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Deliberative Institutions and Optimality

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Abstract
We derive necessary and sufficient conditions to achieve efficiency in common interest deliberative games. Our model explicitly characterizes a large class of deliberative institutions where privately informed agents strategically deliberate before taking a decision. Under the model's information structure, the transmission of information may require interpretation from agents with specific knowledge. The dynamics of interpretation are suggestive of a variety of frictions in information transmission. Private information is aggregated, and efficient decisions are taken at equilibrium, if and only if deliberative institutions enable the agents to extend deliberation (consensual extension) and freely interact with one another (freedom of reach). When, instead, these conditions do not hold, deliberation is incomplete and "anything goes": no general conclusion can be drawn as efficiency depends on the details of the deliberative extended-form game. We substantiate some of the implications of this indeterminacy result through detailed examples.

Keywords: Asymmetric Information, Collective Choice, Common Interest Games, Deliberative Institutions, Dynamic Group Communication, Efficient Deliberation.

JEL Classification: D71, D82, D83.

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1 Introduction

Efficient decision-making requires the aggregation of information that is often dispersed among agents - a problem that has long preoccupied economists.[1] A landmark result of modern economic theory shows that, under ideal conditions, competitive markets provide a solution to this problem. Prices help coordinate agents with dispersed information, leading to efficient consumption and production decisions (e.g. Hayek, 1945; Arrow and Debreu, 1954; Hurwicz, 1969). Likewise, the investigation of the limits of this result has long been central to economic research. Outside markets, deliberative institutions complement, or even substitute for, the market's information-aggregation mechanism. In these institutions - including political and governmental bodies, juries, committees, assemblies, panels of experts, and boards - decision-making involves an actual deliberative process and complex interactions among the deliberating agents. The purpose of our analysis is to provide a theoretical foundation of dynamic deliberation in non-market institutions and identify the general conditions that enable agents to aggregate their private information so to produce efficient decisions.

Our paper relates to a large class of studies that explore the impact of deliberation on group decision-making in common interest communication games under asymmetric information. To date, these studies have adopted two main research approaches. A first strand of literature has focused on models that consider abstract environments with no specification of the underlying deliberation mechanism. Classic papers within this approach study how agents are able to reach an agreement by repeatedly interacting and revising their beliefs (e.g. DeGroot, 1974; Aumann, 1976; Geanakoplos and Polemarchakis, 1982). A drawback of these papers, however, is that they overlook the possibility of strategic interaction. Other studies within this approach do consider this strategic dimension (see, for example, the seminal works on the revelation principle in Bayesian games of Myerson, 1982, 1986; Forges, 1986), but do not treat deliberation in purely individualistic terms. Instead, these studies largely focus on the positive analysis of equilibrium existence, while avoiding the question of how agents behave in actual deliberation contexts absent a centralized coordinator (e.g., a trustworthy mediator).

A second strand of literature entails applied studies that focus on agents' strategic

[1]See Smith (1776); Condorcet (1785).

interaction but only in *ad hoc* deliberative contexts. In these studies, the equilibrium characterization and the normative property of deliberation saliently depend on the "details" (i.e., the specific structure) of the model. These details include preferences (e.g. Battaglini, 2002; Austen-Smith and Feddersen, 2006; Breitmoser and Valasek, 2017), voting rules (e.g. Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1998; Coughlan, 2000; Gerardi and Yariv, 2007; Goeree and Yariv, 2011), information structure (e.g. Duggan and Martinelli, 2001; Mathis, 2011; Blume and Board, 2013; Deimen et al., 2015), and length of deliberation (e.g. Van Weelden et al., 2008; Strulovici, 2010; Chan et al., 2018).

Our paper contributes at the intersection of these two sets of studies by providing a general model of dynamic deliberation that is robust to most of the assumption details characterizing the extended-form game. First, since our focus is on information aggregation, we study agents' strategic interaction for a large class of dynamic extended form games that encompass deliberations whose timing is defined exogenously or endogenously. Second, we consider a general information structure where agents' private information can consist in *transferable information* (e.g., a data point), *non-transferable information* (e.g., know-how, language, etc.), or both. Transferable information (TI) can be directly communicated to other agents. Non-transferable information (NTI), instead, is transmittable only indirectly through the interpretation of TI by agents with specific knowledge, while interpretation is often complex and may require repeated agents' interaction.[2] These interpretation dynamics are suggestive of a variety of deliberation frictions including different organizational codes (e.g. Blume and Board, 2013), noise in communication (e.g. Blume et al., 2007), and deliberation with costly delay (e.g. Chan et al., 2018). Third, we consider many different forms of deliberation under specific interaction patterns, including simultaneous or sequential communication, private or public communication, and exogenous or endogenous action sequence. Fourth, we consider almost any possible decision rule governing both the dynamics of communication and how decision outcomes are selected (e.g., consensus, dictatorship, majority, veto). Any mixed selection of these rules constitutes

[2]This information structure represents, for example, the situation where a government sets up a task-force of scientists to anticipate the consequences of a pandemic. The scientists in the task-force need to develop models, acquire and interpret data, and update their initial hypotheses. This task requires specific knowledge, competences, and know-how which cannot be transferred from one scientist to another.

a specific deliberation mechanism.

Our main result is an irrelevance theorem. For any deliberation mechanism, irrespective of its specific configuration, we show that only two conditions are necessary and sufficient for the agents to take the efficient decision outcome at the equilibrium. The first condition requires that the agents can continue deliberation when they all want so (*consensual extension*). The second condition requires that each agent has the freedom to communicate, either directly or indirectly, with every other agent (*freedom of reach*). Under these conditions, we are able to ensure incentive compatibility and we explicitly characterize the equilibrium leading to efficient information aggregation for a very large class of communication games.

We illustrate the intuition behind our main result using two motivating examples, which explore the negative results arising when the theorem's conditions are not met.

Example 1 (*Consensual Extension*)

Two agents $i \in \{1, 2\}$ are required to select a decision outcome $o \in \{0, 1\}$. The outcome is optimal when it matches an unobservable and equiprobable state of Nature $\omega \in \{\omega_0, \omega_1\}$. The agents are required to agree on the decision outcome. They can communicate prior to voting, and repeat so until they reach an agreement with a symmetric vote $d_1 = d_2 \in \{0, 1\}$.[3]

When agents select the efficient decision outcome, they get zero. When, instead, they wrongly select $o = 1$ (resp. $o = 0$) under state ω_0 (resp. ω_1), they get $-q$, with $q = \frac{2}{3}$ (resp. $-(1-q)$).[4]

Before interacting with one another, each agent i privately observes an informative signal $s_i \in \{a, b, c\}$ about the state of Nature in accordance with the signal probability distribution and the corresponding posterior probability, reported in Table 1, that the state is ω_1 .

Suppose both agents receive signal b. From Table 1 and the Bayes' rule (see Appendix C for computational details), we obtain:

$$\mathbb{P}\left(\omega_1 \,|s_i = b\right) \simeq 0.6$$

[3]The treatment of infinite interaction will be addressed later.

[4]This framework replicates the standard Condorcet jury model where q represents the "threshold of reasonable doubt."

Assume that the first action the agents take is voting. Because $\mathbb{P}(\omega_1 | s_i = b) < q$, when both agents vote sincerely,[5] they vote for $o = 0$, reach consensus, and the outcome $o = 0$ is selected. Note that the agents' respective beliefs about the efficiency of outcome $o = 0$ is reinforced after each of them observe the other's first vote (see Appendix C). However, the outcome $o = 0$ is not efficient. Table 2 reports the agents' posterior belief that the state is ω_1 when all signals are complete information.

As

$$\mathbb{P}(\omega_1 | s = (b, b)) = 0.7 > q$$

had the agents shared their private information, they would have selected the efficient outcome $o = 1$.

There are two ways to overcome this problem. In the first place, agents may strategically dissent in the first round of voting to (truthfully) communicate their private information before moving to the second round of voting.[6] Yet, strategic dissent becomes useless under rules that select an outcome anyway (e.g., status quo unanimity rule, or majority rule with an odd number of agents). Under these rules, the alternative way for agents to overcome the above problems is being enabled to consent to extend communication, in spite of the agreement on a given decision outcome.

The possibilities of strategic dissent and communicating beyond consensus on an outcome empower the agents with what we call *consensual extension.*

[5] That the agents vote sincerely or strategically by taking into account the probability of being pivotal for the final decision is irrelevant for this example (see Appendix C).

[6] This result shares Santana (2019)'s idea that dissent can give agents the opportunity to acquire more information. However, unlike in Santana (2019), in our setup, disagreement is the result of the behavior of agents who strategically avoid agreeing in the interest of the group itself, rather than because of their own idiosyncrasies (e.g., credit seeking, noble hypocrisy, congenital stubbornness, or epistemic diversity). In other related works, the deliberation is strategically shortened at the expense of acquiring less information either because agents are impatient (Chan et al., 2018) or because they want to preserve their likelihood to be pivotal for the final decision (Strulovici, 2010).

s_i	$\mathbb{P}(s_i\|\omega_1)$	$\mathbb{P}(s_i\|\omega_0)$	$\mathbb{P}(\omega_1\|s_i)$	Preferred outcome
a	17%	70%	20%	0
b	38%	25%	60%	0
c	45%	5%	90%	1

Table 1: Signal probability distribution.

Decision-relevant information	Posterior belief: $\mathbb{P}(\omega_1\|s)$	Efficient decision
$s = \{a, a\}$	0.06	0
$s = \{a, b\}$	0.27	0
$s = \{a, c\}$	0.69	1
$s = \{b, b\}$	0.70	1
$s = \{b, c\}$	0.93	1
$s = \{c, c\}$	0.99	1

Table 2: Posterior beliefs when all signals are complete information.

Example 2 (***Freedom of reach***)

Three agents, indexed by $i \in \{A, B, C\}$, must identify the length of the segment x in the right triangle in Figure 1 (which is public information).[7] Each agent is endowed with private information that can be *transferable information* (TI), *non-transferable information* (NTI), or both.

Assume agents A (he), B (he), and C (she) respectively know the length of segments a, the length of segment b, and the degrees of angle c. Each of this information is TI. Only C, however, knows trigonometry, a NTI that cannot be directly transmitted to the other agents.

Suppose the agents deliberate according to (i) of Figure 2, that is, a two-channel communication structure where A and C directly report their private information to B, who acts as the decision-maker. Since B does not know trigonometry, he fails to infer the length of x from a, b, and c. Therefore, what we define as *freedom of reach* is violated: A cannot directly (or indirectly) communicate with C, who could use her NTI to better inform B and enable him to make the efficient decision.

When *freedom of reach* is violated, anything can happen, as whether the agents can

[7]We do not explicitly define a utility function here as several configurations are coherent with the setting considered in the example (e.g., agents receive a positive payoff if the decision-maker selects an action that matches the length x and a payoff normalized to zero otherwise).

aggregate the decision-relevant information depends on the details of the communication structure. For example, a one channel communication structure (see (ii) of Figure 2) where C is the decision-maker is more efficient than the two-channel communication structure with B as the decision-maker. Under this different structure, A can directly report his private information to C, who will then be able to correctly assess x.[8]

Conversely, when *freedom of reach* holds, the agents will be able to exploit enough communication channels to fully aggregate their decision-relevant information, regardless of the specific information distribution among the agents.

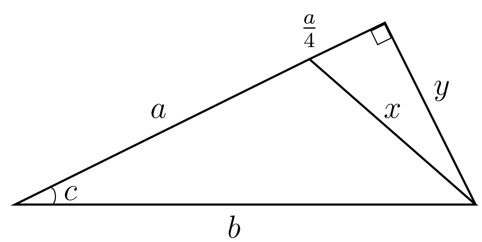

Figure 1: Identifying the length of x from a, b, and c.

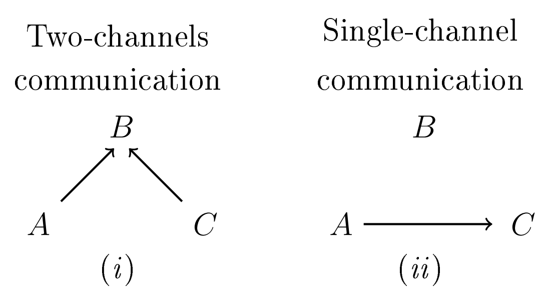

Figure 2: Two-channels communication vs. single channel communication.

Finally, notice that in both examples, once all relevant private information is aggre-

[8]Indeed, combining a and c, C can compute the lengths of y and b to obtain x, by sequentially applying the tangent function, the Pythagorean theorem, and the Al-Kashi's law of cosines:

$$y = \frac{5}{4}a \cdot \tan(c)\,; b = \sqrt{(\frac{5}{4}a)^2 + y^2}; \text{and } x = \sqrt{a^2 + b^2 - 2ab\cos(c)}.$$

gated, the decision rule of any given deliberative institution becomes irrelevant.[9]

***

Our theorem provides the analytical conditions that generalize the intuition illustrated in the two examples above. Under *consensual extension* and *freedom of reach*, the agents know that they will eventually select the efficient outcome. Hence, they may engage in strategic interaction that is incentive compatible, meaning that this interaction serves to aggregate information in the interest of the group. This behavior does not necessarily rule out insincere voting or misreporting. But here strategic behavior becomes instrumental to aggregate decision-relevant information.

Conversely, when one or both of the theorem's conditions do not hold, efficient deliberation rests on the specific structure imposed to the model and the details of the extensive form game. Put differently, deliberative institutions that do not meet these conditions should be considered by policymakers with the utmost caution because, there are cases (e.g., related to the information structure) in which no behavior can guarantee efficient decision-making.

The theorem conditions can fail in several circumstances, including, for example, when (i) the deliberative institutions impose time limits on deliberation (e.g., board meetings), (ii) the interaction patterns exclude some agents from communication (e.g., hierarchies), or, more generally, (iii) there are transaction costs. As a consequence of these frictions, we are unable to provide any general statement on the desirability of specific deliberative institutions, as both the economic analysis and the policy implications of such institutions become context-specific. Outside the theorem conditions, we then argue that deliberation is *incomplete* and "anything goes." This means that while a specific example may show that a deliberative institution selects an outcome that is Pareto-superior to another, a counterexample can prove the reverse.

This indeterminacy has several implications, a few of which are unexpected. First, no communication may (strictly) Pareto-dominate partial or private communication. Depending on the type of the agents who are excluded from communication, partial

[9] In Example 1, the same efficient outcome would be selected independent of whether one agent (i.e., dictatorship) or both (i.e., unanimity) take the decision. Likewise, in the case of Example 2, whether A, B, C, or any coalition of them, take the decision outcome is irrelevant for efficiency because under freedom of reach, the agent who has identified x can report it to the others.

information aggregation can lead either to efficient or inefficient decision-making (see Example 4.1).

Second, conflicts of interest may arise and lead to inefficiency even in a setup where agents have homogeneous preferences (see Example 4.2). Importantly, such inefficiency may arise both off the equilibrium path (e.g., because an agent plays an individual profitable deviation as in Example 4.2) and along the equilibrium path (e.g., Remark 2 of Example 4.2).

Third, communicating less precise information (e.g., opinions, posterior beliefs, preferred outcomes) may be more efficient than reporting primitive information (e.g., data points). As a full interpretation of all decision-relevant information is often too demanding in practice, reporting an aggregate of this information can be a Pareto-superior strategy (see Remark 3 of Example 4.2). This implication complements classic results in information economics showing the reverse, namely that reporting an aggregate view rather than initial information is Pareto inferior.[10]

Finally, when deliberation is incomplete no universal conclusion can be ex-ante stated about the efficiency of a specific deliberative institution. Therefore, social planning should endorse a case-by-case approach to identify desirable matches between deliberative institutions and information configurations. Relatedly, the study of deliberation problems should take into account the institutional details of different deliberative contexts, including, communication in organizations networks, political parties, juries, and so on. In these respects, our result stands as a benchmark for deliberation under ideal conditions.

The remainder of the paper proceeds as follows. Section 2 describes our model. Section 3 illustrates our main result. Section 4 shows what happens outside the ideal scenario described by our theorem. Section 5 discusses our results in light of the relevant literature. Section 6 concludes.

[10]See, e.g., DeGroot (1974); Geanakoplos and Polemarchakis (1982) and Ostrovsky (2012) where the agents repeatedly and truthfully communicate their posteriors, but might not converge to the posterior they would have agreed upon had their initial information been aggregated.

2 Model

This section describes our model. To avoid cumbersome notation, however, some mathematical details are left to Appendix A.

2.1 Basic Setup

A group of $n \geq 2$ agents, indexed by $i \in N \equiv \{1, \ldots, n\}$, is required to choose a decision outcome o in a (finite or infinite) set O with $\#(O) \geq 2$. Agents can fail to select a decision outcome; in such a case a *no-decision* outcome, denoted as $\emptyset$, takes place (e.g., a hung jury in jury trials).

Information and Preferences Identifying the efficient outcome requires the agents to aggregate their private information, which, for each agent $i \in N$, is represented by a type θ_i drawn from a (finite or infinite) set Θ_i. The type profile follows a prior distribution $p \in \Delta(\Theta)$, where $\Theta \equiv \times_{i \in N} \Theta_i$ is the set of type profiles.

Agent i's preferences are represented by a type dependent payoff (von-Neumann Morgenstern utility) function $u_i : (O \cup \{\emptyset\}) \times \Theta \to \mathbb{R}$. To focus on the agents' information problem, we assume they would prefer the same outcome under complete information: for any profile of types $\theta \in \Theta$, there is a unique outcome $o^*(\theta) \in O$, which maximizes $u_i(., \theta)$ for each agent i.[11] The assumption $o^*(\theta) \neq \emptyset$ rules out the trivial case where agents mostly prefer a no-decision outcome.

More generally, we focus on the class of "common interest" preferences under uncertainty defined as follows. Let $\mathcal{N}$ denote the set of groups N composed of n agents where $n \geq 2$ is a positive finite integer, and $\mathcal{O}$ (resp. $\mathcal{T}$) denote the set of possible sets of outcomes O (resp. types profile Θ). For any triple $(N, O, \Theta) \in \mathcal{N} \times \mathcal{O} \times \mathcal{T}$, let $\mathcal{U}^{(N,O,\Theta)}$ denote the set of payoff functions $u \equiv (u_i)_{i \in N}$ for which, for every $\theta \in \Theta$, there is a decision outcome $o^*(\theta) \in O$ that, for all $i \in N$, is the unique maximizer of $u_i(., \theta)$. Therefore, under asymmetric information, the agents may prefer different outcomes, while under complete information they prefer the same outcome (whose

[11] That the agents' divergent view on the preferred decision outcome is reducible to an asymmetric information problem presupposes that the agents act epistemically. This means that when the agents have the same information, they do not have a truth-independent reason to disagree.

payoff strictly Pareto-dominates all other payoffs), but may rank other (i.e., less preferred) outcomes differently. In the following, whenever there is no ambiguity, we lighten notation of this preferences class by denoting $\mathcal{U}^{(N,O,\Theta)}$ with $\mathcal{U}$.

Transferable and Non-Transferable Information The epistemic process leading the agents to extract their private information is similar to assembling a jigsaw puzzle whose pieces are dispersed among the agents. When the puzzle is trivial, one agent can be in charge of collecting all the pieces from the others and then assemble the puzzle on her own. In non-trivial cases, however, the group may need the competence of specific agents to analyze the puzzle pieces (e.g., colors, shapes, etc.). To model this richer environment, we assume that private information can be *transferable* (TI), *non-transferable* (NTI) or both. TI is, for example, data point (e.g., pieces of the puzzle) that can be transmitted by any agent to the others. NTI is, for example, know-how - defined as the theoretical or practical understanding of a subject matter (e.g., the ability to analyze individual puzzle pieces) - which cannot be transmitted to the other agents as such.[12] Nevertheless, NTI can be used in deliberation through interpretation (e.g., identifying the characteristics of the puzzle pieces). We model this distinction by separating the set of types (Θ) into the subset of TI, $\overrightarrow{\Theta}$, and the subset of NTI, $\overset{\times}{\Theta}$, with $\Theta \equiv \overrightarrow{\Theta} \cup \overset{\times}{\Theta}$.[13] Once an agent interprets a piece of TI, she can transmit her interpretation to the other agents. It is also possible that an interpretation requires the NTI of several agents.[14] We assume the agents know to whom to present the TI requiring interpretation.[15]

[12]We can also think of NTI as the set of perspectives and heuristics à la Hong and Page (2001), although in an incomplete information environment.

[13]For example, TI may refer to a document written in a specific language (e.g., foreign, scientific, or technical) and NTI to the knowledge of this language that cannot be taught to others during deliberation, but can be used to translate the document to others. More abstractly, any element of $\overset{\times}{\Theta}$ can be conceived as a list of data points that is too long to be transmitted during deliberation, but where specific entries can be transmitted upon request.

[14]For example, this could be the case of a medical diagnosis requiring multiple competences (e.g., a primary care doctor, a specialist, a radiologist, and a lab technician).

[15]This assumption does not mean that agent i observes agent j's NTI ($\overset{\times}{\theta}_j$), but simply that i knows j's field of expertise (e.g., i knows that j is a doctor but i does not share j's knowledge of medicine). This is without loss of generality in common interest games as the agents have the incentive to truthfully reveal their expertise to

Communication Let $M^{\mathcal{T}}$ be the set of possible messages (i.e., reports) on TI and their interpretations. For any agent $j \in N$ and message m, let $\iota(m|\overset{\times}{\theta}_j)$ denote agent j's transferable and truthful interpretation of m given NTI $\overset{\times}{\theta}_j$ (with the convention that $\iota(\overrightarrow{\theta_j}|\overset{\times}{\theta}_j) = \overrightarrow{\theta_j})$. We require $M^{\mathcal{T}}$ to: (i) contain all possible pieces of TI, i.e., $\overrightarrow{\Theta} \subseteq M^{\mathcal{T}}$ for any $\Theta \in \mathcal{T}$; (ii) be closed under interpretation, i.e., $\iota(m|\overset{\times}{\theta}_j) \in M^{\mathcal{T}}$ for any message $m \in M^{\mathcal{T}}$, agent $j \in N$, set of type profiles $\Theta \in \mathcal{T}$, and any element of NTI $\overset{\times}{\theta}_j \in \overset{\times}{\Theta}_j$;[16] and (iii) be closed under union, i.e., any pair of messages $(m, m') \in (M^{\mathcal{T}})^2$ can be merged into a single message $m'' \equiv (m \cup m') \in M^{\mathcal{T}}$. The latter condition is what allows the agents to communicate indirectly (e.g., this is the case of a firm where information is transmitted from the bottom to the top only indirectly through the managers belonging to each layer of hierarchy). An interpretation can also be interpreted, and so can the "interpreted" interpretation.[17] However, to avoid infinite iterations, the number of required interpretations is bounded and commonly known. Above this upper bound, further interpretations are redundant. Therefore, full interpretation of TI occurs in a finite number of iterations of truthful communication.[18]

Announcing Preferred Outcomes In our model, agents can always announce (spontaneously or when solicited) their preferred outcome, or they can choose to remain silent ($\emptyset$). Formally, $M^{\mathcal{O}}$ denotes the set of agents' announcements with

others.

[16]While $\iota(m|\overset{\times}{\theta}_j)$ denotes a truthful interpretation of m, $M^{\mathcal{T}}$ does not restrict the space of messages to truthful interpretations. Indeed, agent j, upon receiving a piece of TI m (e.g., $m = \overrightarrow{\theta_i}$), might untruthfully provide the other agents with interpretation $\iota(m'|\overset{\times}{\theta}_j)$, where m' corresponds to a different piece of TI (e.g., $m' = \overrightarrow{\theta'_i} \neq \overrightarrow{\theta_i}$). Also observe that $M^{\mathcal{T}}$ is not restricted to the set of type profiles Θ. With such restriction, the agents would be constrained to communicate uninterpreted information.

[17]For example, this is the case of a medical diagnosis requiring sequential interpretations by medical professionals with different expertise.

[18]An equivalent modeling assumption, which would lead the agents to behave with a similar dynamic, is assuming that the agents' messages are either reported or received with noise. In this case, the upper bound on the number of iterations required to interpret TI represents the number of iterations required to eliminate noise. Our results are general enough to be extended to this alternative framework.

$\cup_{O\in\mathscr{O}}O \cup \{\emptyset\} \subseteq M^{\mathcal{O}}$.

Forms of Communication Our model contemplates several forms of communication. The agents can interact according to either an exogenous order of interaction constraints (e.g., round-table public communication) or any endogenous order (e.g., the agents may ask questions to each other). To deal with cases where the group uses specific messages to coordinate on how it intends to deliberate, we define $M^{\neg(\mathcal{T}\vee\mathcal{O})}$ to be the set of possible messages that neither conveys TI (or interpretations of TI) nor announce preferred outcomes.

Finally, let $M \equiv (M^{\mathcal{T}}, M^{\mathcal{O}}, M^{\neg(\mathcal{T}\vee\mathcal{O})})$ be the profile of these set of messages, and $\mathcal{M}$ be the set of all the possible triples.

2.2 Formal Deliberation

Agents select outcomes through *deliberation*.[19] Deliberation is governed by two rules: (i) a *decision* rule D that selects a decision outcome; and (ii) a *continuation* rule C that specifies when deliberation ends (e) or continues ($\neg e$). Note that these rules may be interdependent (e.g., agents' authority to select a decision outcome also enables them to end deliberation). These rules, in conjunction with how the agents interact, characterize the dynamics of an extensive-form game evolving over multiple rounds, indexed by $t \in \{0, 1, \ldots, T\}$, with $T \geq 1$.

Decision Rule A *decision rule* D is defined by a pair (d_{set}, d_{rule}), where d_{set} denotes the set of agents' available "votes,"[20] possibly heterogeneous across agents, and d_{rule} is the aggregation rule mapping any profile of agents' votes $d = (d_1, d_2, ..., d_n) \in d_{set}$ into a probability distribution $d_{rule}(d) \in \Delta(O \cup \{\emptyset\})$ over a possible outcome. We further assume that the agents can deterministically select an outcome by coordinating on a commonly known profile of votes. Formally, for every decision outcome $o \in O$ (and

[19] With the term deliberation, we refer to a wide range of agents' interaction possibilities (without monetary transfers) happening before outcome selection. The agents involved in this procedure may belong to a political assembly, a committee, a court, a corporate board, a jury, a society, or any other group of individuals required to aggregate information and preferences in order to make a collective decision.

[20] "Votes" here have a broad meaning, including actions not restricted to specific outcome selection (see, e.g., Example 4.1).

in case D can render a no-decision, for any outcome $o \in O \cup \{\emptyset\}$), there exists a profile of votes $d^o \in d_{set}$ such that $d_{rule}(d^o)$ corresponds to the Dirac mass at outcome o (when the selection is deterministic, we will simply denote $d_{rule}(d^o) = o$). d_{set} can also include votes not belonging to the set of outcomes such as abstention[21] or a veto power that enables one (or more) agent(s) to reject an outcome.[22] Finally, d_{rule} may also allow agents to have heterogeneous voting power (e.g., one distinguished voter may have the whole voting power, like a dictator, as in senders-receiver games). We denote $\mathcal{D}$ the set of such voting rules D.

Continuation Rule Deliberation evolves over rounds. Whether it continues may depend on the agent's level of agreement about a decision outcome.[23] Let $\mathbb{I}_O$ denote the indicator function, which equals 1 if the agents agree on a decision outcome (i.e., the current votes profile selects an outcome in O), and 0 otherwise.

A *continuation rule* C is a pair (c_{set}, c_{rule}), where c_{set} denotes the set of agents' available choices, and c_{rule} denotes the aggregation rule mapping any profile of agents' choices $c = (c_1, c_2, ..., c_n) \in c_{set}$ and any value $x \in \{0, 1\}$ of $\mathbb{I}_O$ into a probability of ending deliberation $c_{rule}(c, x) \in [0, 1]$. As continuation rules govern deliberation extension decisions, we define the set of agents' choices as $c_{set} \equiv \{e, \neg e, \emptyset\}^N$, where e, $\neg e$, and $\emptyset$ stand for "end deliberation", "do not end or continue deliberation", and "abstain", respectively.[24]

We assume agents cannot be forced to deliberate more when they unanimously agree to end deliberation (i.e., $c_{rule}((e, e, ..., e), .) = 1$). We also assume that the likelihood that deliberation ends is (i) higher when the agents agree on an outcome (i.e., for any $c \in c_{set}$, $c_{rule}(c, 0) \leq c_{rule}(c, 1)$), and (ii) minimal when the agents unanimously agree on continuing deliberation (i.e., for any $x \in \{0, 1\}$, $(\neg e, \neg e, ..., \neg e) \in$

[21] The meaning of abstention is context-specific. For example, in the Council of the European Union, an abstention on a matter decided by unanimity (resp. qualified majority) has the effect of a "yes" (resp. "no") vote.

[22] Veto is a prerequisite for the existence of some committees. This is the case of, for example, the U.N. Security Council. Major powers would not have granted an international body binding legal authority on matters of peace and security unless they were certain that this power would not have prejudiced their interests.

[23] For example, in papal conclaves, deliberation stops as soon as two-third of the voting cardinals converge on a candidate.

[24] To simplify notation, ending (e) and continuing ($\neg e$) deliberation refer both to individual choices and the outcomes of deliberation extension decisions.

$\arg\min_{c \in c_{set}} c_{rule}(c, x)$).[25] Finally, let $\mathcal{C}$ denote the set of continuation rules C satisfying the above conditions.

2.3 Interaction Pattern

Deliberation is not only governed by decision and continuation rules, but also depends on the rules specifying: (i) the agents' interaction order; (ii) the agents' action set (which may vary over time);[26] and (iii) what action each agent observes. These elements, and their possible endogenous dynamics, define an *interaction pattern* F.

In our model, we focus on the set of interaction patterns $\mathcal{F}$, whose elements satisfy the following properties (see Appendix A for a formal definition). First, a decision to end deliberation, according to C, has the effect to either stop the interaction immediately (e.g., the outcome resulting from the previous vote is selected) or bring the agents to a final round of interaction (e.g., the agents agree to engage in a final round of voting). Second, to rule out the unrealistic cases where the agents are exogenously forced to interact forever, the agents are empowered to end their interaction from time to time and cannot perpetually postpone the choice about whether to end deliberation. Third, each agent is required to vote for (or to publicly announce) her preferred outcome from time to time.[27] Fourth, we exclude that the agents use a conventional language made of elements not in $M^{\mathcal{T}}$ to make interpretations of TI.[28] Fifth, each

[25] Note that under (ii), when the minimum likelihood is strictly positive, deliberation can stop prematurely, even if the agents agree to deliberate more. This may happen, for example, when the agents are required to repeatedly vote, either simultaneously or under secret ballot, until they reach consensus. See Example 1.

[26] Whether the round is about communication or voting might change the agents' action set. This set can also vary across agents and time (e.g., this is the case when, at a given round, some agents engage in private communication while others engage in public communication).

[27] Public announcement allows us to rule out situations where the agents use their vote as a communication device to aggregate information rather than preferences. As some agents may be reluctant to announce their preferred outcome, our model enables them to remain silent.

[28] For example, the agents might want to use a conventional language made of votes when the interaction pattern entails sequential voting before communication takes place. More specifically, an agent could provide her interpretation of a piece of TI to other agents in several rounds of public votes. Assume there are only eight possible interpretations $(\iota_1, \iota_2, ..., \iota_8)$, and $d_{set} = \{0,1\}^n$. By using a binary number

agent observes every vote outcome, regardless of possible anonymity, secret ballot or random draw.

2.4 Deliberative Game

The previous characterizations allow us to define a *deliberative game* as an extensive form game with imperfect information
$\Gamma =< N, \Theta, p, O, u, M, D, C, F >$. The game is common knowledge. How the game will be played depends on the elements of the underlying deliberative institution and its timing.

Deliberative Institution A *deliberative institution* is defined by the tuple (N, O, M, D, C, F) composed of: (i) a group of agents $N \in \mathcal{N}$; (ii) a set of outcomes $O \in \mathcal{O}$; (iii) a message space $M \in \mathcal{M}$; (iv) a decision rule $D \in \mathcal{D}$; (v) a continuation rule $C \in \mathcal{C}$; and (vi) an interaction pattern $F \in \mathcal{F}$.[29]

Timing Nature moves first by randomly choosing a profile of types $\theta \in \Theta$. Each agent i privately observes θ_i. The agents then deliberate according to the decision rule D, the continuation rule C, and the interaction pattern F. Once deliberation ends and an outcome $o \in O \cup \{\emptyset\}$ is selected, the agents receive the corresponding payoffs u associated to the realized profile of types θ.

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system, the agent could perfectly communicate her interpretation in three rounds of votes ($2^3 = 8$), by sequentially alternating her vote (e.g., by sequentially expressing the triple (d^1, d^2, d^3) that takes value $(0,0,0)$ to reveal ι_1, $(0,0,1)$ to reveal ι_2, $(0,1,0)$ to reveal ι_3,..., and $(1,1,1)$ to reveal ι_8, respectively). In reality, however, this kind of conventional procedure will rarely, if ever, be sufficiently fine-grained to communicate interpretations.

[29]Our modeling description realistically presupposes that there is no omniscient and omnipotent planner who can (i) select agents with sufficient knowledge expertise so avoid communication on interpretations (i.e., by including Θ and u into the tuple, with the outcome $o^*((\overrightarrow{\theta_j})_{j \in N})$ being both well-defined and matching $o^*((\overrightarrow{\theta_j}, \overset{\times}{\theta_j})_{j \in N})$ for any $\theta \in \Theta$); (ii) select agents with homogeneous preferences (i.e., by including u into the tuple, with $u_i = u_j$ for any pair $(i,j) \in N^2$); or (iii) compel agents to truthfully report their initial private information (i.e., by restricting $M^{\mathcal{T}}$ to $\{\overrightarrow{\theta_i}\}$ for any realized $\overrightarrow{\theta_i}$ at any round where agent i is asked to report his piece of TI).

3 Complete Deliberation

We move from the assumption that by constraining the agents' behavior, the specific rules of each deliberative institution may limit their ability to identify and select efficient outcomes. We thus proceed by formally establishing under what conditions the agents' epistemic pressure can be channeled into a behavior that enables them to aggregate sufficient information and take an efficient decision. In other words, we establish the conditions that *complete deliberation*. As specified below, there are two such conditions.

Freedom of reach When agents' information is transferable and no interpretation is required, the aggregation process can take place for a large set of interaction patterns. For example, any agent can be the collector of the other agents' private information. This does not apply when an agent has TI which interpretation requires other agents' specific NTI. In this case, the interaction pattern must not preclude the agent to report her information to those having the required NTI. Formally, we say that an interaction pattern F enables *agent i to report (directly or indirectly) her relevant information to agent j* whenever: (i) there is a finite positive integer $q \geqslant 2$ and a sequence of agents $(k_1, k_2, ..., k_q) \in N^q$ such that $(k_1, k_q) = (i, j)$; and (ii) agent k_r is allowed to send a message that includes i's report to k_{r+1}, with $1 \leq r \leq q-1$, and where direct reporting corresponds to the case $q = 2$.

Agent i's relevant information may include, of course, her own initial bit of TI, but also all interpreted TI received by other agents during deliberation. If there is no upper bound on the number of times that agent i can report to agent j, we say that *F routinely enables agent i to report her relevant information to agent j*. Finally, we say that the agents are endowed with *freedom of reach* when the latter condition is verified for any pair of agents $(i, j) \in N^2$, and denote $\widetilde{\mathcal{F}}$ as the subset of interaction patterns in $\mathcal{F}$ satisfying *freedom of reach.*[30,31]

[30]Using a graph theory approach, the communication paths between deliberative agents could be represented in the form of a graph where each agent is a node and freedom of reach corresponds to a connected graph.

[31]This condition is similar to that of "fair protocol" à la Parikh and Krasucki (1990) where no agent is blocked from communication.

Consensual Extension We require that deliberation ends endogenously. This means that there should not be obstacles for deliberation to continue when everyone wants to. Formally, let $\widetilde{\mathcal{C}}$ be the set of continuation rules in $\mathcal{C}$ under which deliberation continues if all the agents so agree (i.e., the rules satisfying $c_{rule}((\neg e, \neg e, ..., \neg e), .) = 0$).[32]

Note that not every continuation rule satisfies the above condition. For example, when deliberation ends as soon as the agents reach a formal agreement on a decision outcome, this condition is not satisfied. However, this "obstacle" can be overcome if $c_{rule}((\neg e, \neg e, ..., \neg e), 0) = 0 < c_{rule}((\neg e, \neg e, ..., \neg e), 1)$. In this case, endogenous deliberation extension is preserved when the agents strategically disagree on the decision outcome (see, Example 1). Yet, this strategic behavior is precluded when the voting rule in place is designed so to select a decision outcome independent of the agents' disagreement (e.g., plurality voting, supermajority voting with veto power, etc.).

When deliberation can continue as long as no agent opposes it, we say that deliberation allows *consensual extension*. Formally: (i) $c_{rule}((\neg e, \neg e, ..., \neg e), 0) = 0$ (so that deliberation continues when the agents unanimously want so and there is no agreement on the decision outcome); and (ii) if $c_{rule}((\neg e, \neg e, ..., \neg e), 1) > 0$ then there is a profile of votes $d^{\emptyset} \in d_{set}$ such that $d_{rule}(d^{\emptyset}) = \emptyset$ (meaning that if there is an obstacle for deliberation to continue when everyone so desires, i.e. $C \notin \widetilde{\mathcal{C}}$, the agents can strategically dissent on the decision outcome).

Consensual extension, then, connects decision and continuation rules. Formally, we let $\widetilde{\mathcal{DC}}$ denote the set of pairs of decision and continuation rules in $\mathcal{D} \times \mathcal{C}$ satisfying *consensual extension.* This condition can be seen as minimal in that it imposes nothing regarding the case where agents disagree on whether to end or continue deliberation.

Behavior and solution concept Let $\mathcal{B}$ denote the set of all possible agents' behaviors. An element of this set, B, is a profile of agents' behaviors made of voting, continuation, and communication strategies as well as procedures for updating beliefs

[32]Together with the conditions required by $\mathcal{C}$, the set $\widetilde{\mathcal{C}}$ includes all continuation rules leading to unanimous decisions on continuing or ending deliberation (i.e., all continuation rules satisfying $c_{rule}((e, e, ..., e), .) = 1$ and $c_{rule}((\neg e, \neg e, ..., \neg e), .) = 0$).

(see Appendix A for a formal definition). Finally, we use the ordinary Bayesian-Nash equilibrium, which we refer to as *equilibrium*, as the solution concept.[33]

3.1 Theorem

We now provide a necessary and sufficient condition for agents to achieve the common goal of efficient information aggregation, regardless of the deliberative institution governing their interaction, as long as they adopt an ad-hoc behavior.

Theorem. ***(Irrelevance of Deliberative Institutions)*** *Consider a deliberative institution* $(N, O, M, D, C, F) \in \mathcal{N} \times \mathcal{O} \times \mathcal{M} \times \mathcal{D} \times \mathcal{C} \times \mathcal{F}$. *For any resulting deliberative game* Γ *there is a behavior that always selects the efficient outcome if and only if the agents are enabled with* freedom of reach *and* consensual extension. *Formally,* $\forall(\Theta, p, u) \in \mathcal{T} \times \Delta(\Theta) \times \mathcal{U}$, $\exists B \in \mathcal{B}$ *such that* $o^*(\theta)$ *is selected* $\forall \theta \in \Theta$, *if and only if* $(D, C, F) \in \widetilde{\mathcal{DC}} \times \widetilde{\mathcal{F}}$. *Moreover, such a behavior is an equilibrium.*

Proof. See Appendix B. □

We provide here an intuitive account of the role that the theorem's conditions play in establishing our result. Since the deliberating agents are motivated by epistemic pressure, the conditions enable them to find a behavior that adapts to the constraints of a given deliberative institution, leading them to efficiently aggregate private information.[34]

Concerning *freedom of reach*, there are two cases. The first occurs when information aggregation does not require interpretation of NTI. In this case, *freedom of reach* allows the agents to transmit their information, either directly or indirectly, to whatever agent (*final collector*) who can herd the whole group to select the efficient decision outcome. The second case, instead, involves more complex deliberation dynamics, which

[33] The perfect Bayesian equilibrium solution concept can instead be used whenever this solution is well-defined (e.g., independent types, observed actions, and finite action spaces).

[34] In Appendix B, we formally characterize one possible behavior allowing the agents to overcome any possible constraint imposed by a deliberative institution. Informally, such a behavior provides that: (i) agents truthfully and exhaustively report their relevant information to the agents who are able to interpret such information; (ii) agents extend deliberation as long as no preferred outcome has been identified; and (iii) agents vote sincerely for the efficient outcome once it has been identified.

take place when interpretation through NTI is instrumental to efficient information aggregation. Here *freedom of reach* enables the agents to endogenously determine their real authority, as it facilitates an epistemic hierarchy where those with NTI sit at the upper layers. What counts for information aggregation is the interpretative role of expert agents. Thus, a possible dynamic in which the game could evolve is that "expert agents" behave as *final collectors* of the pieces of dispersed private information and give the group voting recommendations.[35] These real authority dynamics challenge the basic intuition that public deliberation, ideally with no time constraint, is sufficient for efficient information aggregation. Unless the agents with NTI are left free to interact, interpret, and share information with each other, public deliberation does not generally satisfy these requirements. In a small group of agents (e.g., juries, committees, corporate retreats), it is reasonable to assume that *freedom of reach* is satisfied through public speech, but this is not obvious in larger deliberative contexts (e.g., parliaments, shareholder meetings). Although public deliberation gives every agent the opportunity to send messages to the public, it does not generally enable each agent to iteratively communicate with selected agents, a requirement that is necessary for aggregating NTI.

Consensual extension allows the agents to *collectively* behave in equilibrium to exhaust their private information. Under this condition, on the one hand, the agents can *agree* to deliberate as long as there is no more relevant information to aggregate, independent of their agreement on a decision outcome. On the other hand, once an efficient outcome has been identified by at least one agent, the agents can *agree* to stop deliberation. Under rational expectations about equilibrium behaviors, an agent's willingness to stop communication informs the other agents that an efficient outcome has been identified, making further deliberation redundant. Remarkably, at any deliberation extension decision, the agents always behave in unison.

Consensual extension then challenges a second intuition: what matters for efficiency is not agreement on a decision outcome but agreement on communication extension. Indeed, only the agents' agreement on ending communication ensures that all the decision-relevant information has been aggregated. In this sense, agreement on

[35] Of course, once information has been correctly interpreted by agents with NTI and transmitted back to the rest of the group, the identity of final collector(s) – i.e., whether they have NTI or not – is irrelevant. In other words, issues concerning formal authority become irrelevant (Aghion and Tirole, 1997).

communication is purely epistemic, as it is independent of any formal authority specification (i.e., whether at the individual or group level) for extending communication.

Our theorem has other interesting implications concerning agents' behavior. Under *freedom of reach* and *consensual extension,* what one could interpret as agents' "bad behavior" is instead "good" because it is conducive to efficient deliberation. The intuition is straightforward. Once we conceive of a given deliberative institution as imposing a set of constraints on the deliberating agents, the agents may need to coordinate strategically to overcome these constraints. This is the case of, for example, a deliberative institution that imposes the agents to select a decision outcome as soon as they reach consensus on one. In such a case, the agents strategically coordinate to avoid consensus by *voting against their private information.* Similarly, as at some deliberation rounds *reporting information truthfully* might lead the group to take an inefficient decision outcome, the agents may find it profitable to misreport their private information. In such a case, misreporting is instrumental to stimulate epistemic pressure to continue deliberation (and, therefore, to enable the group to reach optimality).

The last part of our theorem states that any behavior that selects the efficient outcome can be supported at equilibrium. The intuition of our proof relies on the martingale property of the agents' rational beliefs. Whenever the agents identify a behavior leading to efficient decision-making but the decision-relevant information has not yet been aggregated, the agents may disagree at an interim stage about the decision outcome to be taken. However, knowing that their further interaction will eventually lead to an efficient decision, no interim disagreement provides the basis for a profitable deviation, unless such disagreement is instrumental to extend deliberation. But in this case, the agents would agree to disagree. Therefore, the prospect of selecting the efficient decision outcome ex post serves as an ex-ante commitment to stick with the behavior leading to efficiency.

Finally, and on a more speculative side, the conditions we have identified are compatible with a system of individual liberties: each agent has access to other agents, but no agent can be forced to communicate if she is not up to, as remaining silent is always an option. The reader can then picture these conditions as characterizing a form of ideal deliberation *à la* Rawls (Rawls, 2009). In such an abstract context, the details of the deliberative institution (i.e., the extensive form of the game), like in the Rawlsian world, are unspecified as they are not essential to efficient deliberation. In practice, however, unless the theorem's conditions are trivially satisfied (such as when information aggregation does not require specific NTI or, if it does, when the group of deliberating agents is small), meeting these conditions is often challenging. Our goal is then to establish a benchmark to assess the optimality of specific deliberative institutions.

4 Incomplete Deliberation

What happens when the theorem conditions do not hold? The answer to this question is that deliberation is *incomplete* and, at the level of generality, the optimality of selected outcomes is not guaranteed, as "anything goes." Outside the theorem, it is not assured that the agents can optimally aggregate their private information. Instead, whether they are able to take an efficient decision comes to depend on the details of the extensive-form game, including the game's underlying information structure. Under incomplete deliberation, the desirability of deliberative institutions thus becomes context specific.

That the theorem conditions do not hold can have several explanations. For example, a given deliberative institution may subject deliberation to time limits (e.g., board meetings) or rigid protocols of interaction (e.g., organizational hierarchies) where some agents are excluded from communication and (or) others are prevented from providing needed interpretations.

The theorem conditions can also be violated when the agents face frictions other than information and strategic constraints (i.e., transaction costs). Although our model does not explicitly formalize these frictions, it does not exclude that some costs may causally prevent the agents from communicating (i.e., their participation constraint is not satisfied), resulting in *freedom of reach* and *consensual extension* to be violated.

For concreteness, consider the case of institutional constraints that limit the agents' *freedom of reach*. This occurs, for example, when the interaction pattern mandatorily requires communication through a form of public speech that prevents the agents with NTI from sharing their interpretations with other agents. This is the case of the Brazilian Supreme Court, which formally excludes the possibility that the justices may react to one another's speech (e.g., by providing a complementary interpretation of what has been related by another justice) (Da Silva, 2013).[36]

Similarly, *freedom of reach* is violated when a decision-maker is required to take a decision in a limited time and cannot wait for the opinion of all experts, although the selection of the efficient outcome would require the aggregation of all experts' opinions.

There are also several cases where *consensual extension* is violated. For example, the rules of the UN Security Council typically require the member States' representatives to adopt resolutions with the widest possible agreement. However, when the state representatives reach unanimous consensus on a resolution, deliberation stops and no formal vote takes place. In this and similar cases where deliberation stops upon unanimous or large majoritarian consensus on a decision outcome,[37] agents may have incentives to behave strategically to the detriment of group decision-making if they anticipate an early communication closure (see Example 4.2).

In the ensuing discussion, we substantiate the "anything goes" claim by identifying information problems that can lead to either efficient or inefficient information aggregation depending on the given deliberative institution and the parameter specification.

Our focus is on information asymmetry problems, in the form of adverse selection. We characterize these issues by means of two numerical illustrations. In the first, we show that the violation of *freedom of reach* can make no communication to (strictly) Pareto-dominate partial or private communication. Depending on which agents are excluded from communication, information aggregation can lead to efficient or inefficient decision-making. Since the agents' set of types (i.e. Θ_i) is a specific detail of the game, whether a deliberative institution is Pareto superior then is context specific.

In the second example, we show that the violation of *consensual extension* induces the agents to untruthfully report their type. However, while under some parameters

[36]During the Court's plenary session, the justices are limited to reading aloud the reports they have drafted beforehand. By constitutional mandate, there cannot be any official or secret meeting among the justices before the Court's session. Instead, their reports are "simultaneously" and independently written. This interaction pattern excludes any possibility for dialogue (e.g., mutual interpretations) among the justices.

[37]Another example that violates consensual extension includes the election of the President of the Italian Republic. Under the Italian Constitution, a candidate is elected through a sequential secret ballot voting system when a two-thirds majority (or a simple majority after the third voting round) of voters converge on a candidate.

the lack of incentive compatibility leads the agents to an inefficient decision outcome, under other specifications the agents take the efficient decision outcome in spite of their untruthful revelation. This finding has important implications regarding the content of agents' communication. Our richer information structure allows us to characterize cases where reporting opinions (posterior beliefs), rather than primitive information, enables the agents to reach Pareto superior outcomes. This implication of incomplete communication challenges some well-known game-theoretic results in information aggregation. These results include, for example, Ostrovsky (2012) where traders are better able to determine the value of a security by pooling their initial information rather than (repeatedly and truthfully) communicating their posteriors (see also, in a more general context, Geanakoplos and Polemarchakis, 1982).

4.1 Example: Negative value of communication

Consider a group composed of two experts E_1 and E_2 and two decision-makers DM_1 and DM_2. Experts have private information that jointly exhausts the underlying state of affairs but do not have decision-making power. Formally, for $i \in \{1,2\}$, let Θ_{DM_i} and $d_{set}^{E_i}$ be singletons, $\Theta_{E_i} = \{\theta_i, \theta_i'\}$, $d_{set}^{DM_1} = \{T, B\}$, $d_{set}^{DM_2} = \{L, R\}$, $O = \{TL, TR, BL, BR\}$ and d_{rule} be the identity function over $d_{set}^{DM_1} \times d_{set}^{DM_2}$. Experts' types are independently drawn and equiprobable.

Players payoffs depend on four possible actions: top (T) or bottom (B), and left (L) or right (R), chosen by DM_1 and DM_2 respectively, and the four possible underlying state of affairs top (θ_1) or bottom (θ_1') row, and left (θ_2) or right (θ_2') column, observed by E_1 and E_2 respectively. DM_1 and DM_2's payoffs are determined by the following four matrices (with DM_1 being the row player and DM_2 the column player).[38] Experts' payoff is equal to the decision-makers average payoff.[39]

[38]For example, $u_i(BL; (\theta_1', \theta_2)) = 3$ (resp. $1, 2$) if $i = DM_1$ (resp. DM_2) .

[39]Other convex combinations of decision-makers' payoffs can equally work. What matters here is that experts and decision-makers' preferences are sufficiently aligned, thus giving experts incentives to truthfully report their private information.

		θ_2		θ'_2	
		L	R	L	R
θ_1	T	4;1.5	1;1	1;0	1.5;1.1
	B	0;1	0;0	0;0	0;1
θ'_1	T	0;1	0;-0.5	0;0	0;0.09
	B	3;1.2	1;0	0.5;-1	1;0.1

The efficient outcome then satisfies

$$o^*(\theta) = \begin{cases} (T, L) & \text{if} \quad \theta = (\theta_1, \theta_2) \\ (B, L) & \text{if} \quad \theta = (\theta'_1, \theta_2) \\ (T, R) & \text{if} \quad \theta = (\theta_1, \theta'_2) \\ (B, R) & \text{if} \quad \theta = (\theta'_1, \theta'_2) \end{cases}$$

Remarkably, the action T (resp. B) is DM_1's strictly dominant strategy under θ_1 (resp. θ'_1) and action L (resp. R) is DM_2's strictly dominant strategy under θ_2 (resp. θ'_2). Under complete information, decision-makers would then play these strategies leading to efficient outcomes with associated expected payoffs (2.375, 0.975).

Assume *consensual extension* and *freedom of reach* hold (i.e., $(D, C, F) \in \widetilde{\mathcal{DC}} \times \widetilde{\mathcal{F}}$). There are several equilibrium behaviors leading to an efficient outcome selection. In one of these, expert E_1 (resp. E_2) truthfully (directly or indirectly) reports her type to decision-maker DM_1 (resp. DM_2), and each decision maker plays the corresponding dominant strategy.

No *freedom of reach* Now suppose *freedom of reach* is violated (i.e., $F \notin \widetilde{\mathcal{F}}$) because expert E_2 and decision-maker DM_2 are both excluded from communication. Expert E_1 sharing his information with decision-maker DM_1 is detrimental to all players.

To see this, observe that without communication, the decision-makers would play the following (average) game, which has a unique equilibrium in strictly dominant strategies (T, L) with associated expected payoff $(1.25, 0.625)$.

	L	R
T	1.25 ; 0.625	0.625 ; 0.4225
B	0.875 ; 0.3	0.5 ; 0.275

Instead, when E_1 truthfully communicates with DM_1, the latter learns whether the actual matrix is at the top (θ_1) or at the bottom (θ_1') row.[40] This reporting creates asymmetric information between the two decision makers. Conjecturing that DM_1 will play his strictly dominant strategy, DM_2 plays the following "expected" game

L	R
$2.125, 0.425$	$1.125, 0.55$

with R being her best response. The corresponding pair of expected payoffs $(1.125, 0.55)$ is, however, (strictly) Pareto-dominated by the one achievable under no communication. Hence, no communication (either imposed at the institutional level or achieved by strategic players in a pooling equilibrium) is preferable to partial (truthful) communication.

Remark 1: Positive value of partial communication It is important to emphasize that we cannot conclude anything about the desirability of an institution that forbids partial (or private) communication. Although the above example shows that no communication leads the agents to a superior outcome payoff, there are counterexamples. To see this, suppose *freedom of reach* is violated (i.e., $F \notin \widetilde{\mathcal{F}}$), but this time expert E_1 and decision-maker DM_1 are excluded from communication. That expert E_2 shares her information with decision-maker DM_2 does improve welfare can be easily checked as the corresponding expected payoffs (strictly) Pareto-dominate those achievable under no communication (see Appendix C).

We now turn to the second example and show how the information aggregation process is jeopardized when agents behave strategically outside the theorem conditions.

[40] The sustainability of truthful reporting at equilibrium is addressed in Appendix C.

4.2 Example: Conflict of interest, misreporting, and opinions

The following example considers adverse selection determined by lack of *consensual extension*.[41] We show that even when agents have homogeneous preferences they may find it profitable to deviate from truthful reporting at the cost of inefficient decision-making.

A decision maker (DM) consults two agents, a quant (Q), and an expert (E), before taking a decision which efficiency depends on an unknown state of Nature. The agents have the same preferences but asymmetric information (TI) and different language skills (NTI). DM speaks English, Q speaks Chinese, and E speaks both English and Chinese. Q receives information in the form of three messages written in Chinese, but the third is also available in English. Nature signals to Q what message to read in accordance with a probability distribution correlated with the realized state. This distribution, as well as the agents' skills, are common knowledge. DM can understand the third message's information content when directly transmitted by Q. This does not happen when Q sends the first or the second message to DM, because DM needs E's NTI to understand it.

When DM has to take a decision after only one round of communication, there is no time for E to intermediate (i.e., to translate) the information from Q to DM. The example shows that, under some parameters, Q finds it profitable to send the third message to DM rather than the actual message that Nature has signaled. This reporting induces DM to choose the outcome that Q thinks is efficient. Such deviation from truthful reporting leads the group to select either an efficient or an inefficient outcome, depending on the parameters. We start with the second case.

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Before implementing an outcome $o \in \{0, 1\}$, the (partially informed) decision maker (DM), the quant (Q), and the expert (E), deliberate according to the interaction pattern described in Fig. 3. What matters here is that Q can report either directly to DM through edge 1, or indirectly with the help of E through edges 2 and 3; and E can report directly to DM through edge 3.

[41] The adverse selection problem we explore in this example can also arise when freedom of reach does not hold. We illustrate this possibility in Appendix C.

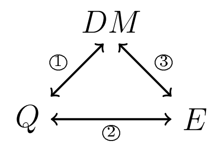

Figure 3: *Freedom of reach*

Under complete information, utility is maximized when DM chooses o matching the unknown and equiprobable state of Nature $\omega \in \{0,1\}$. Matching the decision outcome with the state gives the agents a payoff normalized to zero. When the group wrongly selects $o = 1$ (resp. $o = 0$) when $\omega = 0$ (resp. $\omega = 1$), the agents obtain the negative payoff $-\frac{1}{2}$. Each agent $i \in \{DM, Q, E\}$ privately observes a signal $\theta_i \in \Theta_i$ correlated with state ω. Formally, the agents' utility function writes as $u : \{0,1\} \times \Theta \longmapsto [-\frac{1}{2}; 0]$ with $u(o,\theta) = -\frac{1}{2}\mathbb{P}\left(\omega \neq o \,|\theta\right)$.[42]

Each set Θ_i takes the form of a dataset $\overrightarrow{\Theta_i}$ made of transferable information (TI), i.e. $\overrightarrow{\theta_i}$, revealing information about ω, and non-transferable information (NTI), i.e. $\overset{\times}{\Theta}_i$, enabling agent i to interpret TI. Conditional on the realization of ω, each piece of TI is independently distributed among the agents according to the following information structure.

E can either directly observe the state (ω) or a non-informative signal ($\emptyset$). Formally, $\overrightarrow{\Theta_E} = \{0,1,\emptyset\}$, with $\mathbb{P}\left(\overrightarrow{\theta_E} = \omega \,\middle|\, \overrightarrow{\theta_E} \in \{0,1\}, \omega\right) = 1$. E's TI does not need to be interpreted and is distributed as reported in Table 3, which shows, for each possible $\overrightarrow{\theta_E}$: (i) the unconditional probability to observe $\overrightarrow{\theta_E}$, i.e. $\mathbb{P}\left(\overrightarrow{\theta_E}\right)$; (ii) the probability distribution to observe $\overrightarrow{\theta_E}$ conditional on the realization of each state of nature, i.e. $\mathbb{P}\left(\overrightarrow{\theta_E}\,|\omega\right)$ for any $\omega \in \{0,1\}$; and (iii) the posterior probability that $\omega = 1$ conditional on observing $\overrightarrow{\theta_E}$.

[42] This setup replicates the preferences of the Condorcet jury model, with $\frac{1}{2}$ being the threshold of reasonable doubt.

$\overrightarrow{\theta_E}$	$\mathbb{P}\left(\overrightarrow{\theta_E}\right)$	$\mathbb{P}\left(\overrightarrow{\theta_E}\,\vert\omega=0\right)$	$\mathbb{P}\left(\overrightarrow{\theta_E}\,\vert\omega=1\right)$	$\mathbb{P}\left(\omega=1\,\middle\vert\overrightarrow{\theta_E}\right)$
0	$\frac{1}{3}$	$\frac{2}{3}$	0	0
1	$\frac{1}{3}$	0	$\frac{2}{3}$	1
$\emptyset$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$

Table 3: Signals distribution and posterior beliefs of E. This table is common knowledge of *all* agents.

DM, instead, may observe two different pieces of TI from $\overrightarrow{\Theta_{DM}} = \{\overrightarrow{1_{DM}}, \overrightarrow{2_{DM}}\}$ that are distributed as reported in Table 4.

$\overrightarrow{\theta_{DM}}$	$\mathbb{P}\left(\overrightarrow{\theta_{DM}}\right)$	$\mathbb{P}\left(\overrightarrow{\theta_{DM}}\,\vert\omega=0\right)$	$\mathbb{P}\left(\overrightarrow{\theta_{DM}}\,\vert\omega=1\right)$	$\mathbb{P}\left(\omega=1\,\middle\vert\overrightarrow{\theta_{DM}}\right)$
$\overrightarrow{1_{DM}}$	0.5	0.7	0.3	0.3
$\overrightarrow{2_{DM}}$	0.5	0.3	0.7	0.7

Table 4: Signals distribution and posterior beliefs of DM. This table is common knowledge of *all* agents.

Q may observe three different pieces of TI from $\overrightarrow{\Theta_Q} = \{\overrightarrow{1_Q}, \overrightarrow{2_Q}, \overrightarrow{3_Q}\}$, that are distributed as reported in Table 5, which shows, for each possible $\overrightarrow{\theta_Q}$: (i) the unconditional probability to observe $\overrightarrow{\theta_Q}$, i.e. $\mathbb{P}\left(\overrightarrow{\theta_Q}\right)$; (ii) the probabilities to observe $\overrightarrow{\theta_Q}$ conditional on the realization of each state of Nature, i.e. $\mathbb{P}\left(\overrightarrow{\theta_Q}\,\vert\omega\right)$ for any $\omega \in \{0,1\}$; and (iii) the posterior probability that $\omega = 1$ conditional on observing $\overrightarrow{\theta_Q}$.

$\overrightarrow{\theta_Q}$	$\mathbb{P}\left(\overrightarrow{\theta_Q}\right)$	$\mathbb{P}\left(\overrightarrow{\theta_Q}\,\vert\omega=0\right)$	$\mathbb{P}\left(\overrightarrow{\theta_Q}\,\vert\omega=1\right)$	$\mathbb{P}\left(\omega=1\,\middle\vert\overrightarrow{\theta_Q}\right)$
$\overrightarrow{1_Q}$	**0.35**	0.69	0.01	$\frac{1}{70}$
$\overrightarrow{2_Q}$	**0.35**	0.30	0.40	$\frac{4}{7}$
$\overrightarrow{3_Q}$	**0.30**	**0.01**	**0.59**	$\frac{59}{60}$

Table 5: Signals distribution and posterior beliefs of agent Q. This table is common knowledge of Q and E. DM only knows the probabilities in bold, but does not know whether the rows of the data in gray are presented in this order or in a reverse order (i.e., whether $\mathbb{P}\left(\omega=1\,\middle\vert\overrightarrow{1_Q}\right)$ (resp. $\mathbb{P}\left(\omega=1\,\middle\vert\overrightarrow{2_Q}\right)$) is $\frac{1}{70}$ or $\frac{4}{7}$). Each possibility is assumed to be equally likely.

While the realization of TI is private information for each agent, each agent i's set of TI ($\overrightarrow{\Theta_i}$) and NTI ($\overset{\times}{\Theta}_i$), as well as the probability distribution of types ($\mathbb{P}\left(\overrightarrow{\theta_i}\right)$) are common knowledge. Observe that Q's preferred outcome at this interim stage can be deduced from the posterior belief of the last column: $o = 1$ if $\mathbb{P}\left(\omega = 1 \middle| \overrightarrow{\theta_Q}\right) > \frac{1}{2}$ and $o = 0$ otherwise. Table 5 is common knowledge of Q and E. Both agents possess NTI to interpret Q's TI.

The key assumption of this example is that DM does not have the NTI to perfectly distinguish $\overrightarrow{1_Q}$ from $\overrightarrow{2_Q}$. While she can recognize the first two columns (labels of $\overrightarrow{\theta_Q}$ and the corresponding unconditional probabilities $\mathbb{P}\left(\overrightarrow{\theta_Q}\right)$), she is unable to recognize whether the remaining columns of the rows associated to $\overrightarrow{1_Q}$ and $\overrightarrow{2_Q}$ are presented in the displayed or reversed order. Specifically, we assume that such ordering options are equiprobable.

That E knows all tables implies she is able to "interpret" any Q's signal for DM (e.g., by reporting to DM which data in gray are realized). We assume that the interpretation is possible only when E has received Q's message.[43]

With *consensual extension*, private information can be efficiently aggregated within two rounds of interaction. In the first round, E reports his signal to DM and Q reports his signal to E; and in the second round, E, having both Q and DM's NTI, interprets Q's message for DM. After the second round, whether DM has learned the realized state from E's directly at the first round or has updated her information from Q's interpreted information transmitted by E at the second round, she has all the relevant information to take $o^*(\theta_{DM}, \theta_Q, \theta_E)$ (see Appendix C).

No *consensual extension* Truthful reporting may no longer be an equilibrium strategy without *consensual extension*. To see this, suppose that the interaction pattern allows the agents to interact only for one round, implying that $(D, C) \notin \widetilde{\mathcal{DC}})$.

[43] This assumption prevents E from transferring the interpretation of all three signals to DM before Q speaks and in just one round of communication (thus excluding that DM may directly understand Q's message). This is congruent with many practical contexts (e.g., although an English-Chinese speaker can immediately translate any sentence from one language to another, teaching a language would require much longer time). Alternatively, this assumption could be justified by increasing the number of signals in the example so that the required upfront communication between E and DM on Q's message would become unrealistic (given the excessive number of signals), which we do not do for brevity.

Assume the agents report truthfully and their TI profile is $\overrightarrow{\theta_E} = \emptyset$, $\overrightarrow{\theta_Q} = \overrightarrow{2_Q}$, and $\overrightarrow{\theta_{DM}} = \overrightarrow{1_{DM}}$. From

$$\mathbb{P}\left(\omega = 1 \middle| \overrightarrow{\theta} = (\emptyset, \overrightarrow{2_Q}, \overrightarrow{1_{DM}})\right) = \frac{\mathbb{P}(\emptyset | \omega = 1)\, \mathbb{P}\left(\overrightarrow{2_Q} | \omega = 1\right) \mathbb{P}\left(\overrightarrow{1_{DM}} | \omega = 1\right)}{\sum_{\omega \in \{0,1\}} \mathbb{P}(\emptyset | \omega)\, \mathbb{P}\left(\overrightarrow{2_Q} | \omega\right) \mathbb{P}\left(\overrightarrow{1_{DM}} | \omega\right)} = \frac{4}{11} < \frac{1}{2}$$

we have $o^*(\theta_{DM}, \theta_Q, \theta_E) = 0$. With only one round of (simultaneous) communication, Q is constrained to report directly to DM. Q's report is pivotal for the final decision only when DM has received $\overrightarrow{\theta_E} = \emptyset$. Because Q believes that $o = 1$ is the efficient decision, it can be verified (see Appendix C) that she finds profitable to report $m_Q = \overrightarrow{3_Q}$ rather than the true signal $m_Q = \overrightarrow{2_Q}$ in order to induce DM to implement the outcome $o = 1$. Such a deviation, however, induces DM to take the inefficient decision outcome $o = 1$.

Remark 1: ***Consensual extension vs. Freedom of reach***

A similar example could be obtained where *consensual extension* holds but *freedom of reach* does not. This would simply require that Q is unable to report to DM through E. For the same reasons discussed above, also in this case Q would find it profitable to misreport her private information (see also Appendix C).

Remark 2: ***Efficient misreporting*** The previous example shows that without *consensual extension,* Q misreports and, in turn, DM implements the inefficient decision. The selection of this inefficient outcome, however, depends on the agents' TI distribution. Once we assume a different information structure and types' realization, we can show that Q's misreporting actually brings about an efficient decision outcome that would not be reached under truthful reporting (see Appendix C).

Remark 3: ***Reporting opinions*** When the theorem conditions do not hold, reporting posterior beliefs (*opinions*) rather than initial signals and interpretations (that together constitute *facts*) can actually allow the agents to better aggregate their information. Indeed, while DM does not understand Q's messages communicated in Chinese, she can understand Q's posterior belief (expressed in an intelligible number) and aggregate all decision-relevant information. Therefore, Q could actually report to DM her posterior (i.e., $\mathbb{P}\left(\omega = 1 \middle| \overrightarrow{2_Q}\right) = \frac{4}{7}$), which would lead DM to take the efficient decision outcome (see Table 6 in Appendix C). Although in this case communicating the posterior belief leads to an efficient decision outcome, this is not always true. Counterexamples exist where posterior beliefs convey coarser information, as different facts can lead an agent to form the same posterior (i.e., the function that maps signals in updated beliefs is non-injective).

Remark 4: ***Order of interaction*** When *freedom of reach* does not hold, the order of interaction may also affect efficient information aggregation (see Ottaviani and Sørensen, 2001; Van Weelden et al., 2008). Suppose the agents communicate according to a different interaction pattern, such as that in Fig. 4, where E can communicate with Q (resp. DM) but not vice-versa. Under this pattern, E can no longer interpret Q's information for DM. In this case, Q will still find it profitable to misreport $m_O = \overrightarrow{3}_Q$ to DM, inducing the (inefficient) implementation of $o = 1$. On the other hand, if the position of Q and E was swapped, an efficient decision would be taken.

$$\begin{tikzcd} & DM & \\ Q \arrow[ur, "1"] & & E \arrow[ul, "3"'] \arrow[ll, "2"] \end{tikzcd}$$

Figure 4: Absence of freedom of reach.

5 Connections to the literature

With incomplete deliberation, the efficiency of deliberative institutions depends on both the specific institutional contexts and other modeling details. With this in mind (i.e., that *anything goes*), we now discuss the implications of our research for the related literature.

Deliberating with frictions Our paper is related to a growing literature that focuses on complex forms of deliberation. In these studies, imperfect communication may take several forms such as noisy communication (e.g. Koessler, 2001; Blume et al., 2007; Tsakas and Tsakas, 2019), coarse information partitions (e.g. Battaglini, 2002; Meyer-ter Vehn et al., 2018; Carroll and Egorov, 2019), or language barriers (Blume and Board, 2013; Giovannoni and Xiong, 2019).

In particular, Blume and Board (2013) is closely related to our analysis. Considering a class of common-interest games with a communication stage followed by a decision stage, they show that efficiency losses may arise when agents face "language barriers" (i.e., they are uncertain about the others' language competence). However, the agents may improve efficiency by using messages with "indeterminate meaning" *à la* Lewis (1969). These messages under-specify the states of the world, in the sense that they are indicative of more states (including the realized one). As agents know that these messages are commonly available, they may want to use them to coordinate their actions. This, however, involves additional uncertainty as the agents do not know whether the messages are used for coordination or information purposes.

Our paper complements Blume and Board (2013)'s analysis. In our setup, agents have no uncertainty about their relevant competences or the purpose of their messages. Instead, they face "communication barriers" due to deliberation frictions in their dynamic interaction. To aggregate their decision-relevant information (i.e., TI), agents need to engage in dynamic interpretations requiring specific competence (i.e., NTI). In this context, we find that reporting posteriors - messages that are known to be understood by the receiver - rather than the initial signals and interpretations may improve the efficiency of deliberation.

Voting Our paper also adds to the literature on pre-vote deliberation, which is instrumental for information aggregation (e.g. Austen-Smith, 1990; Coughlan, 2000; Guarnaschelli et al., 2000; Austen-Smith and Feddersen, 2006; Gerardi and Yariv, 2007; Strulovici, 2010; Goeree and Yariv, 2011; Calcagno et al., 2014) In this literature, pre-vote deliberation can solve issues of coordination and lead the group to efficient decision-making under the assumption of perfect communication. In our model, we depart from this assumption and increase the signal space, thus allowing agents to interact along several possible patterns. Our results show that when agents have heterogeneous NTI and their information cannot be trivially aggregated, efficiency is assured only when the theorem conditions hold.

Our analysis also complements Chan et al. (2018). They investigate a deliberation game with agents having both heterogeneous preferences and discount factors but no private information. Their paper shows that voting rules matter as the accuracy of decisions increases in the agents' quorum to approve a decision. However, this comes with a tradeoff. Impatient agents may vote against their preferred outcome to *shorten* deliberation and therefore reduce information acquisition.

On the contrary, our theorem proves an irrelevance result of voting rules in a large class of deliberation games (that includes endogenous interaction patterns) with agents having private information and heterogeneous knowledge, but with no discounting and less preference heterogeneity. When the deliberative institutions allow *consensual extension*, we find that agents may have the incentives to strategically vote against their preferred outcome to *extend* deliberation to aggregate more information.

Our results, however, do not exclude that the quorum to approve a decision can play a role in practice. Indeed, when *freedom of reach* holds, voting mechanisms enabling *consensual extension* (e.g., jury unanimity voting) are in practice superior to voting rules that prevent strategic dissent (e.g., majority voting). Under these latter rules, to enable *consensual extension* in a context where repeated balloting is excluded, the agents should be able to indefinitely postpone a decision vote.

Finally, Chan et al. (2018) show that their set of equilibrium outcomes is also achievable in a modified version of their basic setup where decisions to implement an outcome and to end deliberation are separable and governed by different voting rules. This separation is germane to our model, and we find that it may help to guarantee *consensual extension.*

Communication in networks Our theorem further relates to the literature on strategic communication in networks (e.g. Bala and Goyal, 2000; Hagenbach and Koessler, 2010; Acemoglu and Ozdaglar, 2011; Acemoglu et al., 2011; Galeotti et al., 2013). These papers typically focus on the conditions required for a network to induce asymptotic learning (i.e. information aggregation). In these contexts, *freedom of reach* is a sufficient condition for any given network to enable the agents to aggregate their private information, in this sense resembling the "fair protocol" of Parikh and Krasucki (1990) where no agent is blocked from communication. Conversely, a context without *freedom of reach* is, for example, that of online social networks, where agents are separated into "echo-chambers" that prevent them from interacting (e.g. Del Vicario et al., 2016). Along similar lines, a very recent contribution by Mossel et al. (2020) focuses on social learning communication games and proves an equivalence theorem between herding and information aggregation. The result uses an *ad-hoc* concept of *social learning equilibrium*, which abstracts away from extensive form dynamics of the strategic interaction and focuses on the limit properties of the equilibrium to which the game converges. Conversely, we focus on contexts where agents have payoff externalities and we explicitly characterize the properties of the deliberative institution governing the dynamics of group interaction to reach efficiency.

Organizations Our paper also offers some insights related to organization theory (e.g. Garicano, 2000; Cremer et al., 2007; Chen and Suen, 2019), where agents face a trade-off between the cost of opening additional communication channels and the benefit of information acquisition. See Williamson (1967) and Calvo and Wellisz (1978) for a classic description of the problem. More specifically, *freedom of reach* requires agents to be able to convey their private information to other parts of (i.e., agents in) the organization. While this is trivially satisfied in horizontal communication contexts such as committees and juries, communication might be a challenge in hierarchical communication contexts such as firms. This requirement also relates to several important economic problems such as the organization of the firm's internal communication protocols (Bolton and Dewatripont, 1994; Dewatripont and Tirole, 2005) or the allocation of authority within organizations (Aghion and Tirole, 1997; Calvó-Armengol et al., 2015).

Political theory Finally, our work contributes to the political theory debate on the Aristotelian doctrine of the wisdom of the multitude (Aristotle, 2017; Waldron, 1995). To some, the doctrine provides a normative justification for direct democracy (Kraut et al., 2002; Landemore, 2017; Ober, 2008; Wilson, 2011). This is because a collective decision-making process involving deliberation among diverse participants would have a better-than-random chance of reaching correct outcomes (e.g. Estlund, 2009). Our model put the doctrine into a different and more limited perspective: sharing the same conception of the common good (e.g. MacIntyre, 1988; Ober, 2013) is not always sufficient for direct democracies to take efficient decisions.

6 Concluding remarks

Whether efficient collective decisions are the result of the quality of institutions or individuals' behavior is the subject of a long-standing debate (e.g. Aristotle, 2017; Gaus, 2006; Rawls, 2009; Rousseau, 2018). Our analysis sheds some light on ideal deliberation, using a general model of dynamic and strategic group interaction. Our theorem's conditions requires that deliberative institutions be designed so as to enable individuals to both continue deliberation (i.e., *consensual extension*) and interact with one another (i.e., *freedom of reach*). Under these conditions, what matters for the efficiency of collective decision-making is individuals' behavior and not the details of a deliberative institution. Conversely, for each deliberative institution violating these conditions, we can find an information structure (TI and NTI distribution) such that no behavior allows for efficient decision-making.

We emphasize this inefficiency through specific examples, which help unveiling other findings. We illustrate how (ex-interim) conflicts of interest may lead one or more agents with (ex-ante) homogeneous preferences to behave in a way that is (ex-post) inefficient. Conflict of interest may arise, as in Example 4.2, because two agents observe different signals that make them prefer different outcomes at the interim stage. The same conflict may also arise when the agents observe the same signal (TI) but have different expertise (NTI), so that their interpretations differ. We could also hypothesize that inefficient misreporting may result when the agents prefer the same outcome but have different beliefs about what the others prefer. Ultimately, what are the conditions under which homogeneous preferences guarantee full information sharing at equilibrium is still an open question.

A general theory cannot answer the plethora of questions arising from specific modeling assumptions. For example, we could imagine an extended version of our paper where the distribution of fields of expertise related to NTI is private information. Intuitively, in a common interest game, agents should share this information in a pre-"complete deliberation" game. However, if the agents are not sure that the group has sufficient expertise to correctly interpret this information, ex-interim information asymmetry may distort individual incentives and lead one or more agents to behave opportunistically (e.g., by claiming to have more expertise than they possess), to the detriment of the group. This may be the case when the interaction pattern does not allow the agents to immediately identify who are the agents with the most relevant NTI to interpret TI (as in a large network). Would efficient information aggregation still be possible in this case?

Another open question refers to the possibility of introducing a cost associated to *freedom of reach* and *consensual extension,* to reflect the idea that the larger the group, the more difficult it is to meet these conditions. In this case, there would then be an optimal composition of the deliberating group with a minimum number of experts selected according to the relevance of their NTI expertise that would balance the above costs. However, picturing what would be the deliberative institution leading to second-best efficiency is not obvious. Moreover, how would the answer to these questions change if we allowed agents to have an idiosyncratic bias for one decision alternative? These and other questions cannot be solved within the framework of this paper and remain open. Answering them requires analysis going far beyond the scope of this study.

Finally, we tentatively suggest that the theory of deliberation could relax the Pareto criterion to provide more pragmatic policy implications. This would be similar to what has happened in the theory of incomplete markets, which substituted Pareto efficiency with the less demanding criterion of constrained-Pareto efficiency (e.g. Magill and Quinzii, 1996). For example, a possible direction for future research could consider less demanding efficiency criteria (e.g., ϵ-efficiency) that attempt to internalize the myriad of frictions affecting agents' deliberation. Among other benefits, this approach would be valuable to develop an explicit tradeoff analysis and provide better guidance for institutional design.

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A (Online) Appendix : Definitions

Fictitious player Let Nature, a non-strategic player, select the realization of any random draw. The set of all players (both strategic and non-strategic) writes as $N \cup \{Nature\}$.

Histories We call *histories* the (possibly infinite) collection of partially ordered sequences of moves and we let $\mathcal{H}$ denote this collection. To model uncertainty, we allow for Nature to select actions according to some probability distribution.[44] Any history $h \in \mathcal{H}$ contains the actions previously played, which include agents' past moves as well as any past realization of random draws.

The set $\mathcal{H}$ satisfies the three following standard assumptions. First, $\emptyset \in \mathcal{H}$ (with the interpretation that the empty history is the starting point of the game). Second, if $h \in \mathcal{H}$, then any sub-history $h' \subseteq h$ belongs to $\mathcal{H}$. Third, if an infinite sequence $h_\infty = (a_k)_{k=1}^{\infty}$ of (one-dimensional) actions a_k satisfies $(a_1, a_2, ..., a_q) \in \mathcal{H}$ for every positive integer q, then $h_\infty \in \mathcal{H}$. We say that the sequence $(a_1, a_2, ..., a_q)$ is a history of length q.

A history $(a_1, a_2, ..., a_q) \in \mathcal{H}$ after which no action is taken is *terminal*. Formally, a terminal history is infinite or there is no action a_{q+1} such that $(a_1, a_2, ..., a_{q+1}) \in \mathcal{H}$. A history becomes terminal only after an action to end (e) has been selected according to the continuation rule C. Let $\overline{\mathcal{H}}$ denote the set of terminal histories.

For any $h \in \mathcal{H}$, let h_{-a} denote the sequence of moves in h other than action a (in case there is no such action a, let $h_{-a} \equiv h$; otherwise $h = (h_{-a}, a)$). More generally, any history h can be decomposed in two sub-histories h' and h'' so that $h = (h', h'')$, with the convention that: (i) the order of the elements is preserved (e.g., $h = (x, y, z)$ can be decomposed in $h' = (x, ., z)$ and $h'' = (., y, .)$); and (ii) $h = h'$ implies $h'' = \emptyset$.

Decision and Continuation Rules as functions of histories When the set of agents' available votes are heterogeneous, we write $d_{set} \equiv \prod_{i \in \mathcal{N}} d_{set,i}$.

The order in which the agents play depends on the interaction pattern (see definition below). It may be the case that agents vote sequentially but not contiguously (e.g., one after another, each agent gives a public speech before placing her ballot in an envelope). Consequently, we refine the definition of the aggregation rule $d_{rule} : d_{set} \longmapsto \Delta(O \cup \{\emptyset\})$ with the composite function $d_{rule} \circ d_{\mathcal{H}} : \mathcal{H} \longmapsto \Delta(O \cup \{\emptyset\})$ where $d_{\mathcal{H}} : \mathcal{H} \longmapsto d_{set} \cup \{\emptyset\}$ maps any history $h \in \mathcal{H}$ in the last profile of votes $(d_i)_{i \in N} \subseteq h$ that has not yet been used to select an outcome. We adopt the convention that $d_{\mathcal{H}}(h) = \emptyset$ whenever such a profile does not exist (so that d_{rule} is not defined, with

[44]For example, the profile of types $\theta \in \Theta$ (resp. outcome $o \in O \cup \{\emptyset\}$, ending decision (e)) is selected according to p (resp., $v_{rule}(.)$, $c_{rule}(.)$).

the interpretation that no outcome can be selected as long as one of the required votes is missing). As specified in the model, we assume that the aggregation rule does not vary through deliberation (time or history). Formally, $(d_{rule} \circ d_{\mathcal{H}})(h) = (d_{rule} \circ d_{\mathcal{H}})(h')$ for any pair $(h, h') \in \mathcal{H}^2$ satisfying $d_{\mathcal{H}}(h) = d_{\mathcal{H}}(h')$.

In the same vein, we refine the definition of the aggregation rule $c_{rule} : c_{set} \times \{0,1\} \longmapsto [0,1]$ with the composite function $c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}) : \mathcal{H} \longmapsto [0,1]$ where $\mathbb{I}_{\mathcal{H},O} : \mathcal{H} \longmapsto \{0,1\}$ denote the indicator function, which equals 1 when an outcome in O is selected under history $h \in \mathcal{H}$ if deliberation stops immediately, and 0 otherwise, and $c_{\mathcal{H}} : \mathcal{H} \longmapsto c_{set} \cup \{\emptyset\}$ maps any history $h \in \mathcal{H}$ in the last profile of choices $(c_i)_{i \in N} \subseteq h$ that has not yet been used for a continuation decision. We adopt the convention that: $c_{\mathcal{H}}(h) = \emptyset$ whenever such a profile does not exist; and $(c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h) = (c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h')$ for any pair $(h, h') \in \mathcal{H}^2$ satisfying $c_{\mathcal{H}}(h) = c_{\mathcal{H}}(h')$ and $\mathbb{I}_{\mathcal{H},O}(h) = \mathbb{I}_{\mathcal{H},O}(h')$.

Any continuation rule C that belongs to the set $\mathcal{C}$ satisfies the following three assumptions:

(i) $\forall h \in \mathcal{H}$ such that $c_{\mathcal{H}}(h) = (e, e, ..., e)$ we have $(c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h) = 1$;

(ii) $\forall (h, h') \in \mathcal{H}^2$ satisfying $c_{\mathcal{H}}(h) = c_{\mathcal{H}}(h') \neq \emptyset$, if $\mathbb{I}_{\mathcal{H},O}(h) \leq \mathbb{I}_{\mathcal{H},O}(h')$ then $(c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h) \leq (c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h')$; and

(iii) $\forall (h, h') \in \mathcal{H}^2$ if $c_{\mathcal{H}}(h) = (\neg e, \neg e, ..., \neg e)$, $c_{\mathcal{H}}(h') \neq \emptyset$, and $\mathbb{I}_{\mathcal{H},O}(h) \leq \mathbb{I}_{\mathcal{H},O}(h')$ then $(c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h) \leq (c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h')$.

In the following, we will still refer to d_{rule} and c_{rule} when addressing the properties of these two aggregation rules.

Consensual extension The set $\widetilde{\mathcal{DC}}$ consists in the pairs of (decision and continuation) rules in $\mathcal{D} \times \mathcal{C}$ satisfying:

(i) $\forall h \in \mathcal{H}$ such that $c_{\mathcal{H}}(h) = (\neg e, \neg e, ..., \neg e)$ and $\mathbb{I}_{\mathcal{H},O}(h) = 0$, we have $(c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h) = 0$; and

(ii) If $(c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h) > 0$ when $c_{\mathcal{H}}(h) = (\neg e, \neg e, ..., \neg e)$ and $\mathbb{I}_{\mathcal{H},O}(h) = 1$, then there is $d^{\varnothing} \in d_{set}$ such that $d_{rule}(d^{\varnothing}) = \varnothing$.

Information Partition Deliberation allows agents to update their beliefs about the realized profile of types $\theta \in \Theta$, and to use strategies as functions of past observed actions. Imperfect information implies that agent i does not observe all the elements composing histories (e.g., at the beginning of the interaction, she is uncertain about the realization of the profile of types θ). Let $\mathcal{I}_i$ denote agent i's *information partition* (i.e., a partition of histories at which agent i is supposed to play and that are indistinguishable to her). Let $I_i \in \mathcal{I}_i$ denote agent i's *information set* (i.e., a set of histories, at which agent i is supposed to play, where agent i is informed that some history $h \in I_i$ has occurred but is not informed about the history that has occurred). Congruently, the set of actions available to agent i is the same for any two non-terminal histories that she is unable to distinguish. Formally, for any pair $(h, h') \in I_i^2$, we assume $A(h) = A(h')$, where $A(h)$ denote the set of actions a available after the non-terminal history h (i.e., such that $(h, a) \in \mathcal{H}$). For $I_i \in \mathcal{I}_i$ we denote by $A(I_i)$ the set $A(h)$ for any history $h \in I_i$.

Interaction pattern An *interaction pattern* F is a triple $(\mathcal{H}, P, (\mathcal{I}_i)_{i \in N})$ where: $\mathcal{H}$ is the set of possible histories; P is a *player function* from $\mathcal{H}$ to $N \cup \{Nature\}$ that assigns to every non-terminal history $h \in \mathcal{H} \backslash \overline{\mathcal{H}}$, a player $P(h)$ who takes an action after the history h;[45] and $(\mathcal{I}_i)_{i \in N}$ denotes the collection of player i's information partition. For $I_i \in \mathcal{I}_i$ we denote by $P(I_i)$ the player $P(h)$ for any history $h \in I_i$.

Fix a set of players $N \cup \{Nature\}$ and a tuple (Θ, O, D, C, u, M) in $\mathscr{T} \times \mathcal{O} \times \mathcal{D} \times \mathcal{C} \times \mathcal{U} \times \mathcal{M}$.[46] At each non-terminal history $h \in \mathcal{H} \backslash \overline{\mathcal{H}}$ and player $P(h) \in N$ corresponds an information set $I_{P(h)} \ni h$ and a *set of available actions* $A(I_{P(h)})$ which is either

[45] Notice that this formulation neither excludes the possibility that several players move simultaneously nor rules out that a single player simultaneously selects several actions (e.g., voting and sending different messages to different receivers altogether) after a given history. Imperfect information of our extensive game allows for the decomposition of any simultaneous move into sequential moves where, by convention, to each player or action is attributed a number, and players and actions follow the sequence of these numbers. For instance, a player may move without having observed the move of the previous player with the interpretation that both players move simultaneously.

[46] Observe that any interaction pattern F is linked to the other elements composing the deliberative game Γ. Indeed, the sets Θ, O, d_{set}, c_{set}, and M determine which of the elements can compose a history in $\mathcal{H}$. Furthermore, such a composition is also made of d_{rule} and c_{rule} which determine whether a particular outcome can arise (e.g., not all decision rules allow for the no-decision outcome $\emptyset$). Such a link among several elements composing the extensive-form game is usual (e.g., the dimension of preferences profile always depend on $\#(N)$, as does the dimension of the set Θ, under incomplete information).

$v_{set,P(h)}$, c_{set}, or $M^{\mathcal{T}} \cup M^{\mathcal{O}} \cup M^{\neg(\mathcal{T}\vee\mathcal{O})}$ depending on whether the player has to vote, indicates her willingness to end deliberation, or sends a message.

Any interaction pattern F that belongs to the set $\mathcal{F}$ satisfies the following five assumptions:

(i) every history $h \in \mathcal{H}$ which contains the continuation decision e to end deliberation satisfies that there are a finite positive integer q and a finite sequence of actions $(a_k)_{k=1}^q$ such that $(h, (a_k)_{k=1}^q) \in \overline{\mathcal{H}}$.

(ii) there is a finite positive integer q such that for any history $h \in \mathcal{H}$ of length $q' \geqslant q$, every contiguous subsequence $h' \subseteq h$ of length q contains at least one continuation decision (either e or $\neg e$).

(iii) there is a finite positive integer q such that for any history $h \in \mathcal{H}$ of length $q' \geqslant q$, every contiguous subsequence $h' \subseteq h$ of length q contains at least one profile of actions $(a_i)_{i\in N} \in d_{set}$ (resp. $(a_i)_{i\in N} \in (M^{\mathcal{O}})^n$). Moreover, for any $h \in \mathcal{H}$ containing at least one preferred outcome announcement $a \in M^{\mathcal{O}}$, for any $(i, a', a'', I_i) \in N \times (M^{\mathcal{O}})^2 \times \mathcal{I}_i$, $a' \neq a''$ implies $((h_{-a}, a'), (h_{-a}, a'')) \notin I_i^2$.

(iv) for any $(\theta, h, h', h'') \in \Theta \times \mathcal{H}^3$ satisfying $h = (h', h'')$, where h'' is only made of message(s) in $M^{\mathcal{T}}$, and h allows (resp. h' does not allow) to identify $o^*(\theta)$, there is no finite history $h''' \in \mathcal{H}$ only made of action(s) not in $M^{\mathcal{T}}$ such that (h', h''') allows to identify $o^*(\theta)$.

(v) for any $h \in \mathcal{H}$ containing at least one outcome selected by the composite function $v_{rule} \circ v_{\mathcal{H}}$, let o denote the last such an outcome. For any $(i, o', o'', I_i) \in N \times O^2 \times \mathcal{I}_i$, $o' \neq o''$ implies $((h_{-o}, o'), (h_{-o}, o'')) \notin I_i^2$.

Interpretation *Knowing to whom presenting one's piece of information to be interpreted.* An interpretation may require the participation of several agents. For any profile of types $\theta \in \Theta$ and message m, denote $N(m, \theta)$ the (smallest) set of agents able to contribute to the interpretation of m (with the convention that $N(m, \theta) = \emptyset$ when no interpretation is required, and that $\iota(m|\overset{\times}{\theta}_j) = m$ if $j \notin N(m, \theta)$). To simplify notation, we assume such a set is well-defined (as it would be, for instance, when a single agent is conventionally selected from any subgroup of agents with substitutable NTI). This assumption implies that $N(m, \theta)$ is known by the sender of message m (even when she does not know the whole profile θ).

Full interpretation of TI in a finite number of iterations. Let $\ddot{M}^{\mathcal{T}} \equiv \cup_{l\in\mathbb{N}^*}(M^{\mathcal{T}})^l$

denote the set of all possible report profiles (which may contain several reports from an identical agent). For any profile of types $\theta \in \Theta$, let $\chi_{\theta,\Theta}$ denote a function from $\ddot{M}^{\mathcal{T}}$ into itself, mapping any list of k reports $(m_1, ..., m_k)$, with $k \in \mathbb{N}$, into the corresponding list of truthful and transferable interpretations of such reports $((\iota(m_1|\overset{\times}{\theta}_j))_{j\in N}, ..., (\iota(m_k|\overset{\times}{\theta}_j))_{j\in N})$. For any $q \in \mathbb{N}$, let $\chi^q_{\theta,\Theta}$ denote the $q-$th iterate of $\chi_{\theta,\Theta}$ (i.e., $\chi^{q+1}_{\theta,\Theta} \equiv \chi^q_{\theta,\Theta} \circ \chi_{\theta,\Theta}$, and $\chi^0_{\theta,\Theta} \equiv Id_{\ddot{M}(\Theta)}$ with Id_X denoting the identity function of set X). For any profile of types $\theta = (\overrightarrow{\theta_i}, \overset{\times}{\theta}_i)_{i\in N}$, there is a finite number $\overline{q}$ satisfying $\chi^{q+1}_{\Theta}((\overrightarrow{\theta_i})_{i\in N})) = \chi^q_{\Theta}((\overrightarrow{\theta_i})_{i\in N}))$ for any $q \geq \overline{q}$.

Identifying the efficient outcome in a finite number of iterations. We suppose that for any profile of types $\theta = (\overrightarrow{\theta_i}, \overset{\times}{\theta}_i)_{i\in N}$, there is a finite number $\overline{q}$ such that $\chi^{\overline{q}}_{\Theta}((\overrightarrow{\theta_i})_{i\in N})$ provides sufficient information to allow the group to identify the efficient outcome $o^*(\theta)$.

Behavior and strategies Let a deliberative game $\Gamma =< N, \Theta, p, O, u, D, C, F, M >$. Let $p_i(I_i) \equiv \mathbb{P}(\theta|I_i) \in \Delta(\Theta)$ be agent i's *posterior belief* (on the realized profile of types) given any information set $I_i \in \mathcal{I}_i$. A *behavioral voting* (resp. *continuation*, *communication*) *strategy of player* i is a collection $(\beta(I_i))_{I_i\in\mathcal{I}_i}$ of independent probability measures, where $\beta(I_i)$ is a probability measure over $d_{set,i}$ (resp. c_{set}, $M^{\mathcal{T}} \cup M^{\mathcal{O}} \cup M^{\neg(\mathcal{T}\vee\mathcal{O})}$). Formally, given a game Γ, the set of possible (strategic) behaviors writes as $\mathcal{B} \equiv \times_{i\in N}\mathcal{B}_i$ where $\mathcal{B}_i$ denotes the set of collections $(p_i(I_i), \beta^d(I_i), \beta^c(I_i), \beta^m(I_i))_{I_i\in\mathcal{I}_i}$ with β^d (resp. β^c, β^m) referring to a behavioral voting (resp. continuation, communication) strategy of player $i \in N$.

B (Online) Appendix: Proof of Theorem

Necessity

Proof. Let $(N, O, D, C, F, M) \in \mathcal{N} \times \mathcal{O} \times \mathcal{D} \times \mathcal{C} \times \mathcal{F} \times \mathcal{M}$. Assume, *per contra*, $(D, C, F) \notin \widetilde{\mathcal{DC}} \times \widetilde{\mathcal{F}}$ and $\forall(\Theta, p, u) \in \mathcal{T} \times \Delta(\Theta) \times \mathcal{U}$, $\exists B \in \mathcal{B}$ such that $\forall\theta \in \Theta$, $o^*(\theta)$ is selected.

Assume $(D, C) \notin \widetilde{\mathcal{DC}}$. There are two cases.

First, $\widetilde{\mathcal{DC}} - (i)$ is violated. Formally, there is a history $h \in \mathcal{H}$ such that $c_{\mathcal{H}}(h) = (\neg e, \neg e, ..., \neg e)$, $\mathbb{I}_{\mathcal{H},O}(h) = 0$, and $(c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h) > 0$. From $\mathcal{C} - (ii)$ and $\mathcal{C} - (iii)$, for every $h' \in \mathcal{H}$ satisfying $c_{\mathcal{H}}(h') \neq \emptyset$ we then have $(c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h') > 0$. Let q be such as defined in $\mathcal{F} - (ii)$. From $\mathcal{F} - (ii)$ there is a positive probability that the ending outcome (e) is selected after any history of length q. From $\mathcal{F} - (i)$ there is then a finite positive integer l of actions after which the interaction stops. Hence, there is a positive probability that the interaction stops after any history of length $(q + l)$. Let $(\Theta, p, u) \in \mathcal{T} \times \Delta(\Theta) \times \mathcal{U}$ be such that there is a profile of types $\theta \in \Theta$ that requires more than $(q+l)$ iterations for the outcome $o^*(\theta)$ to be identified. From $\mathcal{F} - (iv)$, there is no sequence of actions $(a_k)_{k=1}^{t}$, with $t \leq q + l$, which allows the group to identify $o^*(\theta)$. Therefore, there is no behavior $B \in \mathcal{B}$ that selects $o^*(\theta)$ in all cases whenever θ is the realized profile of types, a contradiction.

Second, $\widetilde{\mathcal{DC}} - (i)$ holds. From $(D, C) \notin \widetilde{\mathcal{DC}}$, $\widetilde{\mathcal{DC}} - (ii)$ is violated. Formally, there is $h' \in \mathcal{H}$ satisfying $c_{\mathcal{H}}(h') = (\neg e, \neg e, ..., \neg e)$, $\mathbb{I}_{\mathcal{H},O}(h') = 1$, and $(c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h') > 0$; and $d_{rule}(d) \neq \emptyset$ for any $d \in d_{set}$. From $\mathcal{C} - (iii)$, for every $h \in \mathcal{H}$ satisfying $c_{\mathcal{H}}(h) \neq \emptyset$ and $\mathbb{I}_{\mathcal{H},O}(h) = 1$, we have $(c_{rule} \circ (c_{\mathcal{H}}, \mathbb{I}_{\mathcal{H},O}))(h) > 0$. From $\mathcal{F} - (iii)$ there is a finite positive integer r such that for any history $h \in \mathcal{H}$ of length $r' \geqslant r$, every contiguous subsequence $h' \subseteq h$ of length r contains at least one profile of actions $(a_i)_{i \in N} \in v_{set}$. Since $v_{rule}(v) \neq \emptyset$ for any $d \in d_{set}$, the indicator function $\mathbb{I}_O$ has value 1 after any history of length $r' \geqslant r$. A similar argument to the previous case applies then to any profile of types $\theta \in \Theta$ that requires more than $(r + q + l)$ iterations to identify the outcome $o^*(\theta)$, a contradiction.

Assume, instead, $(D, C) \in \widetilde{\mathcal{DC}}$. From $(D, C, F) \notin \widetilde{\mathcal{DC}} \times \widetilde{\mathcal{F}}$ we get $F \notin \widetilde{\mathcal{F}}$. There is then a pair of agents $(i, j) \in N^2$, for whom there is an upper bound $\overline{q}$ on the number of times that agent i can report to agent j. Let $(\Theta, p, u) \in \mathcal{T} \times \Delta(\Theta) \times \mathcal{U}$ be such that there is a profile of types $\theta \in \Theta$ requiring more than $2\overline{q}$ back and forth iterations between agents i and j to identify the outcome $o^*(\theta)$. From $\mathcal{F} - (iv)$, the upper bound $\overline{q}$ precludes the group to identify $o^*(\theta)$. Therefore, there is no behavior $B \in \mathcal{B}$ that allows the group to select $o^*(\theta)$ in all cases whenever θ is the realized profile of types, a contradiction. $\square$

Sufficiency

To prove sufficiency, we characterize a specific behavior that guarantees efficient outcomes selection whenever $(D, C, F) \in \widetilde{\mathcal{DC}} \times \widetilde{\mathcal{F}}$. We proceed in two parts. First, we construct this specific behavior. Second, we establish its efficiency.

Behavior construction

The behavior we construct is symmetric and provides that each agent: (a) reports truthfully and exhaustively all relevant information to any agent who is able to contribute to its interpretation; (b) publicly announces the efficient outcome once identified, otherwise remains silent on outcome identification; (c) requests to continue deliberation as long as no preferred outcome has been announced; (d) votes sincerely in favor of the outcome that has been publicly announced (or, given that the continuation rule allows deliberation to continue if all agents so desire, in favor of the outcome that has been identified), otherwise stays "neutral" (as defined below) to avoid the implementation of a final decision outcome; (e) updates beliefs rationally; and (f) restarts with the identification of the efficient outcome, if behavior is inconsistent (e.g., whenever two agents announce two different efficient outcomes).

This behavior, when played collectively, makes the group interact in an epistemic manner where everyone believes what is reported by the others (both for efficient outcome identification and private information reporting), and ultimately accepts to end deliberation as soon as one agent wants so. This corresponds to the situation where the group is endogenously delegating the decision to the agent who announces at some point to be sufficiently well informed to identify the efficient outcome.

In addition to the general pattern previously described, our behavior includes subtleties allowing the agents to coordinate their moves (e.g., avoiding any *quorum* requirement). Its construction relies on the following additional definitions.

Final collectors. Any interaction pattern $F \in \mathcal{F}$ defines how some agent(s) can be charged to collect interpreted information from the group. Let N^F denote a specific set of final collectors (e.g., under public speech $N^F = N$, and under senders-receiver games $N^F = \{receiver\}$).

Truthful (and exhaustive) reporting. A communication strategy that uses a message in $M^{\mathcal{T}}$ is a *reporting strategy.* A reporting strategy for agent i is *truthful* if $m_i\left(I_i\right)$ is only composed of truthful interpretation $\iota(m|\overset{\times}{\theta}_i)$, where m is either $\overrightarrow{\theta_i}$ or a report received by agent i along history $h \in I_i$, for all $I_i \in \mathcal{I}_i$. Whenever such a composition is made of all truthful interpretations associated with $\overrightarrow{\theta_i}$ and all reports received by agent i along history h, we say the reporting strategy is *truthful and exhaustive.*

Set of candidates for efficient outcome (under truthful reporting). For any agent $i \in N$ and information set $I_i \in \mathcal{I}_i$, let $O^*(I_i)$ be the set of outcomes that are candidates to be efficient when agents report truthfully. This set is made of outcomes $o \in O$ for which there is a profile of types $\tilde{\theta} \in \Theta$ that satisfies $o^*(\tilde{\theta}) = o$ and is compatible with truthful reporting along at least one history $h \in I_i$.

Coordination among agents. The set $\mathcal{D}$ covers a large number of rules. Some of them allow the agents to straightforwardly identify the individual vote corresponding to a specific decision outcome intention (e.g., in criminal cases, a juror can vote either "guilty" or "not guilty" to "convict" or "acquit", respectively). Not all rules offer such an opportunity. To bypass this problem and help the agents to coordinate their votes over outcomes, let us define a function $\kappa : (O \cup \{\emptyset\}) \times \mathcal{D} \longmapsto d_{set}$ that associates to any pair of outcome and decision rule (o, D) a single profile of indications $\kappa(o, D)$ satisfying $d_{rule}(\kappa(o, D)) = o$. From our assumption on $\mathcal{D}$ such a function is well-defined on O (and in case D can render a no-decision, it is well-defined on $O \cup \{\emptyset\}$) as there is always at least one candidate $d^o \in d_{set}$ satisfying $d_{rule}(d^o) = o$. Of course, there may be several such functions (e.g., under majority rule there are different voting profiles compatible with a specific decision outcome). To coordinate their votes, the agents only require to use the same function as a coordination device. Let $\kappa_i(o, D)$ denote the i-th element of $\kappa(o, D)$.

The agents may be required to vote before a decision outcome has been identified. The profile $\kappa^{\emptyset}(D) \equiv \kappa(\emptyset, D)$ can serve to avoid the *quorum* requirement on decision rule D whenever it can render a no-decision $\emptyset$. Otherwise, define $\kappa^{\emptyset}(D)$ as a given profile in the set $d_{set} \setminus \{\cup_{o \in O} \{\kappa(o, D)\}\}$. If this set is empty, let $\kappa^{\emptyset}(D) \equiv \kappa(o_1, D)$ where o_1 is the first element of O. We define $\kappa_i^{\emptyset}(D)$ the agent i's "neutral" vote.

Whenever the interaction pattern F imposes minimal constraints on the group (e.g., an agent, *ad hoc* designated, can endogenously set the order of speech and sequences of votes), the agents use a language made up of messages in $M^{\neg(\mathcal{T} \vee \mathcal{O})}$ to set their order of interaction. To avoid additional notation, we do not further specify the agreed order of interaction and, by a slight abuse of notation, we write $F \in \widetilde{\mathcal{F}}$ whenever the group can (and then sets) an interaction F satisfying the properties of $\widetilde{\mathcal{F}}$.

Finally, we also need to provide the agents with a way to coordinate their behavior when they observe inconsistent moves. The observer of any action that is inconsistent with the current behavior (an action that is "off-the-equilibrium" path) signals to the group, with a message in $M^{\mathcal{O}}$, that the behavior has to be restarted from beginning by publicly announcing an outcome $o \in \mathcal{O}$ which is not in O.

Behavior B^*. The behavior B^* is such that, for any deliberative game

$$\Gamma =< N, \Theta, p, O, u, D, C, F, M >$$

with $(N, O, \Theta, p, u, D, C, F, M)$ in $\mathcal{N} \times \mathcal{O} \times \mathcal{T} \times \Delta(\Theta) \times \mathcal{U} \times \widetilde{\mathcal{DC}} \times \widetilde{\mathcal{F}} \times \mathcal{M}$, at every information set $I_i \in \mathcal{I}_i$, each solicited agent i:

(a) (*reports truthfully and exhaustively*) reports (directly or indirectly) $m(I_i)$, where $m(I_i)$ consists of all truthful interpretations associated with $\overrightarrow{\theta_i}$ and all reports received by agent i along any history $h \in I_i$, to $N(m(I_i), \theta)$ if $N(m(I_i), \theta) \neq \emptyset$; and to N^F otherwise.

(b) (*publicly announces the efficient outcome once identified)* stays silent so long as $\#O^*(I_i) > 1$, otherwise publicly announces $m_i^o = o^*$ where o^* is the unique element of $O^*(I_i)$;

(c) (*requests to continue any valuable deliberation*) requests $c_i = e$ if ending the deliberation would have the effect to select an outcome that either has been publicly indicated or is the unique element of $O^*(I_i)$, otherwise requests $c_i = \neg e$;

(d) (*votes sincerely when the efficient outcome has been publicly announced, or has been identified and the interaction can continue; otherwise votes neutral*) votes $d_i = \kappa_i(o, D)$ when either a unique outcome $o \in O$ has been publicly indicated or $\{C \in \widetilde{\mathcal{C}}$ and o is the unique element of $O^*(I_i)\}$, otherwise votes $d_i = \kappa_i(\emptyset, D)$; and

(e) (*updates beliefs rationally*) updates beliefs according to the Bayes' rule whenever possible; and

(f) (*restarts from the beginning in case of inconsistency)* whenever an action that is inconsistent with the current behavior is observed (e.g., deviations from truthful reporting that produce contradiction in beliefs, public announcement of different outcomes, $O^*(I_i)$ as empty set, etc.), publicly indicates during the next announcement an outcome $o \in \mathcal{O}$ which is not in O; if an outcome $o \notin O$ has been publicly indicated, then restarts from the beginning (i.e., resets the posterior belief to p and behave as if all histories, except the initial observation of θ_i, were "cleared").

Efficient outcomes selection

Proof. Let $(N, O, D, C, F, M) \in \mathcal{N} \times \mathcal{O} \times \widetilde{\mathcal{DC}} \times \widetilde{\mathcal{F}} \times \mathcal{M}$ and $(\Theta, p, u) \in \mathcal{T} \times \Delta(\Theta) \times \mathcal{U}$. Assume the group plays according to behavior B^*. Assume, *per contra*, that there is a profile of types $\overline{\theta} \in \Theta$ such that $o^*(\overline{\theta})$ is not (always) selected.

Observe that under behavior B^* the interaction stops in finite time (i.e., all terminal histories are of finite length). Indeed, from $F \in \widetilde{\mathcal{F}}$ agents benefit from *freedom of reach* and there is no upper bound on the times (so long as the interaction continues) the agents can exchange interpreted information. Moreover, by assumption, full interpretation can always be performed in a finite number of truthful iterations. Under $B^* - (a)$, agents report truthfully and exhaustively, so full interpretation of TI always occurs under sufficiently long interaction. At some point, all agents then report to N^F. By assumption, there is a finite number $\overline{q}$ such that $\chi_{\Theta}^{\overline{q}}((\overrightarrow{\theta_i})_{i \in N})$ provides sufficient information to allow the group to identify the efficient outcome $o^*(\theta)$. This implies that there is a point in time when (a history at which) the efficient outcome is identified by at least one agent $i \in N^F$. From $\mathcal{F} - (iii)$ there is a finite positive integer q such that for any history $h \in \mathcal{H}$ of length $q' \geqslant q$, every contiguous subsequence $h' \subseteq h$ of length q contains at least one profile of actions $(a_i)_{i \in N} \in (M^{\mathcal{O}})^n$. According to $B^* - (b)$, agent i publicly announces $o^*(\theta)$ when requested to do so. From $\mathcal{F} - (iii)$ such an announcement is publicly observed by all agents. From $B^* - (e)$, the group then infers that $o^*(\theta)$ is the efficient outcome (otherwise agent i would have remained silent). From $B^* - (d)$, the group collectively votes $\kappa(o^*(\theta), D)$, which from $D \in \mathcal{D}$ is well-defined, and makes the aggregating rule $d_{rule}(.)$ to select the outcome $o^*(\theta) \in O$. From $\mathcal{F} - (v)$, such an outcome selection is publicly observed by all agents. Beyond that round, further deliberation would not improve information aggregation. From $\mathcal{F} - (ii)$, there is a finite positive integer l such that for any history $h \in \mathcal{H}$ of length $l' \geqslant l$, every contiguous subsequence $h' \subseteq h$ of length l contains at least one continuation decision. From $B^* - (c)$, the group would unanimously request to end deliberation, a request that, from $\mathcal{C} - (i)$ and $\mathcal{F} - (i)$, would have the effect to either immediately stop the interaction or limit the agents to a finite number of actions after which deliberation ends. Finally, observe that $B^* - (f)$ and $\mathcal{F} - (iii)$ guarantee that deliberation ends even after (a finite occurrence of) inconsistent moves (by assumption, an infinite occurrence of actions that are off-the-behavior path is excluded).

At a terminal history, the agents select either an outcome $o \in O$ or the no-decision outcome. Therefore, our contradicting assumption implies that under the profile of types $\overline{\theta}$ there is an outcome $o' \in (O \cup \emptyset) \setminus \{o^*(\overline{\theta})\}$ which is (at least sometimes) selected. From $(D, C) \in \widetilde{\mathcal{DC}}$ deliberation ends only when: at least one agent so requests; or

$C \notin \widetilde{\mathcal{C}}$ and the group, which can frustrate the *quorum* requirement on decision rules (i.e., $\kappa^{\emptyset}(D) = \kappa(\emptyset, D)$), does not do that. From $B^* - (b)$, and $B^* - (c)$ in the first case (resp., $B^* - (d)$ in the second case), an outcome $\tilde{o} \in O$, which is the unique element of $O^*(I_j)$ for a specific agent $j \in N$ and information set $I_j \in \mathcal{I}_j$, is selected. From $F \in \widetilde{\mathcal{F}}$, $B^* - (a)$ and $B^* - (e)$, once the set $O^*(I_j)$ is a singleton, its unique element is necessarily $o^*(\theta)$. Hence $\bar{o} = o^*(\theta)$, a contradiction. *Q.E.D.* $\square$

Equilibrium

Proof. Consider a deliberative game $\Gamma =< N, \Theta, p, O, u, D, C, F, M >$, with (N, O, D, C, F, M) in $\mathcal{N} \times \mathcal{O} \times \mathcal{D} \times \mathcal{C} \times \mathcal{F} \times \mathcal{M}$ and $(\Theta, p, u) \in \mathcal{T} \times \Delta(\Theta) \times \mathcal{U}$. Assume, *per contra*, there is a pair $(B, i) \in \mathcal{B} \times N$ composed of a behavior $B = (B_j)_{j \in N}$ that selects the efficient outcome $o^*(\theta)$ for any $\theta \in \Theta$, and an agent i who has a profitable unilateral deviation B'_i from B_i. Denote by $\widetilde{\mathcal{I}}_i$ the set of information sets I_i under which B'_i has not differed from B_i so far (i.e., such that $B'_i(I'_i) = B_i(I'_i)$ for any information set I'_i that contains a sub-history $h' \subseteq h$, $\forall h \in I_i$). In case there is no such an information set I_i, let $\widetilde{\mathcal{I}}_i = \emptyset$. Under behavior B, i's expected payoff at any information set $I_i \in \widetilde{\mathcal{I}}_i$ (i's *ex-ante* expected payoff whenever $\widetilde{\mathcal{I}}_i = \emptyset$) writes as

$$\sum_{(o,\theta) \in (\mathcal{O} \cup \{\emptyset\}) \times \Theta} u_i(o, \theta) \, \mathbb{P}_B [o \cap \theta \,| I_i]$$

$$= \sum_{\theta \in \Theta} \sum_{o \in \mathcal{O} \cup \{\emptyset\}} \sum_{h \in I_i} u_i(o, \theta) \, \mathbb{P}_B [o \,|\theta, h, I_i] \, \mathbb{P}_B [\theta \cap h | \, I_i] \tag{1}$$

where $\mathbb{P}_B [X \,|Y]$ denotes the belief that the event X occurs given the realization of the event Y and the behavior B (e.g., $\mathbb{P}_B [o \cap \theta \,|I_i]$ denotes i's belief at information set I_i that the types profile is θ and the outcome o will be selected given behavior B). Under the unilateral deviation B'_i, i's expected payoff at information set I_i can be written as

$$\sum_{\theta \in \Theta} \sum_{o \in \mathcal{O} \cup \{\emptyset\}} \sum_{h \in I_i} u_i(o, \theta) \, \mathbb{P}_{(B_{-i}, B'_i)} [o \,|\theta, h, I_i] \, \mathbb{P}_{(B_{-i}, B'_i)} [\theta \cap h | \, I_i] \tag{2}$$

Observe that $\mathbb{P}_{(B_{-i}, B'_i)} [\theta \cap h | \, I_i] = \mathbb{P}_B [\theta \cap h | \, I_i]$ for any pair $(\theta, h) \in \Theta \times I_i$ because B'_i does not differ from B_i in any history occurred under the information set $I_i \in \widetilde{\mathcal{I}}_i$.

The unilateral deviation B_i' is then profitable only if there is a pair $(\overline{\theta}, \overline{h}) \in \Theta \times I_i$ satisfying

$$\sum_{o \in \mathcal{O} \cup \{\emptyset\}} u_i\left(o, \overline{\theta}\right) \mathbb{P}_{(B_{-i}, B_i')}\left[o \left| \overline{\theta}, \overline{h}\right.\right] > \sum_{o \in \mathcal{O} \cup \{\emptyset\}} u_i\left(o, \overline{\theta}\right) \mathbb{P}_{B}\left[o \left| \overline{\theta}, \overline{h}\right.\right] \tag{3}$$

By assumption, the behavior B selects only efficient outcomes. This implies that the RHS of (3) writes as $u_i\left(o^*\left(\overline{\theta}\right), \overline{\theta}\right)$. Hence we have

$$\sum_{o \in \mathcal{O} \cup \emptyset} u_i\left(o, \overline{\theta}\right) \mathbb{P}_{(B_{-i}, B_i')}\left[o \left| \overline{\theta}, \overline{h}\right.\right] > u_i\left(o^*\left(\overline{\theta}\right), \overline{\theta}\right)$$

which, given that $u \in \mathcal{U}$, contradicts the definition of $o^*(.)$. *Q.E.D.* □

C (Online) Appendix: Details of the Examples

C.1 Additional details of Example 1

Note that Bayes' rule gives $\mathbb{P}\left(\omega_1 \left| s_i = b\right.\right) = \frac{\mathbb{P}(b|\omega_1)}{\mathbb{P}(b|\omega_0)+\mathbb{P}(b|\omega_1)} = \frac{0.38}{0.38+0.25} = \frac{38}{63} \approx 0.6$, and $\mathbb{P}\left(\omega_1 \left| s = (b,b)\right.\right) = \frac{\mathbb{P}(b|\omega_1)\mathbb{P}(b|\omega_1)}{\sum_{\omega \in \{\omega_0, \omega_1\}} \mathbb{P}(b|\omega)\mathbb{P}(b|\omega)} = \frac{0.38^2}{0.38^2+0.25^2} = \frac{1444}{2069} \approx 0.7$.

After voting, each agent i's updated belief $\mathbb{P}\left(\omega_1 \left| s_i = b, v_j = 0\right.\right)$ writes as

$$\begin{aligned}
\mathbb{P}\left(\omega_1 \left| s_i = b, s_j \in \{a,b\}\right.\right) &= \frac{\mathbb{P}\left(b \left|\omega_1\right.\right) \sum_{s_j \in \{a,b\}} \mathbb{P}\left(s_j \left|\omega_1\right.\right)}{\sum_{\omega \in \Omega} \mathbb{P}\left(b \left|\omega\right.\right) \sum_{s_j \in \{a,b\}} \mathbb{P}\left(s_j \left|\omega\right.\right)} \\
&= \frac{0.38 \times (0.17 + 0.38)}{0.38 \times (0.17 + 0.38) + 0.25 \times (0.70 + 0.25)} = \frac{22}{47} \approx 0.47
\end{aligned}$$

which reinforces their beliefs supporting the efficiency of $o = 0$. Therefore, the agents would be willing to confirm their votes immediately after the first voting round (i.e., with no intermediate communication).

In this example, any vote is pivotal for the final decision. As discussed in Example 1, the agents, by voting $(v_1, v_2) = (0, 1)$ regardless of their signals, can strategically dissent. This strategy enables them to aggregate their signals before the second round of voting. Notice that, in this example, the agents can also select efficient outcomes through symmetric voting. Indeed, by voting $v_i = 0$ if $s_i = a$ and $v_i = 1$ otherwise, the efficient outcome is either selected at the first round ($o^* = 0$ under (a, a); $o^* = 1$

under (b, b), (b, c), (c, b), and (c, c)), or at the second round. The latter case only occurs when the agents received (a, b), trivially implying that the agents already have all the decision-relevant information. Finally, notice that such a strategy would be inefficient under a different rule such as, for example, the *status quo* unanimity, under which an outcome (e.g., $o = 0$) can be unilaterally implemented by any agent.

C.2 Further details on Example 4.1

Pooling equilibrium As usual in this kind of cheap-talk communication, there exists a pooling equilibrium where DM_1 ignores expert E_1's report, and expert E_1 does not condition her message on her type. Such a pooling equilibrium would not exist if communication were restricted to hard (i.e., verifiable) information.

Truthful reporting at equilibrium In our game, truthful reporting from expert E_1 can be sustained at equilibrium. Indeed, revealing θ_1 (resp. θ_1') makes DM_1 play T (resp. B) which makes DM_2 play R (as previously explained). Therefore, the matrix of expected payoff writes as $\begin{array}{c|c|} & R \\ \hline T & 1.25;1.05 \\ \hline B & 0;0.5 \\ \hline \end{array}$ (resp. $\begin{array}{c|c|} & R \\ \hline T & 0;\text{-}0.205 \\ \hline B & 1;0.05 \\ \hline \end{array}$) under θ_1 (resp. θ_1'), so E_1's incentive constraint to truthfully report to DM_1 holds.

Positive value of partial communication We show that an interaction pattern allowing only DM_2 and E_2 to communicate, but preventing DM_1 and E_1 from doing so, Pareto-dominates the interaction pattern that forbids communication at all. Consider again the game described in Example 4.1. When E_2 reports truthfully to DM_2, by conjecturing that DM_2 will play her strictly dominant strategy, i.e. L (resp. R) when the state is θ_2 (resp. θ_2'), DM_1 plays the following "expected" game

$$\begin{array}{c|c|} \hline T & 1.375 \ ; \ 0.922 \\ \hline B & 1 \ ; \ 0.825 \\ \hline \end{array}$$

with T as best response. The payoffs associated to this outcome, i.e. $(1.375, 0.9225)$, are strictly higher than those achievable under no communication, i.e. $(1.25, 0.625)$, showing that in this case, partial communication dominates no communication.

Also in this case, truthful reporting from E_2 can be sustained at equilibrium. When E_2 reveals θ_2 (resp. θ_2'), this makes DM_2 play L (resp. R) which, in turn, makes DM_1

play T. Therefore, the matrix of expected payoffs writes as $\begin{array}{c|c|c|} & L & R \\ \hline T & 2;1.25 & 0.5;0.25 \\ \hline \end{array}$ (resp. $\begin{array}{c|c|c|} & L & R \\ \hline T & 0.5;0 & 0.75;0.595 \\ \hline \end{array}$) under θ_2 (resp. θ_2'), implying that E_2's incentive constraint to truthfully report to DM_2 holds.

C.3 Further details on Example 4.2

We report here further details on Example 4.2. Table 6 reports the possible DM and Q's signals (columns (1) and (2), respectively); the corresponding posterior beliefs (column (3)) when E has no valuable information (i.e., $\overrightarrow{\theta_E} = \emptyset$) and DM together with Q's signals have been aggregated; and the corresponding efficient outcome (column (4)).

(1)	(2)	(3)	(4)
$\overrightarrow{\theta_{DM}}$	$\overrightarrow{\theta_Q}$	$\mathbb{P}\left(\omega = 1 \middle\vert \emptyset, \overrightarrow{\theta_{DM}}, \overrightarrow{\theta_Q}\right)$	$o^*\left(\emptyset, \overrightarrow{\theta_{DM}}, \overrightarrow{\theta_Q}\right)$
$\overrightarrow{1_{DM}}$	$\overrightarrow{1_Q}$	$\frac{1}{162}$	0
	$\overrightarrow{2_Q}$	$\frac{4}{11}$	0
	$\overrightarrow{3_Q}$	$\frac{177}{184}$	1
$\overrightarrow{2_{DM}}$	$\overrightarrow{1_Q}$	$\frac{7}{214}$	0
	$\overrightarrow{2_Q}$	$\frac{28}{37}$	1
	$\overrightarrow{3_Q}$	$\frac{413}{416}$	1

Table 6: Posterior beliefs under complete information. This table is common knowledge of all agents.

Table 7 reports the possible DM signals and Q's reports (columns (1) and (2), respectively); the corresponding DM's posterior beliefs after one round of truthful communication with Q (column (3)); and the corresponding DM's preferred outcome (column (4)).

(1)	(2)	(3)	(4)
$\overrightarrow{\theta_{DM}}$	m_Q	$\mathbb{P}_{DM}\left(\omega=1 \middle| \emptyset, \overrightarrow{\theta_{DM}}, m_Q\right)$	$o_{DM}\left(\emptyset, \overrightarrow{\theta_{DM}}, m_Q\right)$
$\overrightarrow{1_{DM}}$	$\overrightarrow{1_Q}$	$\frac{41}{272}$	0
	$\overrightarrow{2_Q}$	$\frac{41}{272}$	0
	$\overrightarrow{3_Q}$	$\frac{177}{184}$	1
$\overrightarrow{2_{DM}}$	$\overrightarrow{1_Q}$	$\frac{7}{214}$	0
	$\overrightarrow{2_Q}$	$\frac{287}{584}$	0
	$\overrightarrow{3_Q}$	$\frac{413}{416}$	1

Table 7: DM's post-communication updated beliefs. This table is common knowledge of all agents.

Profitable misreporting of Q We show that when *consensual extension* does not hold, Q finds it profitable to misreport information. Under truthful reporting, DM's posterior belief upon observing $\overrightarrow{\theta_{DM}} \in \overrightarrow{\Theta_{DM}}$ and receiving $m_E \in \left\{0, \overrightarrow{1}\right\}$ writes as $\mathbb{P}\left(\omega=1 \middle| m_E, \overrightarrow{\theta_{DM}}, m_Q\right) = m_E$ for any $m_Q \in \left\{\overrightarrow{1_Q}, \overrightarrow{2_Q}, \overrightarrow{3_Q}\right\}$. Otherwise, for $m_E = \emptyset$ it writes as

$$\mathbb{P}\left(\omega=1 \middle| \emptyset, \overrightarrow{\theta_{DM}}, m_Q\right) = \frac{\mathbb{P}\left(\emptyset \,|\omega=1\right)\mathbb{P}\left(\overrightarrow{\theta_{DM}}\,|\omega=1\right)\sum_{\overrightarrow{\theta_Q}\in\overrightarrow{\Theta_Q}}\mathbb{P}\left(\overrightarrow{\theta_Q}\,|\omega=1\right)\mathbb{P}\left(\overrightarrow{\theta_Q}\,|m_Q,\omega=1\right)}{\sum_{\omega}\mathbb{P}\left(\emptyset\,|\omega\right)\mathbb{P}\left(\overrightarrow{\theta_{DM}}\,|\omega\right)\sum_{\overrightarrow{\theta_Q}\in\overrightarrow{\Theta_Q}}\mathbb{P}\left(\overrightarrow{\theta_Q}\,|\omega\right)\mathbb{P}\left(\overrightarrow{\theta_Q}\,|m_Q,\omega\right)}.$$

Because DM lacks Q's NTI, $\mathbb{P}\left(\overrightarrow{\theta_Q}\,|m_Q,\omega\right)$ satisfies $\mathbb{P}\left(\overrightarrow{1_Q}\,|m_Q,\omega\right) = \mathbb{P}\left(\overrightarrow{2_Q}\,|m_Q,\omega\right)$. These probabilities value $\frac{1}{2}$ if $m_Q \in \left\{\overrightarrow{1_Q}, \overrightarrow{2_Q}\right\}$, and zero otherwise. Similarly, $\mathbb{P}\left(\overrightarrow{3_Q}\,|m_Q,\omega\right) = 1$ if $m_Q = \overrightarrow{3_Q}$, and zero otherwise.

From Table 7, for any $\overrightarrow{\theta_{DM}} \in \overrightarrow{\Theta_{DM}}$, $\mathbb{P}\left(\omega=1 \middle| \emptyset, \overrightarrow{\theta_{DM}}, m_Q\right)$ is lower (resp. greater) than $\frac{1}{2}$ when Q sends $m_Q \in \left\{\overrightarrow{1_Q}, \overrightarrow{2_Q}\right\}$ (resp. $m_Q = \overrightarrow{3_Q}$), which makes DM to implement $o=0$ (resp. $o=1$) Q's expected payoff upon observing $\overrightarrow{2_Q}$ and sending m_Q writes as:

$\sum_{(o,\omega,\overrightarrow{\theta_E},\overrightarrow{\theta_{DM}})} u(o,\omega)\mathbb{P}\left(o \cap \omega \cap \overrightarrow{\theta_E} \cap \overrightarrow{\theta_{DM}} \middle| \overrightarrow{2_Q}, m_Q\right)$ where

$$\begin{aligned}
\mathbb{P}\left(o \cap \omega \cap \overrightarrow{\theta_E} \cap \overrightarrow{\theta_{DM}} \middle| \overrightarrow{2_Q}, m_Q\right) =&\mathbb{P}\left(o \middle| \omega, \overrightarrow{\theta_E}, \overrightarrow{\theta_{DM}}, \overrightarrow{2_Q}, m_Q\right)\mathbb{P}\left(\overrightarrow{\theta_E} \middle| \omega, \overrightarrow{\theta_{DM}}, \overrightarrow{2_Q}, m_Q\right)\\
&\mathbb{P}\left(\overrightarrow{\theta_{DM}} \middle| \omega, \overrightarrow{2_Q}, m_Q\right)\mathbb{P}\left(\omega \middle| \overrightarrow{2_Q}, m_Q\right)\\
=&\mathbb{P}\left(o \middle| \omega, \overrightarrow{\theta_E}, \overrightarrow{\theta_{DM}}, \overrightarrow{2_Q}, m_Q\right)\mathbb{P}\left(\overrightarrow{\theta_E} \middle| \omega\right)\mathbb{P}\left(\overrightarrow{\theta_{DM}} \middle| \omega\right)\mathbb{P}\left(\omega \middle| \overrightarrow{2_Q}\right)
\end{aligned}$$

From $u(1,1) = u(0,0) = 0$, this sum depends on pairs (o,ω) satisfying $o \neq \omega$. Under truthful reporting, $\mathbb{P}\left(o \neq \omega \middle| \omega, \overrightarrow{\theta_E}, \overrightarrow{\theta_{DM}}, \overrightarrow{2_Q}, m_Q\right) = 0$ if $\overrightarrow{\theta_E} \neq \emptyset$. Otherwise, when $\overrightarrow{\theta_E} = \emptyset$, this probability is also zero if $\{\omega = 0$ and $m_Q \in \left\{\overrightarrow{1_Q}, \overrightarrow{2_Q}\right\}\}$ or $\{\omega = 1$ and Q misreports $m_Q = \overrightarrow{3_Q}\}$. Thus, from $u(0,1) = u(1,0) = -\frac{1}{2}$ we have

$$\begin{aligned}
&\sum_{(o,\omega,\overrightarrow{\theta_E},\overrightarrow{\theta_{DM}})} u(o,\omega)\mathbb{P}\left(o \cap \omega \cap \overrightarrow{\theta_E} \cap \overrightarrow{\theta_{DM}} \middle| \overrightarrow{2_Q}, m_Q\right)\\
=&-\frac{1}{2}\left(\mathbb{I}_{\{m_Q \in \{\overrightarrow{1_Q},\overrightarrow{2_Q}\}\}}\mathbb{P}\left(\overrightarrow{\theta_E} = \emptyset \middle| \omega = 1\right)\mathbb{P}\left(\omega = 1 \middle| \overrightarrow{2_Q}\right)\right.\\
&\left.+\mathbb{I}_{\{m_Q = \overrightarrow{3_Q}\}}\mathbb{P}\left(\overrightarrow{\theta_E} = \emptyset \middle| \omega = 0\right)\mathbb{P}\left(\omega = 0 \middle| \overrightarrow{2_Q}\right)\right)
\end{aligned}$$

which equals $-\frac{1}{2} \times \frac{1}{3} \times \frac{4}{7} = -\frac{4}{42}$ when $m_Q = \overrightarrow{2_Q}$, and $-\frac{1}{2} \times \frac{1}{3} \times \frac{3}{7} = -\frac{3}{42}$ when $m_Q = \overrightarrow{3_Q}$. Hence, misreporting is a profitable deviation for Q.

Absence of *freedom of reach* Misreporting can also be profitable when *freedom of reach* does not hold. Consider Example 4.2, but replace the interaction pattern of Fig. 3 with that of Fig. 5, implying that *freedom of reach* is violated. In particular, this interaction pattern while preventing Q from communicating with E, allows Q to directly report to DM.

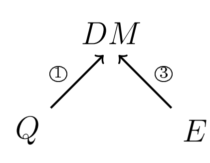

Figure 5: Absence of freedom of reach.

Similar to the previous case, this friction creates an incentive for Q to misreport her information.

Efficient misreporting With different assumptions on the distribution of the agents' type, we show that misreporting is ex-post efficient.

Suppose Q's TI distribution is as in Table 8. Here DM cannot distinguish between $\overrightarrow{2_Q}$ and $\overrightarrow{3_Q}$, but she can identify $\overrightarrow{1_Q}$. Moreover, assume that the TI profile is $\overrightarrow{\theta_E} = \emptyset$, $\overrightarrow{\theta_Q} = \overrightarrow{2_Q}$, and $\overrightarrow{\theta_{DM}} = \overrightarrow{1_{DM}}$. Tables 9 and 10 are the modified version of Tables 6 and 7 respectively.

$\overrightarrow{\theta_Q}$	$\mathbb{P}\left(\overrightarrow{\theta_Q}\right)$	$\mathbb{P}\left(\overrightarrow{\theta_Q}\,\vert\omega=0\right)$	$\mathbb{P}\left(\overrightarrow{\theta_Q}\,\vert\omega=1\right)$	$\mathbb{P}\left(\omega=1\,\middle\vert\,\overrightarrow{\theta_Q}\right)$
$\overrightarrow{1_Q}$	**0.30**	**0.59**	**0.01**	$\frac{1}{60}$
$\overrightarrow{2_Q}$	**0.35**	0.40	0.30	$\frac{3}{7}$
$\overrightarrow{3_Q}$	**0.35**	0.01	0.69	$\frac{69}{70}$

Table 8: Signals distribution and posterior beliefs of agent Q. This table is common knowledge of Q and E. DM only knows what is in bold, but does know whether the rows of the data in gray are presented in this order or are inverted (i.e., whether $\mathbb{P}\left(\omega=1\,\middle\vert\,\overrightarrow{2_Q}\right)$ (resp. $\mathbb{P}\left(\omega=1\,\middle\vert\,\overrightarrow{3_Q}\right)$) is equal to $\frac{3}{7}$ or $\frac{69}{70}$). Each possibility is assumed to be equally likely.

$\overrightarrow{\theta_{DM}}$	$\overrightarrow{\theta_Q}$	$\mathbb{P}\left(\omega=1\,\middle\vert\,\emptyset,\overrightarrow{\theta_{DM}},\overrightarrow{\theta_Q}\right)$	$o^*\left(\emptyset,\overrightarrow{\theta_{DM}},\overrightarrow{\theta_Q}\right)$
$\overrightarrow{1_{DM}}$	$\overrightarrow{1_Q}$	$\frac{3}{416}$	0
	$\overrightarrow{2_Q}$	$\frac{9}{37}$	0
	$\overrightarrow{3_Q}$	$\frac{207}{214}$	1
$\overrightarrow{2_{DM}}$	$\overrightarrow{1_Q}$	$\frac{7}{184}$	0
	$\overrightarrow{2_Q}$	$\frac{7}{11}$	1
	$\overrightarrow{3_Q}$	$\frac{161}{162}$	1

Table 9: Modified version of Table 6.

$\overrightarrow{\theta_{DM}}$	m_Q	$\mathbb{P}_{DM}\left(\omega=1 \middle\vert \emptyset, \overrightarrow{\theta_{DM}}, m_Q\right)$	$o_{DM}\left(\emptyset, \overrightarrow{\theta_{DM}}, m_Q\right)$
$\overrightarrow{1_{DM}}$	$\overrightarrow{1_Q}$	$\frac{3}{416}$	0
	$\overrightarrow{2_Q}$	$\frac{297}{584}$	1
	$\overrightarrow{3_Q}$	$\frac{297}{584}$	1
$\overrightarrow{2_{DM}}$	$\overrightarrow{1_Q}$	$\frac{7}{184}$	0
	$\overrightarrow{2_Q}$	$\frac{231}{272}$	1
	$\overrightarrow{3_Q}$	$\frac{231}{272}$	1

Table 10: Modified version of Table 7.

From Table 9, Q would have the incentive to misreport $\overrightarrow{\theta_Q} = \overrightarrow{1_Q}$. Indeed, notice from Table 7 that, for any $\overrightarrow{\theta_{DM}} \in \overrightarrow{\Theta_{DM}}$, $\mathbb{P}\left(\omega=1 \middle\vert \emptyset, \overrightarrow{\theta_{DM}}, m_Q\right)$ is lower (resp. greater) than $\frac{1}{2}$ when Q sends $m_Q = \overrightarrow{1_Q}$ (resp. $m_Q \in \left\{\overrightarrow{2_Q}, \overrightarrow{3_Q}\right\}$), which results in DM implementing $o=0$ (resp. $o=1$). Thus, Q's expected payoff upon observing $\overrightarrow{2_Q}$ and sending m_Q writes as:

$$\begin{aligned}
&\sum_{(o,\omega,\overrightarrow{\theta_E},\overrightarrow{\theta_{DM}})} u(o,\omega)\mathbb{P}\left(o \cap \omega \cap \overrightarrow{\theta_E} \cap \overrightarrow{\theta_{DM}} \middle\vert \overrightarrow{2_Q}, m_Q\right) \\
=& -\frac{1}{2}\left(\mathbb{I}_{\left\{m_Q \notin \left\{\overrightarrow{2_Q},\overrightarrow{3_Q}\right\}\right\}}\mathbb{P}\left(\overrightarrow{\theta_E}=\emptyset \left\vert \omega=1\right.\right)\mathbb{P}\left(\omega=1 \middle\vert \overrightarrow{2_Q}\right)\right. \\
&\left.+\mathbb{I}_{\left\{m_Q \in \left\{\overrightarrow{2_Q},\overrightarrow{3_Q}\right\}\right\}}\mathbb{P}\left(\overrightarrow{\theta_E}=\emptyset \left\vert \omega=0\right.\right)\mathbb{P}\left(\omega=0 \middle\vert \overrightarrow{2_Q}\right)\right)
\end{aligned}$$

which takes value $-\frac{1}{2} \times \frac{1}{3} \times \frac{4}{7} = -\frac{3}{42}$ when $m_Q = \overrightarrow{1_Q}$, and $-\frac{1}{2} \times \frac{1}{3} \times \frac{3}{7} = -\frac{4}{42}$ when $m_Q = \overrightarrow{2_Q}$, implying that misreporting is a profitable deviation for Q. On the contrary here, misreporting would induce DM to implement the efficient outcome as $o^*(\theta_{DM}, \theta_Q, \theta_E) = 0$.