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Asymmetries of Information in Electronic Systems

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Abstract

We study the efficiency of the equilibrium price in a centralized, order-driven market where asymmetrically informed traders are active for several periods and can observe each other current and past orders, as in electronic systems of trading. We show that the more precise the information the higher the incentive to reveal it in the first trading rounds. On the contrary, strategic competition forces the less informed trader to wait the end of the trading period to reveal his information. This implies that when differences in information quality are very important, the liquidity of the market decreases as we approach the date of public revelation. We are able to show that more transparent markets as the ones organized via electronic systems are not performing better than markets organized on floor trade in terms of revelation of information, due to the oligopolistic behavior of insiders.

Keywords: asymmetric information, liquidity, insider trading, strategic revelation.
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1 Introduction

Electronic quotation systems like the Toronto Stock Exchange CATS (*Computer Assigned Trading System*) have remarkable importance in terms of volume of trade. Their increasing practical relevance has lead microstructure theorists to study *pre-trade* transparency. In this literature, the broad conclusion seems to be that electronic systems bring more informative prices and lower trading costs by allowing for widely spread information about investors' demands (with respect to floor systems). This belief is also shared by some regulators like the US Security and Exchange Commission or the UK Office of Fair Trading.

In this paper we argue that strong asymmetries of information may generate inefficient price dynamics which could explain the observed increase of trading cost before a public announcement (e.g., earnings or dividends). From this result we can then derive conditions under which the high degree of transparency of electronic systems is not beneficial in terms of reduced costs of trading or for informational efficiency. Madhavan, Porter and Weaver (2000) find related empirical evidence for the Toronto Stock Exchange. They analyze the effects of the introduction (on April 12, 1990) of a computerized system called Market by Price (MBP) which dramatically increased the level of pre-trade transparency. These authors observe an increased price volatility (perhaps allowing for an higher informational efficiency) and above all that the cost of trading does not reduce following the reform. Although they give a different interpretation to their empirical findings with respect to the increased competition in information, we stress here the similarity with our theoretical conclusions.

The key to our result is twofold. Firstly the transparency of an electronic quotation system enables traders to observe the identities of the brokers submitting orders and therefore to acquire a private information as soon as it is used by someone to perform a (profitable) trade: this provokes a direct competition among insiders. Secondly we model asymmetries of information between traders in a very natural way that accounts for differences in the *quality* of information which is treated as the main strategic variable.

The issue of information revelation in financial markets has long been studied in competitive markets but it has taken a new start with Kyle (1985). He shows that the optimal behavior of a monopolistic informed trader is to reveal information slowly so as to maintain a constant market depth (until the last few periods). But, as reported by Cornell and Sirri (1992) and Meulbroek (1992), private information can be disseminated among dozens of traders. Thus it makes sense to look at oligopolistic competition among equally informed traders as done by Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993) among others. These models have a Bertrand flavor since they treat information as an *homogeneous* good. Although the ensuing *rat race* yields a quite efficient price dynamic this approach remains unsatisfactory due to the discrepancy existing between the outcomes of the monopoly and duopoly settings.

Traders frequently disagree on the future value of an asset either because they have different private information or because they interpret differently the same piece of news (due to different experience or education). Foster and Viswanathan (1996) pursue this venue and argue that private information is a peculiar good, intrinsically non homogeneous and spread asymmetrically among rational traders.

Since our work has an identical starting point and some similar results we feel the need to highlight the fundamental differences of the two approaches. Firstly we study an *electronic system* where traders directly observe their competitors orders (high transparency) while Foster and Viswanathan (1996)'s model rather applies to a *floor system* since informed traders only observe the order flow (like market makers) and their own orders. Hence from a strategic point of view, informed traders in Foster and Viswanathan (1996) do not compete one against the other but indirectly through the market maker. Secondly we consider *asymmetries in the quality* of information (variance of private signals) while Foster and Viswanathan (1996) use an identical variance of private signals. On the other hand Foster and Viswanathan (1996) consider *asymmetries of opinions* by allowing for any kind of initial correlation structure (agreement or disagreement) while in our model there is always a positive correlation between signals (agreement). Indeed we believe that although traders process the same news differently they should nevertheless agree on the direction of the stock variation (up or down) and disagree on its amplitude. Like Foster and Viswanathan (1996) we treat market makers as market-clearing devices that collect the order book.

In this article we use a two stage game describing a centralized and order-driven market with two insiders having asymmetric private information. At each stage informed traders and liquidity traders simultaneously choose the quantities they trade with competitive market makers. The insiders' information consists of their observation of a signal correlated with the liquidation value of the asset, of the past history of prices and all the individual orders. The signals are independent and have different precisions: we analyze the case of positive correlation between private signals that we believe to be the most realistic. Insiders also observe the amount of liquidity trade present in the market at each period.[1] Market makers set a price and trade the quantity which clears the market. The competition between them makes the equilibrium price equal to the expected value of the asset given the observable order flows (price efficiency). Market makers may be seen as electronic devices that collect the order-book.

We characterize the set of equilibria in pure and linear strategies. A simple backward in-

[1] This is not possible in Kyle (1985) and Holden & Subrahmanyam (1992) where insiders perfectly know the asst value because the price would immediatly incorporate all the information. One can motivate this assumption by invoking the possibility for the traders to estimate the liquidity order flow during the pre-opening period.

duction argument shows that one should always reveal its information in the last period. We then enquire about the incentives to reveal the private information in the first stage. On the one hand, using private information to trade with uninformed agents is beneficial but, by doing this, an insider gives an advantage to its competitor for the last stage and further brings the price closer to its own estimate. Both effects reduce the profit opportunities for the second period.

We show (as in Admati and Pfleiderer (1988)) that the amount of liquidity trade present on the market can be considered as a "cake" to be divided between the insiders. If the second period cake is much larger than the first period one, then traders conceal their information in the first period: the size effect predominates over the duopoly competition. Choosing a constant liquidity volume across periods we obtain more insightful results. If the information is almost equally precise the two traders reveal their signal in the first stage. However, if asymmetries are large, the better informed trader reveals and the opponent conceals in the first period. The better informed player trades more aggressively on his information given that market makers cannot detect perfectly his move. Hence, he acts as a leader in the "information game" and must use his advantage immediately. The less informed trader may conceal his type until the last stage during which he can exploit an informational advantage that was not relevant in the previous stage for the presence of a better informed opponent. The competition between asymmetric agents is not of the "Bertrand" kind because the information released by the traders is non-homogeneous.

To give the better informed trader an incentive to reveal his information in the first period, market makers initially reduce the sensitivity of their pricing rule to the order flow, up to a point that makes not profitable for the lesser informed trader to trade according to his own signal. The depth of the market then reduces with time. The profits of the two competitors are concave in their own precision and reduce to roughly one fifth of the monopolist profit: this reduction is more important for high precisions of private signals. We can then draw the conclusion that in transparent markets there is a low incentive for traders to collect very precise information if they expect that other traders already possess an inside.

The paper is organized as follows: in section 2 we present the two stage game and describe the equilibrium concept. Section 3 tackles the main part of the analysis while in section 4 we present our results on equilibrium revelation of information and on the liquidity and the informativeness of prices in order to point out the differences of our model with the existing literature. Section 5 concludes.

2 A Model of Electronic Trading

2.1 Private Information and Transparency

Consider a market for a risky asset where the exchanges occur between three kinds of agents: informed traders, liquidity traders and market makers. The risky asset has a random liquidation value v distributed with normal law $\mathcal{N}(0,1)$. At the beginning of the first trading round, trader[2] i observes a private signal $s_i = v + \varepsilon_i$ where the error term ε_i has law $\mathcal{N}(0, \tau_i^{-1})$ and τ_i is interpreted as the ex-ante precision with which the trader can guess the true value of v. When τ_i varies in $[1/2, 6]$, the private signal s_i explains from 33% to 86% of the underlying variance. We shall use this range for our later comparative analysis. All three random variables v, ε_i, and ε_j are assumed to be independently distributed.

We model the trading day by two stages of market activity. At stage $t = 1, 2$ the informed traders choose the quantities they trade, $q_{t,i}$ and $q_{t,j}$ knowing their signals s_i and s_j while liquidity traders submit an aggregated order u_t of law $\mathcal{N}(\bar{u}, \tau_t^{-1})$. We normalize their average trade to $\bar{u} = 0$ and assume that liquidity trading is independent of all other random variables.

As we study a market with high transparency, we assume that in period $t = 1, 2$ floor traders i and j observe the orders of each other.[3] As they know the orders identity, they know the exact composition of (at least part of) the book. This degree of transparency in the order flow is present in stock exchanges like the Toronto CATS system. We model market makers as market-clearing devices that electronically collect the order book without making any inference on the actual informational-related part of each individual order; they are just able to extract from the *whole* book what is defined as "public information".

Given the realized order flow $\omega_t = q_{t,i} + q_{t,j} + u_t$, the market maker announces at the end of each stage the price at which all orders are filled. This price is pushed by (potential) competition towards the expected value of the asset given all the public information, including the contemporaneous one. At the end of the second period of trading, the realized liquidation value of the asset is announced and holders of the asset are paid its realized value. We follow the standard assumption (cf. all the literature) according to which market makers price the asset in all periods with a linear rule $p_t(\omega_t) = \mu_t + \lambda_t \omega_t$.

The ex-post stage profit for informed trader i is $q_{t,i}(v - p_t)$ while the ex-interim expectation conditional on the private information $\mathcal{H}_{t,i}$ is $\Pi_{t,i} = E\left[q_{t,i}(v - p_t) \mid \mathcal{H}_{t,i}\right]$. We will consider also the ex-ante profit obtained by integrating $\Pi_{t,i}$ with respect to the joint measure of private signals

[2] *Notation* : i stands for trader 1 or 2 and j for the other trader. Whenever a formula is given for i only, the j formula is obtained by interverting symbols i and j.

[3] Most electronic systems display the 3 or 5 best bid and ask quotes instead of the best ones, along with the dealers identities.

and liquidity trade.

A comment with respect to the literature

We feel the need to comment on the statistical properties of the private informative signals, in order to simplify the comparison of our model with other models of information competition in particular that of Foster and Viswanathan (1996).

The informed trader receives a signal that is distributed as the liquidation value v plus a white noise ε that increases the variance of the information distribution. This feature is peculiar of the model we present. Further, the correlation between signals is always positive (as opposed to Foster and Viswanathan (1996)) since signals are both drawn from a distribution centered on v. In other words, insiders do not have different "opinions" about the true value of the asset, they just receive information that is more or less noisy. Yet as in Foster and Viswanathan (1996), the covariance of the signals with v is the same: $Cov(s_i, v) = Cov(s_j, v) = Var(v)$.

The covariance matrix of the private signals is $\Psi_0 = \begin{pmatrix} 1+\tau_i^{-1} & 1 \\ 1 & 1+\tau_j^{-1} \end{pmatrix}$ where the diagonal terms are different as precisions differ. The expected value of the asset given the observation of the two private signals is $E[v \mid s_i, s_j] = \binom{1}{1}^T \Psi_0^{-1} \binom{s_i}{s_j} = \frac{\tau_i s_i + \tau_j s_j}{\tau_i + \tau_j + 1}$ and a simple average of the signals $\frac{1}{2}(s_i + s_j)$ is not a sufficient statistic for the information known to all informed traders unlike in the approach of Foster and Viswanathan (1996).

This observation implies that the learning process of a market maker is different from that of an insider: the market maker can, at best, infer an average of the signals from the order flow. Yet this statistic has less predictive power than the information known to each single informed trader. This has two important consequences: first, the strategic interaction between informed traders is much more complex than in Foster and Viswanathan (1996). Each insider has to optimally reply to the move of its informed opponent taking into account that the latter and the market makers will derive different statistics from his trade. Both learning processes have to be considered in formulating the optimal informative trade. Secondly, as market makers will not be able to know precisely the individual signals (due to the noise of liquidity traders), even after the second round of trade, the variance of v given the whole history of prices and orders will still be positive.

2.2 Defining an Equilibrium Concept

We formally define the equilibrium concept used in the article and outline the construction of the equilibrium strategies.

Due to the high degree of transparency of the electronic quotation system, insider i who is a floor trader knows the history of all liquidity and informative trades. The order he submits $q_{t,i}$

thus depends on his forecast s_i of the realized value of v, on the trading decisions of j, and on the induced pricing rule of the market makers. Indeed the insider is aware that the market maker adjusts the sensitivity of its pricing rule according to the amount of information forecasted to be released in equilibrium.

Some important restrictive assumptions are necessary in order to be able to solve the model analytically. Firstly, we make the standard assumption that insiders use strategies linear with respect to their private information i.e., $q_{t,i}(\mathcal{H}_{t,i}) = \alpha_{t,i} + \beta_{t,i}E[v \mid \mathcal{H}_{t,i}]$. Excluding partially revealing strategies can be interpreted as an ex-ante commitment to a particular revelation rule. Of course, this implies also that we do not analyze equilibria in mixed strategies.

More fundamental is the Bayesian inference process of insiders and market makers described in Remark 1. It is difficult to construct a Bayesian-Nash equilibrium of the game where the strategy of insider i^{th} at the second stage is required to be optimal not only when insider j plays his optimal strategy in the first period, but also for any arbitrary strategy of j. To construct such an equilibrium, we would have to be able to rule out all possible deviations. Consider for example the following strategy for insider i: in the first stage, he uses his optimal $\beta_{1,i}$, but pretends he received signal $\hat{s}_i \neq s_i$ (the true one). Insider j can detect this individual trade and will wrongly infer $\hat{s}_i$, while the market maker will construct the wrong statistic $E[v \mid \omega_1(q_{1,i}(\hat{s}_i))]$. This deviation could in principle give higher profits to i in the second period, where both his opponents have been mislead. Assuming that the individual traders take the market maker pricing rule as given, this kind of manipulation seems *a fortiori* implausible.

The main problem is that in such an incomplete information game, some signalling activity between the two informed players could arise, and we are not sure that truthful revelation is indeed the only equilibrium, at least if we don't construct beliefs out of the equilibrium path supporting it. A way out to this problem is assuming that any misleading activity is ruled from the strategy space: in this sense, the equilibrium we characterize is restricted to what we call *truthful revelation strategies*.

We denote $\mathcal{H}_t$ the information revealed to market makers at the beginning of stage t and $\mathcal{H}_{t,i}$ the private information of trader i in that stage. Since we consider only full revelation of information or full concealing behavior, the second stage can start with four possible information structures. Strategies in the first stage denoted by $\sigma_{1,i}$ and $\sigma_{1,j}$ are either to reveal R or conceal C. The information sets are therefore:

- ► Only i reveals $(\sigma_{1,i} = R) \Rightarrow \mathcal{H}_{2,i} = \{s_i\}, \mathcal{H}_{2,j} = \{s_i, s_j\}, \mathcal{H}_2 = \{s_i\}$
- ► Only j reveals $(\sigma_{1,j} = R) \Rightarrow \mathcal{H}_{2,i} = \{s_i, s_j\}, \mathcal{H}_{2,j} = \{s_j\}, \mathcal{H}_2 = \{s_j\}$
- ► No revelation $(\sigma_{1,i} = \sigma_{1,j} = C) \Rightarrow \mathcal{H}_{2,i} = \{s_i\}, \mathcal{H}_{2,j} = \{s_j\}, \mathcal{H}_2 = \{\emptyset\}$
- ► Both reveal $(\sigma_{1,i} = \sigma_{1,j} = R) \Rightarrow \mathcal{H}_{2,i} = \{s_i, s_j\}, \mathcal{H}_{2,j} = \{s_i, s_j\}, \mathcal{H}_2 = \{s_i, s_j\}$

Observe that $\mathcal{H}_{2,i}$ is always finer than the observation of the total order flow ω_1 which can therefore be safely ignored in the second stage calculations. In the symmetric revelation case denoted RR, the second period is a Cournot game with complete information i.e., each pair $\{s_i, s_j\}$ defines a proper subgame. The game is solved for any couple of given private signals in the space of linear strategies. When only trader i reveals his information in stage one, denoted RC, trader j behaves as in the previous case, since he knows $\{s_i, s_j\}$. In equilibrium, trader i, despite the fact that he does not know the signal s_j, anticipates the rule $q_{1,j}(s_j, s_i)$ used by his opponent. Hence, we deal with a form of Stackelberg game. When no revelation has occurred, denoted CC, traders play a game with incomplete information on both sides. Each trader has to optimize against a rule and not against a single order.

An obvious consequence of the finite number of stages is that traders have an incentive to use their private information in stage 2, thus ω_2 conveys information about the underlying liquidation value of the asset. Yet, if one or both traders have revealed information during the first stage, then the order flow ω_1 is also an informative statistic for the market makers.

According to our restrictions for the strategy space of players, we analyze the 2×2 matrix game and for any pair of strategies $h \in \{CC, RC, CR, RR\}$, the ex-ante global profit is the sum of profits obtained in each active trading stage $\Pi_i^h = \Pi_{1,i}^h + \Pi_{2,i}^h$.

Definition 1 *A Bayesian-Nash equilibrium in linear, truthfully revealing strategies of the trading game is a vector of strategies (σ_i, σ_j, p) such that:*
1. For any trader $i = 1, 2$, period $t = 1, 2$ and linear strategy $\hat{\sigma}_i = (\hat{\sigma}_{1,i}, \hat{\sigma}_{2,i})$,

$$E\left[\Pi_{t,i}^h(\sigma_i, \sigma_j, p) \mid \mathcal{H}_{t,i}, \omega_{t-1}, \omega_t\right] \geq E\left[\Pi_{t,i}^h(\hat{\sigma}_i, \sigma_j, p) \mid \mathcal{H}_{t,i}, \omega_{t-1}, \omega_t\right]$$

2. For any period $t = 1, 2,$ $\quad p_t = E\left[\widetilde{v} \mid \mathcal{H}_t\right]$

The strategy of player i in the second period is required to (weakly) dominate other possible truthfully revealing strategies conditional on his information. This actually reduces to play a concealing (i.e. constant) order, no matter what the strategy was in the first period.

To construct the equilibrium strategies,[4] we first study the best reply of trader i when trader j conceals his information in stage 1 to show that he ought to reveal even though he is giving an advantage to j for the second stage i.e., $\Pi_i^{RC} > \Pi_i^{CC}$. Thanks to this result, there is always revelation in equilibrium.[5] It is then sufficient to solve the trade-off between concealing and revealing for a trader who faces a (rational) revealing player i.e., determine when $\Pi_i^{CR} \lesseqgtr \Pi_i^{RR}$.

[4] We use backward induction as a solution concept to point out the strategic effect of the information revelation, as in Stackelberg competition. Notice however that, since the strategies we allow are markovian, and the state of the game is the only relevant statistics at the beginning of the second period, this solution concept is equivalent to the dynamic programming method more used in microstructure literature.

[5] Except when the size effect is very large.

In solving the game by backward induction we shall develop the minimal amount of calculation in the body of the text and defer complex computations to the appendices.

3 Analysis of the Trading Game

Our first task in this section is to assess how the revelation of private information in the order flow affects the pricing function used by market makers. It is a relatively easy task in the first period because there is no memory effect as the previous day closing price is assumed to be efficient. Things are more complex for the second period as market makers use ω_2 and ω_1 which potentially contain information.

As market makers use the fixing price p_1 and its underlying information ω_1 to name p_2, insiders may not want to reveal an information that will bring the price too close to their private expectation of the asset value. This behavior is at the heart of our model. We shall therefore analyze the optimal trading strategy when one reveals information immediately or when one decides to wait and this conditional on the decision of the other insider. The next section will then put together these pieces of analysis to determine the Nash equilibrium of the information revelation game.

3.1 Market Makers Pricing Rule

Perfect competition between market makers guarantees that prices $p_1(\omega_1)$ and $p_2(\omega_1, \omega_2)$ convey all the information incorporated in the two respective order flows i.e., $p_1(\omega_1) = E[v \mid \omega_1]$ and $p_2(\omega_1, \omega_2) = E[v \mid \omega_1, \omega_2]$. Notice that in efficient markets, prices are martingales with respect to the public information as $E[p_2 \mid \omega_1] = E[E[v \mid \omega_1, \omega_2] \mid \omega_1] = E[v \mid \omega_1] = p_1$.

We shall later show that the order flows take the following functional forms

$$\omega_1 = \frac{k_i + k_j}{\lambda_1} v + \frac{k_i \varepsilon_i}{\lambda_1} + \frac{k_j \varepsilon_j}{\lambda_1} + \eta_1 u_1 \quad (1)$$

$$\omega_2 = \frac{m_i + m_j}{\lambda_2} v + \frac{m_i \varepsilon_i}{\lambda_2} + \frac{m_j \varepsilon_j}{\lambda_2} + \eta_2 u_2 \quad (2)$$

where k_i, k_j and η_1 depend on the parameters $(\alpha_{i,1}, \beta_{i,1}, \alpha_{j,1}, \beta_{j,1})$ characterizing the first period behavior of insiders and m_i, m_j and η_2 depend similarly on $(\alpha_{i,2}, \beta_{i,2}, \alpha_{j,2}, \beta_{j,2})$.

Since v, $\varepsilon_i, \varepsilon_j$, u_1 and u_2 have been previously assumed to have zero mean, it is also the case for ω_1 and ω_2: notice that u_1 and u_2 have no informational role. Considering the functional form of ω_1 given by (1), we have[6]

$$E[v \mid \omega_1] = \frac{(k_i + k_j)\omega_1}{\lambda_1 Var(\omega_1)} \quad (3)$$

[6] According to the standard projection of normal variables : if $\tilde{x}$ and $\tilde{y}$ are independent and have zero mean, then $E[\tilde{x} / \tilde{x} + \tilde{y}] = \frac{Var(\tilde{x})}{Var(\tilde{x}) + Var(\tilde{y})} (\tilde{x} + \tilde{y})$.

where, by construction, $Var(\omega_1) = \left(\frac{k_i+k_j}{\lambda_1}\right)^2 + \frac{k_i^2}{\lambda_1^2\tau_i} + \frac{k_j^2}{\lambda_1^2\tau_j} + \frac{\eta_1^2}{\tau_1}$.

Using the weak efficiency of price, equation (3) yields $\mu_1 + \lambda_1\omega_1 = \frac{(k_i+k_j)\omega_1}{\lambda_1 Var(\omega_1)}$. Identifying μ_1 and λ_1 in this linear equation, we get $\mu_1 = 0$ and $\lambda_1^2 Var(\omega_1) = k_i + k_j$. Developing the variance, we solve for the inverse of the market depth of the first period, obtaining

$$\lambda_1 = \frac{\sqrt{\tau_1}}{\eta_1}\sqrt{k_i + k_j - (k_i + k_j)^2 - k_i^2\tau_i^{-1} - k_j^2\tau_j^{-1}} \tag{4}$$

The ex-ante variance of exogenous trade (τ_1) positively influence the depth of the market. To find λ_2 and μ_2, we use similar, albeit more complex, techniques in the Appendix. Both values depend on the first period variables k_i, k_j, on the second period ones m_i, m_j, η_2 and on the fundamental parameters of the model $\tau_i, \tau_j, \tau_1$and τ_2.

The variance of the asset conditional on the information revealed in the first stage is $\Sigma_1 \equiv E\left[(v - E[v \mid \omega_1])^2\right]$. It is equal to $1 - Var(E[v \mid \omega_1]) = 1 - \frac{(k_i+k_j)^2}{\lambda_1^2 Var(\omega_1)}$ as the conditional expectation is an orthogonal projector. Developing $Var(\omega_1)$ and applying (4) leads to

$$\Sigma_1 = 1 - \frac{(k_i+k_j)^2}{(k_i+k_j)^2 + k_i^2\tau_i^{-1} + k_j^2\tau_j^{-1} + \lambda_1^2\eta_1^2\tau_1^{-1}} = 1 - k_i - k_j \tag{5}$$

3.2 Both Traders Conceal Information

In this subsection we derive the optimal trading strategies conditional on the decision to conceal information in the first stage. In this *CC* case, trader j places an order $q_{1,j}$ independent of his private signal. As trader i chooses to conceal his own information he places a market order $q_{1,i}$ indifferent to his signal s_i. The first stage order flow $\omega_1 = q_{1,i} + q_{1,j} + u_1$ therefore contains no information on the underlying value of the asset. The equilibrium price is then $E[v \mid \omega_1] = E[v \mid p_0] = p_0$, the closing price of the previous day which is normalized to zero (as it is assumed to be efficient). The zero profit condition for market makers leads to $0 = p_1(\omega)$ so that any $q_{1,i}$ is indeed optimal. The commitment by informed traders to conceal their information drives their first period profits to zero on average. In terms of sensitivity of the order book ω_1 with respect to the private signals, we obtain $k_i = k_j = 0$ and $\eta_1 = \lambda_1 = 1$.

The second stage starts with an empty public information set $\mathcal{H}_2 = \{\emptyset\}$ and private information set $\mathcal{H}_{2,i} = \{s_i\}$. The ex-post profit for trader i of a trade $q_{2,i}$ is $(v - p_2)q_{2,i}$. As it is always optimal to use its information in the last stage of the game, the ex-interim profit is

$$\Pi_{2,i}(\mathcal{H}_{2,i}) = q_{2,i}(s_i)E[v - p_2 \mid \mathcal{H}_{2,i}] \tag{6}$$

Substituting for the market makers strategy $p_2(\omega_2) = \mu_2 + \lambda_2\omega_2$ and using the order flow decomposition $\omega_2 = q_{2,i} + q_{2,j} + u_2$, where u_2 is known to trader i, the expected profit is

$$\begin{aligned}\Pi_{2,i}(\mathcal{H}_{2,i}) &= q_{2,i}E[v - \mu_2 - \lambda_2(q_{2,i} + q_{2,j} + u_2) \mid \mathcal{H}_{2,i}] \\ &= q_{2,i}\left(E[v \mid \mathcal{H}_{2,i}] - \mu_2 - \lambda_2 q_{2,i} - \lambda_2 E[q_{2,j} \mid \mathcal{H}_{2,i}] - \lambda_2 u_2\right)\end{aligned}$$

Using the specific form of trader j's linear strategy $q_{2,j}(\mathcal{H}_{2,j}) = \alpha_{2,j} + \beta_{2,j} E[v \mid \mathcal{H}_{2,j}]$, the first order condition (FOC) for trader i reads[7]

$$\begin{aligned} 2\lambda_2 q_{2,i} &= E[v \mid \mathcal{H}_{2,i}] - \mu_2 - \lambda_2 u_2 - \lambda_2 E[q_{2,j} \mid \mathcal{H}_{2,i}] \\ &= E[v \mid \mathcal{H}_{2,i}] - \mu_2 - \lambda_2 u_2 - \lambda_2 \alpha_{2,j} - \lambda_2 \beta_{2,j} E[E[v \mid \mathcal{H}_{2,j}] \mid \mathcal{H}_{2,i}] \end{aligned} \tag{7}$$

The projection theorem for normal random variables yields $E[v \mid \mathcal{H}_{2,i}] = \frac{\tau_i}{\tau_i+1} s_i$. We then use $E[E[v \mid \mathcal{H}_{2,j}] \mid \mathcal{H}_{2,i}] = \frac{\tau_j}{\tau_j+1} E[v \mid \mathcal{H}_{2,i}]$ to identify the intercept and the slope of the linear strategy: $\alpha_{2,i} = \frac{-\mu_2 - \lambda_2 \alpha_{2,j} - \lambda_2 u_2}{2\lambda_2}$ and $\beta_{2,i} = \frac{1-\lambda_2 \beta_{2,j} \frac{\tau_j}{\tau_j+1}}{2\lambda_2}$. Putting together the symmetric equations for j, we solve the system to get $\alpha_{2,i} = -\frac{u_2}{3} - \frac{\mu_2}{3\lambda_2}$ and $\beta_{2,i} = \frac{(\tau_j+2)(\tau_i+1)}{\lambda_2(3\tau_i\tau_j+4\tau_j+4\tau_i+4)}$. The order flow is therefore

$$\omega_2 = -\frac{2\mu_2}{3\lambda_2} + \frac{\beta_i \tau_i}{\tau_i+1}(v+\varepsilon_i) + \frac{\beta_j \tau_j}{\tau_j+1}(v+\varepsilon_j) + \frac{u_2}{3} \tag{8}$$

and given the assumed form

$$\omega_2 = \frac{m_i + m_j}{\lambda_2} v + \frac{m_i \varepsilon_i + m_j \varepsilon_j}{\lambda_2} + \eta_2 u_2 \tag{9}$$

we can identify the coefficients of the various independent random variables to obtain

$$m_i = \frac{(\tau_j+2)\tau_i}{3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4}, \quad \eta_2 = \frac{1}{3} \quad \text{and} \quad \mu_2 = 0$$

since there is no revelation in stage one.

Using the results for the first period we obtain (cf. Appendix) $\mu_2^{CC} = 0$ and

$$\lambda_2^{CC} = \frac{3\sqrt{\tau_2 Z(\tau_i, \tau_j)}}{3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4} \tag{10}$$

where $Z(\tau_i, \tau_j) = \tau_j \tau_i (2\tau_i\tau_j + 5\tau_i + 5\tau_j) + 4(\tau_i + \tau_j + 1)(\tau_j + \tau_i)$. We also obtain the conditional variance $\Sigma_2^{CC} = 1 - m_j - m_i = \frac{\tau_i\tau_j + 2\tau_j + 2\tau_i + 4}{3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4}$ which is a convex function increasing in both precisions.

We are now able to compute the expected profits of the players. From (8), we derive the optimal orders $q_{2,i}(s_i) = \alpha_{2,i} + \beta_{2,i} \frac{\tau_i s_i}{\tau_i+1} = \frac{1}{3\sqrt{\tau_2}} \left(\frac{(\tau_j+2)\tau_i}{\sqrt{Z(\tau_i,\tau_j)}} s_i - \sqrt{\tau_2} u_2 \right)$ and the equilibrium price is $p_2(s_i, s_j) = \lambda_2^{CC} \frac{u_2}{3} + \frac{(\tau_j+2)\tau_i s_i + (\tau_i+2)\tau_j s_j}{3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4}$. Notice that the equilibrium price function pools all the information as it depends on s_j and s_i.

The ex-interim profit of trader i is the expected profit conditional on its private information s_i. We average p_2 with respect to s_j using $E[s_j \mid s_i] = E[v + \varepsilon_j \mid s_i] = E[v \mid s_i] = \frac{\tau_i s_i}{\tau_i+1}$ (the error terms are independent random variables). Straightforward algebraic manipulations show that $E[v - p_2 \mid s_i] = \lambda_2 q_{2,i}(s_i)$ hence, substituting into (6) we get the traditional Cournot

[7] If traders were manipulating the market makers pricing rule they would take into account the fact that λ_2 is ultimately a function of q_i and q_j (defined by the efficiency condition).

formula for the expected profit conditional on the private signal in the second stage (ex-interim profit).

$$\begin{aligned}\Pi_{2,i}^{CC}(s_i) &= \lambda_2^{CC} q_{2,i}(s_i)^2 \\ &= \frac{\sqrt{Z(\tau_i,\tau_j)}}{3\sqrt{\tau_2}(3\tau_i\tau_j+4\tau_j+4\tau_i+4)}\left(\frac{(\tau_j+2)\tau_i}{\sqrt{Z(\tau_i,\tau_j)}}\, s_i - \sqrt{\tau_2}u_2\right)^2 \end{aligned} \tag{11}$$

To compute the ex-ante payoffs $\Pi_{2,i}^{CC} = E\left[\Pi_{2,i}^{CC}(s_i)\right]$ (prior to the realization of s_i and u_2) we use $E\left[s_i^2\right] = \frac{1+\tau_i}{\tau_i}$, $E\left[u_2^2\right] = \tau_2^{-1}$ and the independence of s_i and u_2. We obtain an increasing and concave function of both precisions τ_i and τ_j

$$\Pi_i^{CC} = \frac{(\tau_j+2)^2(1+\tau_i)\tau_i + Z(\tau_i,\tau_j)}{3\sqrt{\tau_2 Z(\tau_i,\tau_j)}\,(3\tau_i\tau_j+4\tau_j+4\tau_i+4)} \tag{12}$$

We check that $\frac{\partial \Pi_i^{CC}}{\partial \tau_i} > 0$ as it is a polynomial fraction involving only positive coefficients while $\frac{\partial \Pi_i^{CC}}{\partial \tau_j} < 0$ is verified only by a graphical representation. Notice also that a greater precision on information $(\tau_i > \tau_j)$ pays more $\left(\Pi_i^{CC} > \Pi_j^{CC}\right)$ since $\frac{\partial \Pi_i^{CC}}{\partial \tau_i} > \frac{\partial \Pi_j^{CC}}{\partial \tau_i}$ holds when evaluated at $\tau_i = \tau_j$.

3.3 Asymmetric Behavior

We now consider the most interesting case of asymmetric behavior: one player conceals his private information in stage 1 to better use it in stage 2. In the *RC* case, trader i reveals optimally while trader j conceals optimally. Since $\Pi_i^{CR}(\tau_j,\tau_i) = \Pi_j^{RC}(\tau_i,\tau_j)$ we shall compute both traders payoff in the *RC* case to obtain i' payoff in the *CR* case taking care of the reversal of precisions.

We solve the *RC* case by backward induction since in the first stage, the optimal strategy for the trader who reveals depends on the second stage behaviors. At the beginning of stage 2, the public information in the present context is $\mathcal{H}_2 = \{s_i\}$, thus $\mathcal{H}_{2,i} = \{s_i\}$ and $\mathcal{H}_{2,j} = \{s_j, s_i\}$ so that $E\left[E\left[v \mid \mathcal{H}_{2,j}\right] \mid \mathcal{H}_{2,i}\right] = E\left[v \mid \mathcal{H}_{2,i}\right] = \frac{s_i\tau_i}{\tau_i+1}$ and $E\left[E\left[v \mid \mathcal{H}_{2,i}\right] \mid \mathcal{H}_{2,j}\right] = E\left[v \mid \mathcal{H}_{2,j}\right] = \frac{s_i\tau_i+s_j\tau_j}{\tau_j+\tau_i+1}$.

The FOC for trader i is still

$$2\lambda_2 q_{2,i}(\mathcal{H}_i) = E\left[v \mid \mathcal{H}_{2,i}\right] - \mu_2 - \lambda_2 u_2 - \lambda_2 E\left[q_j \mid \mathcal{H}_{2,i}\right] \tag{13}$$

but as trader j knows $\mathcal{H}_{2,i}$, its FOC reads

$$2\lambda_2 q_{2,j}(\mathcal{H}_{2,j}) = E\left[v \mid \mathcal{H}_{2,j}\right] - \mu_2 - \lambda_2 u_2 - \lambda_2 q_{2,i}(\mathcal{H}_{2,i}) \tag{14}$$

Trader i takes into account the fact that he is revealing his information to trader j; he therefore integrates (14) into (13) to find its optimal response:

$$q_{2,i}(s_i) = \frac{s_i\tau_i}{3\lambda_2(\tau_i+1)} - \frac{u_2}{3} - \frac{\mu_2}{3\lambda_2} \tag{15}$$

We can now solve for the more informed trader j substituting (15) in (14)

$$q_{2,j}(s_i,s_j)=\frac{s_i\tau_i+s_j\tau_j}{2\lambda_2(\tau_j+\tau_i+1)}-\frac{s_i\tau_i}{6\lambda_2(\tau_i+1)}-\frac{u_2}{3}-\frac{\mu_2}{3\lambda_2} \tag{16}$$

Observe from (15) that the intensity parameter for trader i is $\beta_i=\frac{1}{3\lambda_2}$ like in the CC case while trader j has a more complex behavior since he puts more weight on his own information. The difference of behavior between traders rests on variations of precision and ex-post signals.

As we did with (8) and (9), we check that $E[p_2 \mid s_j,s_i]=p_2(s_i,s_j)$ and $E[v-p_2 \mid s_i]=\lambda_2 q_{2,i}(s_i)$, thus the ex-interim profits in stage two are

$$\Pi_{2,i}^{RC}(s_i) = \lambda_2 q_{2,i}(s_i)^2 \tag{17}$$

$$\Pi_{2,j}^{RC}(s_j,s_i) = \lambda_2 q_{2,j}(s_i,s_j)^2 \tag{18}$$

Proceeding backward we can now study the first period equilibrium. To comply with our previous notations on the decomposition of the order flow, we write the linear pricing rule of the market makers as $p_1(\omega_1)=\mu_1+\lambda_1(\omega_1-E[\omega_1])$ since in the present case the constant demand of trader j adds a non informative constant term in the first period order flow. The first stage profit for trader i is

$$\Pi_1^{RC}(\mathcal{H}_{1,i})=q_{1,i}(\mathcal{H}_{1,i})E\left[v-\mu_1-\lambda_1(q_{1,i}+q_{1,j}+u_1-E[\omega_1]) \mid \mathcal{H}_{1,i}\right]$$

Solving the two maximization problems (with the same procedure used before) gives

$$q_{1,i} = -\frac{\mu_1}{3\lambda_1}-\frac{u_1}{3}+\frac{s_i\tau_i}{2\lambda_1(\tau_i+1)} \tag{19}$$

$$q_{1,j} = -\frac{\mu_1}{3\lambda_1}-\frac{u_1}{3} \tag{20}$$

Notice that $q_{1,j}$ is independent of v since player j voluntary ignores his private information at stage 1; however it optimally uses the noise trade u_1 by going against it. As trader i is revealing its private information, the order flow $\omega_1=\frac{k_i}{\lambda_1}(v+\varepsilon_i)+\frac{u_1}{3}+\rho_1$ observed by market makers conveys some of this information and we derive $k_i=\frac{\tau_i}{2(1+\tau_i)}$, $k_j=0$, $\eta_1=\frac{1}{3}$ and $\rho_1=-\frac{2\mu_1}{3\lambda_1}$. Recalling that μ_1 is always nil (i.e., yesterday's closing price was efficient) we get $\rho_1=0$ and from (4) we obtain

$$\lambda_1^{RC}=\frac{3}{2}\sqrt{\frac{\tau_1\tau_i}{\tau_i+1}} \tag{21}$$

As expected, λ_1^{RC} is increasing in τ_i because the more precise the signal, the more aggressive the revealing trader and the more aggressive the market makers response which ultimately reduces the depth of the market.

We now compute the expected profits for the players. Integrating (21) into optimal demands (15,16), we get the ex-interim profits

$$\Pi_{1,i}^{RC}(s_i) = \lambda_1^{RC}\left(\frac{s_i\tau_i}{2\lambda_1^{RC}(1+\tau_i)}-\frac{u_1}{3}\right)^2 \tag{22}$$

$$\Pi_{1,j}^{RC}(s_i) = \lambda_1^{RC}\frac{u_1^2}{9} \tag{23}$$

Taking into account $E\left[s_i^2\right]=\frac{1+\tau_i}{\tau_i}$, $E\left[u_1^2\right]=\tau_1^{-1}$ and the independence of s_i and u_2, the ex-ante expectations are

$$\Pi_{1,i}^{RC} = \frac{1}{3\sqrt{\tau_1}}\sqrt{\frac{\tau_i}{\tau_i+1}} \quad (24)$$

$$\Pi_{1,j}^{RC} = \frac{1}{2}\Pi_{1,i}^{RC} \quad (25)$$

As intuition would suggest, there is a first stage advantage to reveal since $\Pi_{1,i}^{RC}>0$ while we saw that $\Pi_{1,i}^{CC}=0$. The revealing trader is also earning more than the concealing one who nevertheless takes advantage of the quality of the information revealed as $\Pi_{1,j}^{RC}$ increases with τ_i.

Having solved the price equilibrium of both stages, we can find out the updating performed by the market makers in the second period. Using the equilibrium values of $k_i, k_j, \eta_1, m_i, m_j$ and η_2 we obtain a fairly complex formula for λ_2^{RC} (cf. (41) in appendix) which is positive like λ_1^{RC} i.e., the market makers are spreading their reactiveness towards adverse selection over the two periods of trade.

The novelty of the RC case with respect to the CC case is the *memory effect* within the second period pricing rule; it is given by

$$\mu_2^{RC}=\frac{4\tau_i+\tau_i\tau_j+3\tau_j+6-2\tau_i^2}{12(\tau_i+\tau_j+1)(\tau_i+1)^2}\left(\tau_i s_i+\sqrt{(\tau_i+1)\tau_i\tau_1}u_1\right)$$

and enters into the second period profits (17) and (18). The ex-interim payoffs are

$$\Pi_{2,i}^{RC}(s_i) = \frac{1}{9\lambda_2^{RC}}\left(\frac{\tau_i s_i}{\tau_i+1}-\lambda_2^{RC}u_2-\mu_2^{RC}\right)^2 \quad (26)$$

$$\Pi_{2,j}^{RC}(s_j,s_i) = \frac{1}{9\lambda_2^{RC}}\left(\frac{3(s_i\tau_i+s_j\tau_j)}{2(\tau_i+\tau_j+1)}-\frac{\tau_i s_i}{2(\tau_i+1)}-\lambda_2^{RC}u_2-\mu_2^{RC}\right)^2 \quad (27)$$

and their ex-ante expectation yields

$$\Pi_{2,i}^{RC} = \frac{G(\tau_i,\tau_j)}{162(\tau_i+\tau_j+1)\sqrt{\tau_2(1+\tau_i)^3Y(\tau_i,\tau_j)}}$$

$$\Pi_{2,j}^{RC} = \frac{H(\tau_i,\tau_j)\sqrt{2}}{162(\tau_i+\tau_j+1)\sqrt{\tau_2(1+\tau_i)^3Y(\tau_i,\tau_j)}}$$

where G and H are polynomial in τ_i and τ_j with positive coefficients. The global ex-ante profit, $\Pi_i^{RC}=\Pi_{1,i}^{RC}+\Pi_{2,i}^{RC}$ is increasing and concave in τ_i and τ_j but barely depends on the latter.

When trader i uses his private information in stage 1, he makes profits but he starts stage 2 with a handicap having made the second period price come closer to its own estimation of v. Whatever the ranking between τ_i and τ_j, trader j starts stage 2 with a better information since it aggregates $\mathcal{H}_{1,i}$ and $\mathcal{H}_{1,j}$. Noticing that hedging and liquidity orders are presumably spread over the day in an equal manner ($\tau_1=\tau_2$), we are now in position to derive a first result.

Lemma 1 *If liquidity trade is constant across stages ($\tau_1 = \tau_2$), it is optimal to reveal its information when the opponent conceals it:* $\Pi_i^{RC} > \Pi_i^{CC}$.

Proof The claim is true if

$$\frac{1}{3}\sqrt{\frac{\tau_i}{\tau_i+1}} = \Pi_{1,i}^{RC} > \Pi_i^{CC} = \Pi_{2,i}^{CC} = \frac{(\tau_j+2)^2(1+\tau_i)\tau_i + Z(\tau_i,\tau_j)}{3\sqrt{Z(\tau_i,\tau_j)}(3\tau_i\tau_j+4\tau_j+4\tau_i+4)}$$

$$\Leftrightarrow (3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)^2\, \tau_i Z - (1+\tau_i)\left((\tau_j+2)^2(1+\tau_i)\tau_i - Z\right)^2 > 0$$

and this τ_j-polynomial has only positive coefficients over the range $\tau_i \geq 1$. Hence $\Pi_i^{RC} > \Pi_i^{CC}$ follows immediately for $\tau_1 = \tau_2$. *Q.E.D.*

Thus, a hit and run strategy (reveal at stage 1 and leave the market) is better than waiting the second period to trade if one guesses that its opponent will trade on information at stage 2, triggering an aggressive reaction from market makers.

Recalling that the precision of the underlying asset v is normalized to unity and using $E[v \mid s_i] = \frac{\tau_i s_i}{1+\tau_i}$, the variance of v not explained by the private signals is $Var(v \mid s_i, s_j) \equiv E\left[(v - E[v \mid s_i, s_j])^2\right] = \frac{1}{\tau_i+\tau_j+1}$ since v, ε_i and ε_j are independent variables. Hence the variance explained by (s_i, s_j) is $\frac{\tau_i+\tau_j}{\tau_i+\tau_j+1}$.

When trader i conceals its information in the first round while j reveals it, the value of private information for trader i passes from $\frac{\tau_i}{1+\tau_i}$ to $\frac{\tau_i+\tau_j}{\tau_i+\tau_j+1}$ since he learns $s_j$while the value of private information for trader j remains $\frac{\tau_j}{1+\tau_j}$. It is immediate to see that the gain for trader i is quickly decreasing with τ_i. In other words, waiting to learn s_j makes sense for trader i only if he is poorly informed. This is what will occur in equilibrium.

3.4 Both Traders Reveal Information

In this final RR case, traders play in the first period as if they were in the second period without any previously revealed information. Accordingly the analysis proceeds in a relative straightforward way. The second stage analysis of the CC case yields immediately the first stage of the RR case by changing τ_2 into τ_1.

We obtain $k_i > 0, k_j > 0$ and $\eta_1 > 0$ i.e., traders use their information against market makers and also trade against the noise traders. The market depth is thus positive ($\lambda_1^{RR} > 0$) and there is no history effect ($\mu_1 = 0$). The expectation of the ex-interim profit is taken from $\Pi_{2,i}^{CC}$ and yields

$$\Pi_{1,i}^{RR} = \frac{(\tau_j+2)^2(1+\tau_i)\tau_i + Z(\tau_i,\tau_j)}{3(3\tau_i\tau_j + 4\tau_j + 4\tau_i + 4)\sqrt{\tau_1 Z(\tau_i,\tau_j)}} \tag{28}$$

As intuition suggest, there is a first period advantage to reveal its information: $\Pi_{1,i}^{RR} > \Pi_{1,i}^{CR}$ (this difference is a polynomial in τ_i and τ_j with positive coefficients). Furthermore, since trader

j is always revealing, the better informed j is, the better it is for trader i to use that public information for himself; this is why $\Pi_{1,i}^{RR} - \Pi_{1,i}^{CR}$ increases with τ_j.

In the second stage, the public information in this case is $\mathcal{H}_2 = \{s_j, s_i\} = \mathcal{H}_{2,i} = \mathcal{H}_{2,j}$, thus $E[v \mid \mathcal{H}_{2,i}] = \frac{s_i\tau_i + s_j\tau_j}{\tau_j+\tau_i+1}$. Using the same resolution method, we find the parameters of the optimal revealing strategies $\alpha_{2,i} = -\frac{u_2}{3} - \frac{\mu_2}{3\lambda_2}$ and $\beta_{2,i} = \frac{1}{3\lambda_2}$. Now, as

$$\omega_2 - E[\omega_2] = \frac{u_2}{3} - \frac{2\mu_2}{3\lambda_2} + (\beta_i + \beta_j)\frac{s_i\tau_i + s_j\tau_j}{\tau_j + \tau_i + 1} = \frac{m_i s_i + m_j s_j}{\lambda_2} + \eta_2 u_2 \tag{29}$$

we are able to identify $m_i = \frac{\tau_i}{3(\tau_j+\tau_i+1)}$ and $\eta_2 = \frac{1}{3}$. This leads to a complex formulae for λ_2^{RR}, μ_2^{RR} and the ex-ante profit $\Pi_{2,i}^{RR}$. The overall expected payoff $\Pi_i^{RR} = \Pi_{1,i}^{RR} + \Pi_{2,i}^{RR}$ is an increasing and concave function of τ_i.

4 Equilibrium Revelation of Information

In this section we characterize the equilibrium of the information revelation game. Before this we restate the "size" effect of liquidity trade already pointed out by Admati and Pfleiderer (1988). As intuition suggests, the bigger the volume of trade present in the market for hedging or other liquidity reasons, the higher the profit insiders can get. Hence, if stage 2 is significantly more liquid than stage 1, traders have an incentive to hide their information in order to exploit it successfully in the last stage.

Lemma 2 *There exists $S(\tau_i, \tau_j)$ such that the unique equilibrium is (C, C) as soon as $\sqrt{\frac{\tau_1}{\tau_2}} > \max\left\{\frac{1}{S(\tau_i,\tau_j)}, \frac{1}{S(\tau_j,\tau_i)}\right\}$.*

Proof Algebraic manipulation enable to show that

$$\Pi_i^{CC}\left(\tau_i, \tau_j, \frac{\tau_2}{\tau_1}\right) > \Pi_i^{RC}\left(\tau_i, \tau_j, \frac{\tau_2}{\tau_1}\right) \Leftrightarrow \sqrt{\frac{\tau_2}{\tau_1}} < S(\tau_i, \tau_j)$$

where $S(\tau_i, \tau_j) = \sqrt{\frac{\tau_i+1}{Z\tau_i}}\left(\frac{(\tau_j+2)^2(1+\tau_i)\tau_i+Z}{(3\tau_i\tau_j+4\tau_j+4\tau_i+4)} - \sqrt{\frac{Z}{Y(1+\tau_i)^3}}\frac{G}{54(\tau_i+\tau_j+1)}\right)$.

Hence it is optimal for i to conceal if j does likewise when the liquidity ratio $\sqrt{\frac{\tau_1}{\tau_2}}$ is larger than $\frac{1}{S(\tau_j,\tau_i)}$ which an increasing concave function. Given our choice of range for precisions τ_i and τ_j, the minimum of $\frac{1}{S}$ is $\frac{1}{S(1/2,1/2)} \simeq 2.11$. Other algebraic manipulations enable to show that

$$\Pi_i^{CR}\left(\tau_i, \tau_j, \frac{\tau_2}{\tau_1}\right) > \Pi_i^{RR}\left(\tau_i, \tau_j, \frac{\tau_2}{\tau_1}\right) \Leftrightarrow \sqrt{\frac{\tau_1}{\tau_2}} > \frac{1}{T(\tau_j, \tau_i)}$$

where $\frac{1}{T(\tau_j,\tau_i)}$, available upon request from the authors, is decreasing in both variables. The maximum is therefore $\frac{1}{T(1/2,1/2)} \simeq 1.65$. It is now clear that for $\sqrt{\frac{\tau_1}{\tau_2}} > \frac{1}{S} > \frac{1}{T}$, (R, R) is not an equilibrium configuration. From the same inequality we deduce that neither (R, C) nor (R, C)

are stable since the revealing trader has an incentive to conceal in the first period. The only possible equilibrium candidate is thus (C, C) and it is indeed an equilibrium. *Q.E.D.*

Roughly speaking if liquidity trade doubles from stage 1 to 2, both traders will prefer to wait to be able to use their private information at the right time. Let us stress that such an inefficient release of information can arise in highly transparency markets if the trading day involves moments of low and high activity. This is the case for instance in European stock exchanges at the opening of American markets.

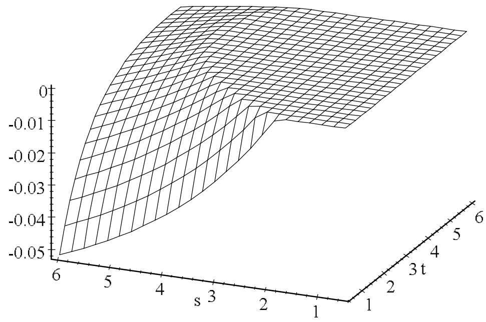
4.1 Asymmetric Information can matter

The previous analysis allows to characterize the equilibrium of our simple trading game. The literature (Kyle (1989), Holden and Subrahmanyam (1992), Foster and Viswanathan (1993) and (1996)) has shown that increasing competition between informed traders leads towards more efficient prices. As this result is obtained by keeping the average liquidity trade volume constant we shall also set $\tau_1 = \tau_2 = 1$ for the rest of the paper.

The following proposition is the central result of the paper; it illustrates when one of the traders prefers not to trade according to his own information in the first period. We shall use the terminology of Foster and Viswanathan (1996) and call this situation the "waiting game".

Proposition 1 *The set of Bayesian-Nash equilibria in truthful revealing strategies is*
► $(\sigma_i, \sigma_j) = (R, R)$ *if* $\tau_j \leq \frac{100+160\tau_i-15\tau_i^2}{100}$ *and* $\tau_i \leq \frac{100+160\tau_j-15\tau_j^2}{100}$
► $(\sigma_i, \sigma_j) = (C, R)$ *if* $\tau_j > \frac{100+160\tau_i-15\tau_i^2}{100}$ *and* $\tau_i \leq \frac{100+160\tau_j-15\tau_j^2}{100}$
► $(\sigma_i, \sigma_j) = (R, C)$ *if* $\tau_j \leq \frac{100+160\tau_i-15\tau_i^2}{100}$ *and* $\tau_i > \frac{100+160\tau_j-15\tau_j^2}{100}$
► *both* (R, C) *and* (C, R) *otherwise*

Proof: By Lemma 1, if a player conceals then the best reply for the other trader is to reveal. Therefore in equilibrium at least one trader (j w.l.o.g.) reveals which leads us to compare Π_i^{RR} and Π_i^{CR}. It is clear that by concealing its information in the first period trader i loses the opportunity of realizing profitable trades, thus $\Pi_{1,i}^{CR} < \Pi_{1,i}^{RR}$ (cf. section 3.4). In the second period the concealing trader i has now a superior information as he "holds" s_i and s_j; thus he earns $\Pi_{2,i}^{CR} > \Pi_{2,i}^{RR}$. The trade-off between revealing and concealing is solved when $\Pi_i^{RR}(\tau_i, \tau_j) = \Pi_i^{CR}(\tau_i, \tau_j)$. As this is polynomial equation of degree 9 in τ_i and τ_j we have to rely on a numerical solution. We approach the inequality $\Pi_i^{CR} \leq \Pi_i^{RR}$ by $\tau_j \leq f(\tau_i) \equiv \frac{100+160\tau_i-15\tau_i^2}{100}$ $(\tau_i \geq \frac{1}{2})$. On figure 1 below we plot $\min\left(0, \Pi_i^{RR} - \Pi_i^{CR}\right)$.



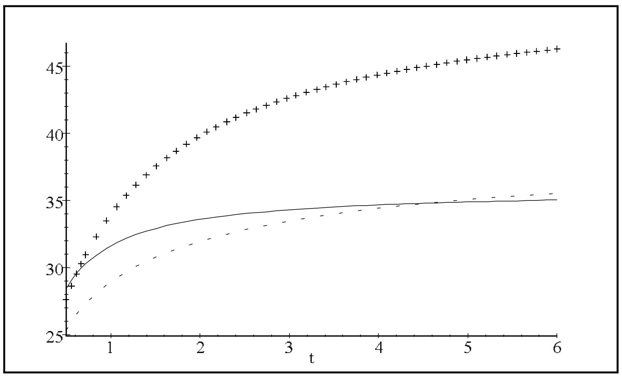
$\min\left(0, \Pi_i^{RR} - \Pi_i^{CR}\right)$ for $\tau_i = t, \tau_j = s$

Figure 1

Roughly speaking, when τ_i is larger than τ_j (i.e., $\tau_j \leq f(\tau_i)$) trader i optimally reveals. For (R, R) to be an equilibrium it is necessary that R is also the optimal strategy for the other trader j i.e., $\tau_i \leq f(\tau_j)$ must hold. If τ_i is too large before τ_j then j prefers to conceal in the first period. Symmetrically if τ_j is large before τ_i it is trader i who prefers to wait.

When both τ_i and τ_j are larger than 5 (the intersection of the two curves $f(\tau_i)$ and $f(\tau_j)$) the information game is a *preemption* game since the first mover forces the other to conceal and obtains a higher payoff $\left(\Pi_i^{RC}\right)$ than if he were the second mover $\left(\Pi_i^{CR}\right)$ as seen on Figure 2 below. There thus exists two asymmetric equilibria (R, C) and (C, R). *Q.E.D.*

As a consequence of the above analysis there is inefficient information disclosure as soon as one trader has a signal explaining more than 82% of the underlying asset variance (cf. $\tau \geq 4.5$ on Figure 2 below). However this case should not be very relevant since information is costly to acquire.



Π_i^{RR} (solid), Π_i^{CR} (dash) and Π_i^{RC} (stars) for $\tau_i = \tau_j = t$

Figure 2

The main practical conclusion we can draw from Proposition 1 is that efficient revelation of information takes place only for similar signal precisions. Otherwise the lesser informed trader prefers to make a low profit in the first period by going only against liquidity traders and be the best informed in the last stage rather than diluting his private information to soon.

The economic interpretation of the inefficient equilibrium (R, C) is that the better informed trader trades more aggressively because market depth is high. Indeed market makers are willing to provide more depth than they otherwise would since they cannot distinguish between orders arising from informed or uninformed traders, and they know that in this equilibrium only one insider uses its signal. By giving a relatively higher depth to the market, they induce some information revelation (in particular, the more precise one), discriminating between insiders since this incentive is not sufficient for the less informed trader to reveal.

This effect is a consequence of the transparent microstructure of the exchanges, and it is not present in previous model as in Foster and Viswanathan (1996), where insiders wait to reveal their signals only when these are negatively correlated among themselves.

4.2 Trading Costs and Efficiency in Equilibrium

The main motivation to study a different microstructure of exchanges and a different information structure from the existing literature relies on the consequences that these two elements induce on the equilibrium liquidity dynamics and the degree of price efficiency.

The liquidity of the market, measured here by the price sensitivity to trade λ gives us an idea of the cost of trading in such a market: a very thin market (high λ) generates a high cost of trading via the effect that individual trading has on price. The degree of price efficiency is given by $\Sigma_t \equiv E\left[(v - p_t)^2 \mid H_t\right]$ the residual variance after the information publicly revealed has been impounded into the price.

We describe the dynamics of λ_t and Σ_t pointing out the difference with the predictions of existing works, especially Holden and Subrahmanyam (1992) and Foster and Viswanathan (1996). With this comparison, we can better understand the effect of the particular asymmetry of information assumed here, as well as the effect of the strategic interaction between insiders in a quotation system with extreme transparency.

4.2.1 Liquidity dynamics

It is immediate to see that whatever the equilibrium, the market depth is never constant across periods as opposed to the monopolist case of Kyle (1985) but similarly to the rest of the literature. Since the degree of correlation between signals and the length of the game are both fixed, we can make some comparative statics analysis changing the precisions τ_i and τ_j.

For τ_i close to τ_j, the unique equilibrium is (R, R). In that case the sensitivity λ of the market makers to the order flow is initially high and then decreases as $\lambda_1^{RR} > \lambda_2^{RR}$.[8] This result is consistent with the behavior of informed traders in equilibrium: adverse selection being higher at period 1 the market makers react with a steeper price function.

When the private signals have different precisions, the player with a better quality of information reveals while the opponent plays an order independent of its own signal (equilibrium (R, C)). For large asymmetries[9] we can show that $\lambda_1^{RC} < \lambda_2^{RC}$ thus the sensitivity factor λ_t increases because there is still a strong adverse selection in the second stage. The intuition here is that market makers price smoothly to incite traders to reveal information and then switch to a more aggressive pricing rule in the second period as there is still an important adverse selection in the market.

An increasing sensitivity factor λ_t is also obtained by Foster and Viswanathan (1996), but only when the information of the insiders is negatively correlated which is not the case here. The reason for this result in Foster and Viswanathan (1996) is that informed trader i makes a small trade in the first period as he thinks that informed trader j is pulling the stock price in the wrong direction and perceive the possibility of making large profits in the future by postponing its trade (that otherwise would pull the price in the "right" direction immediately).

In our model the rationale is different; the player who receives the less valuable information, because he has observed a very noisy signal, prefers to wait to post a trade dependent on that signal, since he knows that in the second period he will be able to update his own information much better than the average (i.e. the market makers). Indeed he will possess two signals over the distribution of v while his opponent has just one. The waiting behavior is then arising from a strategic interaction, and not as a consequence of a statistical characteristic of the model. We may interpret the competition in information as a competition between oligopolist with different cost structures where the most efficient acts as a leader, taking a first mover advantage.

Notice also that $\lambda_1^{RR} > \lambda_1^{RC}$ should hold because when two insiders reveal information (case RR), market makers are more reactive than when only one reveals (case RC). Yet when precisions are highly asymmetric[10] (within the (R, C) equilibrium area) the reverse inequality holds due to the strong competition among insiders. Their trading intensity factor $k_i^{RR} = \frac{\tau_i}{3(\tau_j+\tau_i+1)}$ is low and the overall effect on market makers reactiveness is weaker than if there is only one insider actively using (and revealing) his information because the latter act as an information monopolist and trades more aggressively with $k_i^{RC} = \frac{\tau_i}{2(1+\tau_i)}$ and this causes a very

[8] $\Leftrightarrow \frac{3\sqrt{Z(\tau_i,\tau_j)}}{3\tau_i\tau_j+4\tau_j+4\tau_i+4} > \frac{\sqrt{K(\tau_i,\tau_j)}}{(\tau_i+\tau_j+1)\sqrt{2L(\tau_i,\tau_j)}} \Leftrightarrow 0 < 18\left(\tau_i+\tau_j+1\right)^2 ZL - \left(3\tau_i\tau_j+4\tau_j+4\tau_i+4\right)^2 K$ and this polynomial has only positive coefficients.

[9] The exact equation defining the area are available upon request but an approximation is $\tau_i \geq 3.8$ and $\tau_j \leq 1.35(\tau_i - 3.7)$; it forms an area of the (τ_i, τ_j) space where the equilibrium is (R, C).

[10] $\lambda_1^{RR} < \lambda_1^{RC} \Leftrightarrow 4Z(\tau_i,\tau_j)\left(\tau_i+1\right) - \tau_i\left(3\tau_i\tau_j+4\tau_j+4\tau_i+4\right)^2 < 0 \Leftrightarrow \tau_i \geq 2$ and $\tau_j < \frac{4(\tau_i+1)(\tau_i-2)}{6\tau_i+8-\tau_i^2}$

aggressive response of the market makers so that $\lambda_1^{RC} > \lambda_1^{RR}$.

Using the terminology of Foster and Viswanathan (1996), we can conclude that even if the insiders have the same opinion about the value of an asset, two effects arise: a competition effect ("rat race") and a strategic leadership effect ("waiting game") due to the precision of the information; when the quality of the signals is similar, the first effect prevails, otherwise we can observe the second phenomenon.

4.2.2 The speed of information revelation

The effect of information revelation on λ_t is overall due to the amount of information released to the market.

For asymmetric precisions the equilibrium is $(\sigma_i, \sigma_j) = (R, C)$. The variance of the asset value given the order flow observed in the first period is $\Sigma_1^{RC} = \frac{\tau_i + 2}{2(1+\tau_i)}$. It is independent of τ_j and decreasing in τ_i i.e., the more precise the signal of the revealing insider, the more precise the information contained by the market price and thus the lower the residual variance of v. As Σ_1^{RC} is convex in τ_i, this phenomenon accelerates with the precision τ_i.

When the lesser informed trader prefers to wait (precisions asymmetry), there is less information revelation than if he were to reveal its signal since[11] $\Sigma_1^{RC} > \Sigma_1^{RR}$. This explains why λ_t^{RC} can increase while λ_t^{RR} decreases. In the (R, C) equilibrium there is less revelation of information in the first period when compared to the (R, R) equilibrium; the price sensitivity to the order flow is also lower.

The fact that there is more trading in the first period of the (R, R) equilibrium may explain the higher sensitivity of market makers' pricing function. This implies that in transparent markets such as the CATS system in Toronto, our model predicts that the depth of the market is higher in the beginning of the trading day and lower as we move closer to the date of public information revelation than in less transparent markets such as the NYSE.

In other terms, the market makers in the equilibrium (R, C) are reducing λ_1 since they want to give the better informed player an incentives to reveal his information: in this way, they leave for the second period all the informative trading activity that relies on less precise signals.

The overall picture seems to suggest that an higher transparency microstructure is not beneficial in terms of efficiency of prices. Indeed it creates an additional strategic effect that induces insiders with less precise signals to wait before revealing them. This effect arises only when the difference in information quality is relevant, i.e. in the (R, C) equilibrium. In terms of trading costs, we also find, as Madhavan, Porter and Weaver (2000) that higher transparency does not imply lower costs, given that we use λ_t as a measure of these costs.

[11] $\Sigma_1^{RR} = \frac{\tau_i \tau_j + 2\tau_j + 2\tau_i + 4}{3\tau_i \tau_j + 4\tau_j + 4\tau_i + 4}$ is decreasing convex in both precisions and $\Sigma_1^{RC} - \Sigma_1^{RR}$ is proportional to $\tau_j (\tau_i + 2)^2$.

To end the comparison with the existing microstructure results on private information, we study the profit losses of an information monopolist when a second informed trader enters the market. When information is an homogeneous good the Bertrand competition drives profits to zero. Here the heterogeneity of the signals received by insiders smooth out this competition. Nevertheless, the lower are the duopoly profits, the lower is the incentive for individuals to gather information if at least someone else is already doing it.

We show in the appendix that the optimal strategy for the information monopolist is to reveal its information immediately. The total ex-ante profit (keeping constant liquidity trade between periods) is the increasing concave function

$$\bar{\Pi}(\tau_i) = \frac{1}{2}\sqrt{\frac{\tau_i}{\tau_i + 1}}\left(1 + \frac{4\tau_i^2 + 2\tau_i + 3}{4(\tau_i + 1)\sqrt{1 + \tau_i^2}}\right)$$

If a challenger enters the market with the same precision then the profit of the incumbent drops by at least 44% and up to 60% when the common precision ranges in $\left[\frac{1}{2}; 6\right]$ (the equilibrium being (R, R)).

5 Conclusion

In the studies that address the problem of aggregation of information by equilibrium prices, the role of asymmetries between informed traders has been rarely analyzed. The results of Kyle (1985), Holden and Subrahmanyam (1992) consider the equilibrium in a centralized, order-driven continuous financial market with a monopolist of information or more competitors with the same information. Foster and Viswanathan (1996) study the competition between asymmetrically informed traders, but consider a signal space where insiders receive signals with the same precision but with some positive or negative correlation.

In this article we have shown that the role of asymmetries is crucial in order to assess the efficiency properties of prices. We have studied the strategic interaction between insiders who are able to observe the orders of each other, as in transparent markets such as the Toronto Stock Exchange. In our model this feature can induce a slow process of information revelation if the private signals, even being positively correlated, have highly different precisions. We characterize the linear, pure strategies equilibrium set as a function of the precision of private signals and the volume of liquidity trade present on the market. Keeping aside the influence of the latter we show that the more precise the signal, the higher the incentive to reveal it at the first stage. Yet the optimal response of a lesser informed trader can be to hide its own information during the first stage. Asymmetric equilibria hence arise if the insides have considerably different precision. In the symmetric case, we find the result of Holden and Subrahmanyam (1992) as a special case.

The model presented here has shown that the competition between asymmetrically informed traders has a really different nature from the Bertrand competition taking place between equally informed insiders. In particular, we single out equilibria where the depth of the market decreases as we approach the public announcement of the information. This is due to a purely strategic "waiting" of the lesser informed trader, and not to differences in opinions as in Foster and Viswanathan (1996). The higher transparency structure of electronic exchanges causes then a more sophisticated interaction between traders, and the overall effect on the amount of information incorporated into prices can be negative.

Our result is also supported by the empirical evidence of Madhavan, Porter and Weaver (2000) who observed an increase in trading costs in the Toronto CATS after the reform of 1990 that increased the transparency in that market structure.

6 Appendix

6.1 Market marker pricing rule in the second period

We derive here the market marker pricing rule in the second period depending on the order flow ω_1 and ω_2. Given that the only random variables are $v, \varepsilon_i, \varepsilon_j, u_1$and u_2 we may set

$$\omega_1 = \frac{k_i + k_j}{\lambda_1} v + \frac{k_i \varepsilon_i}{\lambda_1} + \frac{k_j \varepsilon_j}{\lambda_1} + \eta_1 u_1 \quad (30)$$

$$\omega_2 = \frac{m_i + m_j}{\lambda_2} v + \frac{m_i \varepsilon_i}{\lambda_2} + \frac{m_j \varepsilon_j}{\lambda_2} + \eta_2 u_2 \quad (31)$$

for adequate parameters $k_i, k_j, m_i, m_j, \eta_1$ and η_2. Observe that

$$\lambda_2^2 Var(\omega_2) = (m_i + m_j)^2 + m_i^2 \tau_i^{-1} + m_j^2 \tau_j^{-1} + \lambda_2^2 \eta_2^2 \tau_2^{-1} \quad (32)$$

For normal variables the conditional expectation is a linear function of the observations, thus $p_2 = E[v \mid \omega_1, \omega_2] = a_1\omega_1 + a_2\omega_2$ and by the law of iterated expectations, we have

$$E[v \mid \omega_1] = E[E[v \mid \omega_1, \omega_2] \mid \omega_1] = a_1\omega_1 + a_2 E[\omega_2 \mid \omega_1] \quad (33)$$

$$E[v \mid \omega_2] = E[E[v \mid \omega_1, \omega_2] \mid \omega_2] = a_2\omega_2 + a_1 E[\omega_1 \mid \omega_2] \quad (34)$$

To solve (33), we use (30) and (31) to decompose $E[\omega_2 \mid \omega_1]$ into

$$\begin{aligned}
& E\left[\tfrac{m_i+m_j}{\lambda_2} v \mid \omega_1\right] + E\left[\tfrac{m_i\varepsilon_i}{\lambda_2} \mid \omega_1\right] + E\left[\tfrac{m_j\varepsilon_j}{\lambda_2} \mid \omega_1\right] + E[\eta_2 u_2 \mid \omega_1] \\
= & \tfrac{m_i+m_j}{\lambda_2}\tfrac{\lambda_1}{k_i+k_j} E\left[\tfrac{k_i+k_j}{\lambda_1} v \mid \omega_1\right] + \tfrac{m_i}{\lambda_2}\tfrac{\lambda_1}{k_i} E\left[\tfrac{k_i}{\lambda_1}\varepsilon_i \mid \omega_1\right] + \tfrac{m_j}{\lambda_2}\tfrac{\lambda_1}{k_j} E\left[\tfrac{k_j}{\lambda_1}\varepsilon_j \mid \omega_1\right] \\
= & \left(\tfrac{\lambda_1(m_i+m_j)}{\lambda_2(k_i+k_j)}\left(\tfrac{k_i+k_j}{\lambda_1}\right)^2 + \tfrac{\lambda_1 m_i}{\lambda_2 k_i}\left(\tfrac{k_i}{\lambda_1}\right)^2 + \tfrac{\lambda_1 m_j}{\lambda_2 k_j}\left(\tfrac{k_j}{\lambda_1}\right)^2\right)\frac{\omega_1}{Var(\omega_1)} \\
= & \frac{Q\omega_1}{\lambda_1\lambda_2 Var(\omega_1)} \text{ where } Q \equiv (m_i+m_j)(k_i+k_j) + k_i m_i + k_j m_j \quad (35)
\end{aligned}$$

Combining $E[v \mid \omega_1] = \frac{(k_i+k_j)\omega_1}{\lambda_1 Var(\omega_1)}$ and (35), equation (33) becomes

$$\begin{aligned} \frac{k_i+k_j}{\lambda_1}\frac{\omega_1}{Var(\omega_1)} &= a_1\omega_1 + a_2\omega_1\frac{Q}{\lambda_1\lambda_2 Var(\omega_1)} \\ &\Rightarrow a_2 = \frac{\lambda_2(k_i+k_j-a_1\lambda_1 Var(\omega_1))}{Q} \end{aligned} \tag{36}$$

Symmetrically, (34) leads to

$$a_1 = \frac{\lambda_1(m_i+m_j-a_2\lambda_2 Var(\omega_2))}{Q} \tag{37}$$

Solving the system (36-37) gives

$$a_1 = \lambda_1\frac{Q(m_i+m_j)-(k_i+k_j)\lambda_2^2 Var(\omega_2)}{Q^2-(k_i+k_j)\lambda_2^2 Var(\omega_2)} \tag{38}$$

$$a_2 = \lambda_2\frac{(k_i+k_j)\left(Q-(m_i+m_j)\right)}{Q^2-(k_i+k_j)\lambda_2^2 Var(\omega_2)} \tag{39}$$

Now we can easily obtain the linear pricing rule for the market makers at the second stage. Combining $p_2(\omega_1,\omega_2) = \mu_2 + \lambda_2\omega_2$ and (**??**) we obtain $\mu_2 + \lambda_2\omega_2 = a_1\omega_1 + a_2\omega_2$. Identifying the coefficients of this linear equation, we obtain $\mu_2 = a_1\omega_1$and $\lambda_2 = a_2$. Hence from (39) and get

$$(k_i+k_j)\lambda_2^2 Var(\omega_2) = Q^2 - (k_i+k_j)\left(Q-(m_i+m_j)\right) \tag{40}$$

and using (32) we finally obtain

$$(m_i+m_j)^2 + m_i^2\tau_i^{-1} + m_j^2\tau_j^{-1} + \lambda_2^2\eta_2^2\tau_2^{-1} = \frac{Q^2}{(k_i+k_j)} - (Q-(m_i+m_j))$$

$$\Rightarrow \lambda_2 = \frac{\sqrt{\tau_2}}{\eta_2}\sqrt{\frac{Q^2}{(k_i+k_j)} - Q + (m_i+m_j) - (m_i+m_j)^2 - m_i^2\tau_i^{-1} - m_j^2\tau_j^{-1}} \tag{41}$$

Notice that when k_i and k_j tend to zero, Q and $\frac{Q^2}{k_i+k_j}$ tend also to zero, thus (41) is the exact counterpart to (4) the market depth in the first stage. We can also simplify (38) to obtain $a_1 = \lambda_1\left(1-\frac{Q}{k_i+k_j}\right)$ and

$$\mu_2 = a_1\omega_1 = \left(1-\frac{Q}{k_i+k_j}\right)(k_i s_i + k_j s_j + \lambda_1\eta_1 u_1) \tag{42}$$

which is nil whenever no information is revealed in stage one ($k_i = k_j = \lambda_1 = 0$).

Using (41) and (42) we can compute $\Sigma_2 = 1 - Var(a_1\omega_1 + a_2\omega_2)$ as a function $g(m_i, m_j, k_i, k_j, \tau_i, \tau_j)$ available upon request from the authors.

6.2 The Monopoly Benchmark

If the monopoly conceals his information during the first period then $p_1 = p_0 = 0$ and the expected profit is $\bar{\Pi}_1^C = 0$. The trading parameter in the last period obtained from the usual FOC are $\alpha_{2,i} = \frac{-\mu_2 - \lambda_2 u_2}{2\lambda_2}$ and $\beta_{2,i} = \frac{1}{2\lambda_2}$. The order flow is therefore $\omega_2 = -\frac{\mu_2}{2\lambda_2} + \frac{\beta_2 \tau_i}{\tau_i + 1} s_i + \frac{u_2}{2} = \frac{m_i s_i}{\lambda_2} + \eta_2 u_2$. We can identify the coefficients of the various independent random variables to obtain $m_i = \frac{\tau_i}{2(\tau_i+1)}, \mu_2 = 0$ and $\eta_2 = \frac{1}{2}$, $\lambda_2^C = \sqrt{\frac{\tau_2 \tau_i}{\tau_i+1}}$. Hence

$$
\begin{aligned}
q_{2,i}(s_i) &= \alpha_{2,i} + \beta_{2,i} \frac{\tau_i s_i}{\tau_i + 1} = \frac{1}{2\lambda_2^C} \frac{\tau_i s_i}{\tau_i + 1} - \frac{u_2}{2} = \frac{\tau_i s_i}{2\sqrt{\tau_2 \tau_i (\tau_i+1)}} - \frac{u_2}{2} \\
\bar{\Pi}_2^C(s_i) &= \lambda_2^C q_{2,i}(s_i)^2 = \frac{1}{4}\sqrt{\frac{\tau_2 \tau_i}{\tau_i+1}} \left(\frac{\tau_i s_i}{\sqrt{\tau_2 \tau_i (\tau_i+1)}} - u_2 \right)^2 \\
\bar{\Pi}_2^C(\tau_i, \tau_2) &\equiv E\left[\Pi_{2,i}^C(s_i)\right] = \frac{1}{2}\sqrt{\frac{\tau_i}{\tau_2(\tau_i+1)}}
\end{aligned}
$$

Using $E\left[s_i^2\right] = \frac{1+\tau_i}{\tau_i}$, $E\left[u_2^2\right] = \tau_2^{-1}$ and $s_i \perp u_2$. Finally $\bar{\Pi}^C = \bar{\Pi}_1^C + \bar{\Pi}_2^C$.

If the monopoly decides to reveal immediately then $\lambda_1^R = \sqrt{\frac{\tau_1 \tau_i}{\tau_i+1}}$, $\bar{\Pi}_1^R(\tau_i, \tau_1) = \bar{\Pi}_2^C(\tau_i, \tau_1)$ and $\omega_1 = \frac{s_i}{2}\sqrt{\frac{\tau_i}{\tau_1(\tau_i+1)}} + \frac{u_1}{2}$. Hence the best strategy for a monopoly is to reveal immediately in order to benefit from the possibility to trade in two periods. The optimal second period order is $q_{2,i}(s_i) = \frac{1}{2\lambda_2}\left(\frac{\tau_i s_i}{\tau_i+1} - \mu_2 - \lambda_2 u_2\right)$ but we now have $\lambda_2^R = \sqrt{\frac{\tau_2 \tau_i (1+\tau_i^2)}{(\tau_i+1)^3}}$ and the memory effect $\mu_2^R = \frac{1}{2(\tau_i+1)}\left(\frac{\tau_i s_i}{\tau_i+1} + \lambda_1^R u_1\right)$, thus

$$
\begin{aligned}
q_{2,i}(s_i) = \; & \frac{1}{2\lambda_2^R}\left(\frac{(2\tau_i+1)\tau_i s_i}{2(\tau_i+1)^2} - \frac{1}{2(\tau_i+1)}\lambda_1^R u_1 - \lambda_2^R u_2\right) \\
\bar{\Pi}_2^R(s_i) = \; & \lambda_2^R q_{2,i}(s_i)^2 = \frac{1}{4\lambda_2^R}\left(\frac{(2\tau_i+1)\tau_i s_i}{2(\tau_i+1)^2} - \frac{1}{2(\tau_i+1)}\lambda_1^R u_1 - \lambda_2^R u_2\right)^2 \\
\bar{\Pi}_2^R = \; & \frac{1}{4\lambda_2^R}\left(\frac{(2\tau_i+1)^2\tau_i}{4(\tau_i+1)^3} + \frac{\tau_i}{4(\tau_i+1)^3} + \frac{\tau_i(1+\tau_i^2)}{(\tau_i+1)^3}\right) = \frac{\left(4\tau_i^2+2\tau_i+3\right)\sqrt{\tau_i}}{8\sqrt{\tau_2\left(1+\tau_i^2\right)(\tau_i+1)^3}} \\
\bar{\Pi}^R = \; & \frac{1}{2}\sqrt{\frac{\tau_i}{\tau_i+1}}\left(1 + \frac{4\tau_i^2+2\tau_i+3}{4(\tau_i+1)\sqrt{\tau_2(1+\tau_i^2)}}\right)
\end{aligned}
$$

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