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Acquisition, (Mis)use and Dissemination of Information: The Blessing of Cursedness and Transparency

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Acquisition, (Mis)use and Dissemination of Information: The Blessing of Cursedness and Transparency

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Abstract

This paper studies strategic interactions where players observe statistics of others' actions, focusing on: First, the endogeneity of the precision of such aggregate information as signals of the fundamental; and second, agents' well-documented difficulty in making inference based on such signals. We conduct our analysis in a beauty contest game with information acquisition, adapting cursed equilibrium to model agents limited ability to process aggregative information. To discipline information acquisition choices in this setting with incorrect information use, we define a novel notion of *cursed expectations equilibrium with information acquisition:* Agents assess the value of private information according to a *subjective envelope condition,* as they correctly anticipate their actions and (incorrectly) deem them optimal. We show that there is inefficiently low acquisition and use of private information in the rational benchmark due to an information dissemination externality. Despite suboptimal use, cursed agents rely more heavily on their private information which pushes information acquisition towards its efficient level and causes an initial increase in welfare. Transparency crowds out private information but always increases the endogenous precision of the aggregative signal and welfare, while other policy instruments can have paradoxical effects due to their interaction with cursedness. Finally, we explore the behavior and welfare of an atomistic rational agent playing against a cursed crowd and demonstrate that transparency may be an elitist policy.

Keywords: Information Acquisition, Transparency, Cursed Equilibrium, Information Dissemination, Aggregative Information.

JEL Classification: C72, D62, D83, D90, (E50).

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1 Introduction

Many economic decisions are taken in complex environments with interdependent payoffs and uncertainty both about fundamental states and the actions of others. Players receive information not only from public and privately acquired signals about fundamentals but also through aggregate statistics of the actions of other economic agents. Examples of such statistics comprise inflation estimates, the level of activity and number of infections during a pandemic, or transaction prices in a financial market. Learning about fundamentals based on those statistics, however, requires an understanding of how they disseminate the private information contained in the actions of others through equilibrium play. There is ample evidence that agents often fail to make this connection, as indicated by the winners curse and underinference in social learning, for example.[1] It is therefore crucial to take into account both the endogenous dissemination of information as well agents limited grasp in the analysis of such environments, in particular when evaluating policies that target the very availability of such aggregative information.

Towards this end we study a beauty contest game augmented with three keys features whose interaction is the basis of our results: an *aggregative signal*, i.e. a noisy observation of the average action which serves as a signal of endogenous precision about the fundamental; agents' limited understanding of the information contained in such signal, modeled by a notion of *cursed equilibrium* adapted from Eyster and Rabin (2005); and the fact that such biased agents have an *information acquisition* choice to make. We now explore the three features in some more detail.

Aggregative Signals and Transparency Aggregative signals are noisy observations of aggregate moments, like the average action. When used to infer fundamentals, their precision has i) an exogenous component reflecting the dispersion around the moment - arising, say, from errors in processing data and reporting the statistic - and ii) an endogenous component reflecting how informative the moment itself is about the state. In line with policy proposals falling under the banner of transparency, we adopt the moniker of transparency for *the inverse dispersion (precision) of the aggregative signal around the aggregate moment*. In other words, it is the exogenous component i) in the fundamental transposition we were referring to before. For example, a fully transparent financial market would be one where a trader knows the transaction price before submitting his order. A lower level of transparency would correspond to a market where traders have only noisy information about the transaction price, say they observe the current price at another similar market place or in the past.[2]

[1] For the winner's curse, see Kagel and Levin (2002) for a review of the experimental evidence; for social learning, see Weizsäcker (2010).

[2] It is worth noting from the beginning that we assume the aggregative signal to be public, i.e. that all agents in the economy observe the same realization. The situation we have in mind is that of a central authority having exclusive access to the set of actions chosen by each player inside a market. With those data she can perform statistical analysis (difficult because of missing data, imperfect reports, etc) and produce a report which will then be observed without further noise by everyone. Interpreted as the accuracy of the process turning actions into a report, transparency becomes a natural parameter for positive comparative statics as well as policy evaluation.

Cursedness Learning from multiple signals, some of which about endogenous objects, requires considerable skills: beyond mastering the techniques of Bayesian updating, agents need the awareness of operating in rich environment, as well as the ability to put themselves in their coplayers' shoes to make inference from equilibrium statistics. Ample empirical evidence show that many agents do not possess those skills: They suffer from the winner's curse, failing to appreciate that they are more likely to win a common value auction when their private information leads them to overestimate the value of the prize and dismiss market prices. This micro evidence suggests that a complete analysis of economic interactions in which said learning takes place has to accommodate for agents' limited grasp of the information contained in the action of others – and hence in aggregate statistics based on such actions. To this end we adapt cursed equilibrium Eyster and Rabin (2005), a parsimonious solution concept capturing the range from rational to fully cursed agents who fail to take into account that the actions of other players are a result of their private information and hence consider the aggregate outcomes to be uninformative about fundamentals. This concept has been used successfully to account for overbidding in common value auctions and has also found applications to financial markets (Eyster et al., 2019). In our story, cursedness and transparency serve as complementary antagonists: Transparency determines how much aggregative information is available and therefore can be ignored while cursedness represents the degree of ignorance. Without transparency there is no aggregative information and nothing to be cursed about, while at the other extreme, fully cursed completely ignore aggregative information and transparency is without effect.

Private Information Acquisition Aggregative signals disseminate the information contained in individual best response. Their precision ultimately depends on how much private information agents possess. Often times private information is not given exogenously, but has to be acquired at a cost. Then, the precision of private information becomes a function of its perceived equilibrium value – a crucial adjustment channel. As cursed agents misperceive the information environment, one needs to take a stand on how such agents estimate this value, i.e. if and to what extent they are aware of their future misuse. To the best of our knowledge, information acquisition by cursed agents has not been considered in the literature and devising such a notion while attaining the two goals of behavioral plausibility and tractability presents considerable challenges. We propose a notion of cursed expectations equilibrium with information acquisition that obeys the following behavioral assumption: Agents correctly anticipate their expected welfare and play, but they are not meta-rational, i.e. they do not consider their future information use to be erroneous. The latter implies that agents follow a subjective envelope condition which allows for a highly tractable analysis of the information acquisition problem.

We now preview our main results. As agents become more cursed, they increase the use of private information substituting away from the aggregative signal. This is intuitive as cursedness makes them perceive their environment as less informative and they need to rely more on their remaining information sources. The subjective envelope condition implies that the use of private information is a sufficient statistic for its acquisition which, and in the case of linear cost the two are directly proportional; it follows their comparative statics (in particular the fact it increases in cursedness) are the same. This acquisition channel provides a feedback loop: using and acquiring more information, cursed agents disseminate it more effectively. Nevertheless, this effect never dominates and their perceived precision of aggregative information remains decreasing.

If agents were rational, such a feedback loop would generate the famous price paradox: If the aggregative signal is noiseless, it is fully revealing about the state, which in turn causes agents to ignore their private information and to cease acquiring any thereby eliminating the very source of information in the aggregative signal. Therefore, no equilibrium can exist. We show that even an infinitesimal degree of cursedness is sufficient to interrupt this feedback loop and restore existence with a fixed level of private information: Even if the aggregative signal is fully revealing, such agents continue to rely on their private information. While cursed agents continue to use existing private information, its value shrinks to zero for agents that are approximately rational. Therefore, with information acquisition, existence requires either an Inada-type condition on costs, or, in the case of linear costs we embrace throughout the analysis, a sufficiently large degree of cursedness.

Confirming their intuitive dual nature, cursedness and transparency have opposed impacts on equilibrium loadings: For any degree of cursedness, an increase in transparency makes agents substitute from private information to the aggregate signal and crowds out private information acquisition. The crowding out effect never overturns the direct positive effect, and as transparency increases the aggregative signal becomes more informative about the state. This monotonicity does not necessarily hold for other measures of informational efficiency, such as total precision available to agents (that includes a direct account of their acquisition) or the realized covariance between the aggregate action and the state. Along the latter metric, – an indicator of the informational efficiency of the environment – cursedness improves the inflow of private information into the aggregative signal, but it hinders its extraction and thereby reduces the efficiency of dissemination. With information acquisition, these the forces balance exactly and the correlation between the aggregate action and the state is independent of cursedness and transparency. This provides a counterpoint to the observations that naive agents “lean against the wind”, thereby reducing information efficiency: While cursed agents still react less to aggregate information, they inject more.

As for welfare, the presence of an information dissemination externality makes the rational equilibrium loading on (and acquisition of) private information inefficiently low. If agents are cursed enough, however, they may use (and acquire) at or even above the efficient level, though they would simultaneously misuse signals from other sources. The presence of opposing efficiency effects of cursedness is confirmed quantitatively as welfare is non monotonic in the degree of cursedness; in particular, local to rationality the boost from improved dissemination induced by a marginal increment of cursedness is first order relative to misusing away from an efficient level. The fact that cursedness may be a bliss breaks the cursedness/transparency duality as the latter is instead shown to always increase welfare. This result contrasts with the ambiguous impact on welfare of both private acquisition costs and public information: An increase of public fundamental information has two equilibrium effects. First, agents reduce their use and acquisition of private information. Second, agents substitute between sources of public information, away from aggregative information and towards the fundamental source which is made more precise. We show that when strategic complementarities are sufficiently strong, the second effect is particularly important and causes the paradoxical comparative static.

Although their welfare increases with transparency, cursed agents fail to fully reap its benefits. A natural question that arises is how an agent that is able to extract all the information from the environment interacts with a cursed crowd. We address this question by studying the behavior of an atomistic rational agent – such as proverbial smart money in a financial market - facing equilibrium play in a market of cursed agents. Smart money benefits from the increased information acquisition of cursed agent, sometimes even abstaining from acquiring private information itself and living completely parasitically off their information acquisition. However, the motive to imitate the aggregate action forces the rational agent to follow the crowd and distort his actions away from the fundamental. Compared to welfare that would obtain against a rational market, low levels of cursedness are always beneficial for smart money (whose welfare can even exceed first best); however, at high levels of cursedness the imitation effect dominates in a game with sufficiently strong strategic complements, making excessive cursedness harmful. The information parasitism vs co-player miscoordination trade-off creates interesting comparative statics in policy parameters. Smart money always profits from transparency, but can be hurt by more public information and lower acquisition costs, even when they are beneficial for the cursed crowd. This analysis reveals a potential conflict of interest between rational and behavioral agents, possibly pointing at applications to the political economy of financial market regulation which we return to in the concluding remarks.

We conclude the introductory section by discussing the related literature. In Section 2 we present the model. We establish existence and uniqueness of a cursed equilibrium in Section 3, taking the precision of private information as given, and briefly discuss comparative statics results. We introduce information acquisition in Section 4, defining the notion of cursed expectations equilibrium with information acquisition. Section 5 analyses the positive comparative statics of the model. We turn to welfare analysis in Section 6. We analyze the optimal strategy and welfare of an atomistic rational agent in Section 7. Section 8 concludes.

1.1 Related Literature

Conducting our analysis within the workhorse class of linear quadratic models, we connect to a rich theoretical and applied literature. Models within this class (and those that exhibit similar best response structure) are used to investigate questions of information in a wide range of applications, ranging from in business cycles (e.g. Hellwig and Veldkamp, 2009; Angeletos and La, 2010; Benhabib et al., 2015), demand function competition (Vives, 1988, 2017), to political economy (e.g. Shadmehr et al., 2018).

The study of the social value of information in this setting has been initiated by Morris and Shin (2002), who show that more precise public information can reduce welfare in games with strategic complementarities as it leads to excessive coordination. Angeletos and Pavan (2007) characterize the inefficiencies of information use in a general linear-quadratic Gaussian game. Ui and Yoshizawa (2015) classify such games according to the welfare properties of additional public and private information. Colombo et al. (2014) study how private information acquisition affects the value of information, establishing a close link between efficient acquisition and efficient use. All these papers consider exogenous information, i.e. signals about the state. We analyze the value of information in the presence aggregative information, i.e. a signal about the average action which therefore provides information of endogenous precision about the state. Such a signal is studied by Bayona (2018) in a setting akin to Angeletos and Pavan (2007), establishing that this can leads to a dissemination inefficiency in the use of private information.[3] We analyze the interplay with information acquisition – indeed, our rational benchmark nests restrictions of Colombo et al. (2014), Bayona (2018), and their insofar unexplored meet – as well as agents limited understanding of aggregative information. This restriction of our payoff structure to the simple beauty contest game allows us to isolate the novel sources of inefficiency in our setting.[4]

The results of Morris and Shin (2002) have spurred extensive debate in the literature about the desirability of public information in particular in the context of central bank announcements.[5] This discussion has often been couched in the terminology of "anti-transparency" vs. "pro-transparency". This label does not correspond to our usage, as we reserve the word transparency for the precision of the public signal about the aggregate action.[6] Although we would argue that much of the information provided by central banks is aggregative in nature and explore the impact of such transparency at length, we also contribute to the original debate by demonstrating a novel channel based on cursed inference which can render public fundamental information undesirable. The issue of such endogenous information dissemination has been studied in the context of business cycles by Wong (2008) who show that increased transparency can be self-defeating as it reduces the information available to the central bank itself to learn about the state of the

[3] Amador and Weill (2012) show that with such a dissemination externality more public information can cause a decrease in welfare, even without interdependent payoffs.

[4] Indeed, since our specification of the beauty contest does not feature a dependence of individual utility on the variance of others' actions (contrary to Morris and Shin, 2002), it follows from Angeletos and Pavan (2007) and Colombo et al. (2014) that our equilibrium with rational agents has efficient use and acquisition of information in the absence of an aggregative signal. Inefficiencies, therefore, arise only from the presence of aggregative information and from the biased use induced by cursed updating.

[5] Svensson (2006), e.g., argues that the ratio of private to public precision required for the paradoxical welfare result is unreasonably high and Woodford (2005) calls into question the assumptions on strategic complementarity and welfare. The role of these assumptions is clarified and general conditions for such welfare results are given in Angeletos and Pavan (2007) and Ui and Yoshizawa (2015). Cornand and Heinemann (2008) instead analyze the extensive margin in public information provision and show that more precise public information is always desirable if it reaches the optimal fraction of agents.

[6] In the financial economics literature, enhanced transparency is sometimes conceptualized as the sharing of private signals between asymmetrically informed traders. Glosten and Milgrom (1985) observe that such asymmetry leads generically to a positive bid-ask spread and that returns are approximately realizable returns plus what the uninformed anticipate losing to the insiders. Chowdhry and Nanda (1991) consider a Kyle model in which risk-neutral market makers at each location engage in a Bertrand competition in price schedules and show the informativeness of prices increases with the number of markets and with the proportion of liquidity (uninformed) trading accounted for by small traders. Pagano and Volpin (2012) show that in choosing the degree of transparency issuers effectively face a trade-off between primary and secondary market liquidity since the reduction in adverse selection associated with enhanced transparency reduces liquidity in primary markets but increase it (even dramatically) in secondary markets. A notable exeption in this literature is Pagano and Röell (1996) who define transparency as the extent to which market makers can observe the size and direction of the current order flow, a notion that is much closer to that we use in this paper. They find that greater transparency generates lower trading costs for uninformed traders on average, although not necessarily for every size of trade.

economy, a mechanism that has been proposed in Morris and Shin (2005), and individual firms. Amador and Weill (2010) show that through a similar signal jamming channel public information can be welfare decreasing, as it reduces the informativeness of the price system thereby increasing uncertainty about the monetary shock.

Inference from a signal that aggregates information contained in individual best responses is also at the center of the literature on information aggregation in financial markets. Grossman and Stiglitz (1980) prove the impossibility of informationally efficient markets showing that, if prices perfectly revealed private information, then no equilibrium would exist (the price paradox); they also show that the equilibrium informativeness of the price system is unresponsive to changes in transparency: an increase in noise leads to higher returns from information acquisition that exactly offset the direct effect. Vives (2014) shows that the price paradox can be solved even without introducing noise traders provided there is sufficient heterogeneity in traders' valuation; we show that (sufficient) cursedness also solves the paradox in a representative agent model.[7] Vives (2017) shows that in games of demand function competition, there is both an information dissemination externality as well as a pecuniary externality, the latter causing excessive weight on private information.

Cursed equilibrium was proposed by Eyster and Rabin (2005) as a model of underinference from the actions of others to explain the winner's curse as well as trade in settings where BNE would predict a breakdown due to adverse selection. Eyster et al. (2019) apply a cursed analogue to rational expectations equilibrium in a LQG trading game and show that cursed behavior can explain excessive trade volume. We adapt cursed equilibrium to a beauty contest game with endogenous information augmenting it with an information acquisition stage. To the best of our knowledge, the present paper is the first to analyze information acquisition with cursed agents.

- Talk more about evidence.
- KoolMiddeldorpRosenkranz; NgangoueWeizsäcker; etc
- Alternative interpretation of our model: failure of conditional thinking (see above, also Esponda Vespa; game of action schedules in p, market structure (co)determining χ)

2 The Model

The game has two stages: First, agents choose how much private information to acquire. Second, agents play a beauty contest game. We begin our description of the setting with the second stage and treat the game with *exogenous* precision of private information in this and the following Section, and add information acquisition in Section 4.

[7] Although the rational benchmark in our setting does not nest the Grossman and Stiglitz (1980) specification (the aggregative signal is not payoff relevant and the information acquisition choice is continuous), both results reviewed above continue to hold.

Actions and Payoff

There is a unit interval of agents $i \in [0,1]$, playing a simple beauty contest game. Their payoff is given by

$$u(a_i, \bar{a}, \theta) = -(1-r)(a_i - \theta)^2 - r(a_i - \bar{a})^2, \tag{1}$$

where $a_i \in \mathbb{R}$ is the action of player i, $\bar{a} = \int_i a_i \, \mathrm{d}i$ is the average action[8] and $\theta \in \mathbb{R}$ is the state (or fundamental). We allow for both strategic complementarity ($r > 0$), and substitutability ($r < 0$), and assume that complementarity is not too strong ($r < 1$) to ensure existence of unique interior linear equilibrium and a planner solution. This payoff specification belongs to the family of quadratic utilities considered in LQN games where all of (cross) second moments of the joint state, action and average action distribution are considered. Our beauty contest specification would not to generate any inefficiency in use or acquisition in the rational benchmark studied respectively in Angeletos and Pavan (2007) and Colombo et al. (2014): as immediate corollaries of their results we would indeed obtain that the equilibrium is efficient along both channels. We however consider such simple specification to be a strength of our approach, since it allows us to isolate inefficiencies generated by the features specific to our information environment: the dissemination externality of aggregative information, and cursed updating from that source. Clearly, the analysis of the interplay between a richer payoff structure (which broadens the set of economic applications) and our information environment is a natural extension of this paper.

Signals and Inference

The following information structure is common knowledge. The state θ is drawn from the prior distribution $\mathcal{N}(0, \tau_\theta^{-1})$.[9] Agents receive three signals: a private signal $s_i = \theta + z_{s_i} \sim \mathcal{N}(\theta, \tau_s^{-1})$, i.i.d. across agents, a public signal $y = \theta + z_y \sim \mathcal{N}(\theta, \tau_y^{-1})$ about the state and a public aggregative signal $p = \bar{a} + z_p \sim \mathcal{N}(\bar{a}, \tau_p^{-1})$ where τ_p, the precision of the aggregative signal as a signal of $\bar{a}$, is our transparency parameter. We endogenize τ_s in the information acquisition stage.

The optimal action is given by

$$a(s_i, y, p) = \arg\max_{a_i} \mathbb{E}_i \left[u(a_i, \bar{a}, \theta)\right] \tag{2}$$

where $\mathbb{E}_i$ is the expectation operator wrt. agent i's information, including his updating biases. As u is quadratic, (2) takes the linear best response form

$$a_i = (1-r)\,\mathbb{E}_i(\theta) + r\mathbb{E}_i(\bar{a}) \tag{3}$$

Throughout, we focus on linear equilibria. That is, the optimal action rule takes the form

$$a_i = \alpha_0 + \alpha_1 s_i + \alpha_2 y + \alpha_3 p \tag{4}$$

[8] As is customary for simplicity, we assume a SLLN. All statements are to be understood almost surely.

[9] A prior mean of zero is merely a convenient normalization. We insist on a proper prior as we analyze the comparative statics of ex-ante welfare.

for some vector of loadings $\boldsymbol{\alpha}$. Then, we can write the true aggregate action as

$$\bar{a} = \int_0^1 a_i \, \mathrm{d}i = \delta_0 + \delta_1 \theta + \delta_2 y + \delta_3 p \tag{5}$$

with aggregate weights $\boldsymbol{\delta}$. Inspection of equation (5) makes clear that the aggregative signal p provides information of *endogenous precision* about θ. Indeed, under the assumption that $\delta_3 \neq 1$ (and conditionally on y), p is informationally equivalent to

$$\hat{p} = \frac{1-\delta_3}{\delta_1}\left[p - \frac{\delta_2}{1-\delta_3} y\right] = \theta + \frac{1}{\delta_1} z_p \sim \mathcal{N}\left(\theta, \frac{1}{\delta_1^2 \tau_p}\right) \tag{6}$$

The Bayesian posterior on θ can be written based on the three conditionally independent sources $(s, y, \hat{p})$ which determines the posterior on $\bar{a}$ through (5). The precision of the aggregative signal about the state, $\delta_1^2 \tau_p$, depends both on transparency τ_p and on the equilibrium δ_1.

Cursed Equilibrium

As a model of the failure to update from observing the action of others, we adopt cursed equilibrium. In this solution concept, agents are characterized by a parameter χ, the degree of cursedness, that ranges from $\chi = 0$ for rational benchmark to $\chi = 1$ denoting fully cursed behavior. A fully cursed agent fails to perceive any correlation between other agents' actions and their private information. Instead, a cursed agent thinks others play according to the marginal distribution of their actions conditional on his private information. Consequently, according to the beliefs of a fully cursed agent i

$$a_j = \mathbb{E}[a_j | \mathrm{I}_i] = \alpha_0 + \alpha_1 \mathbb{E}[\theta | \mathrm{I}_i] + \alpha_2 y + \alpha_3 p + \alpha_1 \left(s_j - \mathbb{E}[\theta | \mathrm{I}_i]\right) \tag{7}$$

where the α_k are the weights used in the linear strategy of player j. He treats the prediction error $s_j - \mathbb{E}[\theta | \mathrm{I}_i]$ as *independent* of the state. Therefore, in a linear symmetric equilibrium, for a fully cursed agent

$$\bar{a} = \delta_0 + \delta_1 \mathbb{E}[\theta | \mathrm{I}_i] + \delta_2 y + \delta_3 p \tag{8}$$

i.e. that the aggregate statistic is independent of θ conditional on his information. A fortiori, $\bar{a}$ and hence p do not provide additional information about the state.

Partially cursed agents are characterized by an interior level of cursedness $\chi \in (0, 1)$. They form expectations as a convex combination of rational and fully cursed ones, namely

$$\mathbb{E}_\chi[\theta | \mathrm{I}_i] = \chi \frac{\tau_y y + \tau_s s_i}{\tau_\theta + \tau_y + \tau_s} + (1-\chi) \frac{\tau_y y + \tau_s s_i + \delta_1^2 \tau_p \hat{p}}{\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p} \tag{9}$$

$$\mathbb{E}_\chi[\bar{a} | s_i, y, p] = \chi \left(\delta_0 + \delta_1 \frac{\tau_y y + \tau_s s_i}{\tau_\theta + \tau_y + \tau_s} + \delta_2 y + \delta_3 p\right) + (1-\chi)(\delta_0 + \delta_1 \theta + \delta_2 y + \delta_3 p) \tag{10}$$

Note that even cursed agents do all the updating about the state θ and then turns it into a belief about $\bar{a}$ through the equilibrium condition (5): they have "equilibrium awareness".

Where they go wrong is in the under-appreciation of the correlation between their private information and others' actions.

Cursed equilibrium is defined as a solution concept for Bayesian games. Due to our information structure, however, the model described so far is not a Bayesian game, strictly speaking: agents react to a signal that itself is an integral over actions.[10] We therefore adapt cursed equilibrium in a fashion similar to a linear rational expectations equilibrium:[11]

Definition 1. A vector of loadings $(\boldsymbol{\alpha}, \boldsymbol{\delta})$ constitutes a χ-*cursed expectations equilibrium* if $\boldsymbol{\alpha}$ satisfies the best response condition (3)-(4) with expectations formed according to 9-10 given $\boldsymbol{\delta}$; and the aggregate action is consistent with individual actions, $\boldsymbol{\delta} = \boldsymbol{\alpha}$.

3 Equilibrium Analysis

An equilibrium is computed by matching coefficients in the best-response function (3). We arrive at

Proposition 1. *For any τ_s, a unique χ-cursed equilibrium exists. It is given by*

$$\alpha_0 = \delta_0 = 0 \tag{11}$$

$$\alpha_1 = \delta_1 \tag{12}$$

$$\alpha_2 = \delta_2 = \frac{\delta_1^2 \tau_y}{(1-r)\tau_s - \delta_1\left(\tau_\theta + \tau_y\right)} \tag{13}$$

$$\alpha_3 = \delta_3 = 1 - \frac{\delta_1 (1-r)\tau_s}{(1-r)\tau_s - \delta_1\left(\tau_\theta + \tau_y\right)} \tag{14}$$

where $\delta_1 \in [0,1)$ is the unique real solution to

$$\delta_1 = [1 - r + r\delta_1] \frac{\tau_s}{\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p} \left(1 + \chi \frac{\delta_1^2 \tau_p}{\tau_\theta + \tau_y + \tau_s}\right) \tag{15}$$

In (15) the RHS denotes the optimal loading on private information given aggregate δ_1 and hence has a natural interpretation. First, the private signal is valuable for predicting the state, gaining weight $1-r$, as well as the aggregate action to the degree that it reflects the state (conditional on public signals), gaining weight $r\delta_1$. Second, the relative precision of the private signal is the usual Bayesian weight $\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}$. Hence, the private signal is ignored ($\delta_1 = 0$) only if it is pure noise ($\tau_s = 0$). This term also contains the information spillover effect: the more other agents use their private information (higher δ_1), the more can be learned from the aggregative signal, which reduces the weight on the private signal.[12] Third, the final term adjusts this weight as cursed agents fail to understand that the aggregative signal is informative about the state and therefore perceives the private signal to be *relatively*

[10] Alternatively, one could define an analogous game where agents submit reaction functions instead of actions, see Vives (2014).

[11] See also Eyster et al. (2019).

[12] Note that, because of the information use spillover, this is a game of strategic substitutes in δ_1 even for $r \approx 0$. Cursedness mitigates this effect.

more informative. In the extreme case of $\chi = 1$, the agent ignores the aggregative signal and the two final factors simplify to $\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}$, the relative precision of the private signal as if there were no aggregative information.

Cursedness has an impact on the equilibrium[13] only if there is an informative aggregative signal ($\tau_p > 0$); it is a pure updating bias. Absent an aggregative signal, cursed agents correctly understand all remaining signals and act just as rational agents. At first sight, this result may be surprising when compared with the implications of cursed equilibrium in a common value auction. In the auction, there is no aggregative information available to the agent before he chooses his action but still cursedness impacts his choice. This, however, is a natural consequence of the payoff structure: In an auction, the agent considers his payoff conditional on winning the auction, which is exactly such an aggregative conditioning event. In our model, payoffs weight all states equally ex-ante and there is no such "implicit conditioning" embedded in them. Hence, cursed equilibrium coincides with rational equilibrium absent an aggregative signal.

The fully rational case is easily obtained from Proposition 1 but doesn't create simple and immediately interpretable representations. It is analyzed in depth in Bayona (2018). We focus instead on the opposite polar case and obtain

Corollary (**Fully Cursed Equilibrium**). *The fully cursed equilibrium is*

$$\delta_1^{FC} = \frac{(1-r)\tau_s}{\tau_\theta + \tau_y + (1-r)\tau_s} \tag{16}$$

$$\delta_2^{FC} = \frac{\tau_y}{\tau_\theta + \tau_y + (1-r)\tau_s} \tag{17}$$

$$\delta_3^{FC} = 0 \tag{18}$$

The role of strategic substitutability and complementarity is directly apparent in the fully cursed equilibrium. If there are no such strategic interactions, cursed agents weigh the two signals at their (mental) disposal according to their precision. Strategic complementarity shifts weight away from the private signal s_i and towards the public signal, y, while substitutability has the opposite effect.

The fact that the fully cursed equilibrium puts no weight on the aggregative signal deserves a clarification. This does not follow from cursedness alone; indeed even for fully cursed agents, the aggregative signal, p, remains a valid source of the public fundamental signal, y, and of public noise, z_p. As those are relevant for coordination purposes, agents want to incorporate p into their best response as long as other do so, but with a weight lower than theirs. Hence, only as a result of the interplay between equilibrium and cursedness, do we obtain $\delta_3^{FC} = 0$.[14]

[13] Although both cursedness and transparency crucially affects all three loadings, they enter δ_2 and δ_3 only indirectly as summarized by δ_1 in this convenient representation of the equilibrium system.

[14] In a related paper, Vives (2017) considers limited inference, equivalent to fully cursed behavior, in a LQN model of competition in supply schedules with unknown costs. Fully cursed traders in his setting do not ignore the noisy signal of fundamentals, the price, as it is directly payoff relevant.

3.1 The Transparent Limit and the Price Paradox

An important special case is the limit as $\tau_p \to \infty$. In this *transparent limit*, agents observe not just a noisy signal, but the aggregate action itself and can condition their actions on it.

With fully rational agents, this leads to the classic price paradox: Whenever $\delta_1 \neq 0$, knowledge of $\bar{a}$ translates into knowledge of the state which makes it suboptimal to place any weight on the private signal and hence $\delta_1 = \alpha_1 = 0$. If instead $\delta_1 = 0$, agents respond with a positive weight on their private signals $\alpha_1 > 0$, which is inconsistent. Hence, no equilibrium exists.

Cursed agents, however, continue to put a positive weight on the conditionally uninformative signals s_i, y even if they observe a fully informative aggregative signal. Even an infinitesimal amount of cursedness, therefore, resolves the price paradox since it reintroduces residual uncertainty about θ that was wiped out by infinite transparency.[15] Therefore,

Proposition 2. *An equilibrium of the limit game exists if and only if $\chi > 0$. It is given by*

$$\delta_1^\infty = \frac{\chi\tau_s(1-r)}{\tau_\theta + \tau_y + \tau_s(1-r\chi)} \tag{19}$$

$$\delta_2^\infty = \frac{\chi^2(1-r)\tau_y\tau_s}{\left(\tau_\theta + \tau_y + \tau_s(1-r\chi)\right)\left((1-\chi)\left(\tau_\theta + \tau_y\right) + (1-\chi r)\tau_s\right)} \tag{20}$$

$$\delta_3^\infty = \frac{(1-\chi)\left(\tau_\theta + \tau_y + \tau_s\right)}{(1-\chi)\left(\tau_\theta + \tau_y\right) + (1-\chi r)\tau_s} \tag{21}$$

Moreover, the equilibrium of the game with finite τ_p converges to δ^∞ as $\tau_p \to \infty$.

As $\chi \to 0$, we have an exclusive reliance on the fully revealing signal p, as $\delta^\infty \to (0,0,1)$. Close to the rational limit, agents also neglect y in favor of their private signal $\frac{\delta_2^\infty}{\delta_1^\infty} \to 0$, as all coordination can be done by loading on p. As cursedness increases, agents substitute away from this fully revealing source and towards noisy fundamental signals. As shown in Figure 1, substitution towards the private signal dominates and coordination is still almost exclusively achieved through p at low degrees of cursedness, while the public signal gains importance at high degrees of cursedness.

Restoring existence of an equilibrium in the transparent limit reveals a key property of cursed equilibrium. Cursedness is different from dismissing part of the aggregative information, e.g. by scaling down its subjective precision. Indeed, such a scaling is powerless if the precision is infinite. Instead, cursed agents think that for any level of precision the information contained in the aggregative signal is not the whole story.

3.2 Comparative Statics

We study the impact of cursedness and transparency on the equilibrium loadings as well as on $\delta_1^2\tau_p$, the precision of the aggregative signal p as a signal about θ.

[15] Note that cursed agents also remain uncertain about $\bar{a}$: despite being told its realization, they follow updating rule (10) which generates the posterior about $\bar{a}$ from the posterior about θ. Somewhat paradoxically, this consistency of posteriors about $(\bar{a}, \theta)$ delivers this unintuitive implication for posteriors on $(p, \bar{a})$.

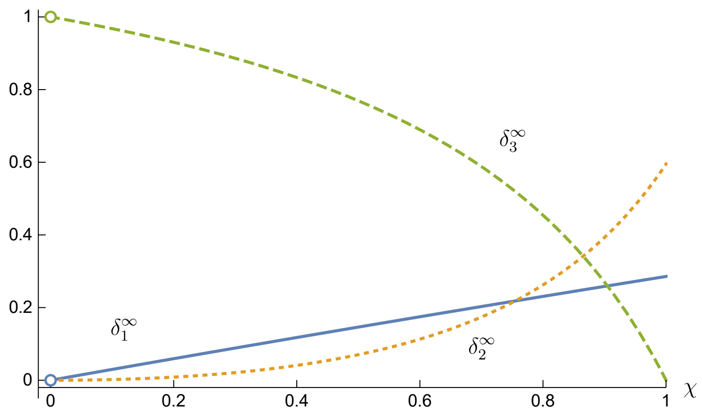

Figure 1: δ^∞ as a function of χ.

Proposition 3. *The comparative statics are given by*

$$\frac{\partial \delta_1}{\partial \chi} \geq 0, \qquad \frac{\partial \delta_2}{\partial \chi} \geq 0, \qquad \frac{\partial \delta_3}{\partial \chi} \leq 0, \qquad \frac{\partial}{\partial \chi}\frac{\delta_1}{\delta_2} \leq 0 \tag{22}$$

$$\frac{\partial \delta_1}{\partial \tau_p} \leq 0, \qquad \frac{\partial \delta_2}{\partial \tau_p} \leq 0, \qquad \frac{\partial \delta_3}{\partial \tau_p} \geq 0, \qquad \frac{\partial}{\partial \tau_p}\frac{\delta_1}{\delta_2} \geq 0 \tag{23}$$

All inequalities being strict if $\tau_p \neq 0$ *and* $\chi \neq 1$. *Furthermore,*

$$\frac{\partial}{\partial \tau_p} \delta_1^2 \tau_p > 0. \tag{24}$$

Cursed agents rely less on the aggregative signal as their behavioral bias makes them underappreciate its information content and substitute towards both private and public information. Even outside the transparent limit, cursedness shifts relative loadings on fundamental information in favor of the public signal ($\frac{\partial}{\partial \chi}\frac{\delta_1}{\delta_2} \leq 0$) irrespective of other parameters, in particular of the degree of complementarities r. This is because the public signal is a closer substitute to the aggregative one as both have a public noise component.

The mirror structure of comparative statics in Proposition 3 confirms the intuition that cursedness and transparency are complementary antagonists:[16] increasing the processing bias has qualitatively the same impact on equilibrium loadings as reducing the amount of information provided by this source. In particular, higher transparency decreases the loading on private information: As the private information of others is disseminated more effectively, I rely less on my own.[17] Nevertheless, this crowding out effect never dominates and the precision of the aggregative signal about the fundamental is always increasing in transparency.

[16] Non-triviality conditions provide a similar intuition; we already noticed that if $\tau_p = 0$, then χ disappears from equilibrium conditions in Proposition 1. Similarly, transparency has no impact on the loadings of the fully cursed equilibrium of Remark 3

[17] This already points to an inefficiency: Making it easier for information to be disseminated, the economy uses less private information and thereby endogenously dampens the impact of transparency.

Proposition 4. *The weight on private information respond to parameter changes as follows:*

$$\frac{\partial \delta_1}{\partial \tau_s} \geq 0, \qquad \frac{\partial \delta_1}{\partial \tau_y} \leq 0, \qquad \frac{\partial \delta_1}{\partial r} \leq 0 \tag{25}$$

The comparative statics of δ_1 are intuitive: Agents will rely more on the private signal as it becomes more precise relative to the public fundamental signal. As complementarities become stronger the public signals become more attractive relative to the private signal as they allow for better coordination with the aggregate action. Consequently, an increase in r decreases the weight on the private signal.

At the heart of the analysis is the loading on private information analyzed above, as it determines the endogenous precision of the aggregative signal. The other two loadings, δ_2 and δ_3 have an immediate interpretation as the weight given by the agent to the public fundamental and aggregative signal, respectively, but have to be interpreted with care. The agent loads on the public signal, for example, both directly through the public signal as well as indirectly through the aggregative signal. Hence, their comparative statics in τ_s, τ_y, r are ambiguous. We can rewrite the agent's equilibrium action in a fundamental representation

$$a_i = \underbrace{\frac{\delta_1 + \delta_2}{1 - \delta_3}}_{\beta} \theta + \delta_1 z_s + \underbrace{\frac{\delta_2}{1 - \delta_3}}_{\gamma_2} z_y + \underbrace{\frac{\delta_3}{1 - \delta_3}}_{\gamma_3} z_p \tag{26}$$

as a linear combination of the state and the shocks. The regression coefficient of the individual action (and hence also the aggregate action) on the state is denoted β. This parameter hence can be interpreted as a measure of informational efficiency of the equilibrium. The weights γ_2, γ_3 on the public shocks differ from the direct loadings on the signals by a factor of $\frac{1}{1-\delta_3}$, since the aggregative signal contains and amplifies both public shocks.[18] Using 13-15 one obtains

Proposition 5. *In equilibrium, the loadings in the fundamental representation (26) are*

$$\beta = 1 - \frac{\delta_1 \tau_\theta}{(1-r)\tau_s}, \qquad \gamma_2 = \frac{\delta_1 \tau_y}{(1-r)\tau_s}, \qquad \gamma_3 = \frac{1-\delta_1}{\delta_1} - \frac{\left(\tau_\theta + \tau_y\right)}{(1-r)\tau_s}.$$

Furthermore,

$$\frac{d\beta}{d\chi} < 0, \qquad \frac{d\beta}{d\tau_p} > 0, \qquad \frac{d\beta}{d\tau_s} > 0, \qquad \frac{d\gamma_2}{d\tau_y} > 0, \qquad \frac{d\gamma_2}{d\tau_s} < 0, \qquad \frac{d\gamma_2}{d\tau_p} < 0.$$

The responsiveness of the action to the state, β, is determined both by the use of private information and by the efficiency of its dissemination. If the precision of private information or the level of transparency increases, it rises unambiguously. An increase in cursedness increases private information use but hampers dissemination as cursed agents fail to learn from the aggregative signal. The latter effect dominates and the responsiveness decreases unambiguously. The comparative statics of γ_2 are intuitive: As the public fundamental

[18] The average action does not contain the agents private noise term z_s, whence the direct loading and the weight on the private shock coincide and we don't introduce a new variable γ_1.

signal becomes relatively more precise, agents substitute towards it.[19] The comparative statics of γ_3 are identical to those of δ_3 and are therefore ambiguous. For instance, consider the effect of an increase in τ_s. If τ_s is low, the information content of the aggregative signal is low as well and increases strongly together with τ_s. Therefore, agents substitute towards the aggregative signal and γ_3 is increasing in τ_s. If τ_s is large, agents already possess precise information about the state and the information content of the aggregative signal reacts relatively little to an increase in τ_s. Therefore, agents substitute away from the noisy aggregative signal and γ_3 is decreasing in τ_s.

4 Information Acquisition

In the first stage, agents simultaneously choose the precision of their private signal, τ_s, at cost $c\tau_s$. The crucial step to study this decision is deriving a representation for ex-ante welfare as a function of τ_s. This is tricky because cursed agents fail to understand the information environment. One natural candidate might be to consider agents as misspecified Bayesians and derive their ex-ante welfare according to these misspecified beliefs. Even though the conditional beliefs of agents are defined not via a misunderstanding of the signal structure, but directly based on a misunderstanding of the equilibrium relationship between others' private information and their play, we can derive such a quasi-Bayesian representation in our setting. This however, has a critical drawback. This representation would imply that – according to their ex-ante beliefs – cursed agents perceive that increased information acquisition by them also leads to more precise information for all other agents in the economy. Consequently, in the eyes of such a representation, the equilibrium responsiveness δ_1 itself is responsive to the individuals information choice or would need to be assumed not to be, leading to questions about higher order beliefs about the updating bias.[20] We consider this implication to be implausible and reject the idea that the quasi-Bayesian representation provides the uniquely plausible candidate as a subjective ex-ante welfare standard for cursed agents.

As an alternative, we define a notion of *cursed expectations equilibrium with information acquisition* based on three principles: First, cursedness is purely a bias of conditional thinking and inference, individuals are correct on average and unconditionally. Therefore, at the information acquisition stage, agents conceptualize their true ex-ante welfare as a function of parameters, their actions and their equilibrium conjecture. Second, cursedness is the result of a systematic tendency and not a systematic but unexpected mistake: agents correctly anticipate their information use, but they do not consider it to be erroneous.

[19] The comparative static of δ_2, by contrast, can be ambiguous. Take for example the impact of τ_s, where we can have $\frac{\partial \delta_2}{\partial \tau_s} > 0$: As δ_1 increases and agents substitute away from aggregative information, they desire to keep a similar loading on public information. Before, this was obtained as a byproduct of aggregative information, but now has to be used directly through δ_2. The transformation to γ_2 neutralizes this composition effect.

[20] Rearranging equation 9, χ- cursed expectations can be written as a Bayesian update from an information structure where the variance of aggregative noise z_p is $\frac{1}{(1-\chi)\tau_p}\left(1 + \frac{\chi \delta_1^2 \tau_p}{(\tau_\theta + \tau_y + \tau_s)}\right)$. This expression depends both on the equilibrium responsiveness δ_1 and on τ_s, the precision of private information acquired by agent i. In other words, when evaluating the impact of higher information acquisition agents would implicitly assume that everybody else also receives a more informative signal.

Therefore, agents believe that the cursed use of information is (individually) optimal and so value private information following a subjective envelope condition. In other words, agents do not try and fix their bias via information acquisition: they consider only the direct impact of more information holding their actions fixed. Third, at odds with the implication of a quasi-Bayesian representation, when evaluating the returns to private acquisition an agent holds coplayers acquisition and use fixed at their equilibrium values. All in all, in a cursed expectations equilibrium with information acquisition agents consider the gradient of true ex-ante welfare taking their future actions and the equilibrium relation as given.

We now proceed towards a formal definition of cursed expectations equilibrium with information acquisition. Assuming linear acquisition cost,[21] the true ex-ante welfare of an agent buying precision τ_s, then playing according to $\boldsymbol{\alpha}$ and facing an equilibrium $\boldsymbol{\delta}$ is given by

$$W(\boldsymbol{\alpha},\boldsymbol{\delta},\tau_s) = \mathbb{E}\left[-(1-r)(a_i-\theta)^2 - r(a_i-\bar{a})^2\right] - c\tau_s \tag{27}$$

$$= -\frac{\alpha_1^2}{\tau_s} - (1-r)\left(\left[\alpha_2+\delta_2\alpha_3\frac{1}{1-\delta_3}\right]^2\frac{1}{\tau_y} + \left[\alpha_3\frac{1}{1-\delta_3}\right]^2\frac{1}{\tau_p} + \left(\alpha_1+\alpha_2+\alpha_3\frac{1}{1-\delta_3}\{\delta_1+\delta_2\}-1\right)^2\frac{1}{\tau_\theta}\right) - c\tau_s \tag{28}$$

It follows that first stage problem reads

$$\max_{\tau_s} W(\boldsymbol{\alpha},\boldsymbol{\delta},\tau_s) \tag{29}$$

Assuming an envelope theorem for $\boldsymbol{\alpha}$, we get the first-order subjective envelope condition

$$\alpha_1^2\frac{1}{\tau_s^2} = c \tag{30}$$

The weight on private information in the best response, α_1, is a sufficient statistic for determining the marginal value of private information, even if agents are cursed. Cursedness only affects the calculus through $\alpha_1(\delta_1)$ and equilibrium. Even though the measure perceived by the agent is distorted by cursedness, he acts optimally according to this subjective representation. This envelope condition follows from the rational choice of a Bayesian agent, both in our setting and in the case without an aggregative signal but a more general payoff structure studied in Colombo et al. (2014). For the cursed case, we include the condition as part of our equilibrium notion.

Definition 2. A tuple $(\boldsymbol{\alpha},\boldsymbol{\delta},\tau_s)$ constitutes a χ-*cursed expectations equilibrium with information acquisition* if $(\boldsymbol{\alpha},\boldsymbol{\delta})$ constitute a χ-cursed expectations equilibrium given τ_s; and (α_1,τ_s) satisfy the subjective envelope condition 30.

[21] Some apologizing about the fact we are not curved but we are pretty straight and proudly so.

The subjective envelope condition 30 and equilibrium consistency give a linear dependence between τ_s and δ_1

$$\tau_s = \frac{\delta_1}{\sqrt{c}} \tag{31}$$

Taking account of endogenous information acquisition in the equilibrium condition 15, we arrive at

$$\delta_1 = [1 - r + r\delta_1] \frac{\delta_1}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)} \left(1 + \chi \frac{\sqrt{c}\delta_1^2 \tau_p}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)}\right) \tag{32}$$

This equation always has a trivial solution at $\delta_1 = 0$, but this solution does not necessarily correspond to an equilibrium of the game. Contrary to the game with exogenous τ_s, with endogenous information acquisition we need to ensure that agents are willing to acquire private information. We require that the best-response weight on private information (RHS) exceeds δ_1 local to $\delta_1 = 0$. This is the case if

$$\sqrt{c} \leq \frac{1 - r}{\tau_\theta + \tau_y} \tag{33}$$

or, equivalently, if the costs of acquiring information are sufficiently small compared to the benefit of information as evaluated at the trivial candidate equilibrium. These benefits depend on the precision of prior and public information $\tau_\theta + \tau_y$ and the relative value of public versus private information, as summarized by $1 - r$.[22] If this condition is not met, we are stuck in a corner solution with zero information acquisition, making the trivial solution of 32 which implicitly encodes complementary slackness into an equilibrium. We now summarize this discussion in the following

Proposition 6. *There exists a unique χ-cursed equilibrium with information acquisition. If $\sqrt{c} < \frac{(1-r)}{\tau_y + \tau_\theta}$, it is given by*

$$\alpha_0 = \delta_0 = 0 \tag{34}$$

$$\alpha_1 = \delta_1 \tag{35}$$

$$\alpha_2 = \delta_2 = \frac{\sqrt{c}\delta_1 \tau_y}{(1 - r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)} \tag{36}$$

$$\alpha_3 = \delta_3 = 1 - \frac{\delta_1 (1 - r)}{(1 - r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)} \tag{37}$$

$$\tau_s = \frac{\delta_1}{\sqrt{c}} \tag{38}$$

and $\delta_1 \in (0, 1)$ is the unique real solution to 32. Otherwise, we have a corner equilibrium with $\delta_1 = \delta_3 = \tau_s = 0$ and $\delta_2 = \frac{\tau_y}{\tau_\theta + \tau_y}$.

[22] If we instead assume convex costs with an Inada-type condition at zero, the analogue of 33 is always satisfied and we have an interior solution.

4.1 The Transparent Limit with Acquisition

An instructive special case is again the transparent limit $\tau_p \to \infty$. In this case, optimal information acquisition 38 implies that τ_s^∞ solves

$$\tau_s \sqrt{c} = \frac{\chi \tau_s (1-r)}{\tau_\theta + \tau_y + \tau_s (1 - r\chi)} \tag{39}$$

This equation always has a trivial solution with $\tau_s^\infty = 0$, which again constitutes an equilibrium only if condition 33 is violated. If the condition is satisfied, there exists a nontrivial solution for fully cursed agents, which again coincides coincides with equilibrium for interior τ_p (as transparency does not affect the fully cursed equilibrium). More generally, an interior limit equilibrium exists if

$$\chi > \sqrt{c} \frac{\tau_\theta + \tau_y}{(1-r)} \tag{40}$$

Contrary to the case without information acquisition, where an infinitesimal amount of cursedness was enough to overcome the price paradox and ensure the existence of a limit equilibrium, here we require a sufficiently large degree of cursedness.[23] If 33 holds but 40 is violated, neither the trivial nor an interior equilibrium exists. This non-existence is due to a problem different from the price paradox.[24] The classic price paradox would persist even if information is provided for free: Private information remains unused due to the abundance of aggregative information, which is of course impossible in equilibrium. Here, (partially) cursed agents are willing to use any private information they have, but the marginal value of acquiring it is smaller then its marginal cost.[25] Therefore they *do not acquire* (and a fortiori *cannot use*) any information, which is impossible in an informative equilibrium.

Proposition 7. *There exists an equilibrium in the transparent limit if and only if* $\chi > \sqrt{c}\frac{\tau_\theta + \tau_y}{(1-r)}$. *It is given by 36-38 with*

$$\delta_1^\infty = \frac{\chi (1-r) - \left(\tau_\theta + \tau_y\right) \sqrt{c}}{(1 - r\chi)} \tag{41}$$

Close to rationality, we now have an existence problem. However, for χ converging to its lower bound $\sqrt{c}\frac{\tau_\theta+\tau_y}{(1-r)}$, we still have an almost exclusive reliance on the aggregative signal $\delta^\infty \to (0,0,1)$. Again, as cursedness increases, the reliance on the aggregative signal vanishes and the loadings on the other two signals increase. Comparing Figure 2 to the case without information acquisition (Fig. 1), however, we see that there is no crossing between δ_1, δ_2 as their ratio is determined independently of χ and can be larger or smaller than one dependent on other parameters.

[23] This does not contradict the sufficiency of condition 33 which holds for any interior τ_p. Here, we are in a situation where $\tau_p = \infty$; the order of limits is relevant, as for every fixed τ_p, the influence of transparency vanishes as δ_1 and τ_s go to zero.

[24] Unless we are in the rational case, when we have nonexistence even fixing τ_s exogenously.

[25] Indeed, existence of the transparent limit is guaranteed if the cost of information acquisition satisfies an Inada condition at zero as the generic condition reads $\sqrt{c'(0)} < \frac{\chi(1-r)}{\tau_\theta+\tau_y}$.

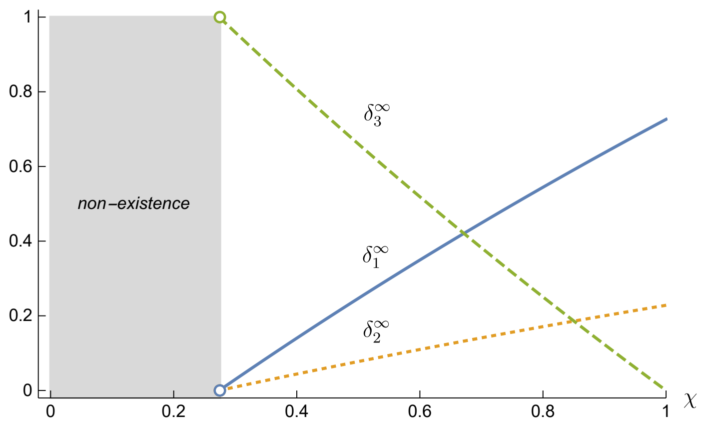

Figure 2: δ^∞ with information acquisition as a function of χ.

5 Comparative Statics with Information Acquisition

The information acquisition channel introduces confounding forces to the comparative statics of Section 3. Consider a parameter change that - holding acquisition fixed - would increase δ_1 (for example, higher cursedness). Since precision of the aggregative signal increases with δ_1, such modification would, among other things increases the amount of information and through a quantity effect depresses the value of private information. But now agents can respond to this shrink in value by reducing their precision τ_s thus further depressing the informativeness of this source and their willingness to use it. This cycle puts downward pressure on the use of private information as δ_1 goes up and produces an attenuating force that has to be quantified in the comparative statics. Immediately by inspecting the envelope condition (30), however, we see that in equilibrium τ_s and δ_1 are tightly linked by a proportionality relation so this feed back loop never dominates. Higher use goes always hand in hand with higher acquisition as δ_1 is a sufficient statistic for the gains from acquiring information. Also, equation (30) implies that the comparative statics of these two variables are proportional.[26]

Proposition 8. *It holds that*

$$\frac{\partial \delta_1}{\partial c} < 0.$$

The comparative statics wrt. other parameters in Propositions 3 and 4 continue to hold with endogenous information acquisition.

The comparative statics of τ_s have the same sign as the comparative statics of δ_1, namely

$$\frac{\partial \tau_s}{\partial c} \leq 0, \qquad \frac{\partial \tau_s}{\partial \tau_y} \leq 0, \qquad \frac{\partial \tau_s}{\partial \tau_p} \leq 0, \qquad \frac{\partial \tau_s}{\partial r} \leq 0 \tag{42}$$

The precision of privately acquired information has the expected comparative statics: it is reduced by both higher cost and more precise alternative sources, irrespective of the

[26]The result that $\frac{d\tau_s}{d\beta} \propto \frac{d\delta_1}{d\beta}$ where we need to pay someone to find a better symbol than β continues to hold with a generic cost of information acquisition subject to a second order condition.

degree of cursedness (at least qualitatively). The effect of cursedness and transparency are preserved qualitatively and amplified: An increase in cursedness, for instance, does not only affect δ_1 through information use, but also causes an increase in information acquisition, further increasing δ_1. In particular, the endogenous precision of the aggregative signal about the state, $\delta_1^2\tau_p$, is still increasing in transparency (see Figure 3). The mirrored roles of transparency and cursedness – providing aggregative information and dampening its processing – are still in place and lead to opposed comparative statics in these two parameters.

As discussed in the case without information acquisition, the comparative statics of δ_2 result are often ambiguous since it captures only part of the fundamental loading on the public fundamental signal. We hence return to the fundamental representation of a_i derived in 26.

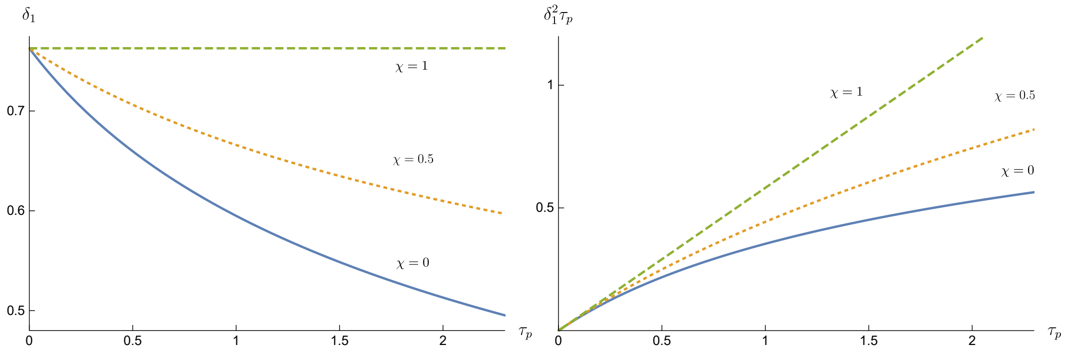

Figure 3: Crowdout vs Reliance on Private Information: The effect of τ_p on δ_1 and $\delta_1^2\tau_P$ for different levels of cursedness.

Proposition 9. *In an equilibrium with information acquisition, the loadings in the fundamental representation (26) are*

$$\beta = 1 - \frac{\sqrt{c}\tau_\theta}{1-r}$$
$$\gamma_2 = \frac{\tau_y\sqrt{c}}{1-r}$$
$$\gamma_3 = \frac{1}{\delta_1}\left(1 - \delta_1 - \frac{\tau_\theta + \tau_y}{1-r}\sqrt{c}\right)$$

The comparative statics of β and γ_2 are immediate. γ_3 is decreasing in cursedness and increasing in transparency.

The loading on aggregative noise, $\gamma_3 = \frac{\delta_3}{1-\delta_3}$, is decreasing in cursedness and increasing in transparency: If they consider it to be less informative, either because they are more cursed or because the environment is less transparent, agents put lower weight on it. The comparative statics of γ_3 in the other parameters $\tau_y, \tau_\theta, r, c$ are instead ambiguous. Take, for example, the comparative static with respect to c. On the one hand, an increase in c reduces information acquisition and makes agents substitute towards the aggregative signal. On the other hand, the reduction in information acquisition and reliance on the private signal removes the very basis of information in p, making it less attractive. Depending on other parameters, either effect can dominate. Similar intuition is the basis for the ambiguous comparative statics also in the other parameters: when they change, the precision of the aggregative signal changes both in level and relative to the precision of other signals. Note, however, that in the transparent limit, the comparative statics of γ_3, δ_3 are unambiguously signed.[27] It is increasing in τ_y, increasing in r, increasing in c. The first two stem from the fact that without additional noise, the aggregative signal is a good source of public information as well: Agents substitute away from δ_1 towards the two public signals and as there is no reason to substitute from p, δ_3 increases as the agent splits his weights optimally.

The state action regression coefficient β is decreasing in costs, the prior precision and the degree of complementarity; it does not depend on either cursedness or transparency. Recall from Proposition 5 that β is increasing in τ_p and decreasing in χ when the acquisition channel is shut down. By allowing τ_s to adjust downward following the reduced perceived value of private information caused by either a more transparent environment or a decrease in cursedness, those effects are neutralized. The resulting invariance property has two consequences of economic relevance: first, we cannot identify the degree of cursedness in a market by just looking at the responsiveness of individual actions to fundamentals; second, transparency is an ineffective tool at increasing the informational efficiency along the β metric.[28]

Since information acquisition stabilizes the factor $\frac{\delta_1}{\tau_s}$ to $\sqrt{c}$, γ_2 is now pinned down by the triplet τ_y, r, c with intuitive comparative statics: agents load more on the public fundamental signal if it is more precise, if private acquisition is more costly, or if the coordination motive is stronger.[29] The degree of cursedness does not affect γ_2, not even indirectly. Even though cursed agents fail to process all information disseminated through the aggregative signal, when they can adjust τ_s, their increased demand for and use of private information precisely offsets the less efficient inference.

While cursedness does not affect the total weight on information obtained through private signals, $\beta - \gamma_2$, it changes the composition: agents substitute from indirect inference of disseminated private information, δ_3, towards information agents have acquired themselves, $1 - \delta_3$. Such decomposition is apparent in the following rewriting of (26)

$$a_i = (\beta - \gamma_2)[(1 - \delta_3)(\theta + z_s) + \delta_3 \theta] + \gamma_2 y + \gamma_3 z_p \tag{43}$$

and is depicted in Figure 4.

[27] But not of δ_2!

[28] The impact of transparency on other "information efficiency" is explored in Section

[29] For δ_2, however, these comparative statics can be obscured. To see this most clearly, consider the transparent limit. If $\chi = 1$, $\delta_2 = \gamma_2$ and we have the above intuitive comparative statics. At the lower bound of χ required for the existence of a limit equilibrium, we have $\delta_3 \to 1$ and the indirect component of the weight on y dominates the consideration. Therefore, the comparative statics of δ_2 itself are reversed.

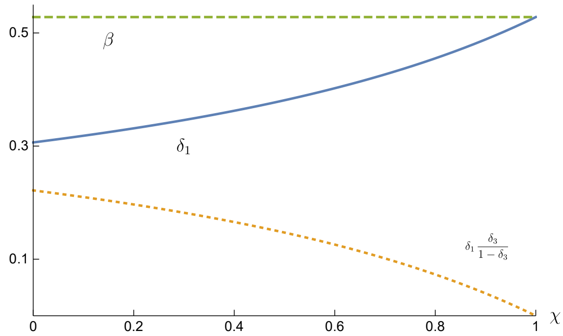

Figure 4: Equilibrium weight on information from private signals: direct and indirect.

5.1 Total Precision

We now address the impact of transparency on informational efficiency of the market.[30] Towards this end we need to establish a metric and quantify the impact on such measure of increased transparency, once the induced crowding out of private information use and acquisition is taken into account. We focus on metrics that try to capture the quantity of information in the market without explicitly linking it to the welfare of agents, which will be the subject of the next section. A natural candidate among those positive metrics is the state/action covariance term β which we showed to be independent of transparency when (and only when) we open the acquisition channel.[31] However β combines acquisition and use of information and therefore it might not be the ideal metric to assess the informativeness of the environment. So we also study the impact of τ_p on $\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p$, the total precision about the state that agents possess at the second stage.[32] We showed that both with and without information acquisition the endogenous precision of the aggregative signal, $\delta_1^2\tau_p$, is increasing in transparency. If the acquisition channel is present, however, private information τ_s also decreases with τ_p thus making the impact on total precision ambiguous at least qualitatively. To quantify the counteracting effects it is convenient to study the following factorization of total precision (obtained by manipulating 32)

$$\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p = \left(\frac{1-r+r\delta_1}{\sqrt{c}}\right)\left(1+\chi\frac{\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s}\right) \tag{44}$$

Recall that δ_1 is decreasing in τ_p while $\delta_1^2\tau_p$ is increasing. Thus the first factor in (44) is increasing in transparency if and only if actions are substitutes ($r < 0$), while the second factor is always increasing, flat only if $\chi = 0$. Therefore in the rational benchmark transparency increases total precision if actions are substitutes and decreases it if actions are

[30]A similar question is addressed in MS05 that show that, by publishing a more detailed statistic a central bank can make such statistic more noisy because of endogenous response by rational agents. Similar question is addressed in Amador and Weill (2010) that also find positive impact of public info. This can be seen as an exercise in evaluating the efficacy of transparency on the "informational efficiency" of a market like in the MIFid banner.

[31]Such unresponsiveness may echo GS point 4 even though the had a different setting.

[32]This has a clear meaning for rational agents, it is more tricky for cursed agents since they do not realize that this is an object: this is why along this measure we sacrifice the "use significance". As for an authority, in same spirit as MS, it would be as if they can take over from the agents but only at the decision stage, so not much meaning.

complements. Moreover, substitute actions induce unambiguously positive impact even for interior degrees of cursedness. In a game of complementarities, however, perverse impact of transparency can still emerge, though cursedness increases the range of parameters where transparency is desirable by scaling up the second factor. This is intuitive since cursedness dampens the crowding out of agents' private information as a response to the increase in transparency. The following Proposition summarizes this discussion.

Proposition 10. *The total precision available the agents is either*

- *increasing in* τ_p *for all* r, *or*
- *increasing in* τ_p *if and only if* $r < R(\chi)$, *for a cutoff* $R(\chi)$ *with* $R(0) = 0$ *and* $R' > 0$.

Since R is a monotonic function, an equivalent restatement of Proposition 10 is that, for each fixed r there exists a threshold level of cursedness $\bar{\chi}$ (trivial for $r \leq 0$) such that transparency increases total precision if and only if $\chi > \bar{\chi}$. In particular, a fully cursed economy always falls into the first bullet point as there is no crowd out of private information acquisition and use. Figure 5 shows, in an economy with substantial cursedness, the effects on total precision of both τ_p and τ_y as a function of the strategic complementarity parameter r.[33] It shows that, when cursedness is large, then public information about the aggregate action becomes more effective at enhancing total precision (relative to information about fundamental) as the crowding out effect is shut down. As agents becomes more cursed, however, this metric is less and less relevant as a welfare measure since they cannot grasp the gains from increased informativeness. The welfare analysis performed in the next section is therefore more suited to tackle efficiency questions.

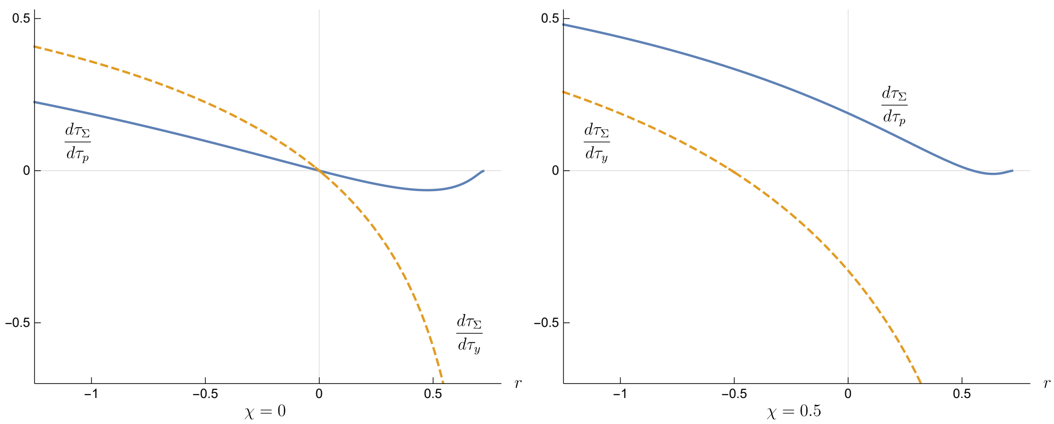

Figure 5: The effect of τ_p, τ_y on τ_Σ in the rational (left) and partially cursed ($\chi = 0.5$, right) model.

6 Welfare

We now turn to normative analysis. First we characterize the first best benchmark use and acquisition of information; then we identify and characterize the inefficiencies of the

[33] As for the impact of public information, the response of total precision has a similar but opposite pattern, as show in Figure 5.

χ−cursed equilibrium with information acquisition introduced in Section 4; finally, we perform welfare comparative statics.

6.1 The Planner Problem

As a welfare benchmark, we consider the problem of a planner who controls both the use of and acquisition of information, but cannot share information across agents.[34] To this end we impose the consistency condition $\boldsymbol{\alpha} = \boldsymbol{\delta}$ in the welfare expression (27) and, with slight abuse of notation, employ $W(\boldsymbol{\delta}, \tau_s) := W(\boldsymbol{\delta}, \boldsymbol{\delta}, \tau_s)$ as the objective function of a planner choosing

$$\left(\boldsymbol{\delta}^\star, \tau_s^\star\right) = \arg\max_{(\boldsymbol{\delta},\tau_s)} W(\boldsymbol{\delta}, \tau_s) \tag{45}$$

where simple calculations show

$$W(\boldsymbol{\delta}, \tau_s) = -\frac{(1-r)}{(1-\delta_3)^2}\left\{\frac{\delta_2^2}{\tau_y} + \frac{\delta_3^2}{\tau_p} + \frac{(1-\delta_1-\delta_2-\delta_3)^2}{\tau_\theta}\right\} - \frac{\delta_1^2}{\tau_s} - c\tau_s \tag{46}$$

We proceed characterizing the solution of (45)

Proposition 11. *The efficient linear action rule satisfies*

$$\delta_2^\star = \frac{\tau_y(1-\delta_1^\star)}{\tau_\theta + \tau_y + \tau_p\delta_1^\star} \tag{47}$$

$$\delta_3^\star = \frac{\delta_1^\star(1-\delta_1^\star)\tau_p}{\tau_\theta + \tau_y + \tau_p\delta_1^\star} \tag{48}$$

$$\left(\frac{\delta_1^\star}{\tau_s^\star}\right)^2 = c \tag{49}$$

where $\delta_1^\star$ is the unique solution of

$$\delta_1 = (1-r+r\delta_1)\,\tau_s^\star \frac{1}{\underbrace{\left(\frac{\tau_\theta+\tau_y+\tau_p\delta_1^2}{\tau_\theta+\tau_y+\tau_p\delta_1}\right)}_{\textit{efficiency wedge}}\left(\tau_\theta+\tau_y+\tau_p\delta_1^2\right)+\tau_s^\star} \tag{50}$$

Condition (50) corresponds to the rational equilibrium condition (15) modified by an efficiency wedge whose interpretation is straightforward: Using public information as the basis of action dilutes the dissemination of private information. The planner internalizes this effect and therefore downweighs public information by the adjustment term $\frac{\tau_\theta+\tau_y+\tau_p\delta_1^2}{\tau_\theta+\tau_y+\tau_p\delta_1} \leq 1$, which would be equal to one only if[35] $\delta_1^\star = 1$ and therefore $\delta_2^\star = \delta_3^\star = 0$, i.e. if the aggregative signal is not polluted by public signals to begin with. This observation has two economically relevant implications. Firstly, the efficient solution features a higher weight on private

[34] This is the benchmark customarily adopted in the literature starting from Angeletos and Pavan (2007). It avoids the unfair comparison with an economy in which agents can also share information: as there are uncountably many, this would coincide with playing a coordination game of complete information which has trivial solution and welfare properties.

[35] We always maintain a non-degeneracy assumption $\tau_p > 0$; if there is no dissemination the model is trivial

information compared to the rational equilibrium, as we formally establish in Proposition 14.[36] Secondly, since one can easily rule out a $(1,0,0)$ rational equilibrium, we conclude there is no efficient equilibrium with $\chi = 0$. As for positive level of cursedness, inefficiency of equilibria is an immediate consequence of the processing bias. Therefore, the equilibrium is always inefficient.

The optimality condition for τ_s is our familiar envelope condition: The use of private information is a sufficient statistic for the gains from acquiring it, even for the planner. Both efficient and equilibrium information acquisition are fully determined by the respective use of private information.

Proposition 12. *There is under-(over)acquisition of private information in equilibrium if and only if there is under- (over)use of private information in equilibrium, i.e.*

$$\text{sgn}\{\tau_s - \tau_s^\star\} = \text{sgn}\{\delta_1 - \delta_1^\star\}. \tag{51}$$

In particular, information acquisition in equilibrium is efficient if an only if information use in equilibrium is efficient.

Before turning to other inefficiencies of equilibrium we record first best comparative statics. Plugging the optimality conditions 47-49 in the welfare expression (46) we get

$$W^\star := W\left(\delta^\star, \tau_s^\star\right) = \max_{\delta_1} -2\sqrt{c}\delta_1 - \frac{(1-r)(1-\delta_1)^2}{\tau_\theta + \tau_y + \delta_1^2 \tau_p} \tag{52}$$

The comparative statics of first-best welfare now follow easily from an envelope argument.

Proposition 13. *We have*

$$\frac{dW^\star}{d\tau_\theta} > 0, \quad \frac{dW^\star}{d\tau_y} > 0, \quad \frac{dW^\star}{d\tau_p} > 0, \quad \frac{dW^\star}{dc} < 0$$

As information is used efficiently by the planner, increasing precision - whatever the source - or lowering acquisition costs always increases first best welfare.

6.2 The Inefficiencies of Equilibrium

In equilibrium, by contrast, information is generally used inefficiently: agents do not internalize the dissemination externality and they are subject to processing bias that makes them misuse available information. Proposition 12 establishes the connection between use and acquisition inefficiencies. We now find conditions for equilibrium to display underacquisition.

Proposition 14. *The rational equilibrium always has inefficiently low information acquisition. There is a cutoff transparency $\overline{\tau}_p$ such that sufficiently cursed agents acquire information above*

[36] The ratio of δ_2 to δ_3 is the same in the efficient action rule and in the rational equilibrium. While agents in the rational equilibrium underuse and underacquire private information, the relative weights on public aggregative and fundamental information are efficient. This changes for the cursed equilibrium, since $\frac{d}{d\chi}\frac{\delta_2}{\delta_3} > 0$ (Proposition (8)). Whenever agents are cursed, they overweigh fundamental information over aggregative information.

$\tau_s^\star$. *It is given by*

$$\overline{\tau}_p = \left(\tau_\theta + \tau_y\right) \frac{1 - 2\delta_1^{\mathrm{FC}}}{\left(\delta_1^{\mathrm{FC}}\right)^3} \tag{53}$$

Since rational agents do not internalize the dissemination externality, the equilibrium with $\chi = 0$ features underacquisition. As τ_s is increasing in χ by Proposition 4, cursedness alleviates this inefficiency. This effect can be strong enough to lead to overacquisition relative to the efficient benchmark if the aggregative signal is sufficiently precise. Intuitively, if transparency exceeds the lower bound 53, then dissemination is so effective that even $\tau_s^\star$ (which is independent of χ) is low compared to the amount of information acquired by the agent in the fully cursed equilibrium (which is by construction independent of τ_p). If transparency exceeds the cutoff, there exists an interior χ such that the equilibrium use and acquisition of private information coincide with the efficient quantity.[37] Even in this case, however, agents misperceive the information environment hence misuse their information at the second stage. To analyze this source of inefficiency, consider the gradient of welfare 46 at equilibrium as we vary the cursedness parameter (see also Figure 6).

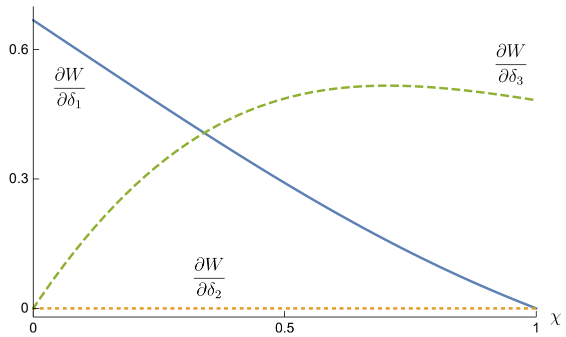

Figure 6: The gradient of W in equilibrium as a function of χ.

Proposition 15. *In a rational equilibrium, δ_3 is conditionally efficient: $\frac{\partial W}{\partial \delta_3}(\boldsymbol{\delta}, \tau_s) = 0$, for $\chi = 0$.*
In a fully cursed equilibrium, δ_1 is conditionally efficient: $\frac{\partial W}{\partial \delta_1}(\boldsymbol{\delta}, \tau_s) = 0$, for $\chi = 1$.
In any equilibrium, δ_2 is conditionally efficient: $\frac{\partial W}{\partial \delta_2}(\boldsymbol{\delta}, \tau_s) = 0$, for all χ.

In a fully rational equilibrium, the only externality is the dissemination of private information. Fixing the use of private information and thereby its dissemination, the other loadings of the equilibrium are conditionally efficient. In a fully cursed equilibrium agents ignore the aggregative signal altogether, so there is no dissemination externality and private information is used efficiently.[38] Independently of the degree of cursedness, there is no externality or misunderstanding in the use of the public fundamental signal. Cursedness

[37] The cutoff is always met ($\overline{\tau_p} < 0$) if incentives for private information acquisition are sufficiently high, namely $\frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{1-r} \leq \frac{1}{2}$.

[38] Again, recall that fully cursed equilibrium coincides at the action stage with fully rational equilibrium in which τ_p is set to zero. Without aggregative signal our model is a special case of Angeletos and Pavan (2007) where payoffs satisfy the conditions for efficient use of information.

shifts the source used to load on this information from p to y, but given δ_3 the loading on y is efficient.[39]

6.3 Equilibrium Welfare Comparative Statics

The sources of equilibrium inefficiency identified in Propositions 14 and 15 provide the bedrock for analyzing the impact of cursedness and changes in the information environment on equilibrium welfare. For expositional compactness we define the equilibrium welfare of agents as a function of the structural parameters of our model. Namely we perform comparative statics on the function

$$W^{EQ}\left(\tau_y, \tau_\theta, \tau_p, c, r, \chi\right) := W\left(\delta^\chi, \tau_s^\chi\right)$$

where we introduce δ^χ, τ_s^χ as shorthand for the equilibrium with χ-cursed agents, clearly as a function of the structural parameters $\left(\tau_y, \tau_\theta, \tau_p, c, r, \chi\right)$.

Cursedness is Bliss

Suppose we start from the rational equilibrium and consider a marginal increase in cursedness. It has two impacts on welfare. First, agents now use their information suboptimally as they underestimate the information contained in p. The associated welfare reduction is however second order as δ is privately optimal in the rational equilibrium. Second, cursed agents acquire and disseminate more information. Since the rational equilibrium features underacquisition, this impact on the dissemination externality has first order effect on welfare. Thus, local to rationality and up to first order, cursedness only has beneficial effects on welfare. When χ is already large, however, marginal increments in cursedness have first order negative effects from additional misuse, while the underacquisition gap is be narrower if existent at all: more cursedness harms agents.

Proposition 16 (Cursedness is Bliss)**.**

$$\left.\frac{dW^{EQ}}{d\chi}\right|_{\chi=0} > 0$$

Furthermore,

$$\left.\frac{dW^{EQ}}{d\chi}\right|_{\chi=1} < 0$$

so any level of cursedness maximizing equilibrium welfare must be interior.

The inefficient use of information dominates close to full cursedness, as is intuitive: fully cursed agents no not use the information contained in the aggregative signal, so providing more disseminated information to them is not very valuable. The shape of welfare as a function of χ and the comparison to efficient welfare is shown in Figure 7. One might wonder if the comparison in the plot holds in general or whether a fully cursed economy

[39] This is the case since absent information dissemination, in.

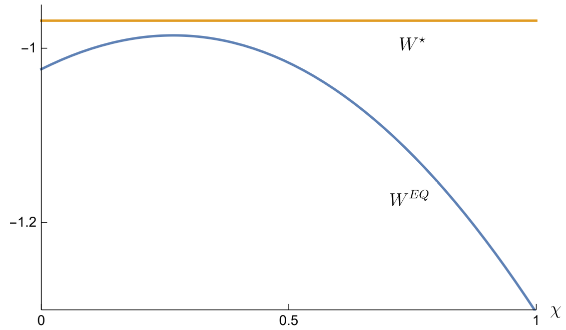

Figure 7: Cursedness is Bliss

can ever outperform full rationality. This cannot happen. Indeed, it is easy to show that in the two extreme cases $\chi \in \{0,1\}$, welfare takes the simple form[40]

$$\mathrm{W}^{\mathrm{EQ}} = -\sqrt{c}\,(1+\delta_1). \tag{54}$$

and it follows from δ_1 comparative statics that the fully cursed equilibrium always has lower welfare than the rational case: Even though acquisition and dissemination of private information are higher, cursed agents are unable to make any use of their aggregative information. The inefficiently low aggregative information provided in the rational equilibrium is preferable to complete ignorance of – albeit plentiful – aggregative information.

Remark (Welfare in the Transparent Limit). Contrary to the efficient solution, which does not admit a transparent limit equilibrium, the welfare formula 46 remains valid. In the transparent limit, which exists for $\chi \geq \sqrt{c\frac{\tau_\theta+\tau_y}{1-r}}$, equilibrium welfare is given by

$$\mathrm{W}^{\mathrm{EQ}} = -\frac{2\sqrt{c}\chi\,(1-r)^2 - c\,(1-2r+r\chi)\left(\tau_\theta+\tau_y\right)}{(1-r)\,(1-\chi r)} \tag{55}$$

It is easy to show that $\frac{\partial \mathrm{W}^{\mathrm{EQ}}}{\partial \chi} < 0$, seemingly overturning the result that cursedness is bliss. This is not the case since the bliss result holds local to $\chi = 0$ but the transparent limit equilibrium only exists for χ sufficiently large.

Precision Comparative Statics

In contrast to the efficient solution (Proposition 13), more information and lower costs do not always increase welfare in equilibrium.

Proposition 17. *If χ is local to either 0 or 1 or τ_p is sufficiently small, W^{EQ} is increasing in τ_y, τ_θ and decreasing in c. If, however, τ_p is sufficiently large, there exist an interior region of χ such that equilibrium welfare is*

- *decreasing in τ_y, τ_θ if strategic complementarities are sufficiently strong ($r > \frac{1}{2}$),*
- *and increasing in c in the game with strategic substitutes ($r < 0$).*

[40] Note that we need to be careful, with these representations, as they cannot be used for comparative statics in χ, though they can be used for the other comparative statics in the extreme cases $\chi \in \{0,1\}$.

Equilibrium welfare is always increasing in τ_p.

When agents are (close to) fully rational or fully cursed, or information dissemination does not play an important role as τ_p is small, the direct effects dominate and more information and lower costs increase welfare. This is apparent from expression 54 as δ_1 is a sufficient statistic for welfare and the positive comparative statics imply the result. The case of $\tau_p = 0$ is equivalent to a rational game without aggregative signal and hence the same results go through.

An increase of τ_y has two equilibrium effects. First, agents reduce their use and acquisition of private information. Second, between sources of public information, y and p, agents substitute towards y and away from p. When strategic complementarities are sufficiently strong, the second effect is particularly important and causes the paradoxical comparative static:[41] For partially cursed agents, the loading on p is already suboptimally low and the a further decrease entails a first-order welfare loss. This effect dominates the welfare calculus for interior but sufficiently large χ, whereas (close to) fully cursed agents already load almost exclusively on y, and thus the beneficial inframarginal effect of an increase in its precision dominates. Further insight is provided by the analysis of the transparent limit. From 55 welfare is decreasing in τ_y (or, τ_θ) iff $1 - 2r + r\chi < 0$, which provides an interior lower bound, $\chi > 2 - \frac{1}{r}$, if $r > \frac{1}{2}$.[42] In this case, the paradoxical comparative static holds for any τ_y. For interior τ_p, however as Figure 8 shows, welfare is decreasing only at low levels of τ_y as the change in equilibrium loadings vanishes for large τ_y.

As for costs, for partially cursed agents an increase has a side-benefit as the reduced reliance on private information cases agents to substitute towards p, whose informativeness they effectively underestimate. The direct effect of higher costs, rendering information acquisition more costly and therefore reducing the information available overall. This effect is mitigated by r: As agents want to anti-coordinate for negative r, the value of information about the state is mitigated, as all agents correctly playing it would lead to high coordination, an undesirable outcome. Hence, the indirect beneficial effect of higher costs can dominate in this case. As we can see in Figure 8, the effect is present for sufficiently small costs, since then the substitution is towards a relatively informative p, whereas if costs are too high, the aggregative signal itself is too noisy.

As other parameters induce ambiguous and paradoxical welfare comparative statics and transparency has an ambiguous effects on the total precision of information available to agents, it might be surprising that it always increases welfare.[43] The key observation to understand this result is that the endogenous precision of the aggregative signal, $\delta_1^2\tau_p$, is increasing in transparency while it is decreasing in the other parameters. This prevents a

[41] A similar result is also obtained in Morris and Shin (2002), but for different reasons. There, all signals are fundamental, but the increased use of public information entails a payoff externality. In our setting, payoffs are such that – absent dissemination externality and cursedness – information use is efficient (Angeletos and Pavan, 2007; Colombo et al., 2014) and hence more precise public information is always welfare improving. Both ingredients are needed to break this result, it continues to hold even if $\tau_p > 0, \chi = 0$, a case not subsumed by the literature.

[42] Incidentally, this is the same threshold that Morris and Shin (2002) obtain for public information τ_y to be welfare reducing.

[43] Similarly, cursedness is a bias of understanding transparency, so it is surprising that τ_y – the precision of a signal all agents understand – turns out ambiguous but τ_p is not.

../../plots/UnituitiveWtau0.pdf

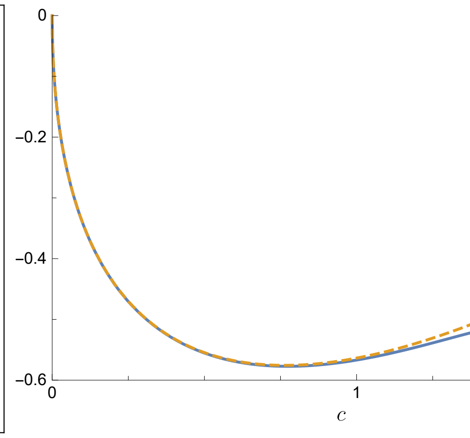

Figure 8: Unintuitive Welfare

negative feedback loop of reduced private information acquisition and dissemination. For partially cursed agents, this increase in effective precision causes a substitution towards the aggregative signal, which is beneficial as well. While cursedness reduces the value aggregative information, it equally dampens its crowding out effect. At worst, in the fully cursed case, transparency is not understood at all hence irrelevant. However, as the fully cursed case makes apparent, there remain unreaped benefits from increased transparency in such economies.

7 Smart Money: Behavior and Policy

In this section, we study the behavior and welfare of *smart money* (with shallow pockets): a fully rational atomistic agent in the model, who understands its structure and is aware that all other agents are χ-cursed.

7.1 Best Response and Information Acquisition

We continue to denote the precision of information acquired by the population as τ_s and denote the precision acquired by the rational agent as τ_s^R. The rational agent takes the equilibrium loadings (and information acquisition) of the cursed crowd as given, and chooses both how much private information to acquire as well as the coefficients in the linear action rule $a_i = \alpha_1 s_i + \alpha_2 y + \alpha_3 p$. Formally, he solves

$$\max_{\boldsymbol{\alpha},\tau_s^R} W\left(\boldsymbol{\alpha}, \delta, \tau_s^R\right)$$

The rational action rule follows immediately from the matching coefficients equations.

Proposition 18. *The action rule of the rational agent is*

$$\alpha_1 = \frac{(1-(1-\delta_1)\,r)\,\tau_s^R}{\tau_\theta + \tau_y + \tau_s^R + \delta_1^2 \tau_p} \tag{56}$$

$$\alpha_2 = \frac{(1-(1-\delta_1)\,r)\,\tau_y + \delta_2\left(r\left(\tau_\theta + \tau_y + \tau_s^R\right) - (1-r)\,\delta_1\,\tau_p\right)}{\tau_\theta + \tau_y + \tau_s^R + \delta_1^2\tau_p} \tag{57}$$

$$\alpha_3 = \frac{\delta_1\,(1-(1-\delta_1)\,r)\,\tau_p + \delta_3\left(r\left(\tau_\theta + \tau_y + \tau_s^R\right) - (1-r)\,\delta_1\,\tau_p\right)}{\tau_\theta + \tau_y + \tau_s^R + \delta_1^2\tau_p} \tag{58}$$

where τ_s^R *solves*

$$\tau_s^R = \frac{\alpha_1}{\sqrt{c}} \tag{59}$$

(with complementary slackness ensuring $\tau_s^R \geq 0$ *when required).*

The weight put by the agent on his private signal, α_1, again is a sufficient statistic for the gains from acquiring information. In this case, this weight is not identical to, but a function of the equilibrium weight δ_1.

Combining (56) and 15 we obtain an equation linking the information acquired by the rational agent and the cursed crowd

$$\frac{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}{\tau_\theta + \tau_y + \tau_s^R + \delta_1^2\tau_p} = 1 + \frac{\chi\delta_1^2\tau_P}{\tau_\theta + \tau_y + \tau_s} \tag{60}$$

As the right hand side always exceeds 1, we immediately get that the rational agent acquires less information. Compared to the cursed crowd, he substitutes private acquisition with a better comprehension of aggregative information. If the crowd is fully cursed, then (60) simplifies to

$$\tau_s^R = \tau_s - \delta_1^2\tau_p \tag{61}$$

that is, his total precision is equal to the total precision *perceived* by the crowd: the rational agent exactly offsets the information he can glean from the aggregative signal.

Clearly, equation (61) holds only if it delivers a positive precision. Otherwise the rational agent will choose $\tau_s^R = 0$ as he is already satiated with he information he can parasitically infer from the aggregative signal (indeed, he would like to sell some private information). Notice that this second contingency will realize at large transparency levels since both τ_s and δ_1 are unresponsive to τ_p in the fully cursed economy: at $\chi = 1$ transparency only serves as a cost-saving device for smart money.

This freeriding intuition generalizes to interior levels of cursedness. If the cursed crowd acquires a strictly positive amount of information, the rational agent may still acquire nothing. The rational agent acquires a strictly positive amount if and only if

$$\tau_\theta + \tau_y \in \left(\frac{(1-r)\left(1 - \frac{1}{\tau_p\sqrt{c}}\right)}{\sqrt{c}}, \frac{1-r}{\sqrt{c}}\right) \tag{62}$$

In other words, we have an inactivity region if $\tau_p > \frac{1}{\sqrt{c}}$; then, smart money acquires private information only if public information is sufficiently (sic!) precise. The logic is as follows. If public fundamental information is noisy the cursed crowd will acquire and use a lot of private information; the rational agent is parasitic on this information, so it is not worthwhile to acquire information himself. As public information becomes more abundant, however, there is less information acquisition and use by the crowd. The aggregative source dries up and smart money needs to supplement it with private information acquisition. Finally, the upper bound on $\tau_\theta + \tau_y$ for the existence of a nontrivial equilibruim is the same for both classes of agents. In the trivial equilibrium, smart money cannot utilize his comparative advantage in understanding the aggregative source, since it is uninformative: it behaves (and has welfare) equal to the crowd.

An immediate consequence of this inactivity region is that τ_s^R is nonmonotonic in τ_y. This contrasts with the unambiguously signed comparative statics for τ_s (Proposition 8). Similarly, the effect of information acquisition costs on τ_s^R is nonmonotonic and we can have an inactivity region (see Figure (9)). Again, a parameters change affects both the availability of aggregative information provided by the cursed crowd and smart money's demand for information overall. The following Proposition summarizes and extends this discussion.

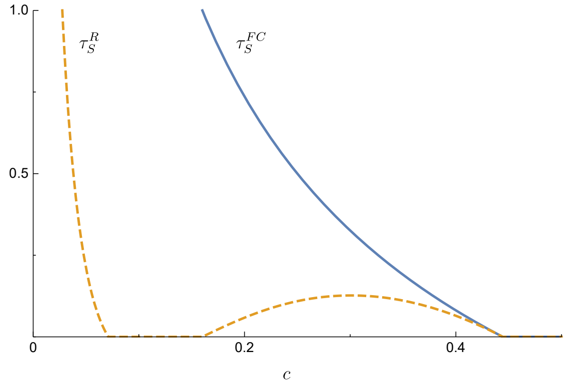

Figure 9: τ_s and τ_S^R as a function of c.

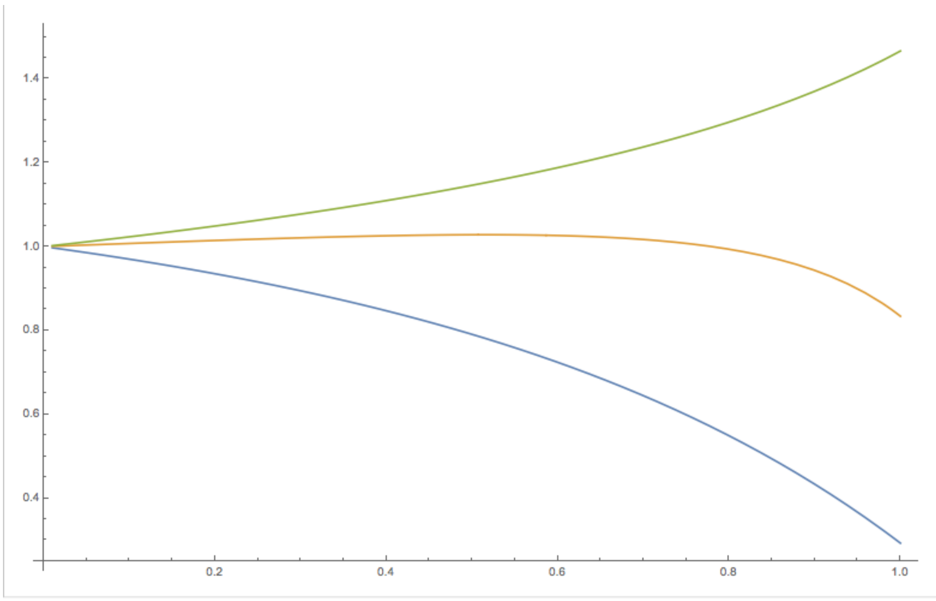

Figure 10: τ_S^R (normalized to 1 at $\chi = 0$) as a function of χ for $r \in \{0.3, 0.75, 0.9\}$.

Proposition 19. *The information acquired by smart money, τ_s^R, has the following properties.*

1. *It is bounded by the precision acquired by the cursed crowd $\tau_s^R \leq \tau_s$. It still explodes as costs vanish but can be zero even when $\tau_s > 0$.*
2. *If τ_p is sufficiently large, then it is*
   - *nonmonotonic in prior and public precision, τ_θ and τ_y, possibly with an interior* activity *region; and*
   - *nonmonotonic in costs c, possibly with an interior* inactivity *region.*
3. *It has ambiguous comparative statics w.r.t. χ as*

$$\frac{d\tau_s^R}{d\chi} \propto r - 2\sqrt{c}\delta_1\tau_p$$

which is not uniformly signed.

The impact of cursedness on the precision of information acquired by the rational agent depends on the nature of strategic interactions (see Figure 10). Take as a benchmark the case of $r = 0$, i.e. all agents simply try to guess the true state, there is no strategic interaction except through the precision of aggregative information. In this case, τ_s^R is decreasing in χ as cursed agents acquire and disseminate more information he can profit from. Strategic substitutes increases this effect: As the population becomes more cursed δ_1 increases which decreases the desire to match the state and therefore the value of private information. With complements, the opposite is the case: A higher δ_1 increases the desire to match θ and therefore - if this motive is sufficiently strong – information acquisition.[44]

Remark (Smart Money in the Transparent Limit). Again, it is instructive to consider to transparent limit. Since $\delta_1^\infty > 0$ whenever a limit equilibrium exists, the rational agent can exactly infer the state. Therefore, he does not acquire or use private information[45] and relies solely on the aggregative signal for information

$$\alpha_1 = \frac{(1-(1-\delta_1)r)\tau_s^R}{\tau_\theta + \tau_y + \tau_s^R + \delta_1^2\tau_p} \to 0; \tau_s^R \to 0 \tag{63}$$

$$\alpha_2 \to -\frac{\delta_2(1-r)}{\delta_1} = \frac{-(1-r)\sqrt{c}\tau_y}{1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)} < 0 \tag{64}$$

$$\alpha_3 =\to \frac{(1-(1-\delta_1)r)-(1-r)\delta_3}{\delta_1} = \frac{1-r\left(1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)}{1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)} > 0 \tag{65}$$

The apparent anti-imitation in $\delta_2 < 0$ allows the rational agent to filter out the over-reliance of the cursed crowd on the public signal.

[44] Finally, note that $\frac{d^2\tau_s^R}{d\chi^2} < 0$. For a region of the parameter space, τ_s^R is hump shaped.

[45] Indeed, notice that the interval (62) vanishes as $\tau_p \to \infty$.

7.2 Welfare

The welfare of the rational agent facing an equilibrium δ^χ is

$$W^R_\chi := \max_{\alpha,\tau_s^R} W\left(\alpha, \delta^\chi, \tau_s^R\right) \tag{66}$$

We have

$$W^{EQ}_\chi = W\left(\delta^\chi, \delta^\chi, \tau_s^\chi\right) \le \max_{\alpha,\tau_s^R} W\left(\alpha, \delta^\chi, \tau_s^R\right) = W^R_\chi \tag{67}$$

where we make the dependence of welfare on the degree of cursedness explicit in the notation. As he comprehends his informational environment, he always obtains a higher welfare than his cursed surroundings. The inequality is strict if $\chi > 0$.

We now ask whether smart money benefits from an increase in the cursedness of the crowd. Local to rationality this is the case since for small positive ϵ

$$W^R_\epsilon > W^{EQ}_\epsilon > W^{EQ}_0 = W^R_0 \tag{68}$$

The central inequality follows since in this region cursedness is bliss (Proposition 16).[46] In a highly cursed environment, however, the impact of cursedness depends on nature of strategic interaction.

Proposition 20. *Suppose parameters are such that* $\tau_s^R > 0$. *Then,*

- *If* $r \le 0$, *then* $\left.\frac{dW^R_\chi}{d\chi}\right|_{\chi=0} \le 0$. *However,*
- *for r sufficiently large,* $\left.\frac{dW^R_\chi}{d\chi}\right|_{\chi=1} \le 0$.

If there are strategic substitutes, smart money always benefits from increased cursedness of the crowd: not only does it free-ride on aggregative information, but the crowd's over-reliance on private signal conforms with the miscoordination desire.[47] In the presence of complementarities, however, informational free-riding and lack of coordination implied by cursed misuse have opposing effects on the smart money's welfare. The latter effect can be overwhelming local to $\chi = 1$ so the smart money would set, if allowed to, an interior level of cursedness of the crowd.

We conclude this section by studying the impact of precision and cost parameters on W^R_χ. If the crowd is close to rational, then policy has an impact similar to that in the rational equilibrium. We focus our analysis on the other extreme case and evaluate the welfare of the smart money facing a fully cursed crowd.[48]

[46] Indeed, the information spillover can be strong enough to make the rational agent in the cursed world better off than first-best welfare, as can be checked for $r = 0$, $\tau_\theta = \tau_y = 0.1$, $\tau_p = 0.19$, $c = 0.03$, where we have $W^R_1 > W^\star$. By continuity, this holds for an open set of parameters.

[47] Since the state action covariance β is invariant in cursedness while the conditional variance of the action increases with δ_1, as cursedness increases the state action correlation decreases. This is harmful if and only if the agent wants to miscoordinate wrt the crowd, i.e. iff $r < 0$. Therefore, only in a game of strategic complements the free-riding and the equilibrium misuse channels deliver opposing impact of cursedeness on the smart money's welfare.

[48] The results extent to an environment with sufficiently cursed agents by continuity.

Proposition 21. W_1^R *has the following properties.*

1. *It is strictly increasing in* τ_p.
2. *It has ambiguous comparative statics with respect to* τ_y. *In particular,*
   - *At the boundary of the activity region, (i.e.* $\tau_y = \frac{(1-r)\left(1-\frac{1}{\tau_p\sqrt{c}}\right)}{\sqrt{c}} - \tau_\theta$*), we have* $\frac{dW_1^R}{d\tau_y} < 0$,
   - *for* τ_y *large (i.e. local to the nontriviality limit* $\tau_y = \frac{(1-r)}{\sqrt{c}} - \tau_\theta$*), we have* $\frac{dW_1^R}{d\tau_y} > 0$.
3. *It is decreasing in costs whenever* $\tau_s^R > 0$*, but it has ambiguous comparative statics with respect to c if* $\tau_s^R = 0$*. In particular,*
   - *If* $\tau_p \geq \frac{1}{\sqrt{c}}$ *and r sufficiently negative, then* $\frac{dW_1^R}{dc} > 0$.

Recall that W_1^{EQ} is independent of transparency as (fully) cursed agents do not respond to τ_p. Therefore, an increase in transparency only affects smart money by providing a more precise aggregative signal, which is clearly strictly beneficial. Recall also that no paradoxical comparative statics in τ_y and c can emerge in a fully cursed environment: more public information and lower cost are always beneficial for the cursed crowd. There is a crowding out effect on the rational agent, however, as public information decreases information acquired and disseminated by the cursed crowd. This is especially harmful if he is largely relying on this source of information, leading to the negative welfare impact at the boundary of the activity region. The detrimental effect of higher costs, by contrast, works through the action externality. Consider a situation where the rational agent does not acquire information himself – i.e. there is no direct effect of higher costs – and aggregate information is relatively abundant as τ_p is large. If costs are higher, cursed agents rely more on y. This makes it easier for the rational agent to miscoordinate with them, which is beneficial in the substitutes setting and can dominate the harm from reduced information dissemination. With complements, however, this increase in coordinated noise exacerbates the trade-off between matching the state and the aggregate action for the rational agent. If the rational agent is acquiring a positive amount of private information, these effects are dampened by the adjustment of τ_s^R and dominated by the direct impact of the change in acquisition costs.

The comparison between Propositions 17 and 21 highlights qualitative differences in the impact of policy on welfare of the cursed crowd and the smart money. Transparency leaves the welfare of the cursed crowd unaffected but has strictly positive (and large) impact for smart money. In an augmented model where the two types of agents compete, potentially already if smart money is not atomistic, this could easily turn into a redistribution result. This observation suggests tat transparency could function as an elitist policy, giving an advantage to sophisticated investors who are able to understand and utilize the often complex information. For public information and lower costs this trade-off is already apparent in the present results, casting doubt on the role of expert lobbying as a source of information on the impact of such policies. A natural extension to study these questions would be a model with different degrees of cursedness, each group having an impact on aggregate outcomes. While the linear structure of the model simplifies aggregation, the correlation between use and acquisition affects the aggregate outcome and introduces nonlinearity. The analysis of such cognitive heterogeneity is beyond the scope of this paper.

8 Conclusion

This paper studies the use of aggregative information in situations with strategic interaction focusing on the interplay of two key aspects: First, that the precision of such aggregate statistics as signals of the fundamental depends on the amount of private information present in individual actions; and second, agents' well-documented difficulty in making inference based on such signals as it requires inferring others' information from their actions.

Due to an information dissemination externality there is inefficiently low acquisition and use of private information in the rational benchmark. Cursed agents rely more heavily on their private information which can push information acquisition at (or even above) its efficient level. While cursedness creates other inefficiencies in information use this effect initially dominates: (a bit of) individual cursedness is a collective blessing. Transparency crowds out the acquisition and use of private information but always increases the endogenous precision of the aggregative signal. This is the main driving force making it the only policy instrument with an unambiguously positive effect on welfare, despite its uncertain effect on some measures of informational efficiency (Section 5) and its redistributive potential (Section 7).

We conduct our analysis in a beauty contest game with information acquisition, adapting a notion of cursed equilibrium to model agents limited understanding of aggregative information. Though parsimonious, the model is sufficiently rich to relate to existing literature and offer alternative explanation of well-established phenomena (solution to the price paradox, detrimental effect of public information, irrelevance of transparency on informational efficiency metrics). Since cursedness significantly alters the positive and normative results in our setting, it would be interesting to extend the analysis to more general payoff specifications as e.g. in Angeletos and Pavan (2007) and more deeply microfounded models yielding reduced forms in similar to this class, as e.g. the business cycle model considered in Colombo et al. (2014) and demand function competition in Vives (2017).

Incorporating information acquisition into a model of cursed equilibrium is the main theoretical contribution of this paper. Doing so requires making an assumption on how such agents asses the value of information. In our notion of *cursed expectations equilibrium with information acquisition*, agents correctly anticipate the equilibrium as well as how they will make use of their information and they (mistakenly) consider this use to be optimal. This assumption is operationalized by a subjective envelope condition, which is highly tractable as it results in a direct proportionality between use and acquisition. While alternative notions don't appear to be attractive in the present setting, the properties and predictive power of such notions across applications of cursed equilibrium (and other behavioral equilibrium notions that do not easily allow a quasi-Bayesian analysis) remains an important question for future research.

A Proofs

A.1 Proofs for Section 3 (Model with fixed τ_s)

Proof of Proposition 1: Recall from the text that $\delta_i = \alpha_i$ and the best response

$$a_i = (1-r)\left(\chi\frac{\tau_s s_i + \tau_y y}{\tau_\theta + \tau_y + \tau_s} + (1-\chi)\frac{\tau_s s_i + \tau_y y + \delta_1^2 \tau_p \frac{1-\delta_3}{\delta_1}\left[p - \frac{\delta_2}{1-\delta_3} y\right]}{\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p}\right)$$
$$+ r\left(\chi(\alpha_0 + \alpha_1 \frac{\tau_s s_i + \tau_y y}{\tau_\theta + \tau_y + \tau_s} + \alpha_2 y + \alpha_3 p) + (1-\chi)(\alpha_0 + \alpha_1 \frac{\tau_s s_i + \tau_y y + \delta_1^2 \tau_p \frac{1-\delta_3}{\delta_1}\left[p - \frac{\delta_2}{1-\delta_3} y\right]}{\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p} + \alpha_2 y + \alpha_3 p)\right)$$
$$a_i = \delta_0 + \delta_1 s_i + \delta_2 y + \delta_3 p$$

Matching coefficients, we get

$$\delta_0 = r\delta_0 \tag{69}$$

$$\delta_1 = (1-r)\left(\chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s} + (1-\chi)\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}\right) + r\left(\chi\delta_1\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s} + (1-\chi)\delta_1\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}\right) \tag{70}$$

$$\delta_2 = (1-r)\left(\chi\frac{\tau_y}{\tau_\theta + \tau_y + \tau_s} + (1-\chi)\frac{\tau_y + \delta_1^2\tau_p\frac{1-\delta_3}{\delta_1}\left[-\frac{\delta_2}{1-\delta_3}\right]}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}\right) \tag{71}$$

$$+ r\left(\chi(\delta_1\frac{\tau_y}{\tau_\theta + \tau_y + \tau_s} + \delta_2) + (1-\chi)\left(\delta_1\frac{\tau_y + \delta_1^2\tau_p\frac{1-\delta_3}{\delta_1}\left[-\frac{\delta_2}{1-\delta_3}\right]}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p} + \delta_2\right)\right) \tag{72}$$

$$\delta_3 = (1-r)\left((1-\chi)\delta_1\frac{\delta_1^2\tau_p\frac{1-\delta_3}{\delta_1}}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}\right) + r\left(\chi\delta_3 + (1-\chi)\left(\delta_1\frac{\delta_1^2\tau_p\frac{1-\delta_3}{\delta_1}}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p} + \delta_3\right)\right) \tag{73}$$

It is easy to see that $\delta_0 = 0$ since $r < 1$. Given δ_1, the latter two equations are linear and we can solve for

$$\delta_2 = (1-r+r\delta_1)\left(\chi\frac{\tau_y}{\tau_\theta + \tau_y + \tau_s} + (1-\chi)\frac{\tau_y - \delta_1\delta_2\tau_p}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}\right) + r\delta_2$$
$$= \frac{(1-r+r\delta_1)\left(\chi\frac{\tau_y}{\tau_\theta+\tau_y+\tau_s} + (1-\chi)\frac{\tau_y}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}\right)}{1-r+(1-r+r\delta_1)(1-\chi)\frac{\delta_1\tau_p}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}} \tag{74}$$

$$\delta_3 = (1-\chi)[(1-r)+r\delta_1]\frac{\delta_1\tau_p(1-\delta_3)}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p} + r\delta_3$$
$$= \frac{(1-\chi)[(1-r)+r\delta_1]\frac{\delta_1\tau_p}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}}{(1-r)+(1-\chi)[(1-r)+r\delta_1]\frac{\delta_1\tau_p}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}} \tag{75}$$

Reformulating the equation for δ_1,

$$\begin{aligned}\delta_1 &= \chi\left[(1-r)+r\delta_1\right]\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}+(1-\chi)\left[(1-r)+r\delta_1\right]\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}\\ &= \left[(1-r)+r\delta_1\right]\tau_s\frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)}\\ &= \left[1-r+r\delta_1\right]\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}\left(1+\chi\frac{\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s}\right)\end{aligned}$$

We arrive at the cubic f by

$$\begin{aligned}\delta_1\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right) &= \chi\left[(1-r)+r\delta_1\right]\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)+(1-\chi)\left[(1-r)+r\delta_1\right]\tau_s\\ \delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right) &= \chi\left\{\left[(1-r)+r\delta_1\right]\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)-\delta_1\tau_s\frac{\tau_\theta+\tau_y+\tau_s}{\tau_\theta+\tau_y+\tau_s}\right\}+(1-\chi)(1-r)(1-\delta_1)\tau_s\\ &= \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left\{(1-r)(1-\delta_1)\left(\tau_\theta+\tau_y+\tau_s\right)+\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p\right\}+(1-\chi)(1-r)(1-\delta_1)\tau_s\\ &= \chi\left\{(1-r)(1-\delta_1)\tau_s+\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p\right\}+(1-\chi)(1-r)(1-\delta_1)\tau_s\\ 0=f(\delta_1) &= \delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[1-r+r\delta_1\right]\delta_1^2\tau_p-(1-r)(1-\delta_1)\tau_s \qquad (76)\\ &= \delta_1\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)-\tau_s\left[1-r+r\delta_1\right]\left\{1+\chi\frac{\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s}\right\}\end{aligned}$$

To show that the solution is unique, note that

$$F(\delta_1)=\delta_1-\left[(1-r)+r\delta_1\right]\tau_s\frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)}$$

evaluated at $\delta_1=0$ is equal to $-\frac{1-r}{\tau_\theta+\tau_y+\tau_s}<0$ and at $\delta_1=1$ it is greater than $1-\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}>0$. Hence, there is at least one root in $(0,1)$. Furthermore, the expression is increasing at a root, as

$$\frac{\partial}{\partial\delta_1}F(\delta_1)=1-r\tau_s\left[\frac{\chi}{\tau_\theta+\tau_y+\tau_s}+\frac{1-\chi}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}\right]+2(1-\chi)\delta_1\tau_p\tau_s\frac{1-r+r\delta_1}{\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)^2}$$

where the first to terms are in sum positive and, at a root, we have $\operatorname{sgn}\{\delta_1\left[(1-r)+r\delta_1\right]\}=1$ whence the final term is also positive.

As a corollary of this argument, we obtain

Corollary 1. *In equilibrium, we have* $(1-r)+r\delta_1\geq 0$, *strictly if* $\delta_1>0$.

Using the rewriting of f

$$\frac{\delta_1}{\tau_s\frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)}}=\left[(1-r)+r\delta_1\right] \qquad (77)$$

we get

$$
\begin{aligned}
\delta_3 &= \frac{\frac{\delta_1}{\tau_s \frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{(\tau_\theta+\tau_y+\tau_s)(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p)}}\delta_1\tau_p}{(1-r)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)+\frac{\delta_1}{\tau_s \frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{(\tau_\theta+\tau_y+\tau_s)(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p)}}\delta_1\tau_p} \\
&\quad \frac{\frac{\delta_1}{\tau_s \frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{(\tau_\theta+\tau_y+\tau_s)(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p)}}\delta_1\tau_p}{(1-r)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)+\frac{\delta_1}{\tau_s \frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{(\tau_\theta+\tau_y+\tau_s)(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p)}}\delta_1\tau_p} \\
&= \frac{\frac{\delta_1}{\tau_s}\delta_1\tau_p\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)\frac{\tau_\theta+\tau_y+\tau_s}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}}{(1-r)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)+\frac{\delta_1}{\tau_s}\delta_1\tau_p\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)\frac{\tau_\theta+\tau_y+\tau_s}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}} \\
&= \frac{\frac{\delta_1}{\tau_s}\delta_1\tau_p\frac{\tau_\theta+\tau_y+\tau_s}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}}{(1-r)+\frac{\delta_1}{\tau_s}\delta_1\tau_p\frac{\tau_\theta+\tau_y+\tau_s}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}} \\
&= \frac{\delta_1^2\tau_p}{(1-r)\tau_s+\delta_1^2\tau_p\left(1+(1-r)\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\right)}
\end{aligned}
$$

and similarly

$$
\begin{aligned}
\delta_2 &= \frac{\frac{\delta_1}{\tau_s \frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{(\tau_\theta+\tau_y+\tau_s)(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p)}}\left(\chi\frac{\tau_y}{\tau_\theta+\tau_y+\tau_s}+(1-\chi)\frac{\tau_y}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}\right)}{1-r+\frac{\delta_1}{\tau_S \frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{(\tau_\theta+\tau_y+\tau_s)(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p)}}(1-\chi)\frac{\delta_1\tau_p}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}} \\
&= \frac{\tau_y\left(\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p\right)}{(1-r)\frac{\delta_1}{\tau_s}\left(\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p\right)+\left(\tau_\theta+\tau_y+\tau_s\right)(1-\chi)\delta_1\tau_p} \\
&= \frac{\delta_1\tau_y\left(\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p\right)}{(1-r)\tau_s\left(\tau_\theta+\tau_y+\tau_s\right)+\delta_1^2\tau_p\left((1-\chi)\left(\tau_\theta+\tau_y\right)+(1-r\chi)\tau_s\right)}
\end{aligned}
$$

Solving for χ from 70 we get

$$
\chi = \frac{\delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-(1-r)(1-\delta_1)\tau_s}{\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}[1+r(\delta_1-1)]\delta_1^2\tau_p}
$$

and plugging this into the above δ_2, δ_3, we obtain the desired expressions.

To see that $\delta_2 \geq 0$, note that

$$
\begin{aligned}
(1-r)\tau_s-\delta_1\left(\tau_\theta+\tau_y\right) &= \delta_1\left(\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}[1+r(\delta_1-1)]\delta_1^2\tau_p+\delta_1\tau_s \\
&= \delta_1^3\tau_p\left[1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[\frac{1+r(\delta_1-1)}{\delta_1}\right]\right]+\delta_1\tau_s \\
&= \delta_1^3\tau_p\left[1-\frac{\chi\tau_\theta+\chi\tau_y+\chi\tau_s+\chi\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\right]+\delta_1\tau_s \geq 0 \qquad (78)
\end{aligned}
$$

□

Proof of Proposition 2: From the equilibrium condition (76) we see

$$\delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p-(1-r)(1-\delta_1)\tau_s=0$$

$$\delta_1^2\tau_p\left(\delta_1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r)+r\delta_1\right]\right)+\delta_1\left(\tau_\theta+\tau_y\right)-(1-r)(1-\delta_1)\tau_s=0$$

so for $\delta_1^2\tau_p$ to go unbounded, we need:

$$\delta_1=\frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)}{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r}+\frac{\xi}{\delta_1^2\tau_p}$$

and

$$\xi+\left(\frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)}{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r}+\frac{\xi}{\delta_1^2\tau_p}\right)\left(\tau_\theta+\tau_y\right)-(1-r)\left(1-\left(\frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)}{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r}+\frac{\xi}{\delta_1^2\tau_p}\right)\right)\tau_s=0$$

$$\xi\left[1+\frac{\tau_\theta+\tau_y+(1-r)\tau_s}{\delta_1^2\tau_p}\right]+\left(\frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)}{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r}\right)\left(\tau_\theta+\tau_y\right)-(1-r)\left(\frac{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}}{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r}\right)\tau_s=0$$

$$\xi=(1-r)\frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left(\tau_\theta+\tau_y\right)-\left(1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\right)\tau_s}{\left(1+\frac{\tau_\theta+\tau_y+(1-r)\tau_s}{\delta_1^2\tau_p}\right)\left(1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r\right)}=0$$

So we just constructed a solution that converges for $\chi>0$. So, in the transparent limit, we have

$$\delta_1^\infty=\frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)}{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r}$$

In the transparent limit

$$\delta_2=\frac{(1-r+r\delta_1)}{\left(\tau_\theta+\tau_y+\tau_s\right)}\frac{\tau_y\left(\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p\right)}{(1-r)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)+(1-r+r\delta_1)(1-\chi)\delta_1\tau_p}\to\frac{\left(1-r+r\delta_1^\infty\right)}{\left(\tau_\theta+\tau_y+\tau_s\right)}\frac{\tau_y\chi\delta_1^\infty}{(1-r)\delta_1^\infty+\left(1-r+r\delta_1^\infty\right)(1-\chi)}$$

$$\delta_2^\infty=\frac{\chi^2(1-r)\tau_y\tau_s}{\left(\tau_\theta+\tau_y+\tau_s(1-r\chi)\right)\left((1-\chi)\left(\tau_\theta+\tau_y\right)+(1-\chi r)\tau_s\right)}$$

It is easy to see that $\frac{\partial\delta_2^\infty}{\partial\chi}\geq 0$

$$\begin{aligned}\delta_3&=\frac{(1-\chi)\left[(1-r)+r\delta_1\right]\delta_1\tau_p}{(1-r)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)+(1-\chi)\left[(1-r)+r\delta_1\right]\delta_1\tau_p}\\&\to\frac{(1-\chi)\left[(1-r)+r\delta_1\right]}{(1-r)\delta_1+(1-\chi)\left[(1-r)+r\delta_1\right]}\\\delta_3^\infty&=\frac{(1-\chi)\left(\tau_\theta+\tau_y+\tau_s\right)}{(1-\chi)\left(\tau_\theta+\tau_y\right)+(1-\chi r)\tau_s}\end{aligned}$$

Which establishes the limit. Existence of a limit equilibrium follows, as all the arguments leading to equilibrium are well defined even taking $\tau_p \to \infty$ assuming $\delta_1 > 0$, which is the case in the solution if and only if $\chi > 0$. □

Proof of Proposition 3: The result follows from implicit differentiation of the equilibrium equation (76). First, let us establish a helpful lemma.

Lemma 1. *In equilibrium, we have $f_\delta > 0$.*

Proof of Lemma: Compute

$$\begin{aligned}
f_\delta &= \tau_\theta + \tau_y + 3\delta_1^2\tau_p - \chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\left[2 + r(3\delta_1 - 2)\right]\delta_1\tau_p + (1-r)\tau_s \\
&= \left(\tau_\theta + \tau_y + 3\delta_1^2\tau_p\left[1 - \chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\left[\frac{1 + r(\delta_1 - 1)}{\delta_1}\right]\right]\right) + \chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\left[1 - r\right]\delta_1\tau_p + (1-r)\tau_s \\
&= \left(\tau_\theta + \tau_y + 3\delta_1^2\tau_p\left[1 - \frac{\chi\tau_\theta + \chi\tau_y + \chi\tau_s + \chi\delta_1^2\tau_p}{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p}\right]\right) + \chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\left[1 - r\right]\delta_1\tau_p + (1-r)\tau_s > 0
\end{aligned}$$

where in the final step we used 77 in the transformation:

$$\begin{aligned}
1 - \chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\left[\frac{1 + r(\delta_1 - 1)}{\delta_1}\right] &= 1 - \chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\left[\frac{1 + r(\delta_1 - 1)}{\left[(1-r) + r\delta_1\right]\tau_s\frac{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p}{\left(\tau_\theta + \tau_y + \tau_s\right)\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right)}}\right] \\
&= 1 - \chi\left[\frac{1}{\frac{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p}{\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right)}}\right] \\
&= 1 - \chi\left[\frac{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p}\right] \\
&= 1 - \frac{\chi\tau_\theta + \chi\tau_y + \chi\tau_s + \chi\delta_1^2\tau_p}{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p} \geq 0
\end{aligned}$$

▲

By implicit differentiation

$$\begin{aligned}
\frac{\partial\delta_1}{\partial\chi} &= -\frac{f_\chi}{f_\delta} = -\frac{-\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\left[(1-r) + r\delta_1\right]\delta_1^2\tau_p}{f_\delta} \\
&\propto \frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\left[(1-r) + r\delta_1\right]\delta_1^2\tau_p \geq 0
\end{aligned}$$

and from 13, 14

$$\begin{aligned}
\frac{\partial\delta_2}{\partial\chi} &= \frac{2\delta_1\tau_y\left((1-r)\tau_s - \delta_1\left(\tau_\theta + \tau_y\right)\right) + \delta_1^2\tau_y\left(\tau_\theta + \tau_y\right)}{\left((1-r)\tau_s - \delta_1\left(\tau_\theta + \tau_y\right)\right)^2}\frac{\partial\delta_1}{\partial\chi} \geq 0 \\
\frac{\partial\delta_3}{\partial\chi} &= -\frac{(1-r)\tau_s\left((1-r)\tau_s - \delta_1\left(\tau_\theta + \tau_y\right)\right) + \delta_1(1-r)\tau_s\left(\tau_\theta + \tau_y\right)}{\left((1-r)\tau_s - \delta_1\left(\tau_\theta + \tau_y\right)\right)^2}\frac{\partial\delta_1}{\partial\chi} \leq 0
\end{aligned}$$

using the fact that $(1-r)\tau_s - \delta_1\left(\tau_\theta + \tau_y\right) > 0$ 78.

Relative size of δ_1 and δ_2:

$$\frac{\partial}{\partial\chi}\frac{\delta_2}{\delta_1} = \frac{\partial}{\partial\chi}\frac{\delta_1\tau_y}{(1-r)\tau_s-\delta_1\left(\tau_\theta+\tau_y\right)} = \frac{\tau_y\left((1-r)\tau_s-\delta_1\left(\tau_\theta+\tau_y\right)\right)+\delta_1\tau_y\left(\tau_\theta+\tau_y\right)}{\left((1-r)\tau_s-\delta_1\left(\tau_\theta+\tau_y\right)\right)^2}\frac{\partial\delta_1}{\partial\chi} \geq 0$$

whence the result in the proposition follows.

τ_p comparative statics: Again, $\frac{d\delta_1}{d\tau_p} \propto -f_{\tau_p}$ and using (77), we have

$$\begin{aligned} f_{\tau_p} &= \delta_1^3 - \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r)+r\delta_1\right]\delta_1^2 \\ &= \delta_1^3\left(1-\chi\frac{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\right) > 0 \end{aligned}$$

From 13 and 14, the comparative statics are immediate. Finally, we have

$$\begin{aligned} \frac{\partial\delta_1^2\tau_p}{\partial\tau_p} &= 2\delta_1\tau_p\frac{\partial\delta_1}{\partial\tau_p}+\delta_1^2 \\ &= -2\delta_1\tau_p\frac{\delta_1^3-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r)+r\delta_1\right]\delta_1^2}{\left(\tau_\theta+\tau_y+3\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[2+r(3\delta_1-2)\right]\delta_1\tau_p+(1-r)\tau_s}+\delta_1^2 \\ &\propto^{f_\delta>0} -2\delta_1\tau_p\left(\delta_1^3-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r)+r\delta_1\right]\delta_1^2\right)+\delta_1^2\left(\left(\tau_\theta+\tau_y+3\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[2+r(3\delta_1-2)\right]\delta_1\tau_p+(1-r)\tau_s\right) \\ &\propto^{\delta_1^2>0} -2\delta_1\tau_p\left(\delta_1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r)+r\delta_1\right]\right)+\left(\left(\tau_\theta+\tau_y+3\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[2+r(3\delta_1-2)\right]\delta_1\tau_p+(1-r)\tau_s\right) \\ &= \left(\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[r\delta_1\right]\delta_1\tau_p+(1-r)\tau_s\right) \\ &= \frac{1}{\delta_1}\left(\delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p-(1-r)(1-\delta_1)\tau_s+(1-r)\tau_s+\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)\delta_1^2\tau_p\right) \\ &= \frac{1}{\delta_1}\left((1-r)\tau_s+\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)\delta_1^2\tau_p\right) > 0 \end{aligned}$$

which establishes the claim.

□

Proof of Proposition 4: Since we have $f_\delta > 0$ (Lemma 1), we get for a generic parameter ν

$$\frac{d\delta_1}{d\nu} = -\frac{f_\nu}{f_\delta} \propto -f_\nu$$

And hence the comparative statics of δ_1 follow immediately, occasionally using (77), from

$$\begin{aligned} f_{\tau_y} &= \delta_1+\chi\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p\frac{\tau_s}{\left(\tau_\theta+\tau_y+\tau_s\right)^2} > 0 \\ &= \delta_1+\frac{\tau_p}{\tau_\theta+\tau_y+\tau_s}\chi\delta_1^3\frac{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p} \end{aligned}$$

$$f_{\tau_s} = -\chi\frac{\tau_\theta+\tau_y}{\left(\tau_\theta+\tau_y+\tau_s\right)^2}\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p-(1-r)(1-\delta_1) < 0$$

$$f_r = \chi \frac{\tau_s (1-\delta_1)}{\tau_\theta + \tau_y + \tau_s} \delta_1^2 \tau_p + (1-\delta_1)\tau_s = \tau_s (1-\delta_1) \left[\frac{\chi \delta_1^2 \tau_p}{\tau_\theta + \tau_y + \tau_s} + 1 \right] > 0$$

The result for $\frac{d\beta}{d\chi}, \frac{d\beta}{d\tau_p}, \frac{d\gamma_2}{d\tau_p}$ is immediate from the comparative statics of δ_1. For $\frac{d\gamma_2}{d\tau_y} \propto \frac{d\delta_1}{d\tau_y}\tau_y + \delta_1$, we get

$$\begin{aligned}
\frac{d\delta_1}{d\tau_y}\tau_y + \delta_1 &= -\frac{f_{\tau_y}}{f_\delta}\tau_y + \delta_1 \\
&\propto^{f_\delta > 0} -f_{\tau_y}\tau_y + f_\delta \delta_1 \\
&= \frac{\delta_1}{\left(\tau_y + \tau_s + \tau_\theta\right)^2} \left[\left(3\delta_1^2 \tau_p + \tau_\theta + (1-r)\tau_s\right)\left(\tau_y + \tau_s + \tau_\theta\right)^2 - \chi \delta_1 \tau_p \tau_s \left((3 - 3r + 4\delta_1)\tau_y + (2 - 2r + 3\delta_1)(\tau_s + \tau_\theta) \right) \right] \\
&\propto \mathrm{A}_1 + \mathrm{A}_2
\end{aligned}$$

where, using 77,

$$\mathrm{A}_1 = \left(\tau_y + \tau_s + \tau_\theta\right)^2 \left[\tau_\theta + (1-r)\tau_s + 3\frac{(1-\chi)\delta_1^2 \tau_p \left(\tau_y + \tau_s + \tau_\theta\right)}{\tau_y + \tau_s + \tau_\theta + \chi \delta_1^2 \tau_p} \right]$$

and

$$\mathrm{A}_2 = \chi \delta_1 \tau_p \left(\tau_y + \tau_s + \tau_\theta\right) \left[(1-r)\tau_s - \frac{\delta_1 \tau_y \left(\tau_y + \tau_s + \tau_\theta + \delta_1^2 \tau_p\right)}{\tau_y + \tau_s + \tau_\theta + \chi \delta_1^2 \tau_p} \right].$$

Now, as clearly $\mathrm{A}_1 > 0$, it remains to show that A_2. We know that in equilibrium, we have from 77

$$\begin{aligned}
\frac{\delta_1}{\tau_s \frac{\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p}{\left(\tau_\theta + \tau_y + \tau_s\right)\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p\right)}} &= [(1-r) + r\delta_1] \\
\iff \frac{\delta_1 \left(\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p\right)}{\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p} &= \tau_s \frac{[(1-r) + r\delta_1]}{\left(\tau_\theta + \tau_y + \tau_s\right)}
\end{aligned}$$

whence

$$\begin{aligned}
\mathrm{A}_2 &\propto (1-r) - \tau_y \frac{[(1-r) + r\delta_1]}{\left(\tau_\theta + \tau_y + \tau_s\right)} \\
&\propto (1-r)(\tau_\theta + \tau_s) - \tau_y r \delta_1 \\
&> (1-r)(\tau_\theta + \tau_s) - \tau_y r \delta_1^{\mathrm{FC}} \\
&= (1-r)(\tau_\theta + \tau_s) - \tau_y r \frac{(1-r)\tau_s}{\tau_\theta + \tau_y + (1-r)\tau_s} = (1-r)\left(\tau_\theta + \tau_s \left(1 - r \frac{\tau_y}{\tau_\theta + \tau_y + (1-r)\tau_s} \right) \right) > 0
\end{aligned}$$

where the last step is immediate if $r < 0$ and equally so if $r \in (0,1)$!

Note that the result for $\frac{d\gamma_2}{d\tau_s}$ and $\frac{d\beta}{d\tau_s}$ follows when we establish $\frac{d}{d\tau_s}\frac{\delta_1}{\tau_s} < 0$:

$$\begin{aligned}
\frac{d}{d\tau_s}\frac{\delta_1}{\tau_s} &= \frac{\frac{d\delta_1}{d\tau_s}\tau_s - \delta_1}{\tau_s^2} \\
&\propto \frac{d\delta_1}{d\tau_s}\tau_s - \delta_1 \\
&\propto^{f_\delta > 0} -f_{\tau_s}\tau_s + f_\delta \delta_1 \\
&= (1-r)(1-\delta_1)\tau_s - \delta_1 \left(\tau_y + \tau_\theta + (1-r)\tau_s\right) + 3\frac{\chi (1-r)\tau_s \delta_1^2 \tau_p}{\tau_y + \tau_\theta + \tau_s} - \frac{\chi (1-r)\tau_s^2 \delta_1^2 \tau_p}{\left(\tau_y + \tau_\theta + \tau_s\right)^2}
\end{aligned}$$

$$-\delta_1^3\tau_p\left(3-4\frac{\chi r\tau_s}{\tau_y+\tau_\theta+\tau_s}+\frac{\chi r\tau_s^2}{\left(\tau_y+\tau_\theta+\tau_s\right)^2}\right)$$

using f to replace $(1-r)(1-\delta_1)\tau_s$, we arrive a decomposition B_1+B_2 where

$$B_1=\frac{\chi\delta_1^2\tau_p\tau_s}{\left(\tau_y+\tau_\theta+\tau_s\right)^2}\left[\delta_1 r\left(\tau_y+\tau_\theta\right)-(1-r)\tau_s\right]$$

$$B_2=-\delta_1\left(2\delta_1^2\tau_p+(1-r)\tau_s\right)+2\frac{\chi\delta_1^2\left[1-r+\delta_1 r\right]\tau_p\tau_s}{\tau_y+\tau_\theta+\tau_s}$$

Note that in B_1 the final term is negative if $r<0$, otherwise, estimate $\delta_1<\delta_1^{FC}$ to arrive at

$$\begin{aligned}\delta_1 r\left(\tau_y+\tau_\theta\right)-(1-r)\tau_s &< \frac{(1-r)\tau_s}{\tau_\theta+\tau_y+(1-r)\tau_s}r\left(\tau_y+\tau_\theta\right)-(1-r)\tau_s\\ &=-\frac{(1-r)^2\tau_s\left(\tau_y+\tau_\theta+\tau_s\right)}{\tau_\theta+\tau_y+(1-r)\tau_s}<0\end{aligned}$$

and therefore $B_1<0$. Plugging 77 into B_2, we arrive at

$$B_2=-\delta_1\left((1-r)\tau_s+2\frac{(1-\chi)\delta_1^2\tau_p\left(\tau_y+\tau_\theta+\tau_s\right)}{\left(\tau_y+\tau_\theta+\tau_s+\chi\delta_1^2\tau_p\right)}\right)<0$$

, whence we have established $\frac{d}{d\tau_s}\frac{\delta_1}{\tau_s}<0$ and therefore $\frac{d\gamma_2}{d\tau_s}<0$ and $\frac{d\beta}{d\tau_s}>0$. □

A.2 Proofs for the Model with Information Acquisition

Proof of Proposition 6: Equation 38 is derived in the text assuming that $\delta_1\geq 0$. We show that there cannot be an equilibrium with $\delta_1<0$ in the supplementary appendix 22. Eq. 34-37 follow immediately from plugging 38 into 11-14. To see that $\grave{f}$ has a unique real solution, note that

$$\grave{f}(\delta_1)=\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\left[1-r+r\delta_1\right]\left(1+\chi\frac{\sqrt{c}\delta_1^2\tau_p}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}\right)$$

$$\grave{f}'(\delta_1)=1+\sqrt{c}2\delta_1\tau_p-r\left(1+\chi\frac{\sqrt{c}\delta_1^2\tau_p}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}\right)-\left[1-r+r\delta_1\right]\left(\chi\frac{2\sqrt{c}\delta_1\tau_p}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}-\chi\frac{\sqrt{c}\delta_1^2\tau_p}{\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2}\right)$$

In equilibrium, we have

$$\begin{aligned}&=1+\sqrt{c}2\delta_1\tau_p-r\left(1+\chi\frac{\sqrt{c}\delta_1^2\tau_p}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}\right)-\left[1-r+r\delta_1\right]\left(\chi\frac{2\sqrt{c}\delta_1\tau_p}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}-\chi\frac{\sqrt{c}\delta_1^2\tau_p}{\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2}\right)\\&=1+\sqrt{c}2\delta_1\tau_p-r\left(1+\chi\frac{\sqrt{c}\delta_1^2\tau_p}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}\right)-\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{1+\chi\frac{\sqrt{c}\delta_1^2\tau_p}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}}\left(\chi\frac{2\sqrt{c}\delta_1\tau_p}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}-\chi\frac{\sqrt{c}\delta_1^2\tau_p}{\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2}\right)\\&=1+\sqrt{c}2\delta_1\tau_p-r\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}-\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}\chi\left(2\sqrt{c}\delta_1\tau_p-\frac{\sqrt{c}\delta_1^2\tau_p}{\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)}\right)\end{aligned}$$

$$=\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)-r\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)\right)+\sqrt{c}\delta_1^2\tau_p\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}\chi}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}+2\sqrt{c}\delta_1\tau_p\left(1-\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}\chi\right)$$

$$=\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)-r\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)\right)+\sqrt{c}\delta_1^2\tau_p\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}\chi}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}+2\sqrt{c}\delta_1\tau_p\left((1-\chi)\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}\right)$$

$$\geq\propto\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)-r\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)\right)+\sqrt{c}\delta_1^2\tau_p\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}\chi$$

$$=(1-r)\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)+\sqrt{c}\delta_1^2\tau_p\chi\left[\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}-r\right]\geq\sqrt{c}\delta_1^2\tau_p\chi[1-r]>0$$

where we used only $\delta_1 \geq 0$ and $\grave{f}(\delta_1)=0$.

If we are in a corner case, we obtain δ_2, δ_3 by plugging $\delta_1=\tau_s=0$ to the original matching coefficients equations 74 and 75 to obtain

$$\delta_2=\frac{\tau_y}{\tau_\theta+\tau_y}$$
$$\delta_3=0$$

as desired. □

Proof of Proposition 7: In the text. □

Proof of Proposition 8: Our system is defined by

$$f(\delta_1,\tau_s)=\delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[1+r(\delta_1-1)\right]\delta_1^2\tau_p-(1-r)(1-\delta_1)\tau_s=0$$
$$g(\delta_1,\tau_s)=\delta_1^2-c(\tau_s)^2=0$$

and by implicit differentiation,

$$\frac{d\delta_1}{dv}=\frac{g_{\tau_s}f_v-f_{\tau_s}g_v}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}$$
$$\frac{d\tau_s}{dv}=\frac{f_\delta g_v-g_\delta f_v}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}$$

First, we establish that the denominator of our implicit derivatives is positive.

Lemma 2. *In equilibrium, we have $g_\delta f_{\tau_s}-g_{\tau_s}f_\delta>0$, and hence*

$$\frac{d\delta_1}{dv}\propto g_{\tau_s}f_v-f_{\tau_s}g_v.$$

Proof of Lemma: Note that

$$f_\delta=\left(\tau_\theta+\tau_y+3\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[2+r(3\delta_1-2)\right]\delta_1\tau_p+(1-r)\tau_s$$
$$f_{\tau_s}=-\chi\frac{\tau_\theta+\tau_y}{\left(\tau_\theta+\tau_y+\tau_s\right)^2}\left[1+r(\delta_1-1)\right]\delta_1^2\tau_p-(1-r)(1-\delta_1)$$

$$g_\delta=2\delta_1>0$$

$$g_{\tau_s} = -2c\tau_s < 0$$

By direct computation

$$\begin{aligned}
g_\delta f_{\tau_s} - g_{\tau_s} f_\delta =& 2\delta_1 \left(-\chi \frac{\tau_\theta + \tau_y}{\left(\tau_\theta + \tau_y + \tau_s\right)^2} \left[1 + r(\delta_1 - 1)\right] \delta_1^2 \tau_p - (1-r)(1-\delta_1) \right) - (-2c\tau_s) \left(\left(\tau_\theta + \tau_y + 3\delta_1^2\tau_p\right) - \chi \frac{\tau_s}{\tau_\theta + \tau_y + \tau_s} \left[2 + r(3\delta_1 - 2 \right.\right. \\
=& 2\delta_1 \left(-\chi \frac{\tau_\theta + \tau_y}{\left(\tau_\theta + \tau_y + \tau_s\right)^2} \left[1 + r(\delta_1 - 1)\right] \delta_1^2 \tau_p - \left[1 + r(\delta_1 - 1)\right] + \delta_1 \right) + 2\frac{\delta_1}{\tau_s} \left(\left(\tau_\theta + \tau_y + 3\delta_1^2\tau_p\right)\delta_1 - 2\chi \frac{\tau_s}{\tau_\theta + \tau_y + \tau_s} \left[1 + r(\delta \right.\right. \\
\geq^{f_\delta > 0}& 2\frac{\delta_1}{\tau_s} \left(-\chi \frac{\tau_\theta + \tau_y}{\tau_\theta + \tau_y + \tau_s} \frac{\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right)\delta_1}{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p} \delta_1^2 \tau_p - \frac{\delta_1 \left(\tau_\theta + \tau_y + \tau_s\right)\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right)}{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p} + \delta_1 \tau_s \right) + 2\frac{\delta_1}{\tau_s} \left(\left(\tau_\theta + \tau_y + 3 \right.\right. \\
=^{mathematica}& 2\delta_1^2 \left\{ -\frac{\tau_y}{\tau_s} - \frac{\tau_\theta}{\tau_s} + \delta_1^2 \tau_p \left(-\frac{1}{\tau_s} + \frac{1}{\tau_\theta + \tau_y + \tau_s} - \frac{1-\chi}{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p} \right) \right\} + 2\frac{\delta_1}{\tau_s} \left(\left(\tau_\theta + \tau_y + 3 \left(1 - \chi \frac{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p} \right) \delta \right.\right. \\
\geq^{r<1}_{(38)}& 2\delta_1^3 \tau_p (1-\chi) \sqrt{c'(\tau_s)} \left\{ \frac{2\delta_1^2 + \sqrt{c}\chi\delta_1^3\tau_p + 4\sqrt{c}\delta_1\left(\tau_\theta + \tau_y\right) + 2c\left(\tau_\theta + \tau_y\right)^2}{\left[\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)\right]\left[\delta_1 + \sqrt{c}\chi\delta_1^2\tau_p + \sqrt{c}\left(\tau_\theta + \tau_y\right)\right]} \right\} \geq 0
\end{aligned}$$

where we can set $r = 1$ as a worst case, for fixed values, the expression is clearly decreasing in r. ▲

Hence, we have

$$\frac{\mathrm{d}\delta_1}{\mathrm{d}\chi} \propto g_{\tau_s} f_\chi - f_{\tau_s} g_\chi = g_{\tau_s} f_\chi = \left(-c''(\tau_s)(\tau_s)^2 - 2c'(\tau_s)\tau_s\right)\left(-\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s} \left[1 + r(\delta_1 - 1)\right] \delta_1^2 \tau_p \right) > 0.$$

For $\frac{d\delta_2}{d\chi}$, using 36

$$\frac{\mathrm{d}\delta_2}{\mathrm{d}\chi} = \frac{\sqrt{c}\tau_y}{(1-r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)} \frac{\mathrm{d}\delta_1}{\mathrm{d}\chi} > 0.$$

Finally, from 37

$$\frac{\mathrm{d}\delta_3}{\mathrm{d}\chi} = -\frac{(1-r)}{(1-r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)} \frac{\mathrm{d}\delta_1}{\mathrm{d}\chi} < 0$$

For the remaining comparative statics let us begin with δ_1. Note that that for all precision parameters, we have $g_x \equiv 0$, and for costs, we have $f_c \equiv 0$.

For the cost parameter

$$\frac{\mathrm{d}\delta_1}{\mathrm{d}c} \propto f_{\tau_s} (\tau_s)^2 < 0$$

as $f_{\tau_s} < 0$. For τ_s, we get

$$\frac{\partial \tau_s}{\partial c} = \frac{1}{c} \frac{\partial \delta_1}{\partial c} - \tau_s c < 0$$

We have

$$\begin{aligned}
\frac{\partial \delta_1}{\partial \tau_y} &\propto g_{\tau_s} f_{\tau_y} - g_{\tau_y} f_{\tau_s} = g_{\tau_s} f_{\tau_y} \\
&= g_{\tau_s} \left[\delta_1 + \chi \frac{\tau_s}{\left(\tau_\theta + \tau_y + \tau_s\right)^2} \left[1 + r(\delta_1 - 1)\right] \delta_1^2 \tau_p \right] < 0
\end{aligned}$$

and

$$\frac{\partial \delta_1}{\partial \tau_\theta} = \frac{\partial \delta_1}{\partial \tau_y} < 0$$

For τ_p, we have

$$\begin{aligned}\frac{\partial \delta_1}{\partial \tau_p} &\propto g_{\tau_s} f_{\tau_p} \\ &= g_{\tau_s}\left[\delta_1^3 - \chi \frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\left[1 + r(\delta_1 - 1)\right]\delta_1^2\right] \\ &= g_{\tau_s}\delta_1^3\left[1 - \chi \frac{\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p\right)}{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p}\right] < 0\end{aligned}$$

and for r, we get

$$\begin{aligned}\frac{\partial \delta_1}{\partial r} &\propto g_{\tau_s} f_r \\ &= g_{\tau_s}(1-\delta_1)\left[\chi \frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\delta_1^2 \tau_p + \tau_s\right] \leq 0.\end{aligned}$$

For δ_2, the comparative static wrt τ_p is immediate.

To see the ambiguity of $\frac{d\delta_2}{d\tau_y}$, consider the transparent limit expression and take the derivative to obtain

$$\begin{aligned}\frac{d\delta_2^\infty}{d\tau_y} &= \frac{\sqrt{c}\left[\chi(1-r) - \sqrt{c}\left(2\tau_\theta + \tau_y\right)\right](1-r\chi)\left(1 - r - \sqrt{c'(\tau_s)}\left(\tau_\theta + \tau_y\right)\right) + \sqrt{c}\tau_y\left[\chi(1-r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)\right](1-r\chi)\sqrt{c'(\tau_s)}}{\left[(1-r\chi)\left(1 - r - \sqrt{c'(\tau_s)}\left(\tau_\theta + \tau_y\right)\right)\right]^2} \\ &\propto \left[\chi(1-r) - \sqrt{c}\left(2\tau_\theta + \tau_y\right)\right]\left(1 - r - \sqrt{c'(\tau_s)}\left(\tau_\theta + \tau_y\right)\right) + \tau_y\left[\chi(1-r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)\right]\sqrt{c'(\tau_s)}\end{aligned}$$

at $\chi = 1$, we get

$$\left[(1-r) - \sqrt{c}\left(2\tau_\theta + \tau_y\right)\right]\left(1 - r - \sqrt{c'(\tau_s)}\left(\tau_\theta + \tau_y\right)\right) + \tau_y\left[(1-r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)\right]\sqrt{c'(\tau_s)} = \left[(1-r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)\right]^2 > 0$$

, whereas at the lower limit $\chi = \frac{\sqrt{c}\left(\tau_\theta + \tau_y\right)}{1-r}$, we have

$$\left[\sqrt{c}\left(\tau_\theta + \tau_y\right) - \sqrt{c}\left(2\tau_\theta + \tau_y\right)\right]\left(1 - r - \sqrt{c'(\tau_s)}\left(\tau_\theta + \tau_y\right)\right) + \tau_y\left[\sqrt{c}\left(\tau_\theta + \tau_y\right) - \sqrt{c}\left(\tau_\theta + \tau_y\right)\right]\sqrt{c'(\tau_s)} = -\sqrt{c}\tau_y\left(1 - r - \sqrt{c'(\tau_s)}\left(\tau_\theta + \tau_y\right)\right) < 0$$

establishing the claim.

To see the other ambiguous comparative statics, we proceed in a very similar fashion. Consider $\frac{d\delta_2}{dc}$; in the transparent limit, for $\chi = 1$ we get $\frac{d\delta_2^\infty}{dc} = \frac{\tau_y}{2\sqrt{c}(1-r)} > 0$, while at the lower bound, $\chi \to \sqrt{c}\frac{\tau_\theta + \tau_y}{1-r}$ we have $\frac{d\delta_2^\infty}{dc} \propto -(1-r)\tau_y\left(\tau_\theta + \tau_y\right) < 0$. For $\frac{d\delta_2}{dr}$; in the transparent limit, for $\chi = 1$ we get $\frac{d\delta_2^\infty}{dc} = \frac{\sqrt{c}\tau_y}{(1-r)^2} > 0$, while at the lower bound, $\chi \to \sqrt{c}\frac{\tau_\theta + \tau_y}{1-r}$ we have $\frac{d\delta_2^\infty}{dr} \propto -c\tau_y\left(\tau_\theta + \tau_y\right) < 0$.

For δ_3, the τ_p comparative statics is immediate as well. To see the ambiguity of others, consider in particular, we can see that δ_3^∞ is increasing in τ_y. Furthermore, it is increasing as $\chi \to 1$ for *any* τ_p, since

$$\begin{aligned}\delta_3 &= 1 - \frac{(1-r)}{1 - r - \sqrt{c}\left(\tau_\theta + \tau_y\right)}\delta_1 \\ \frac{\partial \delta_3}{\partial \tau_y} &\propto -\sqrt{c}(1-r)\delta_1 - (1-r)\left(1 - r - \sqrt{c}\left(\tau_\theta + \tau_y\right)\right)\frac{\partial \delta_1}{\partial \tau_y}\end{aligned}$$

$$\lim_{\chi\to 1}\frac{\frac{\partial\delta_3}{\partial\tau_y}}{\delta_3}\propto\frac{\left(1-\delta_1(1-r)-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)}{1-\frac{\delta_1(1-r)}{1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)}}=1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)>0$$

proving $\frac{\partial\delta_3}{\partial\tau_y}$ converges to 0 from above, hence it is positive around $\chi=1$. However, consider the limit as $r\to 1-\sqrt{c}\left(\tau_\theta+\tau_y\right)$, then

$$\frac{\partial\delta_3}{\partial\tau_y}\to-\frac{1-r}{\tau_\theta+\tau_y}\delta_1<0$$

To see that $\frac{\partial\delta_3}{\partial c}$ is of ambiguous sign, consider the limit as $\sqrt{c}\to\frac{1-r}{\tau_\theta+\tau_y}$.Then, we have

$$\operatorname{sgn}\left\{\frac{\partial\delta_3}{\partial c}\right\}\to\operatorname{sgn}\left\{-(1-r)^3\right\}<0$$

Furthermore, as $c\to 0$, we have

$$\frac{\partial\delta_3}{\partial c}\propto\delta_1^2(1-r)\left((1-\delta_1)\left(\tau_\theta+\tau_y\right)+(1-\chi)\delta_1^2\tau_p\right)>0$$

Following similar arguments, $\frac{\partial\delta_3}{\partial r}$ is ambiguous: In the limit as $\sqrt{c}\to\frac{1-r}{\tau_\theta+\tau_y}$, we have

$$\operatorname{sgn}\left\{\frac{\partial\delta_3}{\partial r}\right\}\to\operatorname{sgn}\left\{-(1-r)^3\right\}<0$$

As $r\to-\infty$, we get

$$\frac{\partial\delta_3}{\partial r}\propto-r(1-\delta_1)\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)\right)^2>0$$

To show that $\frac{\partial}{\partial\tau_p}\delta_1^2\tau_p>0$:

$$\begin{aligned}
\frac{\partial}{\partial\tau_p}\left(\tau_p\delta_1^2\right)&=\delta_1^2+2\delta_1\tau_p\frac{\partial\delta_1}{\partial\tau_p}\\
&=\delta_1^2+2\delta_1\tau_p\frac{g_{\tau_s}f_{\tau_p}}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\\
&=\frac{1}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\left\{\delta_1^2\left(g_\delta f_{\tau_s}-g_{\tau_s}f_\delta\right)+2\delta_1\tau_p g_{\tau_s}f_{\tau_p}\right\}\\
&=\frac{1}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\left\{\delta_1^2\left(g_\delta f_{\tau_s}\right)+\delta_1 g_{\tau_s}\left[2\tau_p f_{\tau_p}-\delta_1 f_\delta\right]\right\}\\
&=\frac{1}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\left\{\delta_1^2\left(g_\delta f_{\tau_s}\right)-g_{\tau_s}\delta_1^2\left[\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r\delta_1^2\tau_p+(1-r)\tau_s\right]\right\}\\
&=\frac{\delta_1^2}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\Bigg\{2\delta_1\left(-\chi\frac{\tau_\theta+\tau_y}{\left(\tau_\theta+\tau_y+\tau_s\right)^2}\left[1+r(\delta_1-1)\right]\delta_1^2\tau_p-(1-r)(1-\delta_1)\right)+\left(2\tau_s c'(\tau_s)+\tau_s^2c''(\tau_s)\right)\Bigg[\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\\
&\geq\frac{\delta_1^2}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\Bigg\{2\delta_1\left(-\chi\frac{\tau_\theta+\tau_y}{\left(\tau_\theta+\tau_y+\tau_s\right)^2}\left[1+r(\delta_1-1)\right]\delta_1^2\tau_p-(1-r)(1-\delta_1)\right)+2\frac{\delta_1}{\sqrt{c'(\tau_s)}}c'(\tau_s)\Bigg[\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\chi\frac{}{\tau_\theta+}\\
&=\frac{2\delta_1^3}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\Bigg\{-\chi\frac{\tau_\theta+\tau_y}{\left(\tau_\theta+\tau_y+\tau_s\right)^2}\left[1+r(\delta_1-1)\right]\delta_1^2\tau_p-(1-r)(1-\delta_1)+\sqrt{c'(\tau_s)}\Bigg[\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r\delta_1^2\tau_p\\
&=\frac{2\delta_1^4}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\left\{1-r+\frac{\sqrt{c'(\tau_s)}\delta_1^2\tau_p(1-\chi r)}{\delta_1+\sqrt{c'(\tau_s)}\left(\tau_\theta+\tau_y\right)}-\frac{\sqrt{c'(\tau_s)}\delta_1^2\tau_p(1-\chi)}{\delta_1+\sqrt{c'(\tau_s)}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}\right\}
\end{aligned}$$

and plugging this expression into mathematica, using (77) twice and then simplifying and then plugging (38) gives something positive. This expression is positive as the last two terms add to something positive, since the last is just the first with the denominator bigger and the numerator smaller since

$$(1-\chi r) > (1-\chi)$$
$$\chi r < \chi$$
$$r < 1$$

From the comparative statics of δ_3, we immediately get the comparative statics if $\gamma_3 = \frac{\delta_3}{1-\delta_3}$ which is an increasing transformation. □

Proof of Proposition 10: Computing

$$\begin{aligned}
\frac{\partial}{\partial \tau_p}\left(\tau_\theta + \tau_y + \tau_s + \tau_p \delta_1^2\right) &= \frac{\partial \tau_s}{\partial \tau_p} + \delta_1^2 + 2\delta_1 \tau_p \frac{\partial \delta_1}{\partial \tau_p} \\
&= \frac{-g_\delta f_{\tau_p}}{g_\delta f_{\tau_s} - g_{\tau_s} f_\delta} + \delta_1^2 + 2\delta_1 \tau_p \frac{g_{\tau_s} f_{\tau_p}}{g_\delta f_{\tau_s} - g_{\tau_s} f_\delta} \\
&= \frac{1}{g_\delta f_{\tau_s} - g_{\tau_s} f_\delta}\left\{-g_\delta f_{\tau_p} + \delta_1^2\left(g_\delta f_{\tau_s} - g_{\tau_s} f_\delta\right) + 2\delta_1 \tau_p g_{\tau_s} f_{\tau_p}\right\} \\
&\propto -g_\delta f_{\tau_p} + \delta_1^2\left(g_\delta f_{\tau_s} - g_{\tau_s} f_\delta\right) + 2\delta_1 \tau_p g_{\tau_s} f_{\tau_p}
\end{aligned}$$

In mathematica, plug in, use (77) wherever apparent, and replace τ_s. This yields a linear equation in r given δ_1, which can be solved for the implicit equation

$$\mathrm{R}_p(\chi) = \chi\left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2 \tau_p\right)}\right)^2 = \chi\left(\frac{\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p}{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2 \tau_p}\right)^2$$

such that $\frac{\partial}{\partial \tau_p}\left(\tau_\theta + \tau_y + \tau_s + \tau_p \delta_1^2\right) > 0$ if $r < \mathrm{R}_p$. To derive the properties of R_p, let us define

$$k(\chi, r) = \chi\left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2 \tau_p\right)}\right)^2 - r = 0$$

To show that $\mathrm{R}_p(\chi)$ is increasing in χ, we need to establish that

$$\mathrm{R}_p'(\chi) = -\frac{k_\chi}{k_r} > 0$$

(we want $k_x > 0, k_r < 0$) By mathematica, it is easy to show that,

$$\begin{aligned}
k_\chi \propto &\left(\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)\right)\left(\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y - \chi\delta_1^2 \tau_p\right)\right) \\
&+ \frac{\partial \delta_1}{\partial \chi} 2\sqrt{c}(1-\chi)\tau_p \delta_1\left(\delta_1 + 2\sqrt{c}\left(\tau_\theta + \tau_y\right)\right)
\end{aligned}$$

Note that $\frac{\partial \delta_1}{\partial \chi} > 0$, and – if the first term is positive – we have $k_\chi > 0$. This is the case for valid parameters. Suppose it is not, i.e.

$$\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y - \chi\delta_1^2 \tau_p\right) < 0$$

$$\tau_p > \frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)}{\sqrt{c}\chi\delta_1^2}$$

But note that $r = \chi\left(\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}\right)^2$ is increasing in τ_p (purely algebraically), so this would imply that

$$\begin{aligned}
r = \chi\left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2\tau_p\right)}\right)^2 &> \chi\left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}{\sqrt{c}\chi\delta_1^2}\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2\frac{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}{\sqrt{c}\chi\delta_1^2}\right)}\right)^2 \\
&= \chi\left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \frac{\delta_1}{\chi\sqrt{c}} + \frac{1}{\chi}\left(\tau_\theta + \tau_y\right)\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \frac{\delta_1}{\sqrt{c}} + \left(\tau_\theta + \tau_y\right)\right)}\right)^2 \\
&= \chi\left(\frac{\left(1 + \frac{1}{\chi}\right)\left(\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)\right)}{2\delta_1 + 2\sqrt{c}\left(\tau_\theta + \tau_y\right)}\right)^2 \\
&= \frac{(\chi+1)^2}{4\chi} = 1 + \frac{(\chi-1)^2}{4\chi} > 1
\end{aligned}$$

a contradiction. Hence we require that τ_p is smaller, otherwise the cutoff is trivial (i.e. greater than one). Hence, whenever we have an interior cutoff, we have $k_\chi > 0$.

It remains to show (to get $\frac{dr}{d\chi} > 0$) that $k_r < 0$. (with liear costs), which is the case, as

$$\begin{aligned}
k_r = \frac{\partial}{\partial r}\left(\chi\left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2\tau_p\right)}\right)^2 - r\right) &= \chi(1-\chi)\,2\sqrt{c}\tau_p\delta_1\frac{\left(\delta_1 + 2\sqrt{c}\left(\tau_\theta + \tau_y\right)\right)\left(\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right)\right)}{\left[\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2\tau_p\right)\right]^3}\frac{\partial\delta_1}{\partial r} - 1 \\
&\leq -1 < 0
\end{aligned}$$

In addition, we have that at the solution to $k = 0$, we always have $k_r < 0$, whence there exists a unique solution and therefore a cutoff $\mathrm{R}(\chi)$, such that $k \geq 0$ iff $r \leq \mathrm{R}(\chi)$, as we wanted to show. In addition, $\mathrm{R}' > 0$,and $\mathrm{R}(0) = 0$.

To see that the cutoff can be trivial, note that

$$\frac{\partial}{\partial\tau_p}\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right) \to 1 - \frac{1-\chi}{2\sqrt{c}\left(\tau_\theta + \tau_y\right)}$$

as $r \to 1 - \sqrt{c}\left(\tau_\theta + \tau_y\right)$, which is of ambiguous sign. □

A.3 Welfare

$$\begin{aligned}
\mathrm{W}(\alpha,\delta) &= \mathbb{E}\left[-(1-r)(a_i-\theta)^2 - r(a_i-\bar{a})^2\right] \\
&= \mathbb{E}\left[-(1-r)(\alpha_1 s_i + \alpha_2 y + \alpha_3 p - \theta)^2 - r(\alpha_1 s_i + \alpha_2 y + \alpha_3 p - \bar{a})^2\right] \\
&= \mathbb{E}\left[-(1-r)\left(\alpha_1(\theta + z_s) + \alpha_2\left(\theta + z_y\right) + \alpha_3\left(\frac{\delta_1+\delta_2}{1-\delta_3}\theta + \frac{\delta_2}{1-\delta_3}z_y + \frac{1}{1-\delta_3}z_p\right) - \theta\right)^2 - r\left(\alpha_1(\theta + z_s) + \alpha_2\left(\theta + z_y\right) + \alpha_3\left(\frac{\delta_1+\delta_2}{1-\delta_3}\right.\right.\right. \\
&= -\frac{1}{(1-\delta_3)^2\tau_\theta\tau_y\tau_s\tau_p}\left\{\left[(\alpha_2(1-\delta_3) + \alpha_3\delta_2)^2 + \delta_2(\delta_2 - 2\alpha_3\delta_2 - 2\alpha_2(1-\delta_3))r\right]\tau_\theta\tau_s\tau_p + \tau_y\left\{\left(\alpha_3^2 - 2\alpha_3\delta_3 r + \delta_3^2 r\right)\tau_\theta\tau_s + \tau_p\left[\left[\right.\right.\right.\right.
\end{aligned}$$

where the last step is acomplished using Mathematica. In the case where $\alpha_i = \delta_i$, we get

$$
\begin{aligned}
W(\delta) &= -\frac{1}{(1-\delta_3)^2\,\tau_\theta\tau_y\tau_s\tau_p}\left\{\delta_2^2(1-r)\tau_\theta\tau_s\tau_p + \tau_y\left\{\delta_3^2(1-r)\tau_\theta\tau_s + (1-\delta_1-\delta_2-\delta_3)^2(1-r)\tau_s\tau_p + \delta_1^2(1-\delta_3)^2\tau_\theta\tau_p\right\}\right\} \\
&= -\frac{(1-r)}{(1-\delta_3)^2}\left\{\frac{\delta_2^2}{\tau_y} + \frac{\delta_3^2}{\tau_p} + \frac{(1-\delta_1-\delta_2-\delta_3)^2}{\tau_\theta}\right\} - \frac{\delta_1^2}{\tau_s} \qquad (79)
\end{aligned}
$$

Proof of Proposition 11: Taking FOC in (45), we obtain

$$
\begin{aligned}
W_{\delta_1} &= 2\frac{(1-r)}{(1-\delta_3)^2}\frac{(1-\delta_1-\delta_2-\delta_3)}{\tau_\theta} - 2\frac{\delta_1}{\tau_s} = 0 \\
W_{\delta_2} &= -\frac{(1-r)}{(1-\delta_3)^2}\left\{2\frac{\delta_2}{\tau_y} - 2\frac{(1-\delta_1-\delta_2-\delta_3)}{\tau_\theta}\right\} = 0 \\
W_{\delta_3} &= -\frac{2(1-r)}{(1-\delta_3)^4}\left\{\frac{\delta_2^2}{\tau_y}(1-\delta_3) + \frac{\delta_3(1-\delta_3)^2 + \delta_3^2(1-\delta_3)}{\tau_p} - \frac{(1-\delta_1-\delta_2-\delta_3)(1-\delta_3)^2 - (1-\delta_1-\delta_2-\delta_3)^2(1-\delta_3)}{\tau_\theta}\right\} = 0
\end{aligned}
$$

Note that the last two equations simplify to a linear system in δ_2, δ_3, which we solve for them as a function of δ_1:

$$
\begin{aligned}
\frac{\delta_2}{\tau_y} - \frac{(1-\delta_1-\delta_2-\delta_3)}{\tau_\theta} &= 0 \\
\frac{\delta_2^2}{\tau_y} + \frac{\delta_3(1-\delta_3) + \delta_3^2}{\tau_p} - \frac{(1-\delta_1-\delta_2-\delta_3)(1-\delta_3) - (1-\delta_1-\delta_2-\delta_3)^2}{\tau_\theta} &= 0
\end{aligned}
$$

which yields

$$
\delta_2 = \frac{\tau_y(1-\delta_1-\delta_3)}{(\tau_\theta + \tau_y)}
$$

$$
\begin{aligned}
\frac{\delta_2^2}{\tau_y} + \frac{\delta_3}{\tau_p} - (1-\delta_1-\delta_2-\delta_3)\frac{(\delta_1+\delta_2)}{\tau_\theta} &= \frac{\tau_y(1-\delta_1-\delta_3)^2}{(\tau_\theta+\tau_y)^2} + \frac{\delta_3}{\tau_p} - \left(1-\delta_1 - \frac{\tau_y(1-\delta_1-\delta_3)}{(\tau_\theta+\tau_y)} - \delta_3\right)\frac{\left(\delta_1 + \frac{\tau_y(1-\delta_1-\delta_3)}{(\tau_\theta+\tau_y)}\right)}{\tau_\theta} \\
&= \frac{\tau_y(1-\delta_1-\delta_3)^2}{(\tau_\theta+\tau_y)^2} + \frac{\delta_3}{\tau_p} - \left(\frac{(1-\delta_1-\delta_3)}{(\tau_\theta+\tau_y)}\right)\left(\delta_1 + \frac{\tau_y(1-\delta_1-\delta_3)}{(\tau_\theta+\tau_y)}\right) \\
&= \frac{\delta_3}{\tau_p} - \left(\frac{(1-\delta_1-\delta_3)}{(\tau_\theta+\tau_y)}\right)\delta_1
\end{aligned}
$$

and hence

$$
\begin{aligned}
\delta_2 &= \frac{\tau_y(1-\delta_1)}{\tau_\theta + \tau_y + \tau_p\delta_1} \\
\delta_3 &= \frac{\delta_1(1-\delta_1)\tau_p}{\tau_\theta + \tau_y + \tau_p\delta_1}.
\end{aligned}
$$

Plugging these two expressions into the first condition, we get

$$
\begin{aligned}
(1-r)\tau_s\frac{\delta_2}{\tau_y} - \delta_1(1-\delta_3)^2 &= 0 \\
(1-r)\tau_s\frac{(1-\delta_1)}{\tau_\theta+\tau_y+\tau_p\delta_1} - \delta_1\left(\frac{\tau_\theta+\tau_y+\tau_p\delta_1^2}{\tau_\theta+\tau_y+\tau_p\delta_1}\right)^2 &= 0
\end{aligned}
$$

$$(1-r)\tau_s(1-\delta_1)\left(\tau_\theta+\tau_y+\tau_p\delta_1\right)-\delta_1\left(\tau_\theta+\tau_y+\tau_p\delta_1^2\right)^2=0$$

$$\delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\underbrace{\left(\frac{\tau_\theta+\tau_y+\tau_p\delta_1^2}{\tau_\theta+\tau_y+\tau_p\delta_1}\right)}_{\text{efficiency adjustment}}-(1-r)\tau_s(1-\delta_1)=0$$

and using the envelope condition we get

$$\delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\underbrace{\left(\frac{\tau_\theta+\tau_y+\tau_p\delta_1^2}{\tau_\theta+\tau_y+\tau_p\delta_1}\right)}_{\text{efficiency adjustment}}-(1-r)\frac{\delta_1}{\sqrt{c}}(1-\delta_1)=0$$

$$\hat{f}(\delta_1)=\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\underbrace{\left(\frac{\tau_\theta+\tau_y+\tau_p\delta_1^2}{\tau_\theta+\tau_y+\tau_p\delta_1}\right)}_{\text{efficiency adjustment}}-(1-r)\frac{1}{\sqrt{c}}(1-\delta_1)=0$$

Lemma 3. *We have $\hat{f}_\delta>0$ for all $\delta_1\in(0,1)$.*

Proof of Lemma:

$$\begin{aligned}
\hat{f}_\delta &= \frac{\left[2\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\delta_1\tau_p\right]\left(\tau_\theta+\tau_y+\tau_p\delta_1\right)-\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)^2}{\left(\tau_\theta+\tau_y+\tau_p\delta_1\right)^2}+(1-r)\frac{1}{\sqrt{c}}\\
&=\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\frac{[2\delta_1-1]\left(\tau_\theta+\tau_y\right)+\delta_1^2\tau_p}{\left(\tau_\theta+\tau_y+\tau_p\delta_1\right)^2}+(1-r)\frac{1}{\sqrt{c}}\\
&=\frac{\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\left(\tau_\theta+\tau_y+\tau_p\delta_1\right)^2}\left[\frac{(1-r)}{\sqrt{c}}\frac{\left(\tau_\theta+\tau_y+\tau_p\delta_1\right)^2}{\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\sqrt{c}}-[2\delta_1-1]\left(\tau_\theta+\tau_y\right)+\delta_1^2\tau_p\right]\\
&\geq\frac{\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\left(\tau_\theta+\tau_y+\tau_p\delta_1\right)^2}\left[\frac{(1-r)}{\sqrt{c}}\left(\frac{\left(\tau_\theta+\tau_y+\tau_p\delta_1\right)^2}{\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}-[2\delta_1-1]\right)+\delta_1^2\tau_p\right]
\end{aligned}$$

and since

$$\begin{aligned}
\frac{\left(\tau_\theta+\tau_y+\tau_p\delta_1\right)^2}{\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}-[2\delta_1-1] &\propto \tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)+\left(\tau_\theta+\tau_y+\tau_p\delta_1\right)^2-2\delta_1\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\\
&=\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)+\left(\tau_\theta+\tau_y\right)^2+2\left(\tau_\theta+\tau_y\right)\tau_p\delta_1+\tau_p\delta_1^2\tau_p-2\delta_1\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\\
&=\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)+\left(\tau_\theta+\tau_y\right)^2+\tau_p\delta_1^2\tau_p-2\delta_1\tau_p\delta_1^2\tau_p\\
&=\tau_p\left(\tau_\theta+\tau_y\right)+\left(\tau_\theta+\tau_y\right)^2+(2-2\delta_1)\tau_p\delta_1^2\tau_p>0
\end{aligned}$$

we obtain $\hat{f}_\delta>0$. ▲

By the Lemma, there is a unique interior solution, as $\hat{f}(0)=\left(\tau_\theta+\tau_y\right)-(1-r)\frac{1}{\sqrt{c}}<0$ by the natural limit and $\hat{f}(1)=\tau_\theta+\tau_y+\tau_p>0$.

Reformulating this expression, we get the desired representation

$$\delta_1\left(\tau_\theta+\tau_y+\frac{\tau_\theta+\tau_y+\delta_1\tau_p}{\tau_\theta+\tau_y+\delta_1^2\tau_p}\tau_s+\delta_1^2\tau_p\right)\left(\frac{\tau_\theta+\tau_y+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\delta_1\tau_p}\right)=(1-r+r\delta_1)\tau_s$$

$$\delta_1 = (1-r+r\delta_1)\tau_s \frac{\tau_\theta+\tau_y+\delta_1\tau_p}{\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\left(\tau_\theta+\tau_y+\frac{\tau_\theta+\tau_y+\delta_1\tau_p}{\tau_\theta+\tau_y+\delta_1^2\tau_p}\tau_s+\delta_1^2\tau_p\right)}$$

$$\delta_1 = (1-r+r\delta_1)\tau_s \frac{1}{\left(\frac{\tau_\theta+\tau_y+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\delta_1\tau_p}\right)\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)+\tau_s}$$

as we needed to show. □

Proof of Proposition 12: Comparing (38) and (49), (51) follows immediately. □

Proof of Proposition 14: Since $\hat{f}_\delta > 0$ (Lemma 3), we know that $\hat{f}(\delta_1) < 0$ implies that there is underacquisition and $\hat{f}(\delta_1) > 0$ implies that there is overacquisition. Plugging the equilibrium δ_1 and using the fact that $f(\delta_1) = 0$

$$\hat{f}(\delta_1) = \left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\underbrace{\left(\frac{\tau_\theta+\tau_y+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\delta_1\tau_p}\right)}_{\text{efficiency adjustment}} - (1-r)\frac{1}{\sqrt{c}}(1-\delta_1) =$$

$$\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)+\delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\left(\frac{\tau_\theta+\tau_y+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\delta_1\tau_p}-1\right)-(1-r)\frac{1}{\sqrt{c}}(1-\delta_1) = using f$$

$$\delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\left(\frac{\tau_\theta+\tau_y+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\delta_1\tau_p}-1\right)+\chi\frac{1}{\sqrt{c}}\frac{1}{\tau_\theta+\frac{\delta_1}{\sqrt{c}}+\tau_y}[1+r(\delta_1-1)]\delta_1^2\tau_p$$

Note that at $\chi = 0$, this expression is negative unless τ_p is zero, whence it is zero, and hence, we have too little δ_1. As δ_1 EQ is increasing in χ, we are below efficient initially, maybe not for χ sufficiently large. There exists a χ with $\delta_1^\chi = \delta_1^{eff}$ iff $\hat{f}(\delta_1^{\chi=1}) > 0$ (by $\hat{f}_\delta > 0$)

$$\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\underbrace{\left(\frac{\tau_\theta+\tau_y+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\delta_1\tau_p}\right)}_{\text{efficiency adjustment}} - (1-r)\frac{1}{\sqrt{c}}(1-\delta_1) \propto$$

$$\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-(1-r)\frac{1}{\sqrt{c}}\left(1-\frac{1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)}{1-r}\right)\left[\tau_\theta+\tau_y+\delta_1\tau_p\right] =$$

$$\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\left(\tau_\theta+\tau_y\right)\left[\tau_\theta+\tau_y+\delta_1\tau_p\right] =$$

$$\left(\tau_\theta+\tau_y\right)^2+2\left(\tau_\theta+\tau_y\right)\delta_1^2\tau_p+\left(\delta_1^2\tau_p\right)^2-\left(\tau_\theta+\tau_y\right)^2-\left(\tau_\theta+\tau_y\right)\delta_1\tau_p =$$

$$2\left(\tau_\theta+\tau_y\right)\delta_1^2\tau_p+\left(\delta_1^2\tau_p\right)^2-\left(\tau_\theta+\tau_y\right)\delta_1\tau_p =$$

$$\delta_1\tau_p\left\{2\left(\tau_\theta+\tau_y\right)\delta_1+\delta_1^3\tau_p-\left(\tau_\theta+\tau_y\right)\right\} =$$

So we get a cutoff in τ_p.

$$\tau_p \geq \frac{\left(\tau_\theta+\tau_y\right)-2\left(\tau_\theta+\tau_y\right)\delta_1}{\delta_1^3}$$

If we have $\delta_1^{\chi=1} \geq \frac{1}{2}$, the fully cursed always overacquires. This gives the final condition

$$\delta_1^{\chi=1} = 1-\frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{1-r} \geq \frac{1}{2}$$

$$\frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{1-r}\leq\frac{1}{2}$$

establishing the Proposition. □

Proof of Proposition 15. Plugging the equilibrium expressions (13),(14) for δ_2,δ_3 into W_{δ_1} and using $f(\delta)=0$ yields

$$W_{\delta_1}=2\frac{(1-\chi)\,\delta_1^3\tau_p\left(\tau_\theta+\tau_y+\tau_s\right)}{(1-r)\,\tau_s^2\left(\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p\right)}$$

which is zero for $\chi=1$.

Plugging the equilibrium expressions for δ_2,δ_3 into W_{δ_2} yields

$$W_{\delta_2}=2\frac{\left[(1-r)\left(\tau_\theta+\tau_y+\tau_s+(1-\chi)\,\delta_1\tau_p\right)+(1-r\chi)\,\delta_1^2\tau_p\right]\left[(1-r)\,\chi\delta_1^2\tau_p\tau_s+(1-r)\,\tau_s\left(\tau_\theta+\tau_y+\tau_s\right)+\delta_1\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+(1-\right.\right.}{(1-r)\,\tau_\theta\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)^2}$$

note that the second factor in the numerator can be written as

$$\left(\tau_\theta+\tau_y+\tau_s\right)\left[\chi\frac{\tau_s}{\left(\tau_\theta+\tau_y+\tau_s\right)}\delta_1^2\tau_p\,(1-r+\delta_1 r)+\delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)+(1-r)\,(1-\delta_1)\,\tau_s\right]$$

where we recognize the factor as $f(\delta_1)=0$, whence $W_{\delta_2}(\delta^{EQ})=0$.

Plugging into W_{δ_3} and simplifying with a heavy use of $f(\delta)=0$, we get

$$W_{\delta_3}=-\frac{2\chi\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)\left[\delta_1\left(\tau_\theta+\tau_y\right)-(1-r)\,\tau_s\right]^2}{(1-r)^2\,\tau_s^3\left(\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p\right)}$$

which clearly is zero in the rational case. □

Proof of Proposition 16: To determine the impact of cursedness, we compute

$$\frac{dW_{eq}}{d\chi}=\frac{\partial W_{eq}}{\partial\delta_1}\frac{d\delta_1}{d\chi}+\frac{\partial W_{eq}}{\partial\delta_2}\frac{d\delta_2}{d\chi}+\frac{\partial W_{eq}}{\partial\delta_3}\frac{d\delta_3}{d\chi}$$
$$\frac{dW_{eq}}{d\chi}|_{\chi=0}=\frac{\partial W_{eq}}{\partial\delta_1}|_{\chi=0}\frac{d\delta_1}{d\chi}|_{\chi=0}$$

by Proposition 15. Therefore, we have

$$\begin{aligned}\frac{dW_{eq}}{d\chi}|_{\chi=0}&=\frac{\partial W_{eq}}{\partial\delta_1}|_{\chi=0}\frac{d\delta_1}{d\chi}|_{\chi=0}\\&=\left\{\frac{(1-r)}{(1-\delta_3)^2}\left\{2\frac{(1-\delta_1-\delta_2-\delta_3)}{\tau_\theta}\right\}-2\sqrt{c}\right\}\frac{d\delta_1}{d\chi}|_{\chi=0}\end{aligned}$$

and applying the rational δ_2,δ_3, $f(\delta_1)=0$ and the envelope condition gives

$$\begin{aligned}\frac{dW_{eq}}{d\chi}|_{\chi=0}&=\left\{\frac{(1-r)}{(1-\delta_3)^2}\left\{2\frac{(1-\delta_1-\delta_2-\delta_3)}{\tau_\theta}\right\}-2\sqrt{c}\right\}\frac{d\delta_1}{d\chi}|_{\chi=0}\\&=\left\{\frac{\left((1-r)\tau_s+\delta_1^2\tau_p\right)^2}{(1-r)\,\tau_s^2}\left\{2\frac{\frac{(1-r)\tau_s-\tau_y\delta_1-\delta_1\left((1-r)\tau_s+\delta_1^2\tau_p\right)}{(1-r)\tau_s+\delta_1^2\tau_p}}{\tau_\theta}\right\}-2\sqrt{c}\right\}\frac{d\delta_1}{d\chi}|_{\chi=0}\end{aligned}$$

$$= 2\left\{\frac{\left((1-r)\tau_s+\delta_1^2\tau_p\right)}{(1-r)\,\tau_s^2}\left\{\frac{(1-r)\,(1-\delta_1)\,\tau_s-\tau_y\delta_1-\delta_1^3\tau_p}{\tau_\theta}\right\}-\sqrt{c}\right\}\frac{\mathrm{d}\delta_1}{\mathrm{d}\chi}|_{\chi=0}$$

$$= 2\left\{\frac{\left((1-r)\tau_s+\delta_1^2\tau_p\right)}{(1-r)\,\tau_s^2}\delta_1-\sqrt{c}\right\}\frac{\mathrm{d}\delta_1}{\mathrm{d}\chi}|_{\chi=0}$$

$$= 2\left\{\frac{\left((1-r)\sqrt{c}+\delta_1\,\tau_p c\right)}{(1-r)}-\sqrt{c}\right\}\frac{\mathrm{d}\delta_1}{\mathrm{d}\chi}|_{\chi=0}$$

$$= 2\left\{\frac{\delta_1\,\tau_p c}{(1-r)}\right\}\frac{\mathrm{d}\delta_1}{\mathrm{d}\chi}|_{\chi=0} > 0$$

which completes the proof for the derivative at $\chi = 0$.

In the fully cursed case, using Proposition 15 we know that

$$\frac{\mathrm{d}W_{eq}}{\mathrm{d}\chi}|_{\chi=1} = \frac{\partial W_{eq}}{\partial\delta_3}|_{\chi=1}\frac{\mathrm{d}\delta_3}{\mathrm{d}\chi}|_{\chi=1}$$

Plugging in the fully cursed weights,

$$\delta_1 = \frac{(1-r)\,\tau_s}{(1-r)\,\tau_\theta+\tau_y+\tau_s}$$

$$\delta_2 = \frac{\tau_y}{\tau_\theta+\tau_y+(1-r)\tau_s}$$

into

$$W_{\delta_3} = -2\,(1-r)\left\{\frac{\delta_2^2}{\tau_y}+\frac{(1-\delta_1-\delta_2)^2}{\tau_\theta}\right\}+(1-r)\left\{2\frac{(1-\delta_1-\delta_2)}{\tau_\theta}\right\}$$

we get

$$W_{\delta_3} = -2\,(1-r)\left\{\frac{\tau_\theta+\tau_y}{\left(\tau_\theta+\tau_y+(1-r)\tau_s\right)^2}-\frac{1}{\tau_\theta+\tau_y+(1-r)\tau_s}\right\}$$

$$= -2\,(1-r)\,\frac{\delta_1}{(1-r)\tau_s}\left\{\frac{\tau_\theta+\tau_y}{\left(\tau_\theta+\tau_y+(1-r)\tau_s\right)}-1\right\}$$

$$= 2\,(1-r)\,\frac{\delta_1}{(1-r)\tau_s}\left\{\frac{(1-r)\tau_s}{\left(\tau_\theta+\tau_y+(1-r)\tau_s\right)}\right\}$$

$$= 2\sqrt{c}\delta_1^{\mathrm{FC}}$$

Therefore, we have

$$\frac{\mathrm{d}W_{eq}}{\mathrm{d}\chi}|_{\chi=1} = \frac{\partial W_{eq}}{\partial\delta_3}|_{\chi=1}\frac{\mathrm{d}\delta_3}{\mathrm{d}\chi}|_{\chi=1}$$

$$= 2\sqrt{c}\delta_1^{\mathrm{C}}\frac{d\delta_3}{d\chi}|_{\chi=1} < 0$$

where $\frac{d\delta_3}{d\chi}|_{\chi=1} < 0$ follows from

$$\frac{d\delta_3}{d\chi}|_{\chi=1} = \left(\frac{\partial\delta_3}{\partial\delta_1}\frac{\partial\delta_1}{\partial\chi}\right)|_{\chi=1}+\frac{\partial\delta_3}{\partial\chi}|_{\chi=1}$$

$$= 0 - \frac{\delta_1^2 \tau_p \left(\tau_\theta + \tau_y + \tau_s\right)}{(1-r)\,\tau_s \left(\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p\right)} < 0$$

so once you are pretty cursed, more is not a bliss anymore. This establishes the result about the derivative at $\chi = 1$. □

Derivation of Formula 54: Plug δs into cursed welfare and get

$$\begin{aligned}
W^C &= -\frac{2(1-r)\sqrt{c} - c\left(\tau_\theta + \tau_y\right)}{1-r} \\
&= -\sqrt{c}\,\frac{2(1-r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)}{1-r} \\
&= -\sqrt{c}\left(1 + \delta_1^C\right)
\end{aligned}$$

In the rational case

$$\begin{aligned}
W^R &= -\frac{2(1-r)\sqrt{c}\delta_1^R + c\left(\tau_\theta + \tau_y + \left(\delta_1^R\right)^2 \tau_p\right)}{1-r} \\
&= -\frac{(1-r)\sqrt{c}\delta_1^R + c\left(\tau_\theta + \tau_y + \delta_1^R \frac{1}{\sqrt{c}}(1-r) + \left(\delta_1^R\right)^2 \tau_p\right)}{1-r} \\
&= -\frac{(1-r)\sqrt{c}\delta_1^R + (1-r)\sqrt{c}}{1-r} \\
&= -\sqrt{c}\left(1 + \delta_1^R\right)
\end{aligned}$$

□

by using f.

Proof of Proposition 17: At $\chi = 0$, $\chi = 1$ we have $W = -\sqrt{c}(1+\delta_1)$. Hence, $\frac{dW}{d\tau} \propto -\frac{d\delta_1}{d\tau}$ and the comparative statics wrt τ_s follow immediately from Proposition 8. For costs, note that in the rational case, direct computation yields

$$\begin{aligned}
\frac{\partial W^{EQ(0)}}{\partial c} &= \frac{1 - r - \sqrt{c}\left(\tau_\theta + \tau_y - \delta_1 \tau_p\right)}{-\sqrt{c}(1-r) - 2c\delta_1 \tau_p} \\
&= -\frac{1 - r - \sqrt{c}\left(\tau_\theta + \tau_y\right) + \sqrt{c}\delta_1 \tau_p}{\sqrt{c}(1-r) + 2c\delta_1 \tau_p} < 0
\end{aligned}$$

which is negative by the natural limit condition. In the fully cursed case, note that

$$\begin{aligned}
\frac{\partial W^{EQ(1)}}{\partial c} &= -\frac{1}{2\sqrt{c}}\left(\frac{2(1-r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)}{1-r}\right) + \sqrt{c}\,\frac{1}{2\sqrt{c}}\,\frac{\left(\tau_\theta + \tau_y\right)}{1-r} \\
&= -\frac{1}{\sqrt{c}}\left(\frac{(1-r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)}{1-r}\right) = -\frac{\delta_1}{\sqrt{c}} < 0
\end{aligned}$$

For $\tau_p = 0$, the rational and (partially) cursed equilibria coincide, hence the above comparative statics prevail, and by continuity, this extends to small but interior τ_p.

To see the paradoxical welfare results consider welfare in the transparent limit. The welfare formula follows immediately by plugging δ^∞ from Prop 7 into welfare, after taking $\tau_p \to \infty$. It has the following comparative statics

Lemma 4. *The welfare in the transparent limit is*

- decreasing in τ_θ, τ_y if and only if $r > 0$ and $\chi \leq \frac{2r-1}{r}$.
- Decreasing in cursedness

$$\frac{\partial W^\infty}{\partial \chi} < 0$$

- Decreasing in costs., unless $r < 0$, when there exists a region, $\chi \in (\sqrt{c}\frac{\tau_\theta+\tau_y}{1-r}, \sqrt{c}(1-2r)\frac{\tau_\theta+\tau_y}{1-r\left(2-r+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)})$ such that costs increase welfare.

Proof of Lemma: To see the comparative static, note that the coefficient of $\tau_\theta + \tau_y$ is $1-2r+r\chi$. Consider the case where $r < 0$, then this expression is negative only for $\chi > 2$, so this case is irrelevant. Instead, with $r > 0$, we get that the impact of τ_θ, τ_y is negative iff $\chi \leq 2 - \frac{1}{r}$.

Furthermore

$$\begin{aligned}\frac{\partial W^\infty}{\partial \chi} &= 2\sqrt{c}\frac{\sqrt{c}r\left(\tau_\theta + \tau_y\right) - (1-r)}{(1-\chi r)^2} \\ &\leq 2\sqrt{c}\frac{r(1-r) - (1-r)}{(1-\chi r)^2} = -2\sqrt{c}\frac{(1-r)^2}{(1-\chi r)^2} < 0\end{aligned}$$

The derivative wrt. costs is

$$\frac{\partial W^\infty}{\partial c} \propto -\frac{\chi}{\sqrt{c}}(1-r)^2 + (1-2r+r\chi)\left(\tau_\theta + \tau_y\right)$$

$$\begin{aligned}-\frac{\chi}{\sqrt{c}}(1-r)^2 + (1-2r+r\chi)\left(\tau_\theta + \tau_y\right) \geq 0 \\ \chi\left[(1-r)^2 - r\sqrt{c}\left(\tau_\theta + \tau_y\right)\right] \leq \sqrt{c}(1-2r)\left(\tau_\theta + \tau_y\right)\end{aligned}$$

Hence, there is two cases we need to consider. First, if $(1-r)^2 - r\sqrt{c}\left(\tau_\theta + \tau_y\right) < 0$: Note that this can only be the case if $r > \frac{1}{2}$, since otherwise by the natural limit condition

$$(1-r)^2 - r\sqrt{c}\left(\tau_\theta + \tau_y\right) \geq (1-r)^2 - r(1-r) = (1-2r)(1-r) > 0$$

. Then, we obtain a lower bound for χ:

$$\chi \geq \sqrt{c}(1-2r)\frac{\tau_\theta + \tau_y}{(1-r)^2 - r\sqrt{c}\left(\tau_\theta + \tau_y\right)}$$

but this bound rules out increasing costs, since

$$\begin{aligned}\sqrt{c}(1-2r)\frac{\tau_\theta + \tau_y}{(1-r)^2 - r\sqrt{c}\left(\tau_\theta + \tau_y\right)} \geq 1 \\ \iff \sqrt{c}(1-r)\left(\tau_\theta + \tau_y\right) \leq (1-r)^2 \\ \iff \sqrt{c}\left(\tau_\theta + \tau_y\right) \leq (1-r)\end{aligned}$$

which is guaranteed by the natural limit condition.

Second, if $(1-r)^2 - r\sqrt{c}\left(\tau_\theta + \tau_y\right) > 0$: we get an upper bound

$$\chi \leq \sqrt{c}(1-2r)\frac{\tau_\theta + \tau_y}{(1-r)^2 - r\sqrt{c}\left(\tau_\theta + \tau_y\right)}$$

. The resulting interval is nontrivial only if $r < 0$, as

$$\begin{aligned}
\sqrt{c}\frac{\tau_\theta + \tau_y}{1-r} &< \sqrt{c}(1-2r)\frac{\tau_\theta + \tau_y}{(1-r)^2 - r\sqrt{c}\left(\tau_\theta + \tau_y\right)} \\
(1-r)^2 - r\sqrt{c}\left(\tau_\theta + \tau_y\right) &< (1-2r)(1-r) \\
r\sqrt{c}\left(\tau_\theta + \tau_y\right) &> r(1-r) \\
r\left[\frac{\sqrt{c}\left(\tau_\theta + \tau_y\right)}{(1-r)}\right] &> 1 \\
\iff r &< 0
\end{aligned}$$

by the natural limit. ▲

The Lemma establishes the paradox cases. By continuity, they hold for sufficiently large τ_p.

It remains to be shown that welfare is always increasing in τ_p. Note that

$$\frac{\mathrm{dW}}{\mathrm{d}\tau_p} = \frac{\partial \mathrm{W}}{\partial \tau_p} + \frac{\partial \mathrm{W}}{\partial \delta_1}\frac{\mathrm{d}\delta_1}{\mathrm{d}\tau_p} + \frac{\partial \mathrm{W}}{\partial \delta_2}\frac{\mathrm{d}\delta_2}{\mathrm{d}\tau_p} + \frac{\partial \mathrm{W}}{\partial \delta_3}\frac{\mathrm{d}\delta_3}{\mathrm{d}\tau_p} + \frac{\partial \mathrm{W}}{\partial \tau_s}\frac{\mathrm{d}\tau_s}{\mathrm{d}\tau_p}$$

Now, we proved that $\frac{\partial \mathrm{W}}{\partial \delta_2} = \frac{\partial \mathrm{W}}{\partial \tau_s} = 0$ for every χ. Hence

$$\frac{d\mathrm{W}}{d\tau_p} = \frac{\partial \mathrm{W}}{\partial \tau_p} + \frac{\partial \mathrm{W}}{\partial \delta_1}\frac{d\delta_1}{d\tau_p} + \frac{\partial \mathrm{W}}{\partial \delta_3}\frac{d\delta_3}{d\tau_p}$$

Simplifying this expression using the representation of $\frac{d\delta_1}{d\tau_p}$ with the noncompactified expressions, we obtain an expression that, after removing clearly signed factors, is proportional to sum of positive addenda plus

$$c\chi\delta_1^4\tau_p^2\left(1+\chi+\chi^2 r - 3\chi r\right)$$

Now, it is clear that we need $\left(1+\chi+\chi^2 r - 3\chi r\right) > 0$, since this term is the dominant term in τ_p and hence we otherwise have a negative expression as $\tau_p \to \infty$. But this turns out to be a nice fact of life: If $\chi \in [0,1]$, $r \leq 1$, then $h(\chi, r) := 1+\chi+\chi^2 r - 3\chi r \geq 0$ (strictly in the interior). To see that, notice that if $r < 0$, then

$$\frac{\partial}{\partial \chi}h(\chi, r) = 1 + 2\chi r - 3r = 1 + r(2\chi - 3) > 0$$

and

$$h(0, r) = 1 > 0$$

On the contrary, if $r \in (0,1)$, then

$$1+\chi+\chi^2 r - 3\chi r > r\left(1+\chi+\chi^2\right) - 3\chi r = r\left(1-2\chi+\chi^2\right) = r(1-\chi)^2 > 0$$

Completing the proof. □

A.4 Proofs for the Rational Agent in The Cursed World

Proof of Proposition 18: This follows immediately from shrewd observation of the derivation of the matching coefficients and g equations. More directly, the welfare of an agent playing loadings $\boldsymbol{\alpha}$ and private precision τ_s^R while the rest of (the average of) others play $\boldsymbol{\delta}$ is

$$W(\boldsymbol{\alpha},\boldsymbol{\delta}) = -\frac{\left\{\left[(\alpha_2(1-\delta_3)+\alpha_3\delta_2)^2+\delta_2(\delta_2-2\alpha_3\delta_2-2\alpha_2(1-\delta_3))r\right]\tau_\theta\tau_s\tau_p+\tau_y\left\{\left(\alpha_3^2-2\alpha_3\delta_3 r+\delta_3^2 r\right)\tau_\theta\tau_s+\tau_p\left[\left[(1-(1-\delta_3)(\alpha_1+\right.\right.\right.\right.}{(1-\delta_3)^2\tau_\theta}$$

By setting

$$\nabla_{\boldsymbol{\alpha}} W(\boldsymbol{\alpha},\boldsymbol{\delta}) = 0$$

we find the best response coefficients as a function of others' loadings. We get

$$(1-\alpha_2-\alpha_3(\delta_1+\delta_2)-\alpha_1(1-\delta_3)-\delta_3-r(1-\delta_1-\delta_2)+\delta_3(\alpha_2+r))\tau_s-\alpha_1(1-\delta_3)\tau_\theta = 0$$

$$(1-\alpha_2-\alpha_3(\delta_1+\delta_2)-\alpha_1(1-\delta_3)-\delta_3-r(1-\delta_1-\delta_2)+\delta_3(\alpha_2+r))\tau_y-(\alpha_2(1-\delta_3)+\delta_2(\alpha_3-r))\tau_\theta = 0$$

$$2\tau_p\tau_\theta(\delta_2(\alpha_2(1-\delta_3)+\delta_2(\alpha_3-r)))+\tau_y\left[-(\delta_1+\delta_2)(1-\alpha_1-\alpha_2-\alpha_3(\delta_1+\delta_2)-(1-\alpha_1-\alpha_2)\delta_3-r(1-\delta_1-\delta_2-\delta_3))\tau_p+(\alpha_3-\delta_3 r)\right.$$

the solution to this system is the $\boldsymbol{\alpha}$ in the proposition, which we can plug back in welfare to obtain the expression

$$\overline{W}(\boldsymbol{\delta}) = -\frac{(1-r)r\left(\delta_3^2\tau_y+\delta_2^2\tau_p\right)}{(1-\delta_3)^2\tau_y\tau_p}-\frac{(1-r)r(1-\delta_1-\delta_2-\delta_3)^2}{(1-\delta_3)^2\tau_\theta}-\frac{(1-(1-\delta_1)r)^2}{\tau_\theta+\tau_y+\tau_s^R+\delta_1^2\tau_p}-c\tau_s^R =$$

$$= \frac{(1-r)r}{(1-\delta_3)^2}\left[\frac{\delta_2^2}{\tau_y}+\frac{\delta_3^2}{\tau_p}+\frac{(1-\delta_1-\delta_2-\delta_3)^2}{\tau_\theta}\right]-\frac{(1-(1-\delta_1)r)^2}{\tau_\theta+\tau_y+\tau_s^R+\delta_1^2\tau_p}-c\tau_s^R$$

Differentiating this equation with respect to τ_s^R, we obtain the final condition.

□

Proof of Proposition 19: To derive equation 60 in the text, note that

$$\frac{(1-(1-\delta_1)r)^2}{\left(\tau_\theta+\tau_y+\tau_s^R+\delta_1^2\tau_p\right)^2}-c = 0$$

$$(1-(1-\delta_1)r)^2 = c\left[\tau_\theta+\tau_y+\tau_s^R+\delta_1^2\tau_p\right]^2$$

using (77) on the LHS we get

$$\left[\frac{\delta_1}{\tau_s\frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)}}\right]^2 = c\left[\tau_\theta+\tau_y+\tau_s^R+\delta_1^2\tau_p\right]^2$$

$$\left[\frac{\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\right] = \left[\tau_\theta+\tau_y+\tau_s^R+\delta_1^2\tau_p\right]$$

$$\frac{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s^R+\delta_1^2\tau_p} = 1+\frac{\chi\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s}$$

From there, $\tau_s \geq \tau_s^{\mathrm{R}}$ is immediate, since

$$\frac{\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p}{\tau_\theta + \tau_y + \tau_s^{\mathrm{R}} + \delta_1^2 \tau_p} = 1 + \frac{\chi \delta_1^2 \tau_p}{\tau_\theta + \tau_y + \tau_s} \geq 1$$

For the fully cursed equilibrium, the above implies

$$\tau_\theta + \tau_y + \tau_s = \tau_\theta + \tau_y + \tau_s^{\mathrm{R}} + \delta_1^2 \tau_p$$

and hence we have

$$\begin{aligned}
\tau_s^{\mathrm{R}} &= \tau_s - \delta_1^2 \tau_p \\
&= \frac{1}{\sqrt{c}} \frac{1 - r - \sqrt{c}\left(\tau_\theta + \tau_y\right)}{1 - r} - \tau_p \left(\frac{1 - r - \sqrt{c}\left(\tau_\theta + \tau_y\right)}{1 - r} \right)^2
\end{aligned}$$

Solving for $\tau_s^{\mathrm{R}} \geq 0$, we get positive information acquisition iff

$$\tau_\theta + \tau_y \in \left(\frac{(1-r)\left(1 - \frac{1}{\tau_p \sqrt{c}}\right)}{\sqrt{c}}, \frac{1-r}{\sqrt{c}} \right)$$

.

Now, for the Proposition, we have established $\tau_s^{\mathrm{R}} \leq \tau_s$. For the limit result, note that

$$\begin{aligned}
\tau_s^{\mathrm{R}} &= \frac{\left(\tau_\theta + \tau_y + \tau_s\right)\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p\right)}{\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p} - \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right) \\
&= \frac{\left(\tau_\theta + \tau_y + \tau_s\right)\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p\right) - \left(\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p\right)\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p} \\
&= \frac{\left(\tau_\theta + \tau_y + \tau_s\right)\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p\right) - \left(\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p\right)\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p} \\
&= \frac{\left(\tau_\theta + \tau_y + \tau_s\right)\tau_s - \chi \delta_1^2 \tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p}
\end{aligned}$$

and that $\tau_s \to \infty$ as $c \to 0$. From there, we have

$$\tau_s^{\mathrm{R}} = \frac{\left(\tau_\theta + \tau_y + \tau_s\right)\tau_s - \chi \delta_1^2 \tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p} \sim \frac{\tau_s^2}{\tau_s} \to \infty$$

We have $\tau_s^{\mathrm{R}} = 0$ for $\tau_\theta + \tau_y \leq \frac{(1-r)\left(1 - \frac{1}{\tau_p \sqrt{c}}\right)}{\sqrt{c}}$ in the fully cursed case, where $\tau_s > 0$.

To see nonmonotonicity in $\tau_\theta + \tau_y$ in the general case, note that as we approach the natural limit, we have

$$\frac{d\tau_s^{\mathrm{R}}}{d\tau_y}\Big|_{\sqrt{c} = \frac{1-r}{\tau_\theta + \tau_y}} = \frac{\tau_\theta + \tau_y}{\sqrt{c}\left(2\tau_\theta + \tau_y\right)} \frac{d\delta_1}{d\tau_y} \propto \frac{d\delta_1}{d\tau_y} < 0$$

and hence, local to this value, we always get a positive τ_s^{R}. However, for $\tau_\theta + \tau_y \leq \chi \frac{1-r}{\sqrt{c}}$ interior, we get that for $\tau_p \to \infty$

$$\tau_s^{\mathrm{R}} = \frac{\left(\tau_\theta + \tau_y + \tau_s\right)\tau_s - \chi \delta_1^2 \tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p} \sim -\delta_1^2 \tau_p < 0$$

which establishes the result. Interior nonmonotonicity follows by continuity.

To see nonmonotonicity in c, note that

1. $\frac{d\tau_s^{\mathrm{R}}}{dc} < 0$ in the natural limit $\sqrt{c} = \frac{1-r}{\tau_\theta+\tau_y}$. To see this, take the representation above, plug 38, take the derivative, set $\sqrt{c} = \frac{1-r}{\tau_\theta+\tau_y}$, $\delta_1 = 0$, then we get

$$\frac{d\tau_s^{\mathrm{R}}}{dc}\Big|_{\sqrt{c}=\frac{1-r}{\tau_\theta+\tau_y}} \to \frac{2(1-r)^4 \frac{\partial\delta_1}{\partial c}}{\left(\tau_\theta+\tau_y\right)^2} \propto \frac{\partial\delta_1}{\partial c} < 0$$

2. $\tau_s^{\mathrm{R}} = 0$ at the natural limit, as $0 \le \tau_s^{\mathrm{R}} \le \tau_s = 0$. Therefore, by (1.) and (2.), $\tau_s^{\mathrm{R}} > 0$ local to this upper bound on costs.
3. $\tau_s^{\mathrm{R}} \to \infty$ as $c \to 0$, as shown above.
4. For any c satisfying the existence of a transparent limit equilibrium ($\sqrt{c} \le \chi\frac{1-r}{\tau_\theta+\tau_y}$), there exists a τ_p sufficiently large such that $\left(\tau_s^{\mathrm{R}}\right)^{\mathrm{FOC}} < 0$. To see this, pick an interior c. Then, $\delta_1^\infty > 0$ and as $\tau_p \to \infty$

$$\left(\tau_s^{\mathrm{R}}\right)^{\mathrm{FOC}} \to -\delta_1^2\tau_p < 0$$

This establishes nonmonotonicity, as τ_s^{R} is large for small c, zero for intermediate c, but nonzero local to the natural limit.

To analyze the derivative in χ, we compute using the noncompact forms

$$\frac{d\tau_s^{\mathrm{R}}}{d\chi} \propto r - 2\sqrt{c}\delta_1\tau_p$$

It is apparent that τ_s^{R} is decreasing for $r \le 0$ and that it is increasing as $r \to 1-\sqrt{c}\left(\tau_\theta+\tau_y\right)$ approaches a strictly positive natural limit, as then $\delta_1 \to 0$. To see that we can have a hump shape, note that δ_1 is increasing in χ and hence

$$\frac{d^2\tau_s^{\mathrm{R}}}{d\chi d\chi} \propto -\frac{d\delta_1}{d\chi} \le 0$$

around $\frac{d\tau_s^{\mathrm{R}}}{d\chi} = 0$, which establishes a hump-shape (but, importantly, not necessarily concavity!). Clearly, all these comparative statics only apply for interior solutions, otherwise $\tau_s^{\mathrm{R}} \equiv 0$ locally. □

Proof of Proposition 20. For χ, we plug the general δ into the welfare expression, take derivative wrt χ, set $\chi = 1$, $\delta_1 = \delta_1^{\mathrm{FC}}$ and we get

$$\begin{aligned}\delta_1 &= 1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{1-r} \\ &= \frac{1}{1-r}\left(1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathrm{W}^{\mathrm{RPC}}}{\partial\chi} &= \frac{2\sqrt{c}(-1+r)\tau_p\left(-1+r+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2\left[r\left(\tau_\theta+\tau_y+\tau_s^{\mathrm{R}}+\tau_p\right)+\tau_p\left(-1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)\right]\left(-1+r\left(1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)\right)^2}{[\cdot]^2\left[1+\sqrt{c}\tau_p\left(-1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2+r^2\left(1+\sqrt{c}\left(\tau_\theta+\tau_y+\tau_p\right)\right)+r\left(-2+2c\tau_p\left(\tau_\theta+\tau_y\right)-\sqrt{c}\left(\tau_\theta+\tau_y+2\tau_p\right)\right)\right]} \\ &\propto -\frac{2\sqrt{c}\tau_p\delta_1^2(1-r)^3\left[r\left(\tau_\theta+\tau_y+\tau_s^{\mathrm{R}}+\tau_p\delta_1\right)-\tau_p\delta_1\right]}{1+\sqrt{c}\tau_p\left(-1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2+r^2\left(1+\sqrt{c}\left(\tau_\theta+\tau_y+\tau_p\right)\right)+r\left(-2-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)+2\tau_p r\left(c\left(\tau_\theta+\tau_y\right)-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)}\end{aligned}$$

As $r \to 1-\sqrt{c}\left(\tau_\theta+\tau_y\right)$ and let us assume that the natural limit occurs for $r>0$

$$\begin{aligned}
&\to -\frac{2\sqrt{c}\tau_p\delta_1^2 r\left(\tau_\theta+\tau_y\right)}{1+\sqrt{c}\tau_p r^2+r^2\left(1+(1-r)+\sqrt{c}\tau_p\right)+r(-2-(1-r))+2\tau_p r\left(\sqrt{c}(1-r)-(1-r)\right)}\\
&\propto -\frac{\delta_1^2}{1+\sqrt{c}\tau_p r^2+r^2\left(1+(1-r)+\sqrt{c}\tau_p\right)+r(-2-(1-r))+2\tau_p r\left(\sqrt{c}(1-r)-(1-r)\right)}\\
&= -\frac{\delta_1^2}{c^{\frac{3}{2}}\left(\tau_\theta+\tau_y\right)^3}\leq 0
\end{aligned}$$

so this is zero anyways as $\delta_1 \to 0$, but we want to know which sign it has locally, so it is negative. If I want to imitate a lot, I want them to be a bit less smart.

Consider now $r \leq 0$. Note that the numerator is always negative. Furthermore, note that the denominator is positive for $r=0$. We will show that it is positive for all $r \leq 0$ and hence the expression is positive for all $r \leq 0$. Note that is a convex quadratic function in r. Minimizing, we find that the minimum occurs at

$$\begin{aligned}
r^* &\propto 2-2c\tau_p\left(\tau_\theta+\tau_y\right)+\sqrt{c}\left(\tau_\theta+\tau_y+2\tau_p\right)\\
\text{denominator} &\propto 1-4c\tau_p\left(\tau_\theta+\tau_y\right)
\end{aligned}$$

Hence, either the minimum or the minimizer is positive. In either case, the denominator is positive for all $r \leq 0$, as it is for $r=0$. □

Proof of Proposition 21: To obtain the welfare of the rational agent in the fully cursed equilibrium, we simply plug action rule (56)-(59) and the equilibrium δs into the welfare equation to obtain, for the unconstrained case

$$\begin{aligned}
W^{\text{RFCU}} &= -\frac{(1-r)r\delta_2^2}{\tau_y\tau_p}-\frac{(1-r)r(1-\delta_1-\delta_2)^2}{\tau_\theta}-\frac{(1-(1-\delta_1)r)^2}{\tau_\theta+\tau_y+\delta_1^2\tau_p}-c\tau_s^{\text{R}}\\
&= -2\sqrt{c}+\frac{-2c^{3/2}(1-r)\tau_p\left(\tau_\theta+\tau_y\right)+c^2\tau_p\left(\tau_\theta+\tau_y\right)^2+c(1-r)\left(\tau_\theta+\tau_y+(1-r)\tau_p\right)}{(1-r)^2}
\end{aligned}$$

as well as

$$W^{\text{RFCC}} = -\frac{(1-r)r\delta_2^2}{\tau_y\tau_p}-\frac{(1-r)r(1-\delta_1-\delta_2)^2}{\tau_\theta}-\frac{(1-(1-\delta_1)r)^2}{\tau_\theta+\tau_y+\delta_1^2\tau_p}$$

For transparency, we obtain by direct computation that
with interior τ_s^{R}: $\frac{\partial W^{\text{RFC}}}{\partial\tau_p}=c\frac{\left(1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2}{(1-r)^2}=c\delta_1^2>0$,
and at corner solutions: $\frac{\partial W^{\text{RFC}corner}}{\partial\tau_p}=\frac{\delta_1^2\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2}{\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)^2}>0$.

For τ_θ,τ_y: Consider the derivative of W^{RFC} and let $\tau_\theta+\tau_y \to \frac{(1-r)\left(1-\frac{1}{\tau_p\sqrt{c}}\right)}{\sqrt{c}}$. Then, we get

$$\begin{aligned}
\frac{\partial W^{\text{RFCC}}}{\partial\tau_y} &= c\frac{1-r-(1-r)2\sqrt{c}\tau_p+2c\tau_p\left(\tau_\theta+\tau_y\right)}{(1-r)^2}=\frac{c}{1-r}\left(1-2\sqrt{c}\tau_p\delta_1^{\text{FC}}\right)\\
&= -\frac{c}{1-r}<0
\end{aligned}$$

Note that this result also holds for W^{RFCC},as the value function is $\mathcal{C}^1$. Taking instead the natural limit (note that local to the natural limit, we always have an interior solution) $\tau_\theta + \tau_y \to \frac{(1-r)}{\sqrt{c}}$ we have

$$\begin{aligned}\frac{\partial W^{RFC}}{\partial \tau_y} &= \frac{c}{1-r}\left(1 - 2\sqrt{c}\tau_p \delta_1^{FC}\right)\\ &\to \frac{c}{1-r}\end{aligned}$$

For c, let us first demonstrate a setting where an increasing in costs is beneficial for the rational agent. Taking the derivative of constrained welfare, let $r \to -\infty$ and hence $\delta_1 = 1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)} \to 1$, $\delta_2 = \frac{\sqrt{c}}{1-r}\tau_y \to 0$. Then,

$$\begin{aligned}W^R|_{\tau_s^R=0} &= -\frac{(1-r)r\left(\delta_3^2\tau_y + \delta_2^2\tau_p\right)}{(1-\delta_3)^2\tau_y\tau_p} - \frac{(1-r)r(1-\delta_1-\delta_2-\delta_3)^2}{(1-\delta_3)^2\tau_\theta} - \frac{(1-(1-\delta_1)r)^2}{\tau_\theta+\tau_y+\tau_s^R+\delta_1^2\tau_p} - c\tau_s^R\\ &= -\frac{(1-r)r\delta_2^2}{\tau_y} - \frac{(1-r)r(1-\delta_1-\delta_2)^2}{\tau_\theta} - \frac{(1-(1-\delta_1)r)^2}{\tau_\theta+\tau_y+\delta_1^2\tau_p}\\ &= -\frac{rc\tau_y}{(1-r)} - \frac{r\tau_\theta c}{(1-r)} - \frac{\left(1-\frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}r\right)^2}{\tau_\theta+\tau_y+\tau_p}\\ &\to c\tau_\theta + \tau_y c - \frac{\left(1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2}{\tau_\theta+\tau_y+\tau_p}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial}{\partial c}\left(c\tau_\theta + \tau_y c - \frac{\left(1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2}{\tau_\theta+\tau_y+\tau_p}\right) &= \tau_\theta + \tau_y - \frac{\left(\tau_\theta+\tau_y\right)}{\sqrt{c}}\frac{\left(1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)}{\tau_\theta+\tau_y+\tau_p}\\ &= \left(\tau_\theta+\tau_y\right)\left(\frac{\tau_\theta+\tau_y+\tau_p-\left(\frac{1}{\sqrt{c}}+\left(\tau_\theta+\tau_y\right)\right)}{\tau_\theta+\tau_y+\tau_p}\right)\\ &= \left(\tau_\theta+\tau_y\right)\left(\frac{\tau_p-\frac{1}{\sqrt{c}}}{\tau_\theta+\tau_y+\tau_p}\right) > 0\end{aligned}$$

since we consider the case $\tau_s^R = 0$, and therefore $\tau_p > \frac{1}{\delta_1\sqrt{c}} = \frac{1}{\sqrt{c}}$.

Note that even in the constrained case we can have $\frac{\partial}{\partial c}W^{RFCC} < 0$. To see this, consider the case $r = 0$. Then

$$W^R|_{r=0} = \max_{\tau_s^R \geq 0} -\frac{1}{\tau_\theta+\tau_y+\tau_s^R+\delta_1^2\tau_p} - c\tau_s^R$$

which is clearly decreasing in c as δ_1 is decreasing.

At an interior solution, we get (relevant for $\tau_p < \frac{1}{\delta_1\sqrt{c}}$)

$$\begin{aligned}\frac{\partial}{\partial c}W^{RFCU} &= -\frac{1}{\sqrt{c}} + \frac{-3\sqrt{c}(1-r)\tau_p\left(\tau_\theta+\tau_y\right) + 2c\tau_p\left(\tau_\theta+\tau_y\right)^2 + (1-r)\left(\tau_\theta+\tau_y+(1-r)\tau_p\right)}{(1-r)^2}\\ &= \frac{\left(\tau_\theta+\tau_y\right)}{(1-r)} - \frac{1}{\sqrt{c}} + \delta_1\tau_p\left(\delta_1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right)\end{aligned}$$

which is always negative by the natural limit condition if $\delta_1 \leq \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}$. In the other case, we have

$$\frac{\left(\tau_\theta+\tau_y\right)}{(1-r)} - \frac{1}{\sqrt{c}} + \delta_1\tau_p\left(\delta_1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right) \leq \frac{\left(\tau_\theta+\tau_y\right)}{(1-r)} - \frac{1}{\sqrt{c}} + \frac{1}{\sqrt{c}}\left(\delta_1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right)$$
$$= -\frac{\delta_1}{\sqrt{c}} + \frac{1}{\sqrt{c}}\left(\delta_1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right) = \frac{1}{\sqrt{c}}\left(-\frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right) < 0$$

concluding the proof. □

B Supplementary Appendix

Proposition 22. *There is no equilibrium with information acquisition and* $\delta_1 < 0$.

Proof. Towards a contradiction, let $\delta_1 < 0$. Then, $\tau_s = -\frac{\delta_1}{\sqrt{c}}$ and $\grave{f}$ reads

$$\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right) + \chi\frac{1}{\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1}\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p + \frac{1}{\sqrt{c}}(1-r)(1-\delta_1) = 0$$

Clearly, for δ_1=0 , the expression is strictly positve, whereas we have

$$\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right) + \chi\frac{1}{\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1}\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p + \frac{1}{\sqrt{c}}(1-r)(1-\delta_1) \to \delta_1^2\tau_p - r\chi r\delta_1^2\tau_p > 0$$

Hence, for an interior solution, we need to have an interior minimum, and therefore

$$2\delta_1\tau_p + \chi\frac{1}{\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)^2}\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p + \chi\frac{1}{\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1}\left[(1-r)+3r\delta_1^2\right]\tau_p - \frac{1}{\sqrt{c}}(1-r) = 0$$

But note that

$$2\delta_1\tau_p + \chi\frac{1}{\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)^2}\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p + \chi\frac{1}{\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1}\left[(1-r)+3\right.$$
$$\frac{1}{\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)^2}\left[2\delta_1\tau_p\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)^2 + \chi\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p + \chi\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)\left[2(1-r)\delta_1+3r\delta_1^2\right]\tau_p - \frac{1}{\sqrt{c}}(1-r)\left(\right.\right.$$
$$\propto 2\delta_1^3\tau_p - 4\delta_1\tau_p\sqrt{c}\left(\tau_\theta+\tau_y\right) + 2\delta_1\tau_p c\left(\tau_\theta+\tau_y\right)^2 + \chi\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p + \chi\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)\left[2(1-r)\delta_1+3r\delta_1^2\right]\tau_p - \frac{1}{\sqrt{c}}(1-r)\left(\right.$$
$$= 2\delta_1^3\tau_p(1-\chi r) - \delta_1^2\left[\frac{1}{\sqrt{c}}(1-r) + \left(4\tau_p - 3\chi r\tau_p\right)\sqrt{c}\left(\tau_\theta+\tau_y\right) + \chi(1-r)\tau_p\right] + \delta_1 2c\left(\tau_\theta+\tau_y\right)\left[(1-r)\left(1+\chi\tau_p\sqrt{c}\right)\tau_p + \tau_p c\left(\tau_\theta+\tau_y\right)\right] -$$

a contradiction. Hence, we cannot have an interior minimum whence $f > 0$ for all $\delta_1 < 0$ and there is no such equilibrium.

□

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