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Abstract
In a recent influential paper Coate et al. (2020) have criticized the standard firm-level approach to market definition in merger review. They argue why a market-level approach to critical loss is more appropriate than a firm-level critical loss analysis. Their conclusion is that under certain plausible demand scenarios - i.e., non-linearity of demand functions - a diversion-based firm-level analysis could easily reach the wrong answer on market definition. We extend their analysis by showing that in standard environments used by the most recent theoretical and empirical academic work on merger analysis (namely CES and logit demand functions), a firm level approach actually leads to an excessively narrow market definition as opposed to a market-level approach, thereby increasing the risk of type I errors.

JEL classification: D43, G34, L4, L13

Keywords: Critical Loss Analysis, Firm- and Market-level approach, Mergers, Non-linear Demand.

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1 Introduction

The first and often fundamental step in reviewing horizontal mergers is 'market definition.' The objective of defining a market — i.e., both along its product and geographic dimensions — is the identification of the edges of competition between firms: the playing field. In a nutshell, the question that is first asked by competition policy authorities when evaluating a potential merger is the following: Would a hypothetical monopolist controlling all the products in the candidate market impose at least a small but significant and non-transitory price increase (SSNIP) on all of its products given the pre-merger prices of the products outside of the candidate market?

By answering this question, competition authorities seek to identify the actual competitors of a potential merged firm capable of constraining its conduct and preventing it from behaving independently of adequate competitive pressure — i.e., the so-called 'relevant market.'

Yet, although the hypothetical monopolist test's idea is extremely simple, its implementation is somewhat controversial. In fact, because of its counterfactual nature, any market definition procedure has to impose some arbitrary choices to resolve (or minimize) potential ambiguities that may bias the definition of the relevant market. As explained by Whinston (2008), knowing the right way to resolve such ambiguities is difficult because, while intuitive, the idea of market definition behind the Merger Guidelines is not based on any explicit model of competition and welfare effects. For this reason, over the years, several attempts, diverging in one or multiple dimensions, have been proposed to define relevant markets.

Harris and Simons (1989) — hereafter H&S — developed the first quantitative method to implement the hypothetical monopolist test with a market-level critical loss analysis. Their study provides an intuitive methodology to identifying the conditions under which it is in the interest of a hypothetical (multiproduct) monopolist to raise prices. Their central idea is that of 'critical loss': the percentage loss in sales necessary to make a given increase of all prices controlled by the hypothetical monopolist unprofitable. Essentially, a critical loss analysis estimates how much the hypothetical monopolist's sales would have to fall in order to render the hypothetical increase of prices unprofitable. Two opposing effects shape the net change in the monopolist's profits. Higher prices hurt profits because sales fall, as some consumers substitute to rival firms' products in response to the increase in prices. However, there is an offsetting positive effect on profits, as the hypothetical monopolist now earns higher margins on all of the remaining sales. If the negative effect on profits is greater than the positive effect, then the price increase will be unprofitable for the hypothetical monopolist, and the relevant market is wider. The relevant market is defined as the narrowest market that is compatible with the hypothetical monopolist being willing to slightly increase its products' prices.

Subsequently, Katz and Shapiro (2002) — hereafter K&S – and O'Brien and Wickelgren (2003) — hereafter O&W — modified H&S's critical loss model to use firm-level demand functions. Specifically, K&S and O&W bounded the hypothetical monopolist's actual loss to the outside good from a price increase of a single product, via the Lerner Index, and reformulated the critical loss analysis in terms of diversion ratios — i.e., a measure of the proportion of sales captured by different substitute products when the price of a product is increased.

Therefore, the fundamental difference between the H&S and the K&S-O&W approaches is how the SSNIP is implemented. The H&S approach intuitively imposes a uniform price increase (i.e., the same SSNIP across all the products in the candidate market): the so-called 'market level' approach. The K&S-O&W methodology focuses on a 'firm-level' approach: the actual loss incurred by the hypothetical monopolist is computed by raising the price of one firm at a time (holding the prices of all other firms under the control of the product monopolist at their pre-SSNIP level), adding up sequentially the diversions to the outside good for each price increase. The advantage of using a firm-level rather than a market-level approach à la H&S is that the former can be adapted to different models of competition.

However, in a recent influential paper, Coate et al. (2020) have pointed out that understanding the precise market conditions under which the analysis developed by K&S and O&W can be safely applied is crucial to avoid errors. Specifically, they develop examples showing that, with non-linear demand functions, a diversion-based or firm-level SSNIP test could easily reach the wrong answer on market definition. The reason is that computing the actual loss by raising one price at a time (holding the other prices at the premerger level) overlooks the potential non-linear effect of price variations on diversion ratios (reflecting the ratio between own and cross elasticities), which may potentially bias results.

In this paper, we further develop the conclusions reached by Coate et al. (2020) by showing that in standard environments used by the most recent theoretical and empirical work on merger analysis (namely CES and logit demand functions)[1], a firm-level approach actually leads to an excessively narrow market definition as opposed to a market-level approach, thereby increasing the risk of type I errors — i.e., the probability of rejecting too many mergers that are potentially consumer welfare enhancing. This stands in sharp contrast with the case of linear demand, often used as a convenient benchmark in merger evaluation. With linear demand, the two approaches are equivalent and deliver the same relevant market, because diversion ratios are constant with respect to prices. By contrast, using the demand functions typically used in the empirical merger literature, those ratios are decreasing in prices: the fraction of consumers that switch from one product (say product i) to an alternative (product j) after a price increase of product i is decreasing in the price of product j. Hence, our results suggest that more care should be taken when choosing how to perform a critical loss analysis. In particular, our main conclusion is that, before deciding the approach to critical loss analysis, more effort should be devoted to understanding the shape of actual demand in more depth. Notably, our results are also supported by the evidence collected in Conlon and Mortimer (2018). In this paper, the authors empirically show that constant diversion is a feature of only the linear demand model. In contrast, most commonly-used models of demand, such as random-coefficients logit or log-linear models, do not feature constant diversion ratios.

The rest of the paper is organized as follows: in Section 2 we briefly summarize the logic and the main conceptual steps behind the critical loss analysis. In Section 3, we review the methodology

[1] See, e.g., Anderson et al. (2020) and Motta and Tarantino (2020) among many others.

behind the critical loss definition. In Section 4 we illustrate the alternative approaches to computing the actual loss and compare the linear-demand benchmark, in which firm-level and market-level are equivalent, with the CES and the logit demand functions, where they are not. Proofs are in the Appendix.

2 Key concepts

The idea behind the hypothetical monopolist test is rather simple. Coate et al. (2020) summarize the procedure (when implemented by a critical loss analysis) as follows. The first step assumes that the hypothetical monopolist controls a candidate set of products (firms) and implements a uniform SSNIP. The second step calculates the critical loss, which is the maximum loss in sales that the hypothetical monopolist could incur (given the size of the uniform SSNIP) before the price increase would reduce its profits. The third step compares this critical loss to an estimate of the actual loss expected from the SSNIP. The actual loss estimate comes from evidence of substitution from products within the candidate market to goods outside of it. If the actual loss is less than the critical loss, the candidate market forms a relevant antitrust product market; otherwise, the test repeats with a broader candidate market.

With symmetric products, which is the case that we shall consider, the main difference between a firm-level approach and a market-level approach is that while in the former case the actual loss incurred by the hypothetical monopolist is computed by raising one price at a time (holding other prices at the pre-SSNIP or premerger level), adding up the diversions to the outside good for each price increase; in the latter, it is obtained by raising all prices at the same time.

3 Critical loss

Following the logic illustrated above, we begin by determining the hypothetical monopolist's critical loss. Consider a market where in the pre-merger scenario there are N symmetric products (firms).

Assume that the hypothetical monopolist sells M of these N goods. The critical loss incurred by the monopolist when it increases by Δp the price of all its M products can be easily defined. Let

$$\pi \triangleq \sum_{i=1}^{M} (p-c)\, q,$$

be the monopolist's (aggregate) profit when setting a symmetric price p for every product $i = 1,..,M$, and where c is the constant marginal cost that is assumed symmetric for all products. Assuming a symmetric system of demand functions, that is

$$q_i\left(p_i, p_{-i}\right) = q_j\left(p_j, p_{-j}\right) \quad \forall p_i = p_j,\ p_{-i} = p_{-j},\ i, j = 1,..,N.$$

Define with

$$q \triangleq q_i\left(p,..,p\right) \quad \forall i = 1,..,N$$

the symmetric level of quantity supplied by the hypothetical monopolist for each product when it charges p for every product $i = 1,..,M$ under its control.

The critical loss associated with a uniform price increase Δp is thus

$$\underbrace{M \Delta p\left(q+\Delta q\right)}_{\text{Benefit of price increase}} = \underbrace{-M\left(p-c\right)\Delta q}_{\text{Cost of price increase}}. \tag{1}$$

The formula for the critical loss is found by dividing both sides of this equation by pq. This gives

$$\frac{\Delta p}{p}\left(1+\frac{\Delta q}{q}\right) = -\frac{p-c}{p} \times \frac{\Delta q}{q}, \tag{2}$$

The critical loss is the percentage reduction in quantity, $-\Delta q/q$, that satisfies condition (2). Solving for the critical loss gives

$$CL = -\frac{\Delta q}{q} \triangleq \frac{\delta}{\delta + m}, \tag{3}$$

where $m \triangleq \frac{p-c}{p}$ represents the margin measured as a percent of the price, and $\delta \triangleq \frac{\Delta p}{p}$ is just the percentage price increase or SSNIP.

Notice that, (3) does not depend on the 'timing' according to which prices are raised — i.e., the whether prices are increased simultaneously (market-level approach) or sequentially (firm-level approach) yields the same critical loss. In contrast, as shown below, with a non-linear demand system — the actual loss depends on the timing according to which the monopolist's prices are raised — i.e., sequential vs simultaneous. Moreover, since p and q are typically evaluated at the pre-merger outcome (p^N and q^N hereafter), it follows immediately that CL does not depend on the definition of the candidate market M.

4 Actual loss

We now turn to study how the methodology adopted in computing the actual loss incurred by the hypothetical monopolist (following a uniform SSNIP) affects the definition of the relevant market. As explained before, we shall consider two alternative approaches, depending on the structure imposed on the competitive process:

- **Firm-level (sequential) approach**: under this approach, the actual loss incurred by the hypothetical monopolist is computed by raising one price at the time (holding all other prices at the pre-SSNIP level), adding up the diversions to the outside good for each price increase.

- **Market-level (simultaneous) approach**: under this approach, the actual loss incurred by the hypothetical monopolist is obtained by raising all prices at the same time.

We will conduct our analysis by first reviewing the benchmark case of linear demand, and then move to the more general case of non-linear demand. To advance the Coate et al. (2020) critique of the firm-level approach and get further insights on the implications on the definition of the relevant market, we consider two different demand specifications commonly employed in the empirical merger literature: CES (constant elasticity of substitution) preferences and a logit model.

In what follows we shall denote by p^N and

$$q^N = q_i\left(p^N, ..., p^N\right), \quad \forall i = 1, .., N,$$

the (symmetric) premerger equilibrium price and quantity, respectively.

For the ease of exposition, we will first define the actual loss under each approach and then apply each expression for every demand specification below.

- Under a market-level approach the actual loss is

$$AL_M^{\text{sim}} = -\frac{\sum_{i=1}^{M}\left[q_i\left(p_1 = p^N + \Delta p, .., p_M = p^N + \Delta p, p_{M+1} = p^N, .., p_N = p^N\right) - q^N\right]}{Mq^N}, \quad (4)$$

  which considers a simultaneous increase of all M prices under the hypothetical monopolist's control by Δp.

- By contrast, under firm-level approach, the actual loss is obtained by adding up sequentially the losses incurred by the hypothetical monopolist when it rises one price at the time (holding other prices at the pre-SSNIP level) — i.e.,

$$AL_M^{\text{seq}} = -\sum_{i=1}^{M}\frac{\Delta q_i(\Delta p) + \Delta q_{-i}\left(\Delta p\right)}{Mq^N}, \quad (5)$$

  where, for every $i = 1, .., M$, the term

$$\Delta q_i\left(\Delta p\right) \triangleq q_i\left(p^N, .., p_i = p^N + \Delta p, .., p^N\right) - q^N$$

  represents the change in quantity of the product whose price has increased by Δp, whereas the term

$$\Delta q_{-i}(\Delta p) \triangleq \sum_{j=1, j\neq i}^{M} \left[q_j \left(p^N, .., p_i = p^N + \Delta p, .., p^N \right) - q^N \right]$$

represents the change in quantity of all products different than i whose price has not increased — i.e., the diversion to products within the candidate market.

4.1 Linear demand: the equivalence result

As a benchmark we first consider the case of a linear demand system. Without loss of generality, suppose that each product available on the market features a (direct) linear demand

$$q_i(p_1, .., p_N) = \frac{1}{N} \left[v - p_i (1+\gamma) + \frac{\gamma}{N} \sum_{j=1}^{N} p_j \right] \quad \forall i = 1, .., N, \tag{6}$$

which is obtained from the utility function of a representative consumer with Shubick-Levitan preferences (see, e.g., Motta, 2004).[2] The parameter $\gamma \geq 0$ defines the degree of product differentiation: the larger γ, the closer substitutes. Notice that in the symmetric premerger equilibrium

$$q^N = \frac{v - p^N}{N},$$

where p^N is defined as the Nash equilibrium of the price setting game between the N competitors in the market.

Market level approach. Consider a uniform SSNIP. Under a market level approach, the actual loss with a linear demand function is

$$AL_M^{\text{sim}} = \frac{N + \gamma (N - M)}{N^2 q^N} \Delta p.$$

[2]Results would not change if we had chosen an alternative linear specification à la Singh-Vives (1984).

Hence, M defines a market if and only if

$$\frac{N+\gamma(N-M)}{N(v-p^N)}\Delta p \leq CL. \tag{7}$$

Since AL_M^{sim} is decreasing in M and CL does not depend on M, because it is evaluated the premerger outcome, the smallest relevant market M^{sim} that is defined with a simultaneous approach and linear demand is obtained by solving (7) as equality.

Firm-level approach. In order to compute the actual loss under a firm-level approach we must first determine $\Delta q_i(\Delta p)$ and $\Delta q_{-i}(\Delta p)$. For every $i = 1, .., N$,with linear demand we have

$$q_i\left(p^N, .., p_i = p^N + \Delta p, .., p^N\right) = \frac{1}{N}\left[v - \frac{N+\gamma(N-1)}{N}\left(p^N + \Delta p\right) + \frac{\gamma(N-1)}{N}p^N\right].$$

Thus,

$$\Delta q_i(\Delta p) = -\frac{\Delta p}{N^2}\left(N+\gamma(N-1)\right). \tag{8}$$

Similarly, for every $j \neq i$

$$q_j\left(p^N, .., p_i = p^N + \Delta p, .., p^N\right) = \frac{1}{N}\left[v - (1+\gamma)p^N + \frac{\gamma(N-1)}{N}p^N + \frac{\gamma}{N}\left(p^N + \Delta p\right)\right].$$

Hence, the change in quantity of the products whose price is held constant at the premerger level is

$$\Delta q_{-i}(\Delta p) = (M-1)\Delta p\frac{\gamma}{N^2}. \tag{9}$$

Combining (8) and (9) with (5), we have

$$AL_M^{\mathrm{seq}} = \frac{N+\gamma(N-M)}{N^2 q^N}\Delta p,$$

which is equal to AL_M^{sim}.

Hence, with linear demand, the actual loss is the same under both approaches, which therefore lead to identifying the same relevant market. Such an equivalence result was emphasized by Coate et al. (2020). Because, linear demand functions feature constant diversion ratios — i.e., diversion ratios,

$$D_{jk} \triangleq -\frac{\frac{\partial q_k(\cdot)}{\partial p_j}}{\frac{\partial q_j(\cdot)}{\partial p_j}} = \frac{\gamma}{N\left(N+\gamma\left(N-1\right)\right)} \quad \forall j,k=1,..,N,$$

which are independent of the level of prices of the products $k = 1, .., N$ that are not increased at a given time. Hence, the two approaches yield necessarily the same relevant market.

In the next section, we show that this is not the case with non-linear demand. In particular, we show that in the CES and logit specification, the sequential approach leads to an excessively narrow market definition compared to the correct simultaneous method.

4.2 Non-linear demand

We now turn to analyze the impact on the definition of the relevant market of the approach followed in computing the actual loss triggered by an increase in the prices under the hypothetical monopolist's control. In doing so, we focus on two broadly used class of demand functions: CES and logit. One reason for this focus is that these functional forms have realistic properties in respect to consumer preferences for many markets compared to linear demand functions.[3] A second reason why these two classes of demand functions are very popular is uncovered through recognizing them as delivering 'aggregative oligopoly games'.[4] The oligopoly problem is in general complex: each firm's action depends on the actions of all other firms. An aggregative game reduces the degree of complexity drastically to a simple problem in two dimensions. Each firm's action depends only on

[3]See, e.g., Helpman and Krugman (1987) and; Melitz (2003) for applications to theories of international trade; Grossman and Helpman, (1993) for applications to endogenous growth; and Fujita, Krugman, and Venables (2001) and Fujita and Thisse (2002) for applications to new economic geography.

[4]Many non-cooperative games in economics are aggregative games, where each player's payoff depends on its own action and an aggregate of all players' actions. Examples abound in industrial organization (oligopoly, contests, R&D races), public economics (public goods provision, tragedy of the commons), and political economy (political contests, conflict models), to name a few.

one variable, the aggregate, yielding a clean characterization of oligopoly equilibria. Aggregative games are the pillar of modern merger analysis (see, e.g., Nocke and Schutz, 2018; Anderson et al., 2020; Motta and Tarantino, 2020, among others).

CES demand functions. The system of Marshallian demand functions when preferences are CES is as follows (see, e.g., Anderson et al., 2020, and Motta and Tarantino, 2020)

$$q_i\left(p_1, .., p_N\right) \triangleq \frac{p_i^{-r-1}}{\sum_{j=1}^{N} p_j^{-r}}, \quad \forall i = 1, .., N, \tag{10}$$

where $r \in [0, 1]$ represents a measure of product substitutability: the larger r the more close substitutes products are. Notably, r also captures an inverse measure of the degree of relative risk aversion and of prudence (Kimball, 1990).[5]

In addition to satisfying the properties needed for aggregative games and being often used in merger simulation analysis, outside of the industrial organization, the CES model is central in theories of international trade (e.g., Helpman and Krugman, 1987; Melitz, 2003), endogenous growth (e.g., Grossman and Helpman, 1993), and new economic geography (e.g., Fujita et al., 2001; Fujita and Thisse, 2002).

Logit demand functions. Under a logit specification the system of Marshallian demand functions is instead as follows

$$q_i\left(p_1, .., p_N\right) \triangleq \frac{\exp\left(\left(s - p_i\right)/\mu\right)}{1 + \sum_{j=1}^{N} \exp\left(\left(s - p_j\right)/\mu\right)}, \quad \forall i = 1, .., N, \tag{11}$$

where s is a quality parameter and μ the degree of preference heterogeneity (see, e.g., Anderson et al., 2020, and Motta and Tarantino, 2020, for applications to merger analysis).

Logit demand functions are typically employed in merger simulations and are the basis of the modern structural empirical industrial organization (see, e.g., Reiss and Wolak, 2007). They are

[5] The prudence index measures the intensity of the precautionary motive just as risk aversion measures the intensity of the desire for insurance.

particularly useful in examining discrete choice models — i.e., circumstances in which consumers choose among a finite set of alternatives. Specifically, discrete choice models statistically relate the choice made by each consumer to its individual attributes (e.g., age, income, gender, etc.) and the attributes of the alternatives available to him/her (e.g., price, quality, etc.) — see, e.g., Reiss and Wolak (2007).

We consider first a market-level approach. Let

$$\chi \triangleq \frac{1 + N \exp\left(\left(s - p^N\right)/\mu\right)}{\exp\left(\left(s - p^N\right)/\mu\right)},$$

then the following is true.

Proposition 1 *Under a market-level approach, the actual loss is*

$$AL_M^{sim} = 1 - \frac{Np^N\left(p^N + \Delta p\right)^{-r-1}}{M\left(p^N + \Delta p\right)^{-r} + (N - M)\left(p^N\right)^{-r}},$$

with a CES demand function, and

$$AL_M^{sim} = 1 - \frac{\exp\left(\left(s - \left(p^N + \Delta p\right)\right)/\mu\right)\chi}{1 + M\exp\left(\left(s - \left(p^N + \Delta p\right)\right)/\mu\right) + (N - M)\exp\left(\left(s - p^N\right)/\mu\right)},$$

with a logit demand function.

Notice that AL_M^{sim} is decreasing in M in both cases (see the Appendix). Hence, under a market-level approach M defines a market if and only if

$$AL_M^{\text{sim}} \leq CL,$$

which implies that $M \geq M^{\text{sim}}$, with M^{sim} being the smallest solution (the narrowest relevant market) of $AL_M^{\text{sim}} = CL$.

Next, consider a firm-level approach.

Proposition 2 *Under a firm-level approach, the actual loss is*

$$AL_M^{seq} = 1 - \frac{(M-1)\left(\left(p^N\right)^{-r} - \left(p^N + \Delta p\right)^{-r}\right) + N p^N \left(p^N + \Delta p\right)^{-r-1}}{\left(p^N + \Delta p\right)^{-r} + (N-1)\left(p^N\right)^{-r}},$$

with a CES demand function, and

$$AL_M^{seq} = 1 - \frac{(M-1)\left(\exp\left(\left(s - p^N\right)/\mu\right) - \exp\left(\left(s - \left(p^N + \Delta p\right)\right)/\mu\right)\right) + \exp\left(\left(s - \left(p^N + \Delta p\right)\right)/\mu\right)\chi}{1 + \exp\left(\left(s - \left(p^N + \Delta p\right)\right)/\mu\right) + (N-1)\exp\left(\left(s - p^N\right)/\mu\right)},$$

with a logit demand function.

Once again, it can be readily shown that AL_M^{seq} is decreasing in M in both cases (see the Appendix). Hence, under a market-level approach, M defines a market if and only if

$$AL_M^{\text{seq}} \leq CL,$$

which implies that $M \geq M^{\text{seq}}$, with M^{seq} being the smallest solution (the narrowest relevant market) of $AL_M^{\text{seq}} = CL$.

We can finally compare AL_M^{seq} with AL_M^{seq}.

Proposition 3 $AL_M^{sim} > AL_M^{seq}$ *with both a CES and a logit demand function. Hence, a firm-level approach defines a narrower relevant market than a market-level approach — i.e.,* $M^{seq} < M^{sim}$.

As explained in Coate et al. (2020) this is because with a non-linear demand, raising one price at a time and adding the effects can only approximate a simultaneous price increases and does not take into proper consideration the actual non-linear effect of the variation of the M prices on diversion ratios. In particular, in both of our examples diversion ratios are decreasing in the prices of the products which are kept constant at a time, which explains why a firm-level approach delivers a narrower market than a market-based approach — i.e., the fraction of consumers that switch from one product (say product i) to an alternative (product j) after a price increase of product i is decreasing in the price of product j. Notably, this result is in line with the empirical evidence found by Conlon and Mortimer (2018) who show that commonly-used models of demand, such as random-coefficients logit or log-linear models, do not feature constant diversion.

Remark. Following Coate et al. (2020)'s core analysis, so far we have considered a version of the firm-level approach to critical loss analysis such that the actual loss incurred by the hypothetical monopolist is computed by raising one price at the time (holding all other prices at the pre-SSNIP level), adding up the diversions to the outside good for each price increase. Hence, when a price is increased in one round, it is returned to the premerger level in the next rounds, where the remaining prices are increased each at a time. Alternatively, one might consider a case 'without return', where once a price is increased, it stays at the post-SSNIP level for the rest of the rounds. This procedure delivers an outcome that is closer in spirit to the market-level approach — i.e., as the number of iterations grows, the actual loss gets closer to that obtained in the market level approach where all prices are increased simultaneously. However, the applicability of this 'without return' procedure is somewhat complicated in practice because, in the case of asymmetric products, its outcome depends on the actual sequence of price increases chosen by the analyst. That is, the relevant market will depend on which prices are increased at the early stage of the procedure, whereby making such a procedure unappealing in practice.

5 Conclusion

Although critical loss analysis is based on simple and intuitive logic, this is an exercise that should be exerted with caution, especially when the stakes are potentially high for consumers. The choice of the approach to the definition of a relevant market in merger analysis is controversial. It typically depends on several factors not observed by analysts – e.g., demand conditions, type of competition, etc. — that should be carefully assessed before taking actions. For this reason, in recent years, many economists have reasonably expressed concerns on rules whose applicability relies on too restrictive assumptions unlikely to be met in reality.

In this paper, we have contributed to this debate by extending the conclusions reached by Coate et al. (2020). In particular, we have shown that in standard environments used by the most recent theoretical and empirical academic work on merger analysis (namely CES and logit demand functions) a firm-level approach to market definition leads to an excessively narrow market definition as opposed to a market-level approach, thereby increasing the risk of type I errors — i.e., the probability of rejecting too early mergers that are potentially consumer welfare enhancing. Hence, our results suggest that more care should be taken when choosing the market definition approach. In particular, our main conclusion is that more effort should be devoted to understanding the exact shape of demand in more depth.

6 Appendix

Proof of Proposition 1. Consider first the CES specification. Substituting the demand system (10) into (4) we have

$$AL_M^{\text{sim}} = -\left(\frac{M\left(p^N + \Delta p\right)^{-r-1}}{M\left(p^N + \Delta p\right)^{-r} + (N-M)\left(p^N\right)^{-r}} - \frac{M}{Np^N} \right) \frac{Np^N}{M}$$

Rearranging,

$$AL_M^{\text{sim}} = 1 - \frac{Np^N\left(p^N + \Delta p\right)^{-r-1}}{M\left(p^N + \Delta p\right)^{-r} + (N-M)\left(p^N\right)^{-r}},$$

which is clearly decreasing in M since $\left(p^N + \Delta p\right)^{-r} < \left(p^N\right)^{-r}$ for $r \in [0,1]$.

Next, consider the logit specification. Substituting the demand system (11) into (4) we have

$$AL_M^{\text{sim}} = -\frac{M\frac{\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)}{1+M\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-M)\exp\left(\left(s-p^N\right)/\mu\right)} - Mq^N}{Mq^N},$$

Rearranging we have

$$AL_M^{\text{sim}} = 1 - \frac{\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)\chi}{1+M\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-M)\exp\left(\left(s-p^N\right)/\mu\right)},$$

which is clearly decreasing in M since $\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right) < \exp\left(\left(s-p^N\right)/\mu\right)$. ■

Proof of Proposition 2. Consider first the CES specification. Substituting the demand system (10) into (5) we have

$$AL_M^{\text{seq}} = -\left(M\frac{\left(p^N + \Delta p\right)^{-r-1}}{\left(p^N + \Delta p\right)^{-r} + (N-1)\left(p^N\right)^{-r}} + M(M-1)\frac{\left(p^N\right)^{-r-1}}{\left(p^N + \Delta p\right)^{r} + (N-1)\left(p^N\right)^{r}} - \frac{M^2}{Np^N} \right) \frac{Np^N}{M}.$$

Rearranging,

$$\begin{aligned}
AL_M^{\text{seq}} &= M - Np^N \frac{\left(p^N + \Delta p\right)^{-r-1} + (M-1)\left(p^N\right)^{-r-1}}{\left(p^N + \Delta p\right)^{-r} + (N-1)\left(p^N\right)^{-r}} \\
&= 1 + M - 1 - Np^N \frac{\left(p^N + \Delta p\right)^{-r-1} + (M-1)\left(p^N\right)^{-r-1}}{\left(p^N + \Delta p\right)^{-r} + (N-1)\left(p^N\right)^{-r}} \\
&= 1 - \frac{(M-1)\left(\left(p^N\right)^{-r} - \left(p^N + \Delta p\right)^{-r}\right) + Np^N\left(p^N + \Delta p\right)^{-r-1}}{\left(p^N + \Delta p\right)^{-r} + (N-1)\left(p^N\right)^{-r}},
\end{aligned}$$

which is clearly decreasing in M since $r < 1$.

Let us now turn to the logit demand system. Substituting the demand system (11) into (5) we have

$$\begin{aligned}
AL_M^{\text{seq}} &= -\frac{M \frac{\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)}{1+\exp((s-(p^N+\Delta p))/\mu)+(N-1)\exp((s-p^N)/\mu)}}{Mq^N} + \\
&\quad -\frac{M(M-1)\frac{\exp\left(\left(s-p^N\right)/\mu\right)}{1+\exp((s-(p^N+\Delta p))/\mu)+(N-1)\exp((s-p^N)/\mu)} - M^2 q^N}{Mq^N}.
\end{aligned}$$

Rearranging,

$$\begin{aligned}
AL_M^{\text{seq}} &= M - \frac{\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right) + (M-1)\exp\left(\left(s-p^N\right)/\mu\right)}{1+\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-1)\exp\left(\left(s-p^N\right)/\mu\right)}\chi \\
&= 1 + M - 1 - \frac{\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right) + (M-1)\exp\left(\left(s-p^N\right)/\mu\right)}{1+\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-1)\exp\left(\left(s-p^N\right)/\mu\right)}\chi \\
&= 1 - \frac{(M-1)\left(\exp\left(\left(s-p^N\right)/\mu\right) - \exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)\right) + \exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)\chi}{1+\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-1)\exp\left(\left(s-p^N\right)/\mu\right)},
\end{aligned}$$

where, since $\Delta p > 0$ and

$$\exp\left(\left(s-p^N\right)/\mu\right) > \exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right),$$

it is immediate to show that AL_M^{seq} is decreasing in M. ■

Proof of Proposition 3. Assume a CES demand function. Since $M \geq 2$ we have

$$\begin{aligned} AL_M^{\text{sim}} &= 1 - \frac{Np^N \left(p^N + \Delta p\right)^{-r-1}}{M \left(p^N + \Delta p\right)^{-r} + (N - M) \left(p^N\right)^{-r}} \\ &> 1 - \frac{(M-1)\left(\left(p^N\right)^{-r} - \left(p^N + \Delta p\right)^{-r}\right) + Np^N \left(p^N + \Delta p\right)^{-r-1}}{M \left(p^N + \Delta p\right)^{-r} + (N - M) \left(p^N\right)^{-r}}. \end{aligned}$$

Now, notice that

$$M \left(p^N + \Delta p\right)^{-r} + (N - M) \left(p^N\right)^{-r} = -M \left(\left(p^N\right)^{-r} - \left(p^N + \Delta p\right)^{-r}\right) + N \left(p^N\right)^{-r},$$

which is decreasing in M since $r \in [0, 1]$. Hence,

$$M \left(p^N + \Delta p\right)^{-r} + (N - M) \left(p^N\right)^{-r} < \left(p^N + \Delta p\right)^{-r} + (N - 1) \left(p^N\right)^{-r},$$

implying that

$$\begin{aligned} AL_M^{\text{sim}} &> 1 - \frac{(M-1)\left(\left(p^N\right)^{-r} - \left(p^N + \Delta p\right)^{-r}\right) + Np^N \left(p^N + \Delta p\right)^{-r-1}}{M \left(p^N + \Delta p\right)^{-r} + (N - M) \left(p^N\right)^{-r}} \\ &> 1 - \frac{(M-1)\left(\left(p^N\right)^{-r} - \left(p^N + \Delta p\right)^{-r}\right) + Np^N \left(p^N + \Delta p\right)^{-r-1}}{\left(p^N + \Delta p\right)^{-r} + (N - 1) \left(p^N\right)^{-r}} \\ &= AL_M^{\text{seq}}. \end{aligned}$$

Hence, because AL_M^{sim} and AL_M^{seq} are both decreasing in M, in contrast to the linear demand case, the market defined when employing a simultaneous method is wider than that obtained with a simultaneous method — i.e., $M^{\text{sim}} > M^{\text{seq}}$.

By the same token, under a logit demand function we have

$$\begin{aligned} AL_M^{\text{sim}} &= 1 - \frac{\exp\left(\left(s - \left(p^N + \Delta p\right)\right)/\mu\right)\chi}{1 + M \exp\left(\left(s - \left(p^N + \Delta p\right)\right)/\mu\right) + (N - M) \exp\left(\left(s - p^N\right)/\mu\right)} \\ &> 1 - \frac{(M-1)\left(\exp\left(\left(s - p^N\right)/\mu\right) - \exp\left(\left(s - \left(p^N + \Delta p\right)\right)/\mu\right)\right) + \exp\left(\left(s - \left(p^N + \Delta p\right)\right)/\mu\right)\chi}{1 + M \exp\left(\left(s - \left(p^N + \Delta p\right)\right)/\mu\right) + (N - M) \exp\left(\left(s - p^N\right)/\mu\right)}. \end{aligned}$$

Next, notice that

$$1+M\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-M)\exp\left(\left(s-p^N\right)/\mu\right)$$
$$=1-M\left(\exp\left(\left(s-p^N\right)/\mu\right)-\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)\right)+(N-M)\exp\left(\left(s-p^N\right)/\mu\right),$$

which decreasing in M. Hence,

$$\begin{aligned}&1+M\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-M)\exp\left(\left(s-p^N\right)/\mu\right)\\&<1+\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-M)\exp\left(\left(s-p^N\right)/\mu\right)\\&=1+\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-1)\exp\left(\left(s-p^N\right)/\mu\right).\end{aligned}$$

As a result we have

$$\begin{aligned}AL_M^{\text{sim}} &> 1-\frac{(M-1)\left(\exp\left(\left(s-p^N\right)/\mu\right)-\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)\right)+\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)\chi}{1+M\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-M)\exp\left(\left(s-p^N\right)/\mu\right)}\\ &> 1-\frac{(M-1)\left(\exp\left(\left(s-p^N\right)/\mu\right)-\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)\right)+\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)\chi}{1+\exp\left(\left(s-\left(p^N+\Delta p\right)\right)/\mu\right)+(N-1)\exp\left(\left(s-p^N\right)/\mu\right)}\\ &= AL_M^{\text{seq}}\end{aligned}$$

Hence, as before, because AL_M^{sim} and AL_M^{seq} are both decreasing in M, even with a logit specification, the sequential approach defines a relevant market narrower than a simultaneous approach. ■

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