\gamma}{1-\gamma}\left(\sum_{j\in\mathcal{I}\setminus\{i,d\}}p_0^*(\tau_j)+p_{d,0}^*(\boldsymbol{f})\right)\right)+$$

$$-\frac{1+(N-1)\gamma}{1+(N-1)\gamma-N\gamma^2}(\tau_i-f_i)\frac{\partial p_0^*(\tau_i)}{\partial\tau_i}+\frac{\gamma}{1+(N-1)\gamma-N\gamma^2}\frac{\partial p_0^*(\tau_i)}{\partial\tau_i}\sum_{j\in\mathcal{I}\setminus\{i,d\}}(\tau_j-f_j)+$$

$$+\frac{\gamma}{1+(N-1)\gamma-N\gamma^2}p_{d,0}^*(\boldsymbol{f})\frac{\partial p_0^*(\tau_i)}{\partial\tau_i}=0.$$

By solving the system of these first-order conditions, we find M's best-reply functions $\tau_{i,0}^*(\boldsymbol{f})$.

Finally, we analyze platforms' problem. When P_i offers a contract f_i to M, it believes that every other platform is offering the equilibrium commission f_0^*. Hence, for every offer f_i, denoting $\boldsymbol{f^*_{-i,0}}\triangleq(f_0^*,\dots,f_0^*)\in\mathbb{R}^{N-1}$, M is expected to set: a price $p_{d,0}^*(f_i,\boldsymbol{f^*_{-i,0}})$ on the direct sale channel; an access price $\tau_{i,0}^*(f_i,\boldsymbol{f^*_{-i,0}})$ on P_i; an access price $\tau_{-i,0}^*(f_i,\boldsymbol{f^*_{-i,0}})$ on every other platform. Hence, P_i's problem is as follows:

$$\max_{f_i} f_iD^i\left(p_{i,0}^*(f_i,\boldsymbol{f^*_{-i,0}}),(N-1)p_{-i,0}^*(f_i,\boldsymbol{f^*_{-i,0}})+p_{d,0}^*(f_i,\boldsymbol{f^*_{-i,0}})\right)$$

where $p_{i,0}^*(f_i,\boldsymbol{f^*_{-i,0}})\triangleq p_0^*(\tau_{i,0}^*(f_i,\boldsymbol{f^*_{-i,0}}))$ and $p_{-i,0}^*(f_i,\boldsymbol{f^*_{-i,0}})\triangleq p_0^*(\tau_{-i,0}^*(f_i,\boldsymbol{f^*_{-i,0}}))$.

Solving the first-order condition

$$\frac{1-\gamma}{1+(N-1)\gamma-N\gamma^2}-\frac{1+(N-1)\gamma}{1+(N-1)\gamma-N\gamma^2}p_{i,0}^*(f_i,\boldsymbol{f^*_{-i,0}})+$$

$$+\frac{\gamma((N-1)p_{-i,0}^*(f_i,\boldsymbol{f^*_{-i,0}})+p_{d,0}^*(f_i,\boldsymbol{f^*_{-i,0}}))}{1+(N-1)\gamma-N\gamma^2}+f_i\Bigg[-\frac{1+(N-1)\gamma}{1+(N-1)\gamma-N\gamma^2}\frac{\partial p_{i,0}^*(f_i,\boldsymbol{f^*_{-i,0}})}{\partial f_i}+$$

$$+\frac{\gamma}{1+(N-1)\gamma-N\gamma^2}\left((N-1)\frac{\partial p_{-i,0}^*(f_i,\boldsymbol{f^*_{-i,0}})}{\partial f_i}+\frac{\partial p_{d,0}^*(f_i,\boldsymbol{f^*_{-i,0}})}{\partial f_i}\right)\Bigg]=0,$$

and imposing symmetry (i.e., $f_i=f_0^*,\tau_{i,0}^*(f^*,\boldsymbol{f^*_{-i,0}})=\tau_0^*,p_{i,0}^*(f^*,\boldsymbol{f^*_{-i,0}})=p_0^*,p_{d,0}^*(f^*,\boldsymbol{f^*_{-i,0}})=$

$p^*_{d,0}$) we obtain the equilibrium values

$$f^*_0 = \frac{1}{\Psi_0}(2(1-\gamma)(1+\gamma(N-1))(2+\gamma(3N-2))(8-\gamma(8-(8-9\gamma)N)),$$

$$p^*_{d,0} = p^M + \frac{1}{2\Psi_0} N\gamma(1+\gamma(N-1))(8-\gamma(24-20N+\gamma(-25+41N-12N^2+3\gamma(N-1)(5N-4))),$$

$$\tau^*_0 = f^*_0 + \frac{1}{2\Psi_0}(32-\gamma(96-144N-4\gamma(27-89N+60N^2)+4\gamma^2(19-82N+121N^2-44N^3)+$$

$$+\gamma^3(-41+161N-334N^2+284N^3-48N^4)))+3\gamma^4(N-1)(-4+9N-18N^2+20N^3),$$

$$p^*_0 = \tau^*_0 + \frac{1}{\Psi_0}(1-\gamma)(1+N\gamma)(8-\gamma(24-20N+\gamma(-25+41N-12N^2+3\gamma(N-1)(5N-4)))),$$

where

$$\Psi_0 \triangleq 64+\gamma(-216+248N-9N\gamma^4(N-1)^2(5N-4)+4\gamma(67-169N+85N^2)+$$

$$-\gamma^2(155-662N+733N^2-192N^3)-6\gamma^3(N-1)(6-40N+47N^2-6N^3)).$$

M's incentive to delist. As in the model with two platforms, assuming that M is allowed to delist from platforms offering excessively high commissions, we must check whether, in each regime k, f^*_k satisfies M's participation constraint — i.e., whether M, who has been offered the commission f^*_k from a platform P_i, does not find it profitable to decline that offer (hence, to shut down that channel), while still accepting the same offer made by the remaining $N-1$ platforms.[21]

To this end, we should determine M's *deviation profit*, that is the profit that M gets by serving $N-1$ platforms, and compare it with the *equilibrium profit*, obtained by accepting all platforms' offers.

Let us examine the regime with platform parity. Considering M's best-reply functions $\tau^*_1(f_i, \boldsymbol{f^*_{-i,1}})$ and $p^*_{d,1}(f_i, \boldsymbol{f^*_{-i,1}})$, computed assuming that the number of active platforms is $N-1$, and substituting the equilibrium value (with N platforms) f^*_1, we obtain that M's optimal choices after refusing P_i's offer are

$$\tilde{p}_{d,1} = p^M + \frac{\gamma(1-\gamma)(2+\gamma(N-1))(N-1)(1+N\gamma)}{(2+\gamma(N-2))\Psi_1},$$

$$\tilde{\tau}_1 = f^*_1 + \frac{1}{2(2+\gamma(N-2))\Psi_1}(16-\gamma(56-64N-4\gamma(17-48N+25N^2)+\gamma^2(34-204N+256N^2-76N^3)+$$

$$-\gamma^3(6-90N+224N^2-159N^3+28N^4)+2\gamma^4N(N-1)(7-31N+20N^2-2N^3)+4N^2(N-2)(N-1)^2\gamma^5)).$$

Accordingly, intermediaries' optimal price becomes[22]

$$\tilde{p}_1 = \left(1 - \frac{1}{2+(N-2)\gamma}\right)\tau + \frac{1-\gamma(1-\tilde{p}_{d,1})}{2+(N-2)\gamma}.$$

[21]Notice that, since we are considering a contract equilibrium, we disregard the possibility of multilateral deviations by M — i.e., that M can refuse more than one offer.

[22]Recall that we assumed that intermediaries observe M's listing decision.

From these values, it is easy to determine M's *deviation profit*. Finally, in Figure 16, we show that M's participation constraint is satisfied when all platforms offer f_1^*.[23] It can be easily checked that, a fortiori, M does not find it profitable to decline P_i's offer f_0^* in the game without platform parity. ■

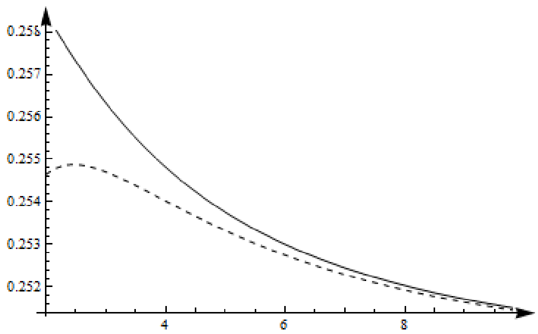


Figure 16: M's equilibrium profit (continuous line) and deviation profit (dashed line) in the game with platform parity agreement as a function of $N \geq 2$. Parameter's value: $\gamma = .5$.

Proof of Proposition 11. Following the same steps of the analysis of Sections 3.1 and 3.2, we can characterize the equilibrium of the game under the two parity regimes, for any given value of $c \in [0, 1-\gamma]$.

Specifically, if a platform parity agreement is in place, platform's commissions are

$$f_1^*(c) = \frac{(4+10\gamma-\gamma^2-10\gamma^3-4\gamma^4)(1-\gamma-c)}{6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4};$$

M sets price

$$p_{d,1}^*(c) = p^M + \frac{\gamma(1+\gamma)(1-\gamma-c)}{2(6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4)}$$

on the direct sale channel, and charges the access price

$$\tau_1^*(c) = f_1^*(c) + \frac{2+9\gamma+11\gamma^2-6\gamma^3-13\gamma^4-4\gamma^5-c(2+5\gamma+\gamma^2-4\gamma^3-2\gamma^4)}{2+9\gamma+11\gamma^2-6\gamma^3-13\gamma^4-4\gamma^5}$$

to the intermediaries, which in turn set

$$p_1^*(c) = \tau_1^*(c) + \frac{(1-\gamma)(1+\gamma)(1+2\gamma)(1-\gamma-c)}{2\left(6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4\right)}.$$

In the absence of platform parity, platforms' commissions are

$$f_0^*(c) = \frac{8(1+\gamma)(4+4\gamma-9\gamma^2)(1-c-\gamma)}{64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4};$$

[23] By using other numerical examples, we checked that M's *deviation profit* is lower than its *equilibrium profit* for many different values of γ.

M's price on the direct channel is

$$p^*_{d,0}(c) = p^M + \frac{\gamma(1+\gamma)(8-9\gamma^2)(1-c-\gamma)}{(1-\gamma)(64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4)},$$

and the access price is

$$\tau^*_0(c) = f^*_0(c) + \frac{32+96\gamma-28\gamma^2-248\gamma^3-5\gamma^4+153\gamma^5-2c(1+\gamma)(8-9\gamma^2)(2+2\gamma-3\gamma^2)}{2(1-\gamma)(64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4)}.$$

Finally, the intermediaries' final price is

$$p^*_0(c) = \tau^*_0(c) + \frac{(1+2\gamma)(8-9\gamma^2)(1-c-\gamma)}{(64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4)}.$$

Clearly, in both parity regimes, equilibrium commissions and prices are all decreasing in c.

M's profits in the two regimes are as follows

$$\pi^M_1(c) \triangleq \pi^M_1 - \frac{c(2-2\gamma-c)(1+\gamma)^2(4+10\gamma-\gamma^2-10\gamma^3-4\gamma^4)}{4(6+15\gamma-\gamma^2-15\gamma^3-6\gamma^4)^2},$$

$$\pi^M_0(c) \triangleq \pi^M_0 - \frac{c(2-2\gamma-c)(1+\gamma)^2(8-9\gamma^2)^2(4+4\gamma-9\gamma^2)}{(1-\gamma)^2(64+152\gamma-28\gamma^2-171\gamma^3-54\gamma^4)^2},$$

where, for every $k = 1, 0$, π^M_k is M's profit when $c = 0$. It is straightforward to see that, in both parity regimes, M's profit is decreasing in c and it can be easily checked that, for every $c \in [0, 1-\gamma]$, $\pi^M_1(c) < \pi^M_0(c)$.

Platforms' equilibrium profits are

$$\pi^P_{i,1}(c) \triangleq \frac{(1-c-\gamma)^2}{(1-\gamma)^2}\pi^P_{i,1},$$

$$\pi^P_{i,0}(c) \triangleq \frac{(1-c-\gamma)^2}{(1-\gamma)^2}\pi^P_{i,0},$$

where, for every $k = 1, 0$, $\pi^P_{i,k}$ is a platform's profit when $c = 0$. It is straightforward to see that, in both parity regimes, platforms' profits are decreasing in c and that $\pi^P_{i,1}(c) > \pi^P_{i,0}(c)$ if and only if $\pi^P_{i,1} > \pi^P_{i,0}$. ■

Proof of Proposition 12. For all $\gamma > \hat{\gamma}$: *(i)* $\pi^P_0(0) < \pi^P_1(0)$, *(ii)* $\pi^P_1(c)$ is decreasing in c, and *(iii)* $\pi^P_0(0) > \pi^P_1(1-\gamma) = 0$. This shows the existence of a threshold $\bar{c} \in (0, 1-\gamma)$ such that $\pi^P_0(0) > \pi^P_1(c)$, for all $c > \bar{c}$. ■

Appendix 2: Second-order conditions

In this Appendix, we derive the second-order conditions for the maximization problems stated in the main analysis.

Multi-product monopolist. The Hessian matrix of the optimization problem of the multi-product monopolist is

$$H(p_A, p_B, p_d) = \begin{pmatrix} -\frac{2(1+\gamma)}{(1-\gamma)(1+2\gamma)} & \frac{2\gamma}{(1-\gamma)(1+2\gamma)} & \frac{2\gamma}{(1-\gamma)(1+2\gamma)} \\ \frac{2\gamma}{(1-\gamma)(1+2\gamma)} & -\frac{2(1+\gamma)}{(1-\gamma)(1+2\gamma)} & \frac{2\gamma}{(1-\gamma)(1+2\gamma)} \\ \frac{2\gamma}{(1-\gamma)(1+2\gamma)} & \frac{2\gamma}{(1-\gamma)(1+2\gamma)} & -\frac{2(1+\gamma)}{(1-\gamma)(1+2\gamma)} \end{pmatrix}$$

whose eigenvalues are

$$\lambda_1 = \lambda_2 = -\frac{2}{1-\gamma} < 0,$$

$$\lambda_3 = -\frac{2}{1+2\gamma} < 0.$$

Hence, second-order conditions are satisfied for all $\gamma \in (0, 1)$.

The wholesale benchmark. It can be easily seen that, in both parity regimes, second-order conditions of intermediaries' and M's problems at the final pricing stage, as well as the second-order condition of platforms' problem, are satisfied for all $\gamma \in (0, 1)$. Second-order conditions of M's maximization problems concerning wholesale prices are as follows.

- In the regime with platform parity, we have

$$\frac{\partial^2 \pi^M}{\partial t^2} = -\frac{2(2+\gamma)(4+7\gamma-10\gamma^2-4\gamma^3)}{(1-\gamma^2)(4+\gamma)^2(1+2\gamma)}$$

  Therefore, the second-order condition is satisfied if and only if

$$\gamma < \overline{\gamma} \approx 0.86.$$

- In the regime without platform parity, we have

$$\frac{\partial^2 \pi^M}{\partial t_i^2} = -\frac{(4-\gamma)(4+9\gamma)}{32(1-\gamma^2)(1+2\gamma)} < 0,$$

  hence, the second-order condition is satisfied for all $\gamma \in (0, 1)$.

The agency model. In both parity regimes, the second-order conditions of M's and the intermediaries' problems at the final pricing stage are satisfied for all $\gamma \in (0, 1)$. The second-order conditions of M's maximization problems concerning access prices and of platforms' problems are as follows.

- In the regime with platform parity, we have

$$\frac{\partial^2 \pi^M}{\partial \tau^2} = -\frac{2(4+10\gamma-\gamma^2-10\gamma^3-4\gamma^4)}{(2+\gamma)^2(1+2\gamma-\gamma^2-2\gamma^3)}$$

which is negative for every

$$\gamma < \frac{1}{8}\left(\sqrt{5}+\sqrt{62+22\sqrt{5}}-5\right) \approx .93.$$

Finally, the second-order condition for the platforms' problem is satisfied if and only if the above condition holds true, since

$$\frac{\partial^2 \pi_i^P}{\partial f_i^2} = -\frac{(1+\gamma)^2}{(4+10\gamma-\gamma^2-10\gamma^3-4\gamma^4)}.$$

- In the regime without platform parity, we have to consider the following Hessian matrix:

$$H(\tau_A,\tau_B) = \begin{pmatrix} -\frac{8+(16-\gamma)\gamma}{8(1+2\gamma)(1-\gamma^2)} & \frac{\gamma(8+17\gamma)}{8(1+2\gamma-\gamma^2-2\gamma^3)} \\ \frac{\gamma(8+17\gamma)}{8(1+2\gamma-\gamma^2-2\gamma^3)} & -\frac{8+(16-\gamma)\gamma}{8(1+2\gamma)(1-\gamma^2)} \end{pmatrix}$$

whose eigenvalues are

$$\lambda_1 = -\frac{1}{1+\gamma} < 0, \quad \lambda_2 = \frac{1}{8}\left(\frac{1}{1-2\gamma^2+\gamma} - \frac{9}{1+\gamma}\right)$$

Thus, the second-order condition is satisfied if and only if

$$\lambda_2 < 0 \quad \Longleftrightarrow \quad \gamma < \frac{2(1+\sqrt{10})}{9} \approx .92$$

Finally, under this condition, also the second-order condition of the platforms' problem is satisfied, since

$$\frac{\partial^2 \pi_i^P}{\partial f_i^2} = \frac{1}{9\gamma^2-4(1+\gamma)} - \frac{1}{4(1-\gamma)} < 0 \quad \forall \gamma < .92.$$