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Signaling Valence in Primary Elections

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Abstract

I build a model of two-stage (primary and general) elections in which primary election candidates differ in terms of a privately observed quality dimension (valence). I show that primary election candidates have the incentive to signal their valence by means of their policy platform choice. There can be two types of separating equilibria in primary elections: an *extremist* equilibrium, in which valent candidates choose more extreme policies than non-valent ones, and a *centrist* one, in which valent candidates instead move close to the incumbent from the opposing party. The ideology of primary elections voters is the main driver of the choice of one versus the other separating strategy. I also study the conditions under which party voters benefit from primaries, as well as those under which primaries increase the probability for a party of winning the general election. Finally, I assess the effects of incumbency advantage/disadvantage, explore alternative patterns of valence observability and extend the model to account for both parties holding primaries.

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Table of contents

1 Introduction

Following the seminal contribution by Stokes (1963), many spatial models of elections also feature a vertical dimension on which politicians can differ, usually referred to as valence. This concept is used to represent a range of qualities (honesty, integrity and charisma among others) that are independent of the ideological positioning of a candidate and are valued by all voters. In an environment with electoral competition along these two dimensions, what correlation should one expect between the valence of candidates and their choice of policy platform? At times in which extremism and polarization are seen as some of the biggest threats to the functioning of democracy, understanding the correlation between valence and ideology becomes ever more important. The existing empirical literature has not provided us with a definitive answer: some studies find evidence of a negative correlation between valence and extremism, but others point in the opposite direction[1]. This suggests that different mechanisms could be at play and calls for a theory capable of reconciling both types of stylized facts.

In order to do this, an institutional element that needs to be included in the picture is that of primary elections. In many democracies, as a matter of fact, primary elections play a key role in the selection of candidates and policy platforms. Moreover, primary elections, and in particular closed primary elections, which only party members can participate in, have been identified by many observers as one of the drivers of growing polarization in American politics[2]. However, also on this issue the empirical literature has left many questions unsettled[3].

[1]Measuring the valence of political candidates is a difficult task. An often used proxy is political experience: Burden (2004) finds that favored candidates are further away from the median voter. He argues that "in many districts the winning candidate is actually further from the center than the loser, but manages victory on the basis of non-ideological criteria that overwhelm the modest effects of ideological proximity". Ansolabehere et al. (2001) find instead that valent candidates tend to be more moderate. Stone and Simas (2010) find significant divergence between candidates: high quality candidates locate closer to their district's preferences, but their main result is that challengers who choose policy platforms further away from their district receive more votes, suggesting that they possess some quality to make up for their extremism. Relatedly, Fiorina (1974) brings attention the puzzle of what he calls a *flip-flopping representation*, in which *extremes are replaced by extremes*, especially in competitive districts where the Downsian forces leading to convergence should be strongest. Evidence in favor of divergence is also provided by Erikson and Wright (1980) and Erikson and Wright (1997).

[2]See for example Fiorina et al. (2006) and Fiorina and Levendusky (2006).

[3]Most studies have focused on the case of the United States, to a good extent due to a matter of data availability: given that primary elections are mandated by law in the US, the only possibility for an empirical investigation is to exploit the staggered introduction of primaries. Using this exogenous variation in the introduction of primaries, Hirano et al. (2010) find little evidence of a positive effect of primary elections on polarization; Cintolesi (2020) instead finds a negative effect. Other studies have instead exploited, rather than the variation between primaries and no primaries, the variation within the system of primary elections, i.e. the so called openness of primaries: McGhee et al. (2014) find that the openness of primary elections has small effects on polarization, whereas Bullock and Clinton (2011) and

The model I construct revolves around the interplay of primary elections and the correlation between valence and policy extremism. The equilibrium behavior I characterize can accommodate both a positive or a negative correlation between valence and ideology and, relatedly, either a positive or a negative effect of primaries on policy extremism. A fundamental ingredient of my model concerns the observability of valence: candidates alone can observe their own valence before the primary election, which only becomes public information before the general election. This captures the increasing availability of information on candidates as the campaign progresses and media scrutiny of candidates increases[4]. Since voters cannot observe valence prior to the primary election, candidates choose their policy platform not only with the goal of capturing the preference of primary and general election voters, but also with the aim of building a reputation for valence. Therefore, primary elections involve signaling: my model is the first one, to the best of my knowledge, to explore this issue.

In order to credibly signal their valence in a separating equilibrium, candidates need to deviate from a *baseline* (first best) party policy platform. This deviation, though, can be in either direction: candidates can credibly signal valence by being more extreme than the party baseline or by moving towards (or even past) the center of the policy spectrum, closer to the positions of voters from the opposing party. The logic behind these two opposing strategies, however, is the same: a primary candidate can credibly signal valence by choosing platforms that non-valent candidates cannot afford to run with. The intuition is simple: choosing an extreme policy platform decreases the probability of winning the general election against the incumbent for any candidate, but it especially hurts non-valent candidates, who cannot make up for their policy platform with their valence. Therefore, choosing an extreme enough policy platform is a credible signal of valence. Similarly, being too close to the incumbent condemns a non-valent candidate, but not a valent one, to a certain electoral defeat. Thus, also moving close to the incumbent can send a credible signal of valence. This captures a realistic feature of political competition, that is the fact that candidates often build a reputation for valence by proposing policies that are innovative for the party, be it because of their extremism or because of their similarity to the policy positions of the opposing party. In the former case, the correlation between valence and ideological extremism is positive, in the latter it is negative.

Gerber and Morton (1998) find significant moderating effects of open primaries. Other studies look at how candidates with different voting records (or policy platforms) fare in primary elections: Brady et al. (2007) find that moderate primary candidates are more likely to lose primary elections, and similarly Burden (2001) and Burden (2004) show that primary election competitiveness leads candidates to diverge further from the center.

[4]In Appendix 9 I show that my results are robust to considering more general patterns of valence revelation.

If both outcomes are possible, what determines when either is realized? In order to exist, a separating equilibrium requires high valence, a small prior probability that primary-candidates are valent, and an incumbent whose policy platform is not too extreme. Concerning the choice between the two possible signaling strategies, the main driver is the ideology of the primary election median voter: the more extreme the primary election median voter, the more likely candidates are to signal valence by choosing extreme platforms. Despite my model not being a model on open versus closed primaries, this feature of the model supports the view that, other things equal, more closed primary elections are more likely to select extreme candidates[5]. Finally, both the signaling strategies that I describe are more likely to take place when the valence of the incumbent is high.

My model can also be used to understand under what circumstances party voters are more likely to benefit from primaries. This is an important question for several reasons: first, in many democracies, primaries are not mandated by law as in the United States, but parties can choose whether or not to hold them. Second, even in systems with mandated primaries such as the United States, there is some institutional flexibility allowing for changes in the degree of primary openness and the influence of party elites over the primary election outcome. Finally, the analysis of this paper can help understand and put into context the historical decision of making primaries mandatory in the United States[6].

Intuitively, I find that party voters are likely to benefit from primaries when valence is high (which can also be interpreted as valence having a large weight in voters' utility function compared to ideology) and when the fraction of valent politicians is low, such that primaries generate a bigger improvement in the selection of politicians. This last factor could depend on party structure, district characteristics, but it can also be interpreted as a measure of the ability of parties to screen potential candidates. Primary voters are also more likely to benefit from primaries when incumbents are not too extreme: this can be interpreted as meaning that primaries are more beneficial in competitive districts rather than districts in which a party enjoys a strong ideological advantage. In this respect, my model predicts that while a valence advantage makes primaries more beneficial, an ideological advantage has the opposite effect. Finally, another interesting implication of my model is that primaries can benefit voters independently of their ideology, which

[5]There is empirical evidence of such an effect, see for example Gerber and Morton (1998) and Bullock and Clinton (2011), but, as for example McGhee et al. (2014) and Casas (2019) argue, there are also elements, empirical and theoretical, pointing in the opposite direction. Refer to Footnote 2 for a further discussion.

[6]See Ware (2002) and the discussion in Section 6.

goes against the prior that primaries disproportionately benefit extreme party voters: if anything, as I discuss in Section 5, there are elements suggesting that moderate electorates might be more likely to benefit from primaries.

The paper is structured as follows: Section 2 reviews the related literature; Section 3 presents the model; Section 4 solves the model; Section 5 provides a welfare analysis and a discussion of equilibrium selection in this model; Section 6 contains a discussion of the institutional implications of the model; Section 7 presents an important extension in which the valence of the incumbent is free to vary; Section 8 presents a summary of the results with a particular focus on the connection with the empirical literature; Section 9 discusses the timing of revelation of valence; Section 10 concludes. In the Appendix, as well as all the proofs, I present the extension of the model allowing both parties to hold primaries. Furthermore, I also relax some important assumptions concerning the distribution of the median voter and the functional form of voters' preferences.

2 Related Literature

My paper is mostly related to two streams of theoretical literature: that on electoral competition with valence and that on primary elections. There are several theoretical papers on electoral competition with valence. Groseclose (2001) and Aragones and Palfrey (2002) mainly deal with the question of equilibrium existence: in both these models, valent candidates choose more moderate policies than non-valent ones.

Adams and Merrill III (2008) constitutes one of the seminal papers in the literature on spatial elections with primaries. In their model, valence has two components: one is common knowledge and party-specific, the other is unobservable even for candidates at the moment of choosing policy-platforms. The possibility of signaling is thus shut down, unlike in my work. In their model, candidates of parties with a valence advantage always choose more extreme platforms[7], whereas my model allows for both positive and negative correlation between valence and extremism.

Another closely related paper is Hummel (2013). In his model, valence is common knowledge and always exactly one of the two primary candidates is valent: therefore, valent primary candidates have all the bargaining power and choose more moderate policies in primary elections, to be more electable in the general election. My models shows that introducing competition among valent primary candidates and making valence unobservable to primary voters can reverse his result.

[7]In their model, primary election voters vote naively, ignoring electability.

Concerning the observability of valence, there are several papers that move away from the assumption of full observability: in Snyder and Ting (2011), valence can be revealed by either the primary or the general election campaign, but policy platforms are fixed, unlike in my model. Also in Kartik and McAfee (2007), valence (character) is unobservable, but only non-valent candidates choose policy platforms strategically, unlike in my model, creating a positive correlation between valence and extremism. Bernhardt et al. (2011) develop a dynamic model of elections in which, just like in my model, the valence of incumbents is observable whereas the valence of challengers is not. In their model, the correlation between valence and extremism is negative for first-term office holders, but it becomes positive as tenure increases, due to selection through rounds of elections. Casas (2019) assumes that policy preferences are not observable, but valence is. In his model, costly endogenous party affiliation leads to a positive correlation between valence and distance from the party mainstream. Finally, Boleslavsky and Cotton (2015) do not study primaries but in their model the extremeness of the policies chosen by parties increases with the informativeness of the general election campaign.

Concerning primaries as an institution, Serra (2011) considers the incentives for party elites to hold primaries, consisting in a trade-off between the valence benefits from an expanded pool of nominees and the costs due to ideological differences with party voters[8]. The choice of elites on whether to hold primaries is also present in Slough et al. (2017), who build a repeated model of elections: in their model, primaries are held when polarization is high and they are preferred by disadvantaged parties.

A positive correlation between valence and extremism also appears in several models with endogenous valence (i.e. valence resulting from players' effort or choices). In Eguia and Giovannoni (2019), a disadvantaged party can tactically choose an extreme platform in order to occupy an ideological space for future elections. In Carrillo and Castanheira (2008), choosing an extreme policy platform acts as a commitment device to invest in valence; similarly, in Serra (2010), candidates can invest in valence to make up for their extreme policy platforms[9]; finally, in Crutzen et al. (2009), primaries provide an incentive to candidates to exert effort, with a signaling component common to my model [10].

[8]In his model, valence is observable before primary elections and there is no uncertainty over the location of the general election median voter. Serra (2013) considers a noisy selection of politicians through primaries.

[9]A positive correlation between charisma and extremism can also be found in Serra (2018): candidates are policy motivated, and therefore charismatic candidates can afford to choose more extreme policy platforms.

[10]There are a number of other models of primary elections that for brevity I did not cover in the main text: Owen and Grofman (2006) study a two-stage spatial model of elections; Grofman et al. (2019) consider both closed and open primaries with candidates of potentially different observable valence and show that primaries benefit the party closer to the general population median; Takayama (2014) presents

3 The Model

Two political parties, L and R (standing for left and right respectively), compete in an election. Party R enters the election with a pre-established incumbent candidate; the L party instead selects a candidate through a primary election. Two pre-candidates (I use this term to distinguish primary election candidates from general election candidates) take part in the L party primary election[11].

A policy platform is a point on the real line, $[-\infty, +\infty]$. The R party incumbent candidate is located at $r \geq 0$. Voters have single-peaked policy preferences with bliss-points distributed on the real line. The bliss-point of the general election median voter, denoted by μ, is uniformly distributed on the $[-b, b]$ interval[12]. Some voters are party voters, that is they also vote in the primary election of the L party, the challenging party. The set of party voters is fixed and I assume that L party voters have bliss-points weakly to the left of $-b$, with the median L party voter taking the (deterministic) value of $m \leq -b$. In other words, there is no overlap between the support of primary voters of the L party and that of the general election median voter. These assumptions simplify the analysis of voting in the primary election, but they are not necessary for the result to hold[13].

Assumption 1. *All primary election voters have bliss-points contained in* $(-\infty, -b]$.

The utility a voter with bliss point x derives from the implementation of a given policy-platform l takes the following additively separable linear form[14]:

$$u_x(l, \theta) = -|l - x| + v(\theta) \tag{1}$$

where $v(\theta)$ indicates the valence of the candidate proposing policy l, which depends on

a model with observable valence in which primaries lead to policy moderation; Serra (2015) shows that policies can converge to the median despite the presence of primaries; Meirowitz (2005) presents a model in which primaries allow candidates to learn the preferences of voters. Finally, Hummel (2010) and Agranov (2016) focus on the flip-flopping of candidates between primary elections and the general election, which is not part of my model, since politicians commit to their platform.

[11]In Appendix B I also consider the case of both parties selecting a candidate through primary elections.

[12]This assumption allows me to work with closed forms, but it is not what drives the results of the model. In Appendix D I consider a more general distribution function for the general election median voter and show that results do not qualitatively change.

[13]In particular, I could allow for an overlap between the set of primary voters and the support of the general election median voter.

[14]The choice of a linear policy loss function is due to tractability reasons and is common to most models of elections with candidates differing in quality, as see for example Aragones and Palfrey (2002), Serra (2011), Hummel (2013). In Appendix E I show that results are robust using more general functional forms.

her type $\theta \in \{A, D\}$ and:

$$v(\theta) = \begin{cases} v \text{ if } \theta = A \\ 0 \text{ if } \theta = D \end{cases} \quad (2)$$

A fraction α of candidates is of type A, standing for advantaged, and the remaining $1-\alpha$ is of type D, i.e. disadvantaged. For now I assume that the party R incumbent has valence equal to $v_r = v$; this captures the fact that incumbency per se gives candidates some valence (name recognition, status of insider, campaign funding opportunities) as well as the fact that non-valent incumbents are often removed by a primary challenge. In Section 7 I consider the case of an incumbent with general valence level q. I assume that pre-candidates know their valence level before the primary election, whereas voters can only observe it before the general election or, as we will see, infer it through candidates signaling in the primary election. As I show in Section 9, the key insights of the model also emerge under a more general setup in which, with some probability, valence is revealed either before the primary election or after the general election[15]. All candidates are purely office motivated and risk neutral: using a convenient normalization, their utility is 1 if they win the general election and zero otherwise.

When voting at the primary election stage, voters are forward-looking: that is to say, they trade-off their policy preference for each pre-candidate with the probability that each of them wins the general election. As a result, voting in the primary election is akin to choosing between lotteries respectively delivering each L party pre-candidate's policy platform in case of victory in the general election and the R party policy platform r in case of defeat. Primary voters vote for the pre-candidate associated with the highest expected utility. I assume that if indifferent between two candidates, a voter breaks the tie in favor of the candidate with the higher valence.

The timing of the game is as follows: first, two pre-candidates, labeled by 1 and 2, are randomly drawn to run for the nomination in the L party. Having observed their own valence, the two pre-candidates simultaneously announce each their policy platform, which remains binding throughout the electoral process. Primary election voters observe the policies of the two primary candidates, update their beliefs on the pre-candidates' types and cast their vote in the primary election. The winner of the primary becomes the L party candidate in the general election and runs against the R party incumbent.

[15] Snyder and Ting (2011) take a similar approach, assuming that valence can be revealed with some probability at each stage of the electoral campaign. In most models of primaries, valence is either known before primary elections, as in Hummel (2013) and Casas (2019), or revealed by the primary election campaign, as in Adams and Merrill III (2008) and Serra (2011); in Kartik and McAfee (2007), valence is private information and never revealed, and in Boleslavsky and Cotton (2015) it is unknown by everyone and revealed by the general election campaign.

Before the general election, valence becomes public knowledge. All voters then cast their vote in the general election, the policy platform of the winner is implemented and payoffs are realized.

Before proceeding I introduce some more of the notation that will be used throughout the paper. Denote by l the policy platform chosen by an L party general election candidate. Given this policy-platform and type $\theta \in \{A, D\}$, the L party candidate has a probability $P_\theta(l)$ of winning the general election. The expected utility a voter with bliss point x derives from an L party candidate of type θ running against incumbent r is thus:

$$\mathbb{E}_\mu[u_x(y, v(y))|l, \theta] = P_\theta(l)u_x(l, \theta) + [1 - P_\theta(l)]u_x(r, v)$$

where $y \in \{l, r\}$ is the policy platform of the election winner and $v(y)$ her valence. The expectation is taken with respect to the realization of μ, the location of the general election median voter. In order to make notation lighter, I introduce function $W_x(l, \theta)$ defined as follows:

$$W_x(l, \theta) = \mathbb{E}_\mu[u_x(y, v_y)|l, \theta] \tag{3}$$

When voting in the primary election, however, voters cannot observe pre-candidates' types. As a result, they form beliefs $\nu(l)$ based on the policy l chosen by a pre-candidate. As usual in Bayesian games, beliefs follow Bayes rule when possible. With this in mind, the expected utility from a pre-candidate becomes $\mathbb{E}_\theta[W_x(l, \theta)] = \mathbb{E}_\theta[\mathbb{E}_\mu[u_x(y, v_y)|l, \theta]]$, that is:

$$\mathbb{E}_\theta[W_x(l, \theta)] = \nu(l)\mathbb{E}_\mu[u_x(y, v_y)|l, A]] + (1 - \nu(l))\mathbb{E}_\mu[u_x(y, v_y)|l, D]] \tag{4}$$

and therefore when comparing two pre-candidates l_1 and l_2, a primary voter located at x votes for l_1 whenever:

$$\mathbb{E}_{\theta_1}[W_x(l_1, \theta_1)] \geq \mathbb{E}_{\theta_2}[W_x(l_2, \theta_2)] \tag{5}$$

Given the behavior of primary election voters, $\mathbb{E}_{\theta_2}[P^{pr}(l_1, l_2(\theta_2))]$ represents the probability of winning the primary election for pre-candidate 1 choosing policy-platform l_1 against pre-candidate 2, taking expectations over the type of pre-candidate 2 and the resulting policy l_2. Conditionally on winning the primary, the probability of winning the general election is $P_{\theta_1}(l_1)$, keeping in mind that valence becomes observable before the general election. The probability of winning office when choosing policy-platform l_1 is therefore:

$$\mathbb{E}_{\theta_2}[P^{pr}(l_1, l_2(\theta_2))]\, P_{\theta_1}(l_1) \tag{6}$$

The equilibrium concept I use in this paper is Perfect Bayesian Equilibrium (PBE). I focus on symmetric pure strategy equilibria, where by symmetry I mean that the choices of a pre-candidate only depend on her type and not on her label as pre-candidate 1 or pre-candidate 2. The following definitions summarize the structure of the game and the concept of Perfect Bayesian Equilibrium.

Definition 1. *The players are the following:* 2 *pre-candidates in the* L *party, indexed by* 1 *and* 2*, and a set of* L *party primary voters*[16]*.*

The strategy S_i *of each pre-candidate* $i \in \{1,2\}$ *is a mapping between their type and a policy platform on the real line, conditional on all the information which is common knowledge*[17]*:*

$$S_i : \theta_i \to \mathbb{R}$$

A pure strategy of a primary voter with bliss-point x*, denoted by* B_x^{pr}*, is a mapping between the voter's bliss point and the policy platforms chosen by pre-candidates on one side and a vote cast for either pre-candidate* 1 *or* 2 *on the other side:*

$$B_x^{pr} : x \times \{l_1, l_2\} \to \{l_1, l_2\}$$

A pure strategy of a general election voter B_x *is a mapping between the policy bliss-point and the platforms and valence levels of candidates on one side and a vote cast for candidate* l *or* r *on the other side:*

$$B_x : x \times \{l, r, v_l, v_r\} \to \{l, r\}$$

Definition 2. *A pure strategy PBE of the primary election game consists of the following elements:*

1. *A belief function* $\nu(l) = Pr(A|l)$*, which associates to each policy choice of pre-candidates from the* L *party a probability of being of the valent type* A*. Beliefs are consistent with Bayes rule on the equilibrium path.*

2. *Given* S_1*,* S_2 *and beliefs* $\nu(\cdot)$*, each primary voter chooses* B_x^{pr} *in order to maximize their expected utility* (4)*. General election voters choose* B_x *in order to maximize* (1)*.*

[16] The R party incumbent is a passive player and takes no action.

[17] Including the policy platform r and type θ_r of the R party incumbent, the distributions of pre-candidate types, the L party median voter location m, the distribution of the general election median voter's bliss point μ and the utility functions of all players

3. *Given B_x^{pr}, B_x, $\nu(\cdot)$ and S_{-i}, each pre-candidate $i \in \{1,2\}$ chooses S_i in order to maximize expected utility as given by condition* (6).

Clearly, this being a signaling game, there is the issue of multiplicity of equilibria: in Section 5 I rank equilibria in terms of welfare of primary election voters and show that only two focal equilibria survive a commonly used selection criterion for signaling games, the D1 criterion introduced by Cho and Kreps (1987).

4 Results

I start the analysis from the general election in which the party R incumbent candidate faces the winner of party L primary. The policy platforms of the two candidates are r and l respectively; their valence levels v_l and v_r are publicly observable. Following previous discussions, $v_r = v$ and $v_l \in \{0, v\}$. The outcome of the general election[18] depends on the comparison between the bliss-point of the median voter μ and that of the voter indifferent between the policy platforms proposed by the two candidates, which I denote by z. Notice that, given the uniform distribution of μ, for all $z \in [-b, b]$ $Pr(\mu \leq z) = \frac{z+b}{2b}$. Therefore, z determines the probability $P_\theta(l)$ of the L party candidate winning the general election conditional on her type θ.

Lemma 1. *Suppose $l \leq r$. The indifferent voter z, denoted by $z_\theta(l,r)$ to make explicit its being a function of l, r and the type θ of the candidate proposing l, takes the following location:*

$$z_\theta(l,r) = \begin{cases} -\infty, & \text{if } v_r > v_l \text{ and } v_r > r - l \\ \frac{l+r+v_l-v_r}{2}, & \text{if } \max\{v_r, v_l\} < r - l \\ +\infty, & \text{if } v_l > v_r \text{ and } v_l > r - l \end{cases} \tag{7}$$

If an indifferent voter z exists, voters with bliss-point $x < z$ vote for l and voters with $x > z$ vote for r. If $x = z$, voters choose the candidate with the highest valence or mix with probability 1/2 *if both have the same valence. Hence:*

$$P_\theta(l) = \begin{cases} 0, & \text{if } z_\theta(l,r) < -b \\ \frac{z_\theta(l,r)+b}{2b}, & \text{if } z_\theta(l,r) \in [-b, b] \\ 1, & \text{if } z_\theta(l,r) > b \end{cases} \tag{8}$$

[18]See Lemma 5 in A for a formal statement.

Given the possibility of corner solutions highlighted in Lemma 1, I make the following assumption:

Assumption 2. *The following restrictions hold: i) $r < 3b$ and ii) $v \leq b$.*

Assumption 2 makes sure that the election between the right party candidate at r and a left party candidate at $-b$ does not lead to certain victory for either of the two candidates. Moreover, notice that $v \leq b$ has a natural interpretation: the voters of the L party always prefer a non valent candidate at $-b$ to a valent one at 0. In other words, L party voters never vote for the R party incumbent.

The following lemma establishes that absent signaling concerns, all voters with bliss-point smaller than $-b$, and hence all L party primary election voters, would choose $-b$ as the optimal policy platform.

Lemma 2. *Fix the probability that a candidate is valent to any value in* $[0, 1]$*. The policy platform maximizing condition* (4) *for voters with bliss-points $x \leq -b$ is $-b$.*

The fact that the optimal location in the primary does not depend on any parameter other than b is a consequence of the uniform distribution in combination with linear policy preferences. However, as I show in Appendix D, the fact that all sufficiently extreme voters have the same optimal location for a primary candidate is a general feature of a model with linear utility.

As a consequence of Lemma 2, suppose that candidates' valence levels were known. As long as pre-candidates choose their policy platform without knowing the valence of their opponent, the outcome of the primary would be $-b$. The same holds for the case of fixed beliefs on the valence of candidates.

The next result establishes that the median voter theorem holds also for the primary election, that is the L party median voter is decisive when comparing two pre-candidates (with potentially different expected valence) in the primary election. Notice that without restrictions on the location of primary election voters, it is possible to construct examples in which a pre-candidate would win the primary election without the support of the median voter in the party.

Proposition 1. *Under Assumption 1, the median voter is always decisive in the primary* *election. Pre-candidate* 1 *wins the primary election with the following probability:*

$$P^{pr}(l_1,l_2) = \begin{cases} 0, & \textit{if } \mathbb{E}_{\theta_1}[W_m(l_1,\theta_1)] < \mathbb{E}_{\theta_2}[W_m(l_2,\theta_2)] \\ 0, & \textit{if } \mathbb{E}_{\theta_1}[W_m(l_1,\theta_1)] = \mathbb{E}_{\theta_2}[W_m(l_2,\theta_2)] \textit{ and } \theta_1 = D, \theta_2 = A \\ 1/2, & \textit{if } \mathbb{E}_{\theta_1}[W_m(l_1,\theta_1)] = \mathbb{E}_{\theta_2}[W_m(l_2,\theta_2)] \\ 1, & \textit{if } \mathbb{E}_{\theta_1}[W_m(l_1,\theta_1)] = \mathbb{E}_{\theta_2}[W_m(l_2,\theta_2)] \textit{ and } \theta_1 = A, \theta_2 = D \\ 1, & \textit{if } \mathbb{E}_{\theta_1}[W_m(l_1,\theta_1)] > \mathbb{E}_{\theta_2}[W_m(l_2,\theta_2)] \end{cases}$$

Notice that when the primary election median voter is indifferent between two pre-candidates, he mixes when the two pre-candidates are of the same type, and always chooses the valent pre-candidate otherwise. This serves the purpose of guaranteeing that the sets of equilibria are closed.

Before moving to the analysis of the primary election in the L party, I provide another preliminary result, which establishes a single crossing property[19]:

Lemma 3. *For any $l_2 > l_1$ such that $P_A(l_1) > 0$ and $P_D(l_2) > 0$,*

$$\frac{P_D(l_1)}{P_D(l_2)} < \frac{P_A(l_1)}{P_A(l_2)}$$

The interpretation of the result in Lemma 3 is that moving from platform l_2 to the more extreme platform l_1, the probability of winning the general election for non-valent pre-candidates decreases proportionally more than for valent pre-candidates. This property plays a key role for the existence of extremist separating equilibria.

4.1 Pooling Equilibria

In a pooling equilibrium, pre-candidates of both types choose the same platform, which I denote by l^{pool}. Since pre-candidates are ex-ante identical, each of them wins the primary election with probability 1/2. In order to minimize the extent of profitable deviations available to pre-candidates and thus describe, without loss of generality[20], the set of all possible pooling equilibria, I fix out of equilibrium beliefs at $\nu(l) = 0$ for all $l \neq l^{pool}$. The existence of a pooling equilibrium requires that, for both types $\theta \in \{D, A\}$ and for all

[19] In Appendix D and E I show that single crossing continues to hold under non-uniform distributions of the general election median voter location as well as under non-linear policy preferences.

[20] See Lemma 6 in Appendix A for a formal statement.

policy platforms l, the following holds:

$$\frac{1}{2}P_\theta(l^{pool}) \geq P^{pr}(l, l^{pool})P_\theta(l) \tag{9}$$

Notice that, by Lemma 2, a majority of voters would choose to locate a pooling candidate at $l = -b$. As a result, if l^{pool} is far enough from $-b$, at some point a majority of voters starts preferring a non-valent candidate at $-b$ to a pooling candidate at l^{pool}. Therefore, $P^{pr}(-b, l^{pool}) = 1$, creating the opportunity for a profitable deviation[21]. As a result, there is an interval of pooling equilibria with bounds $\underline{l}^{pool}$ and $\bar{l}^{pool}$ defined as follows:

$$\mathbb{E}_\theta[W_m(\underline{l}^{pool}, \theta)] = \mathbb{E}_\theta[W_m(\bar{l}^{pool}, \theta)] = W_m(-b, D) \tag{10}$$

The set of existing pooling equilibria is never empty and always contains $l^{pool} = -b$.

Proposition 2. *Policy platform l^{pool} can be part of a pooling equilibrium if and only if, given beliefs $\nu(\cdot)$,*

$$\mathbb{E}_\theta[W_m(l^{pool}, \theta)] \geq W_m(-b, D) \tag{11}$$

This defines an interval $[\underline{l}^{pool}, \bar{l}^{pool}]$ of possible pooling equilibria, with $\underline{l}^{pool} \leq -b \leq \bar{l}^{pool}$.

4.2 Separating Equilibria

I now discuss the existence conditions for separating equilibria. In a pure strategy separating equilibrium, valent pre-candidates choose policy l_A whereas non-valent ones choose policy l_D, so that $\nu(l_A) = 1$ and $\nu(l_D) = 0$ constitute on-path beliefs[22]. In order for no pre-candidate to have a profitable deviation, the following condition must be satisfied for both types $\theta \in \{A, D\}$ and for all possible deviations $l' \in \mathbb{R}$.

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_\theta, l_2(\theta_2))\right]P_\theta(l_\theta) \geq \mathbb{E}_{\theta_2}\left[P^{pr}(l', l_2(\theta_2))\right]P_\theta(l') \tag{12}$$

To put more content into condition (12), the first thing to notice is that in any separating equilibrium, a valent pre-candidate can never lose the primary election against a non-valent one, but can only lose in a tie-break with another valent candidate. That is to say, $P^{pr}(l_D, l_A) = 0$, $P^{pr}(l_A, l_D) = 1$ and $P^{pr}(l_A, l_A) = P^{pr}(l_D, l_D) = 1/2$. This is a consequence of the single-crossing property derived in Lemma 3. The expected probability

[21] If $l^{pool} < -b$, it is immediate to see that a profitable deviation exists, since $P_\theta(-b) > P_\theta(l^{pool})$. If instead $l^{pool} > -b$, we have $P_\theta(-b) < P_\theta(l^{pool})$, making it not automatic for a deviation to be profitable. However, it turns out that this is always the case. See the proof of Proposition 2 for details.

[22] Analogously to the case of pooling equilibria, in order to describe the whole set of possible separating equilibria, I fix, without loss of generality, $\nu(l) = 0$ for all $l \neq l_A$.

for a pre-candidate of each type of winning the primary election can thus be rewritten as:

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_D, l_2(\theta_2))\right] = \frac{1-\alpha}{2} \tag{13}$$

and

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_A, l_2(\theta_2))\right] = 1 - \frac{\alpha}{2} \tag{14}$$

These properties require that valent pre-candidates are preferred to non-valent ones by a majority of primary voters, that is:

$$W_m(l_A, A) \geq W_m(l_D, D) \tag{15}$$

Therefore, a separating equilibrium exists as long as there exist values of l_A satisfying (15) and (12) conditional on $l_D = -b$. Notice that, concerning condition (12), it is enough to check for deviations from l_D to l_A for types $\theta = D$ and from l_A to l_D for $\theta = A$ types[23].

Moreover, thanks to Lemma 2 it can also be established that in any pure strategy separating equilibrium, non-valent pre-candidates have to choose $l_D = -b$. Since l_D is associated with the worst possible belief $\nu(l_D) = 0$, l_D must correspond to the first-best policy platform $-b$.

Lemma 4. *In any separating equilibrium, $l_D = -b$.*

Having pinned down l_D, l_A must lie at the intersection of three sets: $\mathcal{P}^A_{-b}$ contains the points such that condition (15) is satisfied, given $l_D = -b$; $\mathcal{IC}^D_{-b}$ contains the points such that condition (12) is satisfied for $\theta = D$ and $\mathcal{IC}^A_{-b}$ contains the points such that condition (12) is satisfied for $\theta = A$. I start from $\mathcal{IC}^D_{-b}$. Evaluating (12) for $\theta = D$ and $l_D = -b$ yields:

$$\frac{P_D(l_A)}{P_D(-b)} \leq a \tag{16}$$

where $a = \frac{1-\alpha}{2-\alpha}$ denotes the ratio between the expected equilibrium probability of winning the primary elections for a non-valent versus a valent pre-candidate. The set $\mathcal{IC}^A_{-b}$ is derived similarly, rewriting condition (12) as:

$$\frac{P_A(l_A)}{P_A(-b)} \geq a \tag{17}$$

Proposition 3. *A pair (l_D, l_A) can be part of a separating equilibrium as long as $l_D = -b$ and $l_A \in \mathcal{IC}^D_{-b} \cap \mathcal{IC}^A_{-b} \cap \mathcal{P}^A_{-b}$, which describes the points satisfying* (15)*,* (16) *and* (17)*. If*

[23] This is because given $\nu(l) = 0$ for all $l \neq l_A$, any pre-candidate deviating to any policy $l' \notin \{l_A, l_D\}$ would receive a payoff of zero.

$\mathcal{IC}^D_{-b} \cap \mathcal{IC}^A_{-b} \cap \mathcal{P}^A_{-b} = \emptyset$, no separating equilibrium exists.

Solving condition (16) as equality yields an upper bound on l_A, smaller than $-b$, which I denote as l^e_A. Policy platforms below this point are incentive compatible with respect to deviations by non-valent pre-candidates. The intuition is that when the probability of winning the general election with policy-platform l_A becomes sufficiently low compared to that of winning it with platform $-b$, non-valent pre-candidates prefer to reveal themselves as non-valent, win the primary election with a lower probability but be in a better position to win the general election conditional on winning the primary.

Lemma 3 assures that $\mathcal{IC}^D_{-b} \cap \mathcal{IC}^A_{-b} \neq \emptyset$ and in particular $l^e_A \in \mathcal{IC}^A_{-b}$. This means that we can always find a candidate platform l_A which is incentive compatible for both types. However, there might be no such policy that, even if chosen by a valent pre-candidate, the median primary voter prefers to a non-valent pre-candidate at $-b$. This happens if $l^e_A \notin \mathcal{P}^A_{-b}$. In this case, there is no point to the left of $-b$ in the intersection $\mathcal{IC}^D_{-b} \cap \mathcal{IC}^A_{-b} \cap \mathcal{P}^A_{-b}$ and no l_A smaller than $-b$ can be part of a separating equilibrium.

Suppose that $l^e_A \in \mathcal{P}^A_{-b}$. Moving further to the left of l^e_A, the policy chosen by the valent pre-candidate becomes increasingly extreme. At some point, either primary voters start preferring a non-valent candidate at $-b$, or valent candidates themselves start having an incentive to choose the policy platform assigned to non-valent pre-candidates. This defines a lower-bound $\underline{l}^e_A$ to complete the characterization of the interval $[\underline{l}^e_A, l^e_A]$ of values that l_A can take.

Let's now go back to condition (16) and consider policies $l > -b$. Following Lemma 1, $P_D(l) = 0$ for all $l \in [r - v, r + v]$: a non-valent pre-candidate choosing a platform too close to the incumbent is bound to lose the election. As a result, all points in $[r-v, r+v]$ satisfy (16) and are hence in $\mathcal{IC}^D_{-b}$. All these points are more attractive than $-b$ for valent pre-candidates, and hence they satisfy (17) and are part of $\mathcal{IC}^A_{-b}$. Concerning (15), there is an upper bound $\bar{l}^c_A$ at which the median voter would stop supporting a valent pre-candidate in favor of a non-valent one. This upper bound can be larger or smaller than $r - v$. In the former case, the policy-platforms in the interval $[r - v, \bar{l}^c_A]$ are in $\mathcal{P}^A_{-b}$, in the latter case no policy in $[r - v, r + v]$ is in $\mathcal{P}^A_{-b}$. To sum up, $[r - v, \bar{l}^c_A]$ identifies another possible interval of values that l_A can take in a separating equilibrium.

Proposition 4. *If a separating equilibrium exists, l_A is in one of the two intervals $[\underline{l}_A, l^e_A] \subset (-\infty, -b)$ and $[l^c_A, \bar{l}^c_A]$, with $l^c_A = r - v$ and $\bar{l}^c_A \leq r$.*

To sum up, there can exist two types of separating equilibria, which differ in terms of the policy chosen by valent pre-candidates. If $l_A \in [\underline{l}_A, l^e_A]$, valent pre-candidates signal their valence by choosing a more extreme policy than their non-valent counterparts, and therefore I call these equilibria *extremist* separating equilibria. If instead $l_A \in [r - v, \bar{l}_A^c]$, valent pre-candidates choose a policy closer to the incumbent, and also closer to the center of the ideological space: for this reason I call these *centrist* separating equilibria[24].

As Proposition 3 makes clear, a separating equilibrium, unlike a pooling equilibrium, is not guaranteed to exist for all parameter combinations. The next corollary explicitly shows necessary and sufficient conditions for at least one centrist and extremist separating equilibrium to exist. These conditions are nothing other than the existence conditions of the centrist equilibrium with $l_A = l_A^c$ and the extremist with $l_A = l_A^e$. As a matter of fact, if any centrist (respectively extremist) separating equilibrium exists, the centrist (respectively extremist) at l_A^c (respectively l_A^e) exists. As we will see in the next section, these equilibria are also focal in that, together with the pooling equilibrium at $-b$, they make up the set of equilibria that can be welfare dominant for primary election voters.

Corollary 1. *The set of existing centrist (and respectively extremist) separating equilibria is not empty if and only if condition* (15) *is satisfied at* $l_A = l_A^c$ *(and at* $l_A = l_A^e$ *respectively).*

Notice that condition (15) applied to the centrist separating equilibrium does not depend on either m or α, but only on the value of v compared to b and r. The condition for the existence of extremist separating equilibria, on the other hand, depends also on α and m. In particular, evaluating (15) at l_A^e delivers the following condition:

$$m \leq r - |l_A^e - m| - \frac{(b + r - v)^2}{v + a(b + r - v)} \tag{18}$$

which, if $m > l_A^e$, translates in an upper bound on m below which the extremist separating equilibrium at l_A^e exists.

5 Welfare, Comparative Statics and Equilibrium Selection

As it has become clear in Section 4, there are many possible equilibrium outcomes, as is always the case in signaling games. In particular, there is an interval of pooling equilibria and there are two intervals of possible separating equilibria, extremist and centrist. However, each of these three intervals contains a focal equilibrium, that I denote as welfare

[24]In the comparative statics paragraph I provide a more detailed description of the meaning of centrist.

dominant. That is to say, there are parameter combinations for which these equilibria maximize the utility of primary voters out of all existing pure-strategy equilibria.

The characterization of the welfare dominant equilibria of the game follows directly from Lemma 2: comparing two pre-candidates with the same probability of being valent, primary voters prefer the one closer to the first best platform $-b$. Comparing separating equilibria, notice that $l_D = -b$ across all separating equilibria (independently of whether centrist or extremist); in addition to that the probability of selecting a valent pre-candidate is equal to $\alpha(2-\alpha)$ in all separating equilibria. Therefore, comparing the expected utility of any two separating equilibria can be reduced to comparing the expected utility from the policy platforms l_A chosen by valent pre-candidates. Lemma 2 can be applied to this comparison to conclude that the welfare maximizing extremist and centrist separating equilibria are respectively that with $l_A = l_A^e$ and that with $l_A^c = r - v$. An analogous reasoning allows to conclude that the welfare maximizing pooling equilibrium is such that $l^{pool} = -b$.

It is also important to notice that the pooling equilibrium at $l^{pool} = -b$ always exists and, given Corollary 1, the separating equilibria with $l_A \in \{l_A^e, l_A^c\}$ always exist unless no separating equilibrium of this kind exists. This means that the welfare dominant equilibrium of the game is always one of the three focal equilibria I just described.

Proposition 5. *For all parameter combinations, the welfare dominant equilibrium for primary election voters is one of the following three: i) The pooling equilibrium with $l^{pool} = -b$, ii) The extremist separating equilibrium with $l_A = l_A^e$ defined by* (19) *and iii) The centrist separating equilibrium with $l_A = l_A^c = r - v$.*

5.1 Comparative Statics

Having identified the welfare dominant equilibria of the game, I now analyze how the platforms l_A^e and l_A^c depend on the parameters of the model. Let's start from the extremist separating equilibrium: keeping in mind that l_A^e is the value solving condition (16) as equality, we can use expression (8) to write the expression for l_A^e in a closed form:

$$l_A^e = -b - (1-a)(b + r - v) \tag{19}$$

where $a = \frac{1-\alpha}{2-\alpha}$. The comparative statics immediately follow from (19), but the most effective way to glean intuition is to look at condition (16), which reads $\frac{P_D(l_A)}{P_D(-b)} \leq a$. For example, the parameter α appears only on the right hand side of (16): the larger α, the smaller the ratio a, i.e. the less likely non-valent pre-candidates are to win primaries compared to valent pre-candidates. To preserve incentive compatibility, l_A^e moves to the left. Parameters r, b and v instead enter the ratio on the left hand side of (16): an increase in r or b increases the probability of winning the general election with the extremist policy l_A^e proportionally more than with the non-valent policy $-b$. Therefore, incentive compatibility requires a more extreme l_A^e. The effect of an increase in v is the opposite, given the substitutability between r and v in (16). As a result, policy l_A^e is less extreme when valence is higher[25].

A similar analysis can be done for the centrist equilibrium: in that case, policy platform $l_A^c = r - v$ moves to the right as r increases but it moves to the left as v increases (whereas it is not affected by either b or α). Notice that $|l_A^c| \leq |l_A^e|$ (strictly unless $v < b + r$), hence the notion of centrist equilibrium, but there are situations where $l_A^c > 0$ and the centrist equilibrium still exists[26].

To sum up, when the extremist separating equilibrium is the outcome of the game, the valence and extremism of pre-candidates are positively correlated; moreover, a greater extremism of the incumbent has a polarizing effect on the policy-platforms of valent pre-candidates of the challenging party; finally, increasing the valence of the incumbent has a moderating effect. When instead the centrist equilibrium is played, valent candidates track the policy platform of the incumbent, but their platforms move to the left as the incumbent becomes more valent.

Proposition 6. *The extreme separating equilibrium policy l_A^e moves to the left whenever: i) α increases ii) b increases iii) r increases or iv) v decreases. The centrist separating equilibrium policy l_A^c moves to the right as r increases and to the left as v increases.*

Proposition 6 tells us how the policies chosen in each equilibrium change when the parameters of the model change. However, changes in the parameters can also cause the welfare dominant equilibrium to switch kind, either between separating and pooling, or between extremist separating and centrist separating. Consider the comparison between the centrist and extremist separating equilibria. Intuitively, the more extreme primary voters are, the more likely they are to prefer the extremist separating equilibrium. In particular, since the probability of a valent candidate being elected is the same in both equilibria, voters with a bliss point to the left of l_A^e prefer the separating equilibrium which

[25]Notice that the valence v entering condition (16) is the valence of the incumbent.

[26]The fact that $|l_A^c| \leq |l_A^e|$ holds since the following inequality always holds under Assumption 2: $|r - v| \leq |-b - (1 - a)(b + r - v)|$. Moreover, without further parameter restrictions, it is possible for the valent L party pre-candidate in a centrist separating equilibrium to win the general election with certainty against the incumbent. This happens when $\frac{r-v+r}{2} > b$. Intuitively, if the incumbent is to the right of b and valence v is small enough, the centrist pre-candidate can be enough to the right to always win the general election.

delivers a more left-wing expected policy, closer to their bliss-points. This is expressed by the following condition:

$$P_A(l_A^e)l_A^e + (1 - P_A(l_A^e))r \leq P_A(l_A^c)l_A^c + (1 - P_A(l_A^c))r \tag{20}$$

This condition is necessary for a majority of voters to prefer the extremist separating equilibrium, but it is not sufficient, since the median primary election voter might have a bliss-point m to the right of l_A^e. Suppose that this is indeed the case, and that condition (20) is satisfied. Then, the median primary voter prefers the extremist separating equilibrium if:

$$m \leq \frac{l_A^c + l_A^e}{2} - \frac{v}{2}\left[\frac{P_A(l_A^c)}{P_A(l_A^e)} - 1\right] \tag{21}$$

Condition (21) can be interpreted as follows: the first term represents the mid-point between the policy-platform of the valent candidate in the centrist versus the extremist separating equilibrium. The second term, instead, represents the adjustment to take account for the different probabilities of winning the general election at l_A^c and l_A^e. The more likely the centrist candidate is to win the general election compared to the extremist one, the more the threshold moves to the left.

Notice that, in line with the term *extremist*, a necessary condition for the extremist separating equilibrium to be preferred to the centrist separating is, as a consequence of (20), that it delivers a more left-wing expected policy-platform.

The following proposition summarizes these facts into a unique condition, that can be represented as (20) for $m \leq l_A^e$ and (21) if $m > l_A^e$.

Proposition 7. *A majority of primary election voters prefers the extremist to the centrist separating equilibrium if and only if:*

$$m \leq l_A^c - l_A^e - v\left(\frac{P_A(l_A^c)}{P_A(l_A^e)} - 1\right) - |l_A^e - m| \tag{22}$$

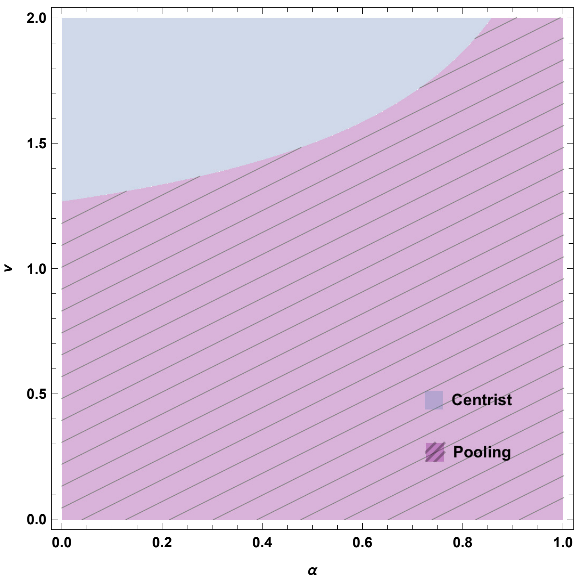
Proposition 7 is interesting since it suggests that changes in the ideology of primary election voters might have large effects on the outcome of primary elections by changing the way valent pre-candidates signal their valence in primary elections. This feature of the model might for example echo the change in electoral narrative that has been seen in recent primary elections in the United States, in which several candidates have, in an increasingly successful manner, chosen radical platforms instead of more traditional centrist ones. My model can reconcile this observation with an environment of office oriented politicians and forward looking primary voters.

Independently of what type of separating equilibrium voters prefer for some given parameters, the ranking between pooling and separating equilibria follows the same pattern. As it can be seen in Figures 1a and 1b, a separating equilibrium is preferred for high values of valence v (with respect to b and r) and low values of the share of valent pre-candidates α. This is connected to the comparative statics described in Proposition 3: when r increases, the separating policy platform l_A (be it extremist or centrist) moves further away from the first-best value $-b$, making it less appealing. The opposite happens after an increase in v, which also makes it more valuable to select a valent pre-candidate. Finally, an increase in α both makes l_A^e more extreme (in case of an extremist separating equilibrium) and makes a pooling equilibrium more appealing, since the risk of drawing a bad candidate decreases.

Proposition 8. *A separating equilibrium is preferred to the pooling when v is large enough (given b and r) and α is small enough. For large enough r the pooling equilibrium is preferred to both separating equilibria.*

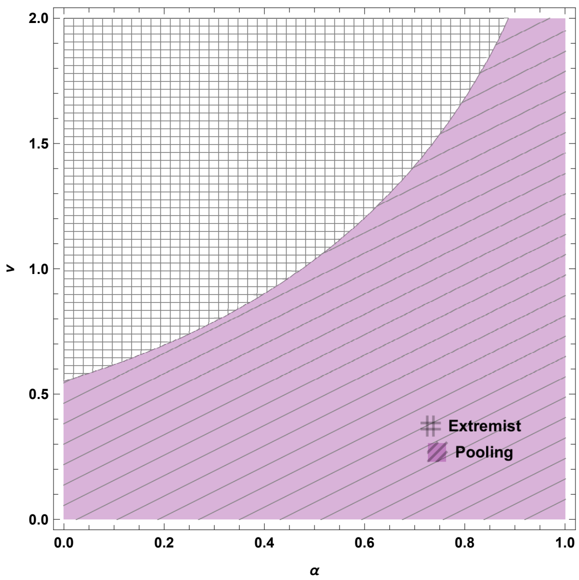
The ranking between separating and pooling equilibrium is also interesting because it can be interpreted as a way to compare nomination through primary elections and direct party nomination, as I will discuss more in depth in Section 6.

Figure 1: Welfare Dominant Equilibria



(a) Moderate Party Median Voter

Figure 1a shows the welfare dominant equilibrium of the game in the (α, v) space for the parameter values: $m = -2$, $b = 2$, $r = 1$



(b) Extreme Party Median Voter

Figure 1b shows the welfare dominant equilibrium of the game in the (α, v) space for the parameter values of Parameter values: $m = -5$, $b = 2$, $r = 1$

5.2 Equilibrium Selection

There are two main sources of equilibrium multiplicity in this game: one is the *classical* multiplicity of signaling games, given by the fact that unrestricted beliefs can lead to a multitude of equilibria. The other has to do with the existence of two different types of separating equilibrium, often also for the same parameter combinations. Moreover, whereas primary election voters prefer one or the other type of equilibrium depending on the circumstances, valent pre-candidates, being purely office motivated, always prefer the centrist separating to the extremist separating equilibrium. This adds a further level of complexity.

In order to take care of the first source of equilibrium multiplicity, I apply a well-known equilibrium selection criterion for signaling games, the D1 criterion introduced by Cho and Kreps (1987). Restricting off-equilibrium beliefs, this eliminates all pooling equilibria as well as all separating equilibria other than the two focal ones with $l_A \in \{l_A^e, l_A^c\}$.

Proposition 9. *The only pure strategy equilibria surviving the D1 criterion are the extremist separating equilibrium with $l_A = l_A^e$ and the centrist separating equilibrium with $l_A = l_A^c$. For some parameter values, both separating equilibria exist; in other cases, no pure strategy equilibrium survives the D1 criterion.*

The D1 criterion, however, cannot take care of the second type of multiplicity: if anything, by restricting beliefs it gives additional leverage to valent pre-candidates, potentially allowing them to exploit their first mover advantage and achieve the centrist separating equilibrium even when against the interests of primary voters. Ultimately, this has to do with competition between valent pre-candidates not being perfect: when α is low, as a matter of fact, valent pre-candidates are almost monopolists and are thus not disciplined to choose the outcome preferred by primary voters.

Therefore, as proposition 10 makes clear, the D1 criterion can fail to select the welfare dominant equilibrium: this happens either when the welfare dominant equilibrium is pooling or, in some circumstances, when it is the extremist separating equilibrium. The intuition is the following: if a majority of primary election voters prefers the centrist equilibrium, the interests of primary voters and valent pre-candidates are aligned. In this case, the D1 criterion selects as the unique equilibrium of the game the centrist separating. When the extremist separating equilibrium is welfare dominant, however, there is a conflict of interests between primary election voters and valent pre-candidates. Since under the D1 criterion, for all $l \in [r - v, r + v]$ beliefs are $\nu(l) = 1$, an extremist separating equilibrium exists only if valent pre-candidates have no incentive to deviate from policy l_A^e to policy $\bar{l}_A^c$, which requires:

$$\frac{2-\alpha}{2} P_A(l_A^e) \geq (1-\alpha) P_A(\bar{l}_A^c) \tag{23}$$

or that all deviations to policies in $[r-v, r+v]$ yield a zero payoff, which happens if a majority of voters prefers the non-valent pre-candidate at $l_D = -b$, that is $W_m(l_A^c, A) < W_m(l_D, D)$.

Proposition 10. *The D1 criterion selects the welfare dominant equilibrium only if this is separating. If the welfare dominant equilibrium is the centrist separating (see Proposition 7), then D1 selects it. If instead the welfare dominant equilibrium is the extremist separating, then D1 selects it if either* (23) *holds or if* $W_m(l_A^c, A) < W_m(l_D, D)$ *(in this case the extremist separating is also the only one selected).*

In Figure 2 I present an example to show that the D1 criterion can fail to select the welfare dominant equilibrium for primary election voters, either because it selects a separating equilibrium instead of the pooling or because it selects the centrist separating instead of the extremist separating[27].

Given this issue with equilibrium selection, are there other arguments in favor of the welfare dominant equilibrium being the outcome of the game, at least when it is separating? A possible answer is that primary election voters have some power to screen candidates and determine which policy-platforms valent pre-candidates are allowed to choose. For example, the support of policies that candidates are allowed to choose could be restricted by imposing an upper bound[28].

[27]Notice that if we were instead interested in the welfare of the average general election median voter, this feature of equilibrium selection would, in most circumstances, be regarded as beneficial.

[28]Possible screening devices might be party interest groups or activists that need to prepare the ground and sponsor a candidate's campaigns to make it visible. Dynamic considerations might also play a role. A deeper analysis of these issues is beyond the scope of this project.

Figure 2: Equilibrium Selected by D1 criterion: Extremist Primary Median Voter

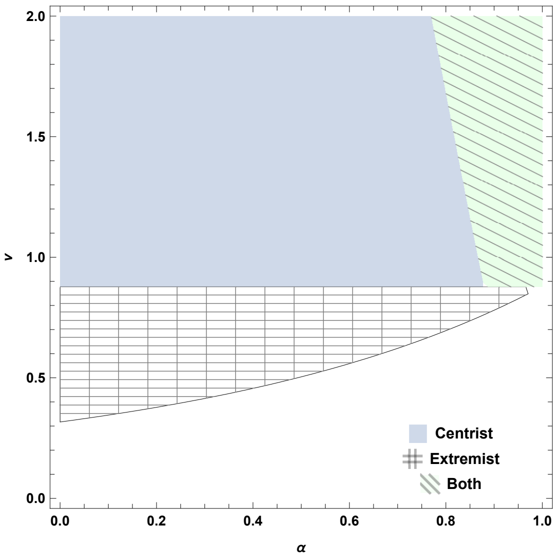


Figure 2 shows, in the (α, v) space, the equilibrium selected by the D1 criterion for parameter values: $m = -5$, $b = 2$, $r = 1$

6 Institutional Analysis

As I anticipated in the previous section, the ranking between separating and pooling equilibrium can be interpreted more broadly as a ranking between primary elections and direct party nomination.

In order for the pooling equilibrium at $l^{pool} = -b$ to be equivalent to the outcome of direct party nomination, two things are needed: i) party elites choose $-b$ as the policy platform under direct party nomination and ii) party elites draw their candidates from the same pool and cannot (or do not want to) select valent candidates. Notice that as long as ii) holds, the welfare ranking between separating and pooling equilibrium determines the smallest set of parameters for which voters prefer primaries to direct party nomination. Adding policy conflict with party elites, as a matter of fact, just increases the scope for primaries to benefit voters.

In Appendix C I briefly consider the scenario in which both i) and ii) do not hold. In particular, I focus on a scenario in which nomination by party elites yields the same selection of valent candidates of a separating equilibrium, but where party elites would choose their own preferred policy platform.

The equilibrium selection through the D1 criterion introduces a further twist on the institutional interpretation of the model: comparing Figure 1b and Figure 2, we can see that there are parameter values for which the welfare dominant equilibrium is the extremist separating, but where the unique equilibrium selected by the D1 criterion is the centrist separating. As a result, there are parameter values for which direct party nomination is preferred in order to achieve *the lesser evil*, that is the pooling equilibrium instead of the centrist separating.

Corollary 2. *Consider the game with equilibrium selection through the D1 criterion. If, as per Proposition 8, the welfare dominant equilibrium is the pooling equilibrium, then direct party nomination is preferred to primaries. If the welfare dominant equilibrium is the centrist separating, then primaries are preferred to direct nomination. If the welfare dominant equilibrium is the extremist separating, things can go both ways.*

To sum up, my model suggests that primary elections should (up to some further caveats due to equilibrium selection) be preferred in the circumstances described in Proposition 8. These are the following: when direct selection is unlikely to result in a valent pre-candidate (low α); when valence is an important dimension for elections (i.e. parties that do not solely revolve around a specific ideology); when the incumbent's policy platform is not too extreme. Interestingly, my model does not suggest that primaries can only benefit parties with relatively extreme voters.

This result suggests that an increased difficulty of identifying valent candidates as well as an increase in the importance of valence might have led parties to hold primaries. Such forces seem to have indeed played a relevant role in the introduction of primaries in the United States at the beginning of the twentieth century. Urbanization and immigration deeply affected social cleavages, which probably made having valent politicians vital in order to keep voters together. Moreover, the same social changes mentioned above are also likely to have broadened the pool of possible politicians, making it harder for party insiders to judge the quality of candidates. The transition from a society in which most party voters would personally know candidates to a much more complex and changing environment is also mentioned by Ware (2002) in his book on the introduction of primary elections. Interestingly, Ware (2002) supports the view of primaries being not the result of a conflict between parties and voters, but the consequence of an evolution of society that made a renewal of the nomination system necessary for the very interests of parties.

With the due reverse causality caveats, this result also seems to be in line with the general observation that primaries are used in the United States, a country in which politics is more personalistic and candidate-centered, with traditionally a larger number of non-professional politicians entering politics from other sectors and where parties are considered to be less ideological than for example in Europe, where primaries are much less used.

Another reason why parties might want to introduce primaries is to increase their probability of winning elections thanks to more valent candidates and, in some circumstances, electorally competitive centrist policies. The following corollary states the conditions under which that happens in my model.

Corollary 3. *When the centrist equilibrium is played, primaries increase the probability of the left party to win the general election. If the extremist equilibrium is played, on the other hand, the probability for the left party to win the general election increases if and only if* $\frac{v}{b+r} \geq \frac{1}{2-\alpha}$.

Notice that this discussion is somewhat connected to that on policy conflict done in Appendix C: office motivated party bosses might have an incentive to allow for primaries as a way not to have to directly propose policies that are in their interest but that, being outside the party mainstream, might attract the criticism of party members.

7 Incumbent Valence

In the baseline model, for simplicity, I assume that the incumbent has the same valence level v as valent pre-candidates in the L party. In this section I briefly show what happens when these two parameters are allowed to differ.

In particular, this serves the purpose of answering the question on the role played by incumbent valence (which can be interpreted as incumbency advantage) for the existence and welfare properties of separating equilibria, and hence primary elections.

To this end, let q be the valence level of the incumbent and v that of a challenger of type $\theta = A$. In terms of policy platforms, the larger is q the more the centrist and extremist separating equilibrium policies l_A^c and l_A^e move close to $-b$. This makes separating equilibria more attractive. Together with the fact that q also makes the incumbent less bad in the eyes of L party voters, this suggests that parties facing a strong incumbent are more likely to resort to the separating equilibria described in my model.

Proposition 11. *Suppose that* $q \leq b + r$. *Fix* b, r *and* m *and consider the* (α, v) *space. The higher is* q, *the larger the set of parameters under which a separating equilibrium (centrist or extremist) is preferred to the pooling equilibrium.*

Notice that given q, a separating equilibrium is more likely for L parties with a valence advantage over the incumbent (i.e. $v > q$), but at the same time given v a separating equilibrium is more likely to be preferred for parties with a valence disadvantage (i.e. $q > v$). In other words, incumbency advantage alone is not the only determinant of whether a separating equilibrium is preferred, but the total level of valence $v+q$ matters, too. In this respect, a separating equilibrium is preferred to a pooling equilibrium for a larger set of parameters when the total amount of valence $v+q$ is larger.

8 Summary of Results

I present a model of a party holding primary elections to select the candidate to challenge an incumbent in a general election. The valence of pre-candidates is private information, but signaling can take place through the choice of policy platforms. I show that two types of separating equilibria exist: a centrist separating equilibrium and an extremist separating equilibrium. In the centrist separating equilibrium, valence and ideological extremism are substitutes, whereas they are complements in the extremist separating equilibrium: this result can help reconcile evidence on the correlation between valence and ideology that points in different directions: my results are compatible both with the view that candidates far from the political center have more successful election performances, as in Burden (2004) or Stone and Simas (2010), and the evidence, provided for example by Ansolabehere et al. (2001), that high quality candidates choose more moderate platforms[29].

Moreover, my model predicts that the more extreme primary election voters are, the more likely it is for the valence and extremism to be positively correlated, as Brady et al. (2007) and Gerber and Morton (1998), among others, suggest[30].

The model also predicts that a separating equilibrium dominates the pooling equilibrium whenever valence is high, when the share of valent pre-candidates is low and when the incumbent's platform is not too extreme: this latter fact suggests that separating strategies are more likely to be seen in parties that do not enjoy a large ideological advantage.

In Section 7 I show that the higher the valence level of the incumbent, the larger the set of parameters for which a separating equilibrium is preferred to the pooling equilibrium. This suggests that pre-candidates aiming to challenge strong incumbents are more likely to resort to a separating strategy compared to pre-candidates aiming to challenge weaker incumbents. This is in line with Londregan and Romer (1993), who conclude that incumbency advantage and platform polarization are positively correlated.

As I discuss in Section 6, the preference for a separating over the pooling equilibrium

[29]For a broader discussion of the empirical literature on valence and ideological positioning, refer to Footnote 1.

[30]See Footnote 2 for a discussion of the empirical literature on primaries and extremism.

can also be interpreted as preference for the institution of primary elections over direct party nomination: in this respect, my model suggests that parties are more likely to resort to primaries when valence is important, valent pre-candidates are scarce and general elections are competitive. This confirms the narrative and stylized facts presented by Ware (2002). Moreover, Snyder and Ting (2011) report that competitive primary elections in the USA are more likely to take place in competitive states rather than in states dominated by one party, consistent with this result. Looking instead at the voluntary introduction of primaries by parties in Latin America, Aragon (2009) finds that the adoption of primaries is positively correlated with the competitiveness of the political environment.

Along with providing results that confirm what some of the empirical literature has concluded on the topics of valence, extremism and primary elections, my analysis will hopefully also be able to guide future empirical investigations on these issues and contribute to settling some of these long-standing puzzles.

9 Timing of Valence Revelation

The assumption that valence is never observable before the primary election but always observable before the general election is clearly a modeling simplification. However, this captures the realistic idea that voters are better able to assess a candidate's valence closer to the election, thanks to an increased salience and the accumulation of media scrutiny of candidates[31]. With this in mind, in this section I discuss the robustness of the model with respect to different patterns of observability of the valence of candidates.

9.1 Revelation Before the Primary Election

The first possibility is that valence might already be known before the primary election. In particular, suppose that, with probability β, the valence of all pre-candidates is revealed before the primary election vote. To make things interesting, suppose that candidates choose a policy platform before knowing whether valence is observable or not. The case $\beta = 0$ is my baseline model, whereas $\beta = 1$ is the full information benchmark. Notice that with full information, Bertrand competition between candidates leads to both types choosing policy $-b$, which is optimal from the perspective of a majority of party voters.

[31]Along with ample anecdotal evidence of this, Nyhan (2015) shows that, in presidential elections, most scandals occur in the few months before the general election. Relatedly (see for example Bernhardt and Ghosh (2020)) negative campaigning is much more prominent in general than primary elections, and also newspaper endorsements often occur shortly before the general election.

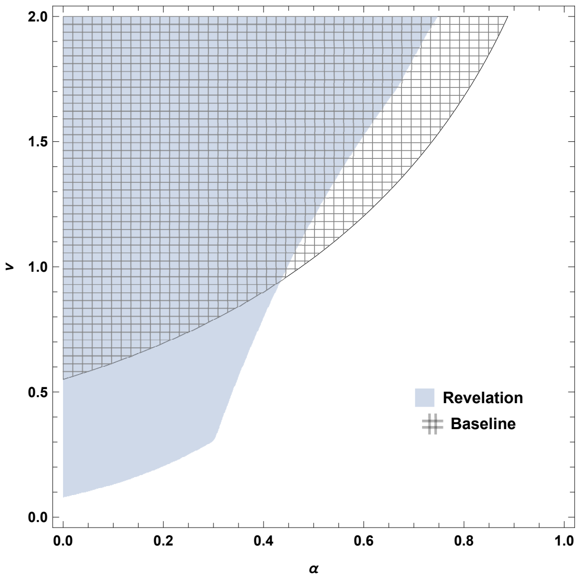
What I want to argue here is that for β not too high, the results of my baseline model go through. When $\beta > 0$, the new possible deviation that we have to account for when considering all equilibria is something that can be called *gambling on valence observability.* If β is high enough, valent pre-candidates have the possibility of deviating out of an equilibrium policy with the purpose of winning the primary election when valence becomes observable. For example, take a centrist separating equilibrium: when β is sufficiently high, valent pre-candidates have the incentive to choose a platform slightly to the left if l_A^c. If valence if revealed before the primary, they win the primary election for sure and suffer only a small loss in terms of general election electability compared to l_A^c. Similarly, starting from an extremist separating equilibrium, for high β valent pre-candidates have two possibly profitable deviations: the first deviation is to the largest platform l such that they can still win the primary against a non-valent opponent and then have a much better chance to win the general election; the second is a deviation to the largest l such that they can win the primary for sure subject to valence becoming observable. An analogous deviation is available from a pooling equilibrium. Moreover, notice that as β increases, the incentive compatibility constraint for non-valent pre-candidates becomes less demanding, facilitating the existence of a separating equilibrium. The reason is that if valence becomes observable before the primary election, deviating to the valent platform l_A results in a certain electoral defeat.

As we can see in Figure 3a, compared to the baseline case, with $\beta > 0$ an extremist separating equilibrium exists for some low values of v and α, due to the improved incentive compatibility constraint, but it ceases to exist for some high values of v and α, due to the deviations by valent pre-candidates.

9.2 No Revelation After the Primary Election

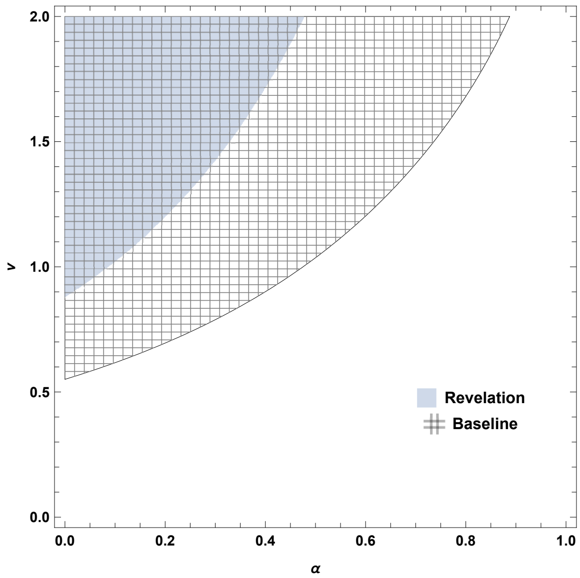
The second possibility is that valence might not be exogenously revealed after the primary election. To this end, suppose that valence is exogenously revealed after the primary election only with probability η. The baseline model would therefore be $\eta = 1$, whereas if $\eta = 0$ valence is never publicly observable before elections. Having $\eta < 1$ clearly makes incentive compatibility with respect to non-valent candidates more demanding. The reason is that with probability $1 - \eta$, a deviation by a non-valent pre-candidate goes undetected (until payoffs are realized after the general election). Therefore, the centrist separating equilibrium only exists for high values of η. The extremist separating equilibrium, instead, turns out to be much more robust as η decreases: the policy l_A^e moves to the left to preserve incentive compatibility, and as η goes to zero it converges

Figure 3: Timing of Valence Revelation



(a) Valence Revelation Before the Primary Election: Extremist Separating Equilibrium

Figure 3a shows the parameter values (α, v) such that the extremist separating equilibrium is welfare dominant for $b = 2$, $r = 1$, $m = -5$ under either the baseline model or the model with $\beta = 0.4$.



(b) Valence Revelation After the Primary Election: Extremist Separating Equilibrium at $\eta = 0.5$

Figure 3b shows the parameter values (α, v) such that the extremist separating is welfare dominant in the baseline model with parameters $b = 2$, $r = 1$, $m = -5$ and in the model with $\eta = 0.5$.

to the value at which valent pre-candidates are indifferent between choosing l_A^e and l_D. When $\eta = 0$, as a matter of fact, both types of pre-candidates receive the same expected payoff in equilibrium. As Figure 3b shows, the extremist separating equilibrium is welfare dominant for a relatively large set of parameters even with a low value of η such as 1/2. Notice that the extremist separating equilibrium exists even at $\eta = 0$ for a quite large set of parameters, but it is never welfare dominant.

9.3 Different Types of Valence

Finally, another possible hypothesis is that valence is always revealed before elections, yet primary and general elections require different types of valence which are not perfectly correlated. This is realistic if we think of valence as the ability to campaign against a specific incumbent, for example. In this setup, observing that a candidate is valent before the primary election allows voters to update their beliefs on the probability that a candidate will be valent in the general election. In light of this, my model can be interpreted as the subgame of a broader candidate selection procedure, in which two pre-candidates with high primary election valence compete to become party nominees. In this setup, the parameter α would measure the correlation between primary and general election valence. In this respect, my model suggests that a separating equilibrium is more likely to occur when α is low, that is to say when the primary and general election race are different from each other.

10 Conclusion

This paper provides a theory of electoral competition in primary elections, focusing on an environment in which the valence of primary election candidates is not observable to voters until after the primary election.

In this setup, I find that office motivated candidates can use their choice of a policy-platform to signal their valence. Interestingly, this can be done using either a sufficiently extreme policy platform, or a platform that is close enough to that of the incumbent from the opposing party. The intuition is that in order to credibly signal valence, a policy-platform must be not worthwhile for non-valent pre-candidates. This can be achieved either with an extreme policy, or with a policy sufficiently close to that of the incumbent. In general, a party with extreme voters prefers candidates to choose the extremist separating strategy, and viceversa for parties with moderate voters.

My model can help provide a theoretical explanation of the mixed empirical findings concerning the divergence of candidates in elections and the effect of primary elections on policy extremism and polarization. Moreover, my results also speak to the question of under what circumstances party voters benefit from primary elections: first of all, I show that primary elections affect the policy platforms of candidates even when there is no policy conflict between parties and voters; the policy/valence trade-off caused by signaling leads voters to benefit from primaries when valence is high, the fraction of valent candidates is low and incumbents are not too extreme. Moreover, signaling is more likely to take place when the incumbency advantage is large.

In the extensions I show that extremist and centrist separating equilibria can take place when both parties hold primaries, and that they are also robust to different specifications of the timing of valence revelation.

In future work, the setup developed in this could be used to answer additional questions: one avenue would be to include other possible ways of signaling valence and study questions related for example to campaign finance regulation; other possibilities would be to study the endogenous revelation of information on candidates or a setup with repeated elections.

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A Proofs

Lemma 5. *The L party candidate, located at $l \leq r$, wins the general election if and only if $u_\mu(l, v(\theta)) \geq u_\mu(r, v_r)$.*

Proof of Lemma 5

Proof. In order to prove this, take $u_x(l, v(\theta)) - u_x(r, v_r)$. This is a continuous function of x. Differentiating with respect to x, we see that this function is constant for $x < l$ and for $x > r$. For $x \in [l, r]$, instead, the derivative is negative. Therefore, there can only be one indifferent voter in $[l, r]$, which I denote as z. To show the if result, suppose $u_\mu(l, v(\theta)) - u_\mu(r, v_r) \geq 0$. Since the derivative of $u_x(l, v(\theta)) - u_x(r, v_r)$ is weakly negative, then $u_x(l, v(\theta)) - u_x(r, v_r) \geq 0$ for all all voters such that $x \leq \mu$, which are a majority. To show the only if part, proceed by contradiction. If the L party candidate wins the election but $u_\mu(l, v(\theta)) < u_\mu(r, v_r)$, then for all $x \geq \mu$, $u_x(l, v(\theta)) < u_x(r, v_r)$. However, this would mean that a majority supports the R candidate, contradicting the fact that the L party candidate wins the election. □

Proof of Lemma 1

Proof. Suppose that $v_r > v_l$ and $v_r - v_l > r - l$. Then $-|x - l| + v_l < -|x - r| + v_r$ for all x. To see this, notice that $-|x - l| - (-|x - r|) \leq r - l$. Therefore, if $v_r - v_l > r - l$ then $-|x - l| + v_l < -|x - r| + v_r$ always holds. Analogously for $v_l > v_r$ and $v_l - v_r > r - l$. Therefore, all voters prefer the candidate at r (l respectively) and the probability for the general election candidate at l of winning the election is 0 (and 1 respectively). Otherwise, to find z, solve the following equation: $-|z - l| + v_l = -|z - r| + v_r$, which yields: $z = \frac{l+r+v_l-v_r}{2}$. Having found z, the probability for the general election candidate of winning the election is $Pr(\mu \leq z) = \frac{z+b}{2b}$. □

Proof of Lemma 2

Proof. Consider $W_x(l, \theta) = P_\theta(l)[u_x(l, v(\theta)) - u_x(r, v)] + u_x(r, v)$, with $P_\theta(l) > 0$, and differentiate with respect to l. First, notice that:

$$\frac{\partial P_\theta(l)}{\partial l} = \frac{1}{4b}.$$

Concerning $u_x(l, v(\theta)) - u_x(r, v)$, this can be rewritten as $r - l + v(\theta) - v$ for $x \leq l$ and as $r + l - 2x + v(\theta) - v$ for $x \in (l, r)$. Therefore, the derivative is -1 for $x \leq l$ and 1 for

$x \in (l, r)$. Putting this together, for $x \leq l$ we have:

$$\frac{\partial W_x(l,\theta)}{\partial l} = \frac{1}{4b}(r - l + v(\theta) - v) - \frac{2b + l + r + v(\theta) - v}{4b} = -\frac{2b + 2l}{4b}$$

which yields $l = -b$. Notice that this does not depend on the valence of candidates, therefore the solution is the same when maximizing $\mathbb{E}_\theta[W_x(l,\theta)]$. Therefore, for all voters such that $x \leq -b$ (i.e. all primary election voters), the optimal platform for a primary election candidate, fixing beliefs $\nu(l)$ to a constant, is $-b$. Notice that the second order condition is negative, so the point found is a maximum. □

Proof of Proposition 1

Proof. I want to show that Assumption 1 is enough to guarantee that the median voter theorem holds for the primary election. First of all, notice that the difference in expected utility from two primary election pre-candidates, represented by condition 5, is continuous in the bliss point of a voter x. Its derivative with respect to x reads:

$$\frac{\partial \left[\mathbb{E}_{\theta_1}\left[W_x(l_1,\theta_1)\right] - \mathbb{E}_{\theta_2}\left[W_x(l_2,\theta_2)\right]\right]}{\partial x} \tag{24}$$

I want to first show that this derivative can change sign. Consider a comparison between two pre-candidates l_1 and l_2, with $l_1 < l_2 < r$. There are four *types* of voters to take into account. For voters such that $x \leq l_1$, the difference in expected utility from the two candidates is constant: therefore, they either all prefer one, or the other, or they are all indifferent. The same holds for voters to the right or r. Concerning voters in $(l_1, l_2]$ and voters in $(l_2, r]$, condition (24) is negative for the former, whereas it can be either negative or positive for the latter. In order to see that the difference in expected utility can be positive for voters in $(l_2, r]$, notice that for these voters, condition (24) can be rewritten, for the case in which the valence of both pre-candidates is known (the argument is unvaried if the valences of pre-candidates are unknown):

$$P_{\theta_1}(l_1)(r + l_1 + v(\theta_1) - v_r) - P_{\theta_2}(l_2)(r + l_2 + v(\theta_2) - v_r) - 2x[P_{\theta_1}(l_1) - P_{\theta_2}(l_2)] \tag{25}$$

This is strictly increasing in x if and only if $P_{\theta_2}(l_2) - P_{\theta_1}(l_1) > 0$. As a result, it is possible for condition (24) to be positive for voters in both tails and negative for those in the middle, so that the median voter theorem would not hold. If the median voter theorem does not hold, then there exists a voter in $x_2 \in (l_2, r]$ who is indifferent between the two pre-candidates. Therefore, in order to show that under Assumption 1 the median voter theorem always holds, I want to rule out that voter x_2 can be a voter in the primary election. First of all, notice that x_2 has to be a voter preferring r to l_2 in the general election. Moreover, notice that x_2 exists if and only if $P_{\theta_2}(l_2) > P_{\theta_1}(l_1)$. Therefore, $x_2 > -b$, otherwise $P_{\theta_2}(l_2) = 0$. This is clearly a sufficient condition, since I did not say anything about the existence of x_1. □

Proof of Lemma 3

Proof. To prove this Lemma it is enough to show that $\frac{2r+b+l_1+v-v_r}{2r+b+l_2+v-v_r}$ is increasing in v. Differentiating with respect to v yields: $\frac{l_2-l_1}{(2r+b+l_2+v-v_r)^2} > 0$, delivering the result. □

Lemma 6. *Consider an equilibrium where a set of policy platforms $\mathcal{L}_{off}$ is off the equilibrium path. If an equilibrium exists under some off-equilibrium beliefs $\nu(l_{off})$ for $l_{off} \in \mathcal{L}_{off}$, then it exists under $\nu(l_{off}) = 0$.*

Proof of Lemma 6

Proof. Suppose that this was not the case. This would imply that there exists an equilibrium which, changing off-equilibrium beliefs from $\nu(l_{off})$ to 0, does not survive. However, this is impossible, since any deviation becomes less profitable by decreasing beliefs from $\nu(l_{off})$ to 0. The reason is that $\mathbb{E}_{\nu(l_{off})} W_m(l_{off}, \theta_{l_{off}}) \geq W_m(l_{off}, D)$, and strictly if $\nu(l_{off}) > 0$. Therefore, if all deviations to off-equilibrium policies were not profitable under $\nu(l_{off})$, they are also not profitable under $\nu(l_{off}) = 0$. Hence, an equilibrium that exists under $\nu(l_{off})$ will also exist under $\nu(l_{off}) = 0$. □

Proof of Lemma 4

Proof. Suppose we have a separating equilibrium in which $l_D^{'} \neq -b$. By Lemma 2, compared to a non-valent pre-candidate at $l_D^{'} \neq -b$, a majority of primary voters prefers a non-valent pre-candidate at $-b$. Deviating away from $l_D^{'}$ therefore results in a probability of winning the primary election of at least $1-\alpha$. Such a deviation is profitable as long as:

$$\frac{1-\alpha}{2}\frac{2b+r+l_d-v}{4b} < (1-\alpha)\frac{2b+r-b-v}{4b}$$

which can be rearranged to:

$$l_D^{'} < r - v$$

Notice that l_D can never be in $[r-v, r+v]$, since a non-valent pre-candidate would never win the general election. Moreover, l_D can also never be to the right of $r+v$, since in that case a deviation to any platform closer $r+v$ would be profitable. □

Proof of Proposition 2

Proof. For the if part, suppose that condition (11) is satisfied at platform l^{pool}. Then, $P^{pr}(-b, l^{pool}) = 0$. Therefore, a pre-candidate deviating to $-b$ would have a payoff of 0. Since by Lemma 2 we have that for all l and $x \leq -b$, $W_x(l, D) \leq W_x(-b, D)$, deviating to any other policy platform delivers a zero payoff. Therefore, condition (9) is satisfied and l^{pool} can be part of a pooling equilibrium. For the only if part, I proceed by contradiction. Suppose that in a candidate pooling equilibrium, condition (11) is not satisfied. Then, $P^{pr}(-b, l^{pool}) = 1$. I want to show that in this case, condition (9) is not satisfied, and therefore there exists a profitable deviation. Condition (9) is violated if: $\frac{P_\theta(-b)}{P_\theta(l^{pool})} \geq \frac{1}{2}$ for at least one type $\theta \in \{A, D\}$. I check it for $\theta = A$. Using Lemma 1 to write the expressions for $P_\theta(\cdot)$ and solving yields $l^{pool} < r$. Therefore, for any $l^{pool} < r$, if (11) does not hold, then (9) does not hold and pre-candidates have a profitable deviation to $-b$. Moreover, for all $l \geq r$, condition (11) is not satisfied, since $W_x(l, \theta) \leq 0$ for all $l \geq r$, neither is condition (9), since $P_\theta(l) < \frac{1}{2}$ for all $l \geq r$, whereas $P_A(-b) > \frac{1}{4}$.

Finally, to see that condition (11) defines an interval $[\underline{l}^{pool}, \bar{l}^{pool}]$, it is enough to solve it as equality using the expressions in Lemma 1. □

Proof of Proposition 3

Proof. By Lemma 4, $l_D = -b$. Following Definition 2 of a Perfect Bayesian Equilibrium, it must be the case that, given the behavior of primary and general election voters, no pre-candidate has a profitable deviation.

I start by showing that condition (15) is necessary for an equilibrium. Suppose we have a separating equilibrium with (l_D, l_A) in which (15) is not satisfied. Then, $\mathbb{E}_{\theta_2}[P^{pr}(l_D, l_2(\theta_2))] = 1 - \frac{1-\alpha}{2} = \frac{1+\alpha}{2}$. This makes a deviation by a valent pre-candidate to $l_D = -b$ profitable as long as:

$$\frac{\alpha}{2} P_A(l_A) < \frac{1+\alpha}{2} P_A(-b) \tag{26}$$

This can be solved to yield: $l_A < \frac{b+r}{\alpha} - b$, where $\frac{b+r}{\alpha} - b \geq r$. Therefore, for all $l_A < r$, in a separating equilibrium condition (15) has to be satisfied. Finally, notice that we cannot have $l \geq r$ in a separating equilibrium, since valent pre-candidates would have a profitable deviation to $-b$. To see this, notice that the probability of winning the general election with any policy $l \geq r$ is at most 1/2, whereas the probability of winning the general election at $-b$ is at least 1/4. Therefore:

$$\frac{\alpha}{2}\frac{1}{2} < \frac{1+\alpha}{2}\frac{1}{4} \Leftrightarrow \alpha < 1,$$

which is always satisfied. Therefore, in a separating equilibrium:

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_D, l_2(\theta_2))\right] = \frac{1-\alpha}{2}$$

and

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_D, l_2(\theta_2))\right] = 1 - \frac{1-\alpha}{2} = \frac{2-\alpha}{2},$$

which are used in expressions (16) and (17) to get $a = \frac{1-\alpha}{2-\alpha}$.

I now consider condition (16). I first show that it is a necessary condition for a separating equilibrium to exist. Suppose it is violated: then non-valent pre-candidates have a profitable deviation to platform l_A, contradicting the definition of separating equilibrium. To show instead that (16) is sufficient for non-valent pre-candidates to have no profitable deviation, notice that since $\nu(l) = 0$ for all $l \neq l_A$, deviating to any policy platform other than l_A gives a pre-candidate a zero payoff, since for all $l \notin \{l_A, l_D\}$, $W_m(-b, D) > W_m(l, D)$.

Similarly, if (17) is violated, then valent pre-candidates have a profitable deviation to platform l_D, again contradicting the definition of a separating equilibrium. To show that (17) is sufficient to prevent deviations by valent pre-candidates, notice that deviating to any point other than $-b$ would deliver a payoff of zero, since $W_m(-b, D) > W_m(l, D)$ for all $l \neq -b$.

To sum up, given that by definition of separating equilibrium voters and pre-candidates need to have no profitable deviations, conditions (15), (16) and (17) have to be satisfied. By definition, as explained in the text, this means that $l_A \in \mathcal{IC}^D_{-b} \cap \mathcal{IC}^A_{-b} \cap \mathcal{P}^A_{-b}$. □

Proof of Proposition 4

Proof. Condition (16) is satisfied for platforms such that $l \leq l^e_A$, where l^e_A solves condition (16) as equality, and platforms such that $l \in [r-v, r+v]$, since by Lemma 1, $P_D(l) = 0$ for $l \in [r-v, r+v]$, hence (16) is satisfied.

Consider first platforms $l \leq l^e_A$. First, notice that $l^e_A \leq -b$. By Lemma 3, l^e_A also satisfies (17). Solving condition (17) as equality yields a platform $\underline{l}^e_A$, and solving condition (15) as equality gives two solutions, the smaller of which is $\underline{l}^m_A$. Finally, $\max\{\underline{l}^m_A, \underline{l}^e_A\} = \underline{l}_A$. If $\underline{l}_A \leq l^e_A$, there exists an interval of separating equilibria with $l_A \in [\underline{l}_A, l^e_A]$. If instead $\underline{l}_A > l^e_A$, instead, no separating equilibrium exists with $l_A \leq l^e_A$.

Consider now policies $l \in [r-v, r+v]$. Denote by $l^c_A = r - v$. All these policy-platforms also satisfy (17), since $P_A(l) > P_A(-b)$ for all $l \in [r-v, r+v]$. Finally, denote by $\bar{l}^c_A$ the largest of the two solutions to condition (15). If $\bar{l}^c_A < l^c_A$, then no platform in

$[r-v, r+v]$ is in $\mathcal{P}^A_{-b}$, hence no separating equilibrium with $l_A \in [r-v, r+v]$ exists. If instead $\bar{l}^c_A \geq l^c_A$, policy platforms $[l^c_A, \bar{l}^c_A]$ can be part of a separating equilibrium. □

Proof of Corollary 1

Proof. Suppose condition (15) is satisfied at l^e_A. By definition, l^e_A satisfies condition (16). Following Lemma 3, condition (17) is satisfied, too. From Proposition 3, a separating equilibrium with $l_A = l^e_A$ exists. For the only if part, suppose that condition (15) is not satisfied at l^e_A. Since all other values of l_A satisfying condition (16) are smaller than l^e_A, so for these values of l_A, $W_m(l_A, A) < W_m(l^e_A, A)$ and therefore they also fail to satisfy condition (15). Following Proposition 3, no extremist separating equilibrium exists.

I now consider centrist separating equilibria. Suppose condition (15) is satisfied at l^c_A. Since $l^c_A \geq -b$, (17) is satisfied. Given that clearly condition (16) is also satisfied at l^c_A, we can conclude that a centrist separating equilibrium exists. For the only if part, suppose that (15) is not satisfied at l^c_A. Then all other candidate l_A in $(r-v, r+v]$ fail to satisfy (15), given that $W_m(l_A, A) < W_m(l^c_A, A)$. Following Proposition 3, no cenrtrist equilibrium exists. □

Proof of Proposition 5

Proof. This proposition is a direct corollary of Lemma 2. Take the pooling equilibrium. Since for all l^{pool}, $\nu(l^{pool}) = \alpha$, Lemma 2 tells us that the optimal pooling candidate for all party voters is located at $-b$. Similarly, for the separating equilibria, the expected utility from a separating equilibrium can be written as:

$$\alpha(2-\alpha)W_x(l_A, A) + (1-\alpha)^2 W_x(l_D, D) \tag{27}$$

Since $l_D = -b$ in all separating equilibria and the probability of selecting a valent pre-candidate is constant at $\alpha(2-\alpha)$ across all separating equilibria, comparing two separating equilibria with l_A and $l^{'}_A$ reduces to comparing $W_x(l_A, A)$ and $W_x(l^{'}_A, A)$. Since $\nu(l_A) = \nu(l'_A) = 1$ for all feasible values of l_A and l'_A, Lemma 2 applies. Hence, among all extremist separating equilibria, with $l_A \in [\underline{l}_A, l^e_A]$, the welfare maximizing is that at l^e_A since it is closest to $-b$. Similarly, among all centrist separating equilibria, with $l_A \in [l^c_A, \bar{l}^c_A]$, the welfare maximizing is the one with $l_A = l^c_A$. □

Proof of Proposition 6

Proof. For the centrist separating equilibrium, the results follows immediately from $l^c_A = r - v$. For the extremist separating equilibrium, it follows from expression (19). Notice that since $a = \frac{1-\alpha}{2-\alpha}$, $\frac{\partial a}{\partial \alpha} < 0$. □

Proof of Proposition 7

Proof. A majority of primary election voters prefers the extremist to the centrist separating equilibrium if and only if

$$\alpha(2-\alpha)W_m(l_A^c, A) + (1-\alpha)^2 W_m(l_D, D) \le \alpha(2-\alpha)W_m(l_A^e) + (1-\alpha)^2 W_m(l_D, D)$$

which yields:

$$W_m(l_A^c, A) \le W_m(l_A^e, A).$$

The latter condition can be rewritten as:

$$P_A(l_A^c)[r - m - (l_A^c - m)] \le P_A(l_A^e)[r - m - |l_A^e - m|] \tag{28}$$

If (28) holds for some values of $m \ge l_A^e$, then it holds for all $m < l_A^e$, and if it does not hold for any $m \ge l_A^e$, then it does not hold for any $m < r$. Hence, consider $m \ge l_A^e$ and rewrite (28) as:

$$m \le \frac{r + l_A^e}{2} - \frac{v}{2}\frac{P_A(l_A^c)}{P_A(l_A^e)} \tag{29}$$

which is indeed condition (21). □

Proof of Proposition 8

Proof. The following conditions represent the welfare comparisons between equilibria: first of all, condition (28) determines the set of parameters such that the centrist equilibrium is preferred to the extremist separating. Concerning the comparison between the centrist separating and the pooling, the condition reads:

$$\begin{aligned} &\alpha(2-\alpha)P_A(l_A^c)[r - m - (l_A^c - m)] + (1-\alpha)^2 P_D(-b)(r + b - v) \\ &\ge \alpha P_A(-b)(r + b) + (1-\alpha)P_D(-b)(r + b - v) \end{aligned} \tag{30}$$

whereas the condition comparing the extremist separating equilibrium and the pooling reads:

$$\begin{aligned} &\alpha(2-\alpha)P_A(l_A^e)[r - m - |l_A^e - m|] \\ &+ (1-\alpha)^2 P_D(-b)(r + b - v) \ge \alpha P_A(-b)(r + b) + (1-\alpha)P_D(-b)(r + b - v) \end{aligned} \tag{31}$$

where $P_A(l_A^c) = \min\left\{\frac{2b+2r-v}{4b}, 1\right\}$ and $P_A(l_A^e) = \frac{a(b+r)+(1-a)v}{4b}$. Solving conditions (30) and (31) for v, r or b yields bounds above which (in the case of v) or below which (in the case of r and b) every separating equilibrium is preferred to the pooling. □

Definition 3. *In order to apply to my game the D1 criterion introduced by Cho and Kreps (1987), I compare how different types profit after a deviation for all possible mixed responses by primary election voters. In my setup, this means, for a given deviation from the equilibrium path, to look for the type that would profit from a deviation under a lower probability of winning the primary election. The minimum probability of winning the primary election that makes a deviation profitable is defined as follows. For any equilibrium policy platform l^*, a deviation to l' is profitable for a pre-candidate of type θ if the probability of winning the primary election is at least $\pi_\theta(l', l^*)$ defined by:*

$$\pi_\theta(l', l^*)P_\theta(l') > \mathbb{E}_{\theta_2}\left[P^{pr}(l^*, l_2(\theta_2))\right]P_\theta(l^*) \tag{32}$$

Following a deviation from l^* to some l', voters assign probability 1 to the type θ associated to the minimum $\pi_\theta(l', l^*)$.

Proof of Proposition 9

Proof. First, I show that the D1 criterion, as per Definition 3, destroys all pooling equilibria. To do this, notice that, when deviating out of a pooling equilibrium, $\pi_\theta(l', l^*)$ is denoted by:

$$\pi_\theta(l', l^{pool}) = \frac{1}{2}\frac{P_\theta(l^{pool})}{P_\theta(l')}$$

From Lemma 3 it follows immediately that, for any $l' < l^{pool}$, $\pi_A(l', l^{pool}) < \pi_D(l', l^{pool})$. Therefore, any deviation to $l' < l^{pool}$ would be interpreted as coming from a valent type. Given that $P_A(l^{pool}) > 0$ and it is continuous in l, deviating to l' sufficiently close to l^{pool} is profitable and would allow the deviating pre-candidate to win the primary election with probability 1. As a result, no pooling equilibrium is consistent with the D1 criterion.

Second, I show that all separating equilibria of the centrist type except for that at l_A^c are eliminated by the D1 criterion. Since $P_D(l) = 0$ for all $l \in [r - v, r + v]$, a deviation to such platforms can never be profitable for a low type. As a result, $\nu(l) = 1$ for all $l \in [r - v, r + v]$. Suppose that a centrist separating equilibrium was played, in which $l_A = \tilde{l} > r - v$. Any valent pre-candidate deviating to some $l' < \tilde{l}$ would win the primary elections with probability one, since $W_m(l', A) > W_m(\tilde{l}, A) \geq W_m(-b, D)$. Moreover, for l' close enough to $\tilde{l}$, such a deviation is profitable to a valent pre-candidate, by continuity of $P_A(l)$. Therefore, all centrist separating equilibria with $l_A > l_A^c$ are destroyed under the D1 criterion.

Finally, I analyze the case of extremist separating equilibria. I want to show that under the D1 criterion, all such separating equilibria except for the one with $l_A = l_A^e$, are ruled out. Suppose that an extremist separating equilibrium with $l_A = \tilde{l} < l_A^e$ is played. Following the D1 criterion, there is a set of l' such that $\nu(l') = 1$. In particular, following Definition 3, this set includes policies l' such that:

$$\frac{P_D(l')}{P_D(l_D)} \leq a\frac{P_A(l')}{P_A(\tilde{l})} \tag{33}$$

Notice that, for any $\tilde{l} < l_A^e$, a deviation to $l' = l_A^e$ satisfies condition (33). This means that any separating equilibrium with $l_A = \tilde{l} < l_A^e$ cannot survive the refinement applied by D1. Finally, notice that if $l_A = l_A^e$, no such deviation exists, since:

$$\frac{P_D(l')}{P_A(l')} > a\frac{P_D(l_D)}{P_A(l_A^e)} \tag{34}$$

In order to see that condition (34) holds for all $r - v > l' > l_A^e$, notice that (34) is satisfied with equality for $l' = l_A^e$ and that the left hand side of (34) is increasing in l': the result hence follows. This means that the only potentially profitable deviations are those to platforms in $[r - v, r + v]$, which I consider in proposition 10.

□

Proof of Proposition 10

Proof. If the welfare dominant equilibrium is pooling, it cannot be the outcome of the game refined with the D1 criterion. Following Proposition 9, as a matter of fact, all pooling equilibria are destroyed by D1.

Let's now consider the centrist equilibrium. Notice that following the D1 criterion, $\nu(l) = 1$ for all platforms $l \in [r - v, r + v]$, independently of the equilibrium played. Therefore, $\nu(r - v) = 1$. Suppose the centrist separating equilibrium is welfare dominant. I want to show that no profitable deviation to other policy-platforms exists. Following the D1 criterion, there is an upper bound l'_A such that for $l \leq l'_A$, $\nu(l) = 1$. However, $l'_A \leq l_A^e$. To see this, notice that:

$$a = \frac{P_D(l_A^e)}{P_D(l_D)} > a\frac{P_A(l_A^e)}{P_A(l_A^c)} \tag{35}$$

Given that the centrist separating equilibrium is welfare dominant, however, a pre-candidate deviating to $\bar{l}_{D1}$ would win the primary election at most with probability $(1-\alpha)$. However, valent pre-candidates do not find this deviation profitable; given (16), neither do non-valent pre-candidates. Concerning other possible deviations to policies with beliefs restricted by the D1 criterion, notice that deviating to a policy $l \in (r - v, r + v]$

gives a payoff of zero. Finally, deviating to l_D is not profitable following condition (17) and deviating to all policies with unrestricted beliefs is not profitable appropriately fixing beliefs, for example at 0. Hence, the centrist separating equilibrium always exists when it is welfare dominant.

I now want to show that the extremist separating equilibrium does not exist if the centrist separating is welfare dominant. If the extremist separating equilibrium is played, a candidate deviating to $r - v$ would win the primary with probability one, since the centrist separating equilibrium is welfare dominant. This deviation is profitable for valent pre-candidates, since $l_A^c > l_A^e$. Hence, an extremist separating equilibrium cannot exist.

Finally, let's look at the case in which the extremist separating equilibrium at l_A^e is the welfare dominant equilibrium. Notice that under the D1 criterion, $\nu(l) = 1$ for $l \in [r - v, r + v]$. If condition (23) is satisfied,

$$\frac{2-\alpha}{2} P_A(l_A^e) \geq (1-\alpha) P_A(\bar{l}_A^c) \tag{36}$$

and therefore a deviation to policy $\bar{l}_A^c$ defined in Proposition 4 is not profitable for a valent pre-candidate starting from the separating equilibrium at l_A^e. If however, condition (23) is not satisfied, then a profitable deviation to $\bar{l}_A^c$ exists under the D1 criterion the extremist separating equilibrium at l_A^e does not exist despite being welfare dominant. Suppose that (23) is satisfied: is the extremist separating equilibrium at l_A^e the unique equilibrium selected by the D1 criterion? This requires the centrist separating equilibrium at l_A^c not to exist: the most profitable possible deviation out of it is to l'_A, which is defined by the following condition:

$$\frac{P_D(l'_A)}{P_A(l'_A)} = a \frac{P_D(l_D)}{P_A(l_A^c)} \tag{37}$$

Condition (37) states that at point l'_A low types benefit from a deviation out of $-b$ for the same set of mixed best replies as high types deviating out of l_A^c. Therefore, for all points $l \leq l'_A$, $\nu(l) = 1$ when a centrist separating equilibrium is being played. A deviation out of l_A^c to l'_A is profitable if and only if:

$$P_A(l'_A) > \frac{2-\alpha}{2} P_A(l_A^c). \tag{38}$$

□

Proof of Proposition 2

Proof. This result follows directly from Proposition 10 and the equivalence between the pooling equilibrium and direct nomination by party elites. When the pooling equilibrium is welfare dominant, direct party nomination is preferred, since the pooling equilibrium does not survive the D1 criterion. Concerning the second point, this also follows directly from Proposition 10: the D1 criterion always selects the centrist equilibrium when it is welfare dominant, and hence preferred to the pooling equilibrium/direct party nomination. Finally, concerning the third point, again following Proposition 10, it can be the case that the centrist separating equilibrium is selected by the D1 criterion, despite the extremist separating equilibrium being welfare dominant. When this happens, if the pooling equilibrium gives a majority of voters a higher welfare than the centrist equilibrium, then direct nomination is preferred to primaries. □

Proof of Proposition 3

Proof. The probability for the L party candidate of winning the general election under the pooling equilibrium is:

$$\alpha P_A(-b) + (1-\alpha)P_D(-b) \tag{39}$$

whereas the probability of winning the general election under the centrist separating equilbrium is:

$$\alpha(2-\alpha)P_A(l_A^c) + (1-\alpha)^2 P_D(-b) \tag{40}$$

and finally under the extremist separating equilibrium:

$$\alpha(2-\alpha)P_A(l_A^e) + (1-\alpha)^2 P_D(-b) \tag{41}$$

Notice that $P_A(l_A^c) > P_A(-b) > P_A(l_A^e)$. Therefore, the probability of winning the general election under the centrist separating equilibrium, expressed by (40), is unambiguously higher than under the extremist separating expressed by (41) or the pooling, expressed by (39). Concerning the comparison between extremist separating and pooling, subtracting (39) from (41) yields the condition:

$$\frac{v}{b+r} \geq \frac{1}{2-\alpha} \tag{42}$$

□

Proof of Proposition 11

Proof. First of all, notice that in the model with q and v, $l_A^c = r - q$ and $l_A^e = -b - (1-a)(b+r-q)$. The difference in expected utility between the centrist and the pooling equilibrium is:

$$\begin{aligned}&\alpha(2-\alpha)P_A(l_A^c)[r-l_A^c+v-q]+(1-\alpha)^2P_D(-b)(r+b-q)\\&\geq \alpha P_A(-b)(r+b+v-q)+(1-\alpha)P_D(-b)(r+b-q)\end{aligned} \tag{43}$$

whereas the condition comparing the extremist separating equilibrium and the pooling reads:

$$\begin{aligned}&\alpha(2-\alpha)P_A(l_A^e)[r-m-|l_A^e-m|+v-q]\\&+(1-\alpha)^2P_D(-b)(r+b-q)\geq \alpha P_A(-b)(r+b+v-q)+(1-\alpha)P_D(-b)(r+b-q)\end{aligned} \tag{44}$$

Notice that for a given r, b and m, the locus of indifference between the separating and the pooling equilibrium in the (α, v) space is given by either (43) or (44) holding as equalities. Solving these two conditions for v it can be seen that the solution is decreasing in q for any r, b and m. This means that the set of parameters for which a separating equilibrium is preferred to the pooling becomes larger as q increases. □

B Double Primary

In this section I consider an environment in which both parties simultaneously hold primary elections to select their candidates. For simplicity, I model the R party as perfectly symmetric with respect to the L party: the probability of drawing a valent candidate is α for both parties; the primary median voters are located at $m_L \leq -b$ for the L party and $m_R = -m_L \geq b$ for the R party. I still assume that $v \leq b$.

From the point of view of each party, the only difference with respect to the single primary environment is that the policy platform and valence of the opposing party's candidate are uncertain. Therefore, the results of Proposition 1, Lemma 2 and Lemma 3 still hold in this new setup.

In the analysis of the double primary game I focus on symmetric equilibria, that is equilibria in which the the strategies chosen by pre-candidates and voters in equilibrium are invariant to flipping the labels of the two parties. When considering symmetric equilibria in the double primary game, it is useful to have a way to rank equilibria in terms of expected utility they provide to primary election voters in each party. The result is that all voters weakly prefer symmetric equilibria in which candidates are less polarized. Moreover, a crucial variable which positively affects the expected utility from an equilibrium is the probability of electing the valent candidate when a valent candidate faces a non-valent candidate in the general election. As we will see, this implies that centrist separating equilibria are preferred to extremist separating equilibria by all voters, because in a centrist equilibrium a valent candidate, being closer to the center, has a policy advantage on top of the valence advantage when running against a non-valent one in the general election.

Lemma 7. *Consider a symmetric equilibrium with $l_A = -r_A$ and $l_D = -r_D$ and denote by $\sigma_{\theta,\theta'}$ the probability that candidates of two types θ and θ' face each other in the general election. Denote by ϕ the probability that a valent candidate wins the general election when facing a non-valent candidate from the opposing party. The expected utility from such a symmetric equilibrium, for a voter located at x, is:*

$$\sigma_{A,A}[\min\{l_A, x\}+v]+\sigma_{D,D}\min\{l_D, x\}+2\sigma_{A,D}[\phi\min\{l_A, x\}+(1-\phi)\min\{l_D, x\}+\phi v] \quad (45)$$

Lemma 7 has some important consequences: first, when comparing symmetric pooling equilibria, with $l_A = l_D = l^{pool}$, for a given ϕ, the closer l^{pool} is to zero the weakly higher is the expected utility for all voters. The same holds for separating equilibria: fixing $l_D = -b$, the closer to zero l_A the weakly higher expected utility: therefore, centrist equilibria are preferred to extremist equilibria. Moreover, notice that π is also an equilibrium object, since it denotes the probability of electing a valent candidate in a general election between a valent and a non-valent candidate. This provides another reason to prefer centrist to extremist equilibria. In a centrist equilibrium, as opposed to an extremist one, valent candidates have a policy advantage on top of the valence advantage. The same reasoning can be applied to pooling equilibria: in terms of policy, pooling equilibria with l^{pool} closer to zero deliver a weakly higher expected utility. In terms of ϕ, for all pooling equilibria with $l^{pool} < -v/2$ the value of ϕ is constant and equal to $\frac{2b+v}{4b}$, whereas for $l^{pool} \geq -v/2$ it takes the value of 1, since $r^{pool} - l^{pool} \leq v$, making these pooling equilibria preferred from the welfare point of view.

I now discuss the symmetric pooling and separating equilibria of the game. Just like in the single-primary game, there is an interval of pooling equilibria. Whereas the welfare dominant ones for primary election voters are those with $\pi = 1$, i.e. with $l^{pool} \geq -v/2$, these pooling equilibria need not exist (whereas for example the pooling equilibrium at $l^{pool} = -b$ exists for all parameters). Moreover, the pooling equilibrium that maximizes the welfare of primary election voters fixing what the opposing party does is the one at $-b$, following Lemma 2.

A similar argument holds for the centrist separating equilibrium. Whereas the best one in terms of welfare is that where valent pre-candidates locate at $l_A = r_A = 0$, the one existing for a the largest set of parameters is that in which $l^c_{dp} = -\frac{v}{2}$ and $r^c_{dp} = \frac{v}{2}$. This is the centrist separating equilibrium I focus on in the analysis, since in any case any centrist separating equilibrium dominates any extremist separating equilibrium in the double primary game.

Finally, concerning extremist separating equilibria, there is a continuum of extremist separating equilibria, among which the one delivering the highest expected utility to voters (primary and general election voters alike) is the one with the least polarized policies, following Lemma 7. Just like in the single primary game, $l_D = -b$ (and by symmetry $r_D = b$) and $l_A = l^e_{dp}$ defined as:

$$l^e_{dp} = -b - \frac{(1-a)}{1-(1-a)\sigma}[2b - \sigma v] = -b - \underbrace{\frac{1}{(1-\alpha)(2-\alpha)}[2b - \alpha(2-\alpha)v]}_{\equiv \Delta} \qquad (46)$$

and $r_A = r^e_{dp} = -l^e_{dp}$ analogously defined, where $\sigma = \alpha(2-\alpha)$ denotes the probability that the primary of each party selects a valent candidate. Notice that since $v \leq b$ and $\sigma \leq 1$, $2b - \sigma v > 0$, which guarantees that $r^e_{dp} > b$ and $l^e_{dp} < -b$.

Proposition 12. *There is an interval of possible symmetric pooling equilibria with $l^{pool} \in [\underline{l}^{pool}, \bar{l}^{pool}]$. The double primary game also has an interval of possible symmetric centrist separating equilibria in which valent pre-candidates choose platforms $l^c_{dp} \in [-v/2, 0]$ and $r^c_{dp} = -l^c_{dp}$ and non-valent pre-candidates choose platforms $l_D = -b$ and $l_D = b$ respectively. The double primary game also has an interval of possible symmetric extremist separating equilibria with $l_D = -b = -r_D = b$ and $l_A \in [\underline{l}^e_{dp}, l^e_{dp}]$ and $r_A = -l_A$.*

Unlike in the single primary game, in the symmetric double primary game all voters prefer a centrist separating equilibrium, when it exists, to the welfare maximizing extremist separating equilibrium: the intuition has to do with ϕ defined in Lemma 7: in a centrist equilibrium, a valent candidate is more likely to be elected against a non-valent candidate from the opposing party. Moreover, any centrist equilibrium also dominates, in terms of welfare, any pooling equilibrium. Concerning the ranking between pooling and extremist separating equilibria, in the analysis I compare the welfare maximizing extremist equilibrium, with $l_A = l^e_{dp}$, to the pooling at $-b$ as reference point, not least because it exists for all parameter values.

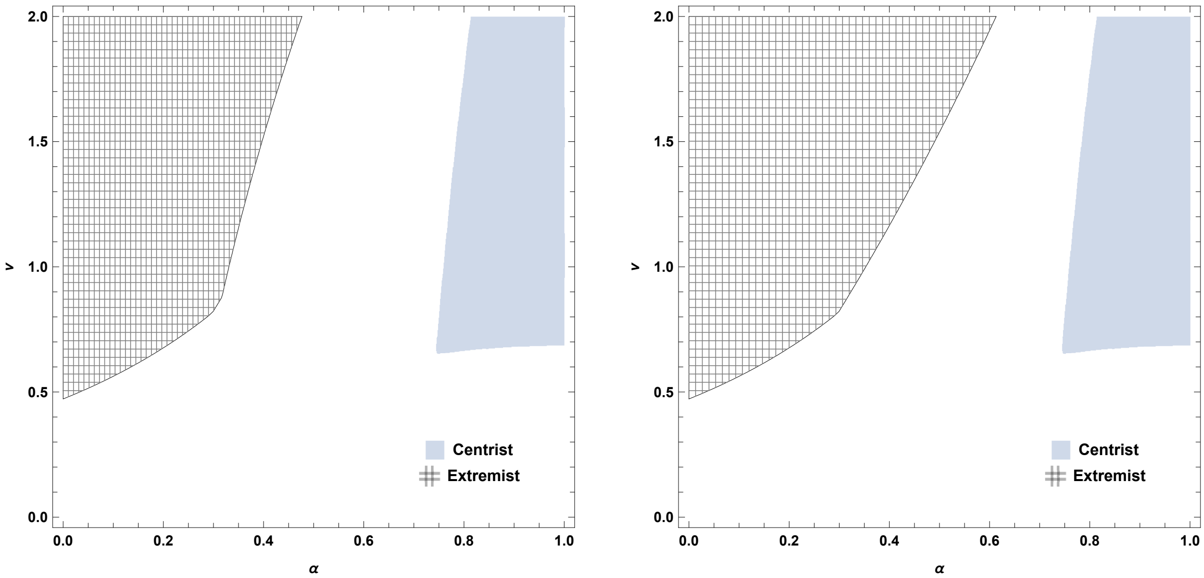
In Figure 4a I display the set of parameters for which a centrist separating equilibrium exists (the set coincides with the parameters for which the centrist separating equilibrium with $l_A = -v/2$ exists) and thus is welfare dominant, and the parameter values for which the welfare maximizing extremist separating equilibrium dominates the pooling at $-b$. In the areas without filling, the pooling at $l^{pool} = -b$ dominates the welfare maximizing separating equilibrium.

Unlike in the single primary case, in the double primary game the symmetric centrist equilibrium exists only under a restricted set of parameters: this has to do with the incentive compatibility constraint for non-valent pre-candidates. Notice that the extremist separating equilibrium is preferred when v is high (relative to b) and α is small, whereas some centrist equilibrium exists and is welfare dominant for high v (relative to b) and large α. Notice that for low values of v, separating equilibria do not exist, similar to what happens in the single primary baseline. However, in the double primary game there are intermediate values of α for which the pooling equilibrium is preferred even under high values of v.

Proposition 13. *The welfare dominant equilibrium of the double primary game is the centrist separating equilibrium, when it exists. If the centrist separating equilibrium does not exist, the extremist separating equilibrium with $l_A = l^e_{dp}$ dominates the pooling equilibrium at $-b$ when it exists and condition* (56) *is satisfied.*

Finally, a few words on equilibrium selection through the D1 criterion. Just like in

Figure 4: Double Primary Game: Symmetric Separating Equilibria



(a) Welfare Dominant Equilibrium

Figure 4a shows the welfare dominant equilibrium of the game in the (α, v) space for parameter values $m = -5$ and $b = 2$. The pooling equilibrium is welfare dominant when no filling is present.

(b) Equilibrium Selected by D1 Criterion

Figure 4b shows the welfare dominant equilibrium of the game in the (α, v) space for parameter values $m = -5$ and $b = 2$.

the single primary case, the D1 criterion destroys all pooling equilibria. Moreover, it allows for centrist or extremist deviations which can destroy the extremist or the centrist separating equilibrium respectively. However, compared to the single primary baseline, it is more unlikely for a centrist deviation to be profitable, whereas an extremist deviation from a centrist separating equilibrium can be profitable. For example, as Figure 4b shows, for $m = -5$ the extremist separating equilibrium is never destroyed by centrist deviations, whereas the centrist separating equilibrium is at times destroyed even if welfare dominant.

B.1 Proofs of Double Primary Game

Proof of Lemma 7

Proof. I consider here voters of the L party primary, for which $x \leq \min\{r_A, r_D\}$ always holds, but the argument is analogous for voters of the R party. A symmetric equilibrium can give rise to three possible scenarios: with probability $\sigma_{A,A}$, depending on the type of equilibrium, two valent candidates are selected, and by symmetry each wins with probability $1/2$. The expected utility conditional on two valent candidates being selected is thus:

$$\frac{1}{2}[-(r_A - x)] + \frac{1}{2}[-|l_A - x|] + v = \begin{cases} x + v \text{ if } x \leq l_A \\ -\frac{1}{2}r_A + \frac{1}{2}l_A + v = l_A + v \text{ if } r_A > x > l_A \end{cases} \tag{47}$$

This can be rewritten as:

$$\frac{1}{2}[-(r_A - x)] + \frac{1}{2}[-|l_A - x|] + v = \min\{l_A, x\} + v \tag{48}$$

The case of two non-valent candidates, which occurs with probability $\sigma_{D,D}$, is analogous, with l_D substituting l_A, r_D substituting r_A, and $v = 0$. Finally, consider the case of one valent and one non-valent candidate selected by the respective primary elections. This happens with probability $\sigma_{A,D} = \sigma_{D,A}$ in each primary election. Putting together the scenarios in which the L party selected a valent candidate and that in which the R party did, which happens with probability $2\sigma_{A,D}$, yields the following expected utility:

$$\begin{aligned} &\frac{1}{2}\{\phi[-|l_A - x| + v] + (1-\phi)[-(r_D - x)]\} \\ &+ \frac{1}{2}\{\phi[-(r_A - x) + v] + (1-\phi)[-|l_D - x|]\} = -\phi \min\{x, l_A\} - (1-\phi)\min\{x, l_D\} + \phi v \end{aligned} \tag{49}$$

Summing up the expected utility from the three scenarios finally yields condition (45).

Notice that this lemma is useful to compare the welfare from different types of equilibria. First of all, given everything else, the smaller l_A and l_D, the weakly higher welfare is for all voters. This can clearly be seen from the fact that the expected utility depends on $\min\{x, l_\theta\}$: therefore, polarization hurts voters with $x > l_\theta$. Second, given everything else, equilibria with higher ϕ are preferred. □

Proof of Proposition 12

Proof. The construction of the separating equilibria of the game is very similar to the single-primary case. The conditions for the L and the R party primaries are completely analogous, given the symmetry of the environment. Therefore, I focus on the conditions for the L party primary here, which allow for a more direct comparability to the single primary case. Consider pooling equilibria first: similar to what happens in the single-primary game, the bounds of the interval of pooling equilibria are defined by:

$$\mathbb{E}_\theta W_m(\underline{l}^{pool}, \theta) = W_m(-b, D) = \mathbb{E}_\theta W_m(\bar{l}^{pool}, \theta) \tag{50}$$

I now analyze separating equilibria. First of all, the result equivalent to that of Lemma 4 goes through: in a pure strategy separating equilibrium, non-valent pre-candidates choose platform $l_D = -b$ in the L party and $r_D = b$ in the R party. Second, in a pure strategy separating equilibrium, primary election voters choose a valent pre-candidate whenever possible. This means that in any separating equilibrium:

$$W_m(l_A, A) \geq W_m(l_D, D) \tag{51}$$

Consider now centrist separating equilibria: in such equilibria, $l_A \in [r_A - v, r_A]$. Using symmetry we get that $l_A \in [-v/2, 0]$ and $-l_A = r_A \in [0, v/2]$. Unlike in the single-primary baseline, where incentive compatibility with respect to types $\theta = D$ holds for all policies $l_A \in [r - v, r]$, in the double primary game this is not the case. The reason is that if the R party candidate is at $r_D = b$, a non-valent pre-candidate choosing platform $r_A - v$ wins with positive probability (higher than when facing the same candidate having chosen platform l_D). Therefore, condition (12) becomes:

$$\begin{aligned} &\frac{1-\alpha}{2}[\alpha(2-\alpha)P_{D,A}(-b, l_A) + (1-\alpha)^2 P_{D,D}(-b, b)] \\ &\geq \frac{2-\alpha}{2}[\alpha(2-\alpha)\underbrace{P_{D,A}(l_A, r_A)}_{=0} + (1-\alpha)^2 P_{D,D}(l_A, b)] \end{aligned} \tag{52}$$

Notice that incentive compatibility for valent pre-candidates is always satisfied. Finally, concerning the extremist separating equilibrium, the upper bound of the interval is given by the incentive compatibility constraint for non-valent pre-candidates, which reads:

$$\begin{aligned} &\frac{1-\alpha}{2}[\alpha(2-\alpha)P_{D,A}(-b, l_A) + (1-\alpha)^2 P_{D,D}(-b, b)] \\ &\geq \frac{2-\alpha}{2}[\alpha(2-\alpha)P_{D,A}(l_A, r_A) + (1-\alpha)^2 P_{D,D}(l_A, b)] \end{aligned} \tag{53}$$

Solving (53) as equality for both the L and the R party yields:

$$l_A \leq -b - (1-a)[b + \underbrace{(1-\sigma)r_D + \sigma(r_A - v)}_{\text{Exp. location of } r}]$$

and analogously that for the right candidate:

$$r_A \geq b + (1-a)[b - \underbrace{(1-\sigma)l_D - \sigma(l_A + v)}_{\text{Exp. location of } l}]$$

Notice that these constraints are the same as those for the single primary election, but instead of r (and respectively l) one has to write the expected location of the opponent, adjusted for valence.

Analogously to the single-primary case, the lower bound of the interval of extremist separating equilibria is either the point l_A where $W_{m_L}(l_A, A) = W_{m_L}(l_D, D)$ or, if larger, in the point where the incentive compatibility constraint for valent pre-candidates holds with equality, i.e.:

$$\begin{aligned} &\frac{2-\alpha}{2}[\alpha(2-\alpha)P_{A,A}(l_A, r_A) + (1-\alpha)^2 P_{A,D}(l_A, b)] \\ &\geq \frac{1-\alpha}{2}[\alpha(2-\alpha)P_{A,A}(-b, r_A) + (1-\alpha)^2 P_{A,D}(-b, b)] \end{aligned} \tag{54}$$

Notice that, by Lemma 3, which goes through also in the double primary equilibrium, at the point where (53) holds with equality condition (54) is strictly satisfied. □

Proof of Proposition 13

Proof. The fact that any centrist separating equilibrium is more efficient than the welfare maximizing extremist equilibrium is a direct consequence of Lemma 7. To see it, take x sufficiently low, in such a way that, policy-wise, a voter is indifferent between the two equilibria (more moderate voters have a further reason to prefer the centrist equilibrium), since $\min\{l_\theta, x\} = x$ for all l_θ. First of all, since we are comparing two separating equilibria, $\sigma_{\theta,\theta'}$ are the same in both equilibria for each combination (θ, θ'). The only other object that changes is therefore ϕ, which I denote as ϕ_c in a centrist and ϕ_e in an extremist separating equilibrium. In a centrist separating equilibrium, $\phi_c \geq \frac{2b+b-v/2+v}{4b}$. In the welfare maximizing extremist equilibrium, instead, $\phi_e = \frac{2b+b-b-\Delta+v}{4b}$. Since $\Delta > 0$ and $v \leq b$, $\pi_c > \pi_e$. Therefore, all voters (also non-primary voters) prefer any centrist separating equilibrium to the welfare maximizing extremist separating. When comparing instead a centrist separating equilibrium with a pooling equilibrium, the best pooling equilibrium is policy-wise equivalent to any centrist separating equilibrium for primary voters of both parties, since all primary voters have $|x| > b$. Given ϕ, a separating equilibrium is more efficient at selecting valent pre-candidates in the primary, which gives an advantage to the separating equilibrium. However, in some pooling equilibria $\phi_{pool} = 1$, which happens when $l^{pool} \geq -v/2$. In the centrist separating equilibrium associated with lowest welfare, on the other hand, $\phi_c = \frac{2b+b-v/2+v}{4b}$. Hence, there is a trade-off, which however always goes in favor of the centrist separating equilibrium. As a matter of fact, the following condition always holds, even for $\phi_{pool} = 1$ and $\phi_c = \frac{2b+b-v/2+v}{4b}$, that is its lower bound in a centrist separating equilibrium:

$$[\alpha(2\alpha)]^2 v + 2\alpha(2-\alpha)(1-\alpha)^2\pi_c v \geq \alpha^2 v + 2\alpha(1-\alpha)\pi_{pool} v \tag{55}$$

Therefore, the centrist separating equilibrium is welfare dominant, if it exists. Concerning the comparison between the welfare maximizing extremist separating equilibrium and the pooling at $-b$, the condition to consider is the folllwing:

$$\begin{aligned}&[\alpha(2-\alpha)]^2[\min\{l_A, m_L\} + v] + (1-\alpha)^4(-b)\\ &\quad + 2\alpha(2-\alpha)(1-\alpha)^2[\pi_e \min\{l_A, m_L\} + (1-\pi_e)(-b) + \pi_e v] \geq m_L\\ &\qquad + \alpha^2 v + 2\alpha(1-\alpha)\frac{2b+v}{4b} v\end{aligned} \tag{56}$$

□

C Policy Conflict

An alternative scenario to that described in Section 6 is one where there is a policy conflict between the elites and party voters, but elites are capable of identifying valent pre-candidates.

In particular, I now assume that, absent primary elections, party elites would draw two pre-candidates from the pool and make one of them run with policy platform $p \neq -b$, after having observed their valence and selected a valent one, if available. In this scenario, the probability of having a valent candidate is the same under a separating equilibrium following primary elections and under direct nomination by the elites: this allows me to isolate the effect of the policy conflict. Voters prefer primaries if the policy distortion associated with a separating equilibrium is smaller than the one due to the policy bias of elites[32]. From the point of view of party elites, instead, direct nomination achieves the first best: therefore, elites would never be in favor of primaries, unless they can directly benefit from primaries, for example in terms of improved reputation for a more transparent selection process. With these additions, my model can thus also shed some light on the circumstances under which party elites would be less averse to allowing for nomination by primary elections. Two situations are of particular interest:

- Party elites are more moderate than the party median voter, but both are moderate. For example, $m = -b$ and $p > m$. When the conditions for a separating equilibrium

[32]If the outcome of primaries is a pooling equilibrium, then voters trade off the valence loss from the pooling equilibrium with the policy bias of party elites.

are met, adopting primaries leads to the centrist separating equilibrium, which, at least in the case of valent candidates, would deliver a policy similar to what would be chosen by party elites.

- The mirror image of the previous case is a situation in which party elites are more extreme than the median voter, but both are relatively extreme $p < m << -b$. If primaries lead to the extremist separating equilibrium, the conflict between elites and voters is partially reconciled and this might make party elites willing to allow for primaries.

Although only suggestive examples, these scenarios might also capture some elements relevant for the introduction of primaries in the United States. For example, Poole (2005) shows that party polarization was close to historical highs when primaries were introduced. This might be evidence in favor of the case of the second example mentioned above.

D General Distribution for Median Voter Location

I now relax the assumption that the median voter preferred policy is distributed uniformly in $[-b, b]$ and analyze the case of a generic distribution $f(\mu)$ which takes positive values on the same support $[-b, b]$ and has a cumulative distribution function $F(\mu)$. I assume the distribution is continuous and has no atoms. Moreover, to simplify the analysis I assume that $f(\mu)$ is symmetric around zero and that is is increasing in $[-b, 0]$ and decreasing in $[0, b]$. Finally, I assume that $\frac{f(\mu)}{F(\mu)}$ is decreasing for all μ in the support $[-b, b]$. Just like in the baseline model, I assume that the right candidate is fixed at r and has valence $v_r = v$. I also keep Assumption 1 and Assumption 2 concerning the location of primary election voters and the value of v.

Since I a am not changing preferences of voters compared to the baseline model, the indifferent voter z given two candidates still follows Lemma 1. Given an indifferent voter z, the probability for the L party candidate to win the general election is $F(z)$, which could be a non-linear function of z. Therefore, changing the distribution of the general election median voter affects the model results through $P_\theta(l) = F(z_\theta)$. In particular, in this Appendix I will study the effects of a change in the distribution of the general election median voter on Lemma 2 and Lemma 3. Given these two building blocks, as a matter of fact, all the rest goes through practically unchanged, at least from the qualitative point of view. I start with presenting the modified version of Lemma 2.

Lemma 8. *Suppose that the L party median voter is sufficiently to the left, in order for condition* (58) *to be satisfied for* $x = m$. *Then, the optimal platform chosen by a majority of primary election voters is denoted by* l^* *and satisfies condition* (59).

Proof of Lemma 8

Proof. Consider a voter with $x < r$. The utility from the policy implemented by the valent candidate at r is: $-(r-x)+v$. Consider now a candidate with policy platform l and valence $v_l \in \{0, v\}$ just like in the baseline model. The utility from this candidate is $-|l-x|+v_l$. Therefore, to look for the optimal location for a candidate we are interested in the interval $[-2b-r+v-v_l, \min\{r-v+v_l, 2b-r+v-v_l\}]$. Denote the difference between the expected utility from the l candidate and the utility from the r candidate as:

$$F\left(\frac{l+r-v+v_l}{2}\right)(r-x-|x-l|+v_l-v) \tag{57}$$

From (57), it is clear that all points such that $l < x$ are dominated by $l = x$. Therefore, focus on the points in the interval $[-2b-r+v-v_l, \min\{r-v+v_l, 2b-r+v-v_l\}]$ such that $l \geq x$, so that the interval of interest becomes $[\max\{x, -2b-r+v-v_l\}, \min\{r-v+v_l, 2b-r+v-v_l\}]$. The solutio l cannot be at $l = -2b-r+v-v_l$, since any point $l \leq r$ such that the l candidate wins with positive probability dominates that policy. Similarly, it cannot be at $r-v+v_l$, since any point to the left at which the l candidate wins with positive probability provides a higher utility. In order to rule out a corner solution at $2b-r+v-v_l$, which requires $r > b$, I check the first order condition of (57) evaluated at $2b-r+v-v_l$. This yields $f(b)(r-b+v_l-v) < 1$ and noticing that $f(b) \leq \frac{1}{2b}$, we obtain $r < 3b+v-v_l$ which is always satisfied. Therefore, the only other possible corner solution to rule out is the one at x. This requires:

$$\frac{f(z(x))}{F(z(x))} \geq \frac{2}{r-x+v_l-v} \tag{58}$$

Notice that the left-hand side of (58) is decreasing in x, whereas the right-hand side is increasing. Therefore, condition (58) defines an upper-bound on x such that the policy platform maximizing (57) is interior. If this condition is satisfied, the optimal platform satisfies:

$$\frac{f(\frac{l^*+r-v+v_l}{2})}{F(\frac{l^*+r-v+v_l}{2})} = \frac{2}{r-l^*+v_l-v} \tag{59}$$

□

Concerning Lemma 3, the result is an immediate consequence of the assumption on

$\frac{f(z)}{F(z)}$.

Lemma 9. *For any* $l_2 > l_1$ *and and* $P_A(l_1) > 0$,

$$\frac{P_D(l_1)}{P_D(l_2)} < \frac{P_A(l_1)}{P_A(l_2)}$$

Proof of Lemma 9

Proof. I want to show that

$$\frac{P_\theta(l_1)}{P_\theta(l_2)} = \frac{F(z_\theta(l_1))}{F(z_\theta(l_2))} = \frac{F(\frac{l_1+r-v+v_l}{2})}{F(\frac{l_2+r-v+v_l}{2})}. \tag{60}$$

is increasing in $v(\theta) = v_l$. Differentiating with respect to v_l yields:

$$\frac{f(z(l_1, v_l))}{F(z(l_1, v_l))} > \frac{f(z(l_2, v_l))}{F(z(l_2, v_l))} \tag{61}$$

which holds for all $l_2 > l_1$ since by assumption $\frac{f(z)}{F(z)}$ is decreasing in z and $z(l, v_l)$ is increasing in l. □

With these results in place we have all it takes to construct a separating equilibrium. Therefore, the next proposition establishes that there exists extremist and centrist separating equilibria that look very similar to those in the baseline model. First of all, in a separating equilibirum, non-valent pre-candidates choose policy $l_D = l^*$. Concerning valent pre-candidates, nothing changes as far as the centrist separating equilibrium is concerned. Concerning the extremist separating equilibrium, again not much changes with respect to the baseline model. The policy platform chosen by valent pre-candidates in the welfare maximizing extremist separating equilibrium is such that condition (12) is satisfied as equality for non-valent pre-candidates, which can be written as:

$$P_D(l_A^e) = aP_D(l^*) \tag{62}$$

Notice that given Lemma 9, the incentive compatibility for valent pre-candidates is automatically satisfied. In order for this extremist separating equilibrium to exist, the only remaining condition is (15), stating that the valent pre-candidate at platform l_A^e is preferred to the non-valent one at l_D (in this case equal to l^*).

E Non-linear Policy Preferences

In this section, I extend the model to allow for a more general functional form to represent policy preferences. To isolate the effect of this change, I keep the distribution of the general election median voter uniform as in the baseline model. Denote the utility a voter with bliss-point x receives from the implementation of policy y as $u_x(y, v_y)$, where $y \in \{l, r\}$, $v_r = v$ and $v_l = v(\theta) \in \{v, 0\}$. I assume that $u_x(y, v_y)$ is single-peaked with bliss-point equal to x. Given two general election candidates running with platforms l and r and denoting by $\delta \equiv v_l - v_r$ the difference in valence between the two candidates, let the function $D(l, r, \delta, x) = u_x(l, v_l) - u_x(r, v)$ denote the difference in utility from candidate l and candidate r for a voter with bliss-point x. Notice that this expression allows for non-linear policy-preferences as well as for potential non-separability between valence and ideology in the utility function $u_x(y, v_y)$. I make the following assumptions on the function $D(l, r, \delta, x)$, denoting by D_j the partial derivative of $D(\cdot)$ with respect to argument j:

- $D(\cdot)$ is continuously differentiable.
- $D_l < 0$ for $l > x$ and $D_l > 0$ for $l < x$ (and $D_l = 0$ $x = l$).
- $D_r > 0$ for $r > x$ and $D_r < 0$ for $r < x$ (and $D_r = 0$ $x = r$).
- $D_{l,l} \leq 0$.
- $D_\delta > 0$
- $D_{x,l} \geq 0$
- In the case of additive separability between δ and the other variables, cross derivatives with respect to δ are null: $D_{\delta,\cdot} = 0$
- $D_x < 0$ and $\lim_{x \to -\infty} D(l, r, \delta, x) = +\infty$ and $\lim_{x \to +\infty} D(l, r, \delta, x) = -\infty$. These assumptions rule out the linear case described in the baseline model.

In this modified model, the indifferent voter, if it exists, is located at $x = z$ satisfying:

$$D(l, r, \delta, z) = 0 \tag{63}$$

Given z, the probability of winning the general election for a candidate with platform l is given by $\frac{z+b}{2b}$.

Lemma 10. *Given two candidates l and r, with $l < r$, there is a unique voter indifferent between the two candidates; the indifferent voter's bliss-point is denoted by z. The following properties hold:*

- $z_l > 0$ *if* $\delta \geq 0$, *but if* $\delta < 0$, *it can be the case that* $z_l < 0$.
- $z_r > 0$ *if* $\delta \leq 0$, *but if* $\delta > 0$, *it can be the case that* $z_r < 0$.
- $z_\delta > 0$.
- $z_{l,l} \leq 0 \Leftrightarrow \frac{D_{l,l}}{D_l} \leq \frac{D_{x,l}}{D_x}$.
- $D_{l,\delta} = 0 \Rightarrow z_{\delta,l} \geq 0$.

Proof of Lemma 10

Proof. The bliss-point of the indifferent voter z is the value of x, if it exists, solving:

$$D(l, r, \delta, x) = 0 \tag{64}$$

Using implicit differentiation:

$$\frac{\mathrm{d}z}{\mathrm{d}\delta} = -\frac{D_\delta}{D_x} > 0 \tag{65}$$

Similarly:

$$\frac{\mathrm{d}z}{\mathrm{d}l} = -\frac{D_l}{D_x} \tag{66}$$

The sign of (66) depends on D_l, which in turn depends on whether $z > l$ or $z < l$. In the former case, $\frac{\mathrm{d}z}{\mathrm{d}l} > 0$, in the latter case $\frac{\mathrm{d}z}{\mathrm{d}l} < 0$. This follows from since $D_l > 0$ for $x \geq l$. Concerning r,

$$\frac{\mathrm{d}z}{\mathrm{d}r} = -\frac{D_r}{D_x} \tag{67}$$

If $r \geq z$, then $D_r > 0$ and $-\frac{D_r}{D_x} > 0$. To calculate $z_{l,\delta}$, notice that:

$$\frac{\mathrm{d}^2 z}{\mathrm{d}\delta \mathrm{d}l} = -\frac{[D_{\delta,l} D_x - D_\delta D_{x,l}]}{D_x^2} = \frac{D_\delta D_{x,l}}{D_x^2} \geq 0 \tag{68}$$

Finally, $z_{l,l}$ is derived by:

$$\frac{\mathrm{d}^2 z}{(\mathrm{d}l)^2} = -\frac{[D_{l,l} D_x - D_l D_{x,l}]}{D_x^2} \tag{69}$$

from which we obtain the condition $\frac{D_{l,l}}{D_l} \leq \frac{D_{x,l}}{D_x}$ in order for (69) to be negative. It can be checked that this condition holds for example for the case of quadratic preferences. □

I now proceed to presenting the analogous results to Lemma 2 and Lemma 3 in this modified environment.

Lemma 11. *The platform a voter with bliss-point x would pick for a candidate of the L party, when interior, is denoted by l^* and satisfies the following condition:*

$$-\frac{D_x(l^*)}{D(l^*)} = \frac{1}{z(l^*) + b} \tag{70}$$

The platform l^* is increasing in x as long as $\frac{D_{x,x}}{D_x} \leq \frac{D_x}{D}$.

Proof of Lemma 11

Proof. The first order condition of the maximization problem of a primary election voter with bliss-point x is the following:

$$-\frac{D_l(l^*)}{D(l^*)} = \frac{z_l(l^*)}{z(l^*) + b} \tag{71}$$

Using the fact that $z_l(l^*) = -\frac{D_l(l^*,z)}{D_x(l^*,z)}$ and substituting this into (71) yields (70). □

Notice that in the case of quadratic policy preferences, for example, $D_{x,x} = 0$ and therefore the optimal platform l^* is increasing in x. Concerning the modified version of Lemma 3, we can show that the same result goes through:

Lemma 12. *For additively separable valence,*

$$\frac{P_D(l_1)}{P_D(l_2)} < \frac{P_A(l_1)}{P_A(l_2)}$$

for any $l_2 > l_1$ such that $P_A(l_1) > 0$, $P_D(l_2) > 0$, $z(l_2) > z(l_1)$.

Proof of Lemma 12

Proof. Notice that $P_\theta(l_1) = \frac{z(l,r,\delta(\theta))+b}{2b}$. Differentiating $\frac{P_\theta(l_1)}{P_\theta(l_2)}$ with respect to δ yields:

$$\frac{z_\delta(l_1)(z(l_2) + b) - z_\delta(l_2)(z(l_1) + b)}{(z(l_2) + b)^2} \tag{72}$$

which is positive if and only if:

$$\frac{z_\delta(l_1)}{z(l_1) + b} > \frac{z_\delta(l_2)}{z(l_2) + b} \tag{73}$$

This can be rewritten as:

$$\frac{z_\delta(l_1)}{z_\delta(l_2)} > \frac{z(l_1)+b}{z(l_2)+b} \tag{74}$$

which can be further rearranged to:

$$\frac{D_x(l_2)}{D_x(l_1)} > \frac{z(l_1)+b}{z(l_2)+b} \tag{75}$$

Notice that the left-hand side is larger than one, since $D_{x,l} \geq 0$, whereas the right-hand side is lower than one as long as $z(l_2) > z(l_1)$. Therefore, as long as $z(l_2) > z(l_1)$ single crossing holds. □

Having these ingredients in place, the characterization of separating equilibria does not change significantly. Non-valent pre-candidates choose $l_D = l_m^*$, that is the policy preferred by the primary median voter. In the best separating equilibria (centrist or extremist), valent pre-candidates, instead, choose the policy satisfying the incentive compatibility constraint for non-valent pre-candidates. There are two such policies, the centrist one and the extremist one:

Lemma 13. *In a separating equilibrium, non-valent pre-candidates choose platform $l_D = l_m^*$ defined by condition (77). Valent pre-candidates, instead, choose the policy platform satisfying:*

$$\frac{z(l_A, r, -v)+b}{z(l_D, r, -v)+b} = a \tag{76}$$

where $a \equiv \frac{1-\alpha}{2-\alpha}$.

Proof. Evaluating (70) at $x = m$ we obtain $l_m^* = l_D$:

$$-\frac{D_x(l_m^*, m)}{D(l_m^*, m)} = \frac{1}{z(l_m^*)+b} \tag{77}$$

For $l_A = l_D$, clearly we have that $\frac{z(l_A,r,-v)+b}{z(l_D,r,-v)+b} = 1 > a$. For sufficiently low l_A, instead, $z(l_A, r, -v) = -b$ and so $\frac{z(l_A,r,-v)+b}{z(l_D,r,-v)+b} = 0 < a$. Given the continuity of $z(\cdot)$, therefore, a solution to (76) exists. In order to see that (76) can have at most two solutions, notice that $z_l(l, r, \delta)$ can change sign at most once, at the point $\tilde{l}$ such that $z_l(\tilde{l}, r, \delta) = \tilde{l}$, if $\delta < 0$, so there are at most two solutions, which correspond to the welfare maximizing extremist and centrist separating equilibria. □