CSEF
Centre for Studies in Economics and Finance

WORKING PAPER NO. 548

Tying in Evolving Industries, When Future Entry Cannot be Deterred

Chiara Fumagali and Massimo Motta

October 2019

University of Naples Federico II
University of Salerno
Bocconi University, Milan

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS – UNIVERSITY OF NAPLES
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unina.it
ISSN: 2240-9696

WORKING PAPER NO. 548

Tying in Evolving Industries, When Future Entry Cannot be Deterred

Chiara Fumagalli[*] and Massimo Motta[**]

Abstract
We show that the incentive to engage in exclusionary tying (of two complementary products) may arise even when tying cannot be used as a defensive strategy to protect the incumbent's dominant position in the primary market. By engaging in tying, an incumbent firm sacrifices current profits but can exclude a more efficient rival from a complementary market by depriving it of the critical scale it needs to be successful. In turn, exclusion in the complementary market allows the incumbent to be in a favorable position when a more efficient rival will enter the primary market, and to appropriate some of the rival's efficiency rents. The paper also shows that tying is a more profitable exclusionary strategy than pure bundling, and that exclusion is the less likely the higher the proportion of consumers who multi-home.

Keywords: Inefficient foreclosure, Tying, Scale economies, Network Externalities

JEL Classification: K21, L41

Acknowledgement: Comments by Yossi Spiegel and two anonymous referees as well as by participants to CRESSE Conference 2019 are gratefully acknowledged. Chiara Fumagalli acknowledges financial support from Baffi-Carefin Centre and IGIER (Università Bocconi). Massimo Motta gratefully acknowledges financial aid from the Spanish Agencia Estatal de Investigación (AEI) and FEDER (project ECO2016-76998-P).

[*] Università Bocconi, CSEF and CEPR. E-mail: chiara.fumagalli@unibocconi.it

[**] ICREA-Universitat Pompeu Fabra and Barcelona Graduate School of Economics, and CEPR. E-mail: massimo.motta@upf.edu

Table of contents

1 Introduction

It is well known that, when market structure evolves over time, tying can have an anti-competitive effect.[1] In particular, consider an incumbent firm that is monopolist in one market (that we denote as the "primary" market P) and faces actual or potential competition in the market for a complementary product (the "secondary" market, S). Consumers need both products in order to enjoy utility. If only the secondary market is under the threat of entry, while the primary market is a safe monopoly, the incumbent has no incentive to use tying to exclude more efficient rivals from the secondary market: it is more profitable to accommodate entry and use the price of the primary product to extract (some of) the rival's efficiency rents. Hence the incumbent would have the ability but not the incentive to leverage its dominance to the secondary market by means of tying. However, if current entry (or expansion of the rival) in the secondary market paves the way for future entry in the primary market, the incentive to engage in exclusionary tying arises. As shown by Carlton and Waldman (2002), tying represents a defensive strategy that enables the incumbent to protect its dominant position in the primary market.

In this paper we extend Carlton and Waldman (2002) and show that the incentive to engage in exclusionary tying still arises even when the incumbent's dominant position in the primary market cannot be protected from future entry in P. To see why, suppose there exist scale economies on the supply side in the secondary market. Tying limits significantly the entrant's sales of the secondary product in the initial period, when the incumbent's position in the primary market remains unchallenged. If scale economies are sufficiently important, this denies the entrant profits that are crucial to cover its entry cost, and discourages its entry in the secondary market altogether. As a consequence the incumbent will be in a favorable position in the future, when entry in the primary market will occur (irrespective of lack of entry in the secondary market): being the unique producer of the secondary product, the incumbent will not only benefit from market power in the secondary market, but it will also appropriate some of the rival's efficiency rents in the primary market. Therefore, it will earn higher profits relative to the counterfactual in which there is no tying and it faces competition both in the primary and secondary market in the future period.

Of course, this exclusionary strategy comes at the cost of sacrificing profits in the initial period, when tying prevents the incumbent from extracting efficiency rents from the rival in the secondary market. Tying turns out to be a profitable exclusionary practice if the rival's advantage (and hence its efficiency rents) in the primary market is large enough.

The same insight also applies to the extension developed in Carlton and Waldman (2002), in which tying does not aim to protect the incumbent's dominant position in the primary market (because that position is assumed to be safe) but to monopolise a newly emerging market complementary to the secondary one i.e., in their terminology, to "swing" its monopoly from one market to another over time. Indeed, our paper shows that the incentive to engage in tying exists even when the new market cannot be monopolised because a more efficient competitor will enter for sure: by excluding the rival in the secondary market, tying allows the incumbent to be in the position to appropriate some of the efficiency rents that entry in the new market will create.

A policy implication of this result is that entry in the secondary market needs not be a pre-

[1] It is well-known that tying may also have pro-competitive effects (see e.g. Fumagalli et al., 2018: 353-358). They are not the focus of this paper, but the reader should be aware that, when dealing with tying cases, one has to take into account both anti-competitive effects and pro-efficiency gains.

condition for future entry in the primary market (or in a newly emerging market) to have an anti-competitive rationale for tying. Indeed, it would be sufficient that future entry in the primary (or new) market is reasonably *likely*, irrespective of whether it would occur anyway or it depends on successful entry in the secondary market.

Our paper also shows that integration of the entrants (i.e. the entrants belonging to the same group/firm) limits the scope for inefficient exclusion. In our model lack of entry in the secondary market harms period-2 consumers (who will have to pay a higher price for the monopolised secondary product) and the new competitor in the primary market (whose profits will be squeezed by the incumbent's control of the secondary product). If those agents could take part in period-1 contracting, tying would not manage to exclude: they would be willing to compensate the entrant in the secondary market to ensure that it manages to cover the entry cost so as to benefit from lower prices and higher profits in the second period.[2] Integration of the entrants makes it harder for the incumbent to exclude even when payments across time are not feasible, because the integrated firm internalizes the beneficial effect that entry in the secondary market exerts on the profits of the future entrant in the primary market.

The case in which future entry in the primary market cannot be deterred allows us to highlight the difference between tying and pure bundling. Pure bundling refers to the case in which a firm only offers the bundle as a package and sells none of the products on a stand-alone basis. Tying, instead, refers to the case in which the sale of one product (the tying product) is conditional upon the purchaser also buying some other product (the tied product), but the tied product is also available as stand-alone product.[3] In the existing literature on exclusion whether the dominant firm engages in tying or pure bundling does not make a difference, with the exception of Choi and Stefanadis (2006). Instead, it does make a difference in our model, where the incumbent always prefers tying over bundling because of the higher flexibility ensured by tying. More precisely, tying and bundling are equivalent in period 1, when the incumbent's monopoly position in the primary market is unchallenged: in both cases consumers cannot obtain the primary product on a stand-alone basis, and thus the entrant does not manage to sell its secondary product (or its sales are substantially limited). However, tying and bundling are not equivalent in period 2, when there is scope for entry in both markets: first, under bundling the incumbent cannot use the price of the secondary product to compete with the rivals. Then, E_S makes higher profits in period 2 if it enters the secondary market and this makes exclusion harder to achieve. Second, when entry in the secondary market is discouraged, the fact that under tying product S is available on a stand-alone basis enables E_P to sell its higher quality product, thereby allowing the incumbent to exploit its monopoly position in the secondary market to appropriate part of E_P's efficiency rents. Under bundling, instead, E_P does not manage to sell its primary product in the absence of entry in the secondary market, and the incumbent loses the possibility to extract efficiency rents in period 2. Overall, bundling is not only less capable than tying of excluding the more efficient rival from the secondary market, but also less profitable when it manages to exclude.[4]

[2]In the terminology of Bernheim and Whinston (1998) ours is a setting with "non-coincident markets effects".

[3]In other words, given the two products P and S, under pure bundling consumers can only buy PS; under tying of P with S they can buy PS but also S alone.

[4]Flexibility explains why tying (or partial mixed bundling, as denoted in their paper) might be preferred to bundling also in Choi and Stefanadis (2006). In their paper the incumbent faces sequential entrants; each entrant observes the realization of own production costs of the two complementary products and then decides whether to enter both markets (when it turns out to be more efficient than the incumbent in the production of both components), only one or none. The paper shows that tying is a preferred option for the incumbent when the threats of entry in the two markets

Finally, another contribution of this paper is that we allow for the possibility that some consumers are *multi-homers*, i.e. they can add another secondary product to the system composed of the incumbent's primary and secondary products. We show that the anti-competitive concern of tying (or bundling) is the more severe the lower the share of multi-homers. We believe this helps shed light on an issue which is central in high-profile antitrust cases such as *US v. Microsoft* and the recent European Commission's *Google Android* (in both cases, it is controversial whether a consumer who finds a pre-installed piece of software would limit herself to keeping this default option or would be ready to download additional software, thereby multi-homing).

We also study a variant of the model in which scale economies in the secondary market arise on the demand side because of the existence of network externalities, and show that the mechanism described above extends to this case. By engaging in tying the incumbent limits the rival's sales of the secondary product in the initial period and hence prevents the rival from achieving the network size that is critical to make the quality of its product superior to that of the incumbent's. In turn this allows the incumbent to earn higher profits in the future, despite the rival's entry in the primary market.

In the model with network externalities the presence of non-negative price constraints is crucial for exclusion. When negative prices are not feasible, in the absence of tying competition in the secondary market is softened, which prevents the equilibrium price in that market from decreasing too much. In turn, in the initial period when its dominant position is still unchallenged, this prevents the incumbent from using the price of the primary product to extract a sufficient share of the rents that the more efficient rival in the secondary market will make in the future period. For this reason engaging in tying and excluding the rival in the secondary market may turn out to be the optimal strategy for the incumbent.[5]

This paper is related to the literature on the leverage theory of tying. Economic theory has moved into three main directions to show why it may be profitable for a firm that is dominant in one market to engage in exclusionary tying or bundling. One strand of the literature relies on imperfect rents extraction, i.e. it identifies some reasons why the incumbent may fail to appropriate, through the price of the primary (or core) product, enough efficiency rents either currently or in the future. Those reasons include regulation of the price of the primary product, the impossibility to set negative prices (Choi and Jeon, 2019), and restrictions to the set of feasible contracts (Carlton and Waldman 2012; Greenlee et al. 2008). Another strand of the literature identifies the circumstances under which tying (or bundling) allows the monopolist in a market to commit to aggressive behavior which discourages entry in the adjacent market, as in Whinston (1990) and Hurkens et al. (2016), Choi (1996, 2004). The third strand of the literature relaxes the assumption that the incumbent's dominant position in the core market is safe as in Choi and Stefanadis (2001) and Carlton and Waldman (2002). As discussed above, our paper is closely related to Carlton and Waldman (2002) and contributes to this literature by showing that the incentive to engage in exclusionary tying also arises when it does not

differ significantly, i.e. when it is more likely that an entrant turns out to be more efficient than the incumbent in the production of component i than of component j. The advantage of tying is that it accommodates entry and allows the incumbent to appropriate part of the first entrant's efficiency rents when the first entrant turns out to be more efficient than the incumbent only in component i, which is relatively likely. The drawback of tying is that, by accommodating entry in the first period, it makes it more likely that the incumbent will be completely displaced in the second period, which occurs when the second entrant turns out to be more efficient than the incumbent in component j. This outcome, however, is relatively unlikely.

[5] See also Carlton and Waldman (2002) and Choi and Jeon 2019 for the role of non-negative price constraints for exclusion when tying involves two-sided markets.

represent a defensive strategy that is, when entry in the primary market cannot be deterred.[6] Our paper is also related to Choi and Stefanadis (2001). Their model and ours share the fact that the incumbent engages in bundling (or tying) so as to make entry in *both* the complementary markets less likely, a scenario in which the incumbent is completely displaced. However, the two models differ in the mechanism through which bundling (or tying) leads to exclusion. In our model tying, by limiting significantly the entrant's sales of the secondary product in the first period, when the incumbent still monopolises the primary market, deters entry altogether: since those sales are crucial to achieve efficient scale, the rival will not enter in the second period either, despite the presence of the independent rival in the primary market. Instead, in Choi and Stefanadis (2001) there is a single period in which both independent entrants take their respective entry decisions, conditional on the success of their $R\&D$ investment. Bundling, by allowing an independent rival to sell its product only if the other component is available separately, makes entry in a given market profitable only when the $R\&D$ processes of both firms are successful. In turn, this reduces the marginal returns of each firm's $R\&D$ investment relative to the case in which the incumbent does not engage in bundling, thereby inducing the entrants to invest less and making the scenario in which both entrants are active less likely.[7]

Our paper is also related to the literature on exclusionary conduct in general, beyond tying. In this perspective, it is probably closest to another paper of ours (Fumagalli and Motta, 2018) in which we examine the incentives for a vertically integrated firm to engage in refusal to supply so as to monopolise the downstream market, either to protect the upstream monopoly, or as a way to extract more rents from an upstream entrant.

An antitrust case that might be interpreted in the light of our model is *Genzyme*.[8] At the time of the decision Genzyme was the unique producer of Cerezyme, then the only drug available for the treatment of the Gaucher disease (a rare metabolic disorder). For home patients the drug needed to be administered by specialised nurses or doctors. Up until 2000 Genzyme had relied on an independent company, "Healthcare at Home", as an exclusive provider of delivery and homecare services for Cerezyme. Later, it started operating its own company of delivery and homecare services, "Genzyme Homecare", and sold to the National Health Service (NHS) the drug together with the additional services at the same price at which it sold the drug only to "Healthcare at Home".[9]

The Office of Fair Trading (OFT) found that by this practice Genzyme had engaged in anti-competitive behavior, bundling and margin squeeze, two separate abuses (see Paragraph 293 of the OFT Decision). (We discuss the case here as bundling, while in Fumagalli, Motta and Calcagno, 2018 we present it through the lenses of margin squeeze and vertical foreclosure. Note that the practice at issue is the same, which can be interpreted in two different ways.)

[6] Technically, what drives the difference between our paper and Carlton and Waldman (2002) is that we assume that the cost of entering the primary market are sufficiently low, so that entry cannot be deterred.

[7] Note that bundling (or tying) in both models entails a profit sacrifice: in Choi and Stefanadis (2001) bundling impedes sole entry in one of the two markets, a situation in which the incumbent could appropriate the efficiency rents generated in that market through the pricing of the complementary product that remains under its monopoly power. Similarly, in our model, tying prevents rents extraction from the more efficient rival in the secondary market in period 1, when the incumbent's position in the primary market is still unchallenged. However, contrary to bundling, tying allows the incumbent to extract efficiency rents in period 2 when the more efficient rival in the primary market will operate. (See the discussion above on the difference effects of tying and bundling in our model.)

[8] *Genzyme*, Decision CA98/93/03 of the UK Office of Fair Trading, 27 March 2003.

[9] In order to continue offering the delivery and homecare services, "Healthcare at Home" had to first purchase Cerezyme from Genzyme and then agree with the NHS on a price for the provision of the drug and additional services.

While it is uncontroversial that Genzyme's behaviour left no potential scope for competition in the market for delivery and homecare services,[10] the OFT and the Competition Appeal Tribunal (CAT) differ in the assessment of the extent to which it might have prevented further entry in the market for the supply of drugs for the Gaucher disease. (The OFT argued that monopolisation of the home delivery service would have raised barriers to entry into the drug market.[11] Instead, the CAT was more skeptical about the potential for foreclosure in the drug market.[12])

Based on the available evidence we cannot take a position on whether Genzyme's conduct would limit future entry in the market for the supply of drugs for the Gaucher desease. However, our paper highlights that Genzyme's incentives to exclude "Healthcare at Home" would exist even if lack of competition in the market of homecare services did *not* discourage future entry in market for the drug. In that case Genzyme's conduct would not be rationalised by the intent to protect its dominant position in the drug market; but by the aim to gain a favorable position in the complementary market of the homecare services so as to appropriate some of the efficiency rents produced by future entry in the drug market.

The paper proceeds as follows. Section 2 analyses the baseline model with supply-side scale economies in the secondary market. Section 3 studies the case in which there exist network externalities in the secondary market. Section 4 concludes the paper.

2 The baseline model

There are two complementary products: the primary one, denoted by P, and the secondary one, denoted by S. The two products can be used only in conjunction with one another. Consumers combine the two products in fixed proportions on a one-to-one basis to form a final product (or system). The system that combines the incumbent's primary and secondary products gives consumers value U, whereas buying either product alone would give zero utility.

We assume there is a current potential entrant in the secondary market, E_S. In the secondary market, entry is possible both in the first period and in the second period. In the primary market a potential entrant, E_P will materialise in the second period with probability $p \in (0, 1]$. Hence, in the first period the incumbent enjoys a safe monopoly position in the primary market. Both entrants and the incumbent, I, have zero variable costs of production, but the product of each entrant produces extra-utility $\delta_i < U$ with $i = P, S$ relative to the incumbent's (primary or secondary) product.

Entry requires a fixed and sunk cost f_P for the primary market,[13] and f_S for the secondary

[10] The decision of the OFT and the Judgment of the Competition Appeal Tribunal (CAT) argue that the administration of the drug at home is a delicate phase. Once patients build a trustful relationship with an existing homecare provider, they are very reluctant to switch to a different provider and it is hard for new homecare providers to find their way into the market. Expert witness testimony supports this view (See Paragraph 635 of the CAT judgment).

[11] "Since the supply of the Cerezyme is effectively tied to homecare services provided by "Genzyme Homecare", a new competitor would face the additional hurdle of persuading the patient to switch not only to a new drug, but also to a new homecare services provider." (Paragraphs 331-334 of the OFT Decision.)

[12] "Our overall conclusion, on the balance of the evidence, is that if Genzyme were to succeed in monopolising the supply of homecare services, that would probably have some adverse effect on the ability of a new treatment for the Gaucher desease to establish itself in the UK over a reasonable time, but the additional foreclosure effect in this latter market is unlikely to be as great as that suggested by the OFT in the decision." (see paragraph 639 of the CAT Judgment – *Genzyme v The Office of fair Trading*, Case No. 1016/1/03 [2004] CAT.)

[13] Allowing for $f_P = 0$ would not change the results. It would only make firm E_P indifferent between entering the market or not when the incumbent engages in pure bundling.

market, where:

$$0 < f_P \leq \frac{\delta_P}{2} \tag{A1}$$

and

$$0 \leq f_S \leq \frac{\delta_S(2+p)}{2}. \tag{A2}$$

As will be clearer from the analysis below, the upper bounds on fixed costs reflect the fact that, when the incumbent is the monopolist seller of one of the two products, while a more efficient rival operates in the complementary market, the price game exhibits a continuum of equilibria. These equilibria differ in how much of the surplus produced by the rival's product the incumbent manages to extract through the price of the monopoly product. At one extreme there exists an equilibrium in which the incumbent appropriates entirely the surplus of the more efficient rival; at the other extreme there exists an equilibrium in which the incumbent does not appropriate any of that surplus. For simplicity, and in line with Carlton and Waldman (2002), we focus on the equilibrium in which the incumbent appropriates half of the surplus. This equilibrium could also be interpreted as the outcome of a Nash bargaining game in which the incumbent and the entrants are given equal weights.

Firm I, being the incumbent, has already paid its entry costs before the game starts. As will be clear from the analysis below, these assumptions imply that E_P will always enter in the second period if it materialises, and that E_S would enter if I did not engage in bundling or tying. Note also that the assumption that entry in P is possible means that we are in an environment where Carlton and Waldman's exclusionary theory of tying does not apply: in their model, the rationale for exclusion is the protection of the primary market, which here cannot be protected by assumption.

Another important assumption that differentiates our setting from Carlton and Waldman (2002) concerns the possibility for consumers to buy and use another secondary product, when the incumbent engages in tying. Carlton and Waldman assume that a consumer who bought the system including the incumbent's (primary and secondary) products cannot also use the secondary product of the rival, for instance because of incompatibility design. Instead, we assume that a proportion β of consumers have zero disutility from adding another secondary product to their system once they already have I_P and I_S. The remaining $1-\beta$ fraction of consumers has instead an arbitrarily large disutility from doing so, and therefore will not add E_S to their system even if E_S was sold at zero price. For instance, the secondary product can be one type of application (e.g., maps, instant messaging, browser) available for a mobile phone: β would then be the proportion of people who have no difficulty of searching for an alternative application and who have enough storage capacity in their mobile phone to be able to download and use it. We call them "multi-homers".

The game is as follows.

> At period 0, firm I decides whether it wants to sell the two products separately, tie them, or sell them as a pure bundle. Tying implies that I sells the primary product always bundled with the secondary product, while the secondary product can also be sold separately. Pure bundling implies that the two products can only be sold together. The decision to engage in pure bundling or tying is irreversible.[14]

[14] In this baseline model the irreversibility assumption is crucial: if the firm offering the bundle could easily undo the tying decision, then entry in the secondary market could not be discouraged. This is because the entrant would decide to enter even if the incumbent engages in tying: it would anticipate that following entry the incumbent would undo the tying decision to be able to extract some of the entrant's efficiency rents. (See footnote 18). However, tying does not need to be irreversible in the model of Section 3 in which the incumbent and the rivals are already in the market and

At period 1, (i) E_S decides whether it wants to enter the secondary market, and accordingly pay f_S, or not; (ii) price choices are made by active firms; (iii) buyers decide, and (iv) transactions are made and profits realised.

At period 2, (i) if it materialises (which occurs with probability p), E_P decides whether it wants to enter the primary product market, and accordingly pay f_P, or not; if E_S has not entered the secondary market in period 1, it has another chance to do so; (ii) price choices are made by active firms; (iii) buyers decide, and (iv) transactions are made and profits realised. We assume no discounting between the two periods.

As usual, we solve the game by backward induction. We first consider the case where the incumbent chooses to sell the two products separately (Section 2.1), and then we will move to the case of tying (Section 2.2.1) and pure bundling (Section 2.2.2) where we will also analyse under which conditions engaging in these two practices is more profitable than independent sales. In Section 2.3 we will study the case in which the entrants belong to the same company.

2.1 Independent sales

In this Section we focus on the case in which the incumbent commercializes the two products independently. The following Lemma describes the continuation equilibrium in this case.

Lemma 1. *If the incumbent does not engage in tying, the continuation equilibrium of the game is such that entry by E_S takes place in the first period, and entry by E_P follows in the second period (if E_P materialises). Equilibrium total payoffs (expected at the beginning of the game) are as follows:*

$$\pi_I^* = (U + \frac{\delta_S}{2})(2-p); \quad \pi_{E_S}^* = \frac{\delta_S}{2} + p\delta_S + (1-p)(\frac{\delta_S}{2}) - f_S; \quad \pi_{E_P}^* = p(\delta_P - f_P) \quad CS^* = pU \tag{1}$$

Proof. See Appendix A.1 □

To see the intuition behind this result, consider that, in period 2, E_P always enters the primary market – if it materialises – irrespective of the entry decision of E_S. More precisely, if also E_S is in the market, consumers will have access to the superior quality products of both new entrants and, when price competition will take place, each entrant will appropriate its efficiency advantage: $\pi_{E_i}^2 = \delta_i$ with $i = P, S$. By assumption $A1$, E_P's second-period profits will be large enough to cover the entry cost f_P. If E_S is not in the market, the incumbent will be the sole seller of the complementary secondary product and will manage to squeeze partially E_P's margin, appropriating part of the increase in buyers' surplus produced by E_P's higher quality product.[15] E_P's profits will be lower than in the previous case (they will amount to $\pi_{E_P}^2 = \frac{\delta_P}{2}$) but, by assumption $A1$, they are still large enough to cover f_P.

In the first period the incumbent monopolises the primary market, and it will continue doing so also in the second period, if E_P does not materialise. In that case the incumbent partially squeezes E_S's margin, should E_S be in the market. E_S would, then, earn $\frac{\delta_S}{2}$ per period. If, instead, E_P does

scale economies arise from the demand side. Note also that consumers can "de facto" undo the bundle if they can add another complementary product to the bundled system. As explained above, we allow for this possibility by assuming that a proportion β of consumers, denoted as "multi-homers", can do that. The paper shows that, if the proportion of multi-homers is large enough, exclusion does not occur at the equilibrium.

[15] In the Appendix we characterise all the equilibria of the price game when the incumbent is the sole seller of one of the two products and faces a more efficient rival in the complementary market.

materialise, E_S earns $\frac{\delta_S}{2}$ in the first period, but it earns more, namely δ_S, in the second period when E_P enters the market and competition between primary producers allows it to appropriate all of its rents.

Overall, by entering the secondary market in period 1, E_S expects to make total profits $\pi_{E_S}^{*TOT} = \frac{\delta_S}{2} + p\delta_S + (1-p)\frac{\delta_S}{2}$. By assumption $(A2)$ those profits cover the entry cost f_S.

Note that since products are sold independently consumers can mix-and-match them without constraints, and the existence of multi-homers does not play any role.

2.2 Tying and bundling

2.2.1 Tying

Let us now study the case where there is tying: I always sells its primary product together with the secondary product, but also sells the secondary product independently.

Lemma 2. *If the incumbent engages in tying the continuation equilibrium of the game is as follows:*

(i) If either the entry cost in the secondary market is sufficiently low, i.e. $f_S \leq p\delta_S$, or $f_S > p\delta_S$ and the share of multi-homers is sufficiently large, i.e. $\beta \geq \beta^*(f_S)$, then E_S enters in the first period, and E_P enters in the second period (if it materialises). Equilibrium total payoffs are as follows:

$$\pi_I^* = (U + \beta\frac{\delta_S}{2})(2-p); \quad \pi_{E_S}^* = \beta\frac{\delta_S}{2} + p\delta_S + (1-p)\beta\frac{\delta_S}{2} - f_S; \quad \pi_{E_P}^* = p(\delta_P - f_P) \quad CS^* = pU \tag{2}$$

(ii) If the entry cost in the secondary market is high enough, i.e. $f_S > p\delta_S$ and the share of multi-homers is sufficiently low, i.e. $\beta < \beta^*(f_S)$, E_S does not enter the secondary market. E_P enters the primary market in period 2 if it materialises. In this case equilibrium total payoffs are as follows:

$$\pi_I^* = U + U + p\frac{\delta_P}{2}; \quad \pi_{E_S}^* = 0; \quad \pi_{E_P}^* = p(\frac{\delta_P}{2} - f_P) \quad CS^* = 0 \tag{3}$$

The threshold $\beta^*(f_S) \in (0, 1]$ and is increasing in f_S.

Proof. See Appendix A.1 □

When the incumbent engages in tying, and E_P is in the market, the post-entry profits in period 2 are the same as in the case of independent sales.

In particular, we show more extensively in the Appendix that when both E_P and E_S compete with the bundle, no equilibrium exists in which, given the total price $p_{E_S} + p_{E_P} = \delta_S + \delta_P$, either entrant appropriates more than its efficiency advantage. E_S cannot appropriate more than δ_S because under tying the incumbent sells the secondary product on a stand-alone basis, on top of the bundle, and could profitably deviate by increasing p_{I_s}. E_P, in turn, cannot appropriate more than δ_P because the presence of multi-homers creates the scope for profitable deviations by the incumbent, who could profitably increase the bundle price. Therefore, $\pi_{E_P}^2 = \delta_P$ and $\pi_{E_S}^2 = \delta_S$.

Importantly, when E_S is not in the market, the availability of the incumbent's secondary product on a stand-alone basis makes it possible for consumers to combine I_S with the higher-quality product offered by E_P. Then, under tying the incumbent still manages to partially appropriate the rival's efficiency advantage and $\pi^2_{E_P} = \frac{\delta_P}{2}$. Hence, assumption $(A1)$ ensures that in period 2 E_P always enters the primary market, irrespective of the entry decision of E_S.

Tying, however, affects the profits that E_S makes when the incumbent monopolises the primary market. In that case a consumer who purchases the primary product from the incumbent obtains also the incumbent's secondary product. Price discrimination allows the incumbent to extract all of the surplus U that single-homers enjoy when they purchase the bundle, and part of E_S's efficiency gain through the price set for multi-homers (i.e. those consumers who, once obtained the primary and the secondary product from the incumbent, can add the entrant's secondary product).[16] Hence, E_S manages to sell only to a fraction β of consumers and to earn $\frac{\delta_S}{2}$ only from them. As a consequence, tying reduces the profits that E_S makes in period 1, and in period 2 when E_P does not materialise, and affects its entry decision. Indeed, as stated in Lemma 2, if the entry costs are large enough and the fraction β of multi-homers is sufficiently small, total profits are insufficient to cover the entry costs and E_S will decide not to enter the secondary market.[17]

Next we analyse under which conditions the incumbent decides to engage in tying.

Period 0: Tying decision

As shown earlier, if the incumbent chooses not to engage in tying, entry will occur in the secondary market in period 1 and in the primary market in period 2 (if E_P materialises). The incumbent extracts some of E_S' efficiency advantage, in period 1 and in period 2 when E_P does not materialise, but makes zero profits in period 2 when also E_P enters the market. The incumbent's total profits in the absence of tying are:

$$\pi_I^{TOT(Independent\ Sales)} = U + \frac{\delta_S}{2} + (1-p)(U + \frac{\delta_S}{2}) \tag{4}$$

Consider now the case in which the incumbent chooses tying and E_S enters anyway – this occurs either when $f_S \leq p\delta_S$ or when $f_S > p\delta_S$ and $\beta \geq \beta^*(f_S)$ – followed by E_P in period 2, if it materialises. In this case tying is clearly unprofitable. Tying does not discourage E_S's entry; hence, the incumbent still makes zero profits in period 2 when E_P is also active and both the more efficient entrants exert competitive pressure. Moreover, when E_P is not active, i.e. in period 1 and in period 2 when it does not materialise, tying prevents the incumbent from extracting from single-homers some of the surplus created by the activity of the more efficient rival E_S. The incumbent's total profits are:

$$\pi_I^{TOT(Tying|Entry\ by\ E_S)} = U + \beta\frac{\delta_S}{2} + (1-p)(U + \beta\frac{\delta_S}{2}) \tag{5}$$

and are clearly lower than $\pi_I^{TOT(Independent\ Sales)}$.

Instead, when entry costs are large enough (i.e. $f_S > p\delta_S$) and the proportion of multi-homers

[16] Allowing for price discrimination increases the profits that the incumbent earns when engaging in tying. We will show that, nonetheless, if E_S enters the market in period 1, tying is not optimal for the incumbent.

[17] Exclusion is feasible because the entrant in the secondary market has no more than two periods to cover its entry costs. This assumption aims at capturing dynamic market structures where the products evolve quickly. If one allows period 2 to last longer, i.e. assumes that the new entrants can be both active in more than one period, it would be easier for the entrant in the secondary market to cover fixed costs and more difficult for the incumbent to exclude through tying. If, instead, period 2 is less important for the the entrant in the secondary market because of discounting of future profits, exclusion becomes easier.

is sufficiently low (i.e. $\beta < \beta^*(f_S)$), tying allows the incumbent to preserve its monopoly position in the secondary market and, as a consequence, to be in a favourable position in period 2, when efficient entry in the primary market will occur: being the unique producer of the secondary product, the incumbent will be able to extract some of E_P's efficiency advantage. The incumbent's total profits are:

$$\pi_I^{TOT(Tying|No\ entry\ by\ E_S)} = U + p(U + \frac{\delta_P}{2}) + (1-p)U \tag{6}$$

The comparison between (6) and (4) highlights that, when E_P materialises, tying (when it discourages E_S' entry) allows the incumbent to earn higher profits in period 2 relative to the case of independent sales in which entry in the secondary market combined with entry in the primary market prevents the incumbent from extracting any rents ($U + \frac{\delta_P}{2} > 0$). However, this comes at the cost of sacrificing profits in period 1 and in period 2 when E_P does not materialise, because in those cases tying prevents the incumbent from appropriating some of E_S's efficiency advantage ($U < U + \frac{\delta_S}{2}$).[18]

Tying turns out to be profitable for the incumbent when the expected increase in future profits dominates the profits' sacrifice which occurs when E_P is not in the market. As the following expression indicates, this is the case when the probability that E_P materialises is sufficiently large.

$$p(U + \frac{\delta_P}{2}) > \frac{\delta_S}{2} + (1-p)(\frac{\delta_S}{2}) \Leftrightarrow p > \frac{2\delta_S}{2U + \delta_S + \delta_P} \equiv p^* \tag{7}$$

Note that the incentive to engage in exclusionary tying is the stronger the higher the incumbent's efficiency disadvantage in the primary market (because this increases the benefit of extracting some of the rival's efficiency rents) and the lower the incumbent's efficiency disadvantage in the secondary market (because this decreases the profits' sacrifice).

Of course tying reduces consumers' surplus, because lack of entry in the secondary market makes them pay a higher price in period 2 for the combination of the primary and secondary product. Tying also reduces total welfare because the secondary product is produced by the inefficient incumbent.

The following Proposition summarises the analysis:

Proposition 1. ***Profitability of tying and welfare effects***

(i) The incumbent chooses to engage in tying rather than independent sales if tying excludes E_S, i.e. if the entry cost is large enough ($f_S > p\delta_S$) and the share of multi-homers is sufficiently low ($\beta < \beta^*(f_S)$), and if future gains exceed losses suffered when E_P is not in the market, i.e. if the probability that the more efficient entrant in the primary market materialises is large enough ($p > p^*$).

(ii) When it arises at equilibrium, tying excludes the more efficient producer of the secondary product thereby reducing consumer surplus and total welfare.

2.2.2 Pure bundling

Let us consider the case in which the incumbent engages in pure bundling, i.e. it does not sell the secondary product on a stand-alone basis, but it commits to sell the two products only together.

[18] The profit sacrifice in period 1 suggests why the irreversibility of the tying decision is crucial to exclude. If the incumbent could easily undo the tying decision, then the entrant would enter the market in period 1, even when it observes that the incumbent engaged in tying in the earlier period. This is because the entrant would anticipate that if it entered the market, the incumbent would undo tying to be able to appropriate the rival's efficiency rents.

Lemma 3. *When the incumbent engages in pure bundling, the continuation equilibrium of the game is as follows:*

(i) If either $f_S \leq p(\delta_S + \frac{\delta_P}{2})$ or $f_S > p(\delta_S + \frac{\delta_P}{2})$ and $\beta \geq \beta^{*b}(f_S)$, then entry by E_S takes place in the first period, followed by by E_P in the second period (if E_P materialises). Equilibrium total payoffs are as follows:

$$\pi_I^* = (U+\beta\frac{\delta_S}{2})(2-p); \quad \pi_{E_S}^* = \beta\frac{\delta_S}{2}+p(\delta_S+\frac{\delta_P}{2})+(1-p)\beta\frac{\delta_S}{2}-f_S; \quad \pi_{E_P}^* = p(\frac{\delta_P}{2}-f_P) \quad CS^* = pU \tag{8}$$

(ii) If $f_S > p(\delta_S + \frac{\delta_P}{2})$ and $\beta < \beta^{*b}(f_S)$ firm E_S does not enter the secondary market and firm E_P does not enter the primary market in period 2. In this case equilibrium total payoffs are as follows:

$$\pi_I^* = U + U; \quad \pi_{E_S}^* = 0; \quad \pi_{E_P}^* = 0 \quad CS^* = 0 \tag{9}$$

The threshold $\beta^{*b}(f_S) \in (0, 1]$, is increasing in f_S and $\beta^{*b}(f_S) < \beta^*(f_S)$.

Proof. See Appendix A.1. □

By comparing Lemma 3 and Lemma 2 two features emerge. First, under pure bundling the condition such that E_S's entry is discouraged is more stringent than in the case of tying: the lower bound of the entry cost is higher (i.e. $p(\delta_S + \frac{\delta_P}{2}) > p\delta_S$) and the upper bound of the share of multi-homers is smaller (i.e. $\beta^{*b}(f_S) < \beta^*(f_S)$). This is because under pure bundling the incumbent is deprived of a tool to compete in the secondary market, the individual price of the secondary product. This allows E_S to earn higher profits in period 2 if it enters the market relative to the case of tying, and makes exclusion harder.

Secondly, pure bundling, differently from tying, discourages also period-2 entry in the primary market when it excludes E_S from the secondary market. The fact that I's secondary product is not available on a stand-alone basis prevents the more efficient rival from selling its primary product in period 2 when E_S is excluded from the market, which in turn discourages E_P's entry. As a consequence, when it manages to exclude, pure bundling is less profitable than tying because it prevents the incumbent from extracting E_P's efficiency rents in period 2.

The condition for bundling to be chosen at equilibrium is, therefore, more difficult to be satisfied as compared to the case of tying:

$$\pi_I^{TOT(Bundling|No\ entry\ by\ E_S)} = U+U > (U+\frac{\delta_S}{2})(2-p) = \pi_I^{TOT(Independent\ Sales)} \Leftrightarrow p > \frac{2\delta_S}{2U+\delta_S} \equiv p^{*b} \tag{10}$$

where $p^{*b} > p*$.

Since bundling is less capable of excluding than tying and it is also less profitable when it manages to exclude, the incumbent never prefers bundling over tying, as stated by the following Proposition:

Proposition 2. ***Choice between tying and bundling.***
The incumbent never finds pure bundling more profitable than tying.

Proof. Follows from the discussion above. □

2.3 Integrated entrants

We have thus far assumed that the entrants E_S and E_P are two different companies. In this Section we assume, instead, that they are integrated, and affiliates of the same company E. The following Proposition states the results of the analysis in this case.

Proposition 3. ***Integrated entrants***
When the entrants belong to the same company, the incumbent is less capable of excluding from the secondary market through the use of tying. The incumbent's incentive to engage in tying is the same as in the case of separated entrants.

Proof. See Appendix A.1 □

To see the intuition note first that the fact that firm E is an integrated company does not affect the equilibrium prices in the various market structures and the profits that the incumbent earns with or without tying. Hence, the incumbent's incentive to engage in tying does not change when the entrants belong to the same company.

However, integration changes firm E's entry decision. The intuition is that, when deciding whether to enter a market, say market S, on top of market P, firm E takes into account not only the profits that it earns in market S, but also the increase in profits in the complementary market, due to the fact that simultaneous entry in both markets prevents the incumbent from extracting the efficiency rents of the firm active in market P. That increase in profits is not internalised when the entrants do not belong to the same company. This makes firm E more willing to enter also market S (or also market P) than separate entrants.

Hence, absent tying, the result of Lemma 1 is *a fortiori* valid when the entrants are integrated and entry in market S occurs in period 1, followed by entry in market P in period 2 (if E_P materialises).

Instead the condition such that tying discourages entry in market S becomes more stringent than in the case of separation: the lower bound of the entry cost f_S is higher and the upper bound of the share of multi-homers is smaller (see Lemma 6, in Appendix A.1, thereby reducing the incumbent's capability of excluding the more efficient rival in the secondary market.

3 The model with network externalities

In this Section we analyse a model where scale economies arise on the demand-side due to the existence of network externalities in the secondary market. To focus away from the supply side, we assume that all firms have zero entry costs. Therefore, differently from Carlton and Waldman (2002) where entry in the primary market entails positive fixed costs, in our setting entry cannot be discouraged either in the primary or the secondary market. We assume that E_S operates in the secondary market since the first period, whereas E_P starts operating in the primary market only in the second period. (For simplicity we assume here that E_P materialises with $p = 1$.)

We assume there are $N_1 = N$ consumers who can make purchases in the first period, and $N_2 = N$ in the second period. For simplicity, all of them consume only at the end of period 2.[19]

[19]This could also be rationalised by saying that the good is going to be sold only in periods 1 and 2, but then is consumed for an infinite number of periods in the future. As long as the discount factor is large enough, period 1 consumption will become irrelevant.

Consumption of the secondary product generates network externalities. We model network effects as in Katz and Shapiro (1986) and Carlton and Waldman (2002). The system given by the primary product and the secondary product will give the following utilities, depending on the number of users and which particular version of the (primary or secondary) product is used.

$$V(I_P, I_S) = U + v(n_{I_S}); \quad V(I_P, E_S) = U + \delta_S + v(n_{E_S});$$
$$V(E_P, I_S) = U + \delta_P + v(n_{I_S}); \quad V(E_P, E_S) = U + \delta_P + \delta_S + v(n_{E_S}),$$

where $v(n_j)$ is an increasing and concave function in the total number of users of the secondary product n_j, with $j = I_S, E_S$. We shall assume that:

$$\delta_S < v(2N) - v(N). \tag{A1'}$$

As will be clearer below, this hypothesis amounts to saying that, despite its intrinsic advantage δ_S, selling to a single cohort of consumers is not enough for E_S to be preferred to I_S. In particular, if (A1') did not hold, then even if I_S sold to all consumers in the first period, it would never be able to win second period customers when competing with E_S. As a consequence, the exclusionary mechanism identified in the analysis below would not apply: firm I will not have the ability to raise its second period profits by selling more of its secondary product in the first period.

Finally, we follow Katz and Shapiro (1986) and most of the literature on network industries, by assuming that consumers of a certain cohort (a cohort corresponding to a period) choose as if they were able to coordinate. In formal terms, we restrict attention to coalition-proof perfect Nash equilibria.

The game is as follows.

At period 0, firm I decides whether it wants to sell products P and S as a tying — that is, selling I_P only with I_S, but also allowing consumers to buy I_S separately — or not.

At period 1, (i) prices are set by I and E_S; (ii) first-period buyers decide.

At period 2 also E_P operates. Then, (i) all firms set prices; (ii) buyers decide.

At period 3, all consumption takes place and profits are realised.

We will solve the model under the assumption that prices cannot be negative. This is indeed a crucial assumption: if firms could set negative prices exclusion would not occur at equilibrium, as also acknowledged by Carlton and Waldman (2002) and Choi and Jeon (2019). The intuition is that, absent tying, the possibility to set negative prices intensifies competition in period 1: at equilibrium it is the more efficient rival E_S that secures the first cohort of consumers (and, because of the network externalities, also the second cohort), by offering them a negative price. The negative price transfers to the first-cohort of consumers some (or all) of the surplus that the more efficient rival obtains in the second period by selling its higher quality product. Such a surplus is however appropriated by the incumbent through the price of the primary product, which in the first period is still a monopoly. In sum, when negative prices are feasible, the incumbent appropriates not only the rival's efficiency rents generated in the first period but also some (or all) of the efficiency rents generated in the second period. Those rents cannot be appropriated when the rival is excluded, which makes tying sub-optimal. Differently stated, the non-negative price constraints, by softening price competition in the first period, is a source of imperfect rents extraction which creates the incumbent's incentive to engage in tying to exclude. (See also the discussion at the end of Section 3.3).

Opportunism and adverse selection by consumers may justify the assumption that firms cannot set negative prices. Firms set negative prices on a given product when its purchase generate extra-rents, for instance on other products, like in two-sided markets. In our model if the first cohort of consumers chooses a given product also the second cohort will do, because of the presence of network externalities. Then, the firm offering that product will obtain additional rents in the second period. If the mere purchase of the product generates the extra-rents, then negative price are not problematic. The problem arises when the fact that consumers obtain the product does not necessarily imply that extra-rents are generated, for instance because consumers need to use the good/service for extra-rents to be produced. For instance, if the provider of a search engine paid consumers whenever they search on the internet, consumers would engage in random search all day long, but extra-revenues from advertisers would not be generated because consumers would not pay attention to what they are searching for. In our model, when a negative price is offered, it might be the case that individuals who do not enjoy any utility from the consumption of the product buy it to obtain the transfer. However, if they do not actually use the product, those consumers will not contribute to the creation of network externalities and to the extra-rents. In some cases, instruments may be available which could solve or relax the opportunism problems above while effectively offering negative prices. For instance, cashback reward programs that offer rebates on cardholders' purchases can be considered a negative price. However, cashback rewards are conditional on the user of the card to spend a pre-defined amount of money with the card. The negative price is therefore conditional on use of the product and on the generation of extra-rents (through interchange fees), and is feasible. In sum, the plausibility of the non-negative price constraint depends on the characteristics of the product and of the market. Moreover, it is more likely that negative prices are feasible when buyers are firms rather than final consumers because it is easier to monitor the latter and make sure that opportunistic behavior does not limit the use of negative prices.

We solve the game by backward induction starting from the case where the incumbent, in period 0, chooses to sell the two products independently.

3.1 Independent sales

When the incumbent sells the two products independently, the continuation equilibrium of the game is the one described by the following Lemma:

Lemma 4. *Independent sales*
If the incumbent does not engage in tying, the continuation equilibrium of the game is such that E_S sells the secondary product both in period 1 and in period 2. Equilibrium prices and quantities are as follows:

$$p_{I_S}^{1*} = 0, \quad p_{E_S}^{1*} = \delta_S/2; \quad p_{I_P}^{1*} = U + v(2N) + \frac{\delta_S}{2}; \quad q_{I_S}^{1*} = 0, \quad q_{E_S}^{1*} = N$$

$$p_{I_S}^{2*} = 0, \quad p_{E_S}^{2*} = \delta_S + v(2N) - v(N), p_{I_P,2}^{*} = 0, \quad p_{E_P}^{2*} = \delta_P, \quad q_{I_S}^{2*} = 0 = q_{I_P,2}^{*}, \quad q_{E_S}^{2*} = N = q_{E_P}^{2*}$$

Equilibrium payoffs are given by:

$$\pi_I^{IS*} = N[U+v(2N)+\frac{\delta_S}{2}];\ \ \pi_{E_S}^{IS*} = N\frac{\delta_S}{2}+N[\delta_S+v(2N)-v(N)];\pi_{E_P}^{IS*} = N\delta_p;\ \ CS^{IS*} = N[U+v(N)].$$

Proof. See Appendix A.2. □

Lemma 4 states that, absent tying, the incumbent manages to extract part of E_S's efficiency advantage in the first period through the price of the primary product, but makes no profits in the second period when the more efficient producer of the primary product starts operating.

The intuition is the following. In the second period asymmetric Bertrand competition leads E_P, who offers a higher quality product, to sell to all the consumers setting a price equal to its efficiency advantage δ_P.

In the secondary market, the existence of network externalities implies that the choice of period-1 consumers determines which firm offers the higher utility product in period 2. In particular, if all period-1 consumers chose the incumbent's product, then in the second period the network effect would outweigh the incumbent's intrinsic quality disadvantage and, at equal prices, second-period consumers prefer to join the incumbent's network than that of the rival: $v(2N) - \delta_S > v(N)$ by assumption $(A1')$. At equilibrium, the entrant sets price equal to (zero) marginal cost and the incumbent sells to all period-2 consumers setting a price $p_{I_S}^{*2} = v(2N) - \delta_S - v(N)$. Conversely, if all period-1 consumers chose the rival's product, in period 2 it is E_S that will offer the higher utility product and will capture the whole market setting the equilibrium price $p_{E_S}^{*2} = v(2N) + \delta_S - v(N)$.

In period 1 firms anticipate that who sells to the first cohort of consumers will also sell to the second one. Hence, the total network effect is the same irrespective of the firm that captures period-1 consumers; however the incumbent has to compensate for its intrinsic quality gap and has to discount by the amount δ_S the rival's price so as to secure the first cohort of consumers. Moreover, in period 1 the incumbent is the unique seller of the primary product and, given the price paid by consumers for the secondary product, it can extract through the price of the primary product the remaining surplus left to consumers.

Consider, then, the following candidate equilibrium prices: $p_{E_S}^1 = \delta_S$, $p_{I_S}^1 = 0$ and $p_{I_P}^1 = U + v(2N)$. If the incumbent deviated and offered a discount slightly larger the δ_S, i.e. $p_{I_S}^{1'} = -\varepsilon$, it would secure both cohorts of consumers and it would earn $(U + v(2N))N$ in period 1, and $(v(2N) - v(N) - \delta_S)N$ in period 2. If not, it would lose sales in the second period, but it could still use the price of the primary product to extract consumers' surplus is period 1 earning $(U + v(2N) + \delta_S - p_{E_S}^1 = (U + v(2N) + \delta_S - \delta_S)N$ in period 1. When $p_{E_S}^1 = \delta_S$, the deviation would be profitable, because the rival's price is sufficiently high and the rents extracted from period-1 consumers would be insufficient to compensate for the ones lost in period 2. However, the *non-negative price-constraint* bites and the incumbent cannot engage in such a deviation. Therefore, the proposed one is an equilibrium.

There exist other equilibria in which the incumbent sets a higher price for the primary product and extracts some (or all) of the rival's efficiency advantage: $p_{E_S}^{*1} = \alpha\delta_S$, $p_{I_S}^{*1} = 0$ and $p_{I_P}^{*1} = U + v(2N) + (1 - \alpha)\delta_S$, with $\alpha \in [0, 1]$. Consistently with the analysis of Section 2, Lemma 4 indicates the equilibrium in which the incumbent appropriates half of the rival's efficiency advantage in period 1.

3.2 Tying

In this Section we consider the case in which I engages in *tying*: I_P is sold exclusively together with I_S, whereas I_S can also be sold independently. Even thought the proof is slightly involved (see the Appendix), the intuition of the continuation equilibrium is simple.

Tying forces period-1 consumers who buy the primary product from the incumbent to buy also the secondary product from it. Single-homers have no other choice, whereas multi-homers can decide whether or not to add the rival's secondary product to the bundle, potentially affecting the choice of period-2 consumers through the network effects. Indeed, if their proportion is sufficiently high, multi-homers anticipate that by adding the rival's secondary product they will also induce second-period consumers to choose E_S: the network that they create together is large enough to make the rival's secondary product gain superior quality. Hence, multi-homers add E_S to the bundle in period 1. The incumbent extracts from the sales to multi-homers part of the rival's efficiency rents in period 1 (through the price of the bundle), but it will not sell any product in period 2, when also the more efficient rival in the primary market will be active.[20] Conversely, in period 1, multi-homers will not add the rival's secondary product when their proportion is low enough. In this case it is the incumbent's secondary product that turns out to have superior quality. The incumbent sells the bundle to all period 1 consumers and sells I_S to period-2 consumers who will combine it with E_P. The following Lemma reports total equilibrium payoffs in the two cases.

Lemma 5. ***Equilibrium payoffs if the incumbent does engage in tying.***
There exists a threshold level of the proportion of multi-homers $\beta^* \in (0,1)$ such that:

(i) If $\beta < \beta^*$, the incumbent sells the bundle to period-1 consumers (and multi-homers do not add E_S's secondary product) and sells its secondary product to period-2 consumers. Equilibrium payoffs are given by:

$$\pi_I^{TOT*} = N[U+v(2N)]+N[v(2N)-v(N)-\delta_S],\ \ \pi_{E_S}^{t*} = 0;\ \ \pi_{E_P}^{t*} = N\delta_P;\ \ CS^{t*} = N[U+\delta_S+v(N)]$$

(ii) If $\beta \geq \beta^*$, the incumbent sells the bundle to period-1 consumers (and multi-homers do add E_S's secondary product), while it does not sells in period 2 when consumers choose the combination E_P, E_S. Equilibrium payoffs are given by:

$$\begin{aligned}
\pi_I^{TOT*} &= (1-\beta)N[U+v((1-\beta)N)] + \beta N[U + \frac{1}{2}v(2N) + \frac{1}{2}(\delta_S + v(\beta N + N))] \\
\pi_{E_S}^{TOT*} &= \begin{cases} \beta N\frac{1}{2}[\delta_S + v(\beta N + N) - v(2N)] + N\frac{1}{2}[\delta_S + v(\beta N + N) - v(2N)] & if \quad \beta \in [\beta^*, \widehat{\beta}] \\ \beta N\frac{1}{2}[\delta_S + v(\beta N + N) - v(2N)] + N\frac{1}{2}[\delta_S + v(\beta N + N) - v(2N)] + N\frac{1}{2}\widehat{p}_{E,S}(\beta) & if \quad \beta > \widehat{\beta} \end{cases} \\
\pi_{E_P}^{TOT*} &= N\delta_P;\ \ CS^{t*} = N[U+v(2N)]
\end{aligned}$$

Proof. See Appendix A.2 □

[20] As we did in Section 2.2.1, we allow the incumbent to discriminate the price between single-homers and multi-homers. Price discrimination increases the profits that the incumbent earns when it engages in tying. Nonetheless, as we will show in Proposition 4, when the proportion of multi-homers is large enough the incumbent's optimal choice is not to engage in tying.

3.3 The tying decision

We can now proceed to the initial stage of the game, where the incumbent decides whether to engage in tying or not. When the proportion of multi-homers is sufficiently small (i.e. when $\beta < \beta^*$), the incumbent faces a trade-off. Tying, by forcing all period-1 consumers to buy the incumbent's secondary product, will prevent the rival from gaining sufficient network size, thereby inducing also period-2 consumers to buy I_S and allowing the incumbent to make positive profits in period 2. Instead, absent tying both cohorts of consumers would buy E_S's secondary product and the incumbent would make zero profits in period 2. However, tying sacrifices profits in period 1, because it prevents the incumbent from extracting part of the rival's quality advantage δ_s. The increase in period-2 profits prevails when the rival's efficiency advantage is small enough, as stated in Proposition 4.

Instead, when the proportion of multi-homers is sufficiently large (i.e. when $\beta \geq \beta^*$) engaging in tying is not optimal for the incumbent. The reason is that forcing period-1 consumers to purchase the incumbent's secondary product does not prevent the rival from gaining sufficient network size because period-2 consumers and period-1 multi-homers still purchase the rival's secondary product. As a consequence, irrespective of tying the incumbent will not sell in period 2. On top of that, with tying the incumbent sacrifices profits in period 1, because it sets a lower price in period 1 both for single-homers and multi-homers than in the case of independent sales. The willingness to pay of single-homers is small because they are stuck with the low-quality network product; moreover, under tying the quality advantage that the incumbent extracts from multi-homers is lower than absent tying. The reason is that under tying the incumbent extracts the quality advantage of E_S's network product when multi-homers together with single-homing consumers join E_S's network relative to the case in which both cohorts of consumers join the incumbent's network; instead without tying the incumbent extracts the full quality advantage of E_S's network product when both cohorts of consumers join E_S's rather than the incumbent's network.

Proposition 4 summarises the results of the analysis:

Proposition 4. *The incumbent engages in tying if (and only if) $\beta < \beta^*$ and $\delta_s < \frac{2}{3}(v(2N) - v(N))$.*

Proof. Follows immediately from the comparison of the incumbent's profits in Lemma 4 and 5. □

Note that in the model with network externalities in period 2 E_S is in the market and exerts competitive pressure even though the incumbent engages in tying. Then, contrary to the fixed costs model, the incumbent is not able to extract E_P's efficiency advantage in period 2.

The role of the non-negative price constraints. Consider the case in which the incumbent engages in independent sales. Absent the non-negative price constraint, the price configuration $p^1_{E_S} = \delta_S$, $p^1_{I_S} = 0$ and $p^1_{I_P} = U + v(2N)$ discussed in Section 3.1 would not be an equilibrium: as mentioned above, the incumbent would have an incentive to deviate, slightly decreasing the price of the secondary product. Indeed, the incumbent would have an incentive to deviate whenever the rival's price is sufficiently high, i.e. $p^1_{E_S} \geq \widehat{p}^1_{E_S}$, where $\widehat{p}^1_{E_S} < \delta_S$ is the rival's price such that:

$$(U + v(2N))N + (v(2N) - v(n) - \delta_S)N = (U + v(2N) + \delta_S - p^1_{E_S})$$

Instead, for any price $p^1_{E_S} < \widehat{p}^1_{E_S}$, for the incumbent it would be more profitable to let the rival sell to both cohorts of customers. Equilibria would involve $p^{*1}_{E_S} \in [-\delta_S - (v(2n) - v(N)), \widehat{p}^1_{E_S}]$. Note that in some of these equilibria firm E_S would set a negative price for the secondary product and the incumbent, through the price of the primary product, would appropriate not only the rival's efficiency rents in the first period, but also some (or all) of the rents that the rival can produce in period 2. These equilibria cannot arise when negative prices are unfeasible.

Note also that all the equilibria that would arise when firms can set negative prices would entail incumbent's profits that are higher than $(U + v(2N))N + (v(2N) - v(n) - \delta_S)N$. As shown in this Section, these are the highest profits that the incumbent can make by engaging in tying. Therefore, when negative prices are feasible, independent sales are always more profitable than tying.

Instead, when firms cannot set negative prices tying may turn out to be profitable. The non-negative price constraint softens price competition under independent sales, thereby limiting the efficiency rents that the incumbent manages to extract through the price of the primary product, and creating the incentive to exclude. (See also Choi and Jeon, 2019).

4 Conclusions

In this paper, we have extended Carlton and Waldman (2002)'s analysis of exclusionary tying in dynamic industries by showing that a monopolistic incumbent may have an incentive to tie its complementary products so as to deter entry in a secondary product, even when its dominant position in the primary market cannot be protected. Indeed, by engaging in tying, the incumbent sacrifices current profits, but it can exclude a more efficient rival from a complementary market by depriving it of the critical scale it needs to be successful (economies of scale may arise on the supply-side, like in our base model, or on the demand-side, like in our extension with network effects). In turn, monopolising the complementary market allows the incumbent to be in a favorable position when a more efficient rival will enter the primary market, allowing it to appropriate some of the latter's efficiency rents.

We have also showed that tying is a more profitable exclusionary strategy than pure bundling, and that exclusion is the more likely the higher the proportion of consumers who single-home (that is, who will not buy another complementary product if the complementary product of the incumbent is already sold to them as a bundle).

In the paper the efficiency gap between the incumbent and the rivals is exogenously given. In a possible extension of the model, one could study the incumbent's incentives to invest in the quality of its products or in the efficiency of its production processes. If the incumbent expects that other market participants are better at creating efficiencies in the primary market, it will have no incentive to invest in that market. Rather, it will prefer to act so as to dominate the complementary market – by investing in the efficiency of its own secondary product or by engaging in exclusionary practices (or both) – and be in the position to partially appropriate those efficiencies, when new rivals will enter the primary market. Analogously, if the incumbent expects rivals to better equipped to produce efficiencies in the secondary market, it will have the incentive to focus its innovation effort in the primary market, so as to make its dominant position in that market persist and be able to extract the efficiencies that new entry brings into the secondary market.

References

[1] Bernheim, B.D., and M.D. Whinston. 1998. "Exclusive Dealing", *Journal of Political Economy.* 106: 64-103.

[2] Carlton, D.W. and M. Waldman. 2002. "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries." *Rand Journal of Economics.* 33: 194-220.

[3] Carlton, D.W. and M. Waldman. 2012. "Upgrades, Switching Costs and the Leverage Theory of Tying", *Economic Journal.* 122:675-726.

[4] Choi, J. P. 1996. "Preemptive R&D, Rent Dissipation, and the Leverage Theory.", *Quarterly Journal of Economics.* 110: 1153-81.

[5] Choi, J. P. 1996. "Tying and Innovation: A Dynamic Analysis of Tying Arrangements", *Economic Journal.* 114: 83-101.

[6] Choi, J. P. and C. Stefanadis. 2001. "Tying, Investment and the Dynamic Leverage Theory." *Rand Journal of Economics.* 32: 52-71.

[7] Choi, J. P. and C. Stefanadis. 2006. "Bundling, Entry Deterrence and Specialist Innovators." *Journal of Business.* 79: 2575-2594.

[8] Choi, J. P. and D-S. Jeon. 2020. "A Leverage Theory of Tying in Two-Sided Markets with Non-Negative Price Constraints." *American Economic Journal: Microeconomics*, *forthcoming.*

[9] Fumagalli, C. and M. Motta. 2018. "Dynamic vertical foreclosure". CEPR DP No. 12498.

[10] Fumagalli, C., M. Motta and C. Calcagno. 2018. *Exclusionary Practices.* Cambridge University Press.

[11] Grenlee, P., D. Reitman and D.S. Sibley. 2008. "An antitrust analysis of bundled loyalty discounts." *International Journal of Industrial Organization.* 26: 1132-52.

[12] Hurkens, S., D.-S. Jeon and D. Menicucci. 2016. "Dominance and Competitive Bundling." *American Economic Journal: Microeconomics*, 11 (3): 1-33.

[13] Katz, M. L. and C. Shapiro. 1986. "Technology Adoption in the Presence of Network Externalities." *Journal of Political Economy.* 94: 822-41.

[14] Whinston, M.D. 1990. "Tying, Foreclosure and Exclusion." *American Economic Review.* 80: 837-59.

A Appendix

A.1 Proofs of the baseline model

Proof of Lemma 1

Proof. Let us proceed by backward induction starting from period 2.

Period 2

We need to distinguish two cases, one where E_S entered the secondary market in period 1; the other where it did not.

E_S **entered the secondary market in period** 1

If E_P materialises and enters the primary market in period 2, then each product is sold by two competing firms. Bertrand competition leads to $p^*_{E,P} = \delta_P$, $p^*_{I,P} = 0$ in the primary market and to $p^*_{E,S} = \delta_S$, $p^*_{I,S} = 0$ in the secondary market, with the entrants selling in each market.[21]

The agents' post-entry payoffs in the second period, gross of the entry costs, are:

$$\pi^{*2}_{I,2} = 0; \quad \pi^{*2}_{E_S} = \delta_S; \quad \pi^{*2}_{E_P} = \delta_P; \quad CS^*_2 = U - \delta_P - \delta_S \tag{11}$$

Since $f_P \leq \frac{\delta_P}{2}$ by assumption, E_P enters the primary market in period 2 if it materialises and if E_S entered the secondary one in period 1.

If E_P does not materialise, the incumbent will monopolise the primary market also in period 2. Since the incumbent's products are sold independently, consumers can freely combine the incumbent's primary product with the secondary of either the incumbent or the entrant. There are multiple equilibria in the price game: in the secondary market $p^*_{I,S} = 0, p^*_{E,S} = (1-\alpha)\delta_S$, with $\alpha \in [0, 1)$, and with E_S selling to consumers; in the primary market the incumbent sets the price $p^*_{I,P} = U + \delta_S - (1-\alpha)\delta_S = U + \alpha\delta_S$. Given $p^*_{I,P}$, E_S cannot increase its profits by increasing $p_{E,S}$ because the total price of the system would exceed $U + \delta_S$ and consumers would not buy either product. Similarly, given $p^*_{E,S}$ the incumbent cannot increase its profits by increasing $p_{I,P}$. The incumbent cannot increase its profits either by decreasing $p_{I,S}$. Note that E_S would fully appropriate its efficiency advantage δ_S only when $\alpha = 0$, with the equilibrium price in the secondary market being δ_S. Instead, when $\alpha > 0$ the entrant's margin in the secondary market is squeezed, which allows the incumbent to set a higher price for the primary product, thereby extracting a share α of the increase in buyers' surplus generated by the activity of the more efficient rival. Without loss of generality, we follow the assumption made in Carlton and Waldman (2002) that $\alpha = 1/2$, that is, $p^*_{I,S} = 0, p^*_{E,S} = \delta_S/2$ and $p^*_{I,S} = U + \delta_S/2$.

The agents' post-entry payoffs in the second period, gross of the entry costs, are:

$$\pi^{*2}_{I,2} = U + \delta_S/2; \quad \pi^{*2}_{E_S} = \frac{\delta_S}{2}; \quad CS^*_2 = 0 \tag{12}$$

E_S **did not enter the secondary market in period** 1

Consider the case in which E_P materialises and expects that E_S does not enter the secondary market in period 2 either. In this case the incumbent monopolises the secondary market in the second period. Applying the above logic, and assuming that $\alpha = 1/2$, the incumbent manages to appropriate half of the rival's efficiency advantage: $p^*_{I,P} = 0, p^*_{E,P} = \delta_P/2$ and $p^*_{I,S} = U + \delta_P/2$. Consequently, if only E_P enters in period 2 the agents' post-entry payoffs in the second period are:

$$\pi^*_{I,2} = U + \delta_P/2; \quad \pi^*_{E_P,2} = \delta_P/2; \quad \pi^*_{E_S,2} = 0; \quad CS^*_2 = 0 \tag{13}$$

Since $f_P \leq \frac{\delta_P}{2}$ by assumption, E_P enters the primary market in period 2 if it materialises and if it expects E_S not to enter the secondary market in period 2.

Applying the analysis above we can conclude that E_P enters in period 2 if it materialises and expects E_S to enter the secondary market in period 2.

[21] Here and throughout the analysis we will disregard equilibria that would be eliminated under standard refinement criteria such as Pareto-dominance or trembling hands.

Hence, if E_P materialises, entering the market in period 2 is a dominant strategy. If the entry cost in the secondary market is low enough, i.e. if $f_S \leq \delta_S$, also E_S enters the market in period 2. If, instead, $f_S \in (\delta_S, \frac{\delta_S(2-p)}{2}]$, only E_P enters the market in period 2.

If E_P does not materialise, payoffs in the second period are given by (12). E_S enters the secondary market in period 2 if (and only if) $f_S \leq \frac{\delta_S}{2}$.

Period 1

Let us move to the first period. If E_S enters the market in period 1, then it will earn profits $\frac{\delta_S}{2}$ in period 1, when I monopolises the primary market. E_S expects to make the same profits also in period 2, when E_P does not materialise. Instead, when E_P materialises, E_S expects to earn profits δ_S in period 2, because E_P will decide to enter the primary market. Total post-entry profits $\pi^{*TOT}_{E_S} = \frac{\delta_S}{2} + p\delta_S + (1-p)\frac{\delta_S}{2}$ are sufficient to cover entry costs f_S by assumption $A2$.

If, instead, E_S decides not to enter in period 1, it will never enter the secondary market when entry costs are sufficiently large ($f_S > \delta_S$), it will enter in period 2 when entry costs are low ($f_S \leq \frac{\delta_S}{2}$), it will enter in period 2 only if E_P materialises for intermediate entry costs ($f_S \in (\frac{\delta_S}{2}, \delta_S]$). Entering the market in period 1 gives higher expected profits in all the cases. □

Proof of Lemma 2

Proof. Let us start from period 2.

Period 2

E_S entered the secondary market in period 1

Imagine that E_P materialises and enters the primary market in period 2. The incumbent sells the bundle of the two products at a price $\tilde{p_I}$, and the secondary product at the stand-alone price p_{I_s}. Consumers can then choose between buying the combinations (I_P, I_S), (E_P, E_S), or (E_P, I_S). Multi-homers can also add E_S to the bundle, therefore obtaining (I_P, I_S, E_S). The price equilibrium is such that the entrants appropriate their respective efficiency advantage:

$$\tilde{p_I}^{*,2} = 0; \quad p^{*,2}_{I_S} = 0; \quad p^{*,2}_{E_S} = \delta_S; \quad p^{*,2}_{E_P} = \delta_P \tag{14}$$

with all the consumers purchasing from the entrants. It is easy to show that the incumbent cannot profitably deviate changing $\tilde{p}_I$,p_{I_s} or both. Given $p^*_{E_P}$ and $\tilde{p_I}^*$, E_S would lose all of its customers if it increased p_{E_S}. Similarly, given $p^*_{E_S}$ and $\tilde{p_I}^*$, E_P would lose all of its customers if it increased p_{E_P}.

Note that equilibria in which E_S extracts more than δ_S do not exist. This is due to the incumbent's ability to sell the secondary product individually under tying. Consider a candidate equilibrium in which:

$$\tilde{p_I} = 0 = p_{I_S}; \quad p_{E_S} \in (\delta_S, \delta_S + \delta_P]; \quad p_{E_P} = \delta_P + \delta_S - p_{E_S} < \delta_P,$$

with the incumbent not selling. Consumers would enjoy utility U purchasing either the combination (E_P, E_S) or the bundle, whereas they would enjoy utility $U + \delta_P - p_{E_P} > U$ by purchasing the combination (E_P, I_S). As a consequence the incumbent would have an incentive to deviate and set $p^{'}_{I_S} = \delta_P - p_{E_P} - \varepsilon > 0$ for ε sufficiently low. It would attract users that combine its secondary product with the entrant's primary product and it would make positive profits. The deviation would be profitable.

Likewise equilibria in which E_P extracts more than δ_P do not exist. This is due to the existence of multi-homers that can add E_S to the bundle. Consider a candidate equilibrium in which:

$$\tilde{p_I} = 0 = p_{I_S}; \quad p_{E_P} \in (\delta_P, \delta_S + \delta_P]; \quad p_{E_S} = \delta_P + \delta_S - p_{E_P} < \delta_S,$$

with the incumbent not selling. Consumers would enjoy utility U by purchasing either the combination (E_P, E_S) or the bundle, and utility lower than U by purchasing the combination (E_P, I_S). However, multi-homers would enjoy utility $U + \delta_S - p_{E_S} > U$ by adding E_S to the incumbent's bundle (I_P, I_S). As a consequence the incumbent would have an incentive to deviate and set $\tilde{p_I}' = \delta_S - p_{E_S} - \varepsilon > 0$ for ε sufficiently low. It would attract multi-homers and it would make positive profits. The deviation would be profitable.

The post-entry payoffs in the second period are:

$$\pi_I^{*,2} = 0; \quad \pi_{E_S}^{*,2} = \delta_S; \quad \pi_{E_P}^{*,2} = \delta_P; \quad CS_2^* = U \tag{15}$$

Since $f_P \leq \frac{\delta_P}{2}$ by assumption, E_P enters the primary market in period 2 if it materialises and if E_S entered the secondary market in period 1.

Let us consider now the case in which E_P does not materialise. The incumbent monopolises the primary market also in period 2. Since the incumbent has engaged in tying, if a consumer buys I_P that consumer is also getting I_S. Then, E_S will not be able to sell to single-homers consumers; it will sell to multi-homers who are willing to add E_S to the incumbent's bundle as long as $p_{E_S} \leq \delta_S$ and $U + \delta_S - \tilde{p}_I - p_{E_S} \geq 0$. Since we allow the incumbent to discriminate the price of the bundle across multi-homing and single-homing consumers, it will extract all of the surplus U that single-homers enjoy when they purchase the bundle, and will extract part of E_S's efficiency gain through the price set for multi-homers: $\tilde{p}_I^{*L,1} = U$, $\tilde{p}_I^{*H,1} = U + \alpha\delta_S$, $p_{E_S}^{*,1} = (1-\alpha)\delta_S$.

The post-entry payoffs in the second period (focusing on the case in which $\alpha = 1/2$) are:

$$\pi_I^{*,2} = (1-\beta)U + \beta(U + \frac{\delta_S}{2}); \quad \pi_{E_S}^{*,2} = \beta\frac{\delta_S}{2}; \quad CS_2^* = 0 \tag{16}$$

E_S did not enter the secondary market in period 1

Consider the case in which E_P materialises and expects E_S not to enter the secondary market in period 2 either. Tying does not prevent the incumbent from selling the secondary product, that it monopolises, on a stand-alone basis. Then consumers can combine the incumbent's secondary product with the higher quality primary product of the entrant. This allows the incumbent to extract half of the entrant's efficiency advantage in the primary market: $\tilde{p_I}^* = U$, $p_{E_P}^* = \frac{\delta_P}{2}$, $p_{I_S}^* = U + \frac{\delta_P}{2}$. Consequently, if E_P materialises and enters the primary market in period 2 the agents' post-entry payoffs in the second period are:[22]

$$\pi_I^{*,2} = U + \frac{\delta_P}{2}; \quad \pi_{E_P}^{*,2} = \frac{\delta_P}{2}; \quad CS_2^* = 0. \tag{17}$$

Since $f_P \leq \frac{\delta_P}{2}$ by assumption $A1$, E_P enters the primary market in period 2 if it materialises and

[22]Consider the following candidate equilibrium: $\tilde{p_I} = U$, $p_{E_P} = (1-\alpha)(U + \delta_P)$, $p_{I_S} = \alpha(U + \delta_P)$. The incumbent could deviate and set $p_{I_S}' > \alpha(U + \delta_P)$. Consumers would buy the bundle from the incumbent whose profits would amount to U. The deviation profit is larger than the candidate equilibrium profit as long as $U > \alpha(U + \delta_P)$. When $\alpha = 1/2$ this inequality becomes $U > \delta_P$ which holds by assumption. Then, the proposed one is not an equilibrium.

if it expects E_S not to enter the secondary market in period 2.

Assume now that E_P materialises and expects E_S to enter the secondary market in period 2. By applying the analysis above one can conclude that E_P enters in period 2.

Hence, if E_P materialises, entering the primary market in period 2 is a dominant strategy.

As a consequence, if E_P materialises and the entry cost in the secondary market is low enough, i.e. if $f_S \leq \delta_S$, both E_S and E_P enter the market in period 2. If, instead, $f_S \in (\delta_S, \frac{\delta_S(2-p)}{2}]$, only E_P enters the market in period 2.

If E_P does not materialise, payoffs in the second period are given by (16). E_S enters the secondary market in period 2 if (and only if) $f_S \leq \beta\frac{\delta_S}{2}$.

Period 1

In the first period E_P is not on sale and, as discussed above, under tying E_S will sell only to multi-homers if it enters the market in period 1. The incumbent will extract all of the surplus U that single-homers enjoy when they purchase the bundle, and will extract half of E_S's efficiency gain through the price set for multi-homers: $\tilde{p}_I^{*L,1} = U$, $\tilde{p}_I^{*H,1} = U + \frac{\delta_S}{2}$, $p_{E_S}^{*,1} = \frac{\delta_S}{2}$.

Hence, if it enters the secondary market in period 1, firm E_S will earn profits $\beta\frac{\delta_S}{2}$ in period 1. E_S expects to make the same profits also in period 2, when E_P does not materialise. Instead, when E_P materialises, E_S expects to earn profits δ_S in period 2, because E_P will enter the primary market. Total post-entry profits are given by $\pi_{E_S}^{*TOT} = \beta\frac{\delta_S}{2} + p\delta_S + (1-p)\beta\frac{\delta_S}{2}$. If $f_S \leq p\delta_S$ firm E_S enters the secondary market in period 1 for any $\beta \in [0, 1]$. (When $\beta = 0$, it is indifferent between entering in period 1 or 2.) If, instead, $f_S > p\delta_S$, there exists a threshold level of multi-homing consumers $\beta^*(f_S)$, with $\beta^*(f_S) \in (0, 1]$ and increasing in f_S such that firm E_S enters E_S in period 1 if (and only if) $\beta \geq \beta^*(f_S)$. (Recall that when $\beta = 1$, $\pi_{E_S}^{*TOT} = \frac{\delta_S}{2} + p\delta_S + (1-p)\frac{\delta_S}{2} \geq f_S$ by assumption $A2$, whereas when $\beta = 0$, $\pi_{E_S}^{*TOT} = p\delta_S < f_S$.) □

<u>Proof of Lemma 3</u>

Proof. **Period 2**

E_S entered the secondary market in period 1

Imagine that E_P materialises and enters the primary market in period 2. The incumbent sells only the bundle of the two products at a price $\tilde{p_I}$. Consumers can then choose between the combinations (I_P, I_S) or (E_P, E_S). Multi-homers can also add E_S to the bundle, therefore obtaining (I_P, I_S, E_S). One can check that the price configuration such that the entrants appropriate their respective efficiency advantage is an equilibrium:

$$\tilde{p_I}^{*,2} = 0; \quad p_{E_S}^{*,2} = \delta_S; \quad p_{E_P}^{*,2} = \delta_P \tag{18}$$

with all the consumers purchasing from the entrants.

However, differently from the case of tying, equilibria in which E_S extracts more than δ_S do exist:

$$\tilde{p_I}^{*,2} = 0; \quad p_{E_S}^{*,2} \in (\delta_S, \delta_S + \delta_P]; \quad p_{E_P}^{*,2} = \delta_P + \delta_S - p_{E_S} < \delta_P,$$

Since the incumbent does not sell the secondary product on a stand-alone basis, it cannot use the price of the secondary product to profitably deviate from the equilibrium. Among these equilibria we select the one in which E_S appropriates half of E_P's efficiency advantage, in line with the assumption made so far with respect to the share of rents going to I when there is only the secondary product entrant.

Instead, equilibria in which E_P extracts more than δ_P do not exist. Similarly to what we already discussed for the case of tying, the existence of multi-homing consumers that can add E_S to the bundle creates the scope for profitable deviations.

The post-entry payoffs in the second period are:

$$\pi_I^{*,2} = 0; \quad \pi_{E_S}^{*,2} = \delta_S + \frac{\delta_P}{2}; \quad \pi_{E_P}^{*,2} = \frac{\delta_P}{2}; \quad CS_2^* = U \tag{19}$$

Since $f_P \leq \frac{\delta_P}{2}$ by assumption, E_P enters the primary market in period 2 if it materialises and if E_S entered the secondary market in period 1.

If, instead, E_P does not materialise, the post-entry payoffs in period 2 are the same as in the case of tying:

$$\pi_I^{*,2} = (1-\beta)U + \beta(U + \frac{\delta_S}{2}); \quad \pi_{E_S}^{*,2} = \beta\frac{\delta_S}{2}; \quad CS_2^* = 0 \tag{20}$$

E_S did not enter the secondary market in period 1

Consider the case in which E_P materialises and it is the only firm to enter the market in period 2. Pure bundling, differently from tying, implies that E_P will not be able to sell. Then if E_P only enters in period 2 the incumbent will extract the entire consumers' surplus through the price of the bundle and the agents' post-entry payoffs in the second period are:

$$\pi_I^{*,2} = U; \quad \pi_{E_P}^{*,2} = 0; \quad CS_2^* = 0 \tag{21}$$

Since $f_P > 0$ by assumption, E_P does not enter the primary market if it expects E_S not to enter the secondary market in period 2.

Consider now the case in which E_S only enters the (secondary) market in period 2. The agents' post-entry payoffs are the ones indicated in (20).

Finally, if both E_S and E_P enter the market in period 2, post-entry profits are the ones indicated in (15).

Entry in period 2, when E_P materialises and E_S did not enter the secondary market in period 1, depends on the size of the entry cost in the secondary market:

- If $f_S \leq \beta\frac{\delta_S}{2}$, entry in period 2 is a dominant strategy for firm E_S. Since E_P expects E_S to enter for sure, it will also enter the market in period 2 (recall that $f_P \leq \frac{\delta_P}{2}$ by assumption).

- If $f_S \in (\beta\frac{\delta_S}{2}, \delta_S + \frac{\delta_P}{2}]$, each firm finds it profitable to enter the market if it expects the entrant in the complementary market to take the same decision. In this case there exists multiple equilibria: either both E_S and E_P enter the market in period 2 or none of them does.

- If $f_S > \delta_S + \frac{\delta_P}{2}$, for firm E_S not entering the market is a dominant strategy. Expecting E_S not to enter, also E_P finds it optimal not to enter. There is a unique equilibrium with no firm entering the market in period 2.

Consider now the case in which E_P does not materialise. The payoffs in period 2 are given by (20). E_S enters the secondary market in period 2 if (and only if) $f_S \leq \beta\frac{\delta_S}{2}$.

Period 1

In the first period E_P is not on sale. Equilibrium prices in period 1 in the case in which E_S enters the market are given by: $\tilde{p}_I^{*L,1} = U$, $\tilde{p}_I^{*H,1} = U + \frac{\delta_S}{2}$, $p_{E_S}^{*,1} = \frac{\delta_S}{2}$.

Hence, if E_S enters the secondary market in period 1, it earns profits $\beta\frac{\delta_S}{2}$ by selling to multi-homers in period 1 and in period 2 if E_P does not materialise; moreover it earns $\delta_S + \frac{\delta_P}{2}$ in period 2, when also E_P will enter the (primary) market. Entry in the secondary market occurs in period 1 if total post-entry profits $\pi_{E_S}^{*TOT} = \beta\frac{\delta_S}{2} + p(\delta_S + \frac{\delta_P}{2}) + (1-p)\beta\frac{\delta_S}{2}$ cover the entry cost f_S.

It is easy to see that if $f_S \leq p(\delta_S + \frac{\delta_P}{2})$, firm E_S enters the secondary market in period 1 for any $\beta \in [0,1]$.[23] If, instead, $f_S > p(\delta_S + \frac{\delta_P}{2})$, there exists a threshold level of multi-homers $\beta^{*b}(f_S)$, with $\beta^{*b}(f_S) \in (0,1]$ and increasing in f_S such that firm E_S finds it profitable to enter E_S in period 1 if (and only if) $\beta \geq \beta^{*b}(f_S)$. (Recall that when $\beta = 1$, $\pi_{E_S}^{*TOT} = p\frac{\delta_S}{2} + \delta_S + p\frac{\delta_P}{2} \geq f_S$ by assumption $(A2)$, whereas when $\beta = 0$, $\pi_{E_S}^{*TOT} = p(\delta_S + \frac{\delta_P}{2}) < f_S$). Since $\pi_{E_S}^{*TOT\ Bundling} > \pi_{E_S}^{*TOT\ tying}$, $\beta^{*b}(f_S) < \beta^{*}(f_S)$.

□

Proof of Proposition 3.

Proof. When the entrants are integrated equilibrium prices in the various market structures are the same as in the case of separated entrants. Here, therefore, we analyse the entry decisions, starting from the case in which the incumbent sells the two products separately.

Independent sales

Period 2

E_S entered the secondary market in period 1

If E_P materialises, and enters market P, total post-entry profits in period 2 of firm E are given by $\pi_E^2 = \pi_{E_S}^2 + \pi_{E_P}^2 = \delta_S + \delta_P$ (see (11)) gross of the entry costs. If E_P does not enter, $\pi_E^2 = \frac{\delta_S}{2}$ (see (12)). Hence, firm E decides to enter the primary market in period 2 if (and only if) $f_P \leq \delta_P + \frac{\delta_S}{2}$. Note that, when deciding whether to enter the primary market in period 2, firm E takes into account the increase in profits in the secondary market due to lack of rents extraction by the incumbent. Then the upper bound of the entry cost f_P such that entry occurs in the primary market is higher than in the case of separated entrants. Since $f_P \leq \frac{\delta_P}{2}$ by assumption $(A1)$, E_P enters the primary market in period 2 if it materialises and if E_S entered the secondary market in period 1.

E_S did not enter the secondary market in period 1

Consider the case in which E_P materialises. Firm E must decide whether to enter both markets, earning $\pi_E^2 = \delta_S + \delta_P$ post-entry, gross of the entry costs (from (11)); to enter only market $i = P, S$, earning $\pi_E^2 = \frac{\delta_i}{2}$ (from (13) and (12)); not to enter at all, earning 0 profits.

Since $f_P \leq \frac{\delta_P}{2}$ by assumption $(A1)$, E's entry in P alone is always profitable. Firm E enters also the secondary market if (and only if) $f_S \leq \delta_S + \frac{\delta_P}{2}$.

[23] Note that, differently from the case of tying, when $\beta = 0$ firm E_S strictly prefers to enter the secondary market in period 1 over entry in period 2. In both cases E_S does not collect any revenue in period 1, but entering earlier avoids coordination failures in period 2 that might arise when the two entrants take their entry decisions simultaneously.

If E_P does not materialise, payoffs in the second period are given by (12). E_S enters the secondary market in period 2 if (and only if) $f_S \leq \frac{\delta_S}{2}$.

Period 1

If E enters the secondary market in period 1, then it also enters market P in period 2 (if E_P materialises) and it earns expected total profits $\pi_E^{*TOT} = \frac{\delta_S}{2} + p(\delta_S + \delta_P) + (1-p)\frac{\delta_S}{2}$ (gross of the entry costs).

If it does not enter the secondary market in either period then it will enter market P in period 2 (if E_P materialises) and its expected total profits amount to $\pi_E^{*TOT} = p\frac{\delta_P}{2}$.

Firm E decides to enter the secondary market in period 1 if (and only if) $f_S \leq \frac{\delta_S}{2} + p(\delta_S + \frac{\delta_P}{2} + (1-p)\frac{\delta_S}{2})$, which is always satisfied by assumption ($A2$). Also in this case the upper bound of the entry cost f_S such that entry occurs in the secondary market is period 1 is higher than in the case of separated entrants. This is because firm E takes into account that, by entering the secondary market, it will also increase the profits that it will earn in the primary market in period 2 by preventing the incumbent from extracting efficiency rents from E_P.

In sum, the results of Lemma 1 are confirmed when the entrants belong to the same company.

Next, let us consider the case in which the incumbent engages in tying.

Tying

Period 2

E_S **entered the secondary market in period** 1

Imagine that E_P materialises and enters the primary market in period 2. The post-entry payoffs in the second period are given by (15) and firm E's total post-entry profit is $\pi_E^2 = \delta_S + \delta_P$. If E_P does not enter, $\pi_E^2 = \beta\frac{\delta_S}{2}$ (see (16)). Hence, firm E decides to enter the primary market in period 2 if (and only if) $f_P \leq \delta_P + \delta_S(1 - \frac{\beta}{2})$. Note that under tying firm E's incentive to enter market P in the second period is stronger than under independent sales, because absent entry in P firm E sells the secondary product only to multi-homers. Since $f_P \leq \frac{\delta_P}{2}$ by assumption $A1$, E_P enters the primary market in period 2 if it materialises and if E_S entered the secondary market in period 1.

E_S **did not enter the secondary market in period** 1

Consider the case in which E_P materialises. Firm E must decide whether to enter both markets, earning $\pi_E^2 = \delta_S + \delta_P$ post-entry ross of the entry costs (see (15)); to enter only market P, earning $\pi_E^2 = \frac{\delta_P}{2}$ (see (17)); to enter only market S earning $\pi_E^2 = \beta\frac{\delta_S}{2}$ (see (16)); not to enter at all, earning 0 profits.

Since $f_P \leq \frac{\delta_P}{2}$ by assumption $A1$, E's entry in P alone is always profitable. Firm E enters also the secondary market if (and only if) $f_S \leq \delta_S + \frac{\delta_P}{2}$.

If E_P does not materialise, payoffs in the second period are given by (16). E_S enters the secondary market in period 2 if (and only if) $f_S \leq \beta\frac{\delta_S}{2}$.

Period 1

If firm E enters the secondary market in period 1 then it also enters market P in period 2 (if E_P materialises) and it earns expected total profits $\pi_E^{*TOT} = \beta\frac{\delta_S}{2} + p(\delta_S + \delta_P) + (1-p)\beta\frac{\delta_S}{2}$ (gross of the entry costs).

If it does not enter the secondary market in either period then it will enter market P in period 2 (if E_P materialises) and its expected total profits amount to $\pi_E^{*TOT} = p\frac{\delta_P}{2}$.

Firm E decides to enter the secondary market in period 1 if (and only if) $f_S \leq \beta\frac{\delta_S}{2} + p(\delta_S + \frac{\delta_P}{2} + (1-p)\beta\frac{\delta_S}{2})$. It is easy to see that if $f_S \leq p(\delta_S + \frac{\delta_P}{2})$ firm E_S enters the secondary market in period 1 for any $\beta \in [0,1]$. (When $\beta = 0$, it is indifferent between entering in period 1 or 2.) If, instead, $f_S > p(\delta_S + \frac{\delta_P}{2})$, there exists a threshold level of multi-homing consumers $\beta^{*i}(f_S)$, with $\beta^{*i}(f_S) \in (0,1]$ and increasing in f_S such that firm E_S enters E_S in period 1 if (and only if) $\beta \geq \beta^{*i}(f_S)$.

Precisely because firm E internalises that entry in S increases the profits earned in market P, the lower bound of the entry cost f_S such that entry in S is discouraged is higher than in the case of separated entrants, and the upper bound of the share of multi-homers is smaller.

The continuation equilibria following the incumbent's decision at period 0 is summarised by the following Lemma:

Lemma 6. *If E_S and E_P belong to the same company, when the incumbent engages in tying the continuation equilibrium of the game is as follows:*

> *(i) If either the entry cost in the secondary market is sufficiently low, i.e. $f_S \leq p(\delta_S + \frac{\delta_P}{2})$, or $f_S > p(\delta_S + \frac{\delta_P}{2})$ and the share of multi-homers is sufficiently large, i.e. $\beta \geq \beta^{*i}(f_S)$, then entry by E_S takes place in the first period, and entry by E_P follows in the second period (if E_P materialises). Equilibrium total payoffs are as follows:*

$$\pi_I^* = (U+\beta\frac{\delta_S}{2})(2-p); \quad \pi_{E_S}^* = \beta\frac{\delta_S}{2} + p\delta_S + (1-p)\beta\frac{\delta_S}{2} - f_S; \quad \pi_{E_P}^* = p(\delta_P - f_P) \quad CS^* = pU \tag{22}$$

> *(ii) If the entry cost in the secondary market is high enough, i.e. $f_S > p(\delta_S + \frac{\delta_P}{2})$ and the share of multi-homers is sufficiently low, i.e. $\beta < \beta^{*i}(f_S)$, firm E_S does not enter the secondary market. Firm E_P enters the primary market in period 2 if it materialises. In this case equilibrium total payoffs are as follows:*

$$\pi_I^* = U + U + p\frac{\delta_P}{2}; \quad \pi_{E_S}^* = 0; \quad \pi_{E_P}^* = p(\frac{\delta_P}{2} - f_P) \quad CS^* = 0 \tag{23}$$

> *The threshold $\beta^{*i}(f_S) \in (0,1]$ and is increasing in f_S and $\beta^{*i}(f_S) < \beta^*(f_S)$.*

□

A.2 Proofs of the model with network externalities

<u>Proof of Lemma 4</u>

Proof. **Period** 2

In period 2 E_P will also be active in the primary market. Bertrand competition between I_P and E_P will drive equilibrium prices of the primary product down to:

$$p^*_{I_P,2} = 0, \quad p^*_{E_P,2} = \delta_P, \tag{24}$$

with all consumers buying the rival's primary product (i.e. $q^*_{E_P,2} = N, q^*_{I_P,2} = 0$). As for the secondary product, the existence of network externalities implies that consumers' choice will also depend on the choice of consumers in the first period. Define M_E and M_I as the number of consumers who in $t = 1$ bought the secondary product from E_S and I respectively. Then, the period-2 consumers will prefer to buy E_S over I_S if $V(E_P, E_S) - p_{E_S,2} \geq V(E_P, I_S) - p_{I_S,2}$ that can be written as (recall that they will coordinate on the best outcome):

$$\delta_S + v(M_E + N) - p_{E_S,2} \geq v(M_I + N) - p_{I_S,2}$$

Firm E_S wins competition for period-2 consumers if its intrinsic advantage combined with the size of its network makes the quality of its product $\delta_S + v(M_E + N)$ superior to that of the incumbent's product $v(M_I + N)$. This is more likely to be the case the higher the number of period-1 consumers who bought the rivals' secondary product M_E and the stronger E_S's advantage δ_S. Otherwise the incumbent will sell to period-2 consumers. Equilibrium prices and payoffs are summarized as follows (we omit the proof since this is just a simple asymmetric Bertrand game):

(i) If $\delta_S \geq v(M_I + N) - v(M_E + N)$, equilibrium prices and quantities in the secondary market are given by:

$$p^*_{I_S,2} = 0, \quad p^*_{E_S,2} = \delta_S + v(M_E + N) - v(M_I + N); \quad q^*_{I_S,2} = 0, \quad q^*_{E_S,2} = N$$

(ii) If $\delta_S < v(M_I + N) - v(M_E + N)$, equilibrium prices and quantities in the secondary market are given by:

$$p^*_{E_S,2} = 0, \quad p^*_{I_S,2} = v(M_I + N) - \delta_S - v(M_E + N); \quad q^*_{I_S,2} = N, \quad q^*_{E_S,2} = 0$$

In case (i), equilibrium payoffs in period 2 are given by, respectively:

$$\pi^*_{I_S,2} = \pi^*_{I_P,2} = 0, \;\; \pi^*_{E_S,2} = N[\delta_S + v(M_E + N) - v(M_I + N)]; \;\; \pi^*_{E_P,2} = N\delta_P; \;\; CS^*_2 = N[U + v(M_I + N)]$$

In case (ii) they are given by:

$$\pi^*_{I_S,2} = N[v(M_I + N) - \delta_S - v(M_E + N)], \;\; \pi^*_{I_P,2} = 0, \;\; \pi^*_{E_S,2} = 0; \;\; \pi^*_{E_P,2} = N\delta_P; \;\; CS^*_2 = N[U + \delta_S + v(M_E + N)]$$

Period 1

Recall that in period 1 the primary product is sold only by the incumbent. Consumers at period 1 will anticipate that their choice will affect period-2 consumers' choice. If they choose I_S, then at the beginning of period 2 $M_I = N$ and $M_E = 0$. From assumption (A1') it follows that case (ii) above applies: $\delta_S < v(2N) - v(N)$. In turn, this means that period-2 consumers will all buy I_S. Then, the individual surplus of period-1 consumers is $CS(I_P, I_S)_1 = U + v(2N) - p_{I_P,1} - p_{I_S,1}$.

If instead period-1 consumers choose E_S, then at the beginning of period 2 it will be $M_I = 0$ and $M_E = N$. From assumption (A1') it follows that case (i) above applies: $\delta_S \geq v(N) - v(2N)$. In turn, this means that period-2 consumers will all buy E_S. In this case period-1 consumers will derive surplus $CS_1(I_P, I_S) = U + \delta_S + v(2N) - p_{I_O,1} - p_{E_S,1}$.

Hence, period-1 consumers will buy I_S as long as $p_{I_S,1} < p_{E_S,1} - \delta_S$ and $CS(I_P, I_S) \geq 0$. In other words, in order to attract period-1 consumers, the incumbent must offer a discount slightly larger than δ_S to compensate users for its intrinsic quality disadvantage.

When competing for period-1 consumers firms anticipate that who sells to period-1 consumers will also sell to period-2 consumers. If firm E_S sells the secondary product to period-1 consumers at the price $p_{E_S,1}$, its total profits would be:

$$p_{E_S,1}N + (\delta_S + v(2N) - v(N))N$$

Hence, the minimum price that firm E_S is willing to offer is $\overline{p}_{E_S,1} = -\delta_S - (v(2n) - v(N)) < 0$.

If the incumbent sells the secondary product to period-1 consumers at the price $p_{I_S,1}$, it will set a price of the primary product that allows it to extract from consumers their remaining surplus from the system: $p_{I_P,1} = U + v(2N) - p_{I_S,1}$. It would earn $(p_{I_S,1}N + (U + v(2N) - p_{I_S,1})N = (U + v(2N))N$ in period 1 and $(v(2N) - v(N) - \delta_S)N$ in period 2. If the incumbent does not sell its secondary product in period 1, it will not sell it in period 2 too. However, the incumbent sells the primary product to period 1 consumers: if it expects firm E_S to set the price $p_{E_S,1}$ for the secondary product, the incumbent optimally sets $p_{I_P,1} = U + v(2N) + \delta_S - p_{E_S,1}$ for the primary product. This shows that, given the rival's price $p_{E_S,1}$ the incumbent finds it optimal to undercut and sell the secondary product to period-1 consumers as long as the rival's price is sufficiently high because in that case letting the rival sell the secondary product does not allow the incumbent to extract sufficient rents from period-1 consumers:

$$\begin{aligned}\Delta\pi_I = (U + v(2N))N + (v(2N) - v(n) - \delta_S)N - (U + v(2N) + \delta_S - p_{E_S,1}) \geq 0 \\ \Leftrightarrow p_{E_S,1} \geq \delta_S + \delta_S - (v(2N) - v(N)) \equiv \widehat{p}_{E_S,1}.\end{aligned}$$

Note that, by assumption $A1'$, $\widehat{p}_{E_S,1} < \delta_S$. This implies that when $p_{E_S,1} = \delta_S$, the incumbent has an incentive to offer a discount slightly larger than δ_S and sell the secondary product to period-1 consumers. However, the non-negative price-constraint bites and the incumbent cannot make such a counteroffer.

Similarly to the case analysed in Section 2.1, there is a continuum of Nash equilibria with firm I selling the primary product at a price $p_{I_P,1} = U + v(2N) + \alpha\delta_S$ and E_S selling the secondary product at $p_{E_S,1} = (1-\alpha)\delta_S$, with $\alpha \in [0,1]$. Note that the non-negative price constraint bites also in this case: if $p_{I_p,1} \in (U + v(2N) + \delta_S, U + v(2N) + \delta_S + (\delta_S + v(2N) - v(N))]$ firm E_S would find it profitable to set a negative price for its secondary product $p_{E_S,1} \in [-\delta_s - (v(2N) - v(N)), 0)$. The incumbent would extract through the price of the primary product not only E_S's quality advantage δ_S generated in period 1, but also the rents that E_S will earn in period 2. However, the non-negative price constraint prevents these equilibria from arising.

Among the equilibria, as in the previous sections, we select the one where the incumbent and the rival split equally E_S's efficiency rents, namely $\alpha = 1/2$. We can then summarise the analysis as follows:

Price equilibrium in period 1 with no tying

In period 1 E_S sells the secondary product. Equilibrium prices and quantities are as follows:

$$p^*_{I_S,1} = 0, \quad p^*_{E_S,1} = \delta_S/2; \quad p^*_{I_P,1} = U + v(2N) + \frac{\delta_S}{2}; \quad q^*_{I_S,1} = 0, \quad q^*_{E_S,1} = N$$

The corresponding equilibrium payoffs in period 1 are given by:

$$\pi^*_{I,1} = N[U + v(2N) + \frac{\delta_S}{2}]; \quad \pi^*_{E_S,1} = N\frac{\delta_S}{2}; \quad CS^*_1 = 0.$$

□

Proof of Lemma 5

Proof. **Last stage**
When the incumbent engages in tying, multi-homers who want to combine the incumbent's primary product and E_S's secondary product have to buy the bundle from the incumbent and add also E_S's secondary product. We did not consider this situation in the case of no tying analysed in Section 3.1 because in that case a consumer who wants to combine the incumbent's primary product and E_S's secondary product can buy I_P from the incumbent independently from I_S.

Let us denote with M_I the number of consumers who bought only the incumbent's secondary product in the previous periods (and, therefore, that will use I_S in the last stage), with M_E those who bought only E_S's secondary product (and, therefore, who will use E_S in the last stage), and as M_{IE} the multi-homers who added E_S to the incumbent's system. At the last stage the latter consumers have the possibility to choose which secondary product to use and will compare the overall utility that each product generates. They will use E_S secondary product iff:

$$v(M_E + M_{IE}) + \delta_s > v(M_I + M_{IE}) \tag{25}$$

Period 2
In the second period, both E_S and E_P are in the market. The choice of period-2 consumers depends on the choice of period-1 consumers. Recall that in period 1 E_P's primary product is not available. Hence, in period 1 consumers buy either only the bundle or the bundle and E_S's secondary product. Then, given M^1_I, M^1_{IE} and $M^1_E = 0$, and given prices $\tilde{p_I}$, p_{IS}, p_{ES} and p_{EP}, period 2 consumers decide which combination of products to buy anticipating the choice of the multi-homers that installed both E_S and I_S.

Period-2 consumers anticipate that if they all choose to buy only the incumbent's secondary product, then the consumers that will have to choose at the last stage are the ones who installed both E_S and I_S in period 1, i.e. $M_{IE} = M^1_{IE}$, whereas $M_E = 0$, and $M_I = 2N - M^1_{IE}$. (We focus here and in what follows on the case in which all consumers buy the bundle at time 1, so that the number of consumers that bought only I_S at time 1 is $M^1_I = N - M^1_{IE}$.) In that case condition 25 becomes $v(M^1_{IE}) + \delta_S > v(2N)$ which is never satisfied from assumption $A1$ and $M^1_{IE} \leq \beta N$. Then, if all period-2 consumers choose to buy only the incumbent's secondary product, then consumers that installed both secondary products will use the one of the incumbent. The utility of period-2 consumers is $U + \delta_P + v(2N) - p_{E_P} - p_{I_S}$ if they choose the combination (E_P, I_S), and $U + v(2N) - p_{E_P} - \tilde{p}_I$ if they buy the bundle.

Instead, if all period-2 consumers choose to buy only E_S's secondary product, then consumers who installed both secondary products will use the one of the entrant. In that case $M_{IE} = M^1_{IE}$, whereas $M_E = N$, and $M_I = N - M^1_{IE}$ so that condition (25) becomes $v(M^1_{IE}+N)+\delta_S > v(N)$ which is satisfied for any $M^1_{IE} \leq \beta N$. The utility of period-2 consumers when they choose the combination (E_P, E_S) is $U + \delta_P + \delta_S + v(N + M^1_{IE}) - p_{E_P} - p_{E_S}$.

Finally, if all period-2 consumers buy the bundle and β of them add E_S's secondary product, $M_{IE} = M^1_{IE} + \beta N$, whereas $M_E = 0$, and $M_I = 2N - M_{IE}$. In this case consumers who installed both secondary products will use E_S iff:

$$\delta_S + v(M^1_{IE} + \beta N) > v(2N). \tag{26}$$

Otherwise, consumers who installed both secondary products will use I_S. Note that from assumption $A1$ it follows that condition (26) is never satisfied if $M^1_{IE} = 0$. Instead, if $M^1_{IE} = \beta N$ there exists a threshold level of the proportion of multi-homing consumers, $\widehat{\beta}$, such that condition (26) is satisfied if (and only if) $\beta > \widehat{\beta}$.

When condition (26) is satisfied, the utility of multi-homing period-2 consumers is given by $U+\delta_S+v(M^1_{IE}+\beta N)-\tilde{p_I}-p_{E_S}$ while that of single-homers is given by $U+v(2N-\beta N-M^1_{IE})-\tilde{p_I}$.

When condition (26) is not satisfied, the utility of multi-homing period-2 consumers is given by $U + v(2N) - \tilde{p_I} - p_{E_S}$ while that of single-homers is given by $U + v(2N) - \tilde{p_I}$.

Let us analyse now price competition between suppliers in period 2. Let us consider first the case in which $\delta_S + v(M^1_{IE} + N) < v(2N)$, so that the superior system is the one in which all period-2 consumers combine the entrant's primary product and the incumbent's secondary one. In this case the unique price equilibrium in period 2 is the one in which each supplier extracts its quality advantage:

$$\tilde{p_I}^* = p^*_{I,s} = v(2N) - v(M^1_{IE} + N) - \delta_s; \quad p^*_{E_s} = 0; \quad p^*_{E_p} = \delta_p.$$

with the incumbent selling I_S and E_P selling the primary product. No supplier has the unilateral incentive to deviate. Period-2 consumers enjoy utility $U + \delta_s + v(M^1_{IE} + N)$.

No equilibrium exists in which the total price for the combination (E_P, I_S) is the same as in the equilibrium described above, but either the incumbent or E_S extracts more than its quality advantage, while the other supplier extracts less. The possibility to combine either I_S with I_P in the bundle, or E_S with E_P creates the scope for profitable deviations.

Namely, no equilibrium exists in which $p_{I_S} + p_{E_P} = \delta_P + v(2N) - v(M^1_{IE} + N) - \delta_S$ but $p_{I_S} > v(2N) - v(M^1_{IE} + N) - \delta_S$ and $p_{E_P} = \delta_P + v(2N) - v(M^1_{IE} + N) - \delta_S - p_{I_s} < \delta_P$. E_S would have an incentive to deviate and set $p^{'}_{E_S} = \delta_P - p_{E_P} - \varepsilon > 0$ for ε sufficiently low. Consumers would decide to combine E_S and E_P and the deviation would be profitable.

Similarly, no equilibrium exists in which $p_{I_S} + p_{E_P} = \delta_P + v(2N) - v(M^1_{IE} + N) - \delta_S$ but $p_{E_P} > \delta_P$ and $p_{I_S} = \delta_P + v(2N) - v(M^1_{IE} + N) - \delta_S - p_{E_P} < v(2N) - v(M^1_{IE} + N) - \delta_S$. The incumbent would have an incentive to deviate and slightly decrease the price of the bundle setting $\tilde{p_I}^{'} = \delta_P + v(2N) - v(M^1_{IE}+N) - \delta_S - p_{E_P} + \varepsilon$, with $\tilde{p_I}^{'} < v(2N) - v(M^1_{IE}+N) - \delta_S$ for ε sufficiently small. Consumers would choose the bundle and the deviation would be profitable.

Let us consider now the case in which $\delta_s + v(M^1_{IE} + N) \geq v(2N)$, so that the superior system is the one in which all period-2 consumers combine the entrant's products (E_P, E_S): at the equilibrium

E_P and E_S will sell. The prices configuration in which each supplier extracts its quality advantage is an equilibrium:

$$\tilde{p_I}^* = 0 = p^*_{I_S}; \;\; p^*_{E_S} = \delta_S + v(M_{IE} + N) - v(2N); \;\; p^*_{E_P} = \delta_P.$$

with the E_S selling the secondary product and E_P selling the primary product. No supplier has the unilateral incentive to deviate and period-2 consumers enjoy utility $U + v(2N)$.

There exists no equilibrium in which E_S extracts more than its quality advantage, i.e. an equilibrium in which $p_{E_S} + p_{E_P} = \delta_P + \delta_S + v(M^1_{IE} + N) - v(2N)$ but $p_{E_S} > \delta_S + v(M^1_{IE} + N) - v(2N)$ and $p_{E_P} = \delta_P + \delta_S + v(M^1_{IE} + N) - v(2N) - p_{E_S} < \delta_P$. The incumbent would have an incentive to deviate and set $p^{'}_{I_S} = \delta_P - p_{E_P} - \varepsilon > 0$ for ε sufficiently small. Consumers would decide to combine I_S and E_P and the deviation would be profitable.

Is there an equilibrium in which E_P extracts more than its quality advantage? Consider a candidate equilibrium in which $p_{E_S} + p_{E_P} = \delta_P + \delta_S + v(M^1_{IE} + N) - v(2N)$ but $p_{E_P} > \delta_P$ and $p_{E_S} = \delta_P + \delta_S + v(M^1_{IE} + N) - v(2N) - p_{E_P} < \delta_S + v(M^1_{IE} + N) - v(2N)$. At the candidate equilibrium, all period-2 consumers would be better off if they could buy the bundle (whose price is zero) so as to have access to I_P and then combine it with E_S's secondary product. The latter's price is low enough to ensure that, if they could do that, they would enjoy greater utility than $U + v(2N)$. However, only a proportion β of period-2 consumers can add E_S to the incumbent's system. Whether they want to do that at the candidate equilibrium prices (and therefore whether there is scope for a profitable deviation for the incumbent) depends on whether condition (26) is satisfied and whether the candidate equilibrium price for E_S is sufficiently low.

Consider first the case in which condition (26) is not satisfied because in period 1 multi-homing consumers did install both secondary products but their proportion is too low (i.e. $M^1_{IE} = \beta N$ but $\beta \leq \widehat{\beta}$).[24] In that case, in period 3 all consumers who installed both secondary products decide to use I_S. Then, by purchasing the bundle and adding E_S, multi-homing period-2 consumers would enjoy utility $U + v(2N) - p_{E_S}$ that is (weakly) lower than the utility they enjoy in the candidate equilibrium. It follows that no profitable deviation exists for the incumbent or E_S and the candidate equilibrium is indeed an equilibrium. To sum up, when $\beta \leq \widehat{\beta}$ the equilibria are as follows:

$$\tilde{p_I}^* = 0 = p^*_{I_S}; \;\; p^*_{E_S} = (1-\alpha)[\delta_S + v(M_{IE} + N) - v(2N)]; \;\; p^*_{E_P} = \delta_P + \alpha[\delta_S + v(M_{IE} + N) - v(2N)].$$

with $\alpha \in [0, 1]$. Consider now the case in which $M^1_{IE} = \beta N$ and $\beta > \widehat{\beta}$ so that condition (26) is satisfied. In that case, in the last stage, all consumers who installed both secondary products decide to use E_S. Then, by purchasing the bundle and adding E_S, multi-homing period-2 consumers enjoy utility $U + \delta_S + v(M^1_{IE} + \beta N) - p_{E_S}$. Condition (26) implies that $U + \delta_S + v(M^1_{IE} + \beta N) - p_{E_S} > U(2N)$ if $p_{E_S} = 0$, whereas $\beta \leq 1$ implies that $U + \delta_S + v(M^1_{IE} + \beta N) - p_{E_S} \leq U(2N)$ if $p_{E_S} = \delta_S + v(M^1_{IE} + N) - v(2N)$. Then, there exists a threshold level of the candidate equilibrium price, $\widehat{p}_{E_S}(\beta) \in (0, \delta_S + v(M^1_{IE} + N) - v(2N)]$, such that $U + \delta_S + v(M^1_{IE} + \beta N) - p_{E_S} > U(2N)$ iff $p_{E_S} < \widehat{p}_{E_S}(\beta)$, with $\widehat{p}_{E_S}(\beta)$ increasing in β. Therefore, no profitable deviation exists when $p_{E_S} \geq \widehat{p}_{E_S}(\beta)$. In that case the candidate equilibrium is an equilibrium.

When, instead, $p_{E_S} < \widehat{p}_{E_S}(\beta)$ the incumbent would have an incentive to deviate and set $\tilde{p_I}^{'} = \delta_S + v(M^1_{IE} + \beta N) - v(2N) - p_{E_S} - \varepsilon > 0$ for ε sufficiently small. A proportion β of consumers

[24] Also when no multi-homing consumer decided to install both secondary products, i.e. when $M^1_{IE} = 0$, condition (26) is not satisfied, but in that case $\delta_s + v(M^1_{IE} + N) \geq v(2N)$ cannot hold by assumption $A1$.

would decide to buy the system (I_P, I_S) from the incumbent and then add E_S, and the deviation would be profitable. In that case the proposed one is not an equilibrium. To sum up, when $\beta > \widehat{\beta}$ the equilibria are as follows:

$$\tilde{p_I}^* = 0 = p^*_{I_S}; \quad p^*_{E_S} = (1-\alpha)[\delta_S + v(M_{IE}+N) - v(2N)] + \alpha \widehat{p}_{E_S}(\beta);$$
$$p^*_{E_P} = \delta_P + \alpha[\delta_S + v(M_{IE}+N) - v(2N) - \widehat{p}_{E_S}(\beta)]$$

with $\alpha \in [0,1]$, $\widehat{p}_{E_S}(\widehat{\beta}) = 0$ and $\widehat{p}_{E_S}(1) = \delta_S + v(M_{IE}+N) - v(2N)$.

As we did earlier when there exist multiple equilibria, we focus on the equilibrium that corresponds to $\alpha = 1/2$.

Lemma 7 summarises the result:

Lemma 7. ***Equilibrium prices at t=2 in case of tying.***
(i) If $\delta_S \geq v(2N) - v(M^1_{IE}+N)$, there exists a threshold level of the proportion of multi-homing consumers, $\widehat{\beta}$ such that:

(i-a) if $\beta > \widehat{\beta}$ there exists a threshold level of the price of E_S's secondary product, $\widehat{p}_{E_S}(\beta)$, such that equilibrium prices and quantities are given by:

$$\tilde{p_I}^* = 0 = p^*_{I_S}; \quad p^*_{E_S} = \frac{1}{2}[\delta_S + v(M_{IE}+N) - v(2N)] + \frac{1}{2}\widehat{p}_{E_S}(\beta);$$
$$p^*_{E_P} = \delta_P + \frac{1}{2}[\delta_S + v(M_{IE}+N) - v(2N) - \widehat{p}_{E_S}(\beta)] \quad q^*_{I_S} = q^*_{I_P} = 0; \quad q^*_{E_S} = q^*_{E_P} = N.$$

with $\widehat{p}_{E_S}(\beta)$ increasing in β, $\widehat{p}_{E_S}(\widehat{\beta}) = 0$ and $\widehat{p}_{E_S}(1) = \delta_S + v(M_{IE}+N) - v(2N)$.

(i-b) If $\beta \leq \widehat{\beta}$ equilibrium prices and quantities are given by:

$$\tilde{p_I}^* = 0 = p^*_{I_S}; \quad p^*_{E_S} = \frac{1}{2}[\delta_S + v(M_{IE}+N) - v(2N)]; \quad p^*_{E_P} = \delta_P + \frac{1}{2}[\delta_S + v(M_{IE}+N) - v(2N)]$$
$$q^*_{I_S} = q^*_{I_P} = 0; \quad q^*_{E_S} = q^*_{E_P} = N.$$

(ii) If $\delta_S < v(2N) - v(M^1_{IE}+N)$, equilibrium prices and quantities are given by:

$$\tilde{p_I}^* = p^*_{I_S} = v(2N) - v(M^1_{IE}+N) - \delta_S; \quad p^*_{E_S} = 0; \quad p^*_{E_P} = \delta_P \quad q^*_{E_S} = q^*_{I_P} = 0; \quad q^*_{I_S} = q^*_{E_P} = N.$$

Note that in the model with fixed costs E_S can decide not to enter the secondary market, so that in period 2 the incumbent can extract rents from E_P. In this variant of the model, instead, E_S is always in the market which prevents the incumbent from extracting rents from E_P when the incumbent sells I_S in period 2. Rents extraction would be possible if we modeled extreme network effects, for instance assuming that by not selling in t=1 means that E_S represents no effective competition in t=2.

Period 1

In the first period E_P is not on sale, and if a consumer buys I_P that consumer is also getting I_S. Consumers with low storage capacity will not buy E_S and hence will use I_S for sure. Instead, consumers with high storage capacity may decide to add E_S to the incumbent's system (I_P, E_S) depending on the price $p_{E,S}$ as well as on their expectations on the decision of period-2 consumers.

Given $\tilde{p}_I$, p_{I_S} and p_{E_S}, if all period-1 consumers buy only the bundle (without adding E_S), then $M^1_{IE} = 0$ and by assumption $A1$ case (ii) of Lemma 7 applies. Then, all period-2 consumers will buy the combination (E_P, I_S). The utility of period-1 consumers is $U + v(2N) - \tilde{p}_I$.

If all period-1 consumers buy the bundle and multi-homers add E_S, then $M^1_{IE} = \beta N$. If $\delta_S + v(\beta N + N) < v(2N)$ then case (ii) of Lemma 7 applies and, again, all period-2 consumers will buy the combination (E_P, I_S) and the multi-homing period-1 consumers that installed both secondary products will use I_S. Their utility is $U + v(2N) - \tilde{p}_I - p_{E_S}$. Note that when $\beta = 0$, assumption $A1$ ensures that $\delta_S + v(\beta N + N) < v(2N)$, whereas when $\beta = 1$ $\delta_S + v(\beta N + N) > v(2N)$. There exists a threshold level of the proportion of multi-homers, β^*, with $\beta^* \in (0, 1)$ and $\beta^* < \widehat{\beta}$, such that $\delta_S + v(\beta N + N) < v(2N)$ if (and only if) $\beta < \beta^*$.

Instead, when $\beta \geq \beta^*$, $\delta_S + v(\beta N + N) \geq v(2N)$, and case (i) of Lemma 7 applies. Then, all period-2 consumers will buy the combination (E_P, E_S) and the multi-homing period-1 consumers that installed both secondary products will use E_S. Their utility is $U + \delta_S + v(\beta N + N) - \tilde{p}_I - p_{E_S}$.

When $\beta < \beta^*$, given $\tilde{p}_I$ consumers are willing to add E_S's secondary product to the incumbent's system as long as $p_{E_S} < 0$. In this case all period-2 consumers will buy the combination (E_P, I_S), so that attracting period-1 multi-homing consumers does not allow firm E_S to sell in period 2. Hence, firm E_S would not be willing to offer a negative price in period 1 even in the absence of the non-negative price constraint. At the equilibrium the incumbent sells the bundle to all period-1 consumers and extracts their entire surplus.

When, instead, $\beta \geq \beta^*$, there exist multiple equilibria. In all of them period-1 consumers buy the bundle from the incumbent and multi-homers add E_S's secondary product. Equilibria differ in the way the incumbent and E_S share the additional utility produced by the use of E_S's secondary product. Also in this case we focus on the case in which that additional utility is shared evenly (i.e. $\alpha = 1/2$).[25] Note that in this case we allow the incumbent to discriminate the price between multi-homers and single-homers in period 1.

Lemma 8 describes period-1 equilibrium prices and quantities following the incumbent's decision to engages in tying.

Lemma 8. ***Equilibrium prices at t=1 in case of tying.***
There exists a threshold level of the proportion of multi-homers, $\beta^* \in (0, 1)$, such that:

(i) If $\beta < \beta^*$, first-period consumers do not add E_S to the incumbent's system. First period equilibrium prices and quantities are:

$$\tilde{p_I}^* = U + v(2N); \quad p^*_{I,S} = 0; \quad p^*_{E,S} = 0 \quad q^*_{I,S} = q^*_{I,P} = N; \quad q^*_{E,S} = 0.$$

(ii) If $\beta \geq \beta^*$, first-period consumers do add E_S to the incumbent's system. First period equilibrium prices and quantities are:

$$\tilde{p_I}^{*H} = U + v(2N) + \frac{1}{2}(\delta_s + v(\beta N + N) - v(2N)); \ \tilde{p_I}^{*L} = U + v((1-\beta)N)$$
$$p^*_{I,S} = 0; \quad p^*_{E,S} = \frac{1}{2}(\delta_s + v(\beta N + N) - v(2N)); \quad q^*_{I,S} = q^*_{I,P} = N; \quad q^*_{E,S} = \beta N.$$

□

[25] As already highlighted in Section 3.1, the non-negative price constraint bites and prevents the incumbent from extracting from E_S also the profits earned in period 2.