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On the Dark Side of Political Stability

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Abstract
We build a simple model highlighting the link between efficient procurement, stability, and competition. Specifically, we characterize the trade-off between political stability and economic efficiency that shapes governments' procurement decisions in hostile or politically unstable environments.

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Table of contents

1. Introduction

2. The model: set up

3. Analysis

4. Conclusions

References

Appendix

1 Introduction

The role of public procurement is widely recognized as a fundamental driver of growth both in developed and developing countries (e.g., Laffont and Tirole, 1993; Hart et al., 1997; Shleifer, 1998). Efficient procurement rules warrant competition and provide an essential safeguard against corruption in the provision of essential private and public goods. Yet, the existing literature has highlighted many possible factors that might distort the functioning of these rules and divert public resources away from the first-best allocation (e.g., Guasch et al., 2008; Bajari et al., 2009). Mispractices such as political capture, corruption, and collusion are well-known examples of factors biasing decision making in the public domain (e.g., Laffont, 1999; Acemoglu, Verdier, 2000; Celentani and Ganuza, 2002; Compte et al., 2005; Lambert-Mogiliansky, Sonin, 2006; Goldman et al., 2013). However, public procurement might also be affected by government activities whose scope is apparently orthogonal to the provision of socially valuable projects. Notably, in developing countries, political

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stability is often a salient element for the design of efficient procurement rules. However, in these countries, such stability is not always granted: governments facing tight budget constraints must divert public resources from socially valued projects to secure political consensus by their armies (see, e.g., Laffont, 2003). Notably, political stability and the military costs needed to guarantee a peaceful environment are elements that standard theory often neglects in the analysis of optimal regulatory and procurement rules. Omitting to consider the military as a key institutional actor in politics and democracy constitutes a significant oversight, especially in developing countries (see, e.g., Agaba and Shipman , 2008, for the case of Uganda).

To fill this gap, in this paper, we build a simple model that provides novel insights on the link between efficient procurement, political stability, and competition. Specifically, we characterize the trade-off between political stability and economic efficiency that shapes governments' procurement decisions in hostile or (politically) unstable environments. We consider an optimal procurement auction à la Myerson (1981) in which a government deals with N firms, each privately informed about its production cost, competing for being selected to carry out a socially valuable project. The government has a limited budget and, in addition to the procurement cost, deals with the army whose task is to secure political stability and protect democracy. The government is uninformed about firms' production costs and the effort that the army exerts to guarantee stability. Therefore, when resources are not sufficient to cover the agency costs associated to both tasks (i.e., when the budget constraint binds), the government faces a dilemma: paying the army a rent in order to guarantee stability and distort economic efficiency away from the second-best, or stick to the optimal second-best procurement auction and bear the risk of a political crisis.

We find the following results. Efficiency is distorted in favor of stability in extremely unstable environments, irrespective of the degree of competition in the procurement (as measured by the number of firms participating in the auction). However, in environments that are not too unstable, it turns out that more competition makes the government relatively more willing to distort public funds in favor of stability at the expense of efficiency. The reason is simple, as the procurement becomes relatively more competitive, the firm that wins the auction and provides the public project obtains a lower rent, which in turn allows the government to leverage public funds in order to secure stability.

This result suggests that although more competition is associated with a more efficient procurement outcome, it may also foster governments' incentive to divert public funds. The lure of large payoffs might be irresistible for politicians, bureaucracy, civil servants, like the army, forcing the government to drain public resources. Hence, we highlight one potential novel dark side of the documented positive relationship between competition and stability (e.g., Francisco and Pontara, 2007): although competition makes democracy easier to maintain, it also requires the society to allocate resources inefficiently.

2 The Model: Set up

Consider a game between a self-interested government (G), an army (A) and N firms, each indexed by $i = \{1, 2, .., N\}$. The government procures a public project, which yields a monetary return B. To do so, it relies on a firm selected through a procurement contract. Firms have *iid* random costs of executing the project, each being denoted by $\theta_i \in [0, 1]$ distributed according to the cdf $H(\theta_i)$, whose density $h(\theta_i)$ is differentiable. As standard, we assume that the (inverse) hazard-rate $\frac{H(\cdot)}{h(\cdot)}$ is increasing.

The army must ensure a stable economic environment, which determines the probability of success of the project — i.e., if the army fails to provide stability, the project value is dissipated. For example, one can think of this event as riots and social unrest that may lead a change in the political leadership of the country, which freezes the on-going projects undertaken by the deposed leadership. To provide stability the army invests effort $e \in \{0, 1\}$ whose cost is

$$\psi(e) = \begin{cases} \psi \text{ if } e = 1 \\ 0 \text{ if } e = 0 \end{cases}$$

The probability of project's success depends on army's unverifiable effort. Specifically, if the army does not exert effort the project value is equal to zero with probability $1 - \rho$ and it equals to $B > 0$ with probability ρ. If, instead, the army exerts effort the project value is equal to B with certainty — i.e., there is no political instability.

The Government has two tasks. First, to obtain political stability, it must provide proper incentives to the army. We assume, for simplicity, that it can pledge to the army a payment T in the event that the project is finalized. Otherwise, the army does not get paid. Second, the government needs to induce firms to truthfully reveal their costs. Following Myerson (1981) we assume that the Government auctions the procurement contract by offering a mechanism

$$\mathcal{M} = \{t_i(\mathbf{m}), \alpha_i(\mathbf{m})\}_{\mathbf{m}\in[0,1]^N}$$

which specifies a probability of winning the auction $\alpha_i(\cdot)$ and a transfer $t_i(\cdot)$ contingent on the profile of reports $\mathbf{m} = (m_1, ..., m_N) \in [0, 1]^N$. For simplicity, we assume that there is no outside funding other than the project's monetary return. Hence, transfers to the army and the firm are paid out of the project value B.[1]

Timing. The timing of the game is as follows.

1 G offers a transfer T to A.

2 A decides whether to exert effort or not.

3 Political uncertainty realizes. If G is still in power, she offers $\mathcal{M}$. Otherwise the game ends and players obtain their outside option normalized to zero.

4 The firm decides whether to accept the offer or reject it. If the contract is accepted, firm i reports m_i to G.

5 The project revenue materializes and payments are made according to the contracts signed.

The equilibrium concept is *Perfect Bayesian Equilibrium* (PBE).

3 Analysis

In this section we derive the equilibrium of the model. We will distinguish two polar cases. First, we characterize the equilibrium in which G provides proper incentives to A in order to exert effort. Then, we compare this outcome with the scenario in which G refuses to pay the army who, accordingly, shirks.

Political Stability. Suppose that G pays the army appropriately to induce effort provision, and firms truthful report their costs. Following Myerson (1981), in a truthful equilibrium, the expected utility of firm i when it reports m_i and experienced a cost θ_i depends of the cost-profile $\boldsymbol{\theta}_{-i} = (..\theta_{i-1}, \theta_{i+1}..) \in [0,1]^{N-1}$ of all the other firms through the mechanism — i.e.,

$$u_i\left(m_i, \theta_i, \boldsymbol{\theta}_{-i}\right) = \alpha_i\left(m_i, \theta_i, \boldsymbol{\theta}_{-i}\right)\left[t_i\left(m_i, \boldsymbol{\theta}_{-i}\right) - \theta_i\right].$$

Hence, in expected terms

$$u_i(m_i, \theta_i) \triangleq \int_{\boldsymbol{\theta}_{-i}} u_i\left(m_i, \theta_i, \boldsymbol{\theta}_{-i}\right) d\mathbf{H}\left(\boldsymbol{\theta}_{-i}\right) = \int_{\boldsymbol{\theta}_{-i}} \alpha_i\left(m_i, \theta_{-i}\right)\left[t_i\left(m_i, \theta_{-i}\right) - \theta_i\right] d\mathbf{H}\left(\boldsymbol{\theta}_{-i}\right).$$

[1] Results carry over if the government is endowed with limited external funds.

Using a standard change of variables, this expression can be rewritten as

$$u_i(m_i,\theta_i) \triangleq t_i\left(m_i\right) - \alpha_i\left(m_i\right)\theta_i,$$

where

$$t_i\left(m_i\right) \triangleq \int_{\boldsymbol{\theta}_{-i}} \alpha_i\left(m_i,\theta_{-i}\right) t_i\left(m_i,\theta_{-i}\right) d\mathbf{H}\left(\boldsymbol{\theta}_{-i}\right),$$

is firm-i's expected payment and

$$\alpha_i\left(m_i\right) \triangleq \int_{\boldsymbol{\theta}_{-i}} \alpha_i\left(m_i,\theta_{-i}\right) d\mathbf{H}\left(\boldsymbol{\theta}_{-i}\right),$$

is firm-i's expected probability of winning the auction, with

$$\mathbf{H}\left(\boldsymbol{\theta}_{-i}\right) \triangleq \prod\nolimits_{j=1,j\neq i}^{N} H\left(\theta_j\right).$$

Equipped with this characterization, we can now turn to analyze each firm's incentive to reveal truthfully its type. Let

$$u_i\left(\theta_i\right) \triangleq \max_{m_i\in[0,1]} u_i\left(m_i,\theta_i\right).$$

The Envelope Theorem implies

$$\frac{du_i}{d\theta_i} = -\alpha_i\left(\theta_i\right),$$

Hence, firm i's expected rent is

$$u_i\left(\theta_i\right) = u_i\left(1\right) + \int_{\theta_i}^{1} \alpha_i\left(x\right) dx. \tag{1}$$

The government chooses the mechanism $\mathcal{M}$ to maximize

$$\int_0^1 \cdots \int_0^1 \sum_{i=1}^{N} \left[B - t_i\left(\boldsymbol{\theta}\right) - T\right] \alpha_i\left(\boldsymbol{\theta}\right) \mathbf{h}\left(\boldsymbol{\theta}\right) d\boldsymbol{\theta} = \sum_{i=1}^{N} \int_0^1 \left[B\alpha_i\left(\theta_i\right) - t\left(\theta_i\right) - T\right] h\left(\theta_i\right) d\theta_i,$$

subject to firm-i's participation constraint $u_i\left(\theta_i\right) \geq 0$, its local incentive compatibility constraint (1), the Army's effort provision condition

$$\sum\nolimits_{i=1}^{N} \int_{\boldsymbol{\theta}} T\alpha_i\left(\boldsymbol{\theta}\right) \mathbf{h}\left(\boldsymbol{\theta}\right) d\boldsymbol{\theta} \geq \frac{\psi}{1-\rho},$$

and the ex-post budget constraint

$$B-\sum\nolimits_{i=1}^{N}\int_{\boldsymbol{\theta}}T\alpha_i\left(\boldsymbol{\theta}\right)\mathbf{h}\left(\boldsymbol{\theta}\right)d\boldsymbol{\theta}-\sum\nolimits_{i=1}^{N}\int_{\boldsymbol{\theta}}t_i(\boldsymbol{\theta})\alpha_i\left(\boldsymbol{\theta}\right)\mathbf{h}\left(\boldsymbol{\theta}\right)d\boldsymbol{\theta}\geq\mathbf{0}.$$

Using standard techniques, this problem boils down to

$$\max_{\alpha_i(\cdot)}\sum_{i=1}^{N}\int_0^1\alpha_i\left(\theta_i\right)\left[B-\theta_i-\frac{H\left(\theta_i\right)}{h\left(\theta_i\right)}-\frac{\psi}{1-\rho}\right]h\left(\theta_i\right)d\theta_i.$$

Optimizing pointwisely with respect to $\alpha_i(\cdot)$, the derivative of the above objective is positive when

$$\underbrace{B}_{\text{Project's benefit}}-\underbrace{\frac{\psi}{1-\rho}}_{\text{Army's rent}}\geq\underbrace{\theta_i+\frac{H\left(\theta_i\right)}{h\left(\theta_i\right)}}_{\text{Firm's information rent}},\tag{2}$$

The interpretation is a follows. The left-hand side represents the benefit of the project net of the rent that must be given up to the army in order to ensure stability. The right-hand side, instead, is the standard virtual cost function — i.e., the production cost θ_i plus the inverse hazard rate $\frac{H(\cdot)}{h(\cdot)}$, which measures the extent of the rent that G must grant to the firm i in order to elicit truthful reporting. Hence:

Proposition 1 *When G induces A to exert effort, there exists a unique threshold $\theta^s\in(0,1)$ that solves (2) with equality such that the optimal procurement auction has the following features:*

$$\alpha_i^s(\theta_i)=1\quad\Leftrightarrow\quad\theta_i\leq\min_{j\neq i}\left\{\theta_j,\theta^s\right\},$$

and 0 otherwise. Moreover, if firm-i's wins the auction then $t_i^s\left(\theta_i\right)=\theta^s$ and 0 otherwise. The threshold θ^s is increasing in B and decreasing in ψ and ρ.

Notice that only the most efficient firm wins the project when it is profitable for G to execute it. However, the need for ensuring political stability exerts a negative externality on the overall profitability of procuring the project. The reason is that the cost of inducing effort provision by A reduces the ex-post surplus that G enjoys from the project. Hence, the optimal threshold θ^s is decreasing in ψ, which reflects a higher cost of inducing effort provision, and ρ, which is instead an inverse measure of the A's ability to ensure stability. In fact, as ρ increases, while there is less need to induce high effort, there is less correlation between A's effort and the state of nature, which worsens the moral hazard problem between G and A.

Political Instability. Consider now the case in which A does not exert effort. In this case G designs the standard second-best mechanism, although there is a chance that political instability wipes out the project. Hence,

Proposition 2 *When G does not induce A to exert effort, the optimal auction has the following features:*

$$\alpha_i^n(\theta_i) = 1 \quad \Leftrightarrow \quad \theta_i \le \min_{j \neq i} \{\theta_j, \theta^n\},$$

and 0 otherwise. Moreover, if firm-i's wins the auction then $t_i^n(\theta_i) = \theta^n$ and 0 otherwise. The threshold $\theta^n > \theta^s$ is the unique solution of

$$B = \theta_i^n + \frac{H(\theta_i^n)}{h(\theta_i^n)},$$

with $\theta^n > \theta^s$.

Clearly, since there are no rents to be given up to A, the auction is second-best efficient — i.e., the allocation rule is less binding compared to when A exerts effort as reflected by the inequality $\theta^n > \theta^s$.

Optimal procurement rule. We can now study the conditions under which G induces A to exert effort and when instead it prefers to bear the risk of instability.

Proposition 3 *The Government prefers to enforce political stability if and only if*

$$\underbrace{\frac{H(\theta^s)}{h(\theta^s)} \int_0^{\theta^s} (1 - H(\theta_i))^{N-1} dH(\theta_i)}_{\text{The bright side of stability}} \ge \underbrace{\rho \frac{H(\theta^n)}{h(\theta^n)} \int_0^{\theta^n} (1 - H(\theta_i))^{N-1} dH(\theta_i)}_{\text{The dark side of stability}}. \tag{3}$$

This condition highlights the first novel result of our paper: the trade-off between efficiency and political stability. When G has limited budget, inducing stability comes at the cost of distorting the procurement stage. Specifically, since the maximal rent that can be paid to the firm winning the auction is lower (compared to the case in which A does not exert effort) G is forced to reduce the probability of the project being realized, as reflected by the fact that $\theta^n > \theta^s$.

A second interesting result concerns the effect of competition in the auction on this trade-off between stability and efficiency. To study the effect of competition on this trade-off, we check how the increase in the number of firms affects condition (3)

Proposition 4 *There exists a threshold $\rho^* \in (0,1)$ such that:*

- *If $\rho \leq \rho^*$ the government always prefers to pay the army and implement stability regardless of N;*
- *If $\rho > \rho^*$, there exists a threshold $N^* > 1$ such that the government prefers to pay the army if and only if $N \geq N^*$.*

This result shows that when ρ is small enough — i.e., when the moral hazard problem that G faces with the army is not too severe — competition is irrelevant for the G's decision. That is, when environment is sufficiently unstable in the absence of control by the army, G will always restore stability by paying the army regardless of the number of firms participating in the procurement auction. By contrast, when the environment is stable enough (i.e., when ρ not too small) G will implement stability and give up the efficient rule only when N is sufficiently large. The reason is that as N grows large the probability that the firm that wins the auction features a cost lower than θ^n increases, whereby reducing the negative impact of securing stability on efficiency of the auction. In other words, as N increases efficiency of the auction becomes less of a concern, whereby fostering G to pay the army a rent in exchange of securing a stable environment.

4 Conclusions

We have examined a public procurement mechanism, where a government tries to finance a socially valuable project in a context characterized by political instability, as for instance, in developing countries. The government faces a trade-off. On one side, it may finance the army and possibly distort efficiency in a perfectly stable environment, or it may not pay the army and accept carrying out the project efficiently in an unstable framework. In this set-up, we focus on the effects of competition at the auction stage. The introduction of a larger number of firms in the pool makes the procurement stage more efficient, and it has the government obtaining a higher surplus, such that more competition seems to induce more stability. Therefore competition seems to increase stability without reducing the efficiency at the procurement level. In this respect, competition makes democracy easier to maintain, but also more costly for the society.

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5 Appendix

Optimal auction. The analysis of the optimal auction follows closely Myerson's (1981), and is omitted for brevity.

Proof of Proposition 3 . Consider first the case in which the army is paid to secure stability. Using the standard dominant-strategy implementation rule $t(\theta_i, \boldsymbol{\theta}_{-i}) = \theta^s$, the agency's expected profit from paying the army is

$$\int_0^1 \dots \int_0^1 \left\{ \int_0^1 [1 - H(\theta_i)]^{N-1} \left[B - t(\theta_i, \boldsymbol{\theta}_{-i}) - \frac{\psi}{1-\rho} \right] dH(\theta_i) \right\} d\mathbf{H}(\boldsymbol{\theta}_{-i}) =$$

$$= \sum\nolimits_{i=1}^{N} \int_0^1 \dots \int_0^1 \left\{ \int_0^{\widetilde{\theta}} [1 - H(\theta_i)]^{N-1} \frac{H(\widetilde{\theta})}{h(\widetilde{\theta})} dH(\theta_i) \right\} d\mathbf{H}(\boldsymbol{\theta}_{-i}) =$$

$$= N \frac{H(\theta^s)}{h(\theta^s)} \int_0^{\theta^s} [1 - H(\theta_i)]^{N-1} dH(\theta_i).$$

Using a similar logic, we can show that if the army is not paid, the agency's expected profit is

$$\rho N \frac{H(\theta^n)}{h(\theta^n)} \int_0^{\theta^n} [1 - H(\theta_i)]^{N-1} dH(\theta_i).$$

Hence, the result. □

Proof of Proposition 4 . The Government prefers to pay the army and maintain a stable environment if (3) holds:

$$\frac{H(\theta^s)}{h(\theta^s)} \int_0^{\theta^s} (1 - H(\theta_i))^{N-1} dH(\theta_i) \geq \rho \frac{H(\theta^n)}{h(\theta^n)} \int_0^{\theta^n} (1 - H(\theta_i))^{N-1} dH(\theta_i). \tag{4}$$

To study the determinants of this inequality, it is useful to begin with $N = 1$, so that (4) becomes

$$\frac{H(\theta^s)^2}{h(\theta^s)} \geq \rho \frac{H(\theta^n)^2}{h(\theta^n)}$$

Let $\rho^* \in (0, 1)$ be the unique solution of

$$\frac{H(\theta^s)^2}{h(\theta^s)} = \rho \frac{H(\theta^n)^2}{h(\theta^n)}.$$

Notice, in fact, that $H(\theta^s)^2/h(\theta^s)$ is decreasing in ρ, while $\rho H(\theta^n)^2/h(\theta^n)$ is increasing in ρ. Furthermore,

$$\lim_{\rho \to 1} \left(\frac{H(\theta^s)^2}{h(\theta^s)} \right) = 0,$$

while

$$\lim_{\rho \to 1} \left(\rho \frac{H(\theta^n)^2}{h(\theta^n)} \right) > 0.$$

Hence, for $N = 1$, G pays the army (stability) if and only if $\rho \leq \rho^*$.

Next, consider the case $N > 1$. Integrating both sides, (4) becomes:

$$\frac{H(\theta^s)}{h(\theta^s)} \left[1 - (1 - H(\theta^s))^N \right] \geq \rho \frac{H(\theta^n)}{h(\theta^n)} \left[1 - (1 - H(\theta^n))^N \right],$$

or, equivalently,

$$\frac{\frac{H(\theta^s)}{h(\theta^s)}}{\frac{H(\theta^n)}{h(\theta^n)}} \geq \rho \frac{1 - (1 - H(\theta^n))^N}{1 - (1 - H(\theta^s))^N}.$$

To show the result, it is sufficient to show that the right-hand side of this inequality is decreasing in N. Differentiating with respect to N and rearranging we have:

$$\frac{\partial}{\partial N} \frac{1 - (1 - H(\theta^n))^N}{1 - (1 - H(\theta^s))^N} = \frac{(1-H(\theta^n))^N \left((1-H(\theta^s))^N - 1\right) \ln(1-H(\theta^n)) - (1-H(\theta^s))^N \left((1-H(\theta^n))^N - 1\right) \ln(1-H(\theta^s))}{\left((1-H(\theta^s))^N - 1\right)^2}.$$

Next, notice that $\theta^s < \theta^n$ implies $1 - H(\theta^n) < (1 - H(\theta^s))$,

$$(1 - H(\theta^n))^N < (1 - H(\theta^s))^N$$

and

$$\ln(1 - H(\theta^n)) < \ln(1 - H(\theta^s)).$$

It then follows that

$$\begin{aligned} &(1 - H(\theta^n))^N \left((1 - H(\theta^s))^N - 1 \right) \ln(1 - H(\theta^n)) + \\ &\quad - (1 - H(\theta^s))^N \left((1 - H(\theta^n))^N - 1 \right) \ln(1 - H(\theta^s)) < \\ &\left((1 - H(\theta^s))^N - 1 \right) \ln(1 - H(\theta^n)) - \left((1 - H(\theta^n))^N - 1 \right) \ln(1 - H(\theta^s)) < \\ &(1 - H(\theta^s))^N \left(\left((1 - H(\theta^s))^N - 1 \right) \ln(1 - H(\theta^s)) - \left((1 - H(\theta^n))^N - 1 \right) \ln(1 - H(\theta^s)) \right) = \\ &\quad (1 - H(\theta^s))^N \ln(1 - H(\theta^s)) \left((1 - H(\theta^s))^N - (1 - H(\theta^n))^N \right) < 0. \end{aligned}$$

Hence,

$$\frac{\partial}{\partial N}\frac{1-(1-H(\theta^n))^N}{1-(1-H(\theta^s))^N} < 0.$$

This implies that if

$$\frac{\frac{H(\theta^s)}{h(\theta^s)}}{\frac{H(\theta^n)}{h(\theta^n)}} > \rho\frac{H(\theta^n)}{H(\theta^s)},$$

or equivalently $\rho < \rho^*$, a fortiori

$$\frac{\frac{H(\theta^s)}{h(\theta^s)}}{\frac{H(\theta^n)}{h(\theta^n)}} > \rho\frac{1-(1-H(\theta^n))^N}{1-(1-H(\theta^s))^N},$$

for any $N > 1$. Otherwise, if

$$\frac{\frac{H(\theta^s)}{h(\theta^s)}}{\frac{H(\theta^n)}{h(\theta^n)}} < \rho\frac{H(\theta^n)}{H(\theta^s)},$$

or equivalently if $\rho \geq \rho^*$, there exists a threshold N^* such that

$$\frac{\frac{H(\theta^s)}{h(\theta^s)}}{\frac{H(\theta^n)}{h(\theta^n)}} \geq \rho\frac{1-(1-H(\theta^n))^N}{1-(1-H(\theta^s))^N}$$

if and only if $N \geq N^*$ because

$$\lim_{N\to+\infty}\frac{1-(1-H(\theta^n))^N}{1-(1-H(\theta^s))^N} = 1 > \rho.$$

To sum up, we have shown that if $\rho \geq \rho^*$, then the Government does not pays the army if and only if N is sufficiently small, and pays the army otherwise. By contrast, if $\rho < \rho^*$ it always pays the army. $\square$