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Financing Start-ups: Advising vs. Competing

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Abstract

High-tech start-ups get external finance and guidance mostly from venture capitalists and/or business angels. We identify a simultaneous double moral hazard for the management style of entrepreneurs and the decision to advise the firm for financiers. We embed this relationship into the financial competition where strategic choices are equity shares, liquidation rights and quality of advising. We show that the financier holds all liquidation rights, that more competition weakly decreases the financier's equity share. Surprisingly, the response in advising quality is non-monotone. In a regime of soft competition, the financier owns the start-up and more competition weakens advising quality. In a regime of acute competition, more competition improves advising quality and lowers the financier's equity share in the start-up. Hence, advising and equity, are substitutes at the industry level once competition effects are taken into account.\bigskip

Keywords: Start-ups, Contract Design, Equity, Oligopoly Competition
JEL Codes: D4, G3, L1, L2

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1 Introduction

The OECD Information Technology Outlook 2000[1] stresses the differing rates of growth for the information technologies (IT) sector among member states and traces one possible origin to differences in corporate banking methods and concentration of financial intermediaries (e.g., comparing the US and the UK with Japan and Germany).

According to the classification of Berger and Udell (1998), businesses less than 2 years old are rather financed by relatives and business angels, while those aged between 3 to 4 years are rather financed by venture capitalists and investment banks. It is indeed well known that high-tech start-ups need large investments to initiate projects offering high potential but also high risk. Due to their lack of reputation, of cash-flow and of collateral, start-ups often cannot access security markets (cf. Petersen and Rajan (1994), (1995)) and must rely instead on financial intermediaries. We shall use the terminology *capitalist* keeping in mind that it applies to both angels and venture capital funds.

The owner-manager of a start-up draws private benefits from the valuable reputation he can obtain by running successfully its project according to its *vision*. This is indeed the case for internet start-ups whose creators acknowledge the peer effect as a motivation for hard work and entrepreneurship.[2] This may lead the entrepreneur to neglect the management of its company and reduce the odds of commercial success. In the traditional corporate finance literature (e.g., Aghion and Bolton (1992)), misbehavior is deterred by the use of collateral and takeover whenever bad news arise. However, the world of high-tech start-ups seems to lack signals[3] correlated to misbehavior that could trigger the lender's intervention (one exception is delays in software development but this is a "soft" information). Rather, success arises suddenly when the firm or its technology (e.g., molecule, software) is sold to a large corporation (e.g., Microsoft, Intel and Cisco) or is agreed by the government.

[1] http://www.oecd.org/dsti/sti/it/prod/it-out2000-e.htm

[2] From interviews lead by J. Tirole and presented at the Internet session of the European Economic Association Congress, Bolzano Sept. 2000.

[3] While investments in intangibles like R&D create market values, key financial variables are often negative. Such anomalous relations are typical of fast-changing, technology-based industries.

Another specificity of the start-ups world is that entrepreneurs often lack of management skills since they graduate mostly in sciences and not in management. Thus, banks, venture capitalists and wealthy investors have a new role to play. They can become the *business angels* of entrepreneurs by providing them support and governance to improve the profitability of their projects. This (costly) decision is the moral hazard issue of financiers. As noted by many different sources like the OECD 2000 IT outlook, Kaplan and Strömberg (2000) and Prowse (1998), the commitment of financiers into start-ups is quite variable in frequency and in quality across regions and also across countries. An angel must decide how much time to devote to a start-up he is funding. Similarly, a venture capital fund must decide how many advisors to recruit to later delegate them in its pool of client start-ups. The investment into the business angel activity thus appears to be a strategic decision in the competition among financiers. We analyze it as a *quality* variable to distinguish it from more traditional pricing variables like financial rights or liquidation rights.

In this paper, we build a principal-agent model of a *high-tech*[4] entrepreneur and its financier depending on the contract they sign but also on the quality of the lender's advising. The entrepreneur can either adopt a visionary management or an obedient one. The financier can decide to monitor the entrepreneur or not. We then embed this relationship into the financial sector to understand how the rivalry among venture capitalists, whatever its origin, affects the equilibrium design of the capitalist's equity participation, its liquidation rights and the quality of advising he provides to the entrepreneur.

In our model, the equilibrium of the principal-agent relationship involves positive frequencies of visionary management and advising, as well as, obedient management and no-advising. A larger equity participation of the capitalist and a better advising quality both contribute to reduce the frequency of advising while the frequency of visionary conduct increases with the equity participation and decreases with the advising quality. Then, we show that in the equilibrium of the financial sector, the liquidation rights are set at the maximum as they enable to reduce the cost of the entrepreneur's moral hazard. Thus, capitalists compete on two dimensions, the equity participation and the advising quality.

[4]We use this terminology to emphasize the difference alluded earlier with more traditionnal entrepreneurs.

Our findings in this respect are that more financial competition weakly decreases the equity participation of capitalists (in equilibrium) and weakly increases the frequency of visionary management. The striking result we obtain is the non-monotonic response in advising quality as competitiveness changes. In a regime of soft competition, capitalists own start-ups (maximal equity share) as a result of the limited wealth of entrepreneurs. In this context, more competition *weakens* advising quality. On the other hand, in a regime of acute competition, more competition *reinforces* advising quality and lowers the equity participation of capitalists. Hence, advising quality and equity participation are substitutes at the industry level, once competition effects are taken into account.

The intuition of this outcome is rooted in the way financial competition crafts the individual contractual relationships. In equilibrium of this financial competition, the marginal rates of substitution between equity share and advising quality are equal for a capitalist and an entrepreneur, if the latter retains some ownership in the start-up. Then, we show that the two variables are negatively linked on the Pareto curve because equity serves to reduces moral hazard. Next, we show that, if the success probability of the start-up is small, the equity share is the main channel of competition transmission.

With these tools in hand we are able to confirm in our model, the intuition according to which, entrepreneurs obtain more utility in equilibrium when financial rivalry increases. This translates de facto into a lower equity participation of the financier and surprisingly into a higher advising quality, as the two variables move on the Pareto curve. Yet, this does not hold when market power is high. Indeed, the equity participation is then so high that the capitalist owns the start-up. In this soft regime, financial competition takes place only on the advising quality which is disliked by entrepreneurs. Thus, more competition translates into lower a quality of advising.

Among recent papers, Repullo and Suarez (2000) analyze an entrepreneur-capitalist relationship with two stages of financing and double moral hazard. The capitalist in their setting is modeled as an in-house manager and not as an adviser delegated by a large financial institution. The emphasis is thus on the optimal claim of this lender which resembles warrants or convertible preferred stocks. Our view of the entrepreneur-capitalist relationship is simpler on several points but endogenizes the involvement into the business angel activity and provide an analysis of the rivalry among capitalists.

While we concentrate on small firms, Aghion et al. (1998) study the complementary problem of public finance under moral hazard for large firms. The firm issues equity to small investors and signals its willingness to effort, instead of being monitored. The optimal contract bears similarities with ours. When the need for external finance is high (our soft competition regime), the business is sold to the financiers and the signaling activity is inefficiently high. When the need for finance is low, the firm keep some of the future profits and tends to shirks more as the need for finance increases.

The paper is organized as follows. Section 2 introduces our model of risky project financing in two stages. Section 3 derives the equilibrium of the second stage and section 4 solves for the first stage perfect equilibrium. Section 5 concludes.

2 A Model of Start-Up

2.1 Financing a risky project

We concentrate on start-ups competing for the development of an innovation (and the reward associated with it). Our model which is inspired by Hölmstrom and Tirole (1997) disregards the issue of adverse selection.[5] We consider a continuum of *identical* risk-neutral entrepreneurs. The cost of development K is larger than the cash-flow w of the start-up (capital brought by the entrepreneur and its relatives). The monetary return in case of success (proceeds from an IPO or from selling the technology to a large firm) is $V_s > K$ while it is only $V_f < K$ in case of failure.

We assume that the cost of issuing public debt is too large for these entrepreneurs. Thus, their only source of financing is venture capital. Without loss of generality, financiers[6] have an equal unbounded access to money at a rate normalized to zero. The project cost K can be partially funded by a venture capitalist who brings an amount L of capital while the entrepreneur pays the

[5] It clearly matters at the development stage where, after an initial success, some information regarding the entrepreneur is revealed (possibly to himself too).

[6] We regroup under this label investment funds, wealthy individuals (the so called angels), lenders (for more information see http://www.vfinance.com).

remaining $K - L$ out of his personal wealth w. We immediately obtain a lower bound on the loan size to be effective: $L \geq K - w$.

Kaplan and Strömberg (2000) report evidence that venture capitalists separately allocate cash-flow rights, voting rights, liquidation rights and other control rights. Hence, in our simple setting, it make sense to distinguish the share α of profits going to the lender when the project succeeds from the liquidation share β which applies when the project fails. Financial payoffs are shown in Table 1 below.

Payoffs	Entrepreneur	Capitalist
Success	$u = (1-\alpha)V_s - K + L + w$	$\pi = \alpha V_s - L$
Failure	$u = (1-\beta)V_f - K + L + w$	$\pi = \beta V_f - L$

Table 1

2.2 A case for Double Moral Hazard

By running a start-up, an entrepreneur gathers a useful experience whose monetary equivalent is denoted B_0. Moreover, he can either be *obedient* or *visionary*. In the former case, he shares time between project development and management. The failure probability is then $\underline{p}$. In the latter case, the entrepreneur concentrates on project development to build a reputation among his peers. He derives an additional private benefit B (cf. Hart and Moore (1994)) but also increases the probability of failure to $\bar{p} > \underline{p}$. We shall later associate management styles and failure probabilities.

As we argued in the introduction, the capitalist is not only a lender but also a business angel for the entrepreneur. Further, their relationship has a timing different from the usual one found in corporate finance. Indeed, the traditional responses to moral hazard do not seem to apply to high-tech start-ups. First, there are no external signals correlated to visionary management that could trigger an intervention (with the associated deterrence effect). Hence, start-ups should be advised from inside, not monitored from outside. Secondly, the commitment to a high frequency of random intervention (a typical remedy of the literature) is not credible in our context. Indeed, inefficient visionary behavior would be eliminated in equilibrium which does not fit stylized facts. Furthermore, capitalists would be tempted to deploy advisors in other activities once they believe that entrepreneurs are obedient.[7]

The natural timing thus appears to be the simultaneity of the management and advising decisions. Still, the capitalist can invest *a priori* into advising by committing to a quality q. It could be the recruitment of more or less advisors, to manage more or less carefully each project. The better the quality, the better the advisor can help the entrepreneur, if the financier decides to advise this particular client. Nonetheless, this effect should be more pronounced when facing a visionary entrepreneur. To simplify the exposition, we assume that advising an obedient entrepreneur has no effect while advising a visionary entrepreneur reduces the probability of failure by $q(\bar{p}-\underline{p})$. Beyond this efficiency effect,[8] we assume that the presence of the advisor has two incentive effects:

▶ it reduces the private benefit by qB (e.g., fame is shared with the business angel)

▶ if the project fails then, as the entrepreneur was visionary and advised, his misbehavior becomes known to other venture capitalist..[9] Then, the entrepreneur will have to pay a risk premium to lenders if he wants to start new projects in the future. The present value of this defaulting cost is denoted F.

Advising a client has a direct cost $\phi^{\pi}(q)$ for the capitalist (advisor's wage) and an opportunity cost $\phi^{u}(q)$ for the entrepreneur (loss of independence as in Dewatripont and Tirole (1994) and Burkhart et al. (1997)). Both functions, defined over the $[0; 1]$ interval, are differentiable, increasing and convex. Letting $q = 0$ stand for no specific investment, an intervention is then a simple audit of the entrepreneur's accounts (only the deterrence effect remains) so that both $\phi^{\pi}(0)$ and $\phi^{u}(0)$ are positive.

[7]The early literature on costly audits assumed a commiment to carry out threats that are not optimal ex-post. A recent stream of literature rejects this inconsistency and tackles the principal-agent interaction as a simultaneous one. Some relevant papers are Gale and Hellwig (1989), Bolton and Scharfstein (1990) and Khalil and Parigi (1998).

[8]Hellman and Puri (2000) observe from their data, that the venture capitalist reduces the time necessary to bring a product to the market, thus increasing the probability of being able to preempt the market.

[9]Prowse (1998) reports the sharing of information and co-investments behavior of angels, although they complain about the low quality of information channels. cf. the new ACE-Net network set-up by the US Small Business Administration.

2.3 Timing of the Finance Competition

We consider the following timing of events describing the market for the financing of risky projects:

▶ Capitalist $i = 1$ to n chooses an advising quality q^i and offers a contract $\left(\alpha^i, \beta^i, L^i\right)$.

▶ Entrepreneurs observe $\left(q^i, \alpha^i, \beta^i, L^i\right)_{i=1}^n$ and decide whether to borrow (or not) and whom from.

▶ Borrowers choose action $\bar{p}$ (visionary) or $\underline{p}$ (obedient) while capitalists simultaneously decide to advise or not each of their clients.

▶ Projects succeeds according to the chosen probability $\bar{p}$ or $\underline{p}$ and payments or liquidation take place.

This two-stage game is solved by backward induction using the concept of Perfect Bayesian Equilibrium (PBE). In the next section, we analyze the advising and obedience game between an entrepreneur and its capitalist; it depends on the advising quality q^i and the contract $\left(\alpha^i, \beta^i, L^i\right)$. Then, we compute the expected utility of entrepreneurs and capitalists conditional on choosing $\left(q^i, \alpha^i, \beta^i, L^i\right)$ and we derive in section 4 the symmetric equilibrium of competition between capitalists.

To guarantee that capitalists are active in equilibrium, the project must have a positive net present value. In our setting, the expected profit has to be larger than the minimal loan size plus the average cost of moral hazard for the capitalist. Then, entrepreneurs are willing to conduct projects if the value of experience is greater than the initial wealth plus the average cost of moral hazard. The corresponding assumptions are

H1 $\underline{p}V_f + (1 - \underline{p})V_s > K - w + \phi^\pi(q)/q$

H2 $B_0 > w + \phi^u(q)/q$

3 The Lender-Borrower Relationship

In this section, we analyze the relationship between a capitalist with advising quality q and one of its customer under the contract (α, β, L).

To make the moral hazard issue relevant, we assume that private benefits are larger than the default cost plus the efficiency effect of advising i.e., $B > (\bar{p} - \underline{p})V_s + \bar{p}F$ (otherwise the entrepreneur would hire himself an advisor). Then, if the capitalist only acts as a lender (no intervention), the entrepreneur adopts a visionary management. We also assume that full control is too costly for the lender[10] i.e., $V_s < \phi^{\pi}(1)$ (best advising quality and systematic advising).

In such a context, the equilibrium of the advising-management game will feature positive frequencies of advising and visionary management. Moreover, both will be influenced by the contract and the advising quality. Indeed, those elements are crucial for trading-off the expected benefit of advising and its cost for the capitalist, and for trading-off the benefit of visionary management and its expected cost for the entrepreneur. Note that, while q is costly for the entrepreneur and the capitalist, it brings benefits too as it influences the desire of the entrepreneur to be benevolent and of the capitalist to advise.

Proposition 1 *In a PBE, the advising-management game following* (q, α, β, L) *has a unique Nash equilibrium in mixed strategies.*

Proof The agent's strategy is to be visionary with probability σ while the capitalist's strategy is to advise with probability τ. The failure probability in case of visionary management is $\bar{p}^{\tau,q} \equiv \bar{p} - \tau q(\bar{p} - \underline{p})$ and we have $(1-\sigma)\underline{p} + \sigma\bar{p}^{\tau,q} = \underline{p} + \sigma(1-\tau q)(\bar{p} - \underline{p})$. The utility of the borrower is

$$\begin{aligned} u(\sigma,\tau) &= B_0 + w + (1-\alpha)V_s - K + L + \left((1-\sigma)\underline{p} + \sigma\bar{p}^{\tau,q}\right)\left((1-\beta)V_f - (1-\alpha)V_s\right) \\ &\quad + \sigma(1-\tau q)B - \sigma\bar{p}^{\tau,q}F - \tau\phi^u(q) \\ &= B_0 + w + (1-\alpha)V_s - K + L - \underline{p}\left((1-\alpha)V_s - (1-\beta)V_f\right) - \tau\phi^u(q) \\ &\quad + \sigma\left[(1-\tau q)B - \bar{p}^{\tau,q}F - (1-\tau q)(\bar{p} - \underline{p})\left((1-\alpha)V_s - (1-\beta)V_f\right)\right] \qquad (1) \end{aligned}$$

The borrower is indifferent between its two actions if and only if the bracketed term of (1) is nil. Developing $\bar{p}^{\tau,q}$ yields

$$\begin{aligned} 0 &= (1-\tau q)B - \bar{p}F + \tau q(\bar{p} - \underline{p})F - (1-\tau q)(\bar{p} - \underline{p})\left((1-\alpha)V_s - (1-\beta)V_f\right) \\ \Leftrightarrow \quad \tau &= \tau^* \equiv \frac{1}{q}\left(1 - \frac{\underline{p}F}{B - (\bar{p} - \underline{p})\left((1-\alpha)V_s - (1-\beta)V_f + F\right)}\right) \qquad (2) \end{aligned}$$

The optimal behavior of the entrepreneur is $\sigma = 1$ if $\tau < \tau^*, \sigma = 0$ if $\tau > \tau^*$ and $\sigma \in [0;1]$ if $\tau = \tau^*$. Similarly, the capitalist's profit is

$$\pi(\sigma,\tau) = \alpha V_s - L + \left((1-\sigma)\underline{p} + \sigma\bar{p}^{\tau,q}\right)(\beta V_f - \alpha V_s) - \tau\phi^{\pi}(q)$$

[10] Otherwise he would own the business and contract the entrepreneur as a scientist.

$$= \quad \alpha V_s - L + \left(\underline{p} + \sigma(\bar{p} - \underline{p})\right)(\beta V_f - \alpha V_s) - \tau\left[\phi^\pi(q) - \sigma q(\bar{p} - \underline{p})(\alpha V_s - \beta V_f)\right] \quad (3)$$

thus, he is indifferent between its two actions if and only if the bracketed term of (3) is nil. We obtain

$$\sigma = \sigma^* \equiv \frac{\phi^\pi(q)}{q(\bar{p} - \underline{p})(\alpha V_s - \beta V_f)} \tag{4}$$

As moral hazard is relevant, we have $B - \bar{p}F > (\bar{p} - \underline{p})((1-\alpha)V_s - (1-\beta)V_f)$ whatever α and β. In turn, this implies $\tau^* > 0$. Hence, if the capitalist chooses a low frequency τ, it is optimal for the entrepreneur to adopt a visionary management. If q is small then $\tau^* > 1$ is possible in (2). In that case, any τ is smaller than τ^* so that $\sigma = 1$ is optimal for the entrepreneur. As q is small, $\sigma^* > 1$ also holds. Thus, any σ is smaller than σ^* so that $\tau = 0$ is optimal. The profit of the capitalist is then $\pi(1,0) = \alpha V_s - L + \bar{p}(\beta V_f - \alpha V_s)$.

Over the domain where $\tau^* < 1$, if $\sigma^* > 1$ then the equilibrium is again $\sigma = 1$, $\tau = 0$. Moving to the domain where $\tau^* < 1$ and $\sigma^* < 1$, the unique equilibrium is $\tau = \tau^*$ and $\sigma = \sigma^*$ yielding the equilibrium profit for the capitalist

$$\begin{aligned}
\pi(q, \alpha, \beta, L) &= \alpha V_s - L + \left(\underline{p} + \sigma^*(\bar{p} - \underline{p})\right)(\beta V_f - \alpha V_s) \\
&= \alpha V_s - L + \underline{p}(\beta V_f - \alpha V_s) - \frac{\phi^\pi(q)}{q} > \pi(1,0)
\end{aligned}$$

because this is exactly equivalent to $(\bar{p} - \underline{p})(\alpha V_s - \beta V_f) > \frac{\phi^\pi(q)}{q} \Leftrightarrow \sigma^* < 1$.

The capitalist who has chosen a low quality q such that the equilibrium is $(\sigma = 1, \tau = 0)$ would be better of choosing a higher quality q' and the same contract (α, β, L) in order to implement the equilibrium (σ^*, τ^*). Yet, this will work only if its clients get the same final utility level or more under the new scheme, for otherwise they could pick another lender. Hypothesis **H1** (positive NPV) tells us that this is possible using L as a transfer variable. Hence, in a PBE, the equilibrium of the advising-management game is never in pure strategies. ■

The effect of the advising quality q on the payoffs of capitalists and entrepreneurs can be summarized by the average cost functions $c^\pi(q) \equiv \frac{\phi^\pi(q)}{q}$ and $c^u(q) \equiv \frac{\phi^u(q)}{q}$. The *ideal* advising quality of capitalists q^π (resp. of entrepreneurs q^u) is defined as the minimum of $c^\pi(q)$ (resp. $c^u(q)$)

over $[0;1]$.[11] The inequality $\phi^\pi(0) > \phi^u(0)$ being the most plausible, we assume that capitalists prefer a higher degree of involvement into the management of the project than entrepreneurs do:

$$\textbf{H3} \quad q^u < q^\pi$$

Using the equilibrium levels of visionary management σ^* and advising τ^*, we obtain the expected utility levels of the entrepreneur u and the expected profit of the capitalist π as

$$\begin{aligned} u\,(q,\alpha,\beta,L) &= w + B_0 + (1-\underline{p})(1-\alpha)V_s + \underline{p}(1-\beta)V_f + L - K - \tau^*\phi^u(q) \\ &= w + B_0 + (1-\underline{p})(1-\alpha)V_s + \underline{p}(1-\beta)V_f + L - K \\ &\quad - \left(1 - \frac{\underline{p}F}{B - (\bar{p}-\underline{p})\big((1-\alpha)V_s - (1-\beta)V_f + F\big)}\right) c^u(q) \end{aligned} \tag{5}$$

$$\begin{aligned} \pi\,(q,\alpha,\beta,L) &= \alpha V_s - L + \big(\underline{p} + \sigma^*(\bar{p}-\underline{p})\big)\,(\beta V_f - \alpha V_s) \\ &= (1-\underline{p})\alpha V_s + \underline{p}\beta V_f - L - c^\pi(q) \end{aligned} \tag{6}$$

The following proposition, whose proof is relegated to the appendix, will permit to focus on advising quality and equity share by deriving the equilibrium value of other variables.

Proposition 2 *In a PBE, an optimal contract minimizes loan size ($L = K - w$) and gives ownership to the capitalist in case of project failure ($\beta = 1$). Advising quality is always chosen between q^u and q^π.*

These results are quite intuitive to understand. If the capitalist does not hold complete liquidation rights ($\beta < 1$), he can increase them and adequately decrease its equity share, so as to maintain constant the entrepreneur's utility. This substitution enables a reduction of moral hazard and thus a larger per-capita profit. Since the capitalist's clients keep the same utility level, its market share is not affected. The switch to complete liquidation rights has therefore yielded a larger total profit. Likewise, if the loan size exceeds the minimal level, the capitalist can substitute the difference by

[11] The minimum $\hat{m}$ of ϕ/m solves $\phi' = \phi/m$. It is positive because $\phi(d) \underset{d\to 0}{\longrightarrow} +\infty$ but $\hat{m} < 1$ because $\phi(1) = \int_0^1 \phi'(x)dx < \phi'(1)$ (by convexity). If for example $\phi^\pi(m) = a_0 + a_1 m^2$ and $\phi^u(m) = b_0 + b_1 m^2$ then $m^u = \sqrt{\frac{b_0}{b_1}}$ and $m^\pi = \sqrt{\frac{a_0}{a_1}}$.

an additional profit share (a reduction of α) which motivates further the entrepreneur i.e., reduce the cost of moral hazard. Finally, the intuition of the last result is that both u and π are increasing with q on $[0;q^u]$, thus the capitalist could commit to the quality q^u, improves its per-capita profit and improve (weakly) its market share as its clients are now better off. The same process would occur if the initial quality was larger than q^π.

We can now write the preferences of the various parties at the first stage where capitalists compete for entrepreneurs. The utility of an entrepreneur and the per-capita profit of a financier are

$$u(q,\alpha) = B_0 + (1-\underline{p})(1-\alpha)V_s - \left(1 - \frac{\underline{p}F}{B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)}\right)c^u(q) \quad (7)$$

$$\pi(q,\alpha) = \underline{p}V_f - K + w + (1-\underline{p})\alpha V_s - c^\pi(q) \quad (8)$$

Note that q and α are goods for the capitalist but anti-goods for the entrepreneur. In the competition among financiers, the advising quality can be chosen between q^u and q^π while the equity share is between 0 and 1. However, it is readily observed that $\pi(q,0) < 0$. Thus, in a perfect equilibrium, the only meaningful constraint for the equity share is

$$\alpha \leq 1 \quad (9)$$

When this constraint is binding the entrepreneur simply becomes a manager working on behalf of the capitalist; his utility is measured by the gains of experience B_0 minus the cost of moral hazard $\frac{B-\bar{p}F}{B-(\bar{p}-\underline{p})F}c^u(q)$.

When the share constraint $\alpha \leq 1$ is not binding, a Pareto optimal pair (q,α) equalizes the marginal rates of substitution $\dfrac{u_\alpha}{u_q}$ and $\dfrac{\pi_\alpha}{\pi_q}$.[12] It is obvious that an equilibrium of the contract competition where the share constraint (9) is not binding must be a Pareto optimum for otherwise, the venture capitalist could attract more entrepreneurs and make more profits with a better designed pair (q,α). Clearly, both parties prefer to retain a maximal equity share, but equity is better handed to the entrepreneur as it reduces the extent of moral hazard ($u_\alpha < 0$ and $u_{\alpha\alpha} > 0$). On Figure 1 below, we display iso-utility (dotted) and iso-profit (plain) curves in the (α, q) space. Starting from

[12] The utilitarian Pareto optimum maximizes $u(q,\bar{\alpha}) + \pi(q,\bar{\alpha}) \propto -\left(1 - \frac{\bar{e}F}{B-(\underline{e}-\bar{e})\left((1-\bar{\alpha})\bar{V}+F\right)}\right)c^u(q) - c^\pi(q)$ thus $\bar{\alpha} = 0$ is optimal while the quality solves $\frac{-c_q^\pi(q^*)}{c_q^u(q^*)} = 1 - \frac{\bar{e}F}{B-(\underline{e}-\bar{e})\left(\bar{V}+F\right)}$.

an optimum A, let us move to point B by decreasing α (keeping q constant). The marginal rate of substitution $\frac{u_\alpha}{u_q}$ increases[13] while $\frac{\pi_\alpha}{\pi_q}$ remains unchanged.[14] Hence, at point B, the entrepreneur values equity shares more than the capitalist. A Pareto improving trade sees the entrepreneur buying more equity (a further decrease in α) from the capitalist in exchange of more control (an increase in q). We deduce from this observation that the contract curve, the set of Pareto optima, is downward sloping in the (α, q) space.

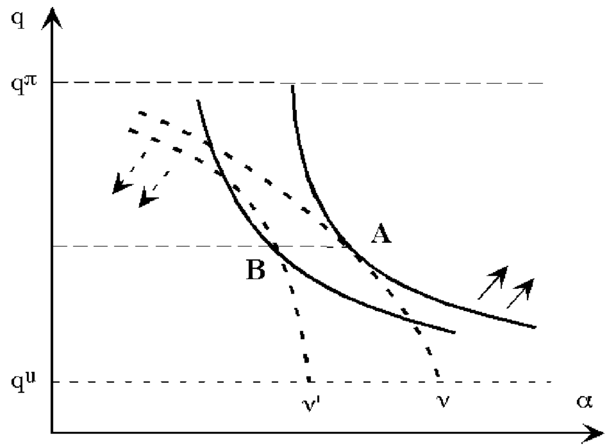

Figure 1

We conclude that, in equilibrium of the financial competition, an increase in control goes along with a lower equity share for the capitalist, if the share constraint (9) is not binding. What remains unclear is the ultimate effect of an efficient substitution from equity to advising quality ($\alpha \searrow, q \nearrow$) for the capitalist and the entrepreneur ? In other words, who gains and who loses ? The answer to this question is fundamental to understand the consequences of the competition among financiers. To derive clear-cut results, we assume that the *absolute* increase of the success probability when switching from visionary to obedient management is small[15] i.e.,

$$\textbf{H4} \quad \bar{p} - \underline{p} < \delta$$

where the threshold δ is determined in the proof of Proposition 3 (cf. appendix).

[13] Indeed, $H1$ implies that $u_\alpha = -(1-\bar{e})\bar{V} - \frac{\bar{e}Fc^u(q)(\underline{e}-\bar{e})\bar{V}}{\left(B-(\underline{e}-\bar{e})\left((1-\alpha)\bar{V}+F\right)\right)^2} < 0$, then we easily get $u_{\alpha\alpha} > 0$, $u_q \propto -c_q^u < 0$ and $u_{q\alpha} < 0$. Together these imply $\partial\left(\frac{u_\alpha}{u_q}\right)/\partial\alpha = \dfrac{u_{\alpha\alpha}u_q - u_\alpha u_{q\alpha}}{(u_q)^2} < 0$.

[14] Indeed $\pi_{\alpha\alpha} = \pi_{q\alpha} = 0$ imply $\partial\left(\frac{\pi_\alpha}{\pi_q}\right)/\partial\alpha = \dfrac{\pi_{\alpha\alpha}\pi_q - \pi_\alpha\pi_{q\alpha}}{(\pi_q)^2} = 0$.

[15] If for instance $\underline{e} = 95\%$ and $\underline{e} - \bar{e} = 2\%$ then the success probablility increases by 40% from the visionary to the obedient management style.

Thanks to this assumption, we can derive the following lemma whose proof has been relegated to the appendix.

Lemma 1 *When the capitalist equity share α decreases and the advising quality q increases, so as to maintain equality of the marginal rates of substitution, the utility of the entrepreneur increases.*

This result means that, on Figure 1 above, the utility level v' is greater than v. In other words, the equity share is the main channel of utility or profit changes, thus the main strategic variable.

4 Competition among Lenders

To analyze how the intensity of competition, whatever its origin, affects the equilibrium design of contracts and the choice of advising quality, we use a flexible model of horizontal differentiation, inspired by the circular city of Salop (1979). Entrepreneurs either bear a cost to visit financial institutions or they are already in relation with a particular one and bear an administrative cost to switch to a new one. These features give capitalists some market power which varies directly with the size of this switching cost.

More specifically, the demand addressed to a capitalist offering an expected utility[16] v while all other capitalists offer v^* is $D(v - v^*)$ where $D(0) = \frac{1}{n}$ (symmetric market shares) and $D'(0) = t$ (switching cost).[17] The parameter $\theta \equiv nt$ is an index of competitiveness in this market for risky loans.

Thanks to Proposition 2, a symmetric PBE is a pair $(q^\theta, \alpha^\theta)$ since other variables are determined by these. The equilibrium utility level of entrepreneurs is denoted $v^\theta \equiv u(q^\theta, \alpha^\theta)$. The default utility for an entrepreneur being $V_f \equiv w$ (no start-up), capitalist i will be active only if its contract pair (q^i, α^i) is such that $0 \leq \alpha^i \leq 1$ and $u(q^i, \alpha^i) \geq V_f$. When capitalist i offers (q^i, α^i) while others capitalists offer $(q^\theta, \alpha^\theta)$, the profit of capitalist i is $\Pi_i(q^i, \alpha^i) \equiv \pi(q^i, \alpha^i) D\left(u(q^i, \alpha^i) - v^\theta\right)$. The aim of this capitalist being to maximize its profit, a symmetric equilibrium is therefore a pair $(q^\theta, \alpha^\theta)$

[16]We shall denote v the levels of utility to avoid confusing with the utility function u.

[17]This formulation is directly inspired by

solving the program

$$\hat{P}(\theta) \equiv \max_{q,\alpha} \Pi_i(q,\alpha) \quad \text{s.t. } u(q,\alpha) \geq V_f, 0 \leq \alpha \leq 1 \tag{10}$$

The system of first order conditions for an interior solution is

$$\left.\frac{\partial \Pi_i}{\partial q}\right|_{v=v^\theta} = 0 \Leftrightarrow \pi_q D(0) + \pi D'(0) u_q = 0 \tag{11}$$

$$\left.\frac{\partial \Pi_i}{\partial \alpha}\right|_{v=v^\theta} = 0 \Leftrightarrow \pi_\alpha D(0) + \pi D'(0) u_\alpha = 0 \tag{12}$$

from which we deduce

$$\frac{u_\alpha}{u_q} = \frac{\pi_\alpha}{\pi_q} \tag{13}$$

$$\theta\pi = \frac{-\pi_q}{u_q} \tag{14}$$

On figure 2 below, we display the decreasing contract curve characterized in Lemma 1. An interior equilibrium is on the contract curve by (13) and its position is determined by (14). The difficult part of the analysis (cf. Proposition 3 below) is to confirm the intuition according to which, less competition yields higher profits for capitalists and lower final utility for entrepreneurs. Combining this result with Lemma 1, we see on Figure 2 below that when the index θ falls, the equilibrium moves down on the contract curve.

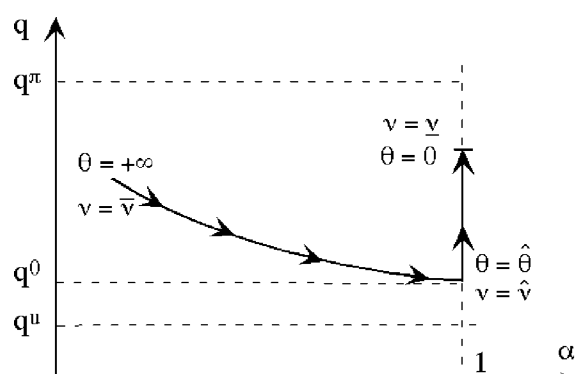

Figure 2

At some value $\hat{\theta}$, the equity share constraint starts to bind. We then show that the only way for capitalists to increase their profits is to move up vertically towards their ideal advising quality q^π. The position of the equilibrium on the vertical line still obeys equation (14) but computed at $\alpha = 1$ instead of being computed on the contract curve.

The following proposition, whose proof has been deferred to the appendix, completely characterize the equilibrium of the market for the financing of high-tech start-ups.

Proposition 3 *There exists an index of competition $\hat{\theta}$ separating two regimes:*

► $\theta \leq \hat{\theta}$ *(soft regime): the capitalist owns the start-up ($\alpha = 1$), advising quality decreases with θ.*

► $\theta > \hat{\theta}$ *(acute regime): the equity share decreases with θ, advising quality increases with θ.*

The proposition has thus shown that a low degree of competition raises the equity participation of capitalists to the point where entrepreneurs become managers of their start-ups. Then, the advising quality is the only strategic choice left to fight for market shares among capitalists. Contrariwise, when competition is fierce enough, capitalists are forced to lower their equity share to attract entrepreneurs and compensate the reduction in their financial stake by choosing an advising quality closer to their ideal.

An important question within our model remains: does more competition yield more visionary management and more advising ?

Corollary 1 *Comparative statics of the advising-obedience game:*

► *The frequency of visionary management σ^θ increases with the index of competition θ.*

► *The frequency of project advising τ^θ increases from 0 to over $\hat{\theta}$ and then decreases.*

► *The advising quality expected ex-ante by entrepreneurs $q^\theta \tau^\theta$ is weakly decreasing with θ.*

Finally, we may reinterpret our findings with respect to the initial (observable) wealth of entrepreneurs. As $\pi(q, \alpha)$ is linear in w, we can solve the equilibrium equation $\theta\pi = \frac{-\pi_q}{u_q}$ keeping θ constant and taking w as an exogenous parameter to obtain the following corollary.

Corollary 2 *For a given θ, there exists a wealth level $\hat{w}$ such that:*

- For $w < \hat{w}$, capitalists own start-ups and advising decreases with wealth.

- For $w > \hat{w}$, the equity participation of capitalists decreases with wealth while advising increases with wealth.

This result is in line with Hölmstrom and Tirole (1997)'s finding regarding the credit crunch that hit Scandinavian firms in 1990-91. Over the lower part of the wealth distribution, richer firms are less monitored than poorer ones. However, our model also suggests that several regimes may coexist which call a careful econometric treatment of international data where various degree of competition are likely to coexist.

5 Conclusion

Our paper offers a contribution, firstly, on the external financing and management of high-tech start-ups and secondly, on the competition among financiers for these high-growth opportunities. We start from two observations: high-tech entrepreneurs are not experienced managers and start-ups display few signals of good or bad management. We alter the traditional corporate finance framework to account for these specificities. Our vision of the lender-borrower relationship with endogenous advising and visionary management seems rather well fitted with stylized facts as we obtain positive frequencies of inefficient management and advising in equilibrium.

Our approach then permits to analyze how venture capitalists or angels compete to fund these start-ups. We also depart from the standard literature by taking into account the multi-dimensionality of capitalists strategies. We point at the relationship between the traditional price variables (equity share, liquidation share,capital investment) and a new and increasingly important qualitative dimension, the advising activity. We show that advising quality and equity share are substitutes at the industry level (in the regime of acute competition).

As for empirical implications, the identification of two possible regimes calls for a prudent analysis of international data since the market concentration and other indicators of rivalry come to play a determinant role (according to our model). Competition in private finance seems to be increasing everywhere in the world. Thus, beyond decreases in the equity shares retained by venture capitalists, we should observe an increase of advising quality if the country under scrutiny is already in a regime of acute financial competition (it should be the case for the US and Europe). In developing countries however, it may well be the case that financial competition lies in the soft regime where capitalists are the owner of start-ups. Advising quality should then decrease until the degree of financial completeness of the country reaches a sufficient level.

Appendix

Proof of Proposition 2

Proof The convexity of c^u and c^π implies that both $\pi(q,\alpha,\beta,L)$ and $u(q,\alpha,\beta,L)$ are increasing with q on $[0;q^u]$ and decreasing on $[q^\pi;1]$. Playing (q,α,β,L) with $q<q^u$ or $q>q^\pi$ is strictly dominated for a capitalist by (q^π,α,β,L) or (q^u,α,β,L). Indeed, each alternative raise the clients utility, thus guarantee that the market share will not fall. Then, as the per-capita profit also increases, the deviation is strictly profitable. We can therefore restrict qualities to lie between q^u and q^π.

Assume $\beta<1$ and consider $\Delta\beta>0$ and $\Delta\alpha=\frac{-\underline{p}V_f}{(1-\underline{p})V_s}\Delta\beta$. With this new contract the profit π remains constant as $\Delta\pi=(1-\underline{p})(-\Delta\alpha)V_s+\underline{p}(-\Delta\beta)V_f=0$ but given that $-V_s\Delta\alpha+V_f\Delta\beta=\frac{V_f}{1-\underline{p}}\Delta\beta>0$ we have

$$\begin{aligned}\Delta u &= \frac{\partial u}{\partial\beta}\Delta\beta+\frac{\partial u}{\partial\alpha}\Delta\alpha\\ &= \underbrace{(1-\underline{p})(-\Delta\alpha)V_s+\underline{p}(-\Delta\beta)V_f}_{=0}+\frac{(\bar{p}-\underline{p})\underline{p}Fc^u(q)V_f}{(1-\underline{p})\left(B-(\bar{p}-\underline{p})\left((1-\alpha)V_s-(1-\beta)V_f+F\right)\right)^2}\Delta\beta>0\end{aligned}$$

Hence the capitalist could alter the initial contract in order to keep its clients and make more per-project profit. The process of increasing the liquidation share β and decreasing the benefit share α will continue until the constraint $\beta=1$ is met.

Payoffs are now

$$\begin{aligned}u(q,\alpha,L) &= w+B_0+(1-\underline{p})(1-\alpha)V_s+L-K-\left(1-\frac{\underline{p}F}{B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)}\right)c^u(q)\\ \pi(q,\alpha,L) &= (1-\underline{p})\alpha V_s+\underline{p}V_f-L-c^\pi(q)\end{aligned}$$

and it is readily observed that $\Delta\alpha>0$ reduces the moral hazard cost so that a compensated $\Delta L=-(1-\underline{p})\Delta\alpha V_s$ keeps π constant while increasing u. Hence an optimal contract has minimal loan size with $L=K-w$. ■

Proof of Lemma 1

Recall that $\pi(q,\alpha)=\underline{p}V_f-K+w+(1-\underline{p})\alpha V_s-c^\pi(q)$ and observe that $u(q,\alpha)=B_0+(1-\underline{p})(1-\alpha)V_s-\left(1-\frac{\underline{p}F}{B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)}\right)c^u(q)$ is bounded over $[q^u;q^\pi]\times[0;1]$ and spans an interval

$[v_1; v_2]$. As $u_\alpha < 0$, the solution $\alpha = \rho(q,v)$ to the equation $v = u(q,\alpha)$ satisfies $\rho_q = -\frac{u_q}{u_\alpha} < 0$ and $\rho_v = \frac{1}{u_\alpha} < 0$ and is bounded over $[q^u; q^\pi] \times [v_1; v_2]$. Let us now introduce

$$\Phi^q(q,\alpha) \equiv -\frac{u_q}{\pi_q} = \left(1 - \frac{\underline{p}F}{B - (\bar{p}-\underline{p})\left((1-\alpha)V_s + F\right)}\right)\frac{c_q^u(q)}{-c_q^\pi(q)} \quad (15)$$

$$\Phi^\alpha(q,\alpha) \equiv -\frac{u_\alpha}{\pi_\alpha} = 1 + \frac{(\bar{p}-\underline{p})\underline{p}Fc^u(q)}{(1-\underline{p})\left(B - (\bar{p}-\underline{p})\left((1-\alpha)V_s + F\right)\right)^2} \quad (16)$$

An interior Pareto optimum yielding a utility level v solves $\Phi^q(q,\rho(q,v)) = \Phi^\alpha(q,\rho(q,v))$. As $\Phi^\alpha_\alpha = \frac{-(\bar{p}-\underline{p})^2 V_s \underline{p} F c^u(q)}{(1-\underline{p})\left(B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)\right)^3} < 0$ is bounded away from zero while $\Phi^q(q,\rho(q,v))$ varies from $+\infty$ to 0 over $[q^u; q^\pi]$ the equation has at least a solution. We assume that it is *unique*, name it $q^*(v)$ and differentiate it.

$$0 = \left(\Phi^q_q - \Phi^\alpha_q\right) q^*_v + \left(\Phi^q_\alpha - \Phi^\alpha_\alpha\right)\left(\rho_q q^*_v + \rho_v\right) = \left(\Phi^q_q - \Phi^\alpha_q\right) q^*_v + \left(\Phi^q_\alpha - \Phi^\alpha_\alpha\right)\frac{1 - q^*_v u_q}{u_\alpha}$$

$$\Rightarrow q^*_v = \frac{\Phi^\alpha_\alpha - \Phi^q_\alpha}{\left(\Phi^q_q - \Phi^\alpha_q\right)u_\alpha - \Phi^q_\alpha u_q} = \frac{\Phi^\alpha_\alpha - \Phi^q_\alpha}{u_\alpha\left(\Phi^q_q - \Phi^\alpha_q - \Phi^q_\alpha \pi_q/\pi_\alpha\right)} \quad (17)$$

using $\frac{\pi_q}{\pi_\alpha} = \frac{u_q}{u_\alpha}$ at the optimum.

We have $\Phi^q_q = \left(1 - \frac{\underline{p}F}{B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)}\right)\frac{c^u_{qq}c^\pi_q - c^\pi_{qq}c^u_q}{-(c^\pi_q)^2} = \Phi^q\left(\frac{c^u_{qq}}{c^u_q} - \frac{c^\pi_{qq}}{c^\pi_q}\right) > 0$, $\Phi^q_\alpha = \frac{(\bar{p}-\underline{p})V_s\underline{p}F}{\left(B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)\right)^2}\frac{c^u_q}{-c^\pi_q} > 0$, $\Phi^\alpha_q = \frac{(\bar{p}-\underline{p})\underline{p}Fc^u_q}{(1-\underline{p})\left(B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)\right)^2} = \Phi^q_\alpha\frac{\pi_q}{\pi_\alpha} > 0$ thus

$$\begin{aligned} q^*_v > 0 \quad &\Leftrightarrow \Phi^q_q > \Phi^\alpha_q - \Phi^q_\alpha\frac{c^\pi_q}{(1-\underline{p})V_s} = 2\Phi^\alpha_q \\ &\Leftrightarrow \Phi^q\left(\frac{c^u_{qq}}{c^u_q} - \frac{c^\pi_{qq}}{c^\pi_q}\right) > \frac{2(\bar{p}-\underline{p})\underline{p}Fc^u_q}{(1-\underline{p})\left(B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)\right)^2} \\ &\Leftrightarrow \frac{c^u_{qq}}{c^u_q} - \frac{c^\pi_{qq}}{c^\pi_q} > \frac{-2(\bar{p}-\underline{p})\underline{p}Fc^\pi_q}{(1-\underline{p})\left(B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)\right)\left(B-\underline{p}F-(\bar{p}-\underline{p})((1-\alpha)V_s+F)\right)} \end{aligned}$$

As $c^u_q(q^u) = c^\pi_q(q^\pi) = 0$, there exists strictly positive constants A^u and A^π such that $\frac{c^u_{qq}}{c^u_q} > A^u$ and $\frac{c^\pi_{qq}}{-c^\pi_q} > A^\pi$ over $[q^u; q^\pi]$. Also, c^π being convex, we have $0 > c^\pi_q > c^\pi_q(q^u)$. The inequality $q^*_v > 0$ therefore holds if

$$0 < (1-\underline{p})\left(A^u + A^\pi\right)\left(B - (\bar{p}-\underline{p})(V_s+F)\right)^2 + 2V_s\underline{p}Fc^\pi_q(q^u)(\bar{p}-\underline{p}) \quad (18)$$

$$\Leftrightarrow \quad \bar{p} - \underline{p} < \gamma \equiv \frac{(1-\underline{p})\left(A^u + A^\pi\right)B(V_s+F) - V_s\underline{p}Fc^\pi_q(q^u) - D}{2(1-\underline{p})\left(A^u + A^\pi\right)(V_s+F)^2} \quad (19)$$

where $D \equiv \sqrt{\left(V_s\underline{p}Fc^\pi_q(q^u) - 2(1-\underline{p})\left(A^u + A^\pi\right)B(V_s+F)\right)V_s\underline{p}Fc^\pi_q(q^u)}$ (we keep only the meaningful root of the second degree equation (18)). In the next proof, we shall introduce the constant $\delta \le \gamma$ that will characterize $H4$. Hence, on the contract curve, quality increases with v while the equity participation $\alpha^*(v) \equiv \rho(q^*(v), v)$ decreases as $\rho_q < 0$ and $\rho_v < 0$. ■

Proof of Proposition 3

We solve the unconstrained problem first and then introduce the constraint in a way that eases the resolution.

Step 1: Solve the program $\tilde{P}(\theta) \equiv \max_{v,\alpha} \Pi_i(q(\alpha, v), \alpha) = D(v - v^{\theta})\, \pi\,(q(\alpha, v), \alpha)$

The FOC of $\tilde{P}(\theta)$ with respect to α is $\frac{\pi_\alpha}{\pi_q} = \frac{u_\alpha}{u_q}$ thus the optimal equity share given v is $\alpha^*(v)$ (cf. Lemma 1). The FOC of $\tilde{P}(\theta)$ with respect to v is $0 = \frac{n}{u_q}\frac{\partial \Pi_i}{\partial q} = \theta\pi + \frac{\pi_q}{u_q}$ and by replacing α with the optimal value $\alpha^*(v)$ we obtain a unique equation[18]

$$\theta\ = H(v) \equiv \left.\frac{-\pi_q}{\pi.\, u_q}\right|_{\substack{\alpha=\alpha^*(v)\\ q=q(\alpha^*(v),v)}} = \left.\frac{-\pi_\alpha}{\pi.\, u_\alpha}\right|_{\substack{\alpha=\alpha^*(v)\\ q=q(\alpha^*(v),v)}} \tag{20}$$

Using the alternative derivation with q and $\alpha = \rho(q, v)$, we can write

$$\frac{1}{H(v)} = \pi\,(q^*, \alpha^*)\, \Phi^\alpha\,(q^*, \alpha^*) \tag{21}$$

Note that $\frac{d\pi}{dv} = \pi_q q_v^* + \pi_\alpha \left(\frac{1-u_q q_v^*}{u_\alpha}\right) = \frac{\pi_\alpha}{u_\alpha} < 0$ by the envelope theorem, while $\frac{d\Phi^\alpha}{dv} = \Phi_q^\alpha q_v^* + \Phi_\alpha^\alpha \alpha_v^* > 0$ as a consequence of $\Phi_q^\alpha > 0$, $q_v^* > 0$, $\Phi_\alpha^\alpha < 0$ and $\alpha_v^* < 0$. Hence to show $H' > 0$ we need to analyze the behavior of the product $\pi\,(q^*, \alpha^*)\, \Phi^\alpha\,(q^*, \alpha^*) = \pi\,(q^*, \alpha^*)\, \Phi^q\,(q^*, \alpha^*)\,.$

Differentiation yields

$$\pi\frac{d\Phi^q}{dv} + \Phi^q\frac{d\pi}{dv} = \pi\left(\Phi_q^q q_v^* + \Phi_\alpha^q \alpha_v^*\right) + \Phi^q\frac{\pi_\alpha}{u_\alpha} = \pi\left(\Phi_q^q q_v^* + \Phi_\alpha^q \alpha_v^*\right) - 1$$

$$\text{thus } H' > 0 \Leftrightarrow \Phi_q^q q_v^* + \Phi_\alpha^q\left(\rho_q q_v^* + \rho_v\right) < \frac{1}{\pi} \Leftrightarrow \Phi_q^q q_v^* + \Phi_\alpha^q \tfrac{1-u_q q_v^*}{u_\alpha} < \tfrac{1}{\pi}$$

$$\Leftrightarrow \frac{\Phi_\alpha^\alpha - \Phi_\alpha^q}{u_\alpha}\, \frac{\Phi_q^q - \frac{\pi_q}{\pi_\alpha}\Phi_\alpha^q}{\Phi_q^q - 2\frac{\pi_q}{\pi_\alpha}\Phi_\alpha^q} + \frac{\Phi_\alpha^q}{u_\alpha} < \frac{1}{\pi} \tag{22}$$

As the LHS of (22) is bounded by $\frac{\Phi_\alpha^\alpha}{u_\alpha}$, a sufficient condition for $H' > 0$ is $\dfrac{\Phi_\alpha^\alpha}{u_\alpha} < \dfrac{1}{\pi}$

$$\Leftrightarrow\quad \Phi_\alpha^\alpha > \frac{u_\alpha}{\pi} = -\Phi^\alpha\frac{\pi_\alpha}{\pi}$$

$$\Leftrightarrow\quad \frac{(\bar{p}-\underline{p})^2 V_s \underline{p} F c^u(q)}{(1-\underline{p})\big(B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)\big)^3} < \frac{\pi_\alpha}{\pi}\left(1 + \frac{(\bar{p}-\underline{p})\underline{p} F c^u(q)}{(1-\underline{p})\big(B-(\bar{p}-\underline{p})((1-\alpha)V_s+F)\big)^2}\right) \tag{23}$$

and a stronger sufficient condition is $\bar{p} - \underline{p} < \delta \equiv \min\left\{\gamma, \frac{(1-\underline{p})}{\pi}\left(B - \gamma(V_s + F)\right)\right\}$. Indeed, on the one hand we have $\bar{p} - \underline{p} < \frac{(1-\underline{p})}{\pi}\left(B - \gamma(V_s + F)\right)$

$$\Rightarrow (\bar{p} - \underline{p})V_s < \frac{\pi_\alpha}{\pi}\left(B - \gamma(V_s + F)\right)$$

[18] We do not need a fixed point argument to derive the equilibrium because the solution of $\tilde{P}(\theta)$ does not depend on the level v^θ.

and on the other hand, $B-(\bar{p}-\underline{p})\left((1-\alpha)V_s+F\right) > B-\gamma\left(V_s+F\right)$

$$\Rightarrow \frac{\pi_\alpha}{\pi}\left(B-(\bar{p}-\underline{p})\left((1-\alpha)V_s+F\right)\right) > \frac{\pi_\alpha}{\pi}\left(B-\gamma\left(V_s+F\right)\right)$$

thus

$$(\bar{p}-\underline{p})V_s < \frac{\pi_\alpha}{\pi}\left(B-(\bar{p}-\underline{p})\left((1-\alpha)V_s+F\right)\right)$$

which is a stronger condition than (23) (obtained by dropping the 1 in the RHS).

Step 2: Solve $P(v) \equiv \max_{q,\alpha} \pi(q,\alpha)$ such that $u(q,\alpha) \geq v$ and $0 \leq \alpha \leq 1$.

We study first the monopoly and the purely competitive markets. Since entrepreneurs have an inelastic demand for one loan, a monopoly maximizes the per-capita payoff $\pi\left(q,\alpha,\beta,L\right)$ under the feasibility constraints ($0 \leq \alpha \leq 1, 0 \leq \beta \leq 1$ and $L \geq K-w$) and the participation constraint $u\left(q,\alpha,\beta,L\right) \geq \underline{v}$. The arguments used in the proof of Proposition 2 apply for the monopoly as well, thus only the equity share constraint $\alpha \leq 1$ remains. The monopoly therefore solves the program $P(\underline{v})$.

The purely competitive outcome, on the other hand, maximizes $u(q,\alpha)$ under the set of constraints $\{\pi(q,\alpha) \geq 0 \text{ and } \alpha \leq 1\}$; by duality[19] this amounts to solve $P(\bar{v})$ for some indirect utility level $\bar{v} > \underline{v}$.

The solution of $P(v)$ is very simple to find using $q^*(v)$ and $\alpha^*(v)$ identified in Lemma 1. Starting from the largest level $\bar{v}$, one decreases v. As long as $v \geq \hat{v} \equiv \alpha^{*-1}(1)$, the contract $(\alpha^v, q^v) \equiv (\alpha^*(v), q\left(\alpha^*(v), v\right))$ is optimal i.e., the equity share increases while the advising quality decreases. For $v = \hat{v}$, the optimal pair is simply $(1, q(1,\hat{v}))$ and as v decreases further the advising quality now increases (direct effect only) since the equity share has reached its maximal value. Let us define $\hat{q} \equiv q^*(\hat{v})$.

Since we assumed that capitalist and entrepreneurs are active in equilibrium, we have $\pi(\hat{q},1) = \underline{p}V_f + (1-\underline{p})V_s - K + w - c^\pi(\hat{q}) > 0$ and $u(\hat{q},1) = B_0 - \frac{B-\bar{p}F}{B-(\bar{p}-\underline{p})F}c^u(\hat{q}) > w$. The first condition means that the participation constraint of the capitalist is not binding at $\hat{q}$. Hence, an increase of entrepreneur utility is possible by increasing q and decreasing α. This implies $\hat{v} < \bar{v}$. Symmetrically, the second condition implies $\hat{v} > \underline{v}$.

[19]Duality applies as the constraint set $u(q,\alpha) \geq 0$ is convex in the (q,α) space (cf. appendix).

The value of $P(v)$ is thus $W(v) \equiv \begin{cases} \pi\left(q(1,v),1\right) & \text{if } v < \hat{v} \\ \pi\left(q(\alpha^*(v),v),\alpha^*(v)\right) & \text{if } v \geq \hat{v} \end{cases}$. The first term is decreasing as $\pi_\alpha > 0$ and $q_v < 0$. The second is also decreasing because its derivative is $\pi_q q_v = \frac{\pi_q}{u_q} < 0$ (envelope theorem). Hence $W(.)$ is decreasing. It is also immediate to observe that both $W(.)$ and $P(.)$ are continuous (including at $\hat{v}$).

Step 3: Solve $\hat{P}(\theta) = \max\limits_{q,\alpha} \Pi_i(q,\alpha)$ s.t. $u(q,\alpha) \geq \underline{v}, \alpha \leq 1$

For $\theta \geq \hat{\theta} \equiv H(\hat{v})$, we set $v^\theta \equiv H^{-1}(\theta) \geq v^0 > \underline{v}$, thus $\alpha^\theta \equiv \alpha^*(v^\theta) \leq 1$ and $q^\theta \equiv q(\alpha^\theta, v^\theta)$ solve $\hat{P}(\theta)$. Yet when $\theta < \hat{\theta}$, the fact that $\alpha^*\left(H^{-1}(\theta)\right) > 1$ means that competition then takes place over a single variable, the advising quality. It varies a priori in $[q\left(\alpha^*(\hat{v}),\hat{v}\right); q^\pi]$. The participation constraint being $u(q,1) \geq \underline{v} \Leftrightarrow q \leq q(1,\underline{v})$, the correct upper bound is thus $\min\left\{q(1,\underline{v}), q^\pi\right\}$.

The profit function is now $\tilde{\Pi}_i(q_i) \equiv D\left(u(q_i,1) - u(\tilde{q},1)\right)\pi(q_i,1)$ where $\tilde{q}$ denotes the equilibrium advising quality. The unique FOC to be satisfied at the symmetric equilibrium is similar but different from (20):

$$\theta = G(q) \equiv \left.\frac{-\pi_q}{\pi.\, u_q}\right|_{\alpha=1} \tag{24}$$

Since $\Phi^q_q < 0$ and $\pi_q > 0$ we have $\Phi^q_q \pi > \Phi^q \pi_q \Leftrightarrow G'(v) < 0$. The candidate Nash equilibrium advising quality is $G^{-1}(\theta)$; it varies from $G^{-1}(0) = q^\pi$ to $G^{-1}(+\infty) = q^u$. The equilibrium is thus $\min\left\{G^{-1}(\theta), q(1,\underline{v})\right\}$.

To summarize, the symmetric equilibrium is $\alpha^\theta = 1$ and $q^\theta = \min\left\{G^{-1}(\theta), q(1,\underline{v})\right\}$ if $\theta < \hat{\theta}$, $\alpha^\theta = \alpha^*\left(H^{-1}(\theta)\right)$ and $q^\theta = q\left(\alpha^\theta, H^{-1}(\theta)\right)$ otherwise. ■

Proof of Corollary 2

As the frequency of visionary management in the symmetric PBE is $\sigma^\theta = \frac{1}{(\bar{p}-\underline{p})V_s}\frac{c^\pi(q^\theta)}{\alpha^\theta}$,we have

$$\frac{d\sigma^\theta}{d\theta} \propto c^\pi_q(q^\theta)\frac{dq^\theta}{d\theta} - \frac{c^\pi(q^\theta)}{\alpha^\theta}\frac{d\alpha^\theta}{d\theta}$$

When $\theta \in \left[0;\hat{\theta}\right[$, $\dfrac{d\alpha^\theta}{d\theta} = 0$ thus $\dfrac{d\sigma^\theta}{d\theta} > 0$ as $c^\pi_q < 0$ and $\dfrac{dq^\theta}{d\theta} < 0$. Over $\left[\hat{\theta};+\infty\right[$, we use $\alpha = \rho(q^*(v),v)$ to obtain $\dfrac{d\alpha^\theta}{d\theta} = \left(\rho_q q^*_v + \rho_v\right)\dfrac{dv^\theta}{d\theta} = \dfrac{1-u_q q^*_v}{u_\alpha}\dfrac{dv^\theta}{d\theta} < 0$ and $\dfrac{dq^\theta}{d\theta} = q^*_v\dfrac{dv^\theta}{d\theta} > 0$, so that

$$\frac{d\sigma^\theta}{d\theta} > 0 \Leftrightarrow q^*_v c^\pi_q(q^\theta) > \frac{c^\pi(q^\theta)}{\alpha^\theta}\frac{1-u_q q^*_v}{u_\alpha}$$

$$\Leftrightarrow \quad q_v^* \alpha^\theta u_\alpha c_q^\pi < (1 - u_q q_v^*)\, c^\pi$$

$$\Leftrightarrow \quad c^\pi > q_v^* \left(\alpha^\theta u_\alpha c_q^\pi + u_q c^\pi \right) = q_v^* \left(u_q c^\pi - \alpha^\theta u_\alpha \pi_q \right)$$

$$\Leftrightarrow \quad c^\pi > q_v^* u_q \left(c^\pi - \alpha^\theta \pi_\alpha \right) \qquad \text{using } \frac{\pi_\alpha}{\pi_q} = \frac{u_\alpha}{u_q}$$

a sufficient condition is thus $q_v^* u_q < 1$

$$\Leftrightarrow \quad \left(\Phi_\alpha^\alpha - \Phi_\alpha^q\right) u_q > u_\alpha \left(\Phi_q^q - \Phi_q^\alpha\right) - \Phi_\alpha^q u_q$$

$$\Leftrightarrow \quad \Phi_\alpha^\alpha u_q > u_\alpha \left(\Phi_q^q - \Phi_q^\alpha\right) \Leftrightarrow \Phi_\alpha^\alpha < \frac{\pi_\alpha}{\pi_q} \left(\Phi_q^q - \Phi_q^\alpha\right)$$

but as $\Phi_\alpha^\alpha < 0$ a stronger sufficient condition is $\Phi_q^q > \Phi_q^\alpha$ which is true as we already proved $q_v^* > 0 \Leftrightarrow \Phi_q^q < 2\Phi_q^\alpha$.

As for advising, we have a simple link between the product $q^\theta \tau^\theta$ and the share α^θ :

$$q^\theta \tau^\theta \equiv 1 - \frac{\underline{p}F}{B - (\bar{p} - \underline{p})\left((1 - \alpha^\theta)V_s + F\right)} \tag{25}$$

Thus, over $\left[0; \hat{\theta}\right[$, we see that $\frac{d\tau^\theta}{d\theta} > 0$ as $\frac{dq^\theta}{d\theta} < 0$ and $\frac{d\alpha^\theta}{d\theta} = 0$. Over $\left[\hat{\theta}; +\infty\right[$, $\frac{d\alpha^\theta}{d\theta} < 0$ implies that the RHS of (25) is decreasing. Combining this with $\frac{dq^\theta}{d\theta} > 0$ over the high regime, yields $\frac{d\tau^\theta}{d\theta} < 0$.

Lastly, it is immediate to see from (25) that the advising quality expected by entrepreneurs $q^\theta \tau^\theta$ is weakly decreasing with competitiveness θ (constant over the soft regime and strictly decreasing otherwise). ■

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