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Competitive Prices in Markets with Search and Information Frictions

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Abstract

This paper introduces directed search into the sequential search model of Diamond (1971) by allowing buyers to observe the distribution of prices charged by two (or more) distinct subgroups of firms in the market. This enables buyers to direct their searches towards the most desirable group of firms. Search within groups remains random since price information about each of the groups is imperfect, as in a standard setup. I find that competitive pricing is then the *unique* equilibrium outcome. This holds even when different buyers observe very different groups of firms and face different and strictly positive levels of search costs. Considering an explicit learning scheme the paper shows that convergence of prices to competitive equilibrium depends crucially on the level of search costs and the number of groups observed by buyers. Lower search costs and a higher number of observed groups generate a higher price elasticity of demand and thereby favor the emergence of competitive prices.

Keywords: Diamond paradox, competitive pricing, random and directed sequential search, equilibrium search model, learning

JEL Classification. NO.: D81, D83

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1 Introduction

With the appearance of the Internet and its increasing importance in coordinating economic transactions, economists have shown a renewed interest in the economic theory of search (e.g. Abrams et. al. (2000), Cason and Friedman (1999), (2000)). Search models seem particularly useful to analyze the economic consequences of the new information technologies since these models take the search and information frictions as their primitives.

Theoretical search models, which can be used to analyze the economic implications of reduced search costs, seem to imply that search frictions are a rather strong impediment to market efficiency, independent from their level. Many search models where buyers search sequentially for low-priced offers robustly generate monopoly price equilibria for any positive level of search frictions (Diamond (1971), Butters (1977), Rob (1985)). Moreover, models which also generate price dispersion (e.g. Burdett and Judd (1983)) face the problem that the dispersed equilibrium is not stable under adaptive learning schemes while the monopoly price equilibrium is indeed so (Hopkins and Seymour (1999)).

The present paper shows that search frictions might not have the severe adverse effects on equilibrium prices previously attributed to them and that market prices can be at the competitive level despite the presence of considerable search and information frictions. In particular, it is shown that in a standard search model with sequentially searching buyers, as the one used by Diamond (1971), competitive prices are the unique equilibrium outcome once one relaxes the assumption that buyers search fully randomly for price offers and allows for directed search.

Furthermore, it is shown that sellers who learn from past experience can coordinate on (almost) competitive prices when buyers direct their searches but will coordinate on monopoly prices when buyers' search is fully random. Convergence to competitive equilibrium, however, is found to depend crucially on the level of search costs and the degree to which buyers can direct their searches. This indicates a new role for search costs and information frictions that is related to the learnability of equilibria whereas the literature has been mostly occupied with the effect of these variables in shaping the equilibrium outcomes.

To model directed search I allow buyers to obtain two (or more) imperfect price signals about the prices charged by two (or more) subsets of firms in the market, which implies that buyers can direct their search efforts towards the subset of firms where search is most promising. I thereby extend the settings considered in the search literature where buyers received just a single imperfect signal about the prices charged by all firms in the market, which implied that buyers' search was undirected or random.

Although buyers can direct their searches, their ability do so is far from perfect. Since the price signals do not reveal which of the firms in a given subset charges which price, buyers must still engage in costly (random) search within any of the subsets.

At the same time, the information structure is flexible enough to bridge the gap all the way to the perfect information structure of a Walrasian market. By increasing the number of subsets one will enable buyers to increasingly direct their searches and thereby reduce the degree of information imperfection in the economy. The limiting case where buyers observe as many subsets as there are firms in the market will generate the Walrasian information structure.

A natural interpretation of the 'more-than-one-price-signal' setup is that buyers possess imperfect price information that allows them to distinguish between the firms of different neighborhoods or different local markets. Alternatively, buyers might receive an additional price signal from a friend living elsewhere who is imperfectly informed about the prices of a different set of firms. For the case of search in the Internet one might think of different signals as buyers' knowledge about prices charged in different virtual market places.

The first part of the paper determines the equilibrium of the model. The main result is as follows: when each buyer observes the distribution of prices from two or more distinct groups of firms in the market and searches sequentially, then the *unique* equilibrium is characterized by competitive pricing. The sets of firms different buyers observe may thereby overlap in arbitrary ways. Moreover, different buyers may face different costs for visiting firms.

For the special case that the subsets of firms from which buyers receive price information contain just a single firm, the result reproduces the one obtained by Burdett and Judd (1983) who showed that competitive pricing is the unique equilibrium outcome when all consumers observe the *actual* prices charged by two (or more) shops.[1]

[1] Note that this particular case implies a special property that does not show up in the more general setup: when buyers observe actual prices of sellers, they will buy from the cheapest store they observe, independently from the level of search costs. Yet, when observing imperfect price signals, buyers' will settle for higher prices the higher the level of search costs.

It may not be surprising that competitive pricing is indeed an equilibrium outcome. The main difficulty lies in showing that it is the *unique* equilibrium.

The intuition underlying the result is roughly as follows. When buyers receive more than one price signal, sellers are confronted with an increase in the price elasticity of demand when compared to the case where buyers sample completely randomly. The increased elasticity is generated by the fact that buyers can avoid buying at high prices by excluding expensive sellers from being searched and direct their searches to other groups of sellers. This puts whole groups of sellers into some kind of collective Betrand competition and leaves competitive prices as the unique equilibrium outcome.

The second part of the paper considers the issue of convergence to equilibrium when firms behave as adaptive price setters who learn from past experience and who occasionally experiment with new prices.

I find that the level of search costs and the number of price signals that buyers observe affect the price-elasticity of demand out of equilibrium and thereby influence whether prices converge to the competitive equilibrium or not. For low to medium levels of search costs prices converge close to the competitive equilibrium. However, when search costs increase beyond a certain threshold prices suddenly jump from an (almost) competitive to an (almost) monopolistic level. The discontinuity arises from the fact that search costs affect the price elasticity in a discontinuous way.

This suggests that the introduction of technologies that reduce search costs from high to moderate levels can have a rather discontinuous effect on the price level and efficiency of the economy. At the same time, it suggests that additional search cost reductions do not generate significant further welfare gains beyond the savings in search cost expenditure. It is up to the reader to judge whether the search cost reductions caused by the new information technologies are rather of the first or of the second type.[2]

Although the model predicts uniformly competitive prices, this result should not be taken too literally. Price dispersion is a fact of life, even in the age of the Internet, and the inability of the present model to account for it is clearly a shortcoming.[3] Experimental research, however, has shown that models that predict uniform prices can nevertheless offer good predictions

[2] Of course, there are many other channels through which the new information technologies might (also) increase efficiency, e.g. by providing consumers access to previously unaccessible market places (Baye and Morgan (2000)).

[3] Check www.cnet.com for real-time price dispersion and see tables 1 in Stigler (1961) or Pratt et. al. (1979) for some more historic evidence.

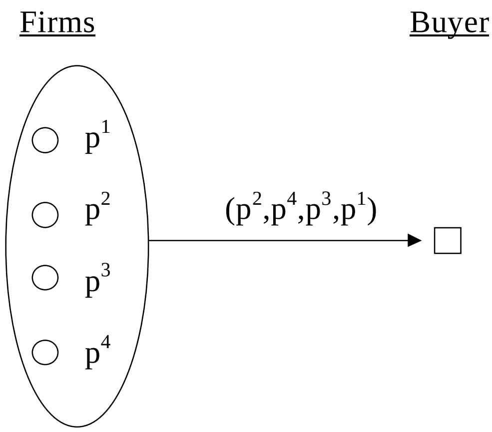


Figure 1: Stigler Information Setting

for the prices that emerge in the laboratory even when these display some dispersion (Grether, Schwartz, and Wilde (1988)).[4]

The paper starts out with a preliminary analysis which mainly reconsiders the intuition underlying the monopoly pricing result emerging in a setting with random sequential search, as first shown by Diamond (1971). Section 3 then presents a search model that allows for various degrees of directed search. Section 4 derives the equilibrium, which is discussed in the following section. Finally, Section 6 studies the convergence to equilibrium. A short conclusion summarizes the findings. Most proofs can be found in the appendix.

2 Preliminary Considerations

The foundation stone of the economics of search was laid by George Stigler back in 1961 when he presented a model where consumers were imperfectly informed about prices. Stigler's setting is illustrated in figure 1. Instead of observing the prices charged by each firm, buyers observe just a list of prices without knowing which firm charges which price, which implies that buyers have to engage in (costly) search for low-priced offers.

Diamond (1971) analyzed the implications of such an information scenario

[4]Price dispersion may also emerge as a result of product heterogeneity, fairness motivated demand-withholding (Ruffle (2000)), or less-than-rational punishment behavior (Davis and Holt (1996)). The present model abstracts from all of these factors.

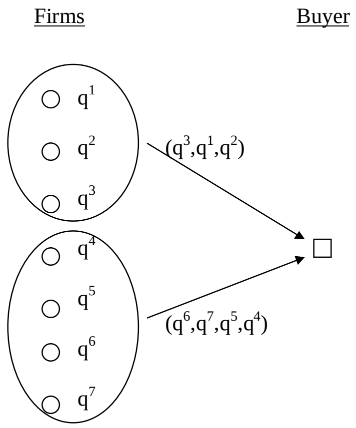


Figure 2: Two Price Signals

for the price setting incentives of sellers and came up with a paradoxical result that reappeared throughout the following literature (e.g. Butters (1977), Burdett and Judd (1983), Rob (1985), Hopkins and Seymour (1999)). When buyers search sequentially and randomly for price offers and when each buyer faces positive search costs, then all firms will eventually charge the monopoly price.[5]

The argument underlying the result is simple: The firm who charges the lowest price can exploit the fact that buyers have to pay some costs to get another price quote and increase its price slightly without loosing any buyers that show up at its door. The only price where firms do not want to raise prices any further is the monopoly price.

The logic that allows firms to exploit the existence of search costs in such an unconstrained way relies, however, on the somewhat unintuitive feature that the number of customers arriving at a firm's door in the first place is independent from the price it charges. This price inelasticity of arrival rates is due to the particular information structure introduced by Stigler. Since buyers have no idea about which price is charged by which firm and therefore sample randomly, the expected costs of finding a firm who lowers its price would be very high, especially when many firms are present in the market. Firms that lower prices, therefore, do not attract more buyers but will only

[5]Without discounting between different search steps and with constant marginal costs for searching firms, sequential search is the optimal search strategy for buyers. Also, in the light of the information structure, searching randomly among firms is as good as any non-random search order.

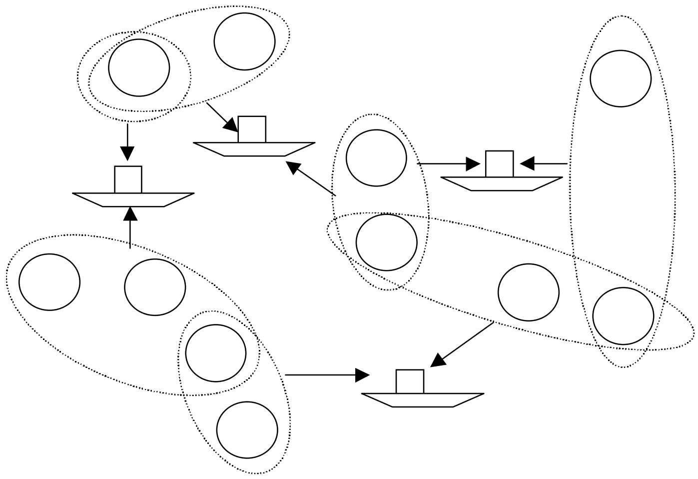


Figure 3: Island Economy with Heterogenous Information Structure

make lower profits per arriving buyer.[6]

Figure 2 shows a different information structure where buyers receive more than one imperfect price signal. One may interpret this as a list of prices from two different shopping malls, e.g. the "West-Town Mall" and the "East-Town Mall". Buyers can now direct their search to the mall that seems most attractive to them, which implies that search decisions affect firms' arrival rates.

Clearly, it cannot be an equilibrium that firms in both malls charge the monopoly price. A price cut by a firm in one of the malls would lead to a better price signal and cause buyers to direct their searches towards this mall: there is a positive probability of accidentally finding the deviating firm and buying at a lower price. Although the firm will capture only part of the increased demand (search within each mall is random), it will be enough to compensate it for the loss it incurs by cutting prices, given that the cut is small enough.

As one can immediately observe, the two local markets are in some kind of Bertrand price competition with each other and the incentives are such that an individual firm is willing to offer the 'club good' to its local market to lower the price and to attract more buyers. As a result, there is a unique

[6]As the number of firms decreases, buyers' expected search length for a deviating low-price firm decreases. The power of the search technology thereby increases and the monopoly price equilibrium eventually breaks down, see Stiglitz (1987).

equilibrium where all firms charge the competitive price.

Clearly, the situation depicted in figure 2 is rather special because all agents are assumed to observe the same set of firms. Figure 3 depicts a situation with a heterogenous information structure. It shows islands that are populated by firms and consumers who navigate through the ocean and search for low prices. Each consumer has information about the prices of neighboring groups of islands. Due to the spatial structure, different consumers observe different islands. Also, note that consumers observe just a subset of firms in the market.

Clearly, when all firms charge competitive prices, it is still the case that no firm has an incentive to increase its price because consumers could simply redirect their searches to other groups of islands. Therefore, competitive pricing is an equilibrium outcome. The difficulty lies in showing that it is also the unique equilibrium.

3 An Equilibrium Search Model with a General Information Structure

There are $J \geq 2$ firms indexed by j. Firms maximize expected profits and sell a homogeneous product that they can produce at constant marginal costs which are normalized to zero. Each firm j simultaneously with all other firms announces a price $p^j \in \Re$ at which it is willing to sell the product. Firms might use mixed strategies where mixing is between an arbitrarily large but finite number of prices.[7] Allowing for mixed strategies is not important for the results and one could easily restrict consideration to pure pricing strategies. Let $d^j(p)$ denote the probability with which seller j announces price p.

There are $I \geq 1$ buyers indexed by i. After sellers have set their prices, buyers receive (imperfect) information about the actual prices quoted by sellers. The details will be described below.

Buyers maximize expected utility and buyer i who is purchasing at a price p that has been found after s searches derives (indirect) utility

$$u^i(p, s) = u(p) - c^i \cdot s$$

where $c^i > 0$ is the disutility that each search causes to i. $u(\cdot)$ is assumed to be strictly decreasing and continuously differentiable in p. Note that buyers

[7] To my knowledge there exists no randomization device that mixes in finite time between infinitely many prices.

are homogeneous with respect to the utility they derive from buying at a particular price but might differ in search costs.

We assume that the above specification gives rise to a demand function $x(p)$ that does not depend on c^i or s. This implies that search costs have no income effect on the demand for the product. Either the budget constraint is unaffected by search costs, e.g. if they represent psychological disutility or the opportunity cost of enjoying leisure, or utility is quasi-linear with the linearity somewhere in the basket of remaining goods.

3.1 Buyers' Price Information

Each buyer i has price information about the group of firms $G^i \subseteq \{1, \ldots, J\}$. For the case that G^i does not contain all existing firms, we assume that i simply ignores the existence of firms that are not contained in G^i.

The observed group of firms G^i is partitioned into n^i subgroups that are denoted by G^i_n $(n = 1, \ldots, n^i)$. The subgroups are mutually exclusive, non-empty, and their union is equal to G^i. Furthermore, buyer i receives a separate list with the actual prices charged by firms in each subgroup G^i_n.[8]

The price information about a subgroup of firms is imperfect in the sense that the price list does not reveal which firm in the group charges which price. This allows buyers to distinguish firms before search starts because firms that belong to different groups differ according to the likelihood with which they charge certain prices. At the same time, firms from the same subgroup look identical at the beginning of search.

The collection of all subgroups of all buyers is denoted by

$$G = \{G^i_n\}^{i=1,\ldots,I}_{n=1,\ldots,n^i} \tag{1}$$

For obvious reasons I require that G is such that each firm j is part of at least one set G^i_n. Moreover, firms are assumed to know G such that the only source of information imperfection in the model consists of buyers being imperfectly informed about the location of prices.[9]

[8] Note that the price list resolves the uncertainty from the mixed pricing strategies of firms. One may interpret this as each firm representing a type with actually many sellers of this type being present. Knowledge about the type's pricing strategy then implies knowledge about the price realization.

[9] Section 6 will consider the case where firms have no information about G.

The information structure outlined above has simple and interesting limiting cases: If each buyer observes just a single group ($n^i = 1$ for all i), which contains all existing firms, then G generates the information structure of the standard random search model as introduced by Stigler (1961).

As the number of subgroups increases, say to $n^i = 2$, buyers start to distinguish firms that belong to different subsets according to the likelihood with which they charge different prices. When the number of sets n^i increases even further, buyers' information gets finer and finer. In the limiting case $n^i = J$, each group contains only one firm and G generates the information structure of the Walrasian market model where buyers know the prices of all firms.

Thus, by varying the number of groups n^i one can generate different degrees of information imperfection in the economy that span the whole range from Stigler's random search environment to the perfect information environment of a Walrasian market model.

3.2 Buyers' Optimal Search Behavior

This section describes buyers' optimal search behavior for the information structures introduced in the previous section. The result is due to Weitzman (1979).

Each buyer calculates a reservation utility R_n^i for each of the subgroups G_n^i from which she receives price information. The reservation utilities rank the groups in terms of their attractiveness and fully characterize the optimal search strategy. Let $R_{\max}^i = \max_n R_n^i$ denote the highest of buyer i's reservation utilities. Buyers search only in the group(s) that achieve reservation utility $R_{\max}^i$ and stop searching if they get an offer p which generates utility $u(p)$ above or equal to $R_{\max}^i$.

For the case that there are several groups that achieve maximum reservations, buyers are assumed to randomize with equal likelihood between these groups.[10]

The reservation utility R_n^i for group G_n^i is implicitly defined by

$$c^i \equiv \sum_{j \in G_n^i} \left[\max \left\{ 0, u(p^j) - R_n^i \right\} \cdot \frac{1}{|G_n^i|} \right] \tag{2}$$

[10] The results do not depend on this assumption as long as there is some positive probability with which each group is searched in the case of a draw.

The reservation utility is the value of a price offer at which the costs of an additional search step (the left-hand side of (2)) exactly balance the expected gains from an additional search step (the right-hand side of (2)). This implies that with positive search costs buyers will settle for prices that are higher than the lowest price in the group, a feature that will turn out to be important later on.

The above sampling rule is a generalization of the optimal choice rule of simpler decision problems.

For a Walrasian information structure where each group G_n^i contains just a single firm j, the reservation utility is given by $R_n^i = u(p^j) - c^i$. The sampling rule, thus, prescribes to buy from the firm offering the lowest price.

Similarly, for the case that search costs are zero, equation (2) implies that the reservation utility is equal to the value of the best offer in the group. The sampling rule, thus, prescribes searching in the group containing the lowest price until this lowest price has been found.

3.3 Firms

This section determines firms' expected profits taking into account buyers' optimal search behavior.

Firms' expected profits can be expressed as the product of the expected number of sales times the profits per sale.

Profits per sale are given by $x(p)p$. In line with Diamond (1971), I assume that $x(p)p$ is continuous and strictly quasi-concave with a maximum at a finite price p^*. That is

$$x(p)p \quad \begin{array}{l} \text{increases for } p < p^* \\ \text{decreases for } p > p^* \end{array}$$

This insures that profits per sale strictly increase as the price increases from the competitive ($p = 0$) to the monopolistic level ($p = p^*$).

The expected number of customers is a little harder to determine. Consider firm j charging price p^j. Optimal search behavior of buyers implies that the following conditions are necessary ones for buyer i to buy from firm j:

- **Condition C1**: The group $G_{n(j)}^i$ must achieve maximum reservation utility (where $n(j)$ denotes the group index that contains firm j).

- **Condition C2**: The indirect utility $u(p^j)$ generated by j's offer must be larger or equal to i's maximum reservation utility $R^i_{\max}$.

If $C1$ failed to hold, then buyer i would prefer to search a different group of firms. Similarly, if $C2$ failed to hold, then i would prefer to search for a better offer whenever finding firm j.

Since $C1$ and $C2$ are only necessary conditions, they do not insure that i actually buys from j:

First, buyer i might see herself confronted with several groups that achieve maximum reservation. Then she might optimally choose to search in another group of firms. Denoting by m^i the number of groups of buyer i that achieve maximum reservation, the probability of seller j's group being chosen is given by $\frac{1}{m^i}$.

Second, there might be several firms that offer above i's reservation utility in $G^i_{n(j)}$. When buyer i first finds one of these other firms, she will be buying there. Let $r^{i,j}$denote the number of firms that offer above i's reservation utility in the group containing firm j. Since i searches randomly for offers within each group, the probability of j actually selling to i is given by $\frac{1}{r^{i,j}}$ in this case.

The combined probability of j actually selling to i when condition $C1$ and $C2$ hold is given $\frac{1}{m^i} \cdot \frac{1}{r^{i,j}}$. This implies that expected profits of firm j that charges price p can be expressed as

$$E\left[\Pi^j(p)\right] = \sum_{i=1}^{I} E\left[I_{\{C1(i,j)\}} I_{\{C2(i,j)\}} \frac{1}{m^i} \frac{1}{r^{i,j}} x(p)p\right] \qquad (3)$$

where $I_{\{C1(i,j)\}}$ $(I_{\{C2(i,j)\}})$ is an indicator function that is equal to one if condition $C1$ $(C2)$ holds for buyer i and firm j and that is zero otherwise.[11]

The term $I_{\{C1(i.j)\}} \frac{1}{m^i}$ in the objective function distinguishes the case where buyers can direct their searches from the case of random search. With random search this term is trivially equal to one, while with directed search its value depends on the prices charged by firms in the respective groups. This ultimately generates an increased price elasticity of demand.

[11]The expectations operator still appears in (3) since other sellers might use mixed pricing strategies which induces a probability distribution over the events on the right-hand side of the equation.

4 Equilibrium

This section introduces the equilibrium concept and determines the equilibria of the model.

The equilibrium concept is standard for imperfect information models up to the requirement that firms have to have positive expected sales, which helps to get rid of uninteresting equilibria where some firms charge non-competitive prices but are neither searched nor selling.[12]

Definition Given J firms, I buyers, and a group structure G, an equilibrium is a collection of pricing strategies $d^j(p)$ for each firm j where

1. The firms anticipate the buyers' search behavior. Each firm's pricing strategy is a best reply to the pricing strategy of other firms, i.e.

$$d^j(p) > 0 \Rightarrow E\left[\Pi^j(p)\right] = \overline{\Pi} \text{ and } \overline{\Pi} \geq E\left[\Pi^j(p)\right] \text{ for all } p$$

   and each price that is charged with positive probability makes positive expected sales.

2. The buyers' searches are optimal given the information they possess.

For the case that buyers cannot direct their search efforts and observe just a single price signal, the present model reproduces the monopoly price equilibrium. The intuition underlying the monopoly pricing result has been discussed in section 2.

Theorem 1 *If $n^i = 1$ for all buyers i, then in the unique equilibrium all firms charge the monopoly price.*

Proof : see Diamond (1971).

Next consider the case where all agents receive more than one price signal and can direct their searches to different subgroups of firms. The following lemma restricts the range of prices firms optimally want to charge.

[12] To see the point assume that there are three sellers with the first two charging the competitive price and the third charging some higher price. These prices constitute an equilibrium price distribution even with *perfect* information. However, all transactions will take place at the competitive price. Requiring positive sales eliminates these kind of equilibria.

Lemma 1 *In equilibrium no firm wants to charge a price below the competitive price or above the monopoly price.*

Proof: If seller j charges $p < 0$, then $E\left[\Pi^j(p)\right] < 0$ and by charging $p = 0$ the seller obtains $E\left[\Pi^j(0)\right] = 0$. If $p > p^*$ then by charging p^* the seller could make a strictly higher profit per sale. At the same time, the firm does not loose any customers by decreasing its price, which implies $E\left[\Pi^j(p^*)\right] > E\left[\Pi^j(p)\right]$.■

In the next theorem I establish the surprising result that a firm that charges a price above the competitive price will not make any sales in equilibrium for the case that buyers can direct their searches. The proof can be found in the appendix.

Theorem 2 *If $n^i \geq 2$ for all buyers i, then in equilibrium no firm charging a price above the competitive price makes any sales.*

The proof shows that if some firm was selling at a non-competitive price than there would exist a firm that would not maximize expected profits. The arguments are roughly as follows. Suppose, for contradiction, that a firm who charges a non-competitive price is selling.

When the firm has customers who are indifferent between several groups of firms and who, consequently, randomize between these groups, it pays to the firm to cut its price slightly to induce a strict preference for its own group, as in simple Bertrand competition.

On the other hand, if each customer strictly prefers the firm's group, then the firm could exploit this strict preference and increase prices a little bit without affecting the customers' ordering of groups. However, two subtleties arise: First, when the firm already charges the monopoly price, a further price increase reduces profits per sale. Second, if the firm has customers who are just indifferent between buying at the offered price and continuing to search, then the price increase leads to a loss of customers and potentially to a decrease in profits.

The first case is easily taken care of. Since customers strictly prefer the firm's group there must be another firm in the group who charges below the monopolistic price.[13] Then just make all arguments for this firm and note that the present case cannot apply anymore.

In the second case, the argument is as follows. When a customer is indifferent between buying and continuing to search, there must be another

[13]Remember that firms in other groups charge at most the monopolistic price.

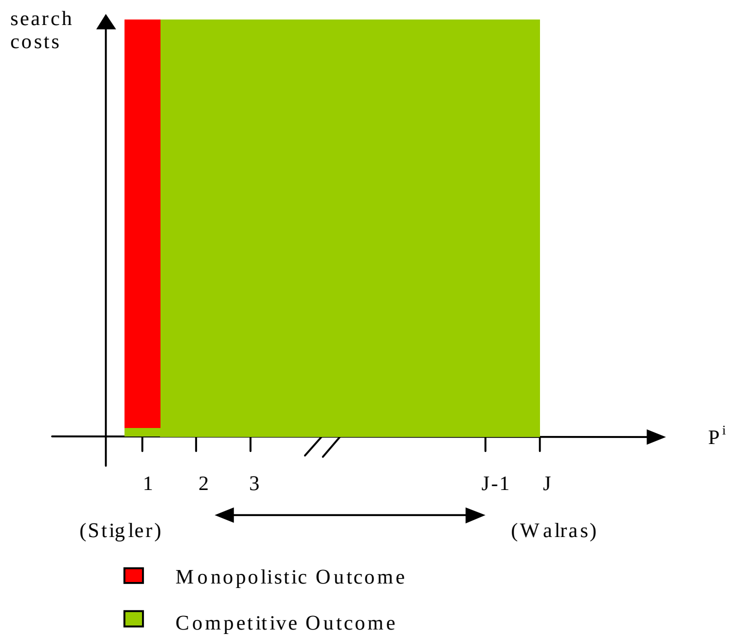


Figure 4: Pricing Results

firm in the same group who charges a price that is at least some $\delta > 0$ lower than the price of the considered firm. This follows from the optimality of reservations prices, since otherwise investing in more search costs could not be optimal. Then make the same arguments for the firm that charges the δ lower price. One either reaches a contradiction with this firm or the present case applies again. Now note that repeated occurrence of the present case will lead, after a finite number of steps, to a firm that must charge below the competitive price, which cannot be optimal, see lemma 1.

Based on theorem 2 it is easy to establish the following result.

Corollary 1 *If $n^i \geq 2$ for all buyers i, then in the unique equilibrium all firms charge competitive prices.*

Proof : The only potential equilibrium is characterized by all firms charging the competitive price, see theorem 2. Deviations to lower prices do not have to be considered, see lemma 1. Deviations to higher prices lead to a loss of all customers and also generates zero profits, which establishes the claim.■

5 Discussion

The important feature that is introduced by buyers' ability to direct search efforts is that the arrival rate of buyers depends on the prices charged by firms. This feature prevents sellers from behaving like local monopolists that fully exploit the fact that an arriving buyers has to invest in additional search costs to get another price quote. A slight price increase, even of the firm charging the lowest price in the market, might now cause a strong decrease in the arrival probability of buyers, since different groups of firms are now competing for buyers.

The competitive outcome puts the Diamond (1971) paradox somewhat into perspective. While Diamond shows that there is a discontinuity in the market outcome when going from zero search costs to $\varepsilon > 0$ search costs, the present result shows that there is an equally large discontinuity when moving from an imperfect information environment with random search ($n^i = 1$) to one with directed search ($n^i \geq 2$), namely one moves back from monopolistic prices right away to competitive prices. Figure 4 illustrates this situation. The x-axis is counting the number of price signals with the far left indicating a random search environment, and the far right indicating a perfect information environment. The y-axis displays the level of search costs. Competitive prices emerge all the way from $n^i = J$ (perfect information) to $n^i = 2$.

Given the reversal in equilibrium prices, there is a natural question to ask: what would happen in a situation in which some buyers are able to direct their searches while others are not? Under the assumption that each firm is observed by at least one buyer who can direct search, the results of theorem 2 go through. This excludes non-competitive prices in equilibrium. At the same time, a situation with competitive prices cannot be an equilibrium either. Firms can exploit buyers that cannot direct their searches by slightly increasing their prices. As a result, no equilibrium price distribution exists for these cases.

Another interesting question concerns the robustness of the result with respect to firms that differ in their marginal cost of production. For simplicity suppose that there are just two types of firms: low-cost and high-cost firms. When agents observe at least two groups that consist solely of low-cost firms, then competition between the low-cost groups will insure that they charge marginal costs. This implies that high-cost firms will be priced out of the market, an even stronger efficiency result. On the other hand, when groups consist of firms with different marginal costs and groups are composed of the same proportions of the respective cost-types, then all firms charging marginal cost is again the unique equilibrium. However, high-cost type firms might now survive when buyers face sufficiently high search costs such that they buy also from firms that offer high marginal cost prices.

Another point worth to be discussed concerns the rather strong assumption that firms know the information structure G in the economy. One might conjecture that such a detailed knowledge about buyers' price information might generate incentives for price discrimination by sellers. However, this is not the case. A model where firms can charge different prices to different buyers can be interpreted like many times the model presented above with just a single buyer in each model. As the main result of this paper then shows, buyers would be charged competitive prices even if we allow for perfect price discrimination.[14]

6 Learning Equilibrium Prices

This section considers the question whether sellers' prices would converge over time to the competitive or monopolistic equilibrium depending on whether buyers can direct search efforts or not. This is not a trivial question since, as the proof of theorem 2 shows, out-of-equilibrium incentives do not always point into the direction of the equilibrium. Moreover, the arguments that have to made to exclude non-competitive equilibria are sufficiently complicated to question agents' ability to coordinate 'through reasoning' on the competitive equilibrium and justify the consideration of learning mechanisms.

To study convergence and learning over time, I consider the following dynamic version of the model. Let time be divided into discrete periods $t = 1, 2, \ldots$ which are called days. In the morning of each day t, all J firms simultaneously commit to a price. Thereafter, a cohort of I new buyers enters the economy. Buyers receive information about the prices posted by sellers in the morning and search optimally for a low-priced offer during the day. At the end of the day all buyers purchase and then leave the economy.

It is assumed that buyers inelastically demand one unit of the product up to a reservation price of $p^* \in \aleph$. Price-inelastic demand makes price increases particularly attractive and, thus, gives competitive prices the lowest chance.

Sellers are assumed to choose their prices from a finite price grid $\{0, 1, 2, ..., p^*\}$ according to a simple learning rule which displays elements of adaptation, experimentation, and inertia (Marimon and McGrattan (1995)):

[14] The same holds of course when discrimination is restricted between different groups of buyers.

- with probability ε^i the seller remains inert and just charges the price of the previous day
- with probability $(1 - \varepsilon^i)$ the seller changes its price as follows:
  - with probability ε^i he experiments and chooses with uniform likelihood one of the prices from the price grid.
  - with probability $(1 - \varepsilon^i)$ the seller chooses the price which generated the highest weighted average profit in the past, where profits that have been made τ periods ago enter with weight β^τ into the average.

Note that all that sellers must observe to implement this rule is their own history of prices and associated profits. In particular, they neither have to know the group structure G nor buyers' demand functions, as assumed in the previous section.

Simulations were run for $n^i = 1$ and $n^i = 2$ with the parameter values shown in table 1. There are many more buyers than sellers to insure that actual profits are roughly in line with expected profits. When $n^i = 2$, there are 5 firms in each group. The allocation of sellers to groups is chosen randomly for each buyer.[15] Sellers are inert in 10% of the cases and experiment with prices in 5% of the times, given that they are not inert. Higher levels of inertia just slow down the speed of convergence without altering the results. The experimentation rate of 5% is relatively high and will act as exogenous noise that impedes complete convergence to the theoretically predicted equilibria, which seems a good test for the robustness of our predictions. Furthermore, we let sellers discount past observations at rate 0.9 to allow them to track changing environments and to increase the speed of convergence. Finally, the monopoly price is set to $p^* = 50$,search costs are set equal to $c = 10$, and utility is simply linear in prices.

[15] However, the random assignment of sellers to groups is done once at the begining of the simulation and then kept unchanged during the course of each simulation.

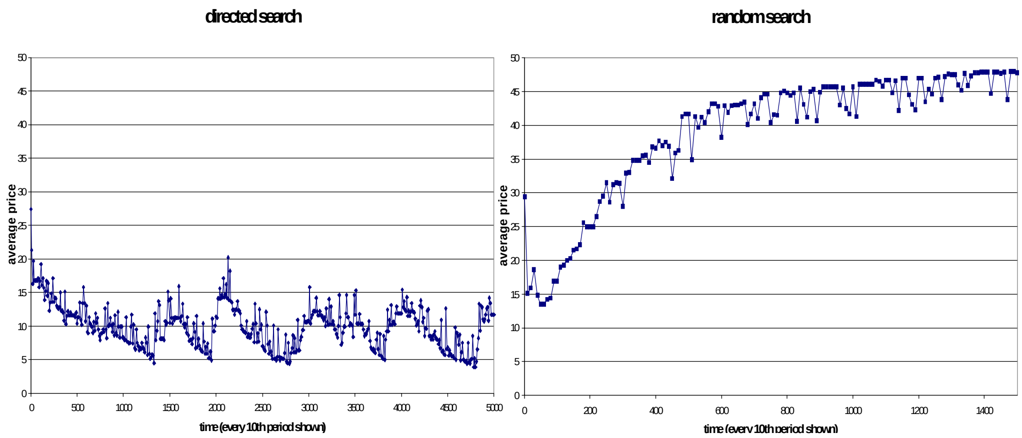


Figure 5: Simulation Results

J	$=$	10
I	$=$	250
ε^i	$=$	10%
ε^e	$=$	5%
β	$=$	0.9
p^*	$=$	50
$u^i(p,s)$	$=$	$-p-c\cdot s$
c	$=$	10

Table1: Parameter Values for Simulation

The panel on the right of figure 5 shows the evolution of the average price charged by firms for a typical simulation outcome for the case that buyers search randomly for price offers ($n^i = 1$ for all i). In period 1 prices are close to 25, the mean of the price range, because sellers initially choose a random price. In the periods immediately following, prices seem to decrease relatively fast but then become stationary, revert direction and slowly start to climb up until they are close to the monopoly price. Once prices are close to the monopoly level, they remain there.

The reason for the observed price swing is readily explained. In period 1 price dispersion is likely to be large due to the randomness of initial prices. Since search costs are quite small relative to the price range, the reservation price of buyers is relatively low such that many high-price sellers are offering above buyers' reservation price and do not make any sales. When experimenting with new prices these sellers quickly find out that lower prices yield positive sales and hence positive profits. Therefore, prices decrease initially. Once prices are such that all firms are selling, experimentation by low-price firms reveals to them that they can raise prices slightly without loosing customers. Prices therefore slowly start to climb up in the manner described by Diamond (1971). Exogenous experimentation prevents full convergence to the monopoly price level.

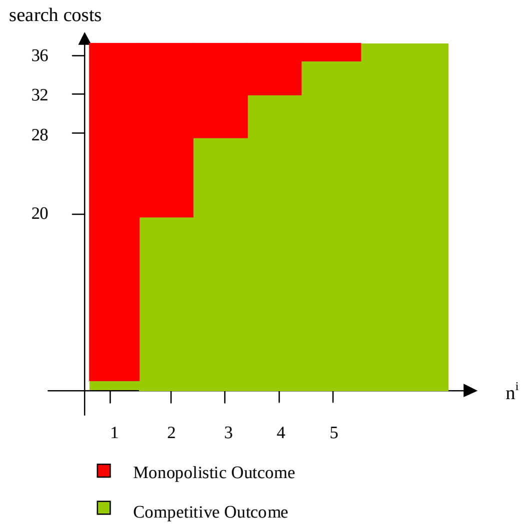
The panel on the left-hand side of figure 5 shows the evolution of the average price for a typical simulation outcome when buyers can direct search efforts ($n^i = 2$ for all i). Prices start close to the mean of the price range and then drop rather fast to a lower level. The reason for the price drop is the same as in the random search case. Yet, prices continue to decrease. Sellers who experiment with prices find out that by lowering the price they can attract more customers and make higher profits despite the price cut. Prices decrease until they reach an average level that approximates the competitive price.[16] Occasional price increases are generated by the fact that some firms (exogenously) experiment with higher prices. This allows other firms to follow and to increase their prices as well without reducing the number of customers. This in turn signals to the first firms that the deviation has been profitable. This process goes on for a while until experimentation with lower prices starts to kick in again.

Checking for the robustness of the simulation results, one finds that the level of search costs affects the pricing outcome in an interesting way for the case that agents can direct their searches.[17] At moderately higher levels of search costs, prices increase somewhat since the price cycles that already show up in figure 5 become more pronounced. However, prices always return to levels close to the competitive price. Surprisingly, when increasing the search costs even more the limit price suddenly switches to the monopoly level. For the parameter values shown in figure 1 this happens at $c = 20$. Increasing the number of groups that buyers observe brings back prices to the monopoly result.

[16] Full convergence is prevented because of two reasons: one is exogenous experimentation with prices, the other is the discreteness of the price grid: Suppose all sellers charge a price of 2. Some seller who lowers its price to 1 doubles the expected number of customers (each buyer observes at most two groups of firms). The seller is therefore at best indifferent between a price of 2 and a price of 1. Convergence to a price level below 2, therefore, cannot be expected with the chosen price grid.

[17] The monopoly pricing result with random search is unaffected by the level of search costs as long as the step size of the price grid is suffiently small compared to the level of search costs.

Figure 6: Simulation Results with Adaptive Price Setters



The relationship between the level of search costs, the number of price signals, and the pricing outcome is illustrated in figure 6.[18]

The results of figure 6 indicate a role for the level of search costs and the number of groups that has not been captured by the equilibrium analysis in section 4. Search costs and the number of groups matter for whether one observes convergence to equilibrium or not. The reason for this result is that these variables influence the price-elasticity of arrival rates of buyers *out of equilibrium*.

Consider a firm who reduces its price, e.g. due to exogenous experimentation. The price cut will lower the buyers's reservation prices and generate additional demand for the firm's group. When search costs are small, the buyers' reservation utilities follow rather closely the utility offered by the price-cutting firm. This implies that the price cut may cause some of the other firms in the *same* group to offer at prices that are now too high for buyers to purchase. This generates additional demand for the price-cutting firm. When search costs are sufficiently high this effect does not show up. Then each seller in the group profits equally from the price cut, which implies a lower price-elasticity of arrival rates of buyers and less incentives for price reductions.

[18]Prices only approximate competitive prices in the light-shades area, similar to the left-hand side panel of figure 5.

Similarly, when buyers observe more groups, the number of firms per group decreases. This implies that buyers' reservation utilities follow much closer the utility offered by a price-cutting firm. Increasing the number of groups has therefore the same effect as decreasing the level of search costs.

The above arguments also explain why prices rather discontinuously switch to monopoly prices: Additional demand for a price-cutting firm will only be generated below a critical level of search costs or number of firms in the group. Above this critical level the elasticity of arrival rates will be equally low, independent from how much these variables exceed the critical level.

Although the level of search costs and the number of groups that buyers observe do not have an effect on the price equilibrium, as long as each buyer observes at least two groups, these variables influence whether one obtains convergence to equilibrium or not. This indicates a new and important role for these variables that is missed by a pure equilibrium analysis.

7 Conclusions

When buyers receive a single imperfect information signal that reveals the distribution of prices charged by firms in the market and when buyers search sequentially, then the unique equilibrium outcome is given by all firms charging the monopoly price.

I showed that this result breaks down when buyers are endowed with slightly more information and can direct their search efforts towards different subgroups of firms. Convergence to equilibrium depends on the level of search costs and the degree of information imperfection in the economy. For low to moderate levels of search costs, sellers can learn to charge prices that approximate the competitive equilibrium.

In future work I will check whether the learning model's prediction about the relation between search costs, information frictions, and the convergence to competitive equilibrium receives support in the experimental laboratory where human subjects play the role of buyers and sellers.

8 Appendix

Lemma 2 *The reservation utility R_n^i is a continuous function of the prices offered by the firms in G_n^i.*

Proof: Since utilities $u(p^j)$ are continuous and strictly decreasing in prices, I consider changes in utilities instead of changes in prices. Defining $u^j = u(p^j)$ I can rewrite the right-hand side of (2) as

$$c^i \equiv G(R_n^i, \{u^j\}_{j \in G_n^i}) \tag{4}$$

Since $G(\cdot, \cdot)$ is continuous, the graph of values $(R_p^i, \{u^j\}_{j|p^i(j)=p})$ fulfilling (4) is closed. Since R_n^i is unique for $c^i > 0$ and given $\{u^j\}_{j \in G_n^i}$, see Weitzman (1979), this establishes the claim.■

Proof of theorem 2: Suppose, for contradiction, that firm j charges a price $p^j > 0$ and is selling. Call buyer i a potential customer of j whenever

$$I_{\{C1(i,j)\}} I_{\{C2(i,j)\}} = 1 \tag{5}$$

Since (5) is a necessary condition for selling, I know that there is at least one potential customer. Now consider the following exhaustive list of cases and subcases:

1. At least one potential customers of firm j is indifferent between the group firm j is part of and other groups of firms.
   (a) j charges above the competitive price
   (b) j charges the competitive price
2. All potential customers of j have a strict preference for the group of firm j.
   (a) There is no potential customer for which $R_{\max}^i = u(p^j)$.
      i. j charges below the monopoly price
      ii. j charges the monopoly price
   (b) There is a potential customer for which $R_{\max}^i = u(p^j)$

Suppose case 1-a holds: Firm j can increase its profits by slightly lowering its price. Customers that were formerly indifferent between different groups will then strictly prefer the group of firm j. This increases firm j's demand and profits by a discrete amount. At the same time the loss per sale from the price reduction can be made arbitrarily small by choosing a sufficiently small price cut.

Case 1-b cannot apply to firm j but will be useful in the latter part of the proof.

Next, suppose case 2-a-i holds: A sufficiently small price increase of firm j will increase its profits. Since buyers' reservation utilities are continuous in sellers prices (see lemma 2 in the appendix) and reservation utilities are discrete random variables which take on a finite number of values, a small enough price increase will not affect the strict preference of the potential customers for j's group, formally: condition $C1(i,j)$ will hold and $m^i = 1$ for every potential customer i even after a sufficiently small price increase of firm j. By the very same logic, condition $C2(i,j)$ will continue to hold for every potential customer for a sufficiently small price increase. Furthermore, the number of firms that offer above i's reservation prices, $r^{i,j}$, stays unchanged for a small enough price increase. As a result, the price increase does not lead to a loss of potential customers or to a decrease in their arrival probabilities. At the same time, it strictly increases profits per sale and thereby expected profits.

Cases 2-a-i and 2-b do not lead to a direct contradiction. The reason is that a small price increase either reduces profits per sale (case 2-a-i) or leads to a loss of potential customers (case 2-b). As the following arguments show, both cases imply that there exists a firm k charging a price $p^k < p^j$ that is also selling in equilibrium.[19] Moreover, for the case 2-b we also have that $p^k < p^j - \delta$ for some $\delta > 0$:

- Consider case 2-a-i: since firms in other groups charge at most the monopoly price (lemma 1) and j's group is strictly preferred by the potential customers of j, the existence of firm k follows.
- Next, consider case 2-b: From the reservation price formula (2) and $R^i_{\max} = u(p^j)$ for some potential customer i it follows that there must be another firm k in the same group as j that charges a price for which

[19] More precisely, there is a firm k charging a price $q^k < q^j$ and there exists a price realization for the mixed strategies of other firm such that k is making positive expected sales.

$u(p^k) \geq u(p^j) + c^i$. Since $c^i > 0$ for all $i = 1, \ldots, I$ and since $u(\cdot)$ is continuously differentiable on $[0, p^*]$, this implies that $p^k \leq p^j - \delta$ for some $\delta > 0$ that does not depend on j or i. The intuition is clear: since buyers face strictly positive search costs there must be a strictly better price out there when buyers optimally reject some price.

For firm k we can distinguish the same list of cases and subcases as for firm j above. One finds that

- Case 1-a: Firm k can increase profits by charging a slightly lower price by the same arguments as put forward for firm j in Case 1-a above.
- Case 1-b: Firm k can make strictly positive expected profits by increasing its price slightly above the competitive level. There is at least one potential customer of k who strictly prefers the group of firm k to other groups (this is the same buyer who strictly preferred the group of firm j in case 2 above). Moreover, this potential customer has a reservation utility strictly below $u(p^k)$ (due to the fact that $u(p^j) < u(p^k)$). By the same arguments as put forward for firm j in case 2-a-i above, k can exploit this buyer's strict preference and charge a sufficiently small but strictly positive price without loosing the customer.
- Case 2-a-i: cannot apply anymore
- Case 2-a-ii: Firm k can increase profits by charging a slightly higher price by the same arguments as put forward for firm j in case 2-a-ii above.

The only case that does not lead to a direct contradiction is case 2-b. However, as before, 2-b implies that there exists a seller l with $p^l < p^k - \delta$ for which the same arguments can be made as for k. Repeated occurrence of case 2-b will lead to a firm must charge below the competitive price, which contradicts equilibrium, see lemma 1.■

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