CSEF
Centre for Studies in Economics and Finance

WORKING PAPER NO. 662

The Limits of Limitless Debt

Kent Osband, Salvatore Capasso, and Valerio Filoso

December 2022

University of Naples Federico II
University of Salerno
Bocconi University, Milan

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS AND STATISTICS – UNIVERSITY OF NAPLES FEDERICO II
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unina.it
ISSN: 2240-9696

CSEF
Centre for Studies in Economics and Finance

WORKING PAPER NO. 662

The Limits of Limitless Debt

Kent Osband[*], **Salvatore Capasso**[†], **and Valerio Filoso**[‡]

Abstract
How worrisome are mounting sovereign debt-to-GDP ratios? Many economists profess little concern. Debt stocks are irrelevant to sustainability in standard macro models, while low real interest rates testify to lender optimism. Furthermore, the debt is mostly in fiat currency, which eases rollover. Yet historical evidence (Reinhart and Rogoff, 2009) shows that high sovereign debt is prone to default and that credit spreads are often trailing indicators. This paper offers a simple way to model the trade-offs. On the one hand, it acknowledges that large debt overhangs tend to raise default risks. On the other hand, it allows sovereigns to roll over debt regardless of long-term fiscal solvency. The combination allows credit spreads to stay very low for decades yet eventually spiral out of control and trigger default. Hence, neither the reassurance of low spreads nor the alarm from growing overhang should automatically prevail. To illustrate the trade-offs, we review the ebb and flow of US sovereign debt burdens since World War II. Between record peacetime debt-to-GDP ratios and weakened fiscal discipline, an exemplary double-or-triple-A credit rating for the US no longer seem justified.

Keywords: bond market, bond interest rate, credit spreads, sovereign debt, sovereign debt default, debt management surprise.

JEL classification: D840, G120, H630.

[*] Institute for Studies on the Mediterranean (ISMed), CNR. E-mail: kent.osband@ismed.cnr.it (*Corresponding author*).

[†] CSEF, University of Naples Parthenope, and Institute for Studies on the Mediterranean (ISMed), CNR. E-mail: capasso@ismed.cnr.it

[‡] University of Naples Federico II, and Institute for Studies on the Mediterranean (ISMed), CNR. E-mail: filoso@unina.it

1 Introduction

Over the past half-century, most leading states have accumulated record levels of debt (Kose et al., 2020; Yared, 2019). Even after netting out debt held by central banks, the ratio of sovereign debt to GDP in 2021 reached 80 percent for the world and 97 percent for G7 countries (IMF, 2022). Twenty years ago, peacetime debt accumulations of these magnitudes were generally considered unattainable, on the grounds that market pressures would force sovereigns to either trim their debt ratios or default. Reality has confounded these expectations.

Naturally, this encouraged a reappraisal. Nowadays, relatively few economists profess much concern about sovereign default risks. Theorists offer three main reasons not to worry. First, the stock of government debt is irrelevant in most macroeconomic models (Bhandari et al., 2017; Barro, 1974); only the net flow of funds matters (Hellwig, 2021). Second, real interest rates are generally low, which suggests that bond markets are optimistic about long-term fiscal sustainability and government commitments to repay (Blanchard, 2019). Third, the debt is mostly in fiat currency, whose issuance imposes few direct costs on the government (Bolton, 2016).

Curiously, none of these rationales command much historical support. Issuers of fiat currencies occasionally default, whether or not they can technically service their debt. Debt markets rarely anticipate either longer-term default or their own abrupt surges in short-term interest rates. And strongly rising debt-to-GDP ratios have often portended difficulties in servicing. Judging from the historical evidence first marshaled by Reinhart and Rogoff's 2009 book *This Time is Different* (2009) and subsequently elaborated by Abbas et al. (2019) and Badia et al. (2022), current debt levels and trends bode significant risks.

In short, fashionable theory calls for calm, while evidence stokes worry. In our view, as explained in Section 2, the current fashion is too complacent. The core theoretical insight worth preserving is that servicing depends far more on rollover than transferring real goods and services. The core insight worth adding is that default risk tends to rise with the stock of outstanding debt.

We are not alone in this view. Romer (2012), Chapter 12.10, Chatterjee and Eyigungor (2012), Ghosh et al. (2013), and Debrun et al. (2019) model default risk as as an increasing function of total debt. Our paper makes three main contributions to this literature. First, we model sovereign debt markets as rational short-term without presuming they are prescient long-term. Second, we show that debt can become unsustainable even when no transversality or no-Ponzi-game conditions are imposed. Third, we offer evidence for downgrading sovereign US debt to single-A or lower credit grade.

For analytic tractability, we model default risk as a positive power function of the debt-to-GDP ratio b. In the simplest case, treated in Section 3, the economy's growth rate g, the risk-free rate r, and the primary surplus q all equal zero. Rollover bolsters the government's reputation for servicing without draining real sovereign resources. In effect, lenders get reassured by their successors. Still, mounting debt feeds back into higher credit spreads, slowly at first but eventually accelerating so fast that default becomes inevitable in finite time. Section 4 illustrates these dynamics through numerical simulations. Using plausible estimates of core parameters, the market might stay calm for many decades and still trigger a dramatic surge in debt burdens and default risks.

Section 5 generalizes the model to allow non-zero r, g, q, and post-default salvage value. It has become fashionable to treat $r < g$ as key to debt sustainability, since that reduces the relative burden of an existing stock of debt. However, consistent with the critiques in Cochrane (2021), Reis (2022), and Brumm et al. (2021),long-term sustainability in our model requires a broader constraint, for which $r < g$ is neither necessary nor sufficient. As our simulations in section 6 show, fiscal laxness can swell a new debt burden much faster than the old burden shrinks.

The main shortcoming of the model is its presumption that parameters are permanently fixed. The application requires qualitative judgments about likely future paths and longer-term averages. To illustrate the challenges facing analysts, section 7 tracks the evolution of US federal debt-to-GDP ratios between two historical peaks. The first stemmed from military spending in World War II; the overhang was scaled back through decades of $r < g$ and modest primary surpluses q. The second stems from decades of deterioration in both $g - r$ and q and shows few signs of aggregate correction. On this basis, we conclude that the US no longer warrants an exemplary credit rating, which is not to claim that default risks are high.

Our results support Reinhart and Rogoff's warning (2009, xxv) that "excessive debt accumulation [...] often poses greater system risks than it seems during a boom". Indeed, they strengthen that warning, as the risks in our models rise through rational calculation rather than the manic delusion that "this time is different". The results call for extra vigilance over fiscal policy without waiting for the market to signal massive discomfort. Managing expectations about short-term interest rates is not enough. Section 8 points to avenues for future research.

2 Does the stock of sovereign debt matter?

Consider a government issuing inflation-indexed debt denominated in its own freely floating currency. A formal default is said to occur if the government fails to repay the debt as originally promised. If it wants, the government can repay any debt by printing more currency, which might stoke extra inflation, and the indexing protects the borrowers if this occurs. Alternatively, the government might choose to issue debt denominated in foreign currency, which simplifies the computation of returns but exposes parties to foreign exchange risk that we will ignore. We impose these restrictions to reduce all risk on real returns to sovereign default alone.

We shall further restrict our focus to a single controllable influence, namely the accumulated stock of sovereign debt. Many economists claim that this has no bearing on default risk, in which case further investigation is moot. Consequently, we will devote this section to sketching and refuting their claims.

The most extreme case for nil influence comes from Modern Monetary Theory (MMT) proponents. Here is a characteristically confident claim:

> A nation that adopts its own floating currency can always afford to put unemployed domestic resources to work. Its government will issue liabilities denominated in its own currency and will service its debt in its own currency. Whether its debt is held internally or externally, it faces no insolvency risk (Mitchell et al., 2019, p. 517).

This is formally correct: a fiat currency issuer can easily print more money. However, governments have plenty of reasons to default other than strict insolvency. Default amounts to a surprise tax imposed on debt holders. The revenues are remarkably easy to collect since they come from not paying what was promised. While default sacrifices longer-term credibility, lenders are often seen as relatively privileged, and governments might decide that the advantages prevail. MMT proponents are not blind to this, as they call for inflationary pressures to be restrained through taxes and recognize the distributional consequences. They just choose to downplay default risks not to undermine their case for fiscal control of fiat money.

Even when sovereign debt isn't treated as inherently worthless, it is tempting to belittle the importance of debt stocks and focus only on flows. According to Diamond (1965), "a fixed absolute amount of debt, in a growing economy, would asymptotically have no effect". According to Blanchard (2019), "public debt may have no fiscal cost". Such claims rest on four core truths:

- Most sovereign debt is effectively repaid by rollover into new debt, which defers pressure on real resources.
- The real growth rate g of GDP often exceeds the real interest rate r on sovereign debt, in which case the relative burden of old debt shrinks at rate $g - r$.
- If the government runs a primary (non-interest) surplus at a fraction q of the debt stock, the aggregate debt-to-GDP ratio b will shrink provided $q > r - g$.
- Governments can temporarily expand b greatly without thwarting long-term fiscal balance.

These truths are easily extended to extremes. If the debt is continually rolled over, no real resources are tapped to service it. In the process, its relative burden diminishes as long g exceeds r. Given an adequate primary surplus, no $r > g$ is worrisome either. The latter can accommodate any interim expansion of b as long as q eventually exceeds $r - g$.

From this perspective, the only threat to debt sustainability is doubt about long-term fiscal balance. Doubt manifests itself as a risk premium c on new debt. A low current c suggests the government's debt path is sustainable. A high c warns the government to manage expectations better, perhaps by pressing its central bank to buy up debt. Either way, debt rollover takes precedence over its nominal costs and obviates concerns about its real costs.

However, default has benefits too. What the government doesn't pay to debt holders, it can pay to others who might need funds more or clamor that they do. Moreover, these benefits tend to scale linearly with the amounts defaulted. In contrast, the costs – including disruption of contracts, restrictions on new borrowing, and higher risk premia – tend to be fixed or scale more slowly than the amounts.

3 Basic model of default-prone debt

Consider a market economy that keeps real output and prices perfectly stable. The government spends exactly what it receives back in taxes, so the primary (non-interest) fiscal surplus is always zero. However, in the past, the government accumulated debt. The government prefers not to redeem the debt, as that might spark inflation or reduce perceived domestic welfare (e.g., perhaps many creditors are foreign or belong to a disliked social group). Yet the government does not wish to default, as that will disrupt the payment system and dry up credit lines that might be needed later. Consequently, the government issues overnight T-bills at whatever rate is needed to roll over the outstanding debt and aims to continue this policy forever.

Fortunately, creditors are risk-neutral and accept waiting. They will gladly hold T-bills as long as expected overnight returns are non-negative. If creditors believe the government will never default, the interest rate for rolling over sovereign debt will equal zero.

Unfortunately, the transactions are tinged with fear. The government fears that private creditors might demand cash for their maturing T-bills, use the cash to buy goods, and upset equilibrium in the real sector. The creditors fear that the government will default. These fears mutually reinforce one another. The more creditors fear default, the more tempted they will be to switch to goods. The more the government fears creditor switch, the more tempted it will be to default preemptively. Let us assume that some fearful equilibrium is reached where perceptions match reality. This equates the perceived hazard rate $h(t)$ to the probability of default at time t given no previous default.

The greater the T-bill stock $B(t)$, the greater the risk of default since more real resources might be drained upon redemption. To reduce clutter, the initial time will be numbered 0, all initial values will be subscripted with 0, and B will be rewritten as a multiple F of B_0. The simplest tractable formulation makes $h(t)$ a power function of F with positive power α:

$$h(t) = h_0 \left(\frac{B(t)}{B_0} \right)^{\alpha} \equiv h_0 (F(t))^{\alpha}. \tag{1}$$

Perceived risk forces the T-bills to offer an extra credit spread $c(t) > 0$. Over the next instant dt, the accrued interest is approximately $c(t)B(t)dt$, while the default risk is approximately $h(t)dt$. If T-bills are worthless after default, the expected rate of return approaches $c(t) - h(t)$. Risk neutrality requires

$$c(t) = h(t). \tag{2}$$

That is, the instantaneous credit spread must equal the hazard rate, absent salvage value after default.

How does $F(t)$ behave before a default occurs? Assuming all payments on T-bills are rolled over into new T-bills,

$$\frac{dF(t)}{dt} = c(t)F(t) \quad \text{or} \quad F' = cF, \tag{3}$$

where the second expression uses $'$ to denote the derivative and drops the time argument.

Combine (1)–(3) to obtain $F' = c_0 F^{\alpha+1}$, whose only feasible solution is

$$F(t) = (1 - c_0 \alpha t)^{-1/\alpha}. \tag{4}$$

For large values of t, the implied $F(t)$ is imaginary or negative. On inspection, F blows up to infinity in a finite time Ω given by

$$\Omega = \frac{1}{c_0 \alpha}. \tag{5}$$

We will call Ω the Debt Apocalypse because no debt gets rolled over past Ω without default. We can confirm this by examining the credit spread, which from (1), (2), (4) and (5) works out to

$$c(t) = \frac{c_0}{1 - c_0 \alpha t} = \frac{1}{\alpha(\Omega - t)}. \tag{6}$$

Result 1 *As $t \to \Omega$, the credit spread, and hazard rate outstrip any finite bound, ensuring default before Ω.*

This certainty of default is puzzling. Since no one should buy T-bills maturing at time Ω, why keep rolling over until then? The core answer is that the buyers in our model do not look that far ahead. They keep shortening the duration of their T-bills so that it never exceeds $\Omega - t$. While such extreme myopia seems to defy rationality, there is no need to worry much about it. In practice, there will be some positive floor on T-bill duration, in which case some high but finite interest rate will generate a zero expected return and some minuscule chance that debt is successfully rolled over past Ω.

How long can debt be rolled over before hazard rates soar? At $t = \frac{1}{2}\Omega$, halfway to the Debt Apocalypse, h only doubles, and another $\frac{1}{4}\Omega$ passes before it doubles again. Many defaults occur well before interest rates get sky-high. The survival probability $S(t)$ satisfies

$$S' = -hS, \tag{7}$$

which together with (2) and (3) imply $(\log S)' = -(\log F)'$. Hence

$$S(t) = \frac{1}{F(t)} = (1 - \alpha c_0 t)^{1/\alpha} = \left(1 - \frac{t}{\Omega}\right)^{1/\alpha}. \tag{8}$$

As shown in Appendix A.1, the mean time until default is $\theta = \alpha\Omega/(\alpha + 1)$. It follows that $c(\theta) = (\alpha + 1)c_0 = 1/\theta$ and $S(\theta) = (\alpha + 1)^{-1/\alpha}$. As we will explain shortly, α is unlikely to be as low as 1 or as high as 4, in which case $2c_0 < c(\theta) < 4c_0$ and $0.5 < S(\theta) < 0.67$. Nevertheless, as Appendix A.2 shows, extreme events tend to dominate.

Result 2 *The expected T-bill stock on default is infinite.*

If we were observing a long sequence of games that roll over debt until it defaults, the average B on default would rise over time. Optimists would praise the apparent progress toward perpetual rollovers of debt without default, while pessimists would lament the naïveté of lenders. All would note the bifurcation between long periods of high credibility with low, slowly accumulating debt and short periods where debt balloons and interest rates soar.

4 Simulations of the basic model

Let us reflect on plausible values for α, the elasticity of default risk to debt. From the sovereign's perspective, the main motive for default is to stem redemptions of debt for real goods and services and their resulting infringements on domestic consumption. Applying standard consumer welfare arguments, the marginal disutility rises more than proportionally to the infringements. Hence, incentives for default scale faster than B, which suggests $\alpha > 1$. The upper bound $\alpha < 4$ cited above reflects skepticism that a doubling of B would increase default risk sixteen-fold.

We can narrow the plausible range further by considering the credit grades that rating agencies use to rank default risks. The eight main letter grades on an ascending scale of risk can be summarized as triple-A, double-A, single-A, triple-B, double-B, single-B, triple-C, and double/single-C. Ratings of triple-B and higher are known as investment grade, while lower ratings are known variously as sub-investment grade, high-yield, or junk.

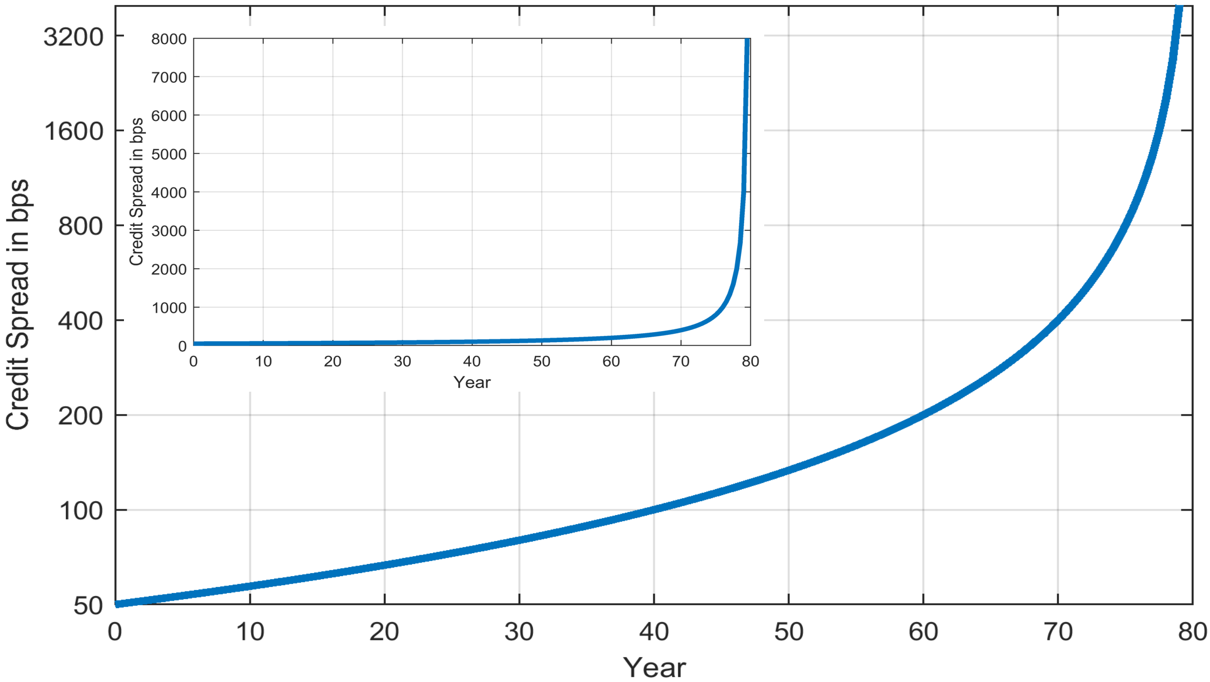
Although sovereign ratings do not explicitly quantify default risks, they are often likened to their corporate analogues, where data is abundant. Corporate default history suggests that default hazard rates are less than 0.01% or one basis point (bp) for triple-A, around 50 bps at the triple-B/double-B boundary, and over 1,000 bps for most triple-C. Each drop in letter grade is associated with roughly quadruple the average default risk; (Osband, 2020) estimates a multiplier of 4.2. By that metric, $\alpha \approx 2$ if doubling debt knocks one letter off the credit grade, while $\alpha \approx 4$ if doubling debt knocks two letters off the credit grade.

Default risks presumably depend far more on the debt-to-GDP ratio b than on the absolute stock B. Reinhart and Rogoff (2009) found that $b > 0.7$ was generally a red flag for default risks. Badia et al. (2022) estimated the same threshold of 0.7 for advanced countries and a lower threshold of about 0.3 for emerging market economies. If we treat $b = 1$ as the average single-B risk and $b = 0.1$ as somewhere between single-A and triple-A risk, then $2 < \alpha < 3$. We will use $\alpha = 2.5$ as a benchmark.

Granted, domestic GDP is an imperfect metric of sovereign debt capacity. When a country's fiat currency serves as an international reserve, the relevant GDP should include some foreign GDP too. Alternatively, we might deflate debt by potential tax revenue or primary surplus, since GDP likely includes many elements the government cannot touch. However, it is hard to estimate potential revenue, which depends on both uncertain capacity and contested political will. Also, Ostry et al. (2010) and Ghosh et al. (2013) found evidence of fiscal fatigue, where the primary surplus stabilizes or declines when b gets high.

Our model finesses the complications. The only debt metric it cares about is F, the ratio of B to B_0, and since that is endogenous we can track fair credit spreads without it. Figure 1 charts the evolution of c starting from $c_0 = 50$ bps. Here $\Omega = 80$ years. The inset plots spreads on a linear scale and make the spreads for the first 70 years look modest. However, the subsequent surge is so steep, reaching 8000 bps at year 79.5, that it dominates the chart.

Figure 1: Credit spreads with $\alpha = 2.5$ and $c_0 = 50$ bps. Linear and log scales.



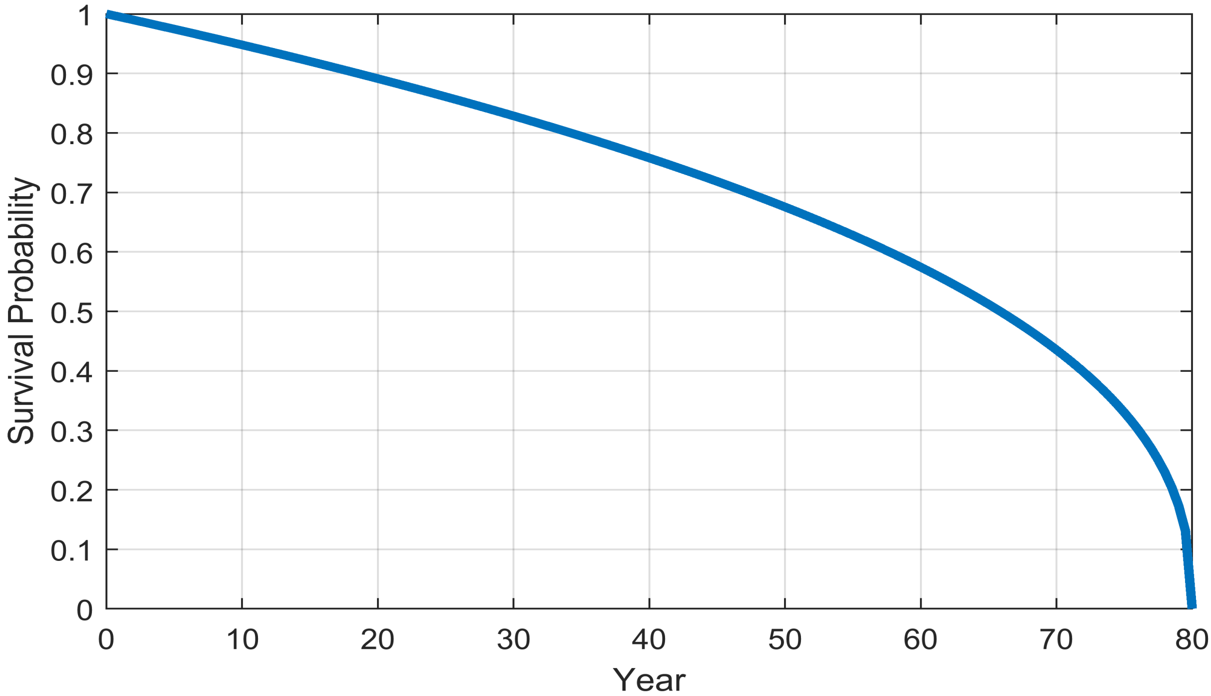
A market that went 75+ years without default would look blasé about rollover most of the time only to panic toward the end. For easier identification, the main chart plots spreads on a logarithmic scale. There it is easy to see that $c = h$ doubles to 100 bps at year 40 and then keeps doubling over a duration that progressively halves: 200 bps at year 60, 400 bps at year 70, 800 bps at year 75, and so on.

For any α and c_0, we can construct a c curve similar to Figure 1. A higher c_0 does not alter that curve; it just shifts the effective starting point to some later time. The core behavior does not change. The credit spread always doubles as the time to Ω halves.

Figure 2 plots survival probability S using the same parameters as Figure 1. There is a 75% chance of rolling over without default for 40 years, a 50% chance of rolling over for 66 years, and a 33% chance of rolling over for 75 years.

If observers of credit spreads and default rates in the first 60 years were unaware of the true drivers, they would be unlikely to guess that the debt is unsustainable long-term. In repeated games, most defaults would occur before debt and credit spreads soar. However, higher α does modify observed behavior. First, it speeds default for any given c_0. Second, it makes the transition to high c from low c more abrupt. When $\alpha = 1$ and $c_0 = 50$ bps, 95% of defaults occur before overnight spreads reach 1000 bps. That share shrinks to 79% for $\alpha = 2$ and 64% for $\alpha = 3$.

Figure 2: Survival probabilities implied by Figure 1.



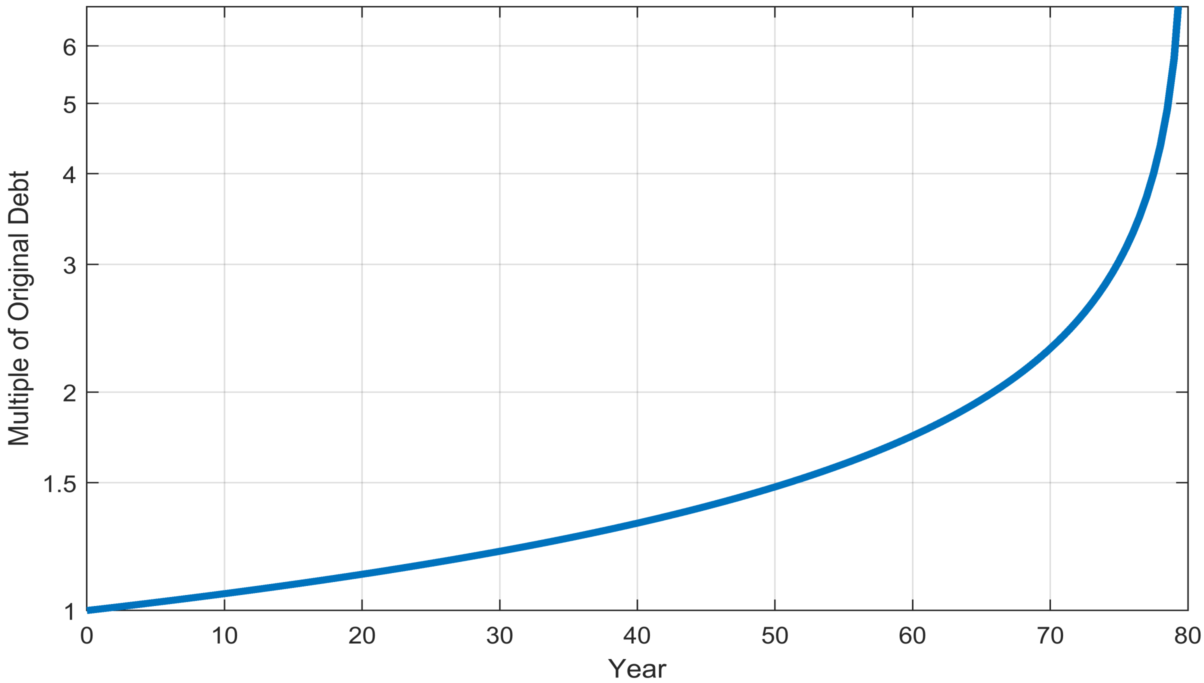
Imagine that a few T-year bonds issued at time t pay a fixed coupon twice a year and redeem the principal at maturity. What annual rate of interest I do they need to offer to sell fairly at par? Net present value calculations require

$$S(t) = S(t+T) + \tfrac{1}{2}I \sum_{k=0}^{2T-1} S(t + \tfrac{1}{2}k),$$

which is readily solved for I. For a 30-year bond issued at time 0, I is 62 bps, only 12 bps more than the initial overnight spread. For a subsequent 30-year bond issued at year 30, I nearly doubles to 119 bps. A 10-year bond issued at year 60 requires $I = 271$ bps, so concerns are clearly mounting. A 5-year bond issued at year 70 requires $I = 539$ bps and a 2-year bond issued at year 75 requires $I = 988$ bps.

Reverting to the initial formulation of all interest repaid immediately through new issuance, Figure 3 tracks F prior to default. F takes 51 years to increase by 50% and 66 years to double. Then growth rapidly accelerates. The next increase of 50% and doubling take 9 years and 11.5 years respectively. To some extent, lenders' rapid enrichment invites default, as the public naturally begrudges the gains and views them as proof that interest rates are extortionate.

Figure 3: Debt ratio F implied by Figure 1.



5 General debt traps

While the preceding simulations warn against complacency about the debt stock and credit spreads, the warnings are exceedingly tame. If the government could raise enough extra net revenue to retire a small share c_0 of maturing debt, B would stabilize; raise any more and debt would eventually vanish. And there's little urgency as spreads will stay tiny for decades. To rouse real alarm, there must be other factors swelling the debt burdens.

To model these factors and potential offsets, this section generalizes the basic model in five ways:

1. Neither creditors nor government worry about B per se but rather about the ratio b of debt to GDP Y. Letting f denote the ratio of b to b_0, we replace (1) with

$$h = h_0 \left(\frac{B/Y}{B_0/Y_0} \right)^{\alpha} \equiv h_0 \left(\frac{b}{b_0} \right)^{\alpha} \equiv h_0 f^{\alpha}. \tag{9}$$

2. GDP grows at constant relative rate g:

$$Y' = gY \tag{10}$$

3. Creditors demand a risk-free return r, which bumps up the interest rate i to r plus the credit spread c:

$$i = r + c. \tag{11}$$

4. If default occurs, creditors lose a fraction η of the T-bills' face value. In an instant dt, the expected rate of return is approximately $(1 - h\,dt)(r + c) - \eta h$. As $dt \to 0$ this should equal r, which modifies (2) to

$$c = \eta h. \tag{12}$$

5. The government runs a primary (non-interest) budget surplus that equals a constant fraction q of the bond stock $B(t)$. While not the most plausible way to depict budget trends, it is the simplest to deal with mathematically, as it lets us replace (3) with

$$F' = B'/B_0 = iF - qF \tag{13}$$

Since $(\log f)' = (\log F)' - (\log Y)'$, it follows from (9) - (13) that

$$f' = \eta h_0 f^{\alpha+1} + (r - g - q)f \equiv c_0 f^{\alpha+1} + \delta f. \tag{14}$$

This differs from (3) in three ways. First, it focuses on changes in relative debt f rather than absolute debt F. Second, it shaves the initial credit spread c_0 to ηh_0 to reflect the salvage value $1 - \eta$. Third, it makes f grow or shrink at a constant relative rate $\delta \equiv r - g - q$ independently of default risk. We call δ the risk-free adjustment.

While (14) is messier than (3), we can establish some key properties by inspection, by noting that f' has the sign of $c_0 f^\alpha + \delta$ and $f_0 = 1$.

Result 3 *Pending default, the debt-to-GDP ratio b grows at an accelerating rate, stabilizes, or shrinks according to the sign of $c_0 + \delta$.*

Appendix A.3 solves the model. For all feasible paths,

$$f(t) = e^{\delta t}\left(\frac{\delta}{\delta + c_0 - c_0 e^{\alpha\delta t}}\right)^{1/\alpha}, \tag{15}$$

$$c(t) = \frac{c_0 \delta e^{\alpha\delta t}}{c_0 + \delta - c_0 e^{\alpha\delta t}}, \tag{16}$$

$$S(t) = \left(1 + c_0 \frac{1 - e^{\alpha\delta t}}{\delta}\right)^{1/(\alpha\eta)}. \tag{17}$$

For $c_0 + \delta > 0$, (15) – (17) apply only until a Debt Apocalypse at time

$$\Omega = \frac{\log(1 + \delta/c_0)}{\alpha\delta}, \tag{18}$$

where b and h grow unbounded and no debt fails to default. For $\eta < 1$, default salvages some value for debt holders and a new cycle starts. For $c_0 + \delta < 0$, b and h approach zero while S approaches $(1 + c_0/\delta)^{1/\alpha}$.

Result 4 *For public debt to be sustainable long-term with primary surplus qB, q must exceed $i_0 - g$ and not just $r - g$.*

If c_0 is small, there is little difference between i_0 and r. However, the distinction does matter when the government contemplates a large jump in debt as an emergency measure, say, to fight a war or to counter a sharp contraction in GDP. In (9), each 1% boost in b raises h by α%. For our benchmark $\alpha = 2.5$, doubling b multiplies h by 5.66. In general, an M-fold boost in debt is sustainable only if $\delta < -cM^{\alpha}$.

Result 5 *A discrete boost in debt can make the stock unsustainable.*

For illustration, suppose a government with a current credit spread on debt of 50 bps and perceived salvage value of 20% pledges to maintain $\delta = -0.015$ long-term, but only after boosting debt to 35% of GDP from the current 20%. If $\alpha = 0$, credit spreads stay constant with a 17% chance of default within the first 30 years and 22% within the first 40 years. Absent default the debt dwindles by 1% per year and reaches 20% of GDP after 56 years. If $\alpha = 2.5$, the initial jump in debt quadruples credit risk to $c_0 = 200$ bps, which is no longer sustainable. Default is certain within $\Omega = 37$ years and there is a 70% chance of default within the first 30 years.

How much of this contrast is driven by the primary surplus being a constant share q of the debt stock B rather than a constant share qb_0 of GDP Y? To investigate this, we need to replace qF in (13) with q, which transforms (14) into

$$f' = c_0 f^{\alpha+1} + (r - g)f - q. \tag{19}$$

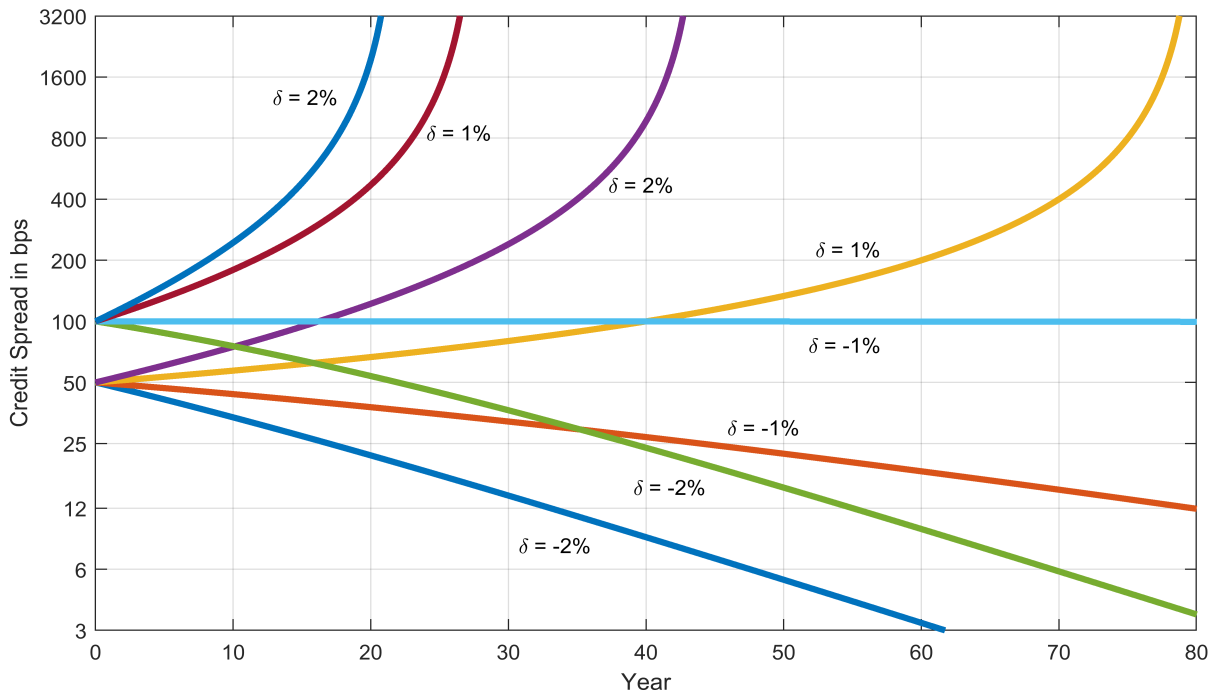
This revises a constant $\delta \equiv r - g - q$ to a variable $r - g - q/f$ that potentially shrinks as f grows or expands as f declines. For some parameter combinations, f will converge to a positive value different from the original, unlike (14) which drives toward either certain default or full cessation of default. While there is no general analytic solution, Appendix A.4 solves (19) for the case $\alpha = 1$.

Result 6 *For a primary surplus of qb_0Y, some negative $(q, i_0 - g)$ combinations will evolve toward a stable positive b. Outside that range, b will either shrink toward 0 or surge toward a Debt Apocalypse.*

6 Simulations of the general model

When $\delta = r - g - q = 0$, the general model in collapses to the basic model, and for tiny c_0 no big risks become apparent for 50 years or more. Accordingly, this section will focus on δ of at least 1% in absolute value. We will start by assuming c_0 of 50 to 100 bps, which for corporate credits would typically be viewed as high double-B, just below investment-grade status. (Osband, 2020). Figure 4 charts credit spreads for the benchmark $\alpha = 2.5$ and four different δ.

Figure 4: Credit spreads on log scale.
Parameters: $\alpha = 2.5$, $\delta \in \{\pm 1\%, \pm 2\%\}$, and $c_0 \in \{50, 100\}$ bps



The flat line in the middle of Figure 4 reflects $c_0 + \delta = 0$ or equivalently $i_0 = g + q$. In the other cases of $\delta < 0$, credit spreads steadily shrink but not quickly. When $\delta = -2\%$, halving a 100 bps spread to 50 bps takes 22 years, and halving it again takes 22 years. When $\delta = -1\%$, halving a 50 bps spread takes 44 years. Recall that major credit grades vary by a factor of roughly four in average default risk. Hence rating upgrades are likely to be slow if the improvement is gauged solely by b.

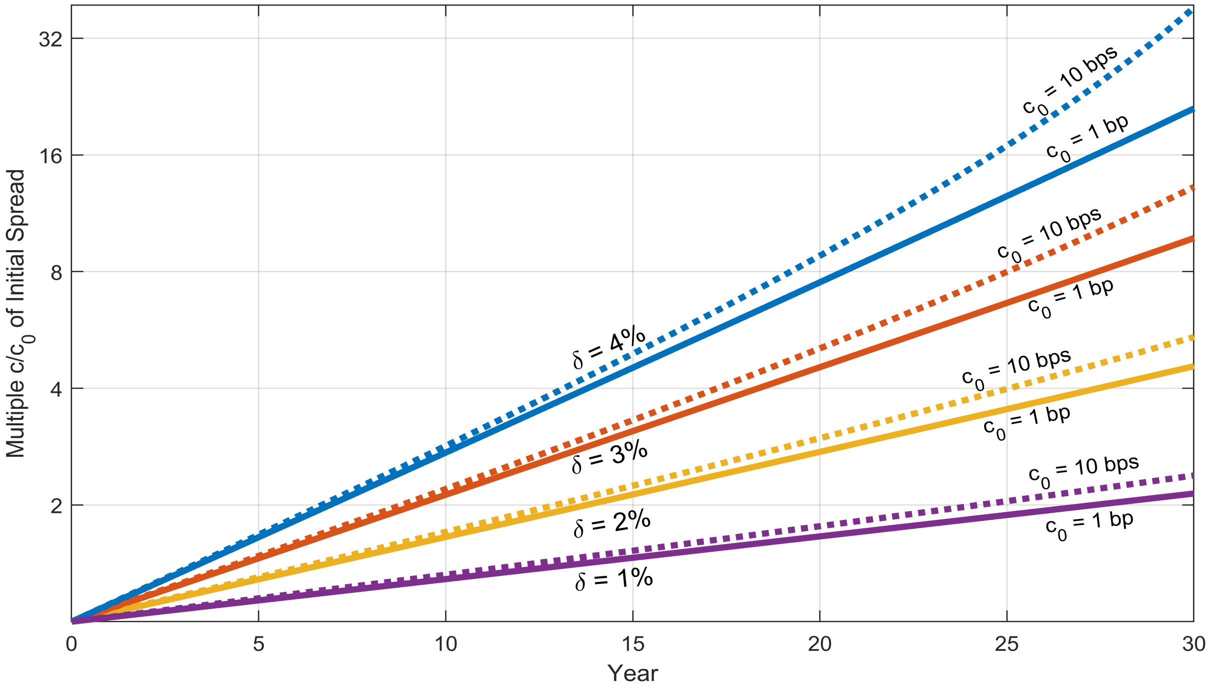
Similarly, rating downgrades are likely to be slow when $c_0 = 50$ bps and $\delta = 1\%$, as it takes 40 years for fair spread to double. Doubling time shrinks to 16 years when $\delta = 2\%$. When $c_0 = 100$ bps, downgrades come due much faster: spread quadruples in 19 years for $\delta = 1\%$ and 14 years when $\delta = 2\%$. At that point, default is likely within a few years absent bailouts or dramatic fiscal tightening.

Let us next consider much tighter initial spreads. For corporate credits, $c_0 = 1$ bp would typically be identified as triple-A grade and $c_0 = 10$ bps assingle-A grade. We will assume that high sovereign ratings estimate risks similarly and defer until later the question of how those estimates are formed. A Taylor series expansion of 16 lets us make the first-pass estimate

$$\log \frac{c(t)}{c_0} \approx (\delta + c_0)\alpha t. \tag{20}$$

For a one-letter drop in major credit grade with quadruple risk, the log difference is about 1.4, so for $\alpha = 2.5$, $q < 150$ bps won't warrant much ratings concern on a 30-year horizon. In contrast, $q > 300$ bps would justify a full letter downgrade within 20 years.

Figure 5: Growth of credit spreads in (16) on log scale.
Parameters: $\alpha = 2.5$, $\delta \in \{1\%, 2\%, 3\%, 4\%\}$, and $c_0 \in \{1, 10\}$ bps



In fact, (20) underestimates the true shifts implied by (16) at longer time periods, as $\log c$ bends upward over time and the convexity increases with α and $\delta + c_0$. Figure 5 displays the true $c(t)/c_0$ out to 30 years for c_0 of 1 or 10 bps and four values of δ. The logarithmic vertical scale reveals both the near log-linearity and the convexity. The solid lines for $c_0 = 1$ bp, are very close to those implied by (20). The dotted lines for $c_0 = 10$ bps bend noticeably higher, and the gap at 30 years for $\delta = 4\%$ is about one-third of a major credit grade wide.

When $\delta = 1\%$, $c(30)$ is only 2.1-2.4 times c_0 for $1 < c_0 < 10$ bps. Letting α vary between 2 and 3 widens the $c(30)/c_0$ range to 1.8-2.9, which still rounds to about half a major credit grade. Hence $q = 1\%$ does not appear to warrant much policy concern for $c_0 < 10$ bps.

When $\delta = 2\%$ for 30 years, the credit grade should drop by just over a letter. While the default risk would remain investment grade, a single-A credit would sink to triple-B, with default risks on a 30-year horizon of over 10%. Similar concerns would be stirred on a 20-year horizon for $\delta = 3\%$ and on a 15-year horizon for $\delta = 4\%$. Thirty years of $\delta = 4\%$ would warrant over a two-letter drop in credit grade, driving us toward the starting points of Figure 4. To be clear, none of these scenarios foretell a high default risk at 30 years, or even of likely default over the subsequent 15 years should b restabilize. However, they warn of too much vulnerability from the stock expansion to keep treating the debt as approximately risk-free.

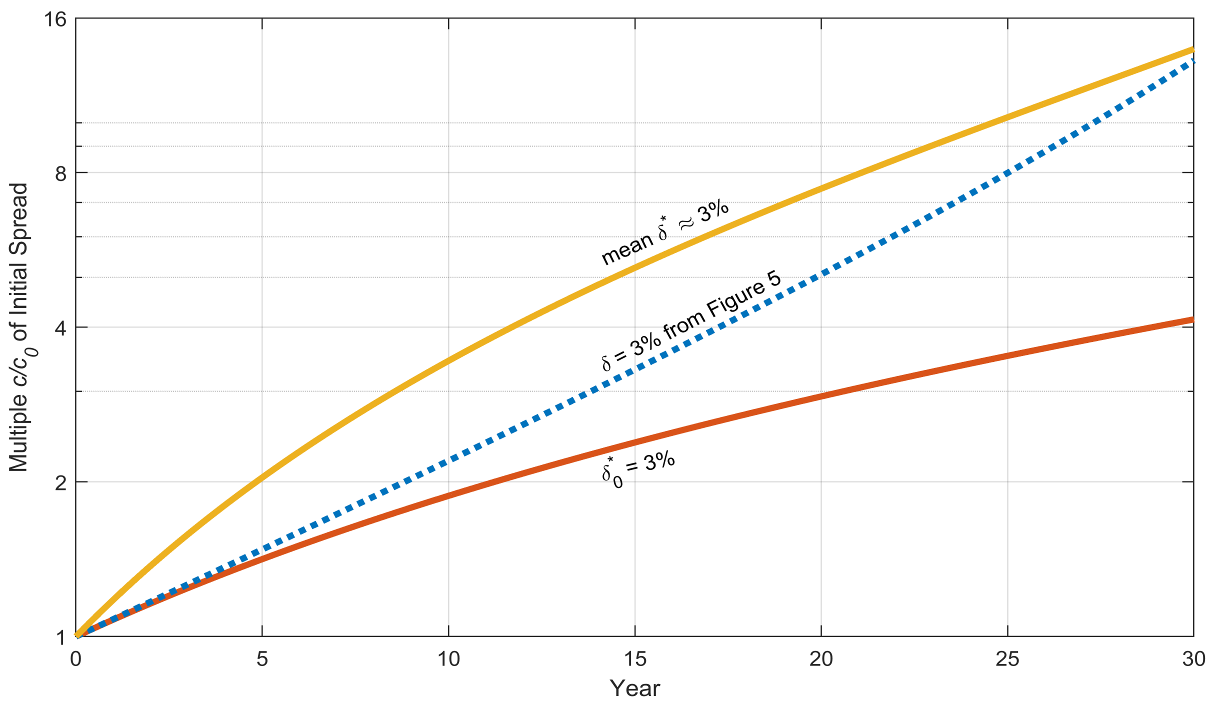
How does making the primary surplus a constant fraction of Y rather than B affect the results? That depends on how we do the conversion. Recall that (19) replaces a constant $\delta = r - g - q$ with a variable $\delta^* = r - g - q^*/f$. While the models behave identically when $q^* = q = 0$, such cases are rare. If we equate q_0^* to a non-zero q, the cumulative fiscal surpluses or deficits can vary enormously between the two models, in which case their spreads will sharply diverge. Alternatively, if we try to match mean q^* to q over T years, the T-year spreads tend to be close but beg questions about which T to choose and why.

For example, suppose the $\delta = 3\%$ curves in Figure 5 reflect $r - g = -2\%$ and $q = -5\%$. Over 30 years b nearly tripled. If we plug those same values into (19), δ^* turns negative by year 30, which keeps b from doubling in that period and significantly retards the growth of c. For more similarity to the $\delta = 3\%$ curves with the same $r - g$, we need $q \approx -8.8\%$, which causes δ^* to shrink from 6.8% to about 1%. These curves are charted in Figure 6.

The curves for other $\delta + c_0 > 0 > q$ combinations usually look qualitatively similar. The log spread for constant surplus to bond ratios tends to be slightly convex, while the log spreads for constant tend to be noticeably concave due to $|q|/f$ decreasing over time. However, when c gets high, this can push spreads toward a Debt Apocalypse and accentuate near-term differences in behavior.

For $\delta + c_0 < 0 < q$, setting $q_0^* = q$ speeds the decline in b, since $|q|/f$ increases over time. Conversely, setting mean $q^* \approx q$ slows the decline. Whether b rises or declines, smaller $|q|$ trims the differences between the two models. In short, the differences between the two models are less important than the aggregate growth of $f = b/b_0$ they imply.

Figure 6: Growth of credit spreads in (19) on log scale.
Parameters: $\alpha = 2.5$, $c_0 = 10$ bps, $r - g = -2\%$, and $\delta^* = q/f + 2\%$



7 Tale of two debt burdens

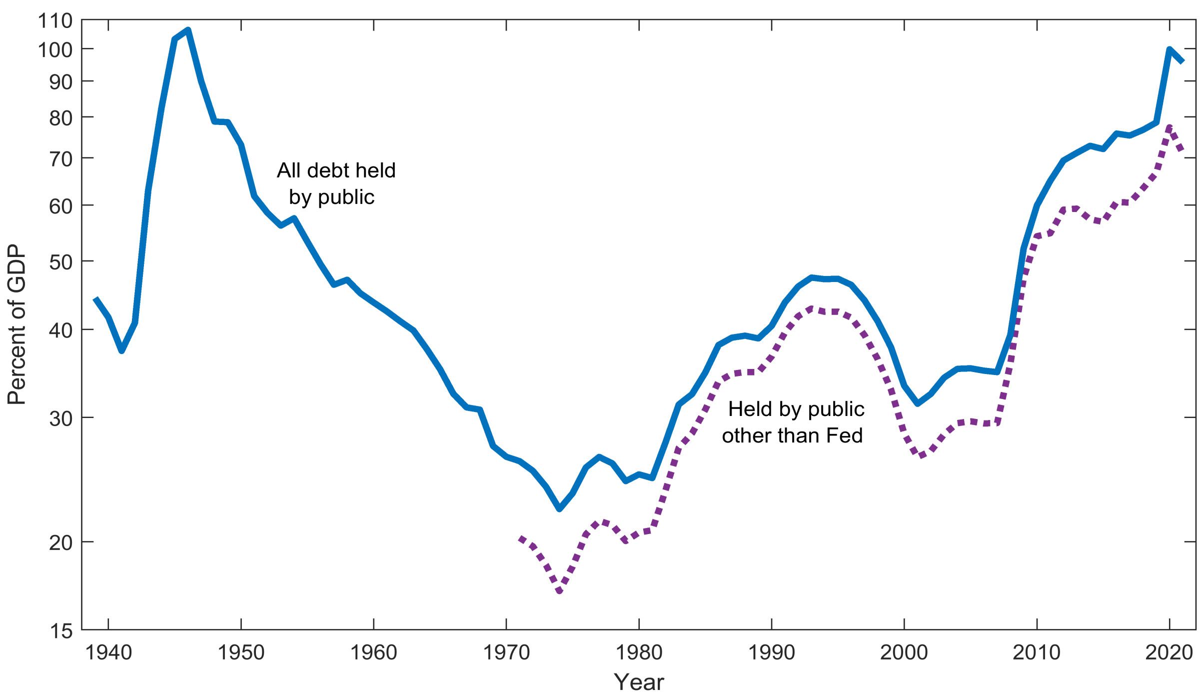
All else being equal, default risks must tend to rise with debt loads; otherwise, no one would worry about debt sustainability. Yet debt loads cannot be the only important influenc; otherwise, they would be far more correlated with credit ratings than they are. One obvious amendment excuses debt expansion during emergencies when coupled with determination to contract debt quickly after. Here rollover is not intended to permanently defer repayments, just to spread them out over time and smooth the associated taxes.

Distinguishing "good" deferrals from "bad" can be difficult. On the one hand, strong growth g can contract relative debt burdens even when payment is permanently deferred, and few would regard that as worrisome. On the other hand, long spells of debt expansion are often depicted as successive emergencies, while promised contractions are repeatedly deferred. Hence credit ratings require judgment calls that are hard to confirm or refute absolutely.

To illustrate the challenge, we tell a tale below of two debt burdens that concern the same country and reach similarly high peaks without significantly affecting its credit rating, yet have very different sources and outlooks. The country is the US, and the burdens can be labeled "World War II" and "now".

Figure 7 depicts US government debt held by the public, which deducts federal intragovernment debt from the gross amounts. Here the public includes the Fed, which arguably could be considered a government agency despite its nominal independence. The dotted line excludes Fed holdings where available. For more information on the data sources used, see Appendix A.5.

Figure 7: Publicly Held US Debt/GDP on log scale.



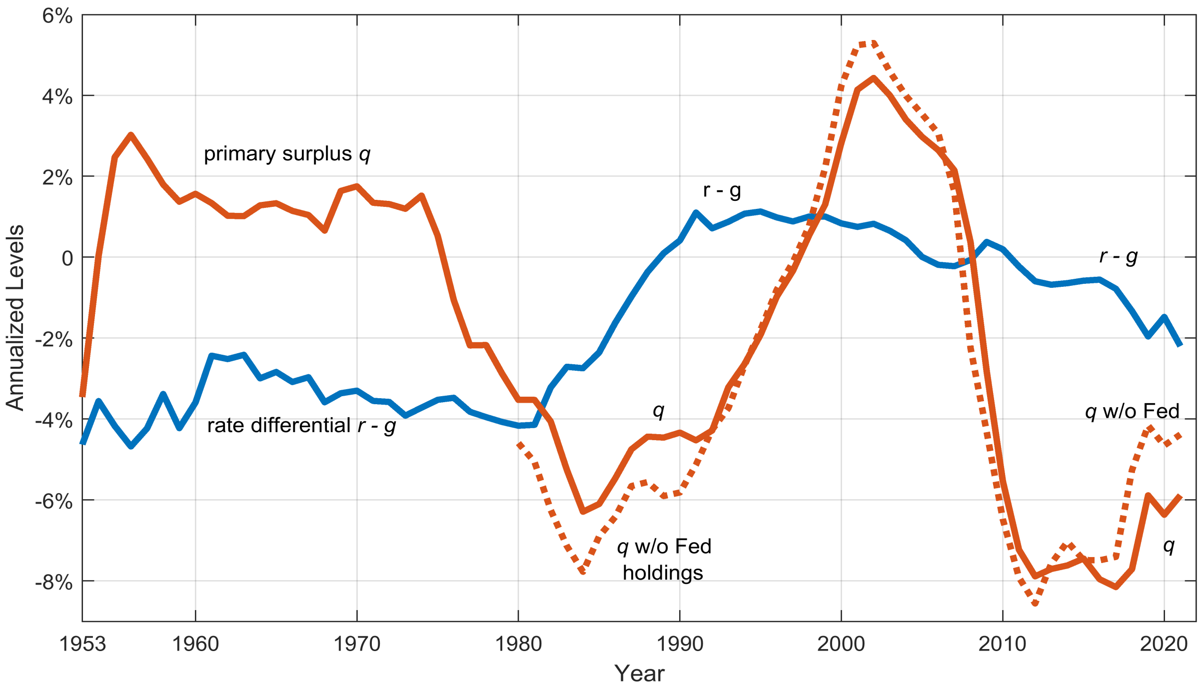
Debt soared from 1941 to 1946 due to military expenditures in World War II but was financed cheaply thanks to financial repression, patriotic fervor, and the expectation of renewed fiscal discipline. In just the next two years, the gross debt-to-GDP ratio fell by 27 percentage points, partly because the primary fiscal balance flipped into surplus but mostly because inflation soared after wartime price controls were removed. Over the next quarter century, δ averaged -4% annually, with $r-g$ accounting for most of it.

In contrast, δ has averaged 3.3% per year since 1980 and 5.0% since 2000. Both $r-g$ and q have contributed to the reversal. The interest rate differential surged on financial liberalization and overshooting inflation expectations but has gradually faded. In 2022 it plunged below -8% due to a big inflation shock. The primary balance oscillated between high deficit to debt shares, high surpluses, and even higher deficits.

Figure 8 displays 10-year moving averages of $r-g$ and q. The signed gap between them is the 10-year moving average δ. Clearly, q has been at least as important as $r-g$ in shaping δ and the path of b. This suggests that the recent emphasis on $r-g$ is misdirected. While r falling below g trims the burden of older debt, its impact is dwarfed by the plunge in q.

Here q is computed as the difference between $r-g$ and b'/b, although imputations from budget balance plus interest paid give basically the same results. Sometimes it is suggested that deficits financed by Fed purchases don't count, so Figure 8 also depicts q without them. Removal reduces average δ to 3.0% since 1980 and 4.4% since 2000, while q oscillates slightly more. Average δ from 2012 through 2021 is trimmed to -2.2% from -3.9% when Fed purchases are excluded.

Figure 8: 10-year Moving Averages of $r-g$ and q for US



In short, the World War II-related debt burden was dominated by a sharp surge and sustained retreat. While the retreat was driven largely by financial repression, it thoroughly justified a top credit rating for the US. Since 1980, the debt-to-GDP ratio has trended upward at an average of 3% per year or higher, albeit with significant multi-year oscillations. The closest analogy in our simulations is the 4%-for-30-years trend in Figure 5, which warranted a two-letter downgrade in credit rating.

The strongest argument against US downgrade is the dollar's status as the world's main money. This spurs foreign demand for Treasuries as reserves and safe-harbor assets. If we expand the denominator in the debt-to-GDP ratio to include a growing share of faster-growing foreign GDP, b's measured rate of growth slows.

Another argument against downgrade is debt markets' apparent lack of concern. Real r has dropped while credit spreads on Treasuries–insofar as we can disentangle them from liquidity premia–seem minuscule. The main caveat is that debt markets are rationally myopic. They naturally find reassurance from past servicing and project that history into the near future. They naturally discount forecasts of future changes that cannot be immediately tested. By overweighting recent experience, rational myopia often gets blindsided by radical change; its saving grace is the rapid correction of demonstrated errors.

How about the prospects for contracting b going forward? On the bright side, q will likely moderate, as some of the deterioration came in response to two emergencies: the financial collapse in 2008/2009 and the Covid-related lockdowns in 2020/2021. Also, the US is extremely wealthy by any historical standard and could tighten its belt without huge privation. However, its prospects for rapidly shrinking b are poor. No armistice slashes fiscal spending and frees up massive resources. There is no political appetite for trimming major entitlements. Most fiscal battles pit lower taxes against higher benefits, as low interest rates and concessions to swing voters have eased the pressure for fiscal discipline. Inflationary shocks can trim b since r will lag the boost to nominal g. However, the only way to keep r much less than g is financial repression, which will rouse a lot of investor opposition and also threatens the dollar's global status.

Few observers expect b to decline without a major fiscal overhaul. The non-partisan Congressional Budget Office (2022, `https://www.cbo.gov/publication/58340`) projects b to average 2.0% annually for the next 30 years. The projection seems to be based entirely on estimated δ, with no provision for increases in the credit spread c. If confidence erodes, as our models predict, it would be hard to avoid a fiscal crisis.

Hence it strikes us as prudent to downgrade sovereign US credit to single-A. Single-A is solidly investment grade; it does not point to significant near-term default risk over a 10-year horizon. However, it does warn of potential deterioration. In contrast, double-A or triple-A ratings are typically considered safe for a generation or longer.

8 Closing remarks

In standard macroeconomic models, the credit spread and default risk of sovereign debt are presumed independent of its absolute stock B or relative stock b. While this can be a reasonable approximation in some contexts, it is badly mistaken in others. In particular, it excuses any debt expansion portrayed as a one-time emergency measure while denying that any debt reduction plan is urgent. Our model of default risk as a power function of b offers a simple, tractable remedy that can handle both long periods of relative calm and shorter periods of rapid deterioration.

Our model also acknowledges that most sovereign debt is rolled over into new debt without exchange for real goods and services. In standard models, rollover is a vote of confidence in long-term sustainability; anyone expecting a Ponzi-like debt bubble should exit. Our model imposes no such restriction. It allows short-term credit spreads to stay low for decades even when agents are perfectly rational, realize the debt is a bubble, and fully count on its eventual collapse.

As noted in the introduction, our results strengthen Reinhart and Rogoff's warnings about the dangers of excessive debt accumulation. Reinhart and Rogoff contended that lenders chronically get lulled into excessive optimism, which leaves open the possibility that a better-educated lending community could forecast much better. Our model shows that unsustainable debt can fester even when lenders are well-informed and fully rational.

However, the model does not fully address Reinhart and Rogoff's concerns, as it rules out any uncertainty about the governing parameters. Once we allow for uncertainty, a long spell of reliable debt servicing can induce excess optimism in rational learners: not excess given the information at hand, but more than than the underlying fundamentals truly justify. We intend to explore the implications in a follow-up paper. For additional context, see Part 3 of Osband (2020), and especially Chapter 16, which reinterprets most of the fickleness noted by Reinhart and Rogoff (2009) as rational responses to evolving risk.

As for our assessment of US creditworthiness, we acknowledge its tentative nature. It hinges on

- a ballpark estimate of α (default sensitivity to debt burden) in the range of 2 to 3,
- a scaling of debt capacity that is roughly proportional to US GDP, and
- an erosion of fiscal discipline.

We concede that the evidence is not clear-cut and welcome counter-arguments. The only notion we wholeheartedly reject is the claim that low credit spreads guarantee sustainability. A debt burden that continually mounts as a share of GDP is like tinder piling up in a forest. Bond markets sometimes act like deer grazing in the clearings and sometimes like forest rangers scanning for fires. Rarely do they react to tinder before it alights. Still, the tinder matters, as it can turn a controllable fire into an all-consuming conflagration.

References

Abbas, S. A., A. Pienkowski, and K. Rogoff (2019). *Sovereign debt: A guide for economists and practitioners.* Oxford University Press.

Badia, M. M., P. Medas, P. Gupta, and Y. Xiang (2022). Debt is not free. *Journal of International Money and Finance 127*, 1–23.

Barro, R. J. (1974). Are Government Bonds Net Wealth? *Journal of Political Economy 82*(6), 1095–1117.

Bhandari, A., D. Evans, M. Golosov, and T. J. Sargent (2017, November). Public Debt in Economies With Heterogeneous Agents. *Journal of Monetary Economics 91*, 39–51.

Blanchard, O. (2019, April). Public Debt and Low Interest Rates. *American Economic Review 109*(4), 1197–1229.

Bolton, P. (2016). Presidential Address. Debt and Money: Financial Constraints and Sovereign Finance. *The Journal of Finance 71*(4), 1483–1510.

Brumm, J., X. Feng, L. J. Kotlikoff, and F. Kubler (2021, October). When Interest Rates Go Low, Should Public Debt Go High? NBER Working Papers 28951, National Bureau of Economic Research, Cambridge, MA.

Chatterjee, S. and B. Eyigungor (2012). Maturity, Indebtedness, and Default Risk. *American Economic Review 102*(6), 2674–99.

Cochrane, J. H. C. (2021, July). $r < g$. Mimeo, available at: `https://tinyurl.com/489vdpbk`.

Debrun, X., J. D. Ostry, T. Willems, and C. Wyplosz (2019). Public Debt Sustainability. In S. A. Abbas, A. Pienkowski, and K. Rogoff (Eds.), *Sovereign Debt: A Guide for Economists and Practitioners*, Chapter 4, pp. 151–191. Oxford, UK: Oxford University Press.

Diamond, P. A. (1965, December). National Debt in a Neoclassical Growth Model. *The American Economic Review 55*(5), 1126–1150.

Ghosh, A. R., J. I. Kim, E. G. Mendoza, J. D. Ostry, and M. S. Qureshi (2013). Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies. *The Economic Journal 123*(566), F4–F30.

Hellwig, K.-P. (2021, May). Predicting Fiscal Crises: A Machine Learning Approach. Working paper 2021/150, International Monetary Fund.

IMF (2022, April). Fiscal Monitor. Fiscal Policy from Pandemic to War. World economic and financial survey 0258-7440, International Monetary Fund, Washington, DC.

Kose, M. A., F. Ohnsorge, and N. Sugawara (2020, September). Benefits and Costs of Debt: The Dose Makes the Poison. Policy Research Working Paper 9166, World Bank, Washington, DC.

Mitchell, W., L. R. Wray, and M. Watts (2019). *Macroeconomics.* London, UK: MacMillan International.

Osband, K. (2020). *Rational Myopia: How Capital Markets Learn.* Mountain Brook, AL: RiskTick.

Ostry, J. D., A. R. Ghosh, J. I. Kim, and M. S. Qureshi (2010, September). Fiscal Space. IMF Staff Position Notes 2010/011, International Monetary Fund.

Reinhart, C. M. and K. S. Rogoff (2009). *This Time is Different.* Princeton, NJ: Princeton University Press.

Reis, R. (2022, Fall). The Fiscal Revenue from Public Borrowing. *Journal of Economic Perspectives 36*(4), 103–124.

Romer, D. (2012). *Advanced Macroeconomics* (5th ed.). New York, NY: McGraw-Hill Education.

Yared, P. (2019, May). Rising Government Debt: Causes and Solutions for a Decades-Old Trend. *Journal of Economic Perspectives 33*(2), 115–40.

A Mathematical appendix

A.1 Mean time θ until default

Integrating by parts and applying (8),

$$\theta = -\int_0^\Omega t\,dS(t) = -tS(t)\Big|_0^\Omega + \int_0^\Omega S(t)dt = 0 - \frac{\alpha\Omega}{\alpha+1}\left(1-\frac{t}{\Omega}\right)^{1+1/\alpha}\Big|_0^\Omega = \frac{\alpha\Omega}{\alpha+1}.$$

A.2 Expected bond stock at time of default

Applying (2), (6), and (7),

$$-\int_0^\Omega F(t)dS(t) = \int_0^\Omega F(t)h(t)S(t)dt = \int_0^\Omega h(t)dt = -\frac{1}{\alpha}\log(\Omega - t)\Big|_0^\Omega = \infty.$$

A.3 Solutions for primary surplus qB

Defining $x \equiv fe^{-\delta t}$ and substituting (14),

$$x' = f'e^{-\delta t} - \delta x = c_0 x^{\alpha+1} e^{\alpha\delta t}. \tag{21}$$

Multiply through by $-\alpha\delta x^{-\alpha-1}$ and integrate both sides with respect to t to obtain

$$\delta x^{-\alpha} = \delta f^{-\alpha} e^{\alpha\delta t} = C - c_0 e^{\alpha\delta t} \tag{22}$$

for some constant C. The initial conditions require $C = \delta + c_0$. Rearrangement yields (15). Substitution into (9) and (12) yields (16).

The log survival rate $\log S$ can be calculated as

$$-\int_0^t h(u)du = -\int_0^t \frac{(c_0/\eta)\delta e^{\alpha\delta u}du}{\delta + c_0 - c_0 e^{\alpha\delta u}} = \frac{\log(\delta + c_0 - c_0 e^{\alpha\delta t}) - \log\delta}{\eta\alpha}$$

which implies (17).

A.4 Solutions for primary surplus qb_0Y

There is no general analytic solution for (19). If $\delta_0 \equiv i_0 - g - q$ is zero, b will stay constant, like in the previous model. For the case $\alpha = 1$ and $\delta \neq 0$,

$$t = \int_0^t du = \int_1^{f(t)} \frac{df(u)}{c_0 f^2(u) + (r-g)f(u) - q}.$$

Define $K \equiv 4c_0 q + (r-g)^2$, $\kappa \equiv \sqrt{|K|}$, and $v \equiv (2c_0 f + r - g)/\kappa$ to transform this into

$$\tfrac{1}{2}\kappa t = \int_{v_0}^{v(t)} \frac{dv(u)}{v^2(u) - \operatorname{sgn}(K)} \equiv V(v(t)) - V_0,$$

where V denotes an indefinite integral and $V_0 \equiv V(v_0)$. Then

$$f(t) = \frac{1}{2c_0}\left(g - r + \kappa V^{-1}(V_0 + \tfrac{1}{2}\kappa t)\right). \tag{23}$$

It follows that δ_0, f', v', $v_0^2 - \mathrm{sgn}(K)$, and dV/dv all have the same sign.

For $K < 0$, which requires $q < -(r-g)^2/(4c_0) \equiv q^*$, V is the inverse tangent function $\tan^{-1}$ plus a constant, so f grows toward a Debt Apocalypse at time

$$\Omega = \frac{\pi - 2\tan^{-1} v_0}{\kappa}.$$

For $K > 0$, which requires $q > q^*$, $V(v) = \frac{1}{2}\log(|1-v|/|1+v|)$ plus a constant. There are three cases to consider:

- $v_0 > 1$: Here V is negative and increases with v and f as long as feasible. Debt Apocalypse looms at time $\Omega = -2V_0/\kappa$ since $f(\Omega)$ includes a multiple of $V^{-1}(0) = \infty$.
- $v_0 < -1$: This requires $-c_0 > i_0 - g > q$. Here a convex V increases without bound, with no Debt Apocalypse, while v is bounded above by -1. Hence f rises at an ever-decreasing rate, converging to $(g - r - \kappa)/(2c_0) \equiv f^*$.
- $|v_0| < 1$: Here $q > i_0 - g$, and f shrinks in finite time to f^* for $q < 0$ and 0 otherwise.

A.5 Data sources on US debt

All data comes from the FRED database maintained by the St. Louis Fed at `fred.stlouisfed.org`. The annual percent change of GDP was taken from A191RP1A027NBEA. The remaining data was reported as a percent of GDP. Yearly data was taken from FYPUGDA188S Gross Federal Debt Held by the Public, FYOIGDA188S Federal Outlays: Interest as Percent of GDP, and FYFSGDA188S Federal Surplus or Deficit as Percent of GDP. Quarterly data was taken from FYGFGDQ188S Federal Debt Held by the Public and HBFRGDQ188S Federal Debt Held by Federal Reserve Banks.

A.6 Symbols used in the text

$_0$	Subscript indicating value at initial time 0
B	Stock of public debt
b	Ratio of B to Y
c	Credit spread
F	Ratio of B to B_0
f	Ratio of b to b_0
g	Growth rate of Y
h	Default hazard rate
I	Annual interest rate
i	Interest rate
q	Primary surplus as share of B or Y
r	Risk-free rate
S	Survival probability
t	Time
Y	GDP
z	Beliefs about h
α	Elasticity of h with respect to B
δ	Risk-free adjustment rate $= r - g - q$
η	Fractional loss in debt value after default
θ	Mean time until default
Ω	Time of Debt Apocalypse