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Promotions and Training: Do Competitive Firms Set the Bar too High?

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Abstract

Firms use promotions to match workers with jobs that fit their ability, but also to provide incentives to exert on-the-job training effort. As promotions make workers more attractive in the labor market, firms will balance productivity and retention costs. I show that if workers exert firm-specific training effort, profit-maximizing firms that cannot commit to promotion rules promote fewer workers than efficient. Differently, if firms can commit to promotion bars, for instance by means of structured managerial practices, they set the bar efficiently. If workers acquire portable training, this directly increases retention costs. Firms that cannot commit to promotion bars will set them inefficiently high. In this case, workers are discouraged from training when competition for talent is fierce. If firms can commit to promotion bars, they set them lower than without commitment providing strong incentives for workers to acquire portable training. However, in this scenario the promotion bar may be set too low compared with the efficient talent allocation.

JEL Classification: D86, M51, M52, M53.

Keywords: Promotions, on-the-job training, poaching, career concerns.

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1 Introduction

In talent-intensive industries, workers' productivity crucially depends on talent and on-the-job training. Firms often provide workers with financial incentives to exert training effort.[1] Nonetheless, in many cases, promotion-based incentives are more important than simple wage raises.[2] Promotions serve two purposes: the provision of incentives for workers to exert training effort and workers' matching with jobs for which they are best suited. This is especially the case in sectors in which the rigidity of wage schemes (for instance, the public sector), does not allow to reward workers executing the same job with too different salaries. However, the productive benefits of promotions contrast with the drawback that it is more expensive to retain promoted workers, who become more attractive in the labor market.[3] This generates a trade-off for firms between productivity and retention costs.

This paper analyzes the relationship between firms' promotion decisions and workers' on-the-job specific and portable training effort in competitive labor markets.After training and promotions, workers may receive job offers by competing firms, thus imposing retention costs on the current employer (see for instance, Lazaer, 1986 and Kim, 2014). Henceforth, competition for workers affects talent allocation within organizations (Waldman, 1984; Greenwald, 1986; Bernhardt, 1995; Mukherjee and Vasconcelos 2018; Picariello, 2019). By affecting talent allocation, firms' competition for talent may also have an impact on workers' incentives to exert training effort.

I study a model featuring learning about workers' talents and unobservable training effort. Firms produce their output by means of two different tasks: one talent-sensitive, the other not. Before being assigned to either task, workers undertake a training stage, whose outcome provides a signal about their talents, based on which firms make their promotion decisions. Specifically, firms infer workers' talent by observing this signal and promote those whose outcome fulfills a minimum cutoff referred to as *promotion bar.*

I consider two cases: one in which the firm does not commit to promotions before workers exert training effort and one in which it does. For instance, such commitment may

[1]Lazear, 2000; Gaynor, et. al, 2004; Kahn, et. al, 2016; Friebel, et. al, 2017 document the effectiveness of pay-for-performance incentive schemes

[2]Baker, Jensen and Murphy (1988), Baker, Gibbs and Holmström (1994a) and (1994b) document empirically the prevalence of promotion-based incentive schemes in hierarchical organizations. McCue (1996) shows that promotions account for around 15% of wage growth in organizations.

[3]For instance because they become more visible as in Milgrom and Oster (1987).

be achieved by adopting structured managerial practices clearly stating the performance standards to obtain promotions or because workers' output and talent are more or less observable and measurable in the economy, so that contingent contracts are enforceable.

Promoted workers acquire the human capital to execute talent-sensitive tasks and can be poached by competing firms. If profit-maximizing firms cannot commit to promotion bars and compete more fiercely for promoted workers than for nonpromoted ones, they set promotion bars inefficiently high. Namely, firms promote fewer workers than is efficient, and the stronger competition for promoted workers, the more severe is this inefficiency.

On-the-job training is either firm-*specific* or *portable* (Groysberg, et al., 2008; Groysberg, 2010). The former enhances workers' productivity only in the current firm, the latter is valuable for any firm in a certain industry, hence it increases workers' attractiveness on the labor market and raises retention costs. Since firm-specific training effort inflates the signal generated at the training stage, it does not affect firms' selection of promotion bars. I show that if firms are willing to promote some workers, effort to acquire specific training monotonically increases with the degree of competition for promoted workers, as this implies higher expected wages conditional on promotion. If instead, firms do not promote workers at all, workers have no incentive to acquire specific training, making effort discontinuous with respect to labor market competition in equilibrium.

Differently, workers' portable training effort, by raising retention costs, affects firms' promotion decisions. When unable to commit ex-ante, profit-maximizing firms set the bar increasingly high with respect to both labor market competition and workers' effort, to shield themselves against aggressive poaching raids. This generates two contrasting effects: on the one hand, high promotion bars imply high expected wages for workers, conditional on promotion, so that they are eager to exert training effort to raise the signal sent to the market; on the other hand, workers are less likely to hit too high promotion bars, hence they have weak incentives to exert effort as on-the-job training is valuable only upon promotion and the cost of it may be sunk. For this reason, portable training effort is nonmonotonic with respect to labor market competition in equilibrium. This provides a novel result, as the existing literature (Becker, 1962; Rosen, 1972; Garmaise, 2011) postulates that workers have strong incentives to exert portable training effort in fiercely competitive environments. However, if profit-maximizing firms react to labor market competition by setting the bar very high, workers have weak incentives to exert effort when competition is fierce, as they have low chances of being promoted.

Finally, I show that if firms commit to promotion bars before workers' training, for instance by adopting structured managerial practices tying promotions to certain performance standards and thus signing long-term agreements with workers, the latter have stronger incentives to exert on-the-job training effort. Inefficiencies in promotions and portable training effort exertion are reduced or reversed with respect to the case without commitment. If workers exert specific training effort, firms commit to promote workers efficiently. If workers exert portable training effort, instead, the optimal promotion bar is nonmonotonic with respect to the degree of competitiveness for promoted workers. Firms internalize the fact that promotion bars affect workers' incentives to exert effort and to work for a certain employer at the hiring stage. If labor market competition is low, workers have weak incentives to exert training effort, as they secure a small share of the realized surplus, hence their expected wages are too low. In this case, firms strengthen incentives by committing to very low promotion bars, setting them below the efficient benchmark. This raises expected wages by making promotions more affordable in expectation, so that workers exert more effort than without commitment at the cost of productive inefficiency. When instead, competition for workers is sufficiently fierce, a worker secures a large share of surplus, so that the market itself provides strong incentives. But in this case, it is not profitable for firms to provide incentives for workers to exert effort, as retention costs would be extremely high, hence they set the bar higher than the efficient benchmark, to shield themselves against too aggressive poaching raids. In this scenario, workers exert inefficiently too much training effort with respect to the efficient benchmark.

To sum up, portability of training effort and firms' inability to commit to promotion bars imply a discouragement effect for workers, who exert low training effort in competitive environments as they expect firms to set the bar for promotion too high. However, if firms can commit to promotion bars before workers' training and labor market competition is not too fierce, promotion bars are set lower than the efficient benchmark. If labor market competition is sufficiently fierce, firms commit to inefficiently high promotion bars, to induce workers to exert less effort, namely, to reduce retention costs. Workers exert more portable training effort when firms commit to promotion bars than when they do not, for any degree of competitiveness.

The structure of the paper is as follows. Section 2 compares its contributions to those in the existing literature. Section 3 lays out the model's features and assumptions. Section 4 derives the efficient promotion rules and training effort to provide a benchmark for the following analyses. Section 4.1 shows a setting in which efficient promotions and effort are attainable. Section 5 studies workers' incentives to exert firm-specific training effort, respectively when promotions rules are set after and before training takes place. Section 6 focuses on the incentives to exert portable training effort, in settings in which promotion rules are set ex-post and ex-ante with respect to workers' training. Section 7 concludes.

2 Related Literature

This paper contributes to the literature in organizational and personnel economics studying the effects of promotion-based incentives in organizations. Specifically, it focuses on the role of promotions in competitive labor markets, both as an incentive for workers to exert training effort and as a device to allocate talent across heterogeneous tasks.

I develop a model featuring uncertain talent and learning, as in career concerns models like Fama (1980), Harris and Holmström (1982), Gibbons and Murphy(1992), Holmström (1999), and Bonatti and Hörner (2017). In my model, workers have career concerns insofar as they exert effort to improve firms' perceptions of their talent, hence their likelihood to obtain promotions. This paper focuses on promotions as a device to provide workers with incentives to exert effort, while in Holmström (1999) (and the successive literature), competitive wages serve this scope. Having promotions instead of wages allows for the analysis of inefficiencies generated by firms and to study workers' reaction to them. In Holmström (1999), the labor market is perfectly competitive at any stage of the game, while in this paper, it is only at the hiring stage and then firms compete differently for promoted and nonpromoted workers. Differently with respect to Holmström (1999), in this paper, when firms are unable to commit to promotion bars, workers' effort results in being nonmonotonic with respect to labor market competition, because the firm takes into account the impact of competitiveness by raising promotion bars. When workers exert portable training effort and competition for promoted workers is fierce, the model I present converges to the one in Holmström (1999) and in both frameworks, workers exert too much effort with respect to the efficient benchmark.

This paper is also related to the literature on job assignment within organizations, including Waldman (1984), Bernhardt (1995) and more recently, Mukherjee and Vasconcelos (2018). These papers share results of inefficient job assignment within organizations with asymmetric learning between firms. They show that if a firm has an informational advantage about the ability of its employees, it may exploit it and allocate them inefficiently across tasks. This is because competing firms perceive task allocation as a signal of workers' talent. Hence, misallocation serves to discourage poaching. In this paper this is also the case, but I show that labor market competition not only impairs efficient talent allocation, but it also affects workers' incentives to exert training effort. Hence, I show an hitherto neglected inefficiency from competition for talent. Becker (1962), Rosen (1972) and Garmaise (2011), show that workers are eager to train in competitive environments. Yet, they did not consider the impact that such competition has on firms' decisions, which in turn affect workers' incentives. Dato, et al. (2019) have provided experimental evidence that employers who can protect themselves against labor market competition by misallocating workers across tasks providing workers with heterogeneous visibility tend to do so.

The first paper analyzing workers' incentives to train over their career is Ben-Porath (1967). Carmichael (1983), studies the impact of workers' seniority and promotion ladders on firms' and workers' investments in specific human capital. Both papers study models featuring asymmetric information (adverse selection) between the firm and its employees. I analyze a model in which both firms and workers are uncertain about the latter's talent. More recently, Ashraf, et. al (2014), study the impact of promotion incentives on workers' job selection. Karachiwalla and Park (2017) provide empirical evidence of how promotions affect teachers' performances in Chinese schools. Brogaard et., al. (2017) show the impact of tenure on finance professors' research performance and risk-taking in terms of projects undergone. Bar-Isaac and Levy (2019) study a model in which promotions provide incentives for workers to exert effort, yet visibility does not depend on task allocation. I study a different model in that I compare different types of training workers may obtain and I study the firm's promotion bar as compared to an efficient benchmark, allowing to analyze the efficiency of certain promotion policies.

Prendergast (1993) studies how promotions incentivize workers to acquire firm-specific human capital, in a setting featuring moral hazard. As in this paper, Prendergast (1993) assumes workers' investment to be unobservable, hence neither firms nor workers can commit respectively to wages and investments. If different tasks in the firm are associated to different wages, then this commitment issue can be solved under certain conditions. However, my model and predictions differ from Prendergast's (1993): I analyze the possibility for workers to exert firm-specific and portable training effort and show differences in the optimal strategies adopted.

Gibbons and Waldman (1999) study wage and promotion dynamics in a setting with job assignments, specific human capital acquisitions and learning about workers' talent. The model in this paper differs from Gibbons and Waldman's (1999) in several respects and in the predictions provided. First, they consider training effort to be costless and not strategically chosen by workers; as Prendergast (1993), they focus their attention on firm-specific human capital, foregoing considerations relative to portable training; third there is completely asymmetric learning about workers' talent, namely it is revealed only to the current employer, while I consider a framework in which promoted workers' talent is revealed to all firms in the industry.

Finally, Ferreira and Nikolowa (2018), study the optimal design of long-term labor contracts in a framework in which workers have preferences over job attributes (such as perks, status, visibility). They study the optimal design of contracts to hire workers at the lowest cost possible. The authors find that the optimal contract is a long term one, in which at the beginning of their careers, workers take less desirable jobs, and later on have a chance of being promoted to more desirable ones. In this paper, I also show that long term contracts including promotion rules provide stronger incentives for workers to exert on-the-job training effort.

3 The Model

Homogeneous price-taking firms hire a continuum of measure 1 of workers with unknown talent each from a perfectly competitive labor market. Firms' production technology is labor-intensive. Namely, firms produce a positive output only if they are able to hire (at least) a worker. Let the output price be normalized to 1. All agents in the model are risk-neutral. In the first stage workers train. For simplicity, I assume that workers do not produce at the training stage.[4] As is explained below in greater detail, the training outcome provides a signal about workers' talent. This signal is observed by everyone in the industry, but may be not verifiable.

After training, workers are either *promoted* and produce via a talent-sensitive technology, or not, and produce by means of a talent-insensitive one.

Finally, assume no discounting across periods, neither for firms nor for workers.

[4] Assuming productive training would not change the qualitative results presented throughout the paper, as long as the output is constant and does not provide further information about workers' talent.

3.1 Tasks, Training and Productivity

Workers' talent is a continuous real-valued random variable to everyone in the economy. Talent η is uniformly distributed over the support $[0\,,\,\bar{\eta}]$,[5] with $\bar{\eta} \geq \frac{3}{2}$ and finite.[6]

At the first stage of the game, workers execute a training task, yielding a signal

$$y_1 = \eta + a \tag{1}$$

where $a \in \mathbb{R}_+$ is training effort and is finite. Effort cost is given by

$$\psi(a) = \frac{a^2}{2}.$$

Training can be either firm-specific or portable and effort is unobservable. In the first case, training effort is productive only in the worker's current firm; in the second case, training effort is productive for any firm in the industry.

Firms cannot observe training effort, but based on the signal received, they form conjectures about it and workers' talent:

$$\mathbb{E}(\eta|y_1) = \hat{\eta} = y_1 - \hat{a}, \quad \forall \hat{\eta} \in [0\,,\,\bar{\eta}]. \tag{2}$$

where the firm expects the workers to exert training effort $\hat{a}$.[7] However, a worker may choose $a \neq \hat{a}$ (namely, deviate from the equilibrium expectation), in order to change the firm's expectation about her talent. If $\hat{\eta} < 0$ or $\hat{\eta} > \bar{\eta}$, equation (2) yields beliefs about talent outside of the feasible range. In these cases, I assume that if $\hat{\eta} < 0$, the firm believes that the worker is not worthy of promotion, whilst if $\hat{\eta} > \bar{\eta}$, the firm believes that the workers is worthy of promotion with probability $\frac{1}{2}$, otherwise, with same probability, the

[5] Assuming the lower bound of the support of η to be negative, would make the quantitative analysis of the model more tedious, without adding any qualitative results to the ones presented throughout the paper.

[6] This assumption makes all the maximization programs concave.

[7] The outcome y_1 resembles the one in Holmström (1999). The main difference is the absence of a white noise error term. Differently with respect to Holmström (1999), the model in this paper consists of two periods and effort is exerted only in the first of these, when workers' talent is still uncertain. Hence, adding a white noise error term in y_1 would not change the qualitative results provided throughout the paper.

worker is not promoted.[8] There exists a cutoff value of η for which even an infinitesimally larger a with respect to $\hat{a}$ yields a signal $\hat{\eta} > \bar{\eta}$, thus generating uncertainty about promotion for the worker. This is given by $E \equiv \bar{\eta} - a + \hat{a} < \bar{\eta}$.

Based on the signal observed (namely on the conjecture $\hat{\eta}$), firms decide upon promotions. Workers who are promoted to the talent-sensitive task A produce

$$y_{2A} = \eta + a \tag{3}$$

whereas, nonpromoted workers execute the routinary task B and produce

$$y_{2B} = x + a, \quad \text{with } x = \mathbb{E}(\eta) = \frac{\bar{\eta}}{2}. \tag{4}$$

A worker with average talent is equally productive in doing both tasks, so that for her $y_{2A} = y_{2B}$.[9] Notice that training effort is productive both if workers are promoted and execute task A and if they are not promoted and execute task B.[10] This assumption is motivated by the fact that training is not intended to be task-specific, but is either firm or industry-specific.

Let $\theta_i \in [0\,,\,1]$ denote the probability that a competing firm tries to poach a worker executing task i. Let $\theta_A > \theta_B$, and to simplify notation, let $\theta_B = 0$ and define θ_A as just θ. [11] As is explained later in greater detail, the assumption that firms compete only for promoted workers affects the results of the model. However, such assumption can be relaxed by allowing firms to compete also for nonpromoted workers. If firms compete more aggressively for promoted workers than for nonpromoted ones (so that $\theta_A > \theta_B$), all qualitative results in this paper are preserved. If instead, competition for nonpromoted

[8]The assumption on beliefs is made for the sake of algebraic simplicity. However, changing this belief with any other possible one, would not change the qualitative results provided throughout the paper. To better understand this assumption, suppose that upon observing a signal $\hat{\eta} > \bar{\eta}$, the firm performs an investigation on the ability of the worker, and that this investigation does not provide results at $t = 2$ with probability $\frac{1}{2}$.

[9]Allowing x to be any value in the interval $[0\,,\,\bar{\eta}]$ would not change the qualitative results provided throughout the paper.

[10]This assumption does not change the qualitative insights of the model, although it allows to rule out less interesting corner results.

[11]Alternatively, one can think of this assumption as stating that nonpromoted workers are *invisible* in the sense of Milgrom and Oster's (1987) assumption.

workers is fiercer than for promoted ones (so that $\theta_A < \theta_B$), the inefficiencies shown throughout the model analysis are reversed.

3.2 Labor Contracts

The firm offers spot wage contracts to workers at each stage. Let w_1 denote the wage offered at the hiring stage, when the labor market is assumed to be perfectly competitive, henceforth, firms compete à la Bertrand to hire employees. Throughout the paper, I consider two possible frameworks: one in which firms can commit to promotion bars (hence, these are set at the hiring stage, before workers' on-the-job training takes place) and one in which this is not the case (hence, they are set after training). Furthermore, workers' talent is assumed to be nonverifiable, so that firms cannot commit to wage contracts contingent on talent.

After promotions, firms pay wages w_{2i} to workers assigned to task i, in order to retain them at the interim stage. Workers cannot commit not to leave the current employer for a competitor. Hence, ex-ante, firms earn expected profit

$$\pi = \mathbb{E}\left[\sum_{i=A}^{B}(y_{2i} - w_{2i})\right] - w_1. \tag{5}$$

Note that, given their production function, the two tasks are substitutes for firms, so that they do not need to promote workers if it is not profitable doing so.

Workers' lifetime expected utility is

$$U(w\,,\,a) = w_1 + \mathbb{E}(w_{2i}) - \psi(a) \text{ for } i = \{A\,,\,B\}. \tag{6}$$

As a result of fierce competition among firms to attract workers, w_1 is set so that workers extract all the expected surplus and firms themselves earn zero profit in expectation (namely, w_1 is driven by the firm's *zero-profit condition*). At the interim stage, the current employer can match possible external bids, thus expected wage conditional on promotion is

$$\mathbb{E}(w_{2A}) = \theta(\eta + \mathbb{I}a) \tag{7}$$

where $\mathbb{I}$ is an indicator function equal to 1 when the training effort exerted is portable and zero otherwise.

Since it is assumed that there is no competition for nonpromoted workers, firms extract all the surplus produced in task B and set $w_{2B} = 0$.

3.3 Time Line

The time line of the model includes five stages:

- $t = 0$ (*hiring stage*), firms offer w_1 to hire workers. If the promotion bar is contractible, it is set.
- $t = 1$ (*training stage*), if workers accept the wage offer, they execute a training task and exert training effort a.
- $t = 2$, training yields a signal y_1. Based on this, firms form a conjecture $\hat{\eta}$ on workers' talent.
- $t = 3$ (*promotion stage*), if the promotion bar is noncontractible, firms set it. Workers who fulfill it are promoted. Nonpromoted workers earn their reservation wage.
- $t = 4$ (*interim poaching* stage), promoted workers may leave the current firm for a competitor if an external bid occurs. The current employer can match outside offers with w_{2A}.
- $t = 5$, production is completed and revenues y_{2i} are produced, for $i = \{A\, ,\, B\}$.

3.4 Equilibrium Concept

The time line of the model features sequential actions and workers have private knowledge of the training effort they exert. The equilibrium concept studied throughout the model is *Perfect Bayesian Nash Equilibrium (PBNE).*

A PBNE is defined by wages w_1 and w_{2i}, a promotion bar $\eta^* \in [0, \infty)$, training effort a and a conjecture about such effort given by $\hat{a}$. Along the equilibrium path, despite the presence of hidden information (about training effort), information will be symmetric, as firms' conjecture $\hat{a}$ will be correct.

Moreover, if the equilibrium promotion bar η^* exceeds the upper bound of the support of workers' talents $\bar{\eta}$, it is equivalent to saying that the firm does not promote any worker.

4 Benchmark

Consider the promotion rule η^o and training effort a^o that maximize expected surplus

$$\mathcal{W} = \int_{\eta^o}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta^o)\bar{\eta}}{2} + a^o - \frac{a^{o2}}{2}. \tag{8}$$

A social planner solves:

$$\underset{\{\eta^o\,,\,a^o\}}{Max} \quad \mathcal{W}$$

Proposition 1. *The total surplus is maximized by a promotion bar* $\eta^o = \frac{\bar{\eta}}{2}$ *and training effort* $a^o = 1$

The proof for this and all other lemmas and propositions are relegated to the Appendix.

This proposition provides a benchmark to be compared with promotion bars and effort exerted in a competitive equilibrium. Notice that for the maximization of total surplus, it makes no difference whether training effort is portable or not.

Intuitively, this result points out that firms' productivity is maximized if all workers with a talent $\eta \geq \frac{\bar{\eta}}{2}$ are promoted to task A. Namely, all workers producing "above average" should be promoted.

4.1 Implementing the Efficient Allocation

Suppose firms can commit to wage contracts contingent on the signal y_1 at the hiring stage. In this case, I assume y_1 to be not only observable in the whole industry, but also verifiable in courts. Suppose at $t = 0$ the firm offers a contract $\{w_1\,,\,w_2\}$, with w_1 defined by the firm's zero-profit condition given perfect labor market competition at the hiring stage, and $w_2 = y_1 = \eta + a$ irrespective of whether the worker is promoted or not. Notice that the wage w_2 is contingent on training effort, although this may be firm-specific. In this case, at the training stage, the worker exerts the training effort that maximizes her expected utility, since w_1 is paid at $t = 0$, so that her maximization program is

$$a^* \in \underset{\{a\}}{argmax} \; \underbrace{\frac{\bar{\eta}}{2} + a}_{\mathbb{E}(w_2)} - \underbrace{\frac{a^2}{2}}_{\psi(a)} \; .$$

Proposition 2. *Assume y_1 is verifiable, then the efficient allocation $\{a^o\,, \eta^o\}$ can be implemented. If the firm offers a contract*

$$w_1 = \mathbb{E}\left[\sum_{i=A}^{B}(y_{2i} - w_{2i})\right]\ ,\ \ w_2 = y_1, \tag{9}$$

in equilibrium workers will exert effort $a^o = 1$ and firms will promote workers fulfilling the threshold $\eta^o = \frac{\bar{\eta}}{2}$. This wage schedule is such that all workers are retained for any θ.

Intuitively, if firms offer a wage w_2 equal to the signal irrespective of whether the worker is promoted or not, the latter has an incentive to acquire training efficiently. Since the wage at $t = 2$ is paid even if the worker is not promoted, when choosing the promotion bar, the firm sets it efficiently, as this will increase its profit for any level of effort of the worker. Specifically, promotion decisions do not affect w_2. Since the firm offers w_1 such that $\mathbb{E}(\pi) = 0$, this result holds both with and without commitment to promotion bars at the hiring stage. Specifically, the assumption that the profit-maximizing firms are able to commit to wages depending on the signal y_1 is sufficient to convey this outcome.

Note that the contract proposed in this section is one of many possible ones to implement the efficient allocation in equilibrium. The only condition needed to attain the efficient allocation is that y_1 is verifiable in courts, so that firms can offer contingent contracts on the signal. In the following sections, I assume y_1 to be observable by firms and workers but nonverifiable in courts, so that firms can only use promotion-based incentives.

5 Firm-Specific Training

Before production, workers undergo a training stage. Training effort is unobservable, hence firms cannot design a contract conditional on a certain level of it. This section studies how promotion decisions affect workers' incentives to exert firm-specific training effort.[12]

In this framework, effort increases y_1 used by firms as a signal of workers' talent to decide whether to promote them or not. Namely, workers exert firm-specific training effort because they are concerned about being promoted, although this will not directly affect expected wages.

[12]This case is similar to a standard moral hazard problem.

5.1 No Commitment to Promotion Bars

Suppose neither promotion bars nor wages contingent on talent are contractible. In this case, the firm sets the promotion bar after workers' training. Recall that workers' training delivers a signal $y_1 = \eta + a$, observed by everyone in the industry. After observing y_1, firms make a conjecture about workers' talent:

$$\mathbb{E}(\eta|y_1) = \hat{\eta} = y_1 - \hat{a}.$$

Recall that if the firm observes an off-equilibrium signal $\hat{\eta} > \bar{\eta}$, it will promote the worker with probability $\frac{1}{2}$.

Solving the model by backward induction, at $t = 3$ firms set a promotion bar η_s^* such that workers are promoted if $\hat{\eta} \geq \eta_s^*$. In order to choose the bar, firms solve the following maximization program

$$\eta_s^* \in \underset{\{\eta\}}{argmax}\ \pi = (1-\theta)\int_{\eta}^{\bar{\eta}} \hat{\eta} f(\hat{\eta}) d\hat{\eta} + \frac{F(\eta)\bar{\eta}}{2} + a$$

delivering the optimal promotion bar without commitment.

Lemma 1. *If firms are unable to commit to promotion bars before workers exert specific training effort, the optimal bar is* $\eta_s^* = \frac{\bar{\eta}}{2(1-\theta)} \geq \eta^o$. *For any* $\theta > \frac{1}{2}$, *firms do not promote any worker as* $\eta_s^* > \bar{\eta}$.

The optimal bar η_s^* has two interesting features: first, it is independent of workers' firm-specific training effort. Since training is only valuable for the current employer, it does not affect directly workers' equilibrium wages.

Second, it is increasing in θ, the degree of competitiveness for promoted workers. Intuitively, firms choose the promotion bar opportunistically, to reduce retention costs.[13] This behavior results in inefficient *underpromotion,* thus fewer workers are promoted with respect to the efficient benchmark, for the firm to extract as much rent as possible. Moreover, as competition for promoted workers exceeds the threshold $\frac{1}{2}$, firms find it profitable to produce only via the routine task, namely, they promote no worker. To see this, consider

[13] Waldman (1984) and Greenwald (1986) feature similar results in a model featuring asymmetric information about workers' productivity. Picariello (2017) features a similar structure to the model presented in this paper with similar results.

two polar cases: let $\theta = 0$, so that promoted workers never receive an outside offer. In this case, the firm cares only about workers' productivity and pays them a fixed wage, thus in equilibrium $\eta_s^* = \eta^o$. Then consider the case in which $\theta = 1$. In this scenario, the optimal promotion bar is $\eta_s^* \to \infty$. Intuitively, in this case, promoted workers would extract all the surplus generated, thus the firm has no incentive to promote. This result provides an important prediction: absent commitment to promotion bars, competition for talent implies inefficient production (namely, inefficient promotions), since firms trade off efficient productivity with higher profits.

At $t = 1$, workers rationally anticipate the firm's optimal promotion rule η_s^* and decide how much firm-specific training effort to exert in order to maximize their expected utility. Recall that workers' lifetime expected utility is

$$U(w\,,\,a) = w_1 + \mathbb{E}(w_{2i}) - \psi(a) \text{ for } i = \{A\,,\,B\}.$$

However, w_1 is paid at $t = 0$, so that it is a constant in the worker's maximization program. Moreover, since $\mathbb{E}(w_{2B}) = 0$, the worker solves

$$a_s^* \in \underset{\{a \geq 0\}}{argmax} \;\; \frac{\theta}{2}\left[\int_{\eta_s^*-a+\hat{a}}^{\bar{\eta}-a+\hat{a}} \Big(\eta + a - \hat{a}\Big) f(\eta) d\eta + \int_{\eta_s^*-a+\hat{a}}^{\bar{\eta}} \Big(\eta + a - \hat{a}\Big) f(\eta) d\eta\right] - \frac{a^2}{2}.$$

Note that workers care about the expected wage they could earn conditional on the signal y_1 and on promotion. This is because every firm in the industry observes y_1, hence workers have an incentive to exert specific training effort in order to increase the value of the signal they send to the market. In this way, they hope to earn higher wages upon promotion (although, along the equilibrium path, every firm will correctly conjecture workers' effort and be able to tell what their actual talent is). This maximization program yields the following result:

Proposition 3. *The optimal firm-specific training effort when firms cannot commit to promotion bars is:*

$$a_p^* = \begin{cases} \frac{\theta}{2} & \text{if } \theta \in [0\,,\,\frac{1}{2}], \\ 0 & \text{if } \theta \in (\frac{1}{2}\,,\,1]. \end{cases}$$

This proposition shows that optimal specific training effort is discontinuous. Workers have an incentive to exert positive effort as long as firms have an incentive to promote some employees and there is some labor market competition for them (namely, if $0 < \theta \leq \frac{1}{2}$). For values of $\theta \leq \frac{1}{2}$, workers have an increasing incentive to exert effort as labor market competition increases, since this results in higher expected wages conditional on promotion. Specifically, since specific effort does not affect these wages (hence, the promotion bar), workers try to increase their likelihood of promotion by inflating y_1.

However, if $\theta > \frac{1}{2}$, as stated in *lemma 1,* firms find it profitable not to promote any worker. In this case, workers have no reason to exert specific training effort, as they anticipate that the cost of their investment will be sunk. The same outcome realizes if $\theta = 0$, yet the driving mechanism is different: although firms promote workers efficiently in this scenario, the latter are locked in as no firm would bid an offer for them upon promotion. Hence, workers are indifferent between being promoted and not, thus they exert no training effort.[14]

5.2 Commitment to Promotion Bars

Consider now the case in which firms can commit to promotion bars before workers' training. At $t = 0$, firms offer contracts $\{w_1\,,\,\eta_S^{**}\}$ to hire workers. Let η_s^{**} denote the contractible promotion bar and a_s^{**} the training effort workers exert in equilibrium. Since the firm produces via a labor-intensive technology, it needs necessarily to hire some workers in order to produce a positive output. Should the firm fail to hire any worker, it would produce no output and earn no profit.

Solving the model by backward induction, the firm anticipates that workers' effort decision is independent of the promotion bar:

$$a_s^{**} = \frac{\theta}{2}. \tag{10}$$

Promotion bars change the wages firms offer and workers accept the offer providing them the highest lifetime expected utility as of $t = 0$

$$U(w\,,\,a) = w_1 + \mathbb{E}(w_{2i}) - \psi(a). \tag{11}$$

[14] Effort discontinuity results from the fact that I assume talent to be uniformly distributed. Such assumption allows to rule out the role of the distribution of talent itself in workers' choice of training effort, thus implying that firm-specific effort only depends on the competition for promoted workers. This also allows to stress uniquely the impact of portability of the acquired training on the results shown later on.

Since the labor market is perfectly competitive, w_1 equates the firm's expected profit. The latter chooses the promotion bar by solving

$$\underset{\{\eta_s^{**}\}}{Max} \;\; U = \int_{\eta_s^{**}}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta_s^{**})\bar{\eta}}{2} + a_s^{**}\left(1 - \frac{a_s^{**}}{2}\right).$$

Proposition 4. *When workers exert firm-specific training effort, and firms can commit to promotion bars at the hiring stage, the optimal effort is*

$$a_s^{**} = \frac{\theta}{2} \tag{12}$$

firms, instead, commit to the efficient promotion bar

$$\eta_s^{**} = \frac{\bar{\eta}}{2} = \eta^o. \tag{13}$$

Intuitively, at $t = 0$ firms know that promotion bars affect both retention costs (namely, w_2) and wages they can offer to attract workers (namely, w_1). Profit maximizing firms prefer being productive and to do so, they need to hire workers, so they commit to the promotion bar that maximizes workers' lifetime expected utility as of $t = 0$. As a result, in equilibrium, firms commit to the surplus maximizing promotion bar, although workers' effort decision is independent of it.

Since specific training effort does not affect promotion bars, but only the signal y_1, workers' optimal effort only depends on the degree of competitiveness for promoted workers, as it raises expected wages upon promotion.

Differing from the case without commitment, firms commit to promote a constant positive share of workers for any level of labor market competition. Hence, workers have an incentive to exert positive training effort in equilibrium, for any $\theta > 0$.

Note that, for any value of θ, effort is inefficiently low, yet for any $\theta > \frac{1}{2}$, firms' ability to commit to promotion bars, allows for increasingly more efficient training effort with respect to the case without commitment, since workers have an incentive to exert strictly positive training effort.

6 Portable Training

Consider now the case in which workers exert portable training effort, increasing their productivity in any firm in the industry. In this framework, workers' effort affects firms' choice of the promotion bar, as it directly increases retention costs for any degree of labor market competition.

6.1 No Commitment to Promotion Bars

If firms cannot commit either to wages contingent on talent, or to promotion bars, the latter are set after workers' training. After having observed the signal y_1, firms promote workers whose conjectured talent fulfills

$$\hat{\eta} = y_1 - \hat{a} \geq \eta_p^*$$

where η_p^* denotes the noncontractible optimal promotion bar. At $t = 3$, firms set a bar for every possible level of training effort in order to maximize their expected profit:

$$\eta_p(a_p) \in \underset{\{\eta\}}{argmax} \;\; \pi = (1-\theta)\int_{\eta}^{\bar{\eta}} (\eta + a_p) f(\eta) d\eta + F(\eta)\left(\frac{\bar{\eta}}{2} + a_p\right).$$

Lemma 2. *If firms cannot commit to promotion bars and workers exert portable training effort, the profit-maximizing promotion bar is*

$$\eta_p(a_p) = \frac{\bar{\eta}}{2(1-\theta)} + \frac{\theta a_p}{1-\theta} \geq \eta_s^* \geq \eta^o$$

for any effort $a_p \geq 0$ *and* $\theta > 0$.

Firms' best reaction is a promotion bar $\eta_p(a_p)$, increasing with workers' portable training effort and with the probability for promoted workers to receive an offer from a competing firm, θ.

As in Section 5.1, firms cope with a trade off between productivity and high retention costs. For this reason, when unable to commit to bars, they promote fewer workers with respect to the efficient benchmark, in order to reduce retention costs. Both competition for promoted workers and portable training effort increase such costs, hence, promotion bars. Intuitively, if θ increases, promoted workers are more likely to receive an offer from a competing firm, hence their expected wage increases. Furthermore, since workers' training effort is portable, it also increases their expected wage upon promotion, for any θ, as it makes them more productive within the whole industry. These two effects, combined together, imply a stronger inefficiency in promotions without commitment to bars. Furthermore, if competition for promoted workers is too fierce, firms may find it profitable not to promote workers at all.

At $t = 1$, workers take into account the firm's reaction to their training and exert the portable training effort that maximizes their expected utility

$$U(w\,,\,a) = w_1 + \mathbb{E}(w_{2i}) - \psi(a) \text{ for } i = \{A\,,\,B\}.$$

At the training stage, w_1 has already been paid, hence is a constant for the worker, whilst $\mathbb{E}(w_{2B}) = 0$. So the worker's maximization program is

$$a_p^* \in \underset{\{a_p \geq 0\}}{argmax}\ \frac{\theta}{2}\Bigg\{\int_{\eta_p(a_p)-a_p+\hat{a}}^{\bar{\eta}-a_p+\hat{a}} (\eta + a_p - \hat{a})f(\eta)d\eta + \big[F\big(\bar{\eta}-a_p+\hat{a}\big) - F\big(\eta_p(a_p) - a_p + \hat{a}\big)\big]a_p +$$

$$+\int_{\eta_p(a_p)-a_p+\hat{a}}^{\bar{\eta}} (\eta + a_p - \hat{a})f(\eta)d\eta + \big[1 - F\big(\eta_p(a_p) - a_p + \hat{a}\big)\big]a_p\Bigg\} - \frac{a_p^2}{2}$$

and provides the following results:

Proposition 5. *If firms cannot commit to promotion bars, in equilibrium:*

1. *workers' portable training effort is*

$$a_p^*(\theta) = \frac{\bar{\eta}\theta\big[3(1-\theta)^2 - 1\big]}{(2\bar{\eta} - \theta)(1-\theta)^2 + 2(2-\theta)\theta^2} \tag{14}$$

which is nonmonotonic with respect to θ and zero for $\theta = 0$ or θ sufficiently large.

2. *The profit-maximizing promotion bar is*

$$\eta_p^*(\theta) = \frac{\bar{\eta}}{(1-\theta)}\left[\frac{1}{2} + \frac{\theta^2\big[3(1-\theta)^2 - 1\big]}{(2\bar{\eta}-\theta)(1-\theta)^2 + (2-\theta)2\theta^2}\right] \tag{15}$$

which is larger than η^o for any θ and larger than $\bar{\eta}$ for any $\theta > \frac{1}{2}$.

When workers exert portable training effort, firms raise promotion bars as a response to such effort. This happens because training effort and competition for promoted workers raise retention costs at the interim stage. If firms compete too fiercely for promoted workers (namely, if $\theta \geq \frac{1}{2}$), the former will not promote any worker in equilibrium, thus preferring to produce only by means of routine jobs.[15]

Workers rationally anticipate that in equilibrium, firms will increase the promotion bar when they exert more effort. This implies that portable training effort is driven by two contrasting forces. On the one hand, as θ increases, the expected wage conditional on promotion increases. Thus, workers want to increase the signal y_1 to improve their likelihood of promotion. I refer to this as the *"expected wage effect"*. On the other hand, firms react more fiercely to workers' effort when θ is high, since fierce competition imposes a strong externality on firms promoting workers, as it implies high retention costs. Thus, the firm shields itself against competition by setting the promotion bar so high that workers are discouraged from exerting effort, as their chances of obtaining a promotion are low and the cost of effort will most likely be sunk. I refer to this as the *"discouragement effect"*.

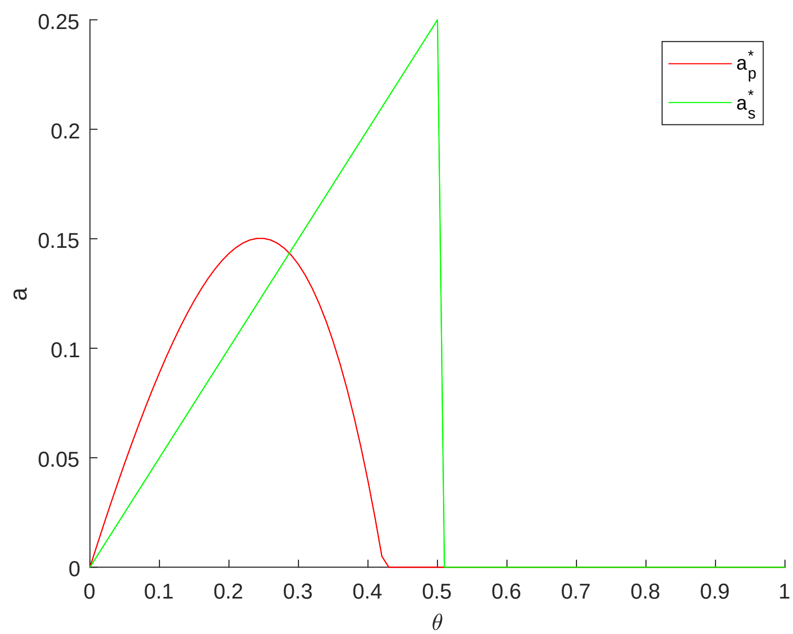

Figure 1: Portable training effort ($\bar{\eta} = 4$).

[15]As in Section 5.1, if $\theta \to 1$, firms set $\eta_p^* \to \infty$.

This implies that optimal effort is nonmonotonic with respect to the degree of competition for promoted workers: for low values of θ, the expected wage effect dominates and effort is increasing up to a cutoff value $\hat{\theta}$; as θ exceeds $\hat{\theta}$, the discouragement effect dominates and effort decreases towards zero, as shown in Figure 1. The figure also shows that portable training effort is always below the efficient benchmark ($a^o = 1$).

Figure 1, compares portable and specific training effort in the absence of commitment to promotion bars. When competition for promoted workers is low, workers exert more portable effort than the specific one, because the former not only inflates the signal y_1, but also raises expected wages conditional on promotion. However, as competition becomes fiercer, the discouragement effect implies that portable training effort falls toward zero, whilst specific training effort increases up to when firms do not promote any worker. When $\theta \geq \frac{1}{2}$, workers do not exert either firm-specific or portable training effort.

This result provides a novel prediction on portable training effort in competitive labor markets. The existing literature (Becker, 1962; Rosen, 1972; Garmaise, 2011) postulates that workers are eager to exert portable training effort in more competitive environments, as it increases their expected wage. The model presented hereby, instead, predicts that in highly competitive labor markets, workers may not be willing to exert portable training effort due to the impact of competition on firms' promotion decisions. The innovation in this model with respect to the existing literature is that not only workers, but also firms react to labor market competition. This implies that in highly competitive environments, firms set promotion bars very high in order to lower retention costs, thus making promotions less likely for workers who would then have no incentive to exert training effort, as its costs will most likely be sunk.

6.2 Commitment to Promotion Bars

Consider now the case in which firms commit to promotion bars at the hiring stage. At $t = 0$, firms offer contract $\{w_1\,,\,\eta_p^{**}\}$ to hire workers. Let η_p^{**} denote the optimal promotion bar and a_p^{**} optimal training effort in this framework.

Solving the problem by backward induction, first consider workers' choice of effort $a_p(\eta_p)$ that maximizes their expected utility at $t = 1$. Since w_1 is constant at this stage and $\mathbb{E}(w_{2B}) = 0$, workers' maximization program is

$$a_p(\eta_p) \in \underset{\{a \geq 0\}}{argmax} \ \frac{\theta}{2}\left\{ \int_{\eta_p - a + \hat{a}}^{\bar{\eta} - a + \hat{a}} (\eta + a - \hat{a}) f(\eta) d\eta + \big[F\big(\bar{\eta} - a + \hat{a}\big) - F\big(\eta_p - a + \hat{a}\big)\big] a + \right.$$

$$\left. + \int_{\eta_p - a + \hat{a}}^{\bar{\eta}} (\eta + a - \hat{a}) f(\eta) d\eta + \big[1 - F\big(\eta_p - a + \hat{a}\big)\big] a \right\} - \frac{a^2}{2}$$

Lemma 3. *If firms commit to promotion bars at $t = 0$, workers' optimal effort is*

$$a_p(\eta_p) = \frac{\theta(3\bar{\eta} - 2\eta_p)}{2\bar{\eta} - \theta}$$

for any promotion bar η_p.

The higher the promotion bar η_p, the lower is workers' effort, as a consequence of the discouragement effect.

Given perfect labor market competition at the hiring stage, firms earn zero expected profit and commit to the promotion bar that maximizes workers' lifetime expected utility:

$$\underset{\{\eta_p^{**}\}}{Max} \ U = \int_{\eta_p^{**}}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta_p^{**})\bar{\eta}}{2} + a_p^{**}(\eta_p^{**}) \left(1 - \frac{a_p^{**}(\eta_p^{**})}{2}\right).$$

The following proposition describes firms and workers' equilibrium behavior.

Proposition 6. *If firms can commit to promotion bars at $t = 0$ and workers exert portable training effort, in equilibrium:*

1. *the promotion bar is*

$$\eta_p^{**}(\theta) = \frac{\bar{\eta}}{2}\left[\frac{4\bar{\eta}^2 - 12\bar{\eta}\theta(1-\theta) + 5\theta^2}{\theta^2 + 4\bar{\eta}[\bar{\eta} - \theta(1-\theta)]}\right], \tag{16}$$

   nonmonotonic with respect to θ: there exists $\theta^o \equiv \frac{\bar{\eta}}{2\bar{\eta}+1} > 0$ such that $\eta_p^{**} < \eta^o$ for any $\theta \in (0\,,\,\theta^o]$; $\eta_p^{**} > \eta^o$ for any $\theta \in (\theta^o\,,\,1]$ and $\eta_p^{**} = \eta^o$ if $\theta = 0$ or $\theta = \theta^o$;

2. *workers exert effort*

$$a_p^{**}(\theta) = \frac{3\theta\bar{\eta}}{2\bar{\eta} - \theta} - \frac{2\bar{\eta}\Big[\bar{\eta}(4\bar{\eta} - 12\theta(1-\theta) + 5\theta^2)\Big]}{(2\bar{\eta} - \theta)\Big[2\theta^2 + 8\bar{\eta}(\bar{\eta} - \theta(1-\theta))\Big]}. \tag{17}$$

Compare now the equilibrium outcomes with and without commitment to promotion bars.

First, consider the optimal training effort exerted in the two scenarios. *Figure 2* depicts levels of effort a^o, a_s^{**}, a_p^* and a_p^{**} as functions of θ. When the firm commits to promotion bars, workers exert more portable training effort than the firm-specific equivalent (namely, $a_p^{**} \geq a_s^{**}$). This happens because portable training not only increases the signal y_1, but it also directly increases workers' expected wages upon promotion. Namely, portability itself provides incentives to exert training effort, for any degree of competitiveness at the interim stage.

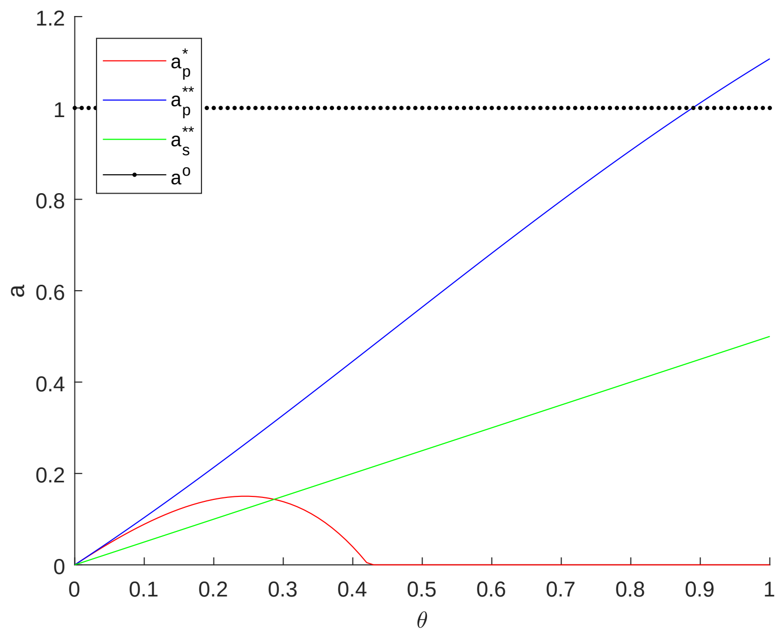

Figure 2: Portable training effort (ex-ante vs ex-post), $\bar{\eta} = 4$

Moreover, the ability of firms to commit to promotion bars, makes workers' portable training effort increasing with θ, and always larger or equal to the one exerted absent commitment to promotion bars. Namely, $a_p^{**} \geq a_p^*$ for any $\theta \in [0\,,\,1]$.

Interestingly, there exists a strictly positive degree of competitiveness for promoted workers θ^*, at which, if firms commit to promotion bars at $t = 0$, workers exert efficient portable training effort. For any θ smaller than θ^*, a_p^{**} is inefficiently low, whilst for any θ larger than θ^* workers exert too much portable effort with respect to the efficient benchmark. As θ increases, workers expect to secure a larger share of the surplus produced, henceforth they have stronger incentives to exert effort: it enhances their likelihood to be promoted, but also the wage they could be offered by competing firms. However, when poaching raids happen more frequently (namely, when the competition for promoted workers is very fierce), workers have an incentive to exert too much effort. This result resembles the one in Holmström (1999), showing that career concerns provide too strong incentives for workers to exert effort.[16] In this paper, promotion bars and competition for talent provide these incentives. However, I show this result in a two-period framework, whereas Holmström (1999) studies an infinitely repeated game.

To sum up, firm's commitment to promotion bars increases efficiency in effort exertion up to a certain cutoff θ^*. When firms compete more aggressively for promoted workers, the latter have too strong incentives to exert effort which in equilibrium exceeds the efficient benchmark. As compared to the case without commitment, when firms commit to promotion bars, workers exert more efficient effort when θ is sufficiently low. Otherwise, when θ is too high, the inefficiency persists but is reversed, as workers exert too much effort. One interesting prediction is that workers should obtain faster promotions in firms where structured managerial practices are in place, or workers' performance is observable across firms. This result makes room for policy discussion: in talent-intensive industries, many labor contracts feature non-compete clauses, preventing workers from accepting job offers from firms competing with the initial one (Samila and Sorenson, 2011; Prescott, et., al., 2016). These clauses lock workers in by endogenously reducing competitiveness. In the model shown hereby, firms could use such covenants in order to reduce workers' portable training effort, leading it to the efficient level, even in highly competitive environments. Clearly, such clauses should not be too binding, otherwise workers would have no incentive to train. The firm should fine tune them in order to achieve efficiency.

Regarding promotion bars, *Figure 3* shows η_p^*, η_p^{**} and η^o as functions of θ. Notice that η_p^* is larger than or equal to η_p^{**} for any θ. Specifically, when committing to a promotion bar at the hiring stage, the firm sets it always below the one it would choose after the training stage. This is because at $t = 0$ the firm internalizes the impact its choice has on workers' effort and on the ability to attract the latter.

The promotion bar η_p^{**} is nonmonotonic with respect to θ. There exists a cutoff value of θ denoted $\theta^o \equiv \frac{\bar{\eta}}{2\bar{\eta}+1}$ such that for any $\theta \in (0\,,\,\theta^o)$, the firm commits to promotion

[16] If workers exert portable training effort and the labor market becomes increasingly competitive, the framework shown in this paper becomes similar to the one in Holmström (1999), where workers are entitled to higher wages, the higher the effort they exert one period before compensation, for any talent. The fact that in Holmström (1999), effort raises upwards workers' expected productivity, translates into higher wages, similar to the model I present, although the driving mechanism is different.

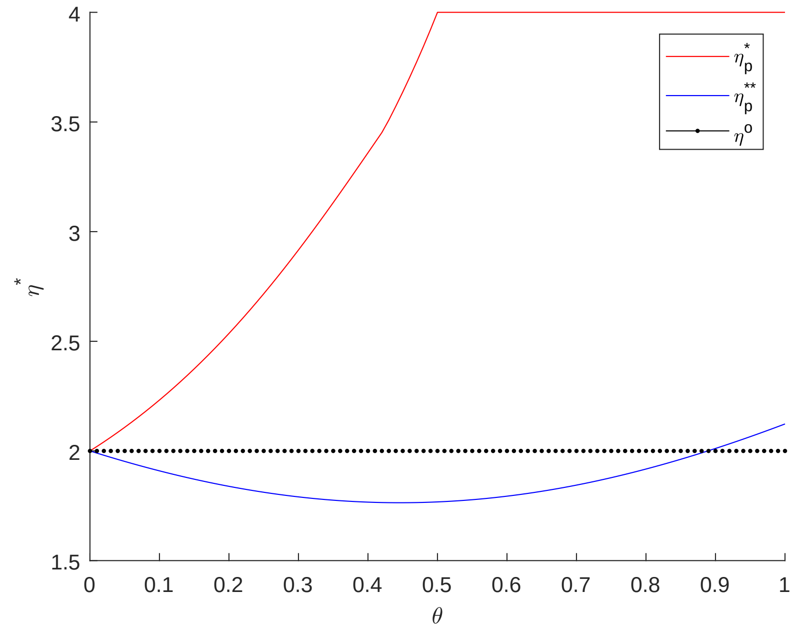

Figure 3: Promotion bars (ex-ante vs ex-post), $\bar{\eta} = 4$.

bars below the efficient benchmark (namely, $\eta_p^{**} < \eta^o$). For any $\theta \in (\theta^o, 1]$, instead the firm commits to promotion bars above the efficient benchmark, yet smaller than the one it would choose without commitment (namely, $\eta_p^* > \eta_p^{**} > \eta^o$). Finally, the firm commits to the efficient promotion bar for $\theta = 0$ or $\theta = \theta^o$.

Intuitively, when competition for promoted workers is low ($\theta < \theta^o$), workers secure a small share of the produced surplus and therefore have little incentive to exert training effort in poorly competitive environments. This is because they know that their effort may increase promotion bars for any degree of competition and make promotions less likely, thus further reducing expected wages. Firms, instead, earn a large share of surplus and would like workers to exert effort, so that they commit to low and more affordable promotion bars. By doing so, firms lower the bar below the efficient benchmark.

However, as competition for promoted workers increases ($\theta > \theta^o$), the labor market itself provides incentives for workers to exert portable training effort. As seen also in *Figure 2*, workers secure a larger share of the surplus produced, thus they have a strong incentive to exert portable training effort. This makes promoted workers' retention more expensive, hence the firm adjusts the promotion bar upwards, even setting it above the efficient benchmark when competition becomes too fierce, in order to shield itself against aggressive poaching raids.

Notice that firms commit to promote a positive share of workers for any θ, differently with respect to the case without commitment (namely, $\eta_p^{**} < \bar{\eta}$). This is because firms anticipate that not promoting workers implies that they will not exert training effort. This, in turn generates low surplus and the inability for firms to attract workers at $t = 0$.

To sum up, when workers exert portable training effort and firms commit to promotion bars at the hiring stage, inefficiencies may persist with respect to the case without commitment. More specifically, when θ is low, firms set the promotion bar below the efficient benchmark, so that the inefficiency in production persists, but is reversed, as for the same region of parameters, without commitment they would set the bar too high with respect to the efficient benchmark. If instead competition for promoted workers is sufficiently fierce, firms set the promotion bar higher than the efficient one. However, the inefficiency is smaller with respect to the case without commitment, as, by committing to promotion bars at $t = 0$, firms take into account the impact their choice has on workers' effort and on their ability to hire them. This result yields the interesting empirical prediction that workers should have faster careers, in firms where structured managerial practices are in place (empirical evidence, such as Lemos and Scur, 2019, show that this is not the case in family firms, for instance), stating clearly what performances will grant a promotion, or in industries where workers' performance is easily observable by other firms which may be able to poach them.

7 Conclusions

Workers' talent and on-the-job training are key inputs in talent-intensive industries, like the financial, the legal and the medical sectors. To provide incentives to exert training effort, firms may reimburse workers' cost of such training. However, if effort is unobservable, its cost cannot be reimbursed. Promotion-based incentive schemes are widespread in organizations (Baker, Jensen and Murphy, 1988), especially in those sectors in which wages are rigid and firms are unable to commit to pay-for-performance schedules. When promotions make workers more attractive in the labor market, firms face a trade-off between productivity and high expected retention cost for promoted workers.

This paper shows the linkage between promotion-based incentive schemes and training effort in competitive labor markets when workers' talent is uncertain at the training stage. Profit-maximizing firms may use promotions to reduce retention costs. Indeed, if firms cannot commit to set promotion bars when hiring workers, they set them higher than the efficient benchmark. This inefficiency becomes stronger as competitiveness for promoted workers increases. In fact, in sufficiently competitive environments, firms prefer not to promote workers at all.

If workers exert firm-specific training effort, their decision does not affect promotion bars, hence, such effort is increasing with the degree of competitiveness for promoted workers, provided that firms promote workers in equilibrium. This happens because specific training effort only serves to improve the signal about workers' talent on which firms base their promotion decisions. In this scenario, when able to commit to promotion bars, firms choose the efficient one since it allows them to offer the highest possible flow of wages to hire workers and be productive. If instead workers exert portable training effort, the latter, together with labor market competition, affect firms' decisions and these will raise promotion bars above the efficient benchmark. Although higher promotion bars raise expected wages conditional on promotion, they also generate a discouragement effect, as workers are less likely to be promoted, hence the optimal effort is nonmonotonic concave with respect to labor market competition. This result differs from those in the existing literature (Becker, 1962; Rosen, 1972; Garmaise, 2011) postulating that workers are eager to exert more training effort in highly competitive environments. In this paper, not only workers, but also firms react to labor market competition. Specifically, they shield themselves against aggressive poaching raids by setting the bar very high. This in turn has an impact on workers' incentives, so that competition affects their choice through two channels: directly raising expected wages, and indirectly lowering their chances of being promoted.

If firms commit to promotion bars before workers' training, when competition for promoted workers is not too high, they reduce the promotion bar below the efficient benchmark, making promotions easier to obtain with respect to the case without commitment at the cost of inefficient production. When competition increases, the firm raises the promotion bar above the efficient benchmark, since in this case, workers have strong market incentives to exert portable training effort, thus increasing retention costs. With commitment to promotion bars, workers exert more effort with respect to the case without commitment, for any level of competition for promoted workers. However, if firms compete too fiercely for promoted workers, the latter exert inefficiently too much effort. This implies that firms' ability to commit to promotion bars before workers exert training effort, does not necessarily implement efficiency (as in the case with firm-specific effort), but it actually reverses such inefficiencies.

In this paper, I assume that firms are homogeneous in their production technology, so that even though the signal workers produce at the training stage is observable by all firms in the market, a worker who is not promoted by her first employer cannot move to a competing firm and be promoted thereby. As a robustness remark, notice that if firms were heterogeneous this would not be the case. However, if only the first employer observes the signal about workers' talent and all other firms observe only promoted workers' productivity, then the qualitative results I have shown throughout the paper hold true.

Furthermore, I study frameworks in which workers exert either firm-specific or portable training effort. However, it would be worth analyzing workers' behavior in a framework in which they can choose a mix of the two. This would allow to understand how do the weights associated with each type of training effort change in frameworks with noncontractible and contractible promotion rules. To do so, one should also change the firms' production function for this analysis to be meaningful. More specifically, the returns to each type of training effort should be modeled in a more structured way.

Another possible venue for future research would be to allow for heterogeneity across firms and workers. Including technological heterogeneities could make workers employed by a certain firm more or less attractive to different competitors. One core assumption of the paper is that the talent-sensitive and routine tasks are substitutes for the productive scope. However, allowing for complementarity between the two tasks, the firm would need always to promote a positive share of workers. If firms are all technologically identical, then the results are qualitatively unchanged with respect to those presented in this paper (although there would not be corner solutions in the equilibria). If instead firms were heterogeneous in the degree of complementarity between the two tasks, then only the firms producing via technologies featuring more complementarity between tasks would be active and able to attract workers.

Workers, instead, may differ for the distribution of their talent. In this scenario, the probability of poaching raids may vary across firms, thus implying different promotion rules across different potential employers. If this is the case, workers may select themselves across employers, depending on their subjective talent distribution. Such result would be even more interesting if workers are risk-averse, as this would also introduce a hedging aspects in job selection. Such approach could generate further testable predictions.

References

[1] Ashraf, Nava, Oriana Bandiera, and B. Kelsey Jack (2014), "No Margin, No Mission? A Field Experiment on Incentives for Public Service Delivery," *Journal of Public Economics,* 120, 1-17.

[2] Autor, David (2001), "Why do Temporary Help Firms Provide Free General Skills Training?" *Quarterly Journal of Economics,* 116 (4), 1409–1448.

[3] Acemoglu, Daron, and Jorn-Steffen Pischke (1998) "Why do Firms Train? Theory and Evidence," *Quarterly Journal of Economics,* 113(February), 79-119.

[4] Baker, George, Michael Gibbs, and Bengt Holmström (1994a), "The Internal Economics of the Firm: Evidence from Personnel Data," *Quarterly Journal of Economics,* 109(4), 881-919

[5] Baker, George, Michael Gibbs, and Bengt Holmström (1994b), "The Wage Policy of a Firm," *Quarterly Journal of Economics,* 109(4), 921-955.

[6] Baker, George, Michael C. Jensen, and Kevin J. Murphy (1988), "Compensation and Incentives: Practice vs Theory," *The Journal of Finance,* 43(3), 593-616.

[7] Bar-Isaac Heski, and Raphael Levy (2019), "Motivating Employees Through Career Paths," *Working Paper.*

[8] Becker, Gary (1962), "Investment in Human Capital: A Theoretical Analysis," *Journal of Political Economy,* 70(5), 9-49.

[9] Ben-Porath, Yoram (1967), "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy,* 75(4), 352-365.

[10] Bernhardt, Dan (1995), "Strategic Promotion and Compensation," *Review of Economic Studies,* 62, 315-339.

[11] Bonatti, Alessandro, and Johannes Hörner (2017), "Career Concerns with Exponential Learning," *Theoretical Economics,* 12, 425-475.

[12] Brogaard, Jonathan, Joseph Engelberg, and Edward van Wesep (2017), "Do Economists Swing for the Fences after Tenure?," *Journal of Economic Perspectives,* forthcoming.

[13] Carmichael, Lorne (1983), "Firm-Specific Human Capital and Promotion Ladders," *The Bell Journal of Economics,* 14(1), 251-258.

[14] Dato, Simon, Andreas Grunewald, and Matthias Kräkel (2019), "Worker Visibility and Firms' Retention Policies," *Working Paper.*

[15] Fama, Eugene (1980), "Agency Problems and the Theory of the Firm," *Journal of Political Economy*, 88(2), 288-307.

[16] Ferreira, Daniel, and Radoslawa Nikolowa (2018), "How to Sell Jobs," *SSRN Working Paper.*

[17] Friebel, Guido, Matthias Heinz, Miriam Krüger, and Nick Zubanov (2017), "Team Incentives and Performance: Evidence from a Retail Chain," *American Economic Review*, 107(8), 2168-2203.

[18] Garmaise, Mark J (2011), "Ties that Truly Bind: Noncompetition Agreements, Executive Compensation, and Firm Investment," *The Journal of Law, Economics, and Organization*, 27(2), 376-425.

[19] Gaynor, Martin, James B. Rebitzer, and Lowell J. Taylor (2004), "Physician Incentives in Health Maintenance Organizations," *Journal of Political Economy,* 112(4), 915-931.

[20] Gibbons, Robert, and Kevin J. Murphy (1992), "Optimal Incentive Contracts in the Presence of Career Concerns: Theory and Evidence," *Journal of Political Economy,* 100, 468-505.

[21] Gibbons, Robert, and Michael Waldman (1999), "A Theory of Wage and Promotion Dynamics inside Firms," *Quarterly Journal of Economics,* 114(4), 1321-1358.

[22] Greenwald, Bruce C. (1986), "Adverse Selection in the Labour Market," *Review of Economic Studies*, 53, 325-347.

[23] Groysberg, Boris, Linda-Eling Lee, and Ashish Nanda (2008), "Can They Take it with Them? The Portability of Star Knowledge Workers' Performance," *Management Science*, 54(7),

[24] Groysberg, Boris (2010), "Chasing Stars. The Myth of Talent and the Portability of Performance," *Princeton University Press.*

[25] Harris, Milton, and Bengt Holmström (1982), "A Theory of Wage Dynamics," *Review of Economic Studies* 49(3), 315-333.

[26] Holmström, Bengt (1999), "Managerial Incentive Problems: A Dynamic Perspective," *Review of Economic Studies*, 66, 169-182.

[27] Karachiwalla, Naureen, and Albert Park (2017), "Promotion Incentives in the Public Sector: Evidence from Chinese Schools," *Journal of Public Economics,* 146, 109-128.

[28] Kahn, Adnan, Asim Khwaja, and Benjamin Olken (2016), "Tax Farming Redux: Experimental Evidence on Performance Pay for Tax Collectors," *The Quarterly Journal of Economics,* 131(1), 219-271.

[29] Kim Jin-Hyuk (2014), "Employee Poaching: Why it can be Predatory," *Managerial and Decision Economics*, 35, 309-317.

[30] Lazear, Edward (1986), "Raids and Offer Matching,', *Research in Labor Economics*, 8, 141-165.

[31] Lazear, Edward (2000), "Performance Pay and Productivity," *American Economic Review,* 90(5), 1346-1361.

[32] Lemos, Renata, and Daniela Scur (2019), "The Ties that Bind: Implicit Contracts and Management Practices in Family-run Firms," *Working Paper*.

[33] McCue Kristin (1996), "Promotions and Wage Growth," *Journal of Labor Economics,* 14(2),

[34] Milgrom, Paul and Sharon Oster (1987), "Job Discrimination, Market Forces, and the Invisibility Hypothesis," *Quarterly Journal of Economics*, 102(3), 453-476.

[35] Mukherjee, Arijit and Luis Vasconcelos (2018) "On the Trade-off between Efficiency in Job Assignment and Turnover: The Role of Breakup Fees," *Journal of Law, Economics and Organization , forthcoming.*

[36] Picariello, Luca (2019), "Organizational Design with Portable Skills," Working Paper.

[37] Prendergast, Canice (1993), "The Role of Promotions in Inducing Specific Human Capital Acquisition," *Quarterly Journal of Economics,* 108(2), 523-534.

[38] Prescott, J.J., Norman D. Bishara, and Evan Starr (2016), "Understanding Noncompetition Agreements: The 2014 Noncompete Survey Project," *Michigan St. L. Review,* 2, 369-464.

[39] Rosen, Sherwin (1972), "Learning and Experience in the Labor Market," *The Journal of Human Resources,* 70(3), 326-342.

[40] Samila, Sampsa, and Olav Sorenson (2011), "Noncompete Covenants: Incentives to Innovate or Impediments to Growth," *Management Science,* 57(3), 425 - 438.

[41] Waldman, Michael (1984), "Job Assignments, Signaling and Efficiency," *RAND Journal of Economics*, 15, 255-267.

Appendix

Proof of Proposition 1

The proof of this proposition is straightforwardly provided by the first-order conditions of the maximization program

$$\underset{\{\eta^o\,,a^o\}}{Max} \quad \mathcal{W} = \int_{\eta^o}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta^o)\bar{\eta}}{2} + a^o - \frac{a^{o2}}{2}.$$

The first-order condition with respect to effort a yields

$$a^o = 1 \tag{18}$$

whilst the first-order condition with respect to the promotion bar η gives

$$\eta^o = \frac{\bar{\eta}}{2}. \tag{19}$$

Proof of Proposition 2

Proof. Throughout this proof, it is assumed that at $t = 0$ firms are able to commit to wages contingent on the signal y_1. At the hiring stage, firms offer wage contracts $\{w_1\,,\,w_2\}$. By the zero-profit condition, the hiring wage is set as $w_1 = \mathbb{E}\left[\sum_{i=A}^{B} (y_{2i} - w_{2i})\right]$, whereas $w_2 = y_1 = \eta + a$.

In this case, at the training stage, the worker exerts the training effort that maximizes her expected utility at the next stage:

$$a^* \in \underset{\{a\}}{argmax} \; \mathbb{E}(w_2) = \frac{\bar{\eta}}{2} + a - \frac{a^2}{2}$$

The first-order condition for an interior optimum of the above maximization program yields

$$a^* = a^o = 1. \tag{20}$$

Since w_2 is paid irrespective of whether the worker is promoted or not, the firm's expected profit is

$$\pi = \int_{\eta^*}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta^*)\bar{\eta}}{2} - w_1 - w_2$$

which is maximized for $\eta^* = \frac{\bar{\eta}}{2} = \eta^o$.

This outcome holds true both if the firm cannot commit to promotion bars at the hiring stage, so that the promotion bar η^* is set after the training effort and if the firm can commit to promotion bars at $t = 0$. In the second case, the firm offers a contract $\{w_1, w_2, \eta^*\}$ at $t = 0$, thus η^* is chosen so as to convince workers to accept the contract ex-ante. Namely, firms maximize workers' lifetime expected utility. Again, since w_2 is independent of the promotion decision, the firm tries to maximize $w_1 = \mathbb{E}\left[\sum_{i=A}^{B} (y_{2i} - w_{2i})\right]$. This procedure yields again the same result as the one shown above.

For which concerns promoted workers' retention at the interim stage, note that $w_2 = \eta + a$ is exactly as much as a worker would be offered by a competing firm with portable effort and $\theta = 1$, as shown in equation (7). Hence, for any $\theta \in [0\,,\,1]$, workers have no incentive to move to another employer. Trivially, the same result holds if workers acquire specific training.

■

Proof of Lemma 1

Proof. The proof for this lemma comes from the firm's profit maximization problem. To choose the promotion bar, the firm solves:

$$\eta_s^* \in \underset{\{\eta\}}{argmax} \;\; (1-\theta) \int_{\eta}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta)\bar{\eta}}{2} + a$$

delivering the first-order condition for an interior solution

$$\frac{f(\eta)\bar{\eta}}{2} - (1-\theta)\eta f(\eta) = 0 \Rightarrow \eta_s^* = \frac{\bar{\eta}}{2(1-\theta)}. \tag{21}$$

Now, $\eta_s^* \leq \bar{\eta}$ for any $\theta \leq \frac{1}{2}$, otherwise it exceeds the upper bound of the support of talent, so that the firm does not promote workers at all. ■

Proof of Proposition 3

Proof. To prove this proposition, recall from Lemma 1 that the firm sets a promotion rule so that a worker producing y_1 is promoted if $y_1 - \hat{a} \geq \eta^* = \frac{x}{1-\theta}$. This condition can be rewritten as $\eta \geq \eta^* - a + \hat{a}$. Recall that the when observing $\hat{\eta} < 0$, the firm does not promote the worker who produced such signal, who then earns zero. If instead the firm observes too high a signal $\hat{\eta} > \bar{\eta}$, the worker who produced such signal is promoted (respectively, not promoted) with probability $\frac{1}{2}$.

Workers exert firm-specific training effort in order to maximize their expected utility, as depicted in equation (5).

Since every firm in the industry observes y_1, a worker exerts firm-specific effort so that

$$a_s^* \in \underset{\{a \geq 0\}}{argmax} \;\; \frac{\theta}{2}\left[\int_{\eta_s^*-a+\hat{a}}^{\bar{\eta}-a+\hat{a}} \Big(\eta + a - \hat{a}\Big) f(\eta) d\eta + \int_{\eta_s^*-a+\hat{a}}^{\bar{\eta}} \Big(\eta + a - \hat{a}\Big) f(\eta) d\eta\right] - \frac{a^2}{2}.$$

However, Lemma 1 shows that firms may not be willing to promote workers if there is too much competition for talent. Thus, I take into account two possible scenarios:

1. If $\theta \leq \frac{1}{2}$, firms promote a positive fraction of workers, so the first-order condition for an internal solution is

$$\frac{\theta}{2}\left(1 + \frac{a - \hat{a}}{\bar{\eta}}\right) - a = 0 \tag{22}$$

   The second-order condition is satisfied if

$$\frac{\theta}{2\bar{\eta}} < 1 \tag{23}$$

   Note that since $\bar{\eta}$ (the upper bound of the support of talent η) is strictly larger than 1, the second-order condition for this program is fulfilled.

   Along the equilibrium path, the firm perfectly anticipates workers' effort, hence $a = \hat{a}$ and the first order condition yields

$$a_s^* = \frac{\theta}{2} \tag{24}$$

2. If $\theta > \frac{1}{2}$, the firm does not promote any workers, so that the first term of workers' maximization program is zero. Given the nonnegativity constraint imposed on effort, in equilibrium the worker exerts no training effort, namely $a_s^* = 0$.

   Notice that such solution also emerges in the case in which $\theta = 0$, for a different reason. In this case, the firm promotes workers efficiently, yet there is no competition in the labor market, hence, workers are indifferent between being promoted and not, so that they have no incentive to exert training effort.

■

Proof of Proposition 4

Proof. By backward induction workers' effort is the same as in the case without commitment, since it does not depend on the promotion bar. This implies that the optimal effort is the same as the one derived in the proof of Proposition 3:

$$a_s^* = a_s^{**} = \frac{\theta}{2} \tag{25}$$

provided that the firm promotes some workers in equilibrium.

Moving backwards to $t = 0$, the firm offers the pair $\{w_1\,,\,\eta_s^{**}\}$ that maximizes workers' lifetime expected utility

$$U(w\,,\,a) = w_1 + \mathbb{E}(w_2) - \psi(a)$$

to ensure activity.

At $t = 0$ the firm has expected profit

$$\mathbb{E}_0(\pi) = \int_{\eta_s^{**}}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta_s^{**})\bar{\eta}}{2} + a_s^* - \mathbb{E}(w_2) - w_1. \tag{26}$$

Since the labor market is perfectly competitive at the hiring stage and firms are unable to commit directly to wages, w_1 is driven by the *zero-profit condition* hence

$$w_1 = \int_{\eta_s^{**}}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta_s^{**})\bar{\eta}}{2} + a_s^* - \mathbb{E}(w_2).$$

Given these wages, the firm chooses the promotion bar solving:

$$\underset{\{\eta_s^{**}\}}{Max} \;\; U = \int_{\eta_s^{**}}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta_s^{**})\bar{\eta}}{2} + a_s^{**}\left(1 - \frac{a_s^{**}}{2}\right)$$

This delivers the first-order condition

$$\eta_s^{**} = \frac{\bar{\eta}}{2} = \eta^o. \tag{27}$$

Since promotion bars do not provide incentives for workers to exert firm-specific training effort, the firm ends up choosing the bar that maximizes productivity. ∎

Proof of Lemma 2

Proof. The proof for this lemma comes from the firm's profit maximization problem:

$$\eta_p(a) \in \underset{\{\eta\}}{argmax} \;\; \pi = (1-\theta)\int_{\eta}^{\bar{\eta}} (\eta + a) f(\eta) d\eta + F(\eta)\left(\frac{\bar{\eta}}{2} + a\right).$$

This yields the first-order condition for an interior solution:

$$f(\eta)\left[\frac{\bar{\eta}}{2} + a - a(1-\theta) - (1-\theta)\eta\right] = 0$$

which can be simplified and deliver the best reaction function

$$\eta_p(a) = \frac{\bar{\eta}}{2(1-\theta)} + \frac{\theta a_p}{1-\theta} \tag{28}$$

for any training effort a_p exerted by the worker.

∎

Proof of Proposition 5

Proof. I prove this proposition in two steps: first, I derive the optimal effort and promotion bar, showing that the latter is larger than the efficient benchmark and exceeds $\bar{\eta}$ if $\theta \geq \frac{1}{2}$; second, I prove the nonmonotonicity of optimal portable training effort with respect to θ.

1. By backward induction, workers know that the firms' best reaction function $\eta_p(a)$ is increasing in their effort. For this reason, at $t = 1$, their maximization program is:

$$a_p^* \in \underset{\{a_p \geq 0\}}{argmax} \ \frac{\theta}{2}\left\{ \int_{\eta_p(a_p)-a_p+\hat{a}}^{\bar{\eta}-a_p+\hat{a}} (\eta + a_p - \hat{a}) f(\eta) d\eta + \int_{\eta_p(a_p)-a_p+\hat{a}}^{\bar{\eta}} (\eta + a_p - \hat{a}) f(\eta) d\eta \right.$$

$$\left. + [F(\bar{\eta} - a + \hat{a}) + 1 - 2F(\eta_p(a_p) - a_p + \hat{a})]a_p \right\} - \frac{a_p^2}{2}$$

The first-order condition for an interior solution, taking into account the fact that the firm correctly conjectures a along the equilibrium path (namely, $a = \hat{a}$), is

$$\frac{\theta}{2\bar{\eta}}\left[3\bar{\eta} - \frac{2\eta_p(a_p)}{1-\theta} + a_p\left(1 - \frac{2\theta}{1-\theta}\right)\right] - a_p = 0 \tag{29}$$

Plugging the firm's reaction function(28) into (29), one gets

$$a_p^*(\theta) = \frac{\bar{\eta}\theta\left[3(1-\theta)^2 - 1\right]}{(2\bar{\eta} - \theta)(1-\theta)^2 + 2(2-\theta)\theta^2}. \tag{30}$$

Plugging the optimal effort (30) into the firm's best reaction function (28), delivers the equilibrium promotion bar without commitment:

$$\eta_p^*(\theta) = \frac{\bar{\eta}}{(1-\theta)}\left[\frac{1}{2} + \frac{\theta^2\left[3(1-\theta)^2 - 1\right]}{(2\bar{\eta} - \theta)(1-\theta)^2 + (2-\theta)2\theta^2}\right] \tag{31}$$

Compare now η_p^* with the efficient benchmark $\eta^o = \frac{\bar{\eta}}{2}$. Notice that η_p^* can be written as

$$\eta_p^* = \frac{\bar{\eta}}{2(1-\theta)} + \frac{\theta a_p^*}{1-\theta} \tag{32}$$

Since $\theta \in [0\,,\,1]$, it is sure that $\frac{\bar{\eta}}{2(1-\theta)} \geq \frac{\bar{\eta}}{2}$, so that the first term in (32) is larger than the efficient promotion bar. The second term is the optimal effort multiplied by $\frac{\theta}{1-\theta}$. These two elements multiplied are by definition larger or equal to zero, thus implying that $\eta_p^{**} \geq \eta^o$, for any $\theta \in [0\,,\,1]$. The promotion bar equates the efficient benchmark only if $\theta = 0$, namely in the absence of labor market competition for promoted workers.

Moreover, note that if $\theta \geq \frac{1}{2}$, $\eta_p^* \geq \bar{\eta}$, as in the case in which workers exert firm-specific training effort. If $\theta = \frac{1}{2}$, the first term of equation (32) is exactly equal to $\bar{\eta}$ and it is summed to something at least equal to zero.

2. To prove that a_p^* is nonmonotonic concave with respect to θ, first notice that a_p^* is continuous, as its denominator is always different from zero. Moreover, one can show that a_p^* has zeros at $\theta = 0$ and $\theta \approx 0.43$.

   Second, take the total differentiation of a_p^*

$$\frac{d(a_p^*)}{d\theta} = \frac{2\bar{\eta}\big[3\bar{\eta}\theta^4 + (3 - 12\bar{\eta})\theta^3 + (19\bar{\eta} - 3)\theta^2 - 12\bar{\eta}\theta + 2\bar{\eta}\big]}{\big[3\theta^3 - (2\bar{\eta} + 6)\theta^2 + (4\bar{\eta} + 1)\theta - 2\bar{\eta}\big]^2}. \tag{33}$$

   To determine the sign of (33), note that the denominator is always positive. This implies that

$$sign\{\frac{da_p^*}{d\theta}\} = -sign\{2\bar{\eta}\big[3\bar{\eta}\theta^4 + (3 - 12\bar{\eta})\theta^3 + (19\bar{\eta} - 3)\theta^2 - 12\bar{\eta}\theta + 2\bar{\eta}\big]\}.$$

   One can see that

$$\lim_{\theta \to 0} \frac{da_p^*}{d\theta} > 0 \text{ and } \lim_{\theta \to 1} \frac{da_p^*}{d\theta} = 0.$$

   By continuity of a_p^* with respect to θ, there exists a positive cutoff value $\hat{\theta}$, such that $\frac{da_p^*}{d\theta} \geq 0$ for any $\theta \leq \hat{\theta}$ and $\frac{da_p^*}{d\theta} < 0$ for any $\theta > \hat{\theta}$, so that optimal effort starts falling up to zero (for $\theta \approx 0.43$).

   Finally, tedious algebra shows that the second derivative of a_p^* with respect to θ is negative for any $\theta \in [0\,,\,1]$, which proves concavity of optimal effort and promotion bar with respect to competition for promoted workers.

■

Proof of Lemma 3

Proof. To prove this lemma, consider workers' best reaction function for any promotion bar η_p. This is derived by solving

$$a_p(\eta_p) \in \underset{\{a \geq 0\}}{argmax}\ \frac{\theta}{2}\Big\{\int_{\eta_p - a + \hat{a}}^{\bar{\eta} - a + \hat{a}} \big(\eta + a - \hat{a}\big) f(\eta) d\eta + \int_{\eta_p - a + \hat{a}}^{\bar{\eta}} \big(\eta + a - \hat{a}\big) f(\eta) d\eta +$$

$$+\big[F(\bar{\eta}-a+\hat{a})+1-2F(\eta_p-a+\hat{a})\big]a\Big\}-\frac{a^2}{2}.$$

The first-order condition for this problem is

$$\frac{\theta}{2}\left[3-\frac{2(\eta_p-a+\hat{a})}{\bar{\eta}}+\frac{a}{\bar{\eta}}\right]-a=0 \tag{34}$$

and the second-order condition is

$$\frac{3\theta}{2\bar{\eta}}<1$$

which is fulfilled, since it is assumed that $\bar{\eta}\geq\frac{3}{2}$.

Along the equilibrium path, the firm holds consistent beliefs, so that $a=\hat{a}$. Thus, worker's best reaction function is

$$a_p(\eta_p)=\frac{\theta(3\bar{\eta}-2\eta_p)}{2\bar{\eta}-\theta}. \tag{35}$$

for any promotion bar $\eta_p\in[0\,,\,\bar{\eta}]$. ■

Proof of Proposition 6

Proof. I prove this proposition in three steps. First, I derive the optimal promotion bar. Second, I show that in equilibrium, the promotion bar is smaller or equal to the efficient benchmark in a certain interval of values of θ, more specifically, it is nonmonotonic with respect to labor market competition for promoted workers. Third, I derive the optimal portable training effort.

1. The firm rationally anticipates workers' reaction specified in equation (35). As in the proof of Proposition 4, at $t=0$ the firm offers a contract $\{w_1\,,\,\eta_p^{**}\}$ to hire workers. Workers are willing to work for the firm offering the contract that maximizes their lifetime expected utility

$$U(w\,,\,a)=w_1+\mathbb{E}(w_2)-\psi(a).$$

The firm's expected profit as of $t=0$ is

$$\mathbb{E}_0(\pi)=\int_{\eta_p^{**}}^{\bar{\eta}}\Big(\eta+a(\eta_p^{**})\Big)f(\eta)d\eta+F(\eta_p^{**})\left(\frac{\bar{\eta}}{2}+a(\eta_p^{**})\right)-w_1-\mathbb{E}(w_2).$$

Since the labor market is perfectly competitive at $t = 0$, w_1 is driven by the *zero-profit condition,* yielding $w_1 = \mathbb{E}_0(\pi)$.

Given these wages, at the hiring stage, the firm commits to the promotion bar solving

$$\underset{\{\eta_p^{**}\}}{Max}\ \ U = \int_{\eta_p^{**}}^{\bar{\eta}} \eta f(\eta) d\eta + \frac{F(\eta_s^{**})\bar{\eta}}{2} + a(\eta_p^{**})\left(1 - \frac{a(\eta_p^{**})}{2}\right).$$

The first-order condition for this maximization program yields

$$\eta_p^{**}(\theta) = \frac{\bar{\eta}}{2}\left[\frac{4\bar{\eta}^2 - 12\bar{\eta}\theta(1-\theta) + 5\theta^2}{\theta^2 + 4\bar{\eta}[\bar{\eta} - \theta(1-\theta)]}\right]. \tag{36}$$

Note that η_p^{**} is continuous with respect to θ as it can be easily shown that its denominator is never equal to zero.

2. Equating η_p^{**} with $\eta^o = \frac{\bar{\eta}}{2}$, one obtains:

$$\theta[(2\bar{\eta} + 1)\theta - \bar{\eta}] = 0$$

which has solution roots $\theta_1 = 0$ and $\theta_2 = \frac{\bar{\eta}}{2\bar{\eta}-1} \equiv \theta^o$. As both roots are real, one can conclude that $\eta_p^{**} < \eta^o\ \forall\theta \in (0\,,\, \theta^o)$, whilst $\eta_p^{**} > \eta^o\ \forall\theta \in (\theta^o\,,\, 1]$.

Note that the total differentiation of the optimal promotion bar with respect to θ yields

$$\frac{d\eta_p^{**}}{d\theta} = -\frac{4\bar{\eta}\left[\theta^2 - (8\bar{\eta}^2 + 4\bar{\eta})\theta + 4\bar{\eta}^2\right]}{\left[(4\bar{\eta} + 1)\theta^2 - 4\bar{\eta}\theta + 4\bar{\eta}^2\right]^2}. \tag{37}$$

The sign of (37) is such that

$$\lim_{\theta\to 0} \frac{d\eta_p^{**}}{d\theta} < 0 \ \text{ and } \ \lim_{\theta\to 1} \frac{d\eta_p^{**}}{d\theta} > 0.$$

Hence, by continuity of η_p^{**}, there exists a value of θ such that the sign of the derivative switches from negative to positive, hence the promotion bar is nonmonotonic with respect to θ.

3. The optimal effort a_p^{**} is straightforwardly derived by plugging equation (36) into the worker's best reaction function (35). This delivers

$$a_p^{**}(x,\theta) = \frac{\bar{\eta}\theta}{2(\bar{\eta}-\theta)}\left\{4 - \left[\frac{4\bar{\eta}\theta^2 + (\bar{\eta}-\theta)(\bar{\eta}-3\theta)}{\bar{\eta}\theta^2 + (\bar{\eta}-\theta)^2}\right]\right\} \geq 0.$$

■