# WORKING PAPER NO. 607

# *Asset Bubbles and Product Market Competition*

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**Abstract**
This paper studies the interactions between asset bubbles and competition. I first document a negative industrylevel relationship between measures of stock market overvaluation and indicators of market power: larger overvaluation is associated with an increase in the number of firms, lower markups and a higher probability of negative earnings. I then construct multi-industry growth model featuring imperfect competition and rational bubbles that sheds light on these findings. By providing an entry or production subsidy, bubbles stimulate competition and reduce monopoly rents. When they are sufficiently large they can, however, lead to excessive entry and competition. I also show that imperfect competition depresses the interest rate, thereby relaxing the conditions for the emergence of rational bubbles.

**Keywords**: Rational Bubbles, Competition, Market Power, British Railway *Mania*, Dotcom Bubble

**JEL Classification:** E44, L13, L16

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# 1 Introduction

*"With valuations based on multiples of revenue, there's ample incentive to race for growth, even at the cost of low or even negative gross margins."*

"Dotcom history is not yet repeating itself, but it is starting to rhyme" *(Financial Times, 12/March/2015)*

Stock markets often experience fluctuations that seem too large to be driven entirely by fundamentals. Major historical events include the Mississippi and the South Sea bubbles of 1720 or the British railway mania of the 1840s. A more recent example is that of the US stock market during the dotcom bubble: between October 1995 and March 2000, the NASDAQ Composite index increased by almost sixfold to then collapse by 77% in the following two years. One common aspect among these episodes is that they appear to be concentrated on a particular market or industry, and to bring about a surge in competition.[1] The dotcom bubble constitutes a good example in this regard. In a period characterized by soaring prices of technology stocks, many internet firms appeared and went public.[2] Furthermore, as the valuation of young firms is typically based on metrics of size (revenues or market shares) and not on earnings, the new *dotcoms* often sought rapid growth and engaged in aggressive commercial practices, such as unusually low penetration prices. For example, some online companies appearing in this period provided their services completely for free (e.g. *Kozmo.com* or *UrbanFetch*) or made money payments to attract consumers (e.g. *AllAdvantage.com*). But even if following unsustainable business models, the new *dotcoms* often posed a threat to incumbents, which were in many cases forced to expand and enter the online market.[3]

The idea that the dotcom bubble was associated with a more competitive market structure is corroborated by indicators of market power. Figure 1 shows average price-cost markups for four high-tech industries that were at the center of the bubble. These are shown against the Shiller CAPE ratio, which is a popular measure for stock market overvaluation. A common pattern can be detected in these four industries – average markups decline from 1995 until the peak of the bubble in 2000/2001, and start increasing after the stock market crash. Note that at the peak of the bubble, the average firm in the four industries charged a price below its variable cost, implying that it exhibited negative earnings.[4] These patterns could be observed for both the full sample of firms and for the set of firms already active in 1995.

[1]For example, the Mississippi and the South Sea bubbles involved two trading companies (the *Compagnie d'Occident* in France and the *South Sea Company* in Great Britain) that engaged in innovative financial schemes (namely the issuance of stocks to finance the acquisition of government debt); the *British railway mania* was an episode that affected the British railway industry; the dotcom bubble was an event concentrated on a group of internet and high-tech industries.

[2]Goldfarb and Kirsch (2008) report that between 1994 and 2001 "approximately 50,000 companies solicited venture capital to exploit the commercialization of the internet"; among these, around 500 companies had an initial public offering.

[3]Some well-known examples involve corporations such as GE or Microsoft. These will be reviewed in Appendix A.

[4]The decline in price-cost markups was accompanied by a marked increase in the fraction of firms reporting negative earnings. At the peak of the bubble in 2000, roughly one half of all firms exhibited negative operating earnings. See Appendix B.1.

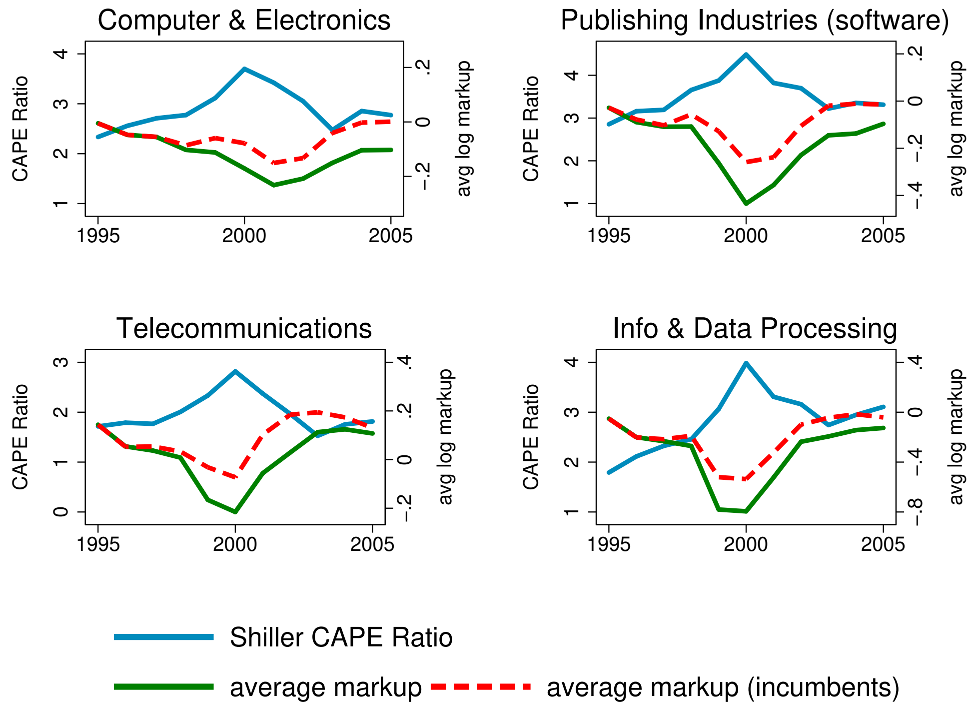



Figure 1: Average markups in the dotcom bubble

This figure shows the Shiller CAPE ratio and average markups for four industries during 1995-2005: 'Computer & Electronic Product Manufacturing' (NAICS 334), 'Publishing Industries (software)' (NAICS 511), 'Telecommunications' (NAICS 517), and 'Information & Data Processing' (NAICS 518-519). The CAPE ratio is the ratio of total stock market capitalization to a 10–year moving average of past earnings (EBITDA); the ratio is in logs and is measured at the beginning of the year (see Appendix B.4 for more details). Markups are the ratio of sales to variable operating costs ('Cost of the Goods Sold' + 'General, Selling and Administrative Expenses'); the ratio is in logs. The group of incumbents includes all firms that already existed in 1995.

Motivated by these observations, I investigate the interactions between asset bubbles and product market competition. I use data from COMPUSTAT to construct industry-level indicators of stock market overvaluation. I show that when stock prices appreciate in a given industry (relative to fundamentals), the number of firms in the industry increases, firms expand and reduce price-cost markups. Furthermore, they appear to be more likely to exhibit negative operating margins. Overall these results suggest that stock market booms are associated with more intense competition in product markets, with firms reducing markups and operating margins.

I then build a model featuring oligopolistic competition, endogenous entry and rational bubbles to shed light on these findings. Bubbles are attached to firms' values and can be either exogenous to production decisions (thus providing their owners with a lump sum rent), or explicit functions of market shares (so that larger firms also generate larger bubbles).[5] In this context, the appearance of a bubble will either subsidize entry or provide firms

[5]Valuation models are often based on metrics of size (such as market shares) and not on profits. The use of such valuation techniques is especially true in the case of young firms: typically they start with low or even negative earnings, which makes it difficult to project future cash

with the incentives to grow and fight for market shares (at the expense of profits), thereby resulting in a more competitive market structure. Insofar as they reduce monopoly rents, bubbles will correct a market failure and will be welfare-improving. However, I show that if they are sufficiently large, bubbles can also generate situations of overinvestment and excessive competition, with too many firms entering and/or charging prices below their marginal cost. The model can therefore explain the prevalence of negative operating margins exhibited by high-tech firms at the peak of the dotcom bubble, or examples of overinvestment in the British railway mania of the 1840s (Appendix A).

I also discuss the impact of imperfect competition on the appearance of rational asset bubbles. I show that imperfect competition depresses the interest rate in general equilibrium, hence relaxing the conditions for the existence of rational bubbles.[6] The intuition is simple: having market power, firms restrict output and investment relative to the social optimum. As a result, both the demand for credit and the interest rate can be sufficiently depressed so that rational asset bubbles become possible even when capital accumulation is dynamically efficient.

To illustrate some of the model's assumptions and predictions, I also provide historical evidence from two famous stock market overvaluation episodes: the British railway mania of the 1840s and the dotcom bubble of the late 1990s. This evidence is interpreted through the lens of the model developed in this paper.

The rest of the paper is organized as follows. Section 2 presents the empirical results. Section 3 describes the model. Section 4 concludes. The historical evidence is provided in Appendix A.

## 1.1 Related Literature

This paper is mostly related to the literature that forms the theory of rational bubbles. Different models have emphasized different aspects of asset bubbles. On the one hand, there are models that view bubbles as assets that provide a store of value. This is the message of the seminal contribution of Samuelson (1958) and Tirole (1985) who show that bubbles may complete intergenerational markets and provide for an efficient intertemporal allocation of consumption. Being a store of value, bubbles can also be a liquidity instrument that helps firms overcome financial frictions as in Caballero and Krishnamurthy (2006), Farhi and Tirole (2012), Hirano and Yanagawa (2017), Kocherlakota (2009) or Miao and Wang (2012). Finally, Ventura (2012) shows that bubbles can increase the return on savings in low productivity countries and act as a substitute for capital flows.

A different strand of the literature has put an emphasis on the appearance of new bubbles: the formation of a new pyramid scheme provides a rent or subsidy that can have economic consequences. Within this category, we find the model of Olivier (2000) who shows that if appearing attached to R&D firms, bubbles can stimulate the

flows from current earnings. See Section 3 for a discussion.

[6] Rational asset bubbles can only emerge in economies in which the steady-state interest rate is lower than the growth rate. On the one hand, they must offer a return that is not lower than the interest rate. On the other hand, bubbles cannot grow faster than the economy (otherwise they can be ruled out with simple backward induction arguments).

invention of new goods and foster economic growth. Martin and Ventura (2012, 2016) argue that the creation of new bubbles allows credit-constrained entrepreneurs to expand borrowing and investment. Tang (2018) studies how the appearance of asset bubbles affects firm dynamics in the presence of credit constraints.[7]

In this paper, I provide a theory of how asset bubbles can be expansionary. My theory will hence be closest in spirit to the recent class of models emphasizing how bubbles can alleviate credit market frictions and be associated with larger investment and output (Farhi and Tirole (2012), Martin and Ventura (2012, 2016), Hirano and Yanagawa (2017), Tang (2018), Ikeda and Phan (2019)). Yet, there are important differences. First, the focus will be on frictions in product markets, not in financial markets. Furthermore, most models featuring asset bubbles and credit market imperfections fail to explain how overvaluation can generate overinvestment, excessive entry and negative earnings. As I argue, these have been important aspects of the dotcom bubble and other historical episodes (Appendix A).[8]

Finally, this paper is related to the vast literature studying the aggregate consequences of market power, which includes the contributions by Rotemberg and Saloner (1986), Chatterjee, Cooper and Ravikumar (1993), Jaimovich and Floetotto (2008) and Etro and Colciago (2010). I provide a theory for countercyclical markups and document a negative correlation between stock market overvaluation and markups. Iraola and Santos (2017) find no contemporaneous correlation between changes in markups and changes in stock prices. However, as my model highlights, it is important to distinguish the overvaluation from the fundamental component of stock prices.[9] One prediction of my model is that imperfect competition depresses the equilibrium interest rate, thereby relaxing the condition for the existence of rational bubbles. A similar result has also been recently emphasized by Ball and Mankiw (2021).

## 2 Stocks Prices and Competition

As Figure 1 suggests, industries that were at the center of the dotcom bubble seemed to have experienced a decline in markups. But was this something particular of these four industries and this period? Or is there a more systematic relationship between stock market overvaluation and markups? In an attempt to answer these questions, I characterize in this section the empirical correlation between indicators of stock market overvaluation and measures of product market competition. I am interested mostly in the following questions: when a bubble

[7]There are several other recent contributions to the theory of rational bubbles. For example, Galí (2014, 2020) and Asriyan et al. (2020) study the interactions between asset price bubbles and monetary policy. Larin (2019) and Guerron-Quintana, Hirano and Jinnai (2020) develop quantitative models to study the impact of asset bubbles on recent US macroeconomic history.

[8]This paper also speaks to the literature describing firm and investor behavior during the British railway mania (Campbell and Turner (2010, 2015), Odlyzko (2010)) and the dotcom bubble (Brunnermeier and Nagel (2004), Pastor and Veronesi (2006), Griffin et al. (2011) and Campello and Graham (2013)).

[9]An increase in overvaluation will drive a negative comovement between stock prices and markups; fundamental/technology shocks that reduce competition will instead drive a positive comovement between markups and stock prices.

appears in an industry (i.e. stock prices appreciate relative to fundamentals), do firms charge on average higher or lower markups? Does the number of active firms increase?

Two observations should be made. First, the results provided in this section will not allow me to establish causality. A direction of causation will be proposed in the model I develop in section 3. Second, to obtain a measure of stock market overvaluation, one has to make assumptions on how to construct fundamentals. Any measure of overvaluation is thus imperfect and open to criticism. While this is an unavoidable limitation, I will be using two indicators that are well known and widely used in the literature.

## 2.1 Data Description

I use annual data from COMPUSTAT for the period 1980-2019.[10] Companies with annual revenue or market capitalization of \$10,000 or less are excluded. Apart from this selection criteria, the panel contains all firm-year observations with nonmissing data on sales, markups (defined below), stock prices, common shares outstanding and industry classification. Firms are grouped in 47 industries (using 3-digit NAICS 2017), covering the manufacturing and service sectors.[11] The full sample includes 158,787 observations, for an average of 3,970 firms per year.

## 2.2 Indicators of Stock Market Overvaluation

**The Shiller CAPE ratio** The first measure of stock market overvaluation I construct is the Shiller CAPE ratio (Shiller (2000)). This is a popular indicator of overvaluation and is simply constructed as the ratio of total stock market capitalization to a 10-year moving average of past earnings (used as a proxy for fundamentals).[12] In this paper, I compute this ratio at the industry level using data from COMPUSTAT. I construct industry indexes for stock market capitalization and earnings, correcting for entry/exit in the dataset (see Appendix B.4 for details). For each industry I then construct

$$\text{cape}_{it} = p_{it} - \bar{e}^{10}_{it}$$

where $p_{it}$ is the real stock price (in logs and measured at the end of year $t$) and $\bar{e}^{10}_{it} \equiv \log\left[\left(E_{it-9} + \cdots E_{it}\right)/10\right]$ is a 10-year moving average of real earnings (in logs). The baseline earnings metric I use is the EBITDA.[13]

**The Campbell and Shiller price-fundamental deviation** Despite being a simple and intuitive indicator of overvaluation, the CAPE ratio has two main disadvantages: it is essentially backward looking (fundamentals are proxied

[10] I choose 1980 as the starting year to guarantee that all industries have at least one active firm throughout the sample.

[11] Appendix B.2 provides details on the industry classification. Industries in primary sectors, utilities and finance are excluded.

[12] Robert Shiller provides a monthly version of this measure for the S&P Composite index, from January 1871 to now; this features a historical maximum in December 1999, at the peak of the dotcom bubble.

[13] In COMPUSTAT, this is equal to 'Sales' minus the sum of 'Cost of the Goods Sold' and 'General, Selling and Administrative Expenses'. In a robustness exercise, I consider different earnings metrics. The results are reported in Appendix B.12.

with past earnings), it does not make use of a formal definition of the fundamental value of an industry (i.e. the discounted value of future dividends). To address these concerns, I construct a second measure of overvaluation based on Campbell and Shiller (1988). This consists in calculating the difference between stock prices and an estimate of the value of discounted future dividends, where the latter are predicted by means of a VAR. This will require more assumptions than the CAPE ratio, but its outcome is consistent with the model of rational bubbles I develop below.

Suppose that an industry $i$ pays a stochastic sequence of dividends $\left\{D_{it+j}\right\}_{j=1}^{\infty}$ to be discounted at rates $\left\{R_{it+j}\right\}_{j=0}^{\infty}$. Then the fundamental value of the industry (i.e. the expected discounted value of its future dividends) can be written as

$$F_t = E_t \left\{ \sum_{k=1}^{\infty} \frac{D_{t+k}}{\prod_{j=0}^{k-1} R_{it+j}} \right\} \tag{1}$$

As shown by Campbell and Shiller (1988), the previous expression can be written in log-linear form as

$$f_{it} \approx d_{it} + \sum_{j=0}^{\infty} \rho_i^j E_t \left\{ \Delta d_{it+j+1} - r_{it+j} \right\} + c_i \tag{2}$$

where $c_i$ is a constant and $\rho_i = \exp\left(\bar{g}_i - \bar{r}_i\right)$, and $\bar{g}_i$ and $\bar{r}_i$ denote the growth rate of dividends and the discount rate along a balanced growth path.[14] Equation (2) can be used to estimate the fundamental $f_{it}$, given assumptions about the required rate of return $r_{it}$. In what follows, I will assume that it can be approximated by a market return $r_t^M$ (e.g. the return on the SP composite index) plus some industry-specific component $\sigma_i$ so that

$$E_t \left\{ r_{it} \right\} = E_t \left\{ r_t^M \right\} + \sigma_i$$

Although $\sigma_i$ is introduced to keep the framework general, below I will be setting $\sigma_i = 0$.[15] As I show in the Appendix B.6, $\sigma_i$ essentially pins down the level of the fundamental, but does not affect its time series patterns. Since in all the regression analysis that follows I exploit within-industry variation (i.e. I control for either firm or industry fixed effects), the exact level of the fundamental will not be important.

Given these assumptions, the right-hand side of (2) can be computed once we estimate a process for $\Delta d_{it+1} - r_t^M$.

[14] In particular, $c_i = (1-\rho_i)^{-1} \log(1/\rho_i) - \log(1/\rho_i - 1)$.

[15] Note that $r_t^M$ is market return (not a risk-free rate), and will incorporate a time-varying risk-premium.

Following Campbell and Shiller (1988) I consider the following VAR

$$\underbrace{\begin{bmatrix} \Delta d_{it+1} - r_t^M \\ p_{it+1} - d_{it+1} \\ \text{cape}_{it+1}^{10} \end{bmatrix}}_{\mathbf{z_{it+1}}} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \Delta d_{it} - r_{t-1}^M \\ p_{it} - d_{it} \\ \text{cape}_{it}^{10} \end{bmatrix}}_{\mathbf{z_{it}}} + \underbrace{\begin{bmatrix} u_{it+1}^1 \\ u_{it+1}^2 \\ u_{it+1}^3 \end{bmatrix}}_{\mathbf{u_{it+1}}} \tag{3}$$

In this VAR all variables are in deviation from their industry-specific mean (hence the exclusion of a constant term). Both the price-dividend ratio $(p_{it} - d_{it})$ and the CAPE ratio $\left(\text{cape}_{it}^{10}\right)$ are included as predictors of discounted dividend growth $\left(\Delta d_{it+1} - r_t^M\right)$. As shown by Campbell and Shiller (1988), the inclusion of the price-dividend ratio is important and theoretically justified: it provides a statistical summary of the information that market participants have about future dividends, and which can be unobservable to the econometrician.[16] Note that equation (3) defines a pooled VAR: it imposes the same matrix of coefficients $A$ across industries. However, as discussed in Queirós (2020), an industry-specific VAR would yield identical results (even if slightly noisier).

The VAR in (3) is estimated with OLS and the results are shown in Table 5 (Appendix B.5). Dividends are the sum of common dividends and net stock repurchases (Appendix B.3), while all other variables are as defined above. Both the price-dividend ratio and the Shiller CAPE ratio are shown to be significant predictors of discounted dividend growth. Given these estimates, we can compute the fundamental defined in (2) as

$$\hat{f}_{it} = d_{it} + \mathrm{e}_1 \, \hat{\mathrm{A}} \left(I_3 - \rho \, \hat{\mathrm{A}}\right)^{-1} \mathbf{z_{it}} + c$$

where $\mathrm{e}_1 = [1 \;\; 0 \;\; 0]'$. Using $\bar{r}^M = 0.071$ (the average annual real return on the S&P composite index in the period), $\sigma_i = 0$ (as explained above) and $\bar{g}_i = 0.03$ (the long-run dividend growth rate), I obtain $\rho = 0.960$ and $c = 4.194$. I then measure the degree of stock market overvaluation as the difference between the log stock price and the log fundamental

$$\text{pdev}_{it} = p_{it} - \hat{f}_{it}$$

I shall refer to this difference as the price-fundamental deviation.

Figure 2 shows the evolution of observed prices and estimated fundamentals in two industries: 'Beverage and Tobacco Manufacturing', and 'Publishing Industries' (which includes software developers). In the first case, stock prices exhibit relatively mild fluctuations around the estimated fundamental. In the second case, it is, however, possible to observe a large price-fundamental deviation around the year 2000 (coinciding with the dotcom

[16] Indeed, in the absence price bubbles, $p_{it}$ is itself the present discounted value of future dividends.

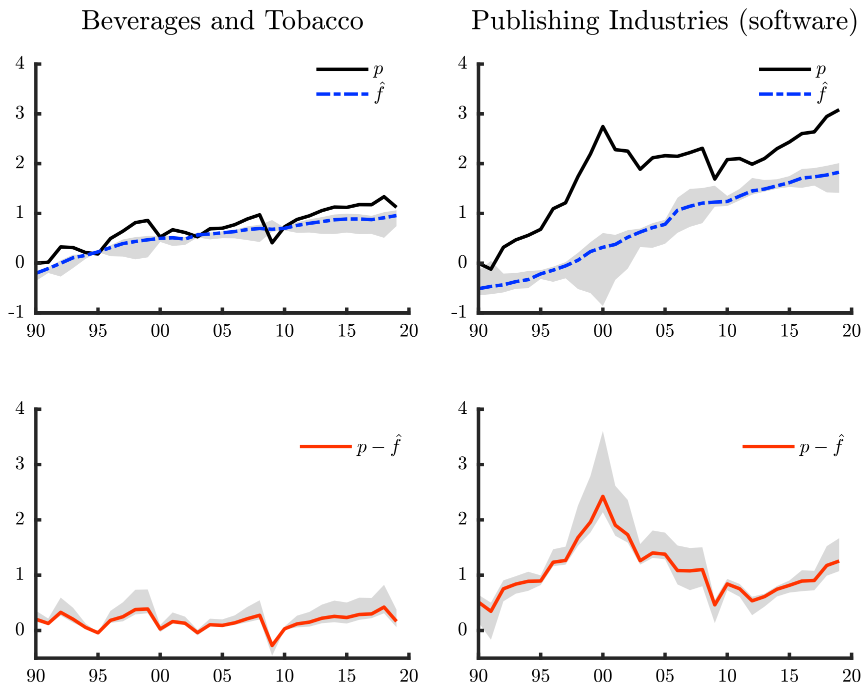



Figure 2: Price-fundamental deviations

This figure shows price-fundamental deviations for two industries: 'Beverage and Tobacco Manufacturing' (NAICS 312) and 'Publishing Industries (software)' (NAICS 511). All variables are measured at the beginning of the year. Fundamentals are estimated using the VAR explained above. Appendix B.3 provides details on variable definitions. Prices and fundamentals are rescaled so that $p_{i,90} = 0$. Shaded areas represent 99% bootstrap confidence bands (see Appendix B.8 for details).

bubble). In Appendix B.7, I provide a comparison between the two measures of stock market overvaluation that I construct. Despite being conceptually different objects (as discussed above), the two measures are shown to be strongly correlated. This fact is, perhaps, not surprising. Even though the two indicators use different measures of fundamentals, these evolve relatively smoothly in comparison to stock prices. In other words, fluctuations in the two indexes are driven mainly by variation in stock prices. (Campbell and Shiller (1988)). I next investigate how these two measures of stock market overvaluation comove with indicators of product market competition.

## 2.3 Stock Market Overvaluation and Competition

**Stock market overvaluation and firm decisions** I am interested in how stock market overvaluation at the end of year $t-1$ affects a firm level outcome (e.g. markups) at $t$. The model I consider is

$$x_{jit} = \lambda_{ji} + \eta_t + \beta \text{ overvaluation}_{it-1} + u_{jit} \tag{4}$$

where $x_{jit}$ is the outcome of interest of firm $j$ in industry $i$ and year $t$ and overvaluation$_{it-1}$ is the indicator for stock market overvaluation in the industry at the end of year $t-1$. $\lambda_{ji}$ and $\eta_t$ represent firm and year fixed effects.

In this section, I report results for three different dependent variables $x_{jit}$ : log sales growth, log markup and

Industry Overvaluation and Firm Outcomes

| VARIABLES | (1) $\log\left(\frac{\text{sale}_{jit}}{\text{sale}_{jit-1}}\right)$ | (2) $\log\left(\frac{\text{sale}_{jit}}{\text{cost}_{jit}}\right)$ | (3) $\mathbb{1}\{\text{earn}_{jit} < 0\}$ | (4) $\log\left(\frac{\text{sale}_{jit}}{\text{sale}_{jit-1}}\right)$ | (5) $\log\left(\frac{\text{sale}_{jit}}{\text{cost}_{jit}}\right)$ | (6) $\mathbb{1}\{\text{earn}_{jit} < 0\}$ |
|---|---|---|---|---|---|---|
| **Shiller CAPE Ratio** | | | | | | |
| $\text{cape}_{it-1}$ | 0.0433*** | -0.0375*** | 0.0395*** | | | |
| | (0.0117) | (0.0126) | (0.0147) | | | |
| **Campbell and Shiller Price-Fundamental Deviation** | | | | | | |
| $\text{pdev}_{it-1}$ | | | | 0.0421*** | -0.0370*** | 0.0357*** |
| | | | | (0.00947) | (0.00973) | (0.0111) |
| Observations | 126,625 | 136,814 | 136,814 | 126,625 | 136,814 | 136,814 |
| R-squared | 0.183 | 0.739 | 0.642 | 0.183 | 0.739 | 0.642 |
| Firm FE | YES | YES | YES | YES | YES | YES |
| Year FE | YES | YES | YES | YES | YES | YES |

**Table 1**
This table reports OLS estimates of equation (4), which regresses a firm level outcome $x_{jit}$ on the degree of overvaluation in the industry at the end of the previous year. Data is from COMPUSTAT. In columns (1) and (4), the dependent variable is the (log) growth rate of sales (COMPUSTAT item #12). Columns (2) and (5) use the (log) markup, defined as the ratio of sales to variable operating costs (COMPUSTAT item #41 + COMPUSTAT item #189). Columns (4) and (6) use a binary variable that takes value one if sales are lower than variable operating costs. The estimation is done for the period 1990-2019. Sales growth and markups are windsorized at the 0.5% and 99.5% percentiles. Standard errors in parentheses are bootstrapped clustered at the industry level. When the Campbell and Shiller price-fundamental deviation is used, the bootstrap also accounts for the fact that $\text{pdev}_{it}$ is a generated regressor (see Appendix B.9 for details). *** p<0.01, ** p<0.05, * p<0.1

a dummy indicator for negative operating earnings. Markups are constructed as the ratio of sales to variables costs, while operating earnings are simply the difference between sales and variable costs. My baseline measure of variable costs is the sum of the 'Cost of the Goods Sold' and 'General, Selling and Administrative Expenses'.[17] The model is estimated with OLS, standard errors are bootstrapped clustered at the industry level. When the Campbell and Shiller price-fundamental deviation is used, the bootstrap also accounts for the fact that $\text{pdev}_{it}$ is a generated regressor (see Appendix B.9 for details). All regression estimates refer to the period 1990-2019.[18]

The results are shown in Table 1. Columns (1) to (3) report the results under the Shiller CAPE ratio. Overall, the results indicate a significant correlation between stock market overvaluation at the end of year $t-1$ and the three outcome variables observed at $t$. In particular, larger overvaluation indicates faster sales growth (column (1)), lower price-cost markups (column (2)) and a larger probability of incurring negative operating earnings (column (3)). All the coefficients are significant at 1%. To get a sense of their economic magnitudes, I consider the effect of a one

[17] This has also been used by Traina (2018). In Appendix B.11, I show that the results are identical when only the 'Cost of the Goods Sold' is used (as in De Loecker, Eeckhout and Unger (2020)).

[18] The two overvaluation indicators that I use require a 10-year moving average of past earnings and are hence not available for the initial years 1980-1989.

standard deviation shock to $\text{cape}_{it}$. This predicts a 2.42 pp increase in sales growth (6.0% of the average firm-level standard deviation), a 2.09 pp decline in the price-cost markup (8.0% of the average firm-level standard deviation) and a 2.20 pp increase in the likelihood of negative operating earnings (12.2% of the average firm-level standard deviation). Columns (4) to (6) repeat the exercise using the Campbell and Shiller price-fundamental deviation. The results are identical and again significant.[19]

These results indicate that, when stock prices appreciate in a given industry (relative to fundamentals), firms exhibit larger sales growth and lower price-cost markups, being also more likely to incur negative operating earnings. This suggests that stock market boom episodes are associated with more intense competition between firms. In Appendix B.10, I show that alternative measures of activity other than sales growth (e.g. employment growth and investment rate) are also positively correlated with the two indicators of stock market overvaluation. While these results are informative about changes in the intensive margin, they are however silent about variation in the number of firms. I next investigate how the number of active firms comoves with industry overvaluation.

**Overvaluation and the number of firms** To conclude, I regress the yearly change in the number of active firms in industry $i$ on the measure of overvaluation at the end of the previous year (as well as industry and year fixed effects)

$$\log\left(\frac{N_{it}}{N_{it-1}}\right) = \lambda_i + \eta_t + \beta \text{ overvaluation}_{it-1} + u_{it} \tag{5}$$

The number of firms in the industry $N_{it}$ is measured in two alternative ways: the number of active firms in COMPUSTAT (which effectively measures the number of public firms in the industry) and the number of active firms for the whole economy reported by the US census of firms (SUBS programme). While the latter is a better indicator of competition in the entire economy, it is available for a shorter time period (1998-2017).

The results are shown in Table 2. Columns (1) and (2) report the results under the Shiller CAPE ratio; column (1) uses the growth in the number of traded firms (i.e. listed in COMPUSTAT) as the dependent variable, whereas column (2) uses the growth in the number of firms in the economy. The results indicate a positive correlation between stock market overvaluation at the end of period $t-1$ and the growth in the number of firms between $t-1$ and $t$. To give a sense of the economic magnitudes, a one standard deviation increase in the Shiller CAPE ratio is associated with a 1.15 pp increase in the growth rate of listed firms (10.9% of the average within-industry standard deviation), and a 0.405 pp increase in the growth rate of firms in the total economy (13.2% of the average within-industry standard deviation). These findings are confirmed with the Campbell and Shiller price-fundamental deviation (columns (3) and (4)). The results are identical and again significant.

[19] Their economic magnitudes are also important. For example, a one standard deviation shock to $\text{pdev}_{it}$ predicts a 2.23 pp increase in sales growth (5.6% of the average firm-level standard deviation), a 1.96 pp decline in markups (7.5% of the average firm-level standard deviation) and a 1.89 pp increase in the likelihood of negative operating earnings (10.5% of the average firm-level standard deviation).

| Industry Overvaluation and the Number of Active Firms | | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| | COMPUSTAT | Total Economy | COMPUSTAT | Total Economy |
| VARIABLES | $\log\left(\frac{N_t}{N_{t-1}}\right)$ | $\log\left(\frac{N_t}{N_{t-1}}\right)$ | $\log\left(\frac{N_t}{N_{t-1}}\right)$ | $\log\left(\frac{N_t}{N_{t-1}}\right)$ |
| Shiller CAPE Ratio | | | | |
| $\text{cape}_{it-1}$ | 0.0209** | 0.00737* | | |
| | (0.00815) | (0.00408) | | |
| Campbell and Shiller Price-Fundamental Deviation | | | | |
| $\text{pdev}_{it-1}$ | | | 0.0236*** | 0.00647** |
| | | | (0.00700) | (0.00321) |
| Observations | 1,363 | 864 | 1,363 | 864 |
| R-squared | 0.338 | 0.300 | 0.339 | 0.300 |
| Industry FE | YES | YES | YES | YES |
| Year FE | YES | YES | YES | YES |

**Table 2**
This table reports OLS estimates of equation (5), which regresses the log growth rate of the number of firms in an industry on the degree of overvaluation at the end of the previous year. Data is from COMPUSTAT and SUBS. Columns (1) and (3) use the number of firms listed in COMPUSTAT (period 1990-2019). Columns (2) and (4) use the number of firms active in the entire economy (from the US census of firms, SUBS programme, period 1998-2017). Standard errors in parentheses are bootstrapped clustered at the industry level. When the Campbell and Shiller price-fundamental deviation is used, the bootstrap also accounts for the fact that $\text{pdev}_{it}$ is a generated regressor (see Appendix B.9 for details). *** $p<0.01$, ** $p<0.05$, * $p<0.1$

**Discussion** In Appendix B.12 I show that the previous findings are robust to alternative versions of the Shiller CAPE ratio.[20] Overall, the results presented in this section suggest that episodes of stock market overvaluation are accompanied by changes in the product market structure: when stock prices appreciate (relative to fundamentals), the number of active firms in an industry increases, firms expand activity, and reduce price-cost markups; as a result of the last fact, they appear to be more likely to incur negative operating earnings. Motivated by these findings, I next build a model to shed light on the relationship between stock market overvaluation, firm entry and product market competition.

# 3 The Model

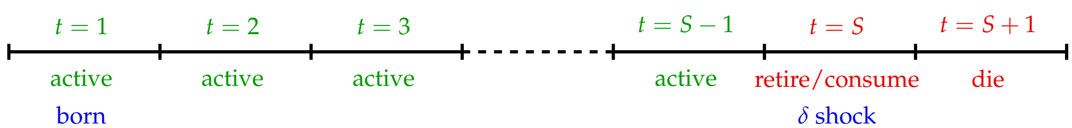
This section presents a multi-industry model with oligopolistic competition and rational bubbles that aims at explaining the evidence of Section 2. It is built upon the overlapping generations model by Diamond (1965), with the important twist that the market structure is endogenous. I then illustrate how rational asset bubbles (when attached to firms' values) can reduce market power. I also discuss how imperfect competition depresses the equilibrium interest rate, thus relaxing the conditions for the emergence of rational bubbles.

[20] In particular, I construct the ratio with alternative metrics of earnings, as well as with sales.

Individuals save during their active life and consume when they die. They can save by either purchasing financial securities (bonds and bubbles) or by building storage. Storage shall be seen as an unproductive investment technology, that can be used in equilibrium when financial securities provide a low return. As we shall see below, the existence of storage can generate a situation of underinvestment, which creates room for expansionary bubbles.

## 3.1 General Setup

**Demographics** Time is discrete and indexed by $t$. The economy is populated by infinitely many overlapping generations. There are two classes of individuals, the *workers* and the *entrepreneurs*. Within each class, a new mass $m$ of agents is born every period and becomes immediately active. Active individuals are subject to a retirement shock, which occurs with probability $\delta$ (independent of age). Agents become inactive upon receiving the retirement shock and die the period after. I normalize $m = \dfrac{\delta}{1-\delta}$ so that each class has a unit mass of active members.




**Labor supply and entrepreneurship** Workers are endowed with a unit labor endowment during their active life, which they supply inelastically. Labor is paid the competitive wage rate $W_t$.

Entrepreneurs run firms in the business sector and can make profits (as we shall see below). Firms are traded in the stock market and may contain a bubble component.

**Preferences and savings decisions** Individuals are risk neutral and have a single consumption opportunity in their period of retirement. Therefore, active individuals save all their wealth, and only consume when they retire.[21] The economy contains two savings options. On the one hand, agents can buy securities in financial markets (corporate bonds and stocks). Holding a financial security from $t$ to $t+1$ yields a gross expected return $R_{t+1}$. I refer to $R_{t+1}$ as the interest rate. On the other hand, they have access to a storage technology with return $r < 1$. Storage must be seen as an inefficient investment opportunity that may nevertheless be used in equilibrium when interest rates are low. It will impose a lower bound on the equilibrium interest rate $R_{t+1}$.

Since individuals face a constant retirement probability $\delta$ (independent of age), aggregate savings consists of a fraction $1-\delta$ of the economy's total assets $A_t$. Total demand for securities is therefore given by

[21] This preference structure greatly simplifies the exposition. It should however be emphasized that it is not crucial for the main results. As I show in section 3.3.3, the limit version of my model with perfect competition yields the same predictions of Tirole (1985).

$$D_t^S = \begin{cases} = (1-\delta) A_t & \text{if} \quad R_{t+1} > r \\ \\ \in [0, (1-\delta) A_t] & \text{if} \quad R_{t+1} = r \end{cases} \tag{6}$$

This equation says that when the equilibrium interest rate $R_{t+1}$ is above $r$, storage is not used and all savings are invested in financial markets. When the interest rate $R_{t+1}$ equals $r$, savers will be indifferent between purchasing financial securities and storing their assets.

**Technology** There is a final good $Y_t$, which is a CES composite of different intermediate varieties:

$$Y_t = \left( \int_0^1 y_{it}^{\rho} \, di \right)^{\frac{1}{\rho}} \tag{7}$$

where $y_{it}$ is the quantity of variety $i \in [0,1]$, $0 < \rho < 1$ and $\sigma \equiv 1/(1-\rho)$ is the elasticity of substitution. The final good is produced in a competitive sector and is chosen as the *numeraire*. The demand for each variety $i$ is given by

$$p_{it} = \left( \frac{Y_t}{y_{it}} \right)^{1-\rho} \tag{8}$$

Entrepreneurs can run one or more firms in the intermediate goods sector. In particular, entrepreneur $j \in [0,1]$ can produce variety $i \in [0,1]$ by combining capital $k_{it}^j$ and labor $l_{it}^j$ through a Cobb-Douglas technology

$$y_{it}^j = \pi_i^j \left( k_{it}^j \right)^{\alpha} \left( l_{it}^j \right)^{1-\alpha} \tag{9}$$

where $\pi_i^j$ is $j$'s time-invariant idiosyncratic productivity in industry $i$. I assume that productivities are distributed in a parsimonious way

$$\pi_i^j = \begin{cases} 1 & \text{if} \quad j = i \quad \text{(leader)} \\ \\ \pi \leq 1 & \text{if} \quad j \neq i \quad \text{(followers)} \end{cases} \tag{10}$$

Therefore, each variety $i \in [0,1]$ can be produced either with productivity $\pi_i^j = 1$ by entrepreneur $j = i$ or with productivity $\pi \leq 1$ by all the others. I will refer to entrepreneur $j$ as the *leader* of industry $i = j$ and to all other entrepreneurs $j \neq i$ as the *followers*. Note that every entrepreneur is a leader in one industry and a follower in all the others.[22]

Labor is hired at the competitive wage $W_t$. Capital needs to be invested one period ahead and fully depreciates

[22] I assume that when one leader retires, she is immediately replaced by a new entrepreneur with the same productivity.

in production. Each unit of capital therefore costs $R_t$. Given these assumptions, entrepreneur $j$ can produce good $i$ with marginal cost $\frac{\theta_t}{\pi_i^j}$, where

$$\theta_t \equiv \left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}$$

is the marginal cost function for a Cobb-Douglas technology with unit productivity. In addition to all variable costs, the production of each variety entails a fixed production cost $c_f \geq 0$ per period (in units of the *numeraire*).

**Financial markets** To finance investment, entrepreneurs may issue one period corporate bonds. Any corporate bond issued in period $t$ must deliver a gross return $R_{t+1}$ in period $t+1$. On top of that, entrepreneurs can also issue stocks. This is another financial instrument that can be used to raise funds in financial markets. However, contrarily to corporate bonds, stocks do not deliver any cash flow or dividend. This assumption is made only for clarity: a stock that is traded at a positive price must be a pure bubble or pyramid scheme.

Bubbles are fully rational, i.e. investors purchase stocks in the expectation that their price appreciate at the interest rate $R_{t+1}$. Let $B_{t+1}^j$ be the bubble attached to a particular firm $j$ at time $t+1$. Such a bubble includes the value of all stocks issued up to $t$, as well as the value of any new stock issued at $t+1$. Let $b_{t+1}^j$ represent the value of the new stocks issued at $t+1$. I assume that when entrepreneur $j$ retires, her firm is liquidated and stops being traded. Given this assumption, we can write the time $t+1$ bubble of firm $j$ as

$$B_{t+1}^j = \begin{cases} \dfrac{R_{t+1}}{1-\delta}\, B_t^j + b_{t+1}^j & \text{if } j \text{ is active} \\[2em] 0 & \text{if } j \quad \text{retires} \end{cases} \tag{11}$$

That is, when entrepreneur $j$ retires (which happens with probability $\delta$), the stock market value of her company becomes zero (i.e. any existing bubble crashes). If the entrepreneur remains active (which happens with probability $1-\delta$), the value of the firm's stocks at $t+1$ has two components. The first component $\left(\frac{R_{t+1}}{1-\delta}\, B_t^j\right)$ is the $t+1$ value of all stocks that were already traded at $t$. The return $\frac{R_{t+1}}{1-\delta}$ compensates for the fact that with probability $\delta$ the firm could have been liquidated. The second component $\left(b_{t+1}^j\right)$ is the value of all new stocks issued at time $t+1$. Note that equation (11) implies that

$$E_t\left\{B_{t+1}^j\right\} = R_{t+1}B_t^j + E_t\left\{b_{t+1}^j\right\}$$

When an entrepreneur issues a new stock or bubble $b_t^j$, she is capturing a rent or subsidy that is provided by the stock market. The existence of such a rent can affect firms' entry and production decisions, as we shall see below.

## 3.2 Equilibrium in the Intermediate Goods Sector

I assume that firms compete à la Cournot: all firms that decide to enter (thus paying $c_f$) will simultaneously announce quantities, taking the output of the other competitors as given.[23] Following Atkeson and Burstein (2008), I assume that firms make sequential entry decisions in reverse order of productivity. I start by describing the industry equilibrium when there are no bubbles.

### 3.2.1 Bubbleless Equilibrium

If entrepreneur $j$ decides to produce in industry $i$, she will choose the amount of output $y_{it}^j$ that maximizes her profits given the output of her competitors. Specifically, she solves

$$\max_{y_{it}^j} \quad \left(p_{it} - \frac{\theta_t}{\pi_i^j}\right) y_{it}^j \qquad \text{s.t.} \quad p_{it} = \left(\frac{Y_t}{y_{it}}\right)^{1-\rho}$$

$$y_{it} = y_{it}^j + y_{it}^{-j}$$

I assume that the market conditions are such that entry is always profitable for the leader. I will denote by $n_{it}$ be the number of followers who decide to enter (to be solved below). When $n_{it} \geq 0$ followers produce, the industry's price and quantity are given by

$$p_{it} = \frac{n_{it} + \pi}{n_{it} + \rho} \frac{\theta_t}{\pi} \quad \text{and} \quad y_{it} = \left(\frac{n_{it} + \rho}{n_{it} + \pi} \frac{\pi}{\theta_t}\right)^{\frac{1}{1-\rho}} Y_t \tag{12}$$

and individual market shares are equal to

$$s_{it}^L \equiv \frac{y_{it}^L}{y_{it}} = \frac{n_{it} - (n_{it} - 1 + \rho)\,\pi}{(1-\rho)\,(n_{it} + \pi)} \quad \text{and} \quad s_{it}^F \equiv \frac{y_{it}^F}{y_{it}} = \frac{\pi - \rho}{(1-\rho)\,(n_{it} + \pi)}$$

Inspecting the last two equations, we can see that the followers will produce a positive amount if and only if $\pi > \rho$ (i.e. when their productivity disadvantage is not too large). Throughout I will assume that $\pi > \rho$, so that any follower who decides to enter can compete against the leader.

**Assumption.** $\pi > \rho$

Given this assumption, it is easy to show that both the equilibrium price $p_{it}$ and individual markups

$$\mu_{it}^L \equiv \frac{p_{it}}{\theta_t} = \frac{1}{\pi} \frac{n_{it} + \pi}{n_{it} + \rho} \quad \text{and} \quad \mu_{it}^F \equiv \frac{p_{it}}{\theta_t / \pi} = \frac{n_{it} + \pi}{n_{it} + \rho}$$

decline in the number of active followers $n_{it}$.

[23] In Appendix D, I show that the main results and intuitions hold under Bertrand competition.

**Equilibrium number of followers** The equilibrium number of followers must be such that (i) all followers that operate do not make a loss, (ii) but if an additional follower were to enter she would incur a loss. Let $\Pi^F(n_{it}, \theta_t, Y_t) \equiv (p_{it} - \theta_t/\pi)\, y_{it}^F$ be the profits made by each follower. This function can be shown to be decreasing in $n_{it}$ (Appendix C.1.1). Denoting by $n_{it}^*$ the equilibrium number of followers we have that

$$\left[\Pi^F(n_{it}^*, \theta_t, Y_t) - c_f\right]\left[\Pi^F(n_{it}^* + 1, \theta_t, Y_t) - c_f\right] \leq 0 \tag{13}$$

Figure 3 below illustrates how the equilibrium number of followers $n_{it}^*$ is determined (given $Y_t$ and $\theta_t$). Each panel shows an equilibrium variable as a function of $n_{it}$. As the number of followers increases, total output $y_{it}$ expands (panel 1) while price-cost markups decline (panel 2). Note that the leader reacts to the entry of additional followers by increasing her output $y_{it}^L$ (panel 1).[24] Finally, panels 3 and 4 show the production profits made by each type, which are a negative function of the number of active followers $n_{it}$. Under the parameters chosen, the production profits are lower than $c_f$ for the first follower so that the industry will consist of a monopoly of the leader. We shall now see what happens when firms can issue bubbly stocks.

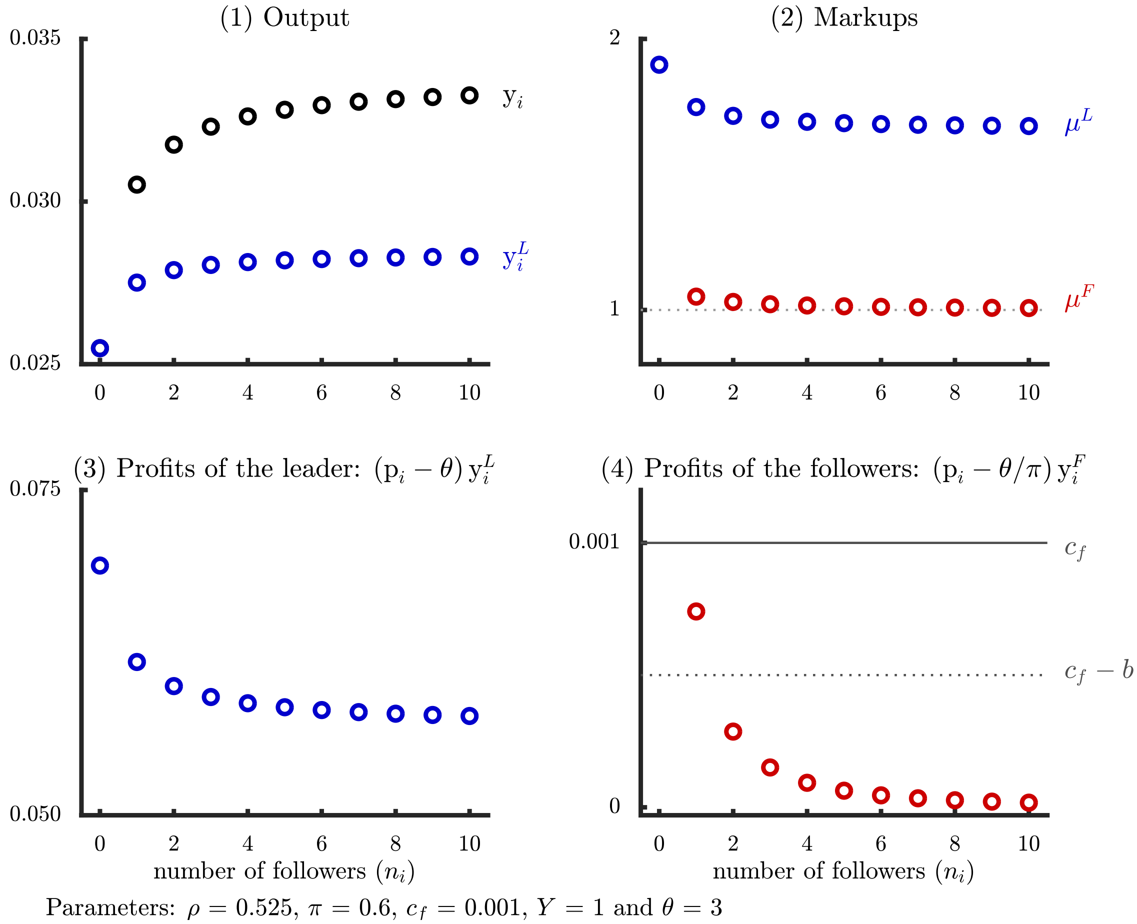



Figure 3: Industry Equilibrium

[24] As shown in Appendix C.1.2, this is the case whenever the productivity gap between leaders and followers is sufficiently large.

### 3.2.2 Bubbly Equilibrium

What happens to the previous equilibrium when firms have the possibility of issuing bubbly stocks? Throughout, I will be making the assumption that firms can only issue stocks if they are active. That is, entrepreneur $j \in [0,1]$ can sell an amount $b_{it}^j$ of new stocks only if she enters and pays the fixed cost $c_f$. I will consider two possibilities regarding investors' beliefs. First, I assume that $b_{it}^j$ is fixed at the firm level (constant firm bubble). Second, I assume that there is instead a fixed new industry bubble $b_{it}$ which is distributed among firms according to market shares (constant industry bubble).

**Constant firm bubbles** Suppose that firm $j$ can issue an exogenous amount $b_{it}^j = b \geq 0$ of bubbly stocks. We can see $b$ as an entry subsidy: upon entering and incurring the fixed cost $c_f$, entrepreneurs are entitled to a rent or subsidy that is provided by the stock market. The equilibrium number of followers will now be determined by

$$\left[\Pi^F\left(n_{it}^*, \theta_t, Y_t\right) - \left(c_f - b\right)\right]\left[\Pi^F\left(n_{it}^* + 1, \theta_t, Y_t\right) - \left(c_f - b\right)\right] \leq 0$$

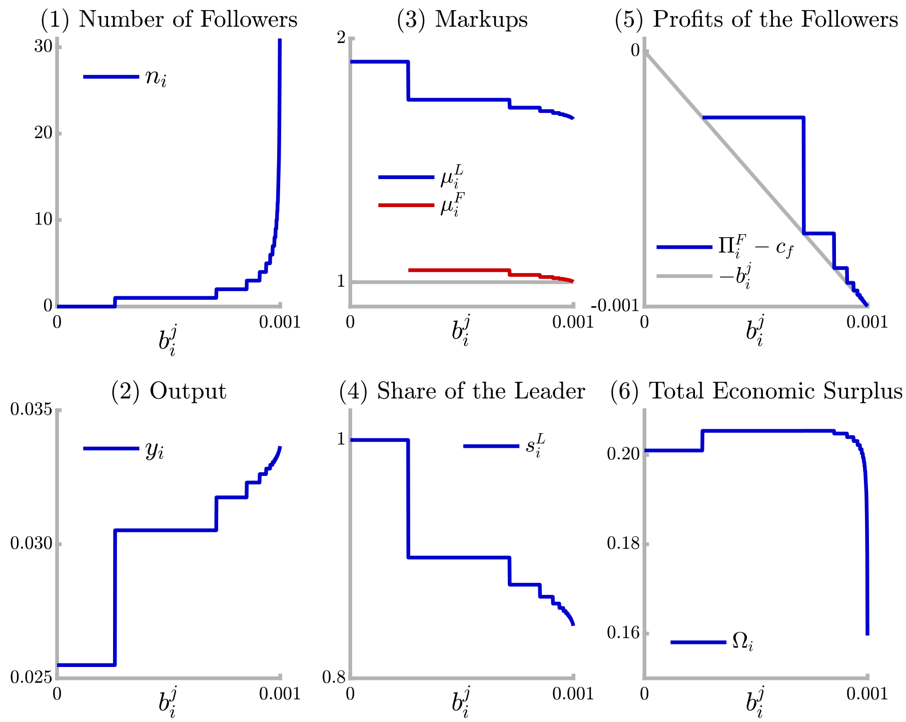
Let us go back to the example of Figure 3. Suppose that all active firms can issue an exogenous amount of bubbly stocks $b$ so that $c_f - b$ takes the value represented by the dotted line. In such a case, when there is one follower producing ($n_{it} = 1$), her production profits more than compensate for the fixed cost net of the bubbly subsidy $c_f - b$. The equilibrium will now consist of a duopoly in which the leader and one follower produce.

Note that as the value of the firm level bubble $b$ increases, more and more firms will be willing to enter. Figure 4 represents some equilibrium variables for this industry as a function of $b$. When $b$ is low, no follower finds entry attractive and the industry remains a monopoly. However, as $b$ rises, the followers will start entering one by one (panel 1). As already discussed, their entry will generate an expansion in total output $y_{it}$ (panel 2) and a decline in price-cost markups $\mu_{it}^j$ (panel 3). Recall that each follower's production profits are insufficient to cover the fixed cost $c_f$, i.e. $\Pi^F\left(n_{it}, \theta_t, Y_t\right) - c_f < 0$. Therefore, each follower is effectively incurring an operating loss, which is financed by the bubbly subsidy $b$ (panel 4). The negative profits $\Pi^F\left(n_{it}, \theta_t, Y_t\right) - c_f$ that each follower makes can therefore be seen as the cost she needs pay to obtain the rent $b$.

Even though the followers will be making an operating loss, their entry will necessarily result in higher consumer welfare, as total output $y_{it}$ increases. To assess the efficiency gains associated with the entry of additional firms, we should evaluate the change in the total industry surplus

$$\Omega_{it} = \underbrace{\int_0^{y_{it}} \left[\left(\frac{Y_t}{x}\right)^{1-\rho} - p_{it}\right] dx}_{\text{consumer surplus}} + \underbrace{\left(p_{it} - \theta_t\right) y_{it}^L - c_f + n_{it}\left[\left(p_{it} - \frac{\theta_t}{\pi}\right) y_{it}^F - c_f\right]}_{\text{producer surplus}}$$

This is a measure of economic efficiency which ignores the private return stemming from the issuance of bubbly stocks. Panel 6 represents the total industry surplus as a function of the firm-level bubble subsidy $b$. Recall that as $b$




Parameters: $\rho = 0.525$, $\pi = 0.6$, $c_f = 0.001$, $Y = 1$ and $\theta = 3$

Figure 4: Industry Equilibrium with Firm Level Bubbles

increases, the number of active followers $n_{it}$ increases and the price $p_{it}$ decreases. This fact results necessarily in higher consumer welfare (the first term in the expression above), but in a lower producer surplus (the second term). When $b$ is small there are few followers producing, so that the increase in consumer welfare exceeds the decrease in producer surplus.[25] As the bubble $b$ becomes large and more firms produce, the increase in consumer welfare resulting from an increase in $b$ is outweighed by the reduction in producer surplus. In the example of Figure 4, the total economy surplus is maximized when one follower produces.

So far, I have assumed that bubbles are fixed at the firm level. Despite being the standard assumption made in the rational bubbles literature, this hypothesis can be problematic for two reasons. First, it seems unrealistic to think that a firm can sell a fixed amount of new stocks $b > 0$ even when producing nothing. Second, as $b \to c_f$, all followers would be willing to enter ($n_{it}^* \to \infty$) and the value of all new bubbles being started would be infinite – an obvious impossibility. Having these observations in mind, I consider a different process for stock market sentiment.

**Constant industry bubbles** Suppose that, instead of emerging at the individual firm level, bubbles emerge at the industry level. In particular, assume that (i) there is a bubble with exogenous size $b_{it} > 0$ appearing in industry $i$ at time $t$ and that (ii) each entrepreneur gets a fraction of this industry bubble corresponding to her market share.

[25] Note that such a positive impact on consumer welfare may come from two channels: (i) the additional output that each follower brings to the market and (ii) the reaction of the leader (who, as we have seen in Figure (3), will produce more output in response to an increase in the number of followers).

According to this formulation, investors' total demand for stocks in industry $i$ exceeds the industry's fundamental value by a fixed amount $b_{it}$. Furthermore, this industry bubble is distributed across firms according to their market shares, so that larger firms also get a larger share in the bubble.[26]

This process is meant to capture one aspect of financial markets – namely the fact that valuation models are often based on multiples of revenues or market shares and not on profits.[27] The use of such valuation techniques is especially true in the case of young firms: they typically start with low or even negative profit margins, which makes it difficult to project future cash flows from current earnings. For instance, Hong, Stein and Yu (2007) provide detailed evidence that equity analysts offering valuations for Amazon in the 1997-1999 period tended to emphasize its growth path (in terms of sales) and highly disregarded operating margins. A well-known consequence of such valuation methods is that they induce firms to boost revenues or market shares, thus disregarding profit margins (Aghion and Stein (2008)). Indeed, as noted in the context of the recent Silicon Valley boom: "*With valuations based on multiples of revenue, there's ample incentive to race for growth, even at the cost of low or even negative gross margins. The many taxi apps and instant delivery services competing for attention, for example, are facing huge pressure to cut prices in the hope of outlasting the competition*".[28]

Taking this process into account, we must reformulate the firms' problem as

$$\max_{y_{it}^j} \quad \left(p_{it} - \frac{\theta_t}{\pi_i^j}\right) y_{it}^j + \frac{y_{it}^j}{y_{it}} b_{it} \qquad \text{s.t.} \quad p_{it} = \left(\frac{Y_t}{y_{it}}\right)^{1-\rho}$$

$$y_{it} = y_{it}^j + y_{it}^{-j}$$

It can be shown that the industry price that is obtained under this problem satisfies

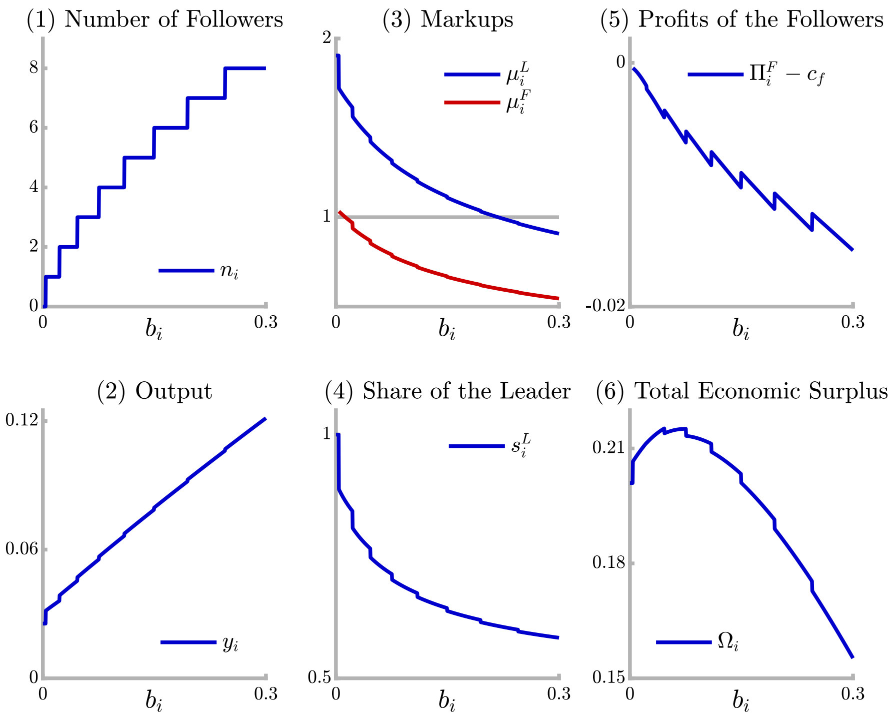
$$p_{it} = \frac{1}{n_{it} + \rho}\left[\theta_t\left(1 + \frac{n_{it}}{\pi}\right) - n_{it}\frac{b_{it}}{y_{it}}\right] \tag{14}$$

Equation (14) establishes a negative relationship between the industry price $p_{it}$ and the size of the industry bubble $b_{it}$ for any positive number of followers $n_{it} > 0$. Note that when no follower operates (i.e. $n_{it} = 0$), changes in the industry bubble $b_{it}$ have no consequence on the industry equilibrium. The intuition is simple: when the leader is a monopolist, she has a constant market share $s_{it}^L = 1$ and appropriates the entirety of $b_{it}$. However, if at least one follower produces, the appearance of an industry bubble will increase firms' desired market shares. As a result,

[26]The literature has considered models where bubbles respond to economic fundamentals. For example, Froot and Obstfeld (1991) construct a simple asset pricing model where rational bubbles are a function of dividends. In their model dividends follow however an exogenous stochastic process, so that bubbles have no impact on firm incentives and behavior.

[27]See, for example, Damodaran (2006).

[28]"Dotcom history is not yet repeating itself but it is starting to rhyme" (03/12/2015), Financial Times




Parameters: $\rho = 0.525$, $\pi = 0.6$, $c_f = 0.001$, $Y = 1$ and $\theta = 3$

Figure 5: Industry Equilibrium with the Industry Bubble

firms will compete more aggressively and increase output. The following proposition summarizes the main results of this process

**Proposition 1.** *Suppose that the number of followers in industry $i$ is constant and equal to $n_{it} \geq 1$. In such a case, as the industry bubble $b_{it}$ increases, all firms increase output $\left(\uparrow y_{it}^j \ \forall j\right)$ and reduce markups $\left(\downarrow \mu_{it}^j \ \forall j\right)$.*

*Proof.* See Appendix C.2.3. ■

The appearance of an industry bubble $b_{it}$ hence leads to an increase in total output and a reduction in cost-price markups even when the number of firms remains fixed (as long as there is at least one follower producing). This happens because the bubble makes firms fight for market shares and hence compete more aggressively. A corollary of this fact is that bubbles can be expansionary even when fixed costs are negligible ($c_f = 0$) and the number of followers is infinity ($n_{it} \to \infty$).

Proposition 1 pertained to an infinitesimal change in the industry bubble $b_{it}$, holding the number of followers $n_{it}$ fixed. However, as $b_{it}$ grows the number of followers deciding to enter will also increase. Figure 5 shows some equilibrium variables as a function of the industry bubble process described in this section. Note that, as they fight for market shares, firms may even find it optimal to charge a markup below one, thereby charging a price below unit cost. This process can therefore rationalize some of the findings of Section 2, namely the fact firms appear more likely to incur negative earnings in periods of high stock market overvaluation. The total economic surplus exhibits the inverted-U shape that we have also identified under the constant firm-level bubble process.[29] Having described the equilibrium of a particular industry, I now solve for the general equilibrium.

## 3.3 General Equilibrium

In this section, I characterize the economy in general equilibrium. I first focus on a static equilibrium, in which I describe aggregate output and factor prices for a given capital stock $K_t$ (the state variable). I then characterize the equilibrium dynamics. Before doing that, I start by defining an equilibrium for this economy. Recall that, despite the existence of individual retirement shocks, there is no aggregate uncertainty.

**Definition 1.** An aggregate equilibrium consists of a sequence for $\{K_t, S_t, B_t\}$, firm policies $\left\{y^j_{it}, k^j_{it}, l^j_{it}, B^j_{it}\right\}$, and a set of active followers $\{n_{it} \geq 0\}_{i \in [0,1]}$ for all $t \geq 0$ such that (i) consumers optimize (active agents save all their wealth, while retired individuals consume all their savings), (ii) firms maximize profits, (iii) active followers break even, and no additional follower can profitably enter, (iv) prices clear all markets, and (v) firm bubbles evolve according to (11).

### 3.3.1 Static Equilibrium

For expositional purposes, I focus on a symmetric equilibrium in which all industries are identical.[30] Suppose all industries are identical and have one leader and $n_t \geq 0$ followers. In such a case, each variety will be characterized by the same level of output $y_{it} = y_t = Y_t$ and hence the same price $p_{it} = 1$. Given a fixed aggregate labor supply $L_t = 1$, we can write aggregate output $Y_t$ as a function of the aggregate capital stock as

$$Y_t = \varphi(n_t)\ K_t^{\alpha}$$

The term $\varphi(n_t)$ can be seen as a measure of aggregate TFP and is equal to

$$\varphi(n_t) = \frac{\pi(1-\rho)(n_t+\pi)}{\pi[(2-\pi)n_t + (1-\rho)\pi] - \rho n_t}$$

It can be shown to be a negative function of $n_t$ when $\pi < 1$ (i.e. when the followers are less productive than the leader, the higher $n_t$, the lower is aggregate TFP). The following proposition summarizes the behavior of $\varphi(n_t)$.

**Proposition 2.** *(Aggregate TFP) Let $\varphi(n_t)$ denote aggregate TFP in a symmetric equilibrium (without bubbles) in which all industries have one leader and $n_t$ followers. We have that*

1. *$\varphi(n_t+1) < \varphi(n_t)$ if and only if $\pi < 1$*

[29] In Appendix C.2.4, I provide a direct comparison between the constant firm bubble and the constant industry bubble processes.

[30] In Appendix C.3.4, I discuss asymmetric equilibria in which industries have different numbers of firms.

2. $\varphi(0) = 1$

3. $\lim_{n_t \to \infty} \varphi(n_t) = \dfrac{\pi(1-\rho)}{\pi(2-\pi) - \rho}$

Note that aggregate TFP is always above $\pi$: even when there are infinitely many followers ($n_t \to \infty$), the leaders always have a non-negligible market share (provided that $\pi < 1$). To obtain the set of active firms, we first need to determine factor prices. Recall that in a symmetric equilibrium, all varieties will have the same price $p_{it} = 1$. We can hence use (12) to determine the aggregate factor cost index

$$\theta(n_t) = \frac{n_t + \rho}{n_t + \pi}\pi \tag{15}$$

The factor cost index can be shown to increase in the number of active followers ($n_t$).

**Proposition 3.** *(Factor Cost Index) Let $\theta(n_t)$ denote the factor cost index in a symmetric equilibrium (without bubbles) with $n_t$ followers in every industry. We have that*

$$\theta(n_t + 1) > \theta(n_t)$$

Proposition 3 states that an increase in the number of active firms will always be associated with higher factor costs. This result is intuitive – the higher is the number of firms operating in every industry, the higher is degree of competition and hence factor demand and factor prices. In the discussion that follows, it will also be convenient to define the aggregate factor share. Let $\sigma(n_t)$ denote the ratio of aggregate labor and capital payments to output

$$\sigma(n_t) := \frac{W_t + R_t K_t}{Y_t}$$

Note that the aggregate profit share (gross of fixed production costs) is given by $1 - \sigma(n_t)$. It is easy to see that the aggregate factor share $\sigma(n_t)$ satisfies

$$\sigma(n_t) = \frac{\theta(n_t)}{\varphi(n_t)}$$

Since $\theta(n_t)$ increases in $n_t$ and $\varphi(n_t)$ decreases in $n_t$, it immediately follows that $\sigma(n_t)$ is a positive function of the number of followers $n_t$. This is again an intuitive result – as the number of firms increases in every industry, competition becomes more intense, so that markups and profit shares decrease and factor income shares increase. The following proposition summarizes the behavior of $\sigma(n_t)$.

**Proposition 4.** *(Aggregate Factor Share) Let $\sigma(n_t)$ denote the aggregate factor share in a symmetric equilibrium (without bubbles) in which there are $n_t$ followers per industry. We have that*

1. $\sigma(n_t + 1) > \sigma(n_t)$

2. $\sigma(0) = \rho$

3. $\lim_{n_t \to \infty} \sigma(n_t) = \dfrac{\pi(2-\pi) - \rho}{1-\rho}$

The aggregate factor share is always lower than one, provided that $\pi < 1$. In other words, if the leaders have a positive productivity advantage over the followers, we have $\sigma(n_t) < 1$ even when $n_t \to \infty$. Note that $\sigma(n_t) = 1$ is only obtained as $n_t \to \infty$ and when $\pi = 1$. In such a case, we achieve a situation of perfect competition – there are infinitely many identical firms, all of which make zero production profits.

Having defined the aggregate factor share, we can determine factor prices

$$\begin{aligned} W_t = & \quad (1-\alpha)\ \sigma(n_t)\ \varphi(n_t)\ K_t^{\alpha} \\ R_t = & \quad \alpha\, \sigma(n_t)\ \varphi(n_t)\ K_t^{\alpha-1} \end{aligned} \tag{16}$$

As equation (16) highlights, the presence of imperfect competition – i.e. the fact that $\sigma(n_t) < 1$ – creates a wedge between factor prices and marginal products. For instance, the interest rate is lower than the marginal product of capital $\dfrac{\partial Y_t}{\partial K_t}$ whenever $\sigma(n_t) < 1$

$$R_t = \alpha\, \sigma(n_t)\ \varphi(n_t)\ K_t^{\alpha-1} < \alpha\, \varphi(n_t)\ K_t^{\alpha-1} = \frac{\partial Y_t}{\partial K_t}$$

As I discuss below, this fact means that the existence of market power will relax the conditions for the existence of rational bubbles.

To conclude, we must define the conditions under which a symmetric equilibrium with $n_t \geq 0$ followers is possible. Note that for such an equilibrium to be possible: (i) the aggregate capital stock must be sufficiently large so that none of the active firms makes a loss but (ii) aggregate capital cannot be too large, so that no additional follower has incentives to enter in any industry. These two conditions define a range of values under which the aggregate capital stock must fall for every $n_t \geq 0$, as stated in the next proposition.

**Proposition 5.** *(Symmetric Equilibrium) A symmetric equilibrium (without bubbles) in which one leader and $n_t \geq 0$ followers produce in every industry is possible provided that*

$$K_t \in \begin{cases} \left[ \left( \dfrac{c_f}{1-\rho} \right)^{\frac{1}{\alpha}}, \overline{K}(0) \right] & \text{if } n_t = 0 \\ \left[ \underline{K}(n_t), \overline{K}(n_t) \right] & \text{if } n_t \geq 1 \end{cases}$$

*where the functions $\underline{K}(n_t)$ and $\overline{K}(n_t)$ are increasing in $n_t$ and satisfy $\underline{K}(n_t) < \overline{K}(n_t) < \underline{K}(n_t+1)$.*

*Proof.* See Appendix C.3.3. ∎

We have fully characterized the static equilibrium of this economy – how aggregate output and factor prices are determined give the state variable $K_t$. We will now characterize the dynamics of $K_t$ (with and without bubbles).

### 3.3.2 Equilibrium Dynamics

Equilibrium in the credit market is obtained by equating aggregate savings to total capital formation storage and bubbly stocks

$$\underbrace{(1-\delta)\left(Y_t - N_t\, c_f + r \cdot S_{t-1}\right)}_{\text{savings}} = \underbrace{K_{t+1}}_{\text{capital}} + \underbrace{S_t}_{\text{storage}} + \underbrace{B_t}_{\text{bubbles}} \tag{17}$$

where $N_t \equiv \int_0^1 (1+n_{it})\, di$ is the aggregate number of firms. To understand equation (17), note that aggregate savings are a fraction $1-\delta$ of net output (total output minus total fixed costs) plus the return on past storage.

When $R_{t+1} > r$, the return on bonds and stocks dominates the one on storage and $S_t = 0$. Therefore, storage is only built when $R_{t+1} = r$. We hence have that

$$S_t \begin{cases} = 0 & \text{if} \quad R_{t+1} > r \\ \in \left[0, (1-\delta)\left(Y_t - N_t\, c_f + r \cdot S_{t-1}\right)\right] & \text{if} \quad R_{t+1} = r \end{cases} \tag{18}$$

A corollary of the previous two equations is that asset bubbles can only be expansionary when $R_{t+1} = r$. In such a case, when a bubble $B_t$ appears, it will crowd out storage $S_t$ and can potentially increase capital $K_{t+1}$. When $R_{t+1} > r$, storage is not used and the appearance of a bubble $B_t$ will necessarily crowd out capital $K_{t+1}$.

Finally, we can determine the law of motion of the aggregate bubble $B_t$ as

$$B_t = R_t \cdot B_{t-1} + \underbrace{\int_0^1 \left(\sum_{j \in \mathcal{I}_{it}} b_{it}^j\right) di}_{\text{new bubbles}} \tag{19}$$

where $b_{it}^j$ is the new stocks issued by firm $j$ in industry $i$ and $\mathcal{I}_{it}$ is the set of active firms in industry $i$. This equation says that the value of all bubbles issued in the past $B_{t-1}$ must provide an average return that is equal to interest rate $R_t$. Note that $B_{t-1}$ hides heterogeneity across individuals stocks. Recall from equation (11) that the stocks of firms that are liquidated provide a zero gross return, whereas stocks of continuing firms grow at rate $\frac{R_t}{1-\delta}$.

Figure 6 illustrates the model dynamics for a particular set of parameters. Given the particular parameter values chosen, no follower finds it profitable to enter when bubbles are not traded (i.e. all industries are a monopoly in a fundamental equilibrium). The black line shows the law of motion when bubbles are not traded. Note that the law

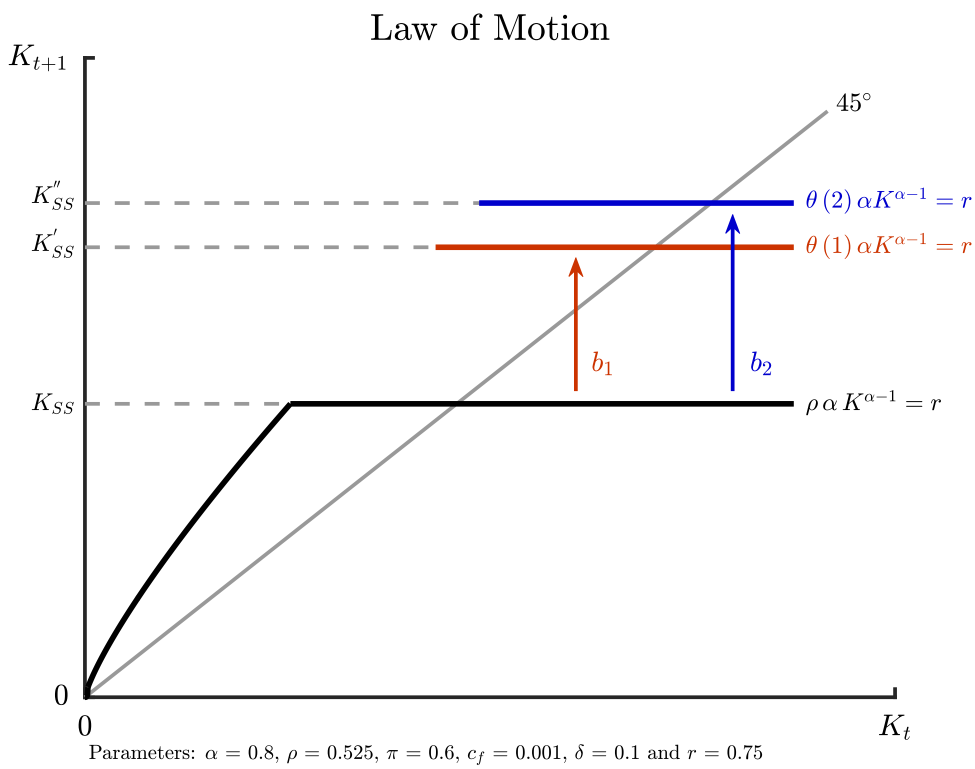



Figure 6: Law of Motion with Bubbles

of motion has two segments: one that is concave, another one that is flat. Equations (17) and (18) help us understand the shape of the law of motion. Because the total return on capital cannot fall short of $r$ (for otherwise storage is preferred), we have a situation of full investment with $K_{t+1} = (1-\delta)\left(\varphi_t K_t^{\alpha} - c_f\right)$ only when $K_t$ is sufficiently small. For large values of $K_t$, total investment is such that

$$\underbrace{\alpha \cdot \rho \cdot (K_{t+1})^{\alpha-1}}_{R_{t+1}} = r$$

where I have used equation (16). For the particular parameter values chosen, the intersection of the law of motion with the 45 degree line occurs in the flat part. Hence the economy, will converge to a steady-state $K_{ss}$ where (i) storage is built and (ii) all industries consist of a monopoly.

Now let us consider how bubbles can affect the law of motion. Suppose that the economy starts at the steady-state $K_{ss}$. However, now firms can issue some constant firm level-bubble $b_1$. If this value is sufficiently large, one follower may be able to enter in each industry. This will make the flat segment in Figure 6 increase (to the red line). This new flat segment is defined by

$$\underbrace{\theta\,(1) \cdot \alpha \cdot \left(K'_{SS}\right)^{\alpha-1}}_{R'_{ss}} = r$$

where $\theta\,(1)$ is the total factor cost index when one follower produces in every industry (as defined in (15)). The previous equation highlights how the entry of one follower can raise aggregate capital. Since we have $\theta(1) > \rho$, it is now possible to keep the aggregate interest rate $R'_{ss} = r$ with $K'_{ss} > K_{ss}$. This happens because the entry of one follower increases competition and hence the total factor share. Note that for a larger bubble $b_2$, then the entry of two followers becomes possible raising the new steady-state further to $K''_{ss}$.[31]

Note that the increase in aggregate capital is only possible insofar as long as agents build storage. If no storage was used, bubbles would simply crowd out capital holdings as equation (17) highlights. In conclusion, when bubbles appear, they can lead to a reallocation of resources from storage to capital (and bubbles). But is such reallocation efficient or can it be inefficient? And under what parameter conditions can bubbles emerge? I shall now address these questions.

#### 3.3.3 Dynamic Inefficiency and Underinvestment

Rational asset bubbles can emerge when the steady-state interest rate is below the economy's growth rate.[32] As shown by Tirole (1985), this condition is satisfied in the standard OLG model if and only if the economy features excessive capital accumulation, so that a reduction in investment can lead to an increase in welfare for all generations. In such a case, capital accumulation is said to be *dynamically inefficient*. If such a result were to hold in my model, bubbles could never appear in a situation of underinvestment. Note however that I depart from Tirole (1985) by assuming imperfect competition in product markets. In this section, I derive the conditions for the existence of rational asset bubbles and ask whether the equivalence of Tirole (1985) is or is not verified.

Let us start by characterizing the steady-state of the bubbleless equilibrium. Note that in a steady-state that features a symmetric equilibrium with $n^*$ followers per industry, the interest rate $R^*$ and the aggregate capital stock $K^*$ are given by

$$R^* = \max\left\{\frac{\alpha\,\sigma(n^*)}{1-\delta}, r\right\}$$

$$K^* = \min\left\{\left[(1-\delta)\,\varphi(n^*)\right]^{\frac{1}{1-\alpha}}, \left[\frac{\alpha\,\theta(n^*)}{r}\right]^{\frac{1}{1-\alpha}}\right\}$$

To understand the previous expression, note that the existence of storage imposes a lower bound on the equilibrium interest rate, which is associated with an upper bound $\left[\frac{\alpha\,\theta(n^*)}{r}\right]^{\frac{1}{1-\alpha}}$ on the equilibrium capital stock. The number of followers $n^*$ will depend on the value of the fixed production cost $c_f$. The following proposition states the conditions for the existence of a steady-state with $n^*$ followers per industry (with and without storage).

**Proposition 6.** *(Steady-State) The economy features a steady-state with $n^*$ followers per industry*

[31] Appendix 3.3 provides details on the exact values that $b_1$ and $b_2$ should take.

[32] Otherwise, bubbles would grow faster than the economy and could not be sustained.

1. ***(Storage)*** *in which storage is built, provided that*

$$c_f \in \left[\underline{\mathfrak{c}}^{ss}\left(n^*, r\right), \overline{\mathfrak{c}}^{ss}\left(n^*, r\right)\right]$$

*and that*

$$\alpha\, \sigma\left(n^*\right) < \left(1-\delta\right) r$$

2. ***(No Storage)*** *in which storage is not built, provided that*

$$c_f \in \left[\underline{\mathfrak{c}}\left(n^*, r\right), \overline{\mathfrak{c}}\left(n^*, r\right)\right]$$

*and that*

$$\alpha\, \sigma\left(n^*\right) > \left(1-\delta\right) r$$

*The thresholds* $\left\{\underline{\mathfrak{c}}^{ss}\left(n^*, r\right), \overline{\mathfrak{c}}^{ss}\left(n^*, r\right)\right\}$ *and* $\left\{\underline{\mathfrak{c}}\left(n^*, r\right), \overline{\mathfrak{c}}\left(n^*, r\right)\right\}$ *are defined in Appendix C.3.8.*

*Proof.* See Appendix C.3.8. ■

Intuitively, storage will be used whenever the capital share in production $\alpha$ or the aggregate factor share $\sigma\left(n^*\right)$ are low (so that the equilibrium interest rate is depressed for any given capital stock $K^*$), when the savings rate $1-\delta$ is high (so that capital is relatively abundant) or when the return on storage $r$ is high.

When are rational asset bubbles possible in this economy? The (gross) interest rate will be below the (gross) growth rate if and only if

$$\frac{\alpha\, \sigma\left(n^*\right)}{1-\delta} < 1$$

Note that this is the condition for the existence of rational asset bubbles.

**Proposition 7.** *(Possibility of Rational Asset Bubbles) Suppose that the economy features one steady-state with $n^*$ followers. Rational asset bubbles can emerge if*

$$1-\delta > \alpha\, \sigma\left(n^*\right)$$

In the particular case of a steady-state where all industries are a monopoly and the leader is the only producer ($n^* = 0$), the above condition becomes $1-\delta > \alpha\, \rho$.

Under what conditions is capital accumulation dynamically inefficient (i.e. the economy overaccumulates capital)? To answer this question, note that the steady-state capital accumulation is dynamically inefficient if the marginal product of capital is below its marginal cost of production (which is one in this model)

$$\left.\frac{\partial Y}{\partial K}\right|_{K=K^*} < 1$$

If the steady-state $K^*$ is such that all savings are converted into capital, such a condition is verified when $1-\delta > \alpha$ (i.e. the savings rate $1-\delta$ exceeds the capital share in production $\alpha$). If, on the other hand, storage is also used, the condition becomes $\sigma(n^*) > r$ (i.e. the factor share $\sigma(n^*)$ is greater than the return on storage $r$). This result is stated in the following proposition

**Proposition 8.** *(Overaccumulation of Capital) Suppose that the economy features one steady-state with $n^*$ followers.*

1. *If storage is not used in such a steady-state, capital accumulation is dynamically inefficient when*

$$1-\delta > \alpha$$

2. *If storage is used in such a steady-state, capital accumulation is dynamically inefficient when*

$$\sigma(n^*) > r$$

How do Propositions 7 and 8 compare? Note that when storage is used, the steady-state interest rate is necessarily below one, so that the condition in Proposition 7 is trivially satisfied. When storage is not used, 7 and 8 coincide if and only if $\sigma(n^*) = 1$, i.e. when there is perfect competition. Recall that the aggregate factor share is equal to one in the limit case where $c_f = 0$ (so that there are infinitely many firms producing) and $\pi = 1$ (so that the leader does not have a productivity advantage over the followers). In such a limit case, the model can be seen as a standard OLG model with perfect markets, so that the result of Tirole (1985) holds – rational asset bubbles are possible if and only if the economy overaccumulates capital. However, if $\sigma(n^*) < 1$ rational asset bubbles can be possible even when investment is dynamically efficient.

Finally, we shall ask under which conditions the economy features underinvestment. Note that underinvestment will arise whenever (i) storage is built despite (ii) there being no overaccumulation of capital (i.e. capital accumulation is dynamically efficient at the margin). This region is of particular interest because, in such a case, the emergence of asset bubbles can potentially lead to an efficient increase in capital accumulation. Proposition 9 below states the conditions under which the economy features a steady-state with underinvestment.

**Proposition 9.** *(Underinvestment) Suppose that the economy features one steady-state with $n^*$ followers. Such a steady-state features underinvestment if*

$$\sigma(n^*) < \min\left\{\frac{1-\delta}{\alpha} r, r\right\}$$

*Proof.* First note that storage is used in a steady-state provided that $\dfrac{\alpha\,\sigma(n^*)}{1-\delta} < r$. Second, note that steady-state capital accumulation is dynamically efficient if $\alpha\,\varphi(n^*)\,(K^*)^{\alpha-1} > 1$. In a steady-state where storage is used, the capital stock is equal to $K^* = \left[\dfrac{\alpha\sigma(n^*)\,\varphi(n^*)}{r}\right]^{\frac{1}{1-\alpha}}$, so that this condition becomes $\sigma(n^*) < r$. ■

This condition says that the economy features underinvestment if the steady-state factor share $\sigma(n^*)$ is low and the return on storage $r$ is high. Note that the particular case where the steady-state consists of a symmetric monopoly across all industries ($n^* = 0$), the condition is $\rho < \min\left\{\frac{1-\delta}{\alpha}r, r\right\}$. When the economy is in a steady-state in which the condition of Proposition 9 is satisfied, the appearance of asset bubbles can be associated with an efficient increase in investment.

### 3.4 Policy and Welfare

The economy studied in this paper is characterized by one main friction – the existence of imperfect competition in product markets. Absent the formation of bubbles, this friction necessarily translates into inefficient outcomes. In partial equilibrium, each industry is characterized by a suboptimal level of output. In general equilibrium, the rental rate and the aggregate supply of capital result in being depressed, which can leave the economy stuck in a low steady-state. In this context, industrial policies that foster competition or increase firms' incentives to produce can mitigate the negative consequences of market power. Consider, for example, a competition authority that wants to increase the leaders' incentives to produce. The authority can grant an *ad valorem* subsidy $\tau \geq 0$ to the leader, to be financed by means of a lump sum tax $\Gamma \geq 0$ so that

$$\tau\, \tilde{p}_{it}\, \tilde{y}_{it} - \Gamma = 0 \tag{20}$$

Note that such a production subsidy would lower each leader's monopoly price to $\tilde{p}_{it} = \frac{1}{(1+\tau)\,\rho}\Theta_t$. Given this price, the competition authority could then pick a combination of policy instruments $(\tau^*, \Gamma^*)$ satisfying (20) so that each leader exactly breaks even, i.e.

$$(\tilde{p}_{it} - \Theta_t)\, \tilde{y}_{it} - c_f = 0$$

In such a case, no follower will enter and the leader will charge a markup $\tilde{\mu}_{it}^{L} = \frac{1}{(1+\tau^*)\,\rho}$, which is just enough to compensate for the fixed production cost $c_f$. Other types of industrial policies could also be implemented. For example, the competition authority could grant an *ad valorem* or lump sum subsidy to the followers, to be financed with a lump sum tax on the leader. If effective, this policy could attract the entry of additional followers who would force the leader to expand; however, this would come at the cost of the duplication of fixed production costs.

Although these policies make sense in the present model, they can be difficult to implement in practice. For example, it may be hard to sustain an *ad valorem* subsidy to a large corporation (i.e. the leader), or to subsidize the entry/activity of unprofitable companies (i.e. the followers). Yet, this paper suggests that financial market sentiment can provide a substitute for this type of industrial policy. When providing an entry subsidy or increasing firms' incentives to compete for market shares, bubbles can improve the workings of product markets and reduce monopoly rents. Under this view, it may be in the interest of regulatory authorities to facilitate firms' access to stock markets, for example by lifting restrictions to initial public offerings or on new equity issuance. In addition, as this paper shows, bubbles can be too large and lead to excessive entry and competition, with firms sometimes finding it optimal to charge prices below marginal cost. In case financial markets are prone to episodes of large bubbles, regulatory authorities may instead want to keep relatively strict norms on initial public offerings and new equity issuance.

Whether regulatory authorities should facilitate or restrict firms' access to stock markets is ultimately a quantitative question: it depends on whether goods markets are characterized by a high or low degree of market power, and on whether stock markets are prone to experience small or large bubbles. Even though a careful assessment of these questions would require a richer model (which is out of the scope of the current paper), I conclude by providing some tentative answers. In particular, I will use the model to characterize the optimal level of bubble creation in one industry. To be clear, this is not meant to be a comprehensive quantitative exercise. To do so one would need to relax a number of assumptions that have kept the model simple and tractable (e.g. absence of sunk entry costs and of product differentiation within an industry). Rather, my goal is to give a sense of the magnitudes implied by the model and show that they can be relevant for policy.

The analysis is conducted in partial equilibrium (i.e. at the industry level). I normalize aggregate variables $(Y = \theta = 1)$ and set $\rho = 2/3$. The latter implies an elasticity of substitution of $\sigma = 1/(1-\rho) = 3$ which is consistent with most estimates in the literature.[33] Given the condition $\pi > \rho$, I set $\pi = 0.75$. This gives a relatively large advantage to the leader and creates a potentially large role for bubbles.[34] Finally, I consider different values of the fixed cost $c_f$ to simulate alternative market structures; these values are such that the industry can have $n^F = 0, 1, 2, 10$ followers in a bubbleless equilibrium.[35] For each market structure, I then compute the optimal level of bubble creation (i.e. the one that maximizes total economic surplus $\Omega_i$). The results are shown in Table 3 below, where each line represents one alternative market structure. In the first line, the leader is a monopolist in the bubbleless equilibrium $\left(n^F = 0\right)$ and charges a markup of $\mu_L = 1.5$. In the fourth line, there are $n^F = 10$ active followers and the leader charges a markup of $\mu_L = 1.34$. Note that given her productivity advantage over the followers $(1 - \pi = 0.25)$, the leader necessarily charges a markup above one. I then compute the optimal level of bubble creation for the two processes of market psychology I consider (constant firm bubble and constant industry bubble). Under the constant firm level bubble process, bubbles can only have a modest role in raising total economic surplus or welfare. The optimal bubble is nonzero only when the industry starts as a monopoly. This bubble is such that only one follower enters every period, thereby raising total industry output by 23%. Note also that this bubble is small – the total value of new bubble creation (by the leader and the entering follower) represents 1% of total

[33]See for example Broda and Weinstein (2006) and Hsieh and Klenow (2014).

[34]In Appendix D.3, I report the results for $\pi = 1$.

[35]To be precise there is a continuum of fixed costs $c_f \in [\underline{c}(n), \overline{c}(n)]$ consistent with $n$ followers, as implied by inequality (13). For each value $n$, I pick the midpoint of the previous interval.

industry revenues. For all alternative market structures, the optimal value of bubble creation is always zero. All in all, these results indicate that a lump sum subsidy to the followers can have a very limited role in raising total industry welfare. Large lump sum subsidies will necessarily increase the degree of competition, but this does not increase the total economics surplus.

The results happen to be more interesting when the industry bubble process is considered. Under this process, the optimal level of bubble creation is always positive and can be large. For example, when the industry starts as a monopoly, optimal bubble creation represents 38% of total industry revenues. The optimal bubble under this process is larger for two reasons. On the one hand, a larger industry bubble is needed to subsidize the entry of the first follower (since this bubble is split according to market shares). On the other hand, for a fixed number of followers $n^F \geq 1$, a larger bubble always makes the leader produce more (Proposition 1); in other words, an industry bubble is characterized by a larger social return. Indeed, even though this bubble only attracts one new follower, it more than doubles total output (which increases by 128%) and implies a substantial reduction of the markup charged by the leader (from 1.5 to 1.14). Note that even when the industry starts with $n^F = 10$ followers, the optimal new bubble represents 10% of total industry revenues. In this case, the optimal bubble is smaller than in the case of $n^F = 0$ for two reasons: (i) the market structure already starts competitive and (ii) because fixed costs are low, a medium/large bubble can easily generate a situation of excessive entry and competition. Being smaller, the optimal bubble gives the leader with lower incentives to expand (his markup declines from 1.34 to 1.23). In Appendix D.3, I report the results for $\pi = 1$ (i.e. when there are no productivity differences between firms). The results are identical. Note that when the leader has no productivity advantage over the followers, there will be less room for market power. As a result, the size of the optimal bubble is lower under $\pi = 1$.

The main takeaway from this exercise is twofold. First, the effects of asset bubbles on competition critically depend on the process of market psychology, i.e. on whether stock market bubbles merely subsidize the entry of new firms, or also provide firms with the incentives to fight for market shares and expand. Note that both processes are not mutually exclusive, and market psychology can combine elements of both. However, and as already discussed, the industry bubble process captures important aspects of valuation techniques, and can explain the coexistence of large stock market values and negative operating margins in the dotcom bubble. Second, bubbles can have a potentially large role in correcting market power, particularly so in uncompetitive industries. There is indeed evidence that large US firms have a considerable degree of market power. For example, De Loecker, Eeckhout and Unger (2020) have recently reported a sales-weighted average markup of approximately 1.6 for US public firms. This markup implies that, under an elasticity of substitution $\sigma = 3$ (as I assumed in this exercise), large US public corporations behave like monopolists. In such a case, this model suggests that asset bubbles can have an important role in promoting product market competition. The optimal level of new bubble creation can be large and as high as 38% of total industry revenues.

Before concluding, I should mention three aspects to be explored in future research. First, my analysis has

| Optimal Bubble Creation | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $b = 0$ | | Firm-Level Bubble | | | | Industry-Level Bubble | | | |
| $n^F$ | $\mu^L$ | $n^F$ | $\mu^L$ | $\frac{b_i}{p_i\, y_i}$ | $\frac{y_i\,(b=b^*)}{y_i\,(b=0)}$ | $n^F$ | $\mu^L$ | $\frac{b_i}{p_i\, y_i}$ | $\frac{y_i\,(b=b^*)}{y_i\,(b=0)}$ |
| 0 | 1.50 | 1 | 1.40 | 0.01 | 1.23 | 1 | 1.14 | 0.38 | 2.28 |
| 1 | 1.40 | 1 | 1.40 | 0.00 | 1.00 | 2 | 1.25 | 0.14 | 1.56 |
| 2 | 1.38 | 2 | 1.38 | 0.00 | 1.00 | 4 | 1.23 | 0.13 | 1.31 |
| 10 | 1.34 | 10 | 1.34 | 0.00 | 1.00 | 19 | 1.23 | 0.10 | 1.15 |

**Table 3**
This table characterizes optimal bubble creation, for the two different processes considered: constant firm-level bubble and constant industry-level bubble. I normalize aggregate variables $Y = \theta = 1$ and set $\rho = 2/3$ and $\pi = 0.75$. I pick four different values for the fixed cost to generate bubbleless equilibria with $n = 0, 1, 2, 10$ followers. These values are $c_f =$ 0.02, 0.002, 0.001, 0.0001.

focused only on product market frictions. If other frictions were to be considered (e.g. financial frictions), asset bubbles could have a larger role in raising output and welfare. Second, I have taken the perspective of a competition authority aiming to maximize total economic surplus $\Omega_i$. To the extent that bubbles also provide a liquidity instrument or store of value, there can be other benefits associated with the issuance of bubbly equity. Finally, this exercise has been centered on a single industry and has taken aggregate variables as given. A careful quantitative general equilibrium analysis would require a more realistic demographic structure and a richer degree of industry heterogeneity. These steps shall be conducted in future research.

# 4 Conclusion

Financial history shows that stock market boom/bust episodes are often an industry phenomenon that can be accompanied by significant changes in the market structure. Motivated by this observation, this paper developed a framework to investigate the interactions between asset bubbles and product market competition. At the heart of the model is the idea that asset bubbles can reduce barriers to entry and force firms to expand, to the ultimate benefit of consumers. An interesting aspect of the theory is that asset bubbles may force (productive) market leaders to expand even when they are attached to potential (unproductive) competitors. This observation helps us think about different questions. For instance, how will a large company react to a bubble on its stock prices? Will Apple lower the price of its *iPhones* if investors suddenly become excited about the company alone and its market value doubles? This paper suggests that it will probably not. Instead, Apple is more likely to expand and cut its profit margins in the presence of a generalized boom in which potential competitors (perhaps smaller and less innovative) can also get overvalued. In such a case, as barriers to entry decrease, Apple may be forced to expand in order to preserve its market share.

The model developed in this paper also gives a novel perspective on famous stock market overvaluation episodes. For instance, it may explain why British railway companies duplicated some of their own lines during the 1840s mania or why large corporations (such as GE) had incentives to adapt their businesses quickly to the internet in the late 1990s. Furthermore, it provides a simple rationale for the low and negative profit margins reported by high-tech firms at the peak of the dotcom bubble. Rather than the realization of a negative technology shock (as argued by Pastor and Veronesi (2006)) this paper suggests that such income losses may have been a rational reaction to an environment characterized by high stock prices. This view seems indeed to receive support from the anecdotal evidence reviewed in Appendix A.

I conclude by pointing to some avenues for future research. The first concerns the role of policy and welfare. This paper provides a stylized model that connects financial and product markets. Its theoretical simplicity allowed me to uncover new mechanisms, but makes it unsuitable for a careful welfare and policy analysis. Although Section 3.4 makes a first step in that direction, a full quantitative analysis is out of the scope of the current paper and is left for future research. Second, by making a connection between the degree of competition in product markets and the interest rate, this paper can also shed light on recent US macroeconomic trends. The last four decades of US history have been characterized by both a steady decline in real interest rates and an increase in market power, evident from an increase in markups (see Hall (2018) and De Loecker, Eeckhout and Unger (2020)) and measures of industry concentration (Autor et al. (2020)). Although there may be different forces contributing to the decline of interest rates, this model suggests that it can be connected to the increase of market power. I believe that a serious assessment of this hypothesis is an important avenue for future research.

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# A Competition in Famous Bubble Episodes

Stock market boom/bust episodes are recurring phenomena in financial history. Famous examples include the Mississippi and the South Sea bubbles of 1720, the British railway mania of the 1840s or more recently the dotcom bubble of the late 1990s. In this section, I provide a brief description of two of these episodes – the British railway mania of the 1840s and the dotcom bubble of the late 1990s – and discuss how they can be reinterpreted in light of the theory developed above.[36]

## A.1 The British Railway Mania of the 1840s

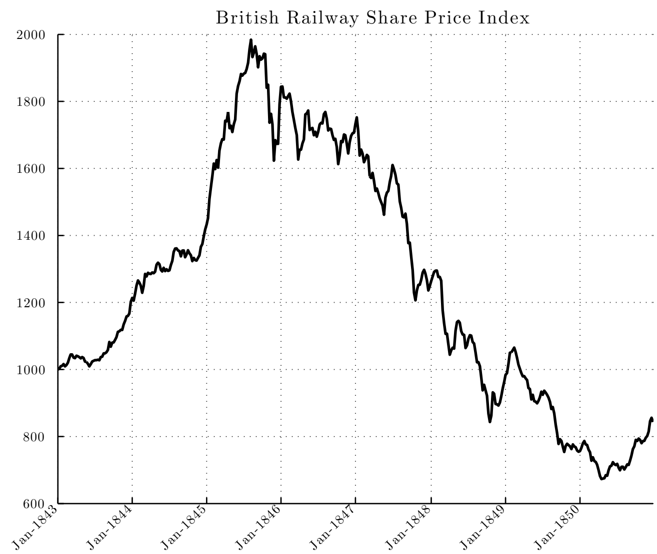



Figure 7: British Railway Share Price Index, 1843-1850
(Source: Campbell and Turner (2015))

The mid 1840s was a period of fast economic growth in Britain: favorable weather conditions (resulting in abundant harvests), together with historically low interest rates made Britain's GDP grow at an average of rate of 4.6% between 1843 and 1845. It was within this environment that a collective enthusiasm about railways emerged. Contrarily to the majority of other countries, where the construction of railway lines was essentially a public investment, the expansion of the British railway system was financed by private companies and individuals. This widespread excitement attracted many new investors to the stock market and triggered a boom in stock prices:

[36]In spite of its relatively short duration, the emergence of the *South Sea bubble* of 1720 also seems to have resulted in larger entry and competition in the British financial industry. As the prices of the *South Sea Company* soared, several joint stock companies started to adopt competing financial schemes in the London stock market. The capital attracted by some of these companies – in particular the recently founded *Royal Assurance Company* and the *London Assurance Company* – posed a direct threat to the *South Sea Company*, which was forced to seek political support from the British Parliament to preserve its *status quo*. The process culminated in the bubble Act of June 1720, which forbade the creation of new unauthorized joint-stock companies (see Garber (1990), Neal (1990) and Harris (1994)).

between January of 1843 and October of 1845, the share prices of railway companies increased by more than 100% (Campbell and Turner (2010)).[37] At the same time, investment shot up: total investment in new railway lines authorized by the British Parliament rose by an average of £4 million per year prior to 1843, to £60 million in 1845 and £132 million in 1846 (Haacke (2004)).[38] Even though not all investments granted parliamentary authorization would ever materialize, total capital formation by railway companies reached £30 million in 1846 and £44 million and in 1847, which represented 5.2% and 7.3% of the British GDP respectively. By comparison, during the dotcom bubble of the late 1990s, total US investment in technological industries reached a maximum of 2.8% of GDP in the year 2000.[39]

Such collective enthusiasm would however cease in the middle of the decade. The escalation of construction costs resulted in substantial calls for capital from railway shareholders and several projects ended up being less profitable than expected.[40] As a result, the share prices of railway companies started to decline and between October 1845 and December 1850 the total stock market capitalization of railway companies decreased by 67% (Campbell and Turner (2010)).

The deteriorating performance of railway companies was ultimately related to an environment dominated by intense competition and, in some cases, overinvestment. Not only new lines opened in relatively unprofitable regions (serving sparsely populated areas) but there were also obvious examples of duplication of railway lines. Situations of line duplication were described (and sometimes harshly criticized) by many contemporary authors. One example, which is described in Cotterill (1849), is the railway line that connected Shrewsbury to Stafford, which opened in 1849 and was in operation until 1966. It was run by The Shropshire Union Railways and Canal Company, founded in 1846:

> *"The Shropshire Union Railway is another instance of the baneful principle [of competition]. It is a line from Shrewsbury to Stafford, joining the Trent Valley; and there being no intermediate traffic, the expenditure of 6 or 700,000l to effect this junction, appears prima facie to be lavish; because, if the Shrewsbury people wish to go to London, there is the Shrewsbury and Birmingham Railway, accommodating at the same time an immense intervening population. If the Shrewsbury people are desirous of moving north, the Shrewsbury and Chester, a line long since in operation, would give ample accommodation. The Shropshire Union to Stafford would therefore appear to be unnecessary and useless. But it is the fruit of competition."*

[37] Individual investors financing railway projects around this time include famous scientists, intellectuals and politicians such as Charles Darwin, Charles Babbage, John Stuart Mill or Benjamin Disraeli (Odlyzko (2010)).

[38] Despite being private investment, the construction of new railway lines required parliamentary authorization. This happened because they often involved processes of land expropriation (Odlyzko (2010)).

[39] Data is from the Bureau of Economic Analysis. The industries considered include Computer and Electronic Products, Publishing Industries, Broadcasting and Telecommunications and Information and Data Processing Services.

[40] *The Economist*, "The Railway Crisis – its Cause and its Cure", 1848/10/21.

Another example involving the duplication of railway lines was the connection between Birmingham and Wolverhampton, described in Martin (1849, p.37). In 1846, the two cities were already connected by the *Grand Junction Railway* (and by water through the *Birmingham Canal*). Still, two other companies – the *London and North Western Railway* and the *Great Western Company* – were granted authorization to build two additional lines between the two cities:

> "*Three years ago, the district between Birmingham and Wolverhampton possessed a double communication for its traffic (...) by means of the Birmingham Canal and the Grand Junction Railway, each connecting the two towns. Additional Railway accommodation was, however, supposed to be desirable, and two Companies presented their rival plans to a* Committee of the House of Commons *for selection. Both Railways are now in the course of formation, traversing a highly valuable and thickly peopled district in* parallel lines (at some points nearly touching each other)*, and each intended to terminate in separate stations in the centres of the two towns. At least four millions of money will thus be unprofitably sunk, in order that three lines of railway and one canal may afford a redundant accommodation to a tract some fourteen miles in length.*"

This example makes the author conclude that "*Monopoly has an ill sound: but, unless it can be proved to be incapable of regulation, we must prefer even monopoly to competition run mad.*"

The idea that the British railway mania was associated with an environment of increased competition is corroborated by indicators of market power. For instance, in their study of competition during the railway *mania*, Campbell and Turner (2015) found that the fraction of lines that enjoyed absolute monopoly fell from 72% in 1843 to 32% in 1850. Furthermore, the *per mile* profits of established companies (i.e. existing in 1843) fell from £1,811 to £1,231 (by 32%). Despite the lower profitability, and confirming some of the anecdotes described above, incumbent companies expanded their capacity quite dramatically: between 1843 and 1850, the milage operated by the average incumbent company grew from 36 to 153 miles.

Why did railway companies expand so quickly? What was behind "*competition run mad*", to use the words of Martin (1849, p.37)? Although different factors may have contributed to the expansion of the British railway system during the 1840s (such as a political environment highly favorable to free markets and competition), these events can be rationalized by the model presented in this paper. As investors perceived railway stocks to be good financial assets (whose price was likely to appreciate in the future), vast amounts of money were poured into the British railway industry. Such high demand for railway shares may have then opened the door to the appearance of new companies and lines that were not profitable from an operating point of view. That the mania was a time characterized by positive sentiment and speculation in railway companies is confirmed by several contemporaneous writers. For instance, keeping his critical view on the events, Martin (1849, p.40) observes that

> "*Men and women, high and low, rich and poor, entered the destructive road of which the gates were so widely opened by the Legislature, in the expectation that all could suddenly become rich; the result to many was, that the*

*rich were impoverished, and persons without a shilling rose on their ruin."*

Seen in this way, the expansion of the British railway system may have been commanded (at least in part) by financial market sentiment. The idea that investor sentiment may drive firms' expansion at the expense of profit margins, and ultimately provide a subsidy to consumers, was a central message of the model presented in this paper. As noted by Jackman (1916, p. 602), "*although many of the railways were not profitable to their owners in yielding large financial returns they may still have been beneficial to the public in providing for the necessities and conveniences of traffic*".

## A.2 The Dotcom Bubble of the Late 1990s

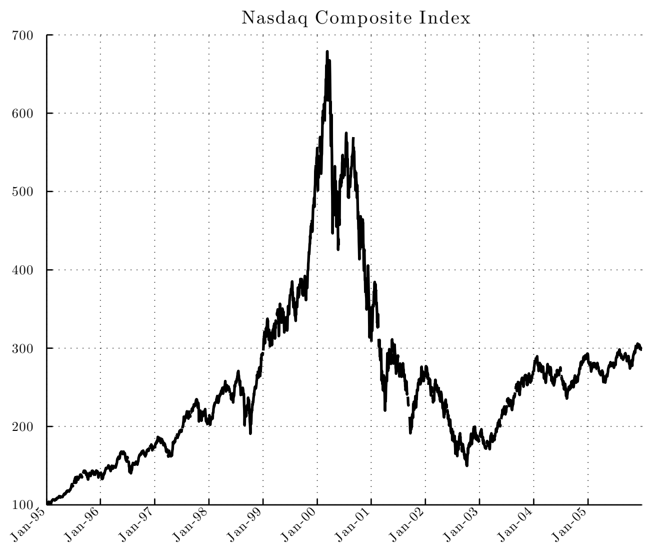



Figure 8: The NASDAQ Composite Index, 1995-2005

Another famous stock market boom and crash would take place in the United States one century and a half later. Associated with the appearance of the internet and in a period marked by low interest rates, the NASDAQ index increased by more than 560% between January 1995 and March 2000 (Figure 8). However, as in the British railway mania of the 1840, such widespread enthusiasm would also cease. Concerns about the persistently negative profitability of the new internet firms and the fact that some were running out of cash marked a turning point in market sentiment. An article published in *Barron's* magazine in March 2000 sounded the alarm: "*An exclusive study conducted for Barron's by the Internet stock evaluation firm Pegasus Research International indicates that at least 51 'Net firms will burn through their cash within the next 12 months. This amounts to a quarter of the 207 companies included in our study.*" And it added "*It's no secret that most Internet companies continue to be money-burners. Of the companies in the Pegasus survey, 74% had negative cash flows. For many, there seems to be little realistic hope of profits in the near term.*" A natural question therefore emerged: "*When will the Internet bubble burst?*".[41] The downturn would start that very same month: between March 2000 and October 2002, the NASDAQ index decreased by 77%.[42]

Behind the poor performance of so many dotcom firms was a search for rapid growth involving aggressive commercial practices – such as extremely low penetration prices, overspending in advertising and excess capacity – and which resulted in low levels of profitability or even extensive losses.[43] For instance, many new companies offered their services at unprofitably low prices or even for free. This was, for instance, common among delivery companies. *Kozmo.com* and *UrbanFetch* were two such examples – offering one-hour delivery services of books, videos, food and other goods totally for free. Many products would even be sold at a discount, gifts were sometimes included and tips were not accepted. None of them survived the stock market crash in 2000. The online music industry also observed many of these practices, with companies such as *CDNow.com*, *Riffage.com* or *Napster* offering downloads or peer-to-peer sharing of music for free.[44] Another example is the software company SunMicrosystems, which decided to enter the office suite market (largely dominated by Microsoft Office) with a software that was made available completely for free (this example is reviewed in more detail below). The pressure for growth was in some cases so high that some companies would actually pay customers to use their services. One well-known example is the advertising company *AllAdvantage.com* (launched in 1999), which has made famous the slogan "*Get Paid to Surf the Web*". Users of *AllAdvantage.com* needed to download a viewbar that displayed advertisements at the bottom of their screens and would be paid $0.5 per each hour logged. Furthermore, members could also invite friends (without any limit) and would receive an additional $0.1 for every hour that person was active. In the first quarter of the year 2000 (which coincided with the peak of the bubble) *AllAdvantage.com* paid a total of $40 million to its members, leading to a loss of $66 million. It also did not survive the market crash and ceased its operations in that same year. Companies that engaged in similar practices include Spedia, Click-Rebates, Jotter Technologies, Radiofreecash and Adsavers.com (Haacke (2004)).

These business strategies were often justified by a first-mover advantage type of argument – most internet businesses were understood to be natural monopolies, where only one firm could ultimately survive. Hence the search for rapid growth and the "*get big fast*" or "*get large or get lost*" mottos. However, these extreme commercial practices were also incited by financial markets. The fact that valuation metrics were often focused on revenue targets or market shares created incentives for rapid growth at the expense of profits (Aghion and Stein (2008)). Indeed, venture capitalists and company executives explicitly admitted their strategies were influenced by financial market sentiment. For instance, Michael Moritz – founder of Sequoia Capital, a venture capital firm that was an

[41] Jack Willoughby, "Burning Up; Warning: Internet companies are running out of cash – fast", *Barron's*, March 20, 2000

[42] Although there is no consensus, a great deal of evidence suggests that technology stocks were overvalued in the late 1990s. See for instance Ofek and Richardson (2002) and Lamont and Thaler (2003).

[43] See Haacke (2004) and Razi, Siddiqui and Tarn (2004) for an analysis of some of the biggest dotcom failures.

[44] See Haacke (2004), p.91 and Razi, Siddiqui and Tarn (2004)

initial funder of Yahoo! – admitted in an interview that "*The world was rewarding us for raising $250 million and penalizing [us for] raising $25 million. Daring to be great overweighted being cautious*".[45] In a similar vein, eToys' founder and CEO Toby Lenk admitted that *"It was the whole land-grab mentality. Grow, grow, grow. Grab market share and worry about the rest later. When you're in that cycle, and less capable people are doing I.P.O.'s, it's like an arms race. If you turn down the gun and put it on the table, all you're doing is letting other people pick it up and shoot you. I made the decisions and I take full responsibility. But there were a lot of amazing forces at work."*[46] Like many other *dotcoms*, eToys would not survive the stock market crash in 2000. Toby Lenk recognizes that the attempt to grow too fast was one of the main reasons behind the failure of eToys: *"We had the capacity for $500 million in revenue but came to a stop at $200 million. That's hard to survive*".

It is therefore interesting to note that, as in the British railway mania 150 years before, the NASDAQ boom of the late 1990s was also associated with rising competitive pressures in product markets, and with situations of excessive investment and low (or even negative) profit margins that became unsustainable once market sentiment reversed. As argued by Varian: "*the driving force behind the rise and fall of the Nasdaq was simple competition.* [...] *in 1999 there was no fundamental scarcity of new business models for dot-coms. The result was an intensely competitive environment, where it has been extremely difficult to make money*."[47]

However, even if lacking market expertise and in many cases investing beyond reasonable levels, many of the new companies posed a competitive threat to incumbents. I next review some examples.

**Sun Microsystems and Microsoft** One significant example in this category is the one involving SunMicrosystems and Microsoft, which is described in Varian (2003). Back in 1999 when the dotcom bubble was about to reach its peak, Sun Microsystems decided to enter the office suite market, which was largely dominated by Microsoft Office. It decided to launch a new office suite called StarOffice and to make it available for free. Besides releasing the software at zero price, SunMicrosystems also promised to make its source code, file formats, and protocols free. This move was seen at that time as a clear attack on Microsoft's dominant position in the market: *"Many in the industry view Sun's move as a direct assault on Microsoft's second most lucrative monopoly*".[48] However, Sun would be severely hit by the stock market crash (its stock price plunged from $63.4 in 8/31/2000 to $3.28 in 11/12/2002).

The threats posed by companies such as SunMicrosystems were recognized by Microsoft in its annual reports. For instance, the 2000 report states that "*Rapid change, uncertainty due to new and emerging technologies, and fierce competition characterize the software industry, which means that Microsoft's market position is always at risk. "Open source"*

[45] See Haacke (2004), p.108

[46] "How to Lose $850 Million – And Not Really Care" (06/09/2002), retrieved from www.nytimes.com/2002/06/09/magazine/ how-to-lose-850-million-and-not-really-care

[47] Hal Varian, "Economic Scene: Comparing Nasdaq and Tulips Unfair to Flowers", The New York Times, 2001/02/08.

[48] Joe Barr, "Is Sun's StarOffice a Microsoft Killer?", 10/08/1999, CNN.com.

*software [...] are current examples of the rapid pace of change and intensifying competition. [...] Competing operating systems, platforms, and products may gain popularity with customers, computer manufacturers, and developers, reducing Microsoft's future revenue*" [Annual Report, 2000, p. 16].

Microsoft also anticipated the necessity to reduce the price of some of its products*: "The competitive factors described above may require Microsoft to lower product prices to meet competition, reducing the Company's net income"* [Annual Report, 2000, p. 17].

**eToys and Toys"R"Us** The retail market for toys experienced considerable action in the late 1990s. Several firms such as eToys, Toysmart, Toytime and Red Rocket appeared as online toy retailers, but went bankrupt in the years 2000 and 2001 as stock prices started to decline. The case of eToys was particularly impressive: it was established in 1997, had its IPO in 1999 and in the same year reached a market capitalization of 8 billion dollars (Shiller (2000)). This value was 33% larger than that of the market leader Toys"R"Us, a well-known company, much larger in terms of size and profitability (see Table 4).

| Firm | Market Value | Sales | Profits |
|---|---|---|---|
| Toys "R" Us | $ 6 billion | $ 11,200 million | $ 376 million |
| eToys | $ 8 billion | $ 30 million | $ -28.6 million |

**Table 4**
Sales and profits refer to 1998, whereas market value refers to 1999

Despite their short existence, the newly founded companies posed a serious competitive threat to Toys"R"Us, which was forced to enter the online market. After a series of unsuccessful experiences with its own website (toysrus.com), it then started a 10-year partnership with Amazon.com in the year 2000.

**GE and the "Destroy Your Business" strategy** The strategic reaction of Toys"R"Us was common among many large, well-established corporations. One well-known example is the "Destroy Your Business" program launched by GE's CEO Jack Welch in 1999. Welch asked all GE's managers to think of possible ways in which Internet startups could challenge their market leadership in different businesses and to adopt effective strategies to avoid such scenarios. The process was focused on adopting the necessary innovations before a new dotcom company appeared and took advantage of such an opportunity. For instance, GE Plastics (a specialized supplier of plastics, established in 1973 as a division of GE), decided to enter the online market in 1997. As part of the "Destroy Your Business" program, GE Plastics e-commerce manager Gerry Podesta and his team decide to equip their website with new tools and functionalities. They got inspiration from car manufacturers' websites, which were developing configuration tools that allowed consumers to customize their cars. A similar scheme was then introduced in the website GE Plastics, allowing potential customers to design their products online, indicating different materials that could be used, their characteristics and cost.

We can also mention the example of several other divisions of GE. GE Medical Systems – a manufacturer of diagnostic imaging systems such as CAT scanners and mammography equipment – launched a platform called iCenter as part of the "Destroy Your Business" initiative. This was an online system designed to monitor GE customers' equipment, collect data and provide customers with information on their relative performance and suggestions on how to improve it. GE Appliances also started using the internet to sell its products. Appliances were traditionally sold through retail stores, but GE feared that such a model could be challenged with the emergence of new internet retailers (which could give preference to appliances from alternative brands). It then developed a point-of-sale system placed in traditional retail stores where customers could make online orders.

# B Data Appendix

## B.1 The Dotcom Bubble: Firms with Negative Operating Earnings

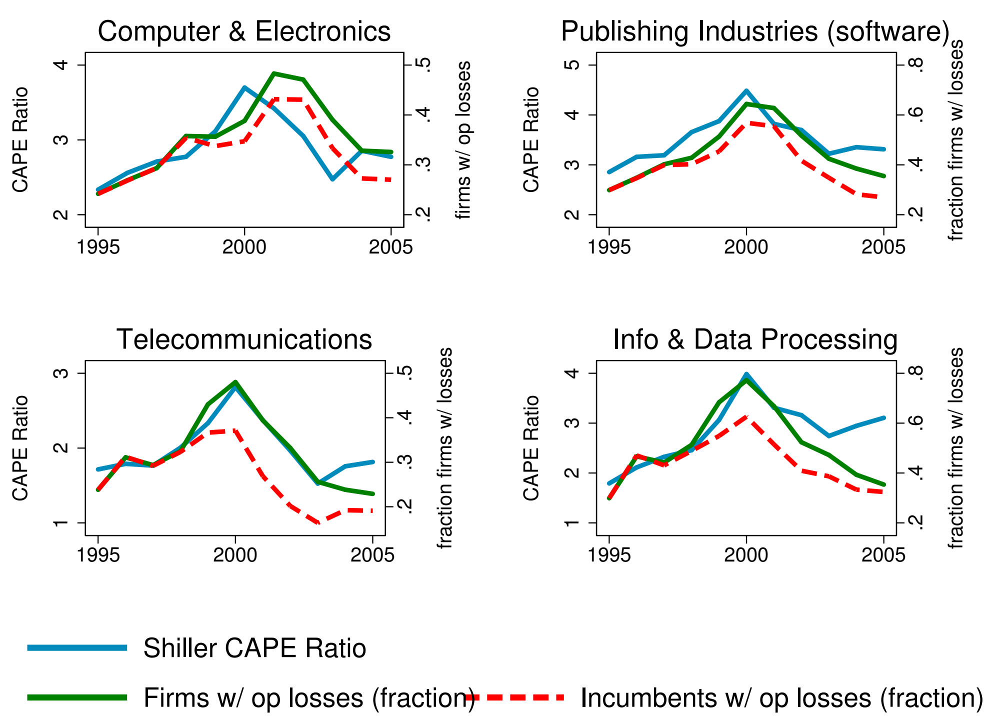



Figure 9: The dotcom bubble: firms with negative earnings

This figure shows the Shiller CAPE ratio and the fraction of firms with negative operating earnings (EBITDA) for four industries during 1995-2005: 'Computer & Electronic Product Manufacturing' (NAICS 334), 'Publishing Industries (software)' (NAICS 511), 'Telecommunications' (NAICS 517), and 'Information & Data Processing' (NAICS 518-519). The CAPE ratio is the ratio of total stock market capitalization to a 10–year moving average of past earnings (EBITDA); the ratio is in logs and is measured at the beginning of the year (see Appendix B.4 for more details on data construction and definitions). The group of incumbents includes all firms that already existed in 1995.

## B.2 Industry Classification

| | | | Number of Firms | | | Market Capitalization | | |
|---|---|---|---|---|---|---|---|---|
| | Name | NAICS | Median | Min | Max | Median | Min | Max |
| 1 | Food Manufacturing | 311 | 105 | 35 | 143 | 399.2 | 101.1 | 844.1 |
| 2 | Beverage and Tobacco Manufacturing | 312 | 54 | 18 | 68 | 593.4 | 159.7 | 1595.8 |
| 3 | Textile Mills and Textile Product Mills | 313-314 | 15 | 5 | 45 | 9.2 | 3.8 | 25.2 |
| 4 | Apparel Manufacturing | 315 | 55 | 13 | 89 | 75.6 | 19.1 | 281.6 |
| 5 | Leather and Allied Product Manufacturing | 316 | 21 | 7 | 30 | 34.1 | 6.1 | 142.4 |
| 6 | Wood Products | 321 | 37 | 16 | 57 | 36.7 | 13.1 | 79 |
| 7 | Paper Products | 322 | 57 | 20 | 99 | 175.1 | 71.5 | 329.3 |
| 8 | Printing Activities | 323 | 28 | 8 | 50 | 17.2 | 8.8 | 23.9 |
| 9 | Chemical Products | 325 | 611 | 241 | 676 | 2573.6 | 506.3 | 4799.6 |
| 10 | Plastics and Rubber Products | 326 | 61 | 13 | 88 | 34.7 | 11.5 | 70.9 |
| 11 | Non Metallic Mineral Products | 327 | 35 | 13 | 59 | 53.5 | 5.9 | 120.3 |
| 12 | Machinery Manufacturing | 333 | 237 | 102 | 350 | 406.2 | 94.8 | 926.5 |
| 13 | Computer and Electronic Products | 334 | 717 | 252 | 972 | 2007.2 | 206.1 | 4673.1 |
| 14 | Electrical Equipment | 335 | 102 | 31 | 130 | 187.3 | 80.9 | 403.9 |
| 15 | Transportation Equipment | 336 | 149 | 66 | 189 | 756.2 | 175.3 | 1796.6 |
| 16 | Furniture | 337 | 32 | 10 | 45 | 18.7 | 3 | 36.6 |

| | | | Number of Firms | | | Market Capitalization | | |
|---|---|---|---|---|---|---|---|---|
| | Name | NAICS | Median | Min | Max | Median | Min | Max |
| 17 | Wholesale Trade (Durables) | 423 | 108 | 53 | 148 | 81.5 | 10.4 | 191.3 |
| 18 | Wholesale Trade (Non-Durables) | 424 | 73 | 32 | 91 | 122.2 | 10.6 | 246 |
| 19 | Retail Trade (Motor Vehicle and Parts) | 441 | 21 | 13 | 30 | 33 | 2.8 | 103.6 |
| 20 | Retail Trade (Furniture) | 442 | 9 | 1 | 20 | 10.6 | 0.4 | 28.3 |
| 21 | Retail Trade (Building Material) | 444 | 8 | 5 | 24 | 118 | 8.7 | 336.8 |
| 22 | Retail Trade (Food and Beverage) | 445 | 30 | 7 | 55 | 142.3 | 43.1 | 1979 |
| 23 | Retail Trade (Health and Personal Care) | 446 | 25 | 6 | 38 | 117.1 | 12.4 | 294.3 |
| 24 | Retail Trade (Clothing) | 448 | 58 | 9 | 84 | 96.2 | 21.9 | 197.5 |
| 25 | Retail Trade (Sporting, Music and Books) | 451 | 18 | 2 | 30 | 12.2 | 4.2 | 23.2 |
| 26 | Retail Trade (General Merchandise) | 452 | 33 | 11 | 56 | 393.8 | 79.2 | 554.5 |
| 27 | Retail Trade (Nonstore Retailers) | 454 | 42 | 16 | 64 | 39.5 | 3.6 | 992.6 |
| 28 | Air Transportation | 481 | 37 | 17 | 43 | 71.3 | 13.8 | 272.7 |
| 29 | Rail Transportation | 482 | 13 | 6 | 21 | 102.2 | 29.4 | 374.9 |
| 30 | Water Transportation | 483 | 31 | 19 | 63 | 56.8 | 4.8 | 122.2 |
| 31 | Truck Transportation | 484 | 30 | 19 | 52 | 25.6 | 2.5 | 82 |
| 32 | Pipeline Transportation | 486 | 24 | 17 | 51 | 176.9 | 10.5 | 548.1 |

| | Name | NAICS | Number of Firms | | | Market Capitalization | | |
|---|---|---|---|---|---|---|---|---|
| | | | Median | Min | Max | Median | Min | Max |
| 33 | Support Activities for Transportation | 488 | 20 | 8 | 25 | 15 | 3 | 34.8 |
| 34 | Couriers and Messengers | 492 | 8 | 1 | 15 | 100.1 | 4.1 | 173.9 |
| 35 | Publishing Industries | 511 | 258 | 76 | 564 | 645.5 | 46.3 | 2046.7 |
| 36 | Motion Pictures, Radio and Television Broadcasting | 512-515 | 103 | 29 | 167 | 538.8 | 107.8 | 1011.2 |
| 37 | Telecommunications | 517 | 160 | 38 | 303 | 1693.6 | 322.6 | 3766.6 |
| 38 | Information and Data Processing Services | 518-519 | 200 | 37 | 318 | 667.1 | 59.4 | 3660.1 |
| 39 | Real Estate | 531 | 231 | 127 | 307 | 265.8 | 12.9 | 1219.8 |
| 40 | Rental and Leasing | 532 | 38 | 14 | 65 | 39.4 | 9.9 | 179.5 |
| 41 | Professional and Technical Services | 541 | 286 | 77 | 482 | 381 | 51.5 | 801.7 |
| 42 | Administrative and Support Services | 561 | 135 | 50 | 219 | 130.1 | 53.6 | 344.7 |
| 43 | Education | 610 | 28 | 16 | 39 | 23.8 | 4.5 | 56.5 |
| 44 | Health Care | 621 | 119 | 39 | 195 | 106 | 20.1 | 219.1 |
| 45 | Arts, Entertainment and Recreation | 713 | 38 | 8 | 53 | 19.3 | 5.5 | 48.7 |
| 46 | Accommodation | 721 | 39 | 18 | 73 | 69.2 | 13.9 | 241.5 |
| 47 | Food Services | 722 | 81 | 32 | 139 | 131.4 | 18.4 | 419.5 |

## B.3 Variable definitions

- Stock market capitalization: end-of-year stock price (COMPUSTAT item #199) times common shares outstanding (COMPUSTAT item #25).
- Dividends: common dividends (COMPUSTAT item #21)
- Net stock repurchases: net purchases of common and preferred stock (COMPUSTAT item #115 - item #108) minus change in preferred stock/redemption value (Δitem #56).
- Sales (COMPUSTAT item #12)
- Cost of the goods sold (COMPUSTAT item #41)
- Selling, General, and Administrative Expense (COMPUSTAT item #189)
- EBITDA: earnings before interest, taxes, depreciation and amortization (COMPUSTAT item #13).[49]
- Employees (COMPUSTAT item #29)
- Property, Plant, and Equipment – Total (Net) (COMPUSTAT item #8)
- Property, Plant, and Equipment – Capital Expenditures (Schedule V) (COMPUSTAT item #30)
- Depreciation (COMPUSTAT item #14)

Nominal variables are deflated by the Consumer Price Index (obtained from the Bureau of Labor Statistics).

[49] In COMPUSTAT, this the same as Operating Income Before Depreciation.

## B.4 Industry Aggregates

Variables such as the Shiller CAPE ratio are constructed with industry-aggregated data. Naturally, one needs to correct for the addition and deletion of firms in the dataset. To this end, I apply a correction factor to all variables. The correction factor is chosen so that changes in aggregate sales cannot be attributable to the addition or deletion of firms in the dataset.[50]

To make it clear, let $\text{sale}_{jt}$ be the sales of firm $j$ in year $t$. Let $S_t^0$ be the set of active firms in a given industry at year $t$ that already existed in $t-1$. Let $S_t^1$ be the set of active firms at year $t$ that are also active in year $t+1$ (in the same industry). The correction factor applied at year $t$ is denoted by $\gamma_t$ and is recursively defined as[51]

$$\gamma_t = \gamma_{t-1} \frac{\sum_{j \in S_t^0} \text{sale}_{jt}}{\sum_{j \in S_t^1} \text{sale}_{jt}}$$

Given this definition, for any firm level variable $x_{jt}$ the corresponding aggregate variable $X_t$ is constructed as

$$X_t = \gamma_t \sum_{j \in S^1} x_{jt}$$

I construct industry aggregates for stock market capitalization, earnings, dividends and net stock repurchases. Aggregate net stock repurchases are set to zero when negative.[52]

[50] An identical methodology underlies the construction of most market aggregates such as the S&P 500 total market capitalization, dividends or earnings.

[51] Note that the set of firms that transition from $t$ to $t+1$ corresponds to the set of firms existing at $t+1$ that were active in the previous period, i.e. $S_t^1 = S_{t+1}^0$.

[52] Even if individual firms are allowed to pay a negative dividend (i.e. they issue new stock in net terms), this is precluded at the industry-aggregated level. This is done to ensure that industry-level dividends are always strictly positive so that a growth rate is well defined.

## B.5 Campbell and Shiller VAR: Estimates

| Pooled VAR: 1990-2019 | | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| VARIABLES | $\Delta d_{it+1} - r^M_t$ | $p_{it+1} - d_{it+1}$ | $\text{cape}^{10}_{it+1}$ |
| $\Delta d_{it} - r^M_{t-1}$ | -0.0962** | 0.112*** | 0.0326*** |
| | (0.0399) | (0.0429) | (0.0126) |
| $p_{it} - d_{it}$ | 0.379*** | 0.641*** | 0.0443*** |
| | (0.0291) | (0.0320) | (0.0118) |
| $\text{cape}^{10}_{it}$ | -0.0850** | -0.148*** | 0.777*** |
| | (0.0430) | (0.0463) | (0.0205) |
| Observations | 1,392 | 1,392 | 1,392 |
| R-squared | 0.239 | 0.342 | 0.643 |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

**Table 5**
This table shows the results for the panel VAR defined in equation (3). Data is from COMPUSTAT and the estimation is done for the period 1990-2019. Industry variables are aggregated from firm data and a correction factor is applied to adjust for listing/delisting. See Appendix B.4 for details.

Table 5 shows the estimate of the VAR defined in equation (2). When looking at the results of column (1) we see that both the price-dividend ratio $(p_{it} - d_{it})$ and the CAPE ratio $\left(\text{cape}^{10}_{it}\right)$ happen to be strong predictors of discounted dividend growth $\left(\Delta d_{it+1} - r^M_t\right)$. As in the results reported by Campbell and Shiller (1988) for the S&P composite index, the price-dividend ratio indicates large future dividend growth, whereas a large CAPE ratio predicts slower dividend growth.

Current dividend growth $\Delta d_{it} - r^M_{t-1}$ also predicts future dividend growth $\Delta d_{it+1} - r^M_t$. However, contrarily to the results of Campbell and Shiller (1988) the two seem to be negatively correlated, i.e. fast dividend growth in the past indicates slower dividend growth in the future. Indeed, when we look at $\Delta d_{it}$ alone, we find that 38 out of 47 industries have a negative first order autocorrelation. This contrasts with the results that would be obtained on economy-aggregated data, where dividend growth exhibits a positive first order autocorrelation.

## B.6 The industry discount factor $\sigma_i$

In the construction of the Campbell and Shiller price-fundamental deviation, I assume that industries dividends are discounted at rate $R_{it} = 1 + r_{it}$ such that

$$E_t\{r_{it}\} = E_t\left\{r_t^M\right\} + \sigma_i$$

where $r_t^M$ denotes a market return (e.g. the return on the SP composite index). In my baseline measure, $\sigma_i$ is set to zero. Figure 10 shows the estimated fundamental and price-fundamental deviations for one particular industry under different values for $\sigma_i$. As one can see, $\sigma_i$ essentially pins down the level of the fundamental (and hence the long-run average of the price-fundamental deviation) but not its time series patterns. In Queirós (2020), I discuss alternative ways of obtaining $\sigma_i$. For example, the theory of rational bubbles states that bubbles cannot be negative; using this condition, one can pin down $\sigma_i$ by imposing a lower bound on the price-fundamental deviation so that $\min_t\left\{p_{it} - \hat{f}_{it}\right\} = 0$ .

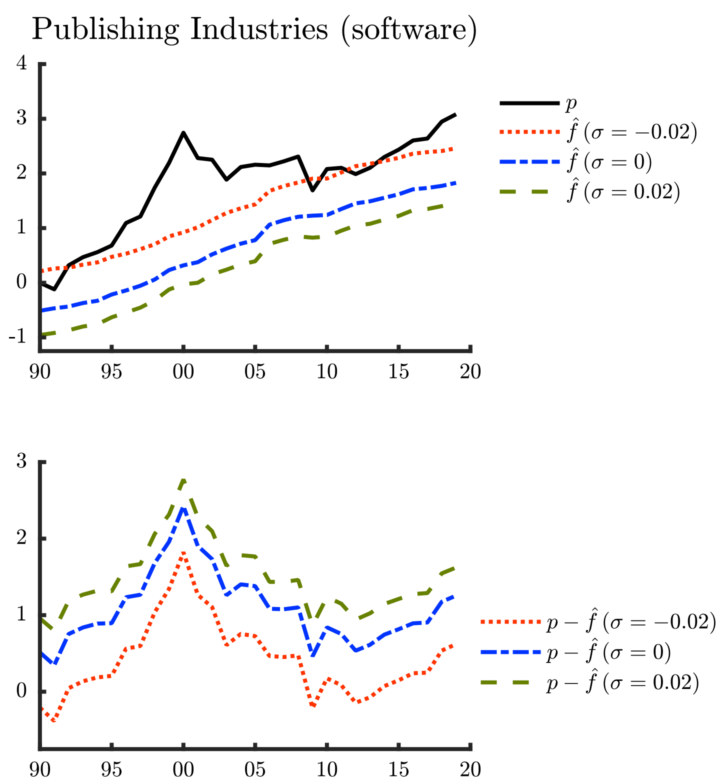



Figure 10: Price-fundamental deviations

This figure shows different estimates for the fundamental of 'Publishing Industries (software)' (NAICS 511) for different values of $\sigma_i$. All variables are measured at the beginning of the year. Appendix B.3 provides details on variable definitions. Prices and fundamentals are rescaled so that $p_{i,90} = 0$.

### B.7 A comparison between the two indicators of stock market overvaluation

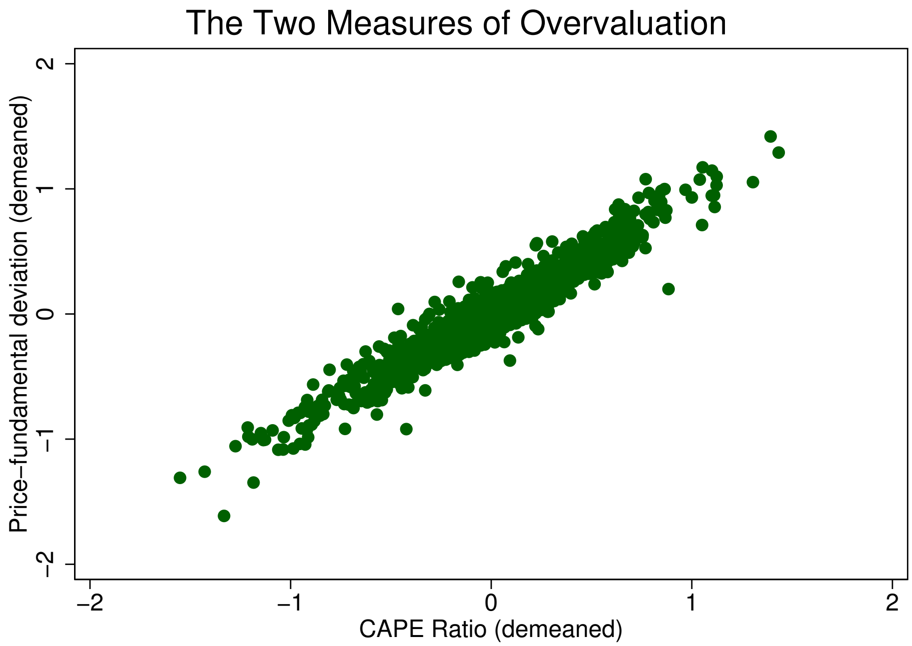



Figure 11: A comparison between the two overvaluation measures

This figure shows the estimate price-fundamental deviations against the Shiller CAPE ratio. Each indicator is in deviation from its industry specific mean. Data is from COMPUSTAT for the period 1990-2019. See the main text and Appendix B.3 for more details on variable construction and definitions.

## B.8 Price-Fundamental Deviation: Standard Errors

To obtain standard errors for the price-fundamental deviation I implement a block bootstrap method (Horowitz (2019)). It allows for both serial and cross-industry correlation in the error terms. The bootstrap procedure is as follows.

1. I use the VAR estimates in Table 5 to obtain the series of error terms for each industry $i$

$$\left\{ \underset{[3\times 1]}{\hat{\mathbf{u}}_{\mathbf{i},0}} , \underset{[3\times 1]}{\hat{\mathbf{u}}_{\mathbf{i},1}} , \underset{[3\times 1]}{\hat{\mathbf{u}}_{\mathbf{i},2}} , \ldots , \underset{[3\times 1]}{\hat{\mathbf{u}}_{\mathbf{it}}} \right\}$$

2. Error terms will be picked in blocks with $n = T^{1/3} = 29^{1/3} \approx 3$ observations (Horowitz (2019)). This allows the error terms to be serial correlated. For example,

$$\mathbf{b}_{\mathbf{i},8} = \{\hat{\mathbf{u}}_{\mathbf{i},8} , \hat{\mathbf{u}}_{\mathbf{i},9} , \hat{\mathbf{u}}_{\mathbf{i},10}\}$$

and

$$\mathbf{b}_{\mathbf{i},10} = \{\hat{\mathbf{u}}_{\mathbf{i},10} , \hat{\mathbf{u}}_{\mathbf{i},11} , \hat{\mathbf{u}}_{\mathbf{i},12}\}$$

are two possible blocks. Note that the blocks can be overlapping.

3. I start by picking a random time period $s \in \{1, 2, \ldots, T\}$ and keep all $N$ blocks starting in that period (where $N$ is the number of industries)

$$\mathbf{B}_{\mathbf{s}} = \left\{ \begin{array}{c} \mathbf{b}_{\mathbf{1},\mathbf{s}} \\ \\ \mathbf{b}_{\mathbf{2},\mathbf{s}} \\ \\ \vdots \\ \\ \mathbf{b}_{\mathbf{N},\mathbf{s}} \end{array} \right\}$$

4. For each industry $i$, I pick a random block $\mathbf{b}_{\mathbf{j},\mathbf{s}}$ (with replacement) from $\mathbf{B}_{\mathbf{s}}$ above. Note that we can have $j = i$ (the block is drawn from the same industry) or $j \neq i$ (the block is drawn from a different same industry).

5. I repeat steps 3 and 4 until I have a new vector of $T = 29$ error terms for each industry.

For example, if in step 3 I first obtain $s = 8$ and then $s = 15$ the new sequence of error terms for the first two industries can be

$$\tilde{\mathbf{u}}_{\mathbf{1}} = \{\underbrace{\hat{\mathbf{u}}_{\mathbf{23},8} , \hat{\mathbf{u}}_{\mathbf{23},9} , \hat{\mathbf{u}}_{\mathbf{23},10}}_{\mathbf{b}_{\mathbf{23},8}} , \underbrace{\hat{\mathbf{u}}_{\mathbf{11},15} , \hat{\mathbf{u}}_{\mathbf{11},16} , \hat{\mathbf{u}}_{\mathbf{11},17}}_{\mathbf{b}_{\mathbf{11},15}} , \ldots\}$$

and

$$\tilde{\mathbf{u}}_2 = \{\underbrace{\hat{\mathbf{u}}_{44,8}\ ,\ \hat{\mathbf{u}}_{44,9}\ ,\ \hat{\mathbf{u}}_{44,10}}_{\mathbf{b}_{44,8}}\ ,\ \underbrace{\hat{\mathbf{u}}_{7,15}\ ,\ \hat{\mathbf{u}}_{7,16}\ ,\ \hat{\mathbf{u}}_{7,17}}_{\mathbf{b}_{7,15}}, \dots\}$$

This procedure allows for both serial correlation (errors are picked in blocks with $T = 3$ observations), and cross sectional dependence (blocks are picked together at the same $s$).

6. Then I use the estimates $\hat{\mathbf{A}}$ in Table 5 and new generated errors to construct a new sample

$$\tilde{\mathbf{y}}_{\mathbf{it}} = \hat{\mathbf{A}}\ \tilde{\mathbf{y}}_{\mathbf{it-1}} + \tilde{\mathbf{u}}_{\mathbf{it}}$$

7. I estimate a VAR on the new generated data $\tilde{\mathbf{y}}_{\mathbf{t}}$ and compute price-fundamental deviations $p_{it} - \tilde{f}_{it}$

8. I do 10,000 repetitions of steps 1 to 7 and obtain the distribution of $p_{it} - \tilde{f}_{it}$ for all industries and time periods.

### B.9 Regression Estimates and Standard Errors

Equations (4) are (5) estimated with OLS and standard errors are bootstrapped clustered at the industry level. The bootstrap is as follows. I generate $T = 400$ samples of data by drawing $n = 47$ industry blocks from the original dataset (with replacement). I reestimate the model for each of the new generated samples and compute the standard deviation of each coefficient estimate.

When the Campbell and Shiller price-fundamental deviation is used as a regressor, I need to correct for the fact that $\text{pdev}_{it}$ is a generated regressor. To do so, when generating a new sample of data, I also pick a new sequence of price-fundamental deviations using the method described in (B.8) (points 1 to 7).

### B.10 Other Firm Level Variables

| Industry Overvaluation, Employment and Investment | | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| VARIABLES | $\log\left(\frac{\text{emp}_{jit}}{\text{emp}_{jit-1}}\right)$ | $\log\left(\frac{\text{inv}_{jit}}{\text{capital}_{jit-1}}\right)$ | $\log\left(\frac{\text{emp}_{jit}}{\text{emp}_{jit-1}}\right)$ | $\log\left(\frac{\text{inv}_{jit}}{\text{capital}_{jit-1}}\right)$ |
| Shiller CAPE Ratio | | | | |
| $\text{cape}_{it-1}$ | 0.0335*** | 0.211*** | | |
| | (0.0101) | (0.0380) | | |
| Campbell and Shiller Price-Fundamental Deviation | | | | |
| $\text{pdev}_{it-1}$ | | | 0.0319*** | 0.189*** |
| | | | (0.00989) | (0.0411) |
| Observations | 113,337 | 120,191 | 113,337 | 120,191 |
| R-squared | 0.214 | 0.413 | 0.214 | 0.413 |
| Firm FE | YES | YES | YES | YES |
| Year FE | YES | YES | YES | YES |

**Table 6**

This table reports OLS estimates of equation (4), which regresses a firm level outcome $x_{jit}$ on the degree of overvaluation in the industry at the end of the previous year. Data is from COMPUSTAT. In columns (1) and (3), the dependent variable is log employment growth (COMPUSTAT item #29). Columns (2) and (4) use the investment rate, constructed as the ratio of capital expenditures (COMPUSTAT item #30) to the value of net capital (COMPUSTAT item #8) at the end of the previous year. The estimation is done for the period 1990-2019. Both dependent variables are windsorized at the 0.5% and 99.5% percentiles. Standard errors in parentheses are bootstrapped clustered at the industry level. When the Campbell and Shiller price-fundamental deviation is used, the bootstrap also accounts for the fact that $\text{pdev}_{it}$ is a generated regressor (see Appendix B.9 for details). *** p<0.01, ** p<0.05, * p<0.1

## B.11 Robustness: Alternative Measures of Variable Costs

In this section, I report the estimates of equation (4) for alternative measures of price-cost markups. In particular, I follow De Loecker, Eeckhout and Unger (2020) and construct markups as the ratio of sales (COMPUSTAT item #12) to the 'Cost of the Goods Sold' (COMPUSTAT item #41).

| Industry Overvaluation and Firm Outcomes | | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| VARIABLES | $\log\left(\frac{\text{sale}_{jit}}{\text{cost}_{jit}}\right)$ | $\mathbb{1}\left\{\text{earn}_{jit} < 0\right\}$ | $\log\left(\frac{\text{sale}_{jit}}{\text{cost}_{jit}}\right)$ | $\mathbb{1}\left\{\text{earn}_{jit} < 0\right\}$ |
| Shiller CAPE Ratio | | | | |
| $\text{cape}_{it-1}$ | -0.0345*** | 0.0167*** | | |
| | (0.0126) | (0.00280) | | |
| Campbell and Shiller Price-Fundamental Deviation | | | | |
| $\text{pdev}_{it-1}$ | | | -0.0346*** | 0.0158*** |
| | | | (0.0105) | (0.00233) |
| Observations | 136,814 | 136,814 | 136,814 | 136,814 |
| R-squared | 0.666 | 0.640 | 0.666 | 0.640 |
| Firm FE | YES | YES | YES | YES |
| Year FE | YES | YES | YES | YES |

**Table 7**
This table reports OLS estimates of equation (4), which regresses a firm level outcome $x_{jit}$ on the degree of overvaluation in the industry at the end of the previous year. Data is from COMPUSTAT. In columns (1) and (3), the dependent variable is the (log) markup, defined as the ratio of sales (COMPUSTAT item #12) to variable operating costs (COMPUSTAT item #41). Columns (2) and (4) use a binary variable that takes value one if sales are lower than variable operating costs. The estimation is done for the period 1990-2019. Markups are windsorized at the 0.5% and 99.5% percentiles. Standard errors in parentheses are bootstrapped clustered at the industry level. When the Campbell and Shiller price-fundamental deviation is used, the bootstrap also accounts for the fact that $\text{pdev}_{it}$ is a generated regressor (see Appendix B.9 for details). *** p<0.01, ** p<0.05, * p<0.1

## B.12 Robustness: Alternative Versions of the Shiller CAPE Index

In this section, I report the estimates of equations (4) and (5) for alternative measures of the Shiller CAPE index. The baseline measure is constructed as

$$\text{cape}_{it} = p_{it} - \bar{e}_{it}^{10}$$

where $p_{it}$ is the real stock price (in logs and measured at the end of year $t$) and $\bar{e}_{it}^{10} \equiv \log\left[\left(E_{it-9} + \cdots E_{it}\right)/10\right]$ is a 10-year moving average of real earnings (in logs). The earnings metric I use is 'EBITDA' (i.e. earnings before interest, taxes, depreciation and amortization, COMPUSTAT item #13).

I this section I construct two alternative measures. The first uses a different earnings metric in the denominator, whereas the second uses sales.

1. The first measure is constructed as

$$\text{cape}'_{it} = p_{it} - \bar{e}'^{10}_{it}$$

where $p_{it}$ is the real stock price (as in the baseline) and $\bar{e}'^{10}_{it} \equiv \log\left[\left(E'_{it-9} + \cdots E'_{it}\right)/10\right]$, with $E'_{it}$ being alternatively constructed as the difference between sales (COMPUSTAT item #12) and the 'Cost of the Goods Sold' (COMPUSTAT item #41).

2. The second measure is constructed as

$$\text{cape}^{\text{sale}}_{it} = p_{it} - \bar{s}_{it}^{10}$$

where $p_{it}$ is the real stock price (as in the baseline) and $\bar{s}_{it}^{10} \equiv \log\left[\left(S_{it-9} + \cdots S_{it}\right)/10\right]$, where $S_{it}$ refers to sales (COMPUSTAT item #12).

| Industry Overvaluation and Firm Outcomes | | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| VARIABLES | $\log\left(\frac{\text{sale}_{jit}}{\text{sale}_{jit-1}}\right)$ | $\log\left(\frac{\text{sale}_{jit}}{\text{cost}_{jit}}\right)$ | $\mathbb{1}\left\{\text{earn}_{jit} < 0\right\}$ | $\log\left(\frac{\text{sale}_{jit}}{\text{sale}_{jit-1}}\right)$ | $\log\left(\frac{\text{sale}_{jit}}{\text{cost}_{jit}}\right)$ | $\mathbb{1}\left\{\text{earn}_{jit} < 0\right\}$ |
| Alternative CAPE Ratio: Sales minus Cost of the Goods Sold | | | | | | |
| $\text{cape}'_{it}$ | 0.0405*** | -0.0309** | 0.0365** | | | |
| | (0.0126) | (0.0120) | (0.0146) | | | |
| Alternative CAPE Ratio: Sales | | | | | | |
| $\text{cape}^{\text{sale}}_{it}$ | | | | 0.0388*** | -0.0276** | 0.0326** |
| | | | | (0.0122) | (0.0129) | (0.0137) |
| Observations | 126,625 | 136,814 | 136,814 | 126,625 | 136,814 | 136,814 |
| R-squared | 0.183 | 0.739 | 0.642 | 0.183 | 0.739 | 0.642 |
| Firm FE | YES | YES | YES | YES | YES | YES |
| Year FE | YES | YES | YES | YES | YES | YES |

**Table 8**

This table reports OLS estimates of equation (4), which regresses a firm level outcome $x_{jit}$ on the degree of overvaluation in the industry at the end of the previous year. Data is from COMPUSTAT. In columns (1) and (4), the dependent variable is the (log) growth rate of sales (COMPUSTAT item #12). Columns (2) and (5) use the (log) markup, defined as the ratio of sales to variable operating costs (COMPUSTAT item #41 + COMPUSTAT item #189). Columns (4) and (6) use a binary variable that takes value one if sales are lower than variable operating costs. The estimation is done for the period 1990-2019. Sales growth and markups are windsorized at the 0.5% and 99.5% percentiles. Standard errors in parentheses are bootstrapped clustered at the industry level. When the Campbell and Shiller price-fundamental deviation is used, the bootstrap also accounts for the fact that $\text{pdev}_{it}$ is a generated regressor (see Appendix B.9 for details). *** p<0.01, ** p<0.05, * p<0.1

## Industry Overvaluation and the Number of Active Firms

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | COMPUSTAT | Total Economy | COMPUSTAT | Total Economy |
| VARIABLES | $\log\left(\frac{N_t}{N_{t-1}}\right)$ | $\log\left(\frac{N_t}{N_{t-1}}\right)$ | $\log\left(\frac{N_t}{N_{t-1}}\right)$ | $\log\left(\frac{N_t}{N_{t-1}}\right)$ |
| **Alternative CAPE Ratio: Sales minus Cost of the Goods Sold** | | | | |
| $\text{cape}'_{it}$ | 0.0209** | 0.00971** | | |
| | (0.00800) | (0.00407) | | |
| **Alternative CAPE Ratio: Sales** | | | | |
| $\text{cape}^{\text{sale}}_{it}$ | | | 0.0232*** | 0.00938** |
| | | | (0.00736) | (0.00430) |
| Observations | 1,363 | 864 | 1,363 | 864 |
| R-squared | 0.338 | 0.302 | 0.340 | 0.302 |
| Industry FE | YES | YES | YES | YES |
| Year FE | YES | YES | YES | YES |

**Table 9**
This table reports OLS estimates of equation (5), which regresses the log growth rate of the number of firms in an industry on the degree of overvaluation at the end of the previous year. Data is from COMPUSTAT and SUBS. Columns (1) and (3) use the number of firms listed in COMPUSTAT (period 1990-2019). Columns (2) and (4) use the number of firms active in the entire economy (from the US census of firms, SUBS programme, period 1998-2017). Standard errors in parentheses are bootstrapped clustered at the industry level. When the Campbell and Shiller price-fundamental deviation is used, the bootstrap also accounts for the fact that $\text{pdev}_{it}$ is a generated regressor (see Appendix B.9 for details). *** $p<0.01$, ** $p<0.05$, * $p<0.1$

# C Model Derivation and Proofs

## C.1 Industry Equilibrium

### C.1.1 Profits

Given aggregate output $Y_t$, a factor cost index $\theta_t$ and $n_{it}$ followers, the leader makes production profits

$$\Pi^L\left(n_{it},\theta_t,Y_t\right) \equiv \left(p_{it}-\theta_t\right)y_{it}^L = \left[\frac{n_{it}-\left(n_{it}-1+\rho\right)\pi}{n_{it}+\pi}\right]^2\left(\frac{n_{it}+\rho}{n_{it}+\pi}\frac{\pi}{\theta_t}\right)^{\frac{\rho}{1-\rho}}\frac{Y_t}{1-\rho}$$

and each follower

$$\Pi^F\left(n_{it},\theta_t,Y_t\right) \equiv \left(p_{it}-\frac{\theta_t}{\pi}\right)y_{it}^F = \left(\frac{\pi-\rho}{n_{it}+\pi}\right)^2\left(\frac{n_{it}+\rho}{n_{it}+\pi}\frac{\pi}{\theta_t}\right)^{\frac{\rho}{1-\rho}}\frac{Y_t}{1-\rho}$$

It is easy to show that $\Pi^F\left(n_{it},\theta_t,Y_t\right)$ is decreasing in $n_{it}$

$$\begin{aligned}
&\frac{\partial\Pi^F\left(n_{it},\theta_t,Y_t\right)}{\partial n_{it}} < 0 \\
\Leftrightarrow\quad & 2\left(\frac{\pi-\rho}{n_{it}+\pi}\right)\frac{-\left(\pi-\rho\right)}{\left(n_{it}+\pi\right)^2}\left(\frac{n_{it}+\rho}{n_{it}+\pi}\frac{\pi}{\theta_t}\right)^{\frac{\rho}{1-\rho}}+\frac{\rho}{1-\rho}\left(\frac{\pi-\rho}{n_{it}+\pi}\right)^2\left(\frac{n_{it}+\rho}{n_{it}+\pi}\frac{\pi}{\theta_t}\right)^{\frac{\rho}{1-\rho}-1}\frac{\pi}{\theta_t}\frac{n_{it}+\pi-\left(n_{it}+\rho\right)}{\left(n_{it}+\pi\right)^2} < 0 \\
\Leftrightarrow\quad & -2\left(\pi-\rho\right)+\frac{\rho}{1-\rho}\left(\frac{\pi-\rho}{n_{it}+\pi}\right)\left(\frac{n_{it}+\rho}{n_{it}+\pi}\frac{\pi}{\theta_t}\right)^{-1}\frac{\pi}{\theta_t}\left(\pi-\rho\right) < 0 \\
\Leftrightarrow\quad & -2+\frac{\rho}{1-\rho}\left(\frac{\pi-\rho}{n_{it}+\rho}\right) < 0 \\
\Leftrightarrow\quad & \left(\pi-\rho\right)\rho < 2\left(1-\rho\right)\left(n_{it}+\rho\right)
\end{aligned}$$

which is always satisfied given than $\pi-\rho<1-\rho$ and $\rho<n_{it}+\rho$.

### C.1.2 The Output of the Leader

When there are $n_{it}$ followers, the leader chooses an output level that is equal to

$$
\begin{aligned}
y_{it}^{L} &= s_{it}^{L} y_{it} \\
&= \frac{n_{it} - (n_{it} - 1 + \rho)\,\pi}{(1-\rho)\,(n_{it} + \pi)} \left( \frac{n_{it} + \rho}{\theta_t} \frac{\pi}{n_{it} + \pi} \right)^{\frac{1}{1-\rho}} Y_t \\
&= \frac{n_{it}\,(1-\pi) + (1-\rho)\,\pi}{n_{it} + \pi} \left( \frac{n_{it} + \rho}{n_{it} + \pi} \right)^{\frac{1}{1-\rho}} \left( \frac{\pi}{\theta_t} \right)^{\frac{1}{1-\rho}} \frac{Y_t}{1-\rho}
\end{aligned}
$$

I want to find a sufficient condition under which the above expression always increases in the number of followers $n_{it} \geq 0$. Although $n_{it}$ is discrete, it will be easy to take it as a continuous variable and compute its partial derivative with respect to $n_{it}$.

$$
\begin{aligned}
& \frac{\partial y_{it}^{L}}{\partial n_{it}} > 0 \\
\Leftrightarrow \quad & \frac{(1-\pi)\,(n_{it} + \pi) - [n_{it}\,(1-\pi) + (1-\rho)\,\pi]}{(n_{it} + \pi)^2} \left( \frac{n_{it} + \rho}{n_{it} + \pi} \right)^{\frac{1}{1-\rho}} + \\
& + \frac{n_{it}\,(1-\pi) + (1-\rho)\,\pi}{n_{it} + \pi} \frac{1}{1-\rho} \left( \frac{n_{it} + \rho}{n_{it} + \pi} \right)^{\frac{1}{1-\rho} - 1} \frac{\pi - \rho}{(n_{it} + \pi)^2} > 0 \\
\Leftrightarrow \quad & (1-\pi)\,(n_{it} + \pi) - [n_{it}\,(1-\pi) + (1-\rho)\,\pi] + \frac{n_{it}\,(1-\pi) + (1-\rho)\,\pi}{n_{it} + \rho} \frac{\pi - \rho}{1-\rho} > 0 \\
\Leftrightarrow \quad & (1-\pi)\,(n_{it} + \pi) > [n_{it}\,(1-\pi) + (1-\rho)\,\pi] \left( 1 - \frac{1}{n_{it} + \rho} \frac{\pi - \rho}{1-\rho} \right) \\
\Leftrightarrow \quad & n_{it} + \pi > \left( n_{it} + \frac{1-\rho}{1-\pi} \pi \right) \left( 1 - \frac{1}{n_{it} + \rho} \frac{\pi - \rho}{1-\rho} \right) \\
\Leftrightarrow \quad & n_{it} + \pi > n_{it} - \frac{n_{it}}{n_{it} + \rho} \frac{\pi - \rho}{1-\rho} + \frac{1-\rho}{1-\pi} \pi - \frac{1}{n_{it} + \rho} \frac{\pi - \rho}{1-\pi} \pi
\end{aligned}
$$

$$\Leftrightarrow \quad \pi > -\frac{n_{it}}{n_{it}+\rho}\frac{\pi-\rho}{1-\rho} + \frac{1-\rho}{1-\pi}\pi - \frac{1}{n_{it}+\rho}\frac{\pi-\rho}{1-\pi}\pi$$

$$\Leftrightarrow \quad \pi\left(1-\frac{1-\rho}{1-\pi}\right) > -\frac{\pi-\rho}{n_{it}+\rho}\left(\frac{n_{it}}{1-\rho}+\frac{\pi}{1-\pi}\right)$$

$$\Leftrightarrow \quad \pi\left(\frac{1-\pi-1+\rho}{1-\pi}\right) > -\frac{\pi-\rho}{n_{it}+\rho}\left(\frac{n_{it}}{1-\rho}+\frac{\pi}{1-\pi}\right)$$

$$\Leftrightarrow \quad \pi\left(\frac{\rho-\pi}{1-\pi}\right) > -\frac{\pi-\rho}{n_{it}+\rho}\left(\frac{n_{it}}{1-\rho}+\frac{\pi}{1-\pi}\right)$$

$$\Leftrightarrow \quad \frac{n_{it}}{1-\rho}+\frac{\pi}{1-\pi} > \frac{\pi}{1-\pi}\left(n_{it}+\rho\right)$$

$$\Leftrightarrow \quad \frac{\pi}{1-\pi}\left(1-\rho\right) > n_{it}\left(\frac{\pi}{1-\pi}-\frac{1}{1-\rho}\right)$$

The above condition is always satisfied provided that

$$\frac{\pi}{1-\pi}-\frac{1}{1-\rho} < 0$$

$$\Leftrightarrow \quad \frac{\pi}{1-\pi} < \frac{1}{1-\rho}$$

$$\Leftrightarrow \quad \pi\left(1-\rho\right) < 1-\pi$$

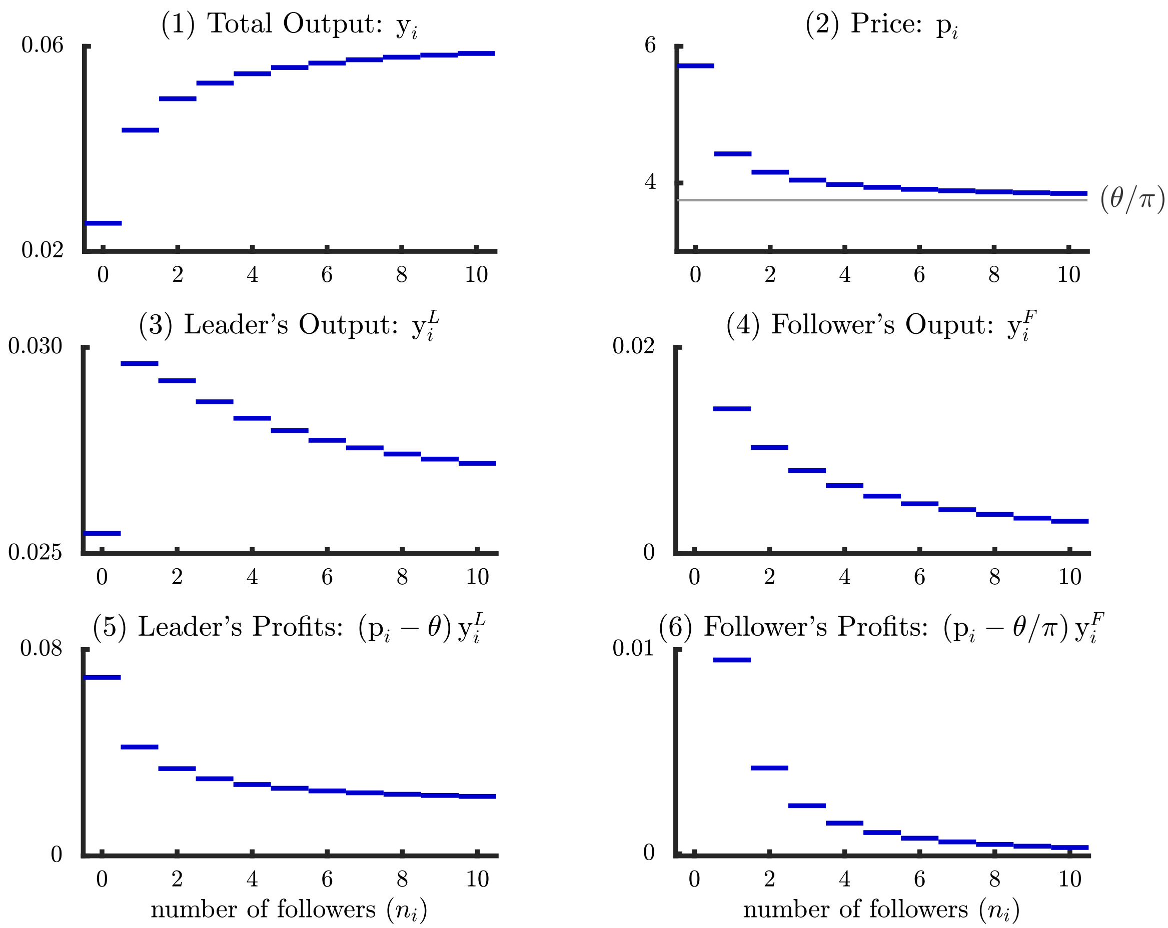
$$\Leftrightarrow \quad \pi\left(2-\rho\right) < 1$$

$$\Leftrightarrow \quad \pi < \frac{1}{2-\rho}$$

Thus, the leader finds it optimal to expand in reaction to the entry of the followers, when the productivity gap is sufficiently large (followers' productivity $\pi$ is low).

**Inverted-U shape** Given the parameters chosen below, the leader chooses a higher level output when the number of followers increases from $n_{it} = 0$ to $n_{it} = 1$; as the number of followers increases further, the leader finds it optimal to contract (see panel 3).

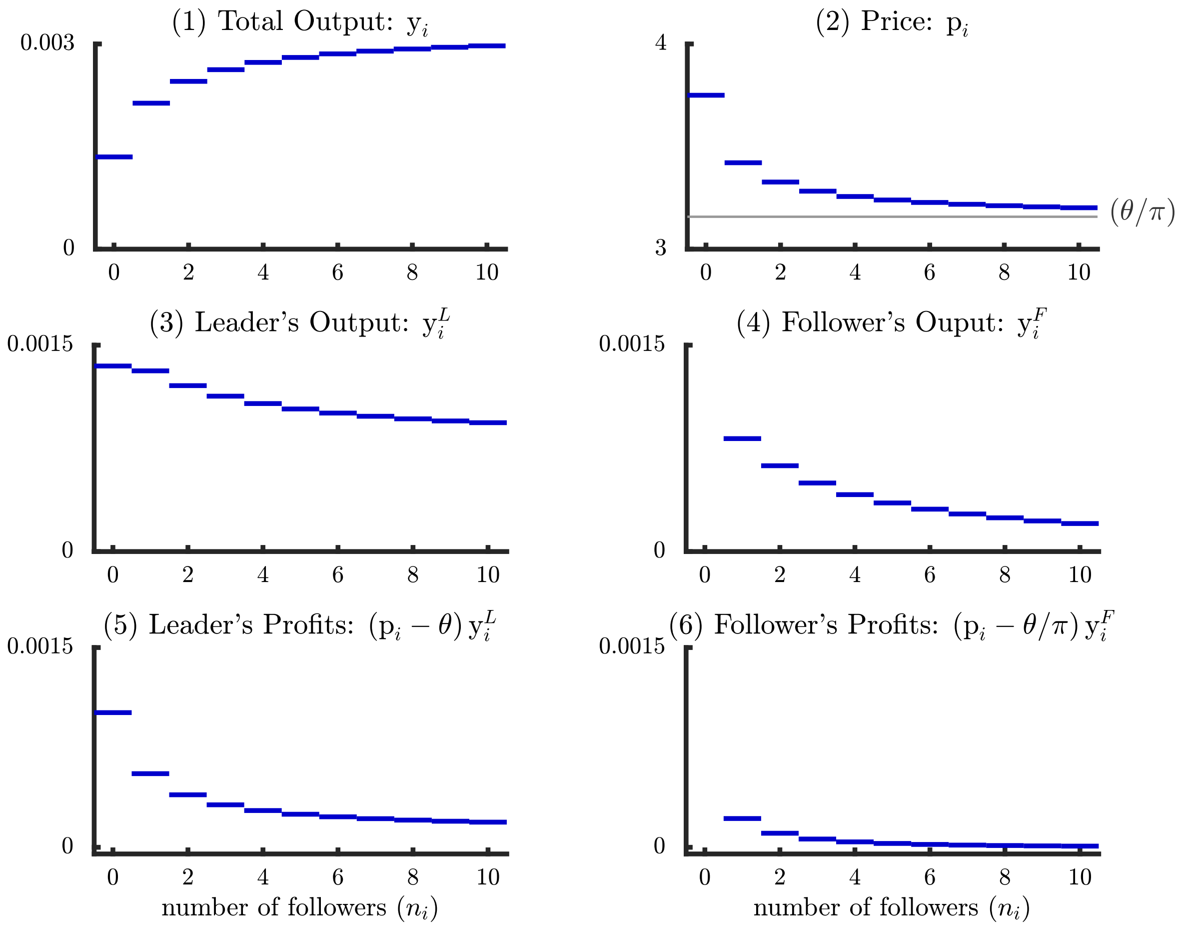
Underlying such disparate reaction is a trade-off faced by the leader. By expanding, the leader can keep a high market share, but at the expense of a lower price $p_{it}$. By contracting, she will lose a larger fraction of the market, but will keep $p_{it}$ relatively high. When the number of followers is low and $p_{it}$ is still high, the benefits of keeping a large market share are high, and the leader ends up producing more. As the number of followers increases and $p_{it}$ becomes sufficiently low, the leader prefers to contract to avoid a further drop in $p_{it}$.




Parameters: $\rho = 0.525$, $\pi = 0.8$, $Y = 1$ and $\theta = 3$

Figure 12: Industry Equilibrium

**Decreasing Output** Given the parameters chosen below, the leader always shrinks in reaction to an increase in the number of followers (see panel (3)).




Parameters: $\rho = 0.8$, $\pi = 0.95$, $Y = 1$ and $\theta = 3$

Figure 13: Industry Equilibrium

## C.2 The Industry bubble

Under the industry bubble process studied in section 3.2 firms solve

$$\max_{y_{it}^{j}} \quad \left[ \left(1+\theta^{j}\right) p_{it} - \frac{\theta_t}{\pi_i^j} \right] y_{it}^{j} + \frac{y_{it}^{j}}{y_{it}} b_i \qquad \text{s.t.} \quad p_{it} = \left(\frac{Y_t}{y_{it}}\right)^{1-\rho}$$

$$y_{it} = y_{it}^{j} + y_{it}^{-j}$$

The solution to this problem yields individual best response functions

$$\left(\frac{Y_t}{y_{it}}\right)^{1-\rho} \left[1 - (1-\rho)\frac{y_{it}^{j}}{y_{it}}\right] = \frac{\theta_t}{\pi_i^j} - \frac{y_{it} - y_{it}^{j}}{y_{it}^{2}} b_{it}$$

### C.2.1 Fixed Number of Followers

When there are $n_{it} \geq 0$ followers, the industry price is characterized by

$$p_{it} = \frac{1}{n_{it}+\rho}\left[\theta_t\left(1+\frac{n_{it}}{\pi}\right) - n_{it}\frac{b_{it}}{y_{it}}\right]$$

and aggregate output is given by

$$y_{it}\,\theta_t\left(1+\frac{n_{it}}{\pi}\right) - (n_{it}+\rho)\ Y_t^{1-\rho}\ y_{it}^{\rho} = n_{it}\ b_{it}$$

From this equation we have that

$$\frac{\partial y_{it}}{\partial b_{it}} = \frac{1}{n_{it}}\left[\theta_t\left(1+\frac{n_{it}}{\pi}\right) - \rho\,(n+\rho)\,p_{it}\right]$$

Furthermore, the leader's market share is equal to

$$s_{it}^{L} = \frac{p_{it} - \theta_t + \dfrac{b_{it}}{y_{it}}}{(1-\rho)\,p_{it} + \dfrac{b_{it}}{y_{it}}}$$

### C.2.2 Determining the Number of Followers

Suppose there are $\tilde{n}_{it} \geq 0$ followers in the bubbleless equilibrium. Let $\underline{b}\,(n_{it})$ denote the minimum industry bubble leading to the existence of $n_{it} > \tilde{n}_{it}$ followers. Such a bubble is implicitly defined by the following two equations

$$y_{it}\,\theta_t\left(1+\frac{n_{it}}{\pi}\right) - (n_{it}+\rho)\;Y_t^{1-\rho}\;y_{it}^{\rho} = n_{it}\;\underline{b}\,(n_{it})$$

$$\underbrace{\frac{1}{n_{it}}\frac{\theta_t - \rho\left(\dfrac{Y_t}{y_{it}}\right)^{1-\rho}}{(1-\rho)\left(\dfrac{Y_t}{y_{it}}\right)^{1-\rho}+\dfrac{b_{it}}{y_{it}}}}_{s_{it}^F}\underbrace{\left\{\left[\left(\frac{Y_t}{y_{it}}\right)^{1-\rho}-\frac{\theta_t}{\pi}\right]y_{it}+\underline{b}\,(n_{it})\right\}}_{\left(p_{it}-\frac{\theta_t}{\pi}\right)y_{it}+\underline{b}(n_{it})} - c_f = 0$$

The first equation defines total industry output as a function of the bubble $\underline{b}\,(n_{it})$, for a given number $n_{it}$ of followers. The second equation states that the production profits of any follower, plus her share on $\underline{b}\,(n_{it})$, should exactly compensate for the fixed cost $c_f$.

### C.2.3 Proof of Proposition 1

To prove Proposition 1 I will show that, for the bubble process described before and given a fixed number of followers $n_{it}$ we have that

1. $\dfrac{\partial y_{it}}{\partial b_{it}} > 0$
2. $\dfrac{\partial y_{it}^{L}}{\partial b_{it}} > 0$
3. $\dfrac{\partial s_{it}^{L}}{\partial b_{it}} < 0$

Points 2 and 3 above imply that all firms expand (if the leader expands while losing market share, it must be the case that the followers also expand). Point 1 will hence follow immediately. It will however be convenient to start the proof by showing the first point.

**1.** $\dfrac{\partial y_{it}}{\partial b_{it}} > 0$

First note that when there is at least one follower, the price will be strictly lower than the leader's monopoly price, i.e. $p_{it} < \dfrac{\theta_t}{\rho}$. In such a case we have that

$$
\begin{aligned}
\frac{\partial y_{it}}{\partial b_{it}} &= \frac{1}{n_{it}}\left[\theta_t\left(1+\frac{n_{it}}{\pi}\right)-\rho\left(n+\rho\right)p_{it}\right] \\
&> \frac{1}{n_{it}}\left[\theta_t\left(1+\frac{n_{it}}{\pi}\right)-\rho\left(n+\rho\right)\frac{\theta_t}{\rho}\right] \\
&= \frac{1}{n_{it}}\left[\theta_t\left(1+\frac{n_{it}}{\pi}\right)-\left(n+\rho\right)\theta_t\right] \\
&= \frac{1}{n_{it}}\left[n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\left(1-\rho\right)\theta_t\right] \\
&> 0 \quad \text{QED}
\end{aligned}
$$

Therefore, given a fixed number of followers $n_{it} \geq 1$, as the industry bubble $b_{it}$ increases, total output $y_{it}$ increases.

**2.** $\dfrac{\partial y_{it}^L}{\partial b_{it}} > 0$

We can write the leader's market share as

$$s_{it}^L = \frac{n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{it}}{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)}$$

We can therefore write the leader's output as

$$y_{it}^L = y_{it}\frac{n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{it}}{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)} \equiv y_{it}\frac{\text{num}_{it}}{\text{den}_{it}}$$

Note that we can write the derivative of the price $p_{it}$ with respect to the industry bubble $b_{it}$ as

$$\frac{\partial p_{it}}{\partial b_{it}} = -\left(1-\rho\right)\frac{p_{it}}{y_{it}}\frac{\partial y_{it}}{\partial b_{it}}$$

Therefore, we have that

$$\begin{aligned}
\frac{\partial y_{it}^L}{\partial b_{it}} &= \frac{\partial y_{it}}{\partial b_{it}}\frac{\text{num}_{it}}{\text{den}_{it}} + y_{it}\frac{\rho\left(1-\rho\right)\frac{p_{it}}{y_{it}}\frac{\partial y_{it}}{\partial b_{it}}\text{den}_{it} - \rho\left(n_{it}+1\right)\left(1-\rho\right)\frac{p_{it}}{y_{it}}\frac{\partial y_{it}}{\partial b_{it}}\text{num}_{it}}{\text{den}_{it}^2} \\
&= \frac{\partial y_{it}}{\partial b_{it}}\frac{\text{num}_{it}}{\text{den}_{it}} + \frac{\rho\left(1-\rho\right)p_{it}\frac{\partial y_{it}}{\partial b_{it}}\text{den}_{it} - \rho\left(n_{it}+1\right)\left(1-\rho\right)p_{it}\frac{\partial y_{it}}{\partial b_{it}}\text{num}_{it}}{\text{den}_{it}^2} \\
&= \frac{1}{\text{den}_{it}}\frac{\partial y_{it}}{\partial b_{it}}\left[n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{it}+\rho\left(1-\rho\right)p_{it}-\rho\left(n_{it}+1\right)\left(1-\rho\right)p_{it}\frac{n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{it}}{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)}\right] \\
&= \frac{1}{\text{den}_{it}}\frac{\partial y_{it}}{\partial b_{it}}\left\{n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{it}+\rho\left(1-\rho\right)p_{it}-\right. \\
&\left. -\frac{\rho\left(n_{it}+1\right)\left(1-\rho\right)p_{it}}{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)}\left[n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{it}\right]\right\} \\
&\frac{1}{\text{den}_{it}}\frac{\partial y_{it}}{\partial b_{it}}\left\{\left[n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{it}\right]\left[1-\frac{\rho\left(n_{it}+1\right)\left(1-\rho\right)p_{it}}{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)}\right]+\rho p_{it}\left(1-\rho\right)\right\}
\end{aligned}$$

Note that

$$\text{den}_{it} = \theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right) = n_{it}\left(\frac{\theta_t}{\pi}-\rho p_{it}\right)+\left(\theta_t-\rho p_{it}\right) > 0$$

Furthermore, we already know from above that

$$\frac{\partial y_{it}}{\partial b_{it}} > 0$$

Therefore, it suffices to show that

$$\left[n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{it}\right]\left[1-\frac{\rho\left(n_{it}+1\right)\left(1-\rho\right)p_{it}}{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)}\right]+\rho p_{it}\left(1-\rho\right)>0$$

$$\Leftrightarrow\quad\left[\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)-\rho\left(n_{it}+1\right)\left(1-\rho\right)p_{it}\right]+\rho p_{it}\left(1-\rho\right)\frac{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)}{n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{it}}>0$$

$$\Leftrightarrow\quad\left[\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)\left(2-\rho\right)\right]+\rho p_{it}\left(1-\rho\right)\frac{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-n_{it}\rho p_{it}-\rho p_{it}}{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-n_{it}\theta_t-\rho p_{it}}>0$$

Since $\rho p_{it}<\theta_t$, the above condition is implied by

$$\left[\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)\left(2-\rho\right)\right]+\rho p_{it}\left(1-\rho\right)\frac{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-n_{it}\theta_t-\rho p_{it}}{\theta_t\left(\frac{n_{it}}{\pi}+1\right)-n_{it}\theta_t-\rho p_{it}}>0$$

$$\Leftrightarrow\quad\left[\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(n_{it}+1\right)\left(2-\rho\right)\right]+\rho p_{it}\left(1-\rho\right)>0$$

$$\Leftrightarrow\quad\theta_t\left(\frac{n_{it}}{\pi}+1\right)-\rho p_{it}\left(2n_{it}+2-\rho n_{it}-\rho-1+\rho\right)>0$$

$$\Leftrightarrow\quad\frac{\theta_t}{\rho p_{it}}\left(\frac{n_{it}}{\pi}+1\right)-\left[n_{it}\left(2-\rho\right)+1\right]>0$$

Note that we have that

$$\begin{aligned}p_{it}&\leq\frac{n_{it}+\pi}{\left(n_{it}+\rho\right)\pi}\theta_t\\ \frac{\theta_t}{\rho p_{it}}&>\frac{\left(n_{it}+\rho\right)\pi}{\rho\left(n_{it}+\pi\right)}\end{aligned}$$

Therefore, the previous condition is implied by

$$
\begin{aligned}
& \frac{(n_{it}+\rho)\,\pi}{\rho\,(n_{it}+\pi)}\left(\frac{n_{it}}{\pi}+1\right)-[n_{it}\,(2-\rho)+1]>0 \\
\Leftrightarrow\quad & \frac{n_{it}+\rho}{\rho\,(n_{it}+\pi)}\,(n_{it}+\pi)-[n_{it}\,(2-\rho)+1]>0 \\
\Leftrightarrow\quad & n_{it}+\rho-\rho\,[n_{it}\,(2-\rho)+1]>0 \\
\Leftrightarrow\quad & n_{it}-\rho n_{it}\,(2-\rho)>0 \\
\Leftrightarrow\quad & n_{it}\,[1-\rho\,(2-\rho)]>0
\end{aligned}
$$

The above condition is always true provided that $\rho < 1$.

**3.** $\dfrac{\partial s^L_{it}}{\partial b_{it}} < 0$

We have that

$$\frac{\partial s^L_{it}}{\partial b_{it}} = \frac{\rho\left(1-\rho\right)\dfrac{p_{it}}{y_{it}}\dfrac{\partial y_{it}}{\partial b_{it}}\mathrm{den}_{it} - \rho\left(n_{it}+1\right)\left(1-\rho\right)\dfrac{p_{it}}{y_{it}}\dfrac{\partial y_{it}}{\partial b_{it}}\mathrm{num}_{it}}{\mathrm{den}^2_{it}}$$

Therefore, we need to show that

$$\frac{\rho\left(1-\rho\right)\dfrac{p_{it}}{y_{it}}\dfrac{\partial y_{it}}{\partial b_{it}}\mathrm{den}_{it} - \rho\left(n_{it}+1\right)\left(1-\rho\right)\dfrac{p_{it}}{y_{it}}\dfrac{\partial y_{it}}{\partial b_{it}}\mathrm{num}_{it}}{\mathrm{den}^2_{it}} < 0$$

$$\Leftrightarrow \quad \mathrm{den}_{it} - \left(n_{it}+1\right)\mathrm{num}_{it} < 0$$

$$\Leftrightarrow \quad \theta_t\left(\frac{n_{it}}{\pi}+1\right) - \rho p_{it}\left(n_{it}+1\right) - \left(n_{it}+1\right)\left[n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t-\rho p_{it}\right] < 0$$

$$\Leftrightarrow \quad \theta_t\left(\frac{n_{it}}{\pi}+1\right) - \left(n_{it}+1\right)\left[n_{it}\theta_t\left(\frac{1}{\pi}-1\right)+\theta_t\right] < 0$$

$$\Leftrightarrow \quad \theta_t\left(\frac{n_{it}}{\pi}+1\right) - \left(n_{it}+1\right)\left[\theta_t\left(\frac{n_{it}}{\pi}+1\right)-n_{it}\theta_t\right] < 0$$

$$\Leftrightarrow \quad -n_{it}\theta_t\left(\frac{n_{it}}{\pi}+1\right) + \left(n_{it}+1\right)n_{it}\theta_t < 0$$

$$\Leftrightarrow \quad n_{it}\theta_t\left[n_{it}-\frac{n_{it}}{\pi}\right] < 0$$

which is always satisfied provided that $\pi < 1$.

### C.2.4 A Comparison Between The Two Processes

To conclude this section, I make a brief comparison between the two bubble processes considered above. Proposition 10 below states two main results.

**Proposition 10.** *Suppose there are two industries, A and B; firms in industry A can issue stocks according to the constant firm level bubble process, whereas firms in industry B issue stocks according to the constant industry bubble. We have that*

1. *if* $\sum_j^{n_A} b_A^j = \sum_j^{n_B} b_B^j$ *, then* $n_A \geq n_B$
2. *if* $n^A = n^B \geq 1$*, then* $y^A < y^B$

The first point of the above proposition says that, if the two industries have the same aggregate industry bubble $b_i$, then industry A must have at least as many followers as industry B. In other words, the size of the aggregate industry bubble $b_i$ leading to the entry of the $n$-th follower is lower under the constant firm level bubble process. To understand this result, suppose that each industry is a monopoly under the bubbleless equilibrium and let $\underline{b}^F$ be the minimum firm level bubble leading to the entry of the first follower in industry A (so that the aggregate industry bubble is equal to $2\,\underline{b}^F$)

$$\left(p_i - \frac{\theta}{\pi}\right) y^F < c_f - \underline{b}^F$$

Now suppose that there is a firm level bubble with size $2\,\underline{b}^F$ in industry B. Clearly, the leader in industry B will produce more than the leader in industry A, implying that the follower in industry B needs to produce more than that of industry A in order to appropriate a bubble with size $\underline{b}^F$. However, as the two firms in industry B produce more, the follower will make lower production profits and will therefore be unwilling to enter the market with an industry bubble with size $2\,\underline{b}^F$

$$\left(\widetilde{p}_i - \frac{\theta}{\pi}\right) \widetilde{y}^F < c_f - \frac{\widetilde{y}^F}{\widetilde{y}^L + \widetilde{y}^F} 2\underline{b}^F$$

The second point says that, when the two industries have the same number of followers, the output in industry B will be strictly larger. This fact can be seen from a direct comparison between equations (12) and (14) and is a consequence of the *fight-for-market-shares* effect emphasized above.

Figure 14 compares three equilibrium variables (the number of followers, total industry output and total industry welfare) in the two industries as a function of the total industry bubble $b_i$. The first panel illustrates the first point of Proposition 10. The second panel shows that, even when industry B features a lower number of followers, its output can still be larger than that of industry A. Such a fact can be explained by the second point of Proposition 10. Note that the maximum level of welfare achieved by industry B is larger than that of industry A. This fact, though not general, can again be linked to the second point of Proposition 10 – industry B can achieve the perfect competition level of output with a fewer number of firms and hence a smaller waste of resources than industry A.[53]

[53] To see why the maximum level of welfare achieved by industry B is not always larger than that of industry A, take the limit case in which $c_f = 0$ and $\pi = 1$ so that the bubbleless equilibrium features perfect competition. In such a case, the appearance of a constant firm level bubble

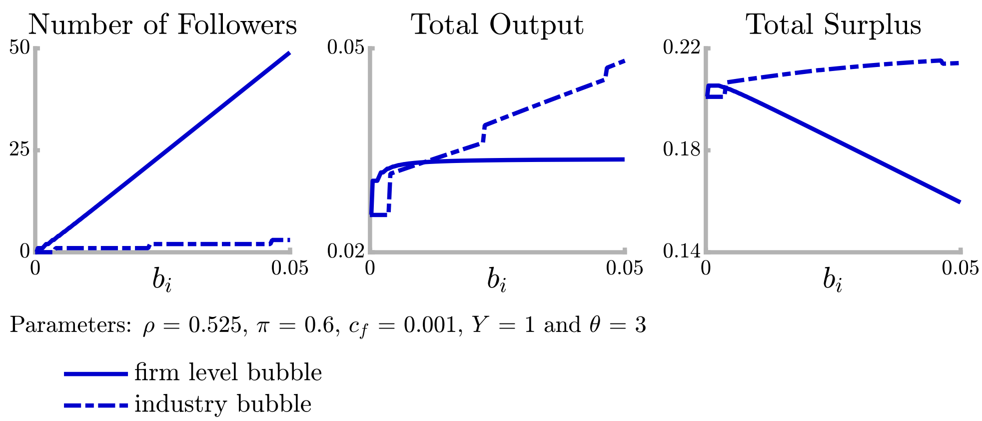



Figure 14: Constant firm level bubble versus constant industry level bubble

## C.3 General Equilibrium with No Bubbles

### C.3.1 Aggregate TFP

Note that we can write aggregate TFP as

$$\varphi(n_t) = \frac{\pi(1-\rho)n_t + (1-\rho)\pi^2}{[\pi(2-\pi)-\rho]n_t + (1-\rho)\pi^2}$$

Note that

$$\pi(1-\rho) < \pi(2-\pi) - \rho \Leftrightarrow \rho(1-\pi) < \pi(1-\pi)$$

which is always satisfied given the assumption that $\rho < \pi$. It then follows immediately that $\varphi(n_t)$ is decreasing in $n_t$.

### C.3.2 Aggregate Factor Share

Note that we can write the aggregate factor share as

$$\sigma(n_t) = \frac{\pi(1-\rho)n_t + (1-\rho)\pi^2}{[\pi(2-\pi)-\rho]n_t + (1-\rho)\pi^2}$$

Note that

$$\pi(1-\rho) < \pi(2-\pi) - \rho \Leftrightarrow \rho(1-\pi) < \pi(1-\pi)$$

which is always satisfied given the assumption that $\rho < \pi$. It then follows immediately that $\varphi(n_t)$ is decreasing in $n_t$.

in industry A will have no impact on welfare. However, as a constant firm level bubble appears in industry B, welfare will necessarily decrease, since firms will charge a price below their marginal cost of production.

### C.3.3 Number of Followers

Suppose that there are $n_t$ followers in every industry, except in industry $i$ (which has $n_{it}$ followers). Since the size of each individual industry is negligible, we have $\theta_t = \frac{n_t+\rho}{n_t+\pi}\pi$ and $Y_t = \varphi(n_t)\ K_t^{\alpha}$. Each follower in this industry $i$ will therefore make profits

$$\Pi^F(n_{it}, n_t) = \left(\frac{\pi-\rho}{n_{it}+\pi}\right)^2 \left(\frac{n_{it}+\rho}{n_{it}+\pi}\frac{n_t+\pi}{n_t+\rho}\right)^{\frac{\rho}{1-\rho}} \frac{\varphi(n_t)\, K_t^{\alpha}}{1-\rho}$$

For a symmetric equilibrium with $n_t$ followers to be possible, we first need that when $n_{it} = n_t$, no follower makes a loss in industry $i$

$$\begin{aligned} & \Pi^F(n_t, n_t) \geq c_f \\ \Leftrightarrow \quad & \left(\frac{\pi-\rho}{n_t+\pi}\right)^2 \frac{\varphi(n_t)\, K_t^{\alpha}}{1-\rho} \geq c_f \\ \Leftrightarrow \quad & K_t \geq \underbrace{\left[\frac{(1-\rho)\, c_f}{\varphi(n_t)}\left(\frac{n_t+\pi}{\pi-\rho}\right)^2\right]^{\frac{1}{\alpha}}}_{\equiv \underline{K}(n_t)} \end{aligned}$$

Second, we need that if an additional follower were to enter industry $i$ (i.e. $n_{it} = n_t + 1$), she could make no gain

$$\begin{aligned} & \Pi^F(n_t+1, n_t) \leq c_f \\ \Leftrightarrow \quad & \left(\frac{\pi-\rho}{n_t+1+\pi}\right)^2 \left(\frac{n_t+1+\rho}{n_t+1+\pi}\frac{n_t+\pi}{n_t+\rho}\right)^{\frac{\rho}{1-\rho}} \frac{\varphi(n_t)\, K_t^{\alpha}}{1-\rho} \leq c_f \\ \Leftrightarrow \quad & K_t \leq \underbrace{\left[\frac{(1-\rho)\, c_f}{\varphi(n_t)}\left(\frac{n_t+1+\pi}{\pi-\rho}\right)^2 \left(\frac{n_t+1+\pi}{n_t+1+\rho}\frac{n_t+\rho}{n_t+\pi}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1}{\alpha}}}_{\equiv \overline{K}(n_t)} \end{aligned}$$

We hence have that a symmetric equilibrium in which one leader and $n_t \geq 0$ followers produce in every industry is possible provided that

$$K_t \in \begin{cases} \left[\left(\frac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}, \overline{K}(0)\right] & \text{if } n_t = 0 \\ \left[\underline{K}(n_t), \overline{K}(n_t)\right] & \text{if } n_t \geq 1 \end{cases}$$

Let us start by analyzing the case in which $n_t \geq 1$. A symmetric equilibrium in which one leader and $n_t \geq 1$ followers produce is possible if $\underline{K}(n_t) \leq K_t \leq \overline{K}(n_t)$. The first inequality requires aggregate capital to be sufficiently large, so that the existing $n_t \geq 1$ followers do not make a loss. The threshold $\underline{K}(n_t)$ defines the aggregate stock of capital at which, in a symmetric equilibrium with one leader and $n_t$ followers, every follower makes exactly zero gains (i.e. her production profits equal the fixed cost $c_f$). The second inequality requires aggregate capital not to be too large, so that there are no profitable entry opportunities for any additional follower. The threshold $\overline{K}(n_t)$ defines the aggregate stock of capital at which, in a symmetric equilibrium with one leader and $n_t$ followers, an additional follower in one industry would make exactly zero gains.

Let us now examine the case in which $n_t = 0$. A symmetric equilibrium with one leader and zero followers in every industry will be possible if $\left(\frac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}} \leq K_t \leq \overline{K}(0)$. The first inequality guarantees that, in a symmetric equilibrium in which only the leaders produce, no leader makes a loss. The threshold $\left(\frac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}$ is the minimum aggregate stock of capital at which all leaders can produce together without making a loss. The second inequality requires that, in such an equilibrium, no follower can make a gain upon entering the market.

### C.3.4 Asymmetric Equilibria

Whenever $K_t \in \left[\overline{K}(n_t), \underline{K}(n_t+1)\right]$ for some $n_t \geq 1$, a symmetric equilibrium with $n_t \geq 1$ followers will not be possible. In such a case, the capital stock is too large to be consistent with $n_t \geq 1$ followers per industry, but too low to sustain the existence of $n_t + 1$ followers. The equilibrium in this context will feature an asymmetry across industries – a fraction $\lambda_t$ of all industries will have $n_t + 1$ followers and a fraction $1 - \lambda_t$ have $n_t$ followers. The share of industries with $n_t + 1$ followers must be such that, in all these industries, followers do not make a loss. In other words, the fraction of industries with $n_t + 1$ followers is pinned down by a zero profit condition

$$\Pi^F(n_t + 1, \theta_t, Y_t) = c_f$$

Aggregate TFP $\varphi(n_t, \lambda_t)$ can be shown to be a negative function of both $n_t$ and $\lambda_t$, whereas $\theta(n_t, \lambda_t)$ and $\sigma(n_t, \lambda_t)$ increase in both $n_t$ and $\lambda_t$.

Aggregate TFP is given by

$$\varphi(n_t, \lambda_t) =$$

$$= \frac{\pi(1-\rho)(n_t+\pi)\left[(1-\lambda_t)\left(\frac{n_t+\rho}{n_t+\pi}\right)^{\frac{\rho}{1-\rho}} + \lambda_t\left(\frac{n_t+1+\rho}{n_t+1+\pi}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1}{\rho}}}{(1-\lambda_t)\left(\frac{n_t+\rho}{n_t+\pi}\right)^{\frac{1}{1-\rho}}\left\{n_t\left[\pi(2-\pi)-\rho\right]+(1-\rho)\pi^2\right\} + \lambda_t\left(\frac{n_t+1+\rho}{n_t+1+\pi}\right)^{\frac{1}{1-\rho}}\left\{(n_t+1)\left[\pi(2-\pi)-\rho\right]+(1-\rho)\pi^2\right\}}$$

The aggregate factor cost index is

$$\theta(n_t, \lambda_t) = \left[(1-\lambda_t)\left(\frac{n_t+\rho}{n_t+\pi}\right)^{\frac{\rho}{1-\rho}} + \lambda_t\left(\frac{n_t+1+\rho}{n_t+1+\pi}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}}$$

The aggregate factor share is equal to

$$\sigma(n_t, \lambda_t) = \frac{\theta(n_t, \lambda_t)}{\varphi(n_t, \lambda_t)}$$

### C.3.5 Transition to Full Monopoly

Suppose that the economy is characterized by a monopoly in every industry. In such a case, the profit share (exclusive of fixed costs) is equal to $1-\rho$, meaning that each leader makes total profits that are equal to $(1-\rho)\,K_t^{\alpha}$. However, such an equilibrium is only feasible if

$$
\begin{aligned}
& (1-\rho)\,K_t^{\alpha}-c_f \geq 0 \\
\Leftrightarrow \quad & K_t \geq \left(\frac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}
\end{aligned}
$$

for otherwise each leader would incur a loss. If the capital stock is relatively small, so that the previous condition is not satisfied, a monopoly is not feasible in all industries. In such a case, production will only take place in a measure $M_t \in (0,1)$ of all industries. Aggregate output is equal to

$$Y_t = M_t^{\frac{1}{\rho}} y_t$$

And each industry's price and output are given by

$$
\begin{aligned}
p_t = & \quad M_t^{\frac{1-\rho}{\rho}} \\
y_t = & \quad M_t^{-1} K_t^{\alpha}
\end{aligned}
$$

We hence have that

$$Y_t = M_t^{\frac{1-\rho}{\rho}} K_t^{\alpha}$$

The number of active industries is pinned down by the no profit condition

$$
\begin{aligned}
& (1-\rho)\,p_t y_t = c_f \\
\Leftrightarrow \quad & (1-\rho)\,M_t^{\frac{1-\rho}{\rho}} M_t^{-1} K_t^{\alpha} = c_f \\
\Leftrightarrow \quad & (1-\rho)\,M_t^{\frac{1-2\rho}{\rho}} K_t^{\alpha} = c_f \\
\Leftrightarrow \quad & M_t = \left(\frac{c_f}{1-\rho} K_t^{\alpha}\right)^{\frac{\rho}{2\rho-1}}
\end{aligned}
$$

Note that $M_t \leq 1$ requires that

$$2\rho - 1 \geq 0$$

$$\Leftrightarrow \quad \rho \geq \frac{1}{2}$$

Aggregate output is hence given by

$$Y_t = \left(\frac{1-\rho}{c_f}\right)^{\frac{1-\rho}{2\rho-1}} K_t^{\frac{\alpha\rho}{2\rho-1}}$$

In other words, when $\rho \geq \dfrac{1}{2}$ and $K_t \leq \left(\dfrac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}$, the economy operates under monopolistic competition. In When $\rho \geq \dfrac{1}{2}$ and $K_t \leq \left(\dfrac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}$ an equilibrium is not possible.

Let us understand this result. Suppose that $K_t \leq \left(\dfrac{c_f}{1-\rho}\right)^{\frac{1}{\alpha}}$. If $\rho \geq \dfrac{1}{2}$, the degree of substitutability across varieties is high. This means that there are weak decreasing returns and each industry will be relatively large. Industries can break even when there are few active industries ($M_t \leq 1$)

If on the other hand $\rho < \dfrac{1}{2}$, the degree of substitutability is low. This means that there are strong decreasing returns and each industry will be relatively small. In such a case, each individual industry can break even only when there is a large number of active industries ($M_t > 1$). Given that each industry is small, a large number of industries is necessary for aggregate output to be high.

### C.3.6 Interest Rate

When there are strong decreasing returns to scale (low $\alpha$), the equilibrium interest rate can monotonically decrease in the aggregate capital stock.

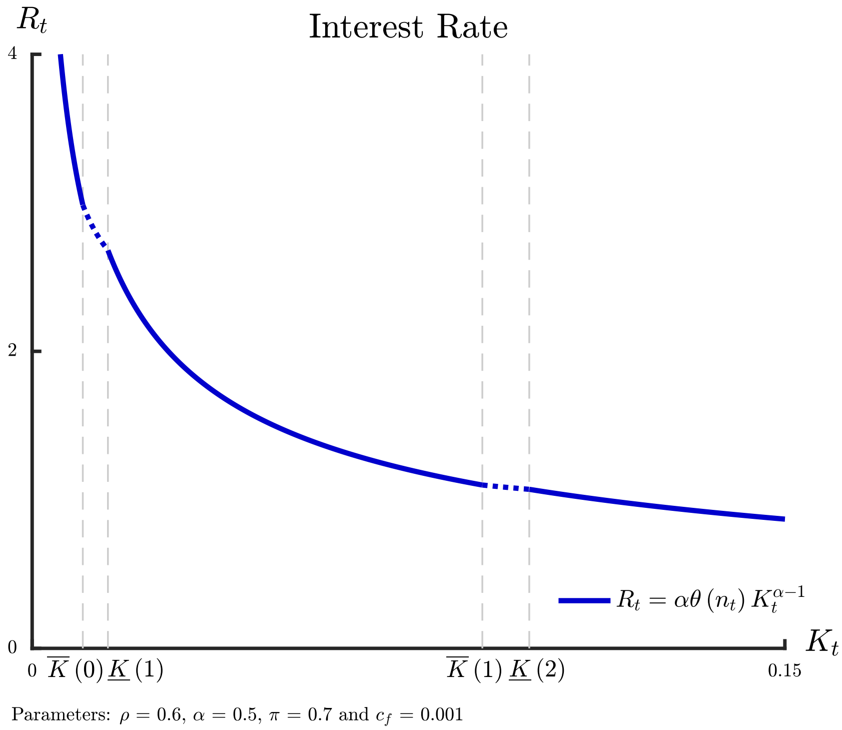
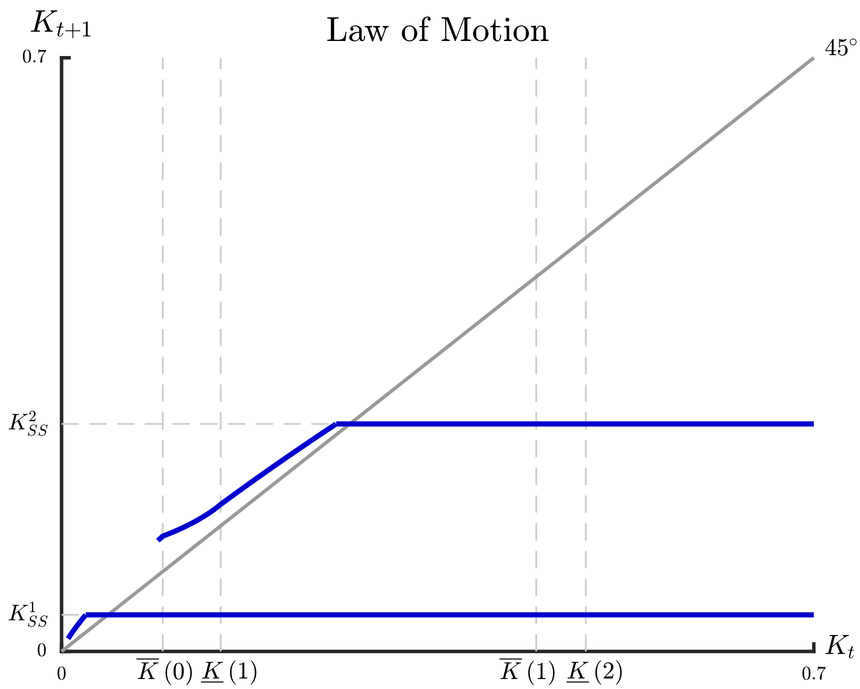
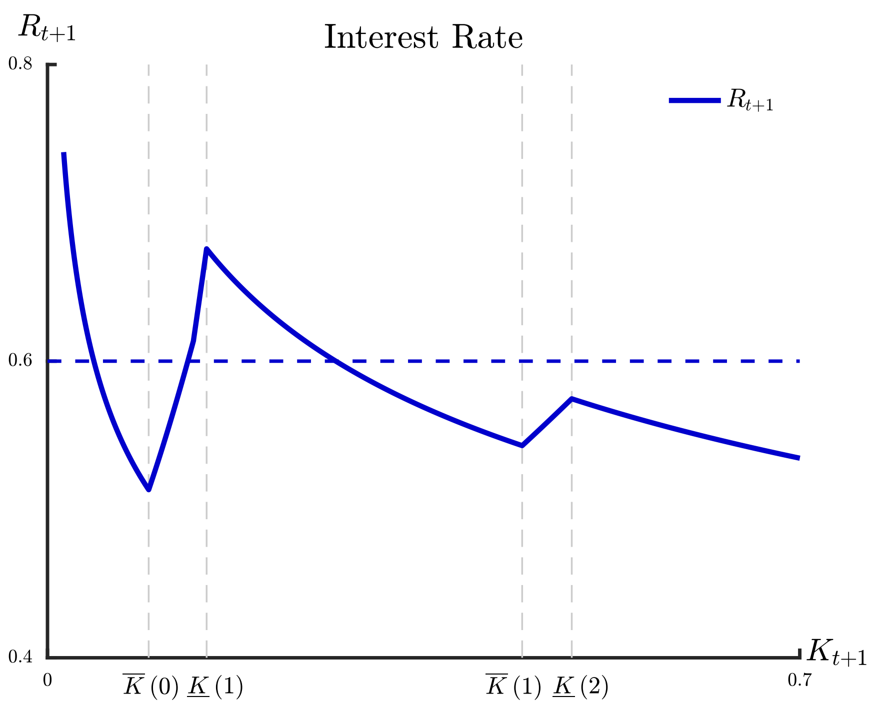



Figure 15: Interest Rate

### C.3.7 Multiple Equilibria with No Bubbles




Parameters: $\rho = 0.4$, $\alpha = 0.8$, $\pi = 0.7$, $c_f = 0.01$ and $\delta = 0.1$




Parameters: $\rho = 0.4$, $\alpha = 0.8$, $\pi = 0.7$, $c_f = 0.01$ and $\delta = 0.1$

Figure 16: Multiple Equilibria with No Bubbles

### C.3.8 Steady-State

**No Storage** Note that in a steady-state where storage is built and there are $n$ followers in all industries, aggregate output is equal to

$$Y = \varphi(n)\left[(1-\delta)\varphi(n)\right]^{\frac{\alpha}{1-\alpha}}$$

In order to have a symmetric equilibrium where $n$ followers produce in every industry we need to have

$$c_f \in \left[\underline{\mathbb{c}}(n^*, r), \overline{\mathbb{c}}(n^*, r)\right]$$

where

$$\underline{\mathbb{c}}(n,r) = \begin{cases} \rho(1-\delta)^{\frac{\alpha}{1-\alpha}} & \text{if } n = 0 \\ \left(\frac{\pi-\rho}{n+\pi}\right)^2 \frac{\varphi(n)\left[(1-\delta)\varphi(n)\right]^{\frac{\alpha}{1-\alpha}}}{1-\rho} & \text{if } n \geq 1 \end{cases}$$

and

$$\overline{\mathbb{c}}(n,r) = \left(\frac{\pi-\rho}{n+1+\pi}\right)^2 \left(\frac{n+1+\rho}{n+1+\pi}\frac{n+\pi}{n+\rho}\right)^{\frac{\rho}{1-\rho}} \frac{\varphi(n)\left[(1-\delta)\varphi(n)\right]^{\frac{\alpha}{1-\alpha}}}{1-\rho}$$

Furthermore, we need that

$$\begin{aligned} & R > r \\ \Leftrightarrow \quad & \alpha\sigma(n)\varphi(n)K^{\alpha-1} > r \\ \Leftrightarrow \quad & \alpha\sigma(n)\varphi(n)\left[(1-\delta)\varphi(n)\right]^{\frac{\alpha-1}{1-\alpha}} > r \\ \Leftrightarrow \quad & \alpha\sigma(n) > (1-\delta)r \end{aligned}$$

**Storage** Note that in a steady-state where storage is built and there are $n$ followers in all industries, aggregate output is equal to

$$Y = \varphi(n)\left[\frac{\alpha\sigma(n)\varphi(n)}{r}\right]^{\frac{\alpha}{1-\alpha}}$$

In order to have a symmetric equilibrium where $n$ followers produce in every industry we need to have

$$c_f \in \left[\underline{\mathbb{c}}^{\mathrm{ss}}(n^*, r), \overline{\mathbb{c}}^{\mathrm{ss}}(n^*, r)\right]$$

where

$$\underline{\mathbb{C}}^{ss} = \begin{cases} \rho\left(\frac{\alpha\rho}{r}\right)^{\frac{\alpha}{1-\alpha}} & \text{if } n = 0 \\ \left(\frac{\pi-\rho}{n+\pi}\right)^{2} \frac{\varphi(n)\left[\frac{\alpha\sigma(n)\varphi(n)}{r}\right]^{\frac{\alpha}{1-\alpha}}}{1-\rho} & \text{if } n \geq 1 \end{cases}$$

and

$$\overline{\mathbb{C}}^{ss}(n,r) = \left(\frac{\pi-\rho}{n+1+\pi}\right)^{2}\left(\frac{n+1+\rho}{n+1+\pi}\frac{n+\pi}{n+\rho}\right)^{\frac{\rho}{1-\rho}} \frac{\varphi(n)\left[\frac{\alpha\sigma(n)\varphi(n)}{r}\right]^{\frac{\alpha}{1-\alpha}}}{1-\rho}$$

Furthermore, we need that

$$\begin{aligned} & (1-\delta)\,Y > K \\ \Leftrightarrow \quad & (1-\delta)\,\varphi(n)\left[\frac{\alpha\sigma(n)\varphi(n)}{r}\right]^{\frac{\alpha}{1-\alpha}} > \left[\frac{\alpha\sigma(n)\varphi(n)}{r}\right]^{\frac{1}{1-\alpha}} \\ \Leftrightarrow \quad & (1-\delta)\,\varphi(n) > \frac{\alpha\sigma(n)\varphi(n)}{r} \\ \Leftrightarrow \quad & (1-\delta)\,r > \alpha\sigma(n) \end{aligned}$$

## C.4 General Equilibrium with Bubbles

### C.4.1 The Firm Level Bubble in General Equilibrium

The minimum bubble that allows $n$ followers to simultaneously enter in all industries, in an equilibrium where storage is built, is given by

$$\underline{b}(n,r) := c_f - \frac{1}{1-\rho}\left(\frac{\pi-\rho}{n+\pi}\right)^{2}\varphi(n)\left[\frac{\alpha}{r}\theta(n)\right]^{\frac{\alpha}{1-\alpha}}$$

Assume that the economy starts at the initial steady-state $K_{SS}$, where all industries consist of a monopoly where the leader is the only producer. Suppose now that firms can issue an amount $b_1 \in [\underline{b}(1,r), \underline{b}(2,r))$ of bubbly stock every period. In such a case, the law of motion will have a new horizontal segment at $K'_{SS} > K^2_{SS}$. The new capital stock is defined by

$$\theta(1)\cdot\alpha\cdot\left(K'_{SS}\right)^{\alpha-1} = r$$

This condition helps us understand how the issuance of bubbly stocks can sustain an expansion in general equilibrium. As firms can issue an amount of bubbly stock $b_1 \in [\underline{b}(1,r), \underline{b}(2,r))$, one follower will be able to enter in every industry. This fact will translate into a higher demand for capital and, because capital supply is infinitely elastic when storage is built, a new equilibrium capital stock $K_{SS}^{'} > K_{SS}$ with the same interest rate $r$. As the amount of bubbly stocks that firms can issue increases further to $b_2 \in [\underline{b}(2,r), \underline{b}(3,r))$, there will exist two followers in every industry. The demand for capital is now even larger and an aggregate equilibrium with storage is now consistent with a larger capital stock $K_{SS}^{''} > K_{SS}^{'}$.

Note that the economy admits one steady-state with storage and in which firms can issue an amount of bubbly stock $b$ in every industry and period if

$$(1-\delta)\underbrace{\varphi(n)\left[\frac{\alpha\theta(n)}{r}\right]^{\frac{\alpha}{1-\alpha}}}_{Y_{SS}} - \underbrace{\left[\frac{\alpha\theta(n)}{r}\right]^{\frac{1}{1-\alpha}}}_{K_{SS}} - \underbrace{\frac{(1+n)\,b}{1-r}}_{B_{SS}} \geq 0 \tag{21}$$

### C.4.2 The Industry Bubble in General Equilibrium

As demonstrated in Appendix C.2.3, $s_t^L$ decreases with $b$ (irrespective of aggregate variables). Furthermore, we can show that there is a negative relationship between the factor cost index $\theta_t$ and the leaders' market share $s_t^L$. To see this, note that in a symmetric equilibrium with $p_{it} = 1$, we have

$$s_t^L = \frac{n_t\left(\frac{\theta_t}{\pi} - \theta_t\right) + \theta_t - \rho}{n_t\left(\frac{\theta_t}{\pi} - \rho\right) + \theta_t - \rho}$$

We can rearrange this expression, to write $\theta_t$ as a function of $s_t^L$

$$\theta_t = \frac{\rho\left[(n_t+1)\,s_t^L - 1\right]}{\left(\frac{n_t}{\pi}+1\right)s_t^L - n_t\left(\frac{1}{\pi}-1\right) - 1}$$

It is easy to see that, in general equilibrium, the factor cost index $\theta_t$ decreases with $s_t^L$

$$
\begin{aligned}
& \frac{\partial \theta_t}{\partial s_t^L} < 0 \\
\Leftrightarrow \quad & \frac{\rho\,(n_t+1)\left[\left(\frac{n_t}{\pi}+1\right)s_t^L - n_t\left(\frac{1}{\pi}-1\right)-1\right] - \left(\frac{n_t}{\pi}+1\right)\rho\left[(n_t+1)\,s_t^L - 1\right]}{\left[\left(\frac{n_t}{\pi}+1\right)s_t^L - n_t\left(\frac{1}{\pi}-1\right)-1\right]^2} \\
\Leftrightarrow \quad & (n_t+1)\left[\left(\frac{n_t}{\pi}+1\right)s_t^L - n_t\left(\frac{1}{\pi}-1\right)-1\right] - \left(\frac{n_t}{\pi}+1\right)\left[(n_t+1)\,s_t^L - 1\right] < 0 \\
\Leftrightarrow \quad & (n_t+1)\left[\left(\frac{n_t}{\pi}+1\right)s_t^L - n_t\left(\frac{1}{\pi}-1\right)-1-\left(\frac{n_t}{\pi}+1\right)s_t^L\right] + \left(\frac{n_t}{\pi}+1\right) < 0 \\
\Leftrightarrow \quad & (n_t+1)\left[-n_t\left(\frac{1}{\pi}-1\right)-1\right] + \left(\frac{n_t}{\pi}+1\right) < 0 \\
\Leftrightarrow \quad & (n_t+1)\left(-\frac{n_t}{\pi}+n_t-1\right) + \left(\frac{n_t}{\pi}+1\right) < 0 \\
\Leftrightarrow \quad & n_t\left(-\frac{n_t}{\pi}+n_t-1\right) + n_t < 0 \\
\Leftrightarrow \quad & n_t^2\left(1-\frac{1}{\pi}\right) < 0 \quad \checkmark
\end{aligned}
$$

To understand this result, note that $s_t^L$ can be seen as a measure of market power. The higher is $s_t^L$, the higher are average profit margins and hence the lower are factor shares. This fact translates into a lower factor price index $\theta_t$.

We hence have that, as the industry bubble $b$ increases, the leaders lose market share and factor prices increase.

**Proposition 11.** *In a symmetric equilibrium in which all industries have $n_t$ followers, as the industry bubble $b$ increases, we have that*

1. *the leaders lose market share, i.e.* $\frac{\partial s_t^L}{\partial b} < 0$
2. *the factor cost index increases, i.e.* $\frac{\partial \theta_t}{\partial b} > 0$

I now characterize the reaction of aggregate output $Y_t$ to a change in the industry bubble $b$. In a symmetric equilibrium in which all industries have $n_t$ followers and an industry bubble with size $b$, aggregate output is implicitly defined by

$$
Y_t\left[\theta_t\left(1+\frac{n_t}{\pi}\right) - (n_t+\rho)\right] - n_t b = 0
$$

Recall that the factor cost index $\theta_t$ increases with $b$. Hence, it is not clear from the previous equation whether $Y_t$ increases or decreases with $b$. If $\theta_t$ reacts negligibly to a change in $b$ $\left(\frac{\partial \theta_t}{\partial b} \to 0\right)$, then $Y_t$ will increase with $b$ $\left(\frac{\partial Y_t}{\partial b} > 0\right)$. To derive some further intuitions, note that when storage is used, the factor cost index can be written as

$$\theta_t = \left\{ \frac{Y_t}{\pi} \left[ 1 - s_t^L \left(1-\pi\right) \right] \right\}^{\frac{1-\alpha}{\alpha}} \frac{r}{\alpha} \tag{22}$$

We therefore have that

$$\underbrace{Y_t \left[ \left\{ \frac{Y_t}{\pi} \left[ 1 - s_t^L \left(1-\pi\right) \right] \right\}^{\frac{1-\alpha}{\alpha}} \frac{r}{\alpha} \left(1 + \frac{n_t}{\pi}\right) - \left(n_t + \rho\right) \right] - n_t b}_{F\left(Y_t, s_t^L, b\right)} = 0$$

We have that

$$\begin{aligned}
& \frac{\partial Y_t}{\partial b} > 0 \\
\Leftrightarrow \quad & -\frac{\frac{\partial F(\cdot)}{\partial b}}{\frac{\partial F(\cdot)}{\partial Y_t}} > 0 \\
\Leftrightarrow \quad & \frac{\partial F(\cdot)}{\partial b} < 0 \\
\Leftrightarrow \quad & Y_t \frac{r}{\alpha} \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} \left\{ \frac{Y_t}{\pi} \left[ 1 - s_t^L \left(1-\pi\right) \right] \right\}^{\frac{1-\alpha}{\alpha}-1} \left[ -\frac{Y_t}{\pi} \left(1-\pi\right) \right] \frac{\partial s_t^L}{\partial b} - n_t < 0 \\
\Leftrightarrow \quad & Y_t \frac{r}{\alpha} \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} \left( \frac{Y_t}{\varphi_t} \right)^{\frac{1-\alpha}{\alpha}-1} \frac{Y_t}{\pi} \left(1-\pi\right) \frac{\partial s_t^L}{\partial b} + n_t > 0 \\
\Leftrightarrow \quad & \varphi_t Y_t \frac{r}{\alpha} \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} \left( \frac{Y_t}{\varphi_t} \right)^{\frac{1-\alpha}{\alpha}} \frac{1-\pi}{\pi} \frac{\partial s_t^L}{\partial b} + n_t > 0 \\
\Leftrightarrow \quad & \varphi_t Y_t \frac{\alpha \varphi_t K_t^{\alpha-1}}{\alpha} \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} \left(K_t^{\alpha}\right)^{\frac{1-\alpha}{\alpha}} \frac{1-\pi}{\pi} \frac{\partial s_t^L}{\partial b} + n_t > 0 \\
\Leftrightarrow \quad & \varphi_t Y_t \frac{\alpha \sigma_t \varphi_t K_t^{\alpha-1}}{\alpha} \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} K_t^{1-\alpha} \frac{1-\pi}{\pi} \frac{\partial s_t^L}{\partial b} + n_t > 0 \\
\Leftrightarrow \quad & \varphi_t \theta_t Y_t \left(1 + \frac{n_t}{\pi}\right) \frac{1-\alpha}{\alpha} \frac{1-\pi}{\pi} \underbrace{\frac{\partial s_t^L}{\partial b}}_{<0} + n_t > 0
\end{aligned}$$

There are different ways to interpret the expression above. One possible interpretation is that we have $\frac{\partial Y_t}{\partial b} > 0$ provided that $\frac{\partial s_t^L}{\partial b}$ is not too negative.[54] This fact is equivalent to requiring that the decline in aggregate TFP resulting from an increase in the followers' market shares is not too high.

Another possible interpretation is that we have $\frac{\partial Y_t}{\partial b} < 0$ when $\frac{1-\alpha}{\alpha}$ is sufficiently high (so that the capital

[54] Indeed, we would always observe $\frac{\partial Y_t}{\partial b} > 0$ if $\frac{\partial s_t^L}{\partial b} = 0$.

elasticity $\alpha$ is low). Recall that the factor cost index $\theta_t$ always increases with $b$ – a higher industry bubble makes firms compete more aggressively and demand more factors of production. When storage is used, capital is infinitely elastic and the interest rate is fixed. In such a case, only the labor price $W_t$ rises after an increase in $b$. The extent to which $W_t$ rises depends however on the labor elasticity $1-\alpha$ – the higher $1-\alpha$, the larger the increase in $W_t$. Indeed, if $1-\alpha$ is sufficiently large, $W_t$ may increase so much (after an increase $b$) that firms can be discouraged from increasing output.

**Steady-State** We can express the aggregate capital stock in a steady-state as

$$K = \left(\frac{Y}{\varphi}\right)^{\frac{1}{\alpha}}$$

Recall that

$$\varphi = \frac{\pi}{1 - s^L (1-\pi)}$$

We hence have that

$$\frac{\partial K}{\partial b} = \frac{1}{\alpha} K^{1-\alpha} \left[ -\frac{1-\pi}{\pi} Y \frac{\partial s^L}{\partial b} + \frac{1 - s^L (1-\pi)}{\pi} \frac{\partial Y}{\partial b} \right]$$

The industry bubble will lead to an efficient expansion provided that

$$\frac{\partial Y}{\partial b} > \frac{\partial K}{\partial b}$$

which is equivalent to

$$\frac{\partial Y}{\partial b} \left( \alpha \varphi K^{\alpha-1} - 1 \right) > \frac{1-\pi}{1 - s^L (1-\pi)} Y \left( -\frac{\partial s^L}{\partial b} \right) \tag{23}$$

This expression defines the conditions under which an increase in the industry bubble $b$ leads to an efficient expansion (for a fixed number of followers in every industry). To derive some intuition, let us focus on a steady-state where capital accumulation is dynamically efficient. The condition for dynamic efficiency can be written as

$$\alpha \varphi K^{\alpha-1} > 1$$

Recall that $\frac{\partial s^L}{\partial b} < 0$. Therefore, the requiring that $\frac{\partial Y}{\partial b} > 0$ is not sufficient to guarantee that $\frac{\partial Y}{\partial b} > \frac{\partial K}{\partial b}$. In words, even if capital accumulation is dynamically efficient, an economic expansion triggered by an increase in $b$ may not always be desirable. Let us understand this result. The condition for dynamic efficiency means that the return to investment exceeds its cost in the current steady-state. This fact implies that, *holding aggregate TFP constant*, if we increase investment by $\varepsilon$, output increases by some amount $\kappa > \varepsilon$. However, as the industry bubble $b$ increases, the leaders lose market share and *aggregate TFP declines*. Therefore, an increase in output resulting from an increase in $b$ comes at the cost of a less efficient allocation of resources, and hence a higher cost of investment. Condition

(23) above states that, for the increase in $b$ to lead to an efficient expansion, the increase in output $\frac{\partial Y}{\partial b}$ needs to be sufficiently large to compensate for the loss in the leaders' market shares, given by $-\frac{\partial s^L}{\partial b}$.

# D The Model with Bertrand Competition

In this section I characterize the model under an alternative market structure: I will assume that firms engage in price competition (Bertrand), instead of competition via quantities (Cournot).

## D.1 Industry Equilibrium

Assume now that there are no fixed costs of production ($c_f = 0$), but that demographics, technology and financial markets are otherwise identical to the framework described in Section 3.1. In particular, each firm can still produce with marginal cost $\frac{\theta_t}{\pi_i^j}$ where

$$\pi_i^j = \begin{cases} 1 & \text{if} \quad j = i \qquad \text{(leader)} \\ \\ \pi \leq 1 & \text{if} \quad j \neq i \qquad \text{(followers)} \end{cases}$$

As before, I shall assume that $\pi > \rho$.

**Bubbleless Equilibrium** In the bubbleless equilibrium, the leader will set a price equal to the followers' marginal cost (limit pricing). We hence have that

$$\begin{aligned} p_{it} &= \frac{\theta_t}{\pi} \\ y_{it} &= \left(\frac{\pi}{\theta_t}\right)^{\frac{1}{1-\rho}} Y_t \end{aligned} \tag{24}$$

As we can see, the leader will charge a markup $\frac{1}{\pi}$ over her marginal cost $\theta_t$. Such a markup decreases in the followers' productivity level $\pi$. Note that in the limit case where $\pi = 1$, the equilibrium of this industry features perfect competition. $\pi$ can therefore be seen as a measure of competition.

**Constant Firm Level Bubbles** If firms can issue a constant amount of stock $b$, there is no impact on the previous equilibrium. The leader will still set a price equal to the marginal cost of the followers, so that the equilibrium is still described by equations (24).

**Constant Industry Bubbles** Suppose now that there is a constant industry bubble equal to $b_i$, which is distributed according to market shares. Will the leader still produce the quantity given by (24)? The answer is no. To see this, note that for any industry output level $y_{it+1}$ such that

$$\frac{\theta_t}{\pi} < p_{it} + \frac{1}{y_{it}} b_i$$

the followers can profitably enter. The reason that their marginal cost of production is still $\frac{\theta_t}{\pi}$, but they now get an additional return of $\frac{1}{y_{it}} b_{it+1}$ per each unit that they sell. Therefore, to prevent the entry of the followers, the leader must set a limit price lower than the followers' marginal cost so that the above condition holds with equality. In this case, the leader will set a limit price that is implicitly defined by

$$p_{it} = \frac{\theta_t}{\pi} - \frac{1}{y_{it}} b_i \tag{25}$$

It is easy to prove that the price implicitly defined by (25) is decreasing in the industry bubble $b_i$.

**Proposition 12.** *The price charged by the leader under Bertrand competition decreases in the size of the industry bubble $b_i$.*

*Proof.* It suffices to show that $y_{it}$ increases in $b_i$. We can write the equilibrium condition as

$$\frac{\theta}{\pi} y_i = y_i^{\rho} \cdot Y^{1-\rho} + b_i$$

Define

$$\Xi(x) = \frac{\theta}{\pi} x - x^{\rho} \cdot Y^{1-\rho}$$

$\Xi$ is increasing in $b_i$. Moreover, we have that

$$\Xi'(x) > 0 \Leftrightarrow \frac{\theta}{\pi} - \rho x^{\rho-1} Y^{1-\rho} > 0 \Leftrightarrow \frac{\theta}{\pi} > \rho x^{\rho-1} Y^{1-\rho} > 0 \Leftrightarrow \theta > \pi\rho x^{\rho-1} Y^{1-\rho} \Leftrightarrow x > \left(\frac{\pi\rho}{\theta}\right)^{\frac{1}{1-\rho}} Y$$

Finally, note that

$$x\,|_{b=0} = \left(\frac{\pi}{\theta}\right)^{\frac{1}{1-\rho}} Y > \left(\frac{\pi\rho}{\theta}\right)^{\frac{1}{1-\rho}} Y$$

∎

As before, as $b_i$ becomes sufficiently large, the leader may even set a price below her marginal cost $\theta_t$. Note however that, contrarily to the model with Cournot competition, under Bertrand competition the leader is always the only producer.

So far I have assumed that all potential market participants (the leader and the followers) can appropriate a fraction of the constant industry bubble (according to market shares). A slight modification to the previous assumption offers however an interesting insight. Suppose that there is still a potential industry bubble but that, contrarily to what I assumed before, $b_i$ can only be shared among the followers (still in proportion to their market shares). The assumption that the leader cannot issue overvalued stocks may be realistic in a world in which investors value change or novelty (and hence penalize incumbents). Suppose then that there is a potential industry bubble $b_i$ and that each individual firm can appropriate a fraction

$$
\begin{aligned}
s_b^L &= 0 \\
s_b^{j,F} &= \begin{cases} 0 & \text{if } y_i^F = 0 \\ \frac{y_i^{j,F}}{y_i^F} b_i & \text{if } y_i^F > 0 \end{cases}
\end{aligned}
$$

Under this alternative assumption, the industry price is still given by equation (25). Output will however be produced by either the leader or the followers, depending on the size of $b_i$. When the size of the potential bubble $b_i$ is sufficiently small, so that the price implied by (25) is not lower than $\theta_t$, the leader will still be the only producer. Note however that, because the followers are inactive, in such a region no bubble actually materializes. In that case, the industry is characterized by a latent bubble – the threat that the followers can issue overvalued stocks (whenever they have a non-negligible market share) will force the leader to set a lower limit price and produce a larger output level. Note that the leader can set a limit price not lower than her marginal unit cost $\theta_t$ provided that

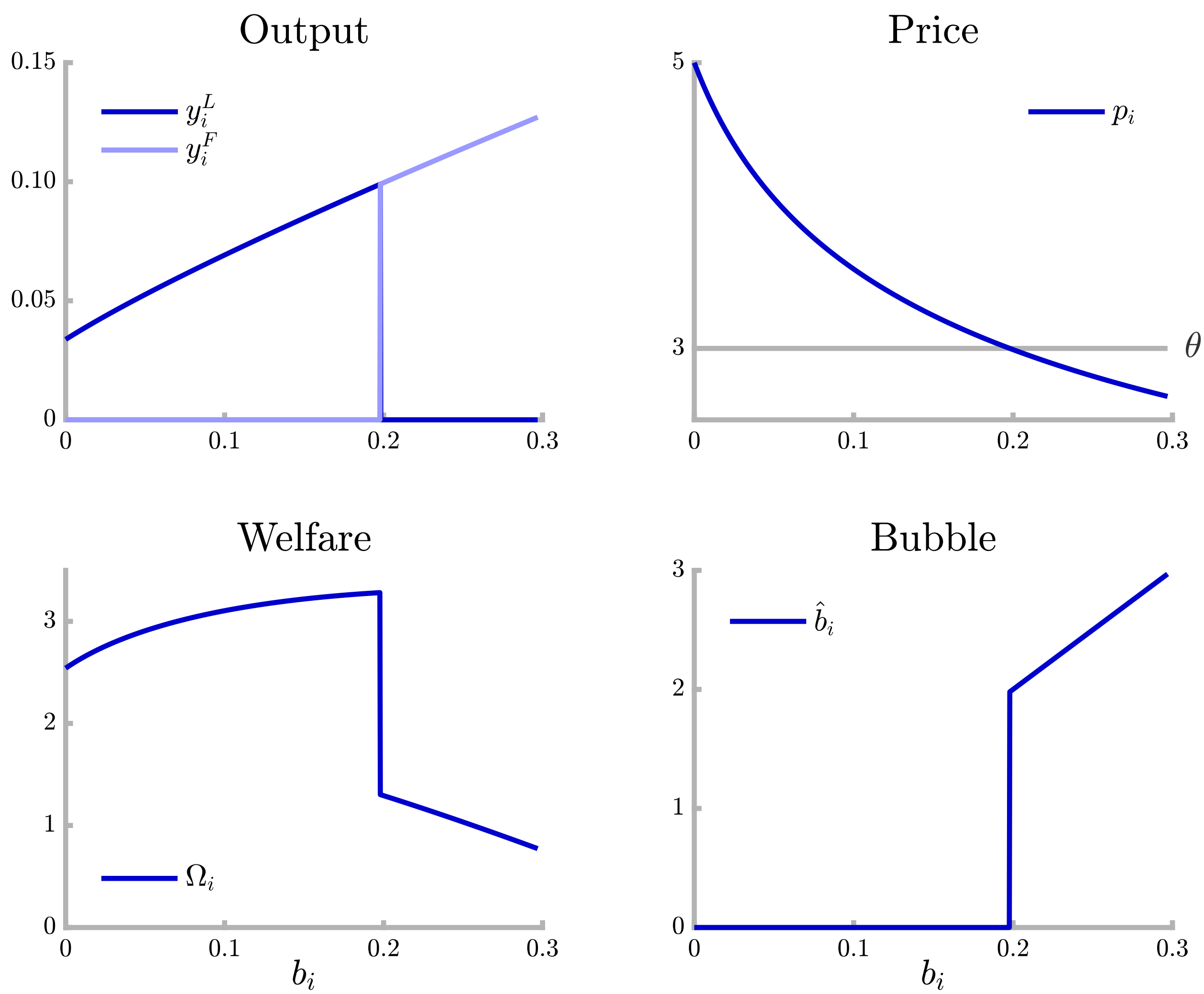
$$
b_i \leq \bar{b}_i := \theta^{-\frac{\rho}{1-\rho}} Y_t \frac{1-\pi}{\pi}
$$

When $b_i = \bar{b}_i$, the leader produces the perfect competition benchmark level of output, so that the maximum level of welfare is obtained. When $b_i > \bar{b}_i$, the leader cannot produce (for otherwise she would incur a loss). In such a case the followers become active and, for that reason, the industry bubble materializes. Figure 17 shows some equilibrium variables as a function of the total industry potential bubble $b_i$.

### D.2 General Equilibrium

**Bubbleless Equilibrium** Under Bertrand competition, the leader is the only producer in a bubbleless equilibrium. All industries will hence behave identically and share the same price $p_{it} = 1$. In such a case, aggregate TFP is constant and equal to $\varphi_t = 1$. Total output $Y_t$ will hence be a simple function of the aggregate capital stock $K_t$

$$
Y_t = K_t^\alpha
$$




Parameters: $\rho = 0.525$, $\pi = 0.6$, $Y = 1$ and $\theta = 3$

Figure 17: The Latent bubble

The aggregate cost index $\theta_t$ will coincide with the aggregate factor share, $\theta_t = \sigma_t$. We have that

$$\sigma_t = \pi$$

The aggregate factor share is equal to the inverse of the leaders' markup $\frac{1}{\pi}$. Note that perfect competition is achieved when $\pi = 1$, so that the leader has no productivity advantage over the followers; in such a case there is a unit factor share $\sigma_t = 1$.

Factor prices will be equal to

$$W_t = \quad (1-\alpha) \; \pi \, K_t^{\alpha}$$

$$R_t = \quad \alpha \, \pi \, K_t^{\alpha - 1}$$

Note that factor prices depend positively on the followers productivity $\pi$ which, as we have just seen, corresponds to the aggregate factor share. The existence of market power creates as before a wedge between factor prices and their aggregate marginal products. For instance, the interest rate will be below the marginal product of capital whenever $\pi < 1$

$$R_t = \alpha\,\pi\,K_t^{\alpha-1} < \alpha\,K_t^{\alpha-1} = \frac{\partial Y_t}{\partial K_t}$$

The equilibrium dynamics will take a simple form. In particular, the economy is characterized by a law of motion

$$K_{t+1} = \begin{cases} (1-\delta)\,K_t^{\alpha} & \text{if } K_t \leq \left[\frac{1}{(1-\delta)}\left(\frac{\alpha\,\pi}{r}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\alpha}} \\ \left(\frac{\alpha\,\pi}{r}\right)^{\frac{1}{1-\alpha}} & \text{if } K_t > \left[\frac{1}{(1-\delta)}\left(\frac{\alpha\,\pi}{r}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\alpha}} \end{cases}$$

This law of motion uniquely pins down the value of $K_{t+1}$ for any given value of $K_t$. When the current capital stock is low enough, all savings $(1-\delta)\ K_t^{\alpha}$ will be converted into capital; the resulting interest rate $\alpha\,\pi\,K_{t+1}^{\alpha-1}$ will not be lower than the return on storage $r$. When the current capital stock is sufficiently high, not all savings can be converted into capital; the capital stock is such that the resulting interest rate $\alpha\,\pi\,K_{t+1}^{\alpha-1}$ is equal to the return on storage $r$.This economy will converge to a steady-state characterized by

$$\begin{aligned} K^* &= \min\left\{(1-\delta)^{\frac{1}{1-\alpha}}, \left(\frac{\alpha\,\pi}{r}\right)^{\frac{1}{1-\alpha}}\right\} \\ R^* &= \max\left\{\frac{\alpha\,\pi}{1-\delta}, r\right\} \end{aligned}$$

Note that Propositions 7 to 9 still hold. We can however make use of the equilibrium result $\sigma^* = \pi$ and rewrite them as an explicit function of the model parameters.

**Proposition 13.** *(Possibility of Rational Asset Bubbles) Rational asset bubbles are possible if*

$$1 - \delta > \pi\alpha$$

**Proposition 14.** *(Overaccumulation of Capital)*

1. *If storage is not used in such a steady-state, capital accumulation is dynamically inefficient when*

$$1 - \delta > \alpha$$

2. *If storage is used in such a steady-state, capital accumulation is dynamically inefficient when*

$$\pi > r$$

**Proposition 15.** *(Underinvestment) The economy features underinvestment in its steady-state if*

$$\pi < \min\left\{\frac{1-\delta}{\alpha}r, r\right\}$$

The economy features underinvestment when the aggregate factor share $\pi$ is low or the return on storage $r$ is high.

### D.3 Optimal Bubble Creation ($\pi = 1$)

Optimal Bubble Creation

| $b = 0$ | | Firm-Level Bubble | | | | Industry-Level Bubble | | | |
|---|---|---|---|---|---|---|---|---|---|
| $n^F$ | $\mu_L$ | $n^F$ | $\mu_L$ | $\frac{b_i}{p_i\, y_i}$ | $\frac{y_i\,(b=b^*)}{y_i\,(b=0)}$ | $n^F$ | $\mu_L$ | $\frac{b_i}{p_i\, y_i}$ | $\frac{y_i\,(b=b^*)}{y_i\,(b=0)}$ |
| 0 | 1.50 | 1 | 1.50 | 0.00 | 1.00 | 1 | 1.14 | 0.23 | 2.89 |
| 1 | 1.20 | 1 | 1.20 | 0.00 | 1.00 | 1 | 1.25 | 0.18 | 1.36 |
| 2 | 1.13 | 2 | 1.13 | 0.00 | 1.00 | 2 | 1.23 | 0.11 | 1.26 |
| 10 | 1.03 | 10 | 1.03 | 0.00 | 1.00 | 10 | 1.23 | 0.02 | 1.06 |

**Table 10**
This table characterizes optimal bubble creation, for the two different processes considered: constant firm-level bubble and constant industry-level bubble. I normalize aggregate variables $Y = \theta = 1$ and set $\rho = 2/3$ and $\pi = 1$. I pick four different values for the fixed cost to generate bubbleless equilibria with $n = 0, 1, 2, 10$ followers. These values are $c_f =$ 0.100, 0.046, 0.029, 0.0025.