CENTRO STUDI IN ECONOMIA E FINANZA

CENTRE FOR STUDIES IN ECONOMICS AND FINANCE

WORKING PAPER NO. 71

Are two investors better than one?

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October 2001

This version March 2002

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Anna Maria C. Menichini* **and Peter Simmons****

Abstract

The paper compares the optimal financial contracts of a firm which has private information over its expost revenues when the finance can be provided by a single or by two groups investors. Costly monitoring can be carried out only by one group of investors. When they are the only investors we use a financial contract with non-contractible monitoring, in which the probabilities of cheating by the entrepreneur/firm and monitoring by investors are mutual best responses. The contract is written by the entrepreneur knowing that this equilibrium will subsequently occur. With a second group of investors who have no monitoring rights, cheating and monitoring probabilities are chosen in a similar way. The non monitoring investors learn the results of any monitoring for free. A main result is that without commitment there is a negative correlation between repayments to the two investor groups: the contract uses the non-monitoring group to smooth out the repayments of the entrepreneur optimally. This reduces his incentive to make false reports and mitigates the investor's incentive to monitor. A second result is that the two investor scenario is Pareto superior to the single investor model. A third result is that the possible extent of this smoothing depends on whether the investors have limited liability; it is found that in some circumstances investors should make repayments to the firm rather than receive them. A further result is that, by restricting to offers coming from the informed party, the three party contract is collusion-proof and renegotiation-proof. Last we show that under limited liability the share of finance provided by the two investors is strictly positive.

Keywords: financial contracts, multiple investors, no commitment

JEL-Class.: D82, D83

♦ The paper has benefited from presentations at Freie Universitat Berlin and Université de Toulouse, and from comments from Helmut Bester, Bruno Biais, Catherine Casamatta, Bruno Jullien, Marco Pagano and Elu Von Thadden. We bear sole responsibility for any error.

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Introduction

The costly state verification literature (Townsend [23], Gale and Hellwig [8]) models the form of loan contract when there are ex-post asymmetry of information and auditing possibilities. In circumstances where the contract can commit the lender to verify particular reported states, the revelation principle is applicable and the optimal contract will involve truth telling. But typically such verification is not optimal for the lender ex-post (Bolton and Scharfstein [3]) so that such contracts will never be carried out unless there is some additional mechanism for ensuring that the contract is implemented such as a third party (Dewatripont [6]) or a multi-period structure (Gale and Hellwig [9]). This is true for both deterministic verification schemes and randomised verification schemes as in Mookherjee and Png [20], Lacker and Weinberg [16]. In response to this several authors have considered the nature of optimal contracts without commitment, either imposing a sequential rationality constraint to ensure that the lender will actually perform the stipulated monitoring (Jost [10]), or removing monitoring from the contract problem leaving it to be determined non-cooperatively at the second stage. Here the contract problem is solved by backward induction (Datta [5], Khalil [11], Khalil and Parigi [13], Choe [4]) and generally randomised behaviour is optimal at the ex-post stage. However none of these papers take account of the problem that ex-post once the firm realises its revenue, either the firm or either group of investors can propose a change in the terms of the contract (changing the repayments) to avoid the monitoring costs. With just a single investor group Krasa and Villamil [15] show that under some circumstances a particular standard debt contract is renegotiation proof.

The literature is in the context of two parties: the firm which is privately informed ex-post about its revenues and a single investor. After the contract has been offered by the firm and accepted by the investor, the firm's revenues are realised and observed only by the firm. Then the firm and the investor simultaneously decide their cheating and monitoring strategies which are generally mixed in this environment.

Of course typically firms have multiple investors. Formal contract theory with three parties either looks at a single principal contracting with two agents and largely considers questions of organisational design (for example the merits of hierarchical organisations are discussed by Melamud, Mookherjee and Reichelstein [19]; Baron and Besanko [2]) or, with two principals and a single agent, looks at issues of collusion and the merits of maintaining a separation between the two principals to avoid capture (Martimort [18]; Laffont and Martimort [17]). These papers do not consider the specific circumstance of a firm with privately observed random revenues, which is acting as agent to various groups of shareholders. Generally they also do not allow for the auditing possibilities that are open to different groups of shareholders.

In this paper we consider a risk neutral firm requiring fixed funding by two risk neutral investors to finance an investment project, whose outcome is the firm's private information. Of the two investors, only one has the possibility of costly monitoring the revenues of the firm. The results of any audit by this investor are made public, so the second non-monitoring investor learns the state of the firm for free. We also suppose that the participation constraints are ex-ante. We then compare the outcomes for the three-party contract with those of a bilateral investor firm contract, which have already been derived by Khalil and Parigi [13] (K&P henceforth).

Our main results are:

i. having two investors is socially preferable to having a single investor;

ii. maximum punishment for audited false reports is generally desirable in both scenarios (this is consistent with Baron and Besanko [1]);

iii. with relatively low observation cost, the contract without commitment involves both randomised verification by investors and randomised cheating by the firm; payments by the firm are set to give the firm zero profit in the bad state of the world but rent in the good state; *payments by the firm to the two investors are negatively correlated* in the sense that in the high state the monitoring investor receives more and the non monitoring investor less than in the low state. The intuition for these results is that the second investor can be used by the firm to reduce the variance of the firms resources across states. This reduced variance mitigates the incentive for the firm to cheat in the high state as well as the incentive to monitor by the auditing group of investors. On both these accounts, the expected level of monitoring costs is reduced by the presence of the second investor. Given that the total investment size is fixed, the deviation from the first best outcome is just the saving in expected monitoring cost induced by the reduction in the probability of monitoring, and so the two investor case dominates the single investor case.

iv. one problem with the K-P solution concept is that it does not allow for an agreement to a change in the terms of the contract once the firm has learned its true revenue state. If the firm knows it has high revenue it could propose a deal to the monitor to share the gain from not monitoring. If we assume that only the firm can make renegotiation offers, the two-investor contract is both collusion-proof and renegotiation-proof.

The plan of the paper is to outline the model in section 1; to analyse its component parts in subsections; in section 2 to derive the various forms of second best behaviour with a single investor and with two investors; in section 3 to derive the shares of investment provided by each investor; in section 4 to offer a welfare comparison of the two organisational forms, in section 5 to introduce collusion and in the last section to conclude.

1 The Model Assumptions

The risk neutral firm requires a fixed investment size $\overline{D}$ which gives it access to a technology in which revenues from debt $\overline{D}$ and state s are given by $y = f(\overline{D}, s) = f_s$ with $s \in \{H, L\}$ and $f_H > f_L$. State s occurs with probability p.

Investors are risk neutral and have access either to the debt contract of the firm or to a safe interest rate of $r-1$. The debt of the firm $\overline{D}$ pays a return of R_s to the monitoring investor and P_s to the non monitoring investor, depending on the state and on whether it is audited. We do not impose any sign restrictions on R_s, P_s. In principle investors could receive a negative payment from the firm in one of the states so long as overall each group of investors earns a zero expected profit from the contract. The debt thus has the form of equity with unlimited liability. We do require the firm to be solvent in each state: $f_s - R_s - P_s \geq 0$. The total investment is shared between the two investor groups with the monitoring investor providing a share α of it and the non monitoring investor providing $(1-\alpha)$. When there is asymmetric information, the monitoring investor can choose to audit any state report of the firm; and, if she does so, she must pay an observation cost of ϕ. We allow the monitoring investor to randomly monitor with an endogenous probability m. Investment has to be applied before the random revenue of the firm is known and payments to investors are made once the uncertainty has been realised. If it is found that the firm has falsely declared a low state of revenue instead of a high true state then, as well as paying the returns due in the high state, it is punished by paying an amount that is specified in the contract to the monitoring investors.

We assume that the fixed financing requirement is such that $f_L < rD$; otherwise the firm could just pay a constant repayment to the investors in each state s and meet their participation constraints, have no reporting incentive problem and face no monitoring. Under this assumption, the second best contract will require either R_s or P_s to vary by state, which in turn induces incentive constraints on the firm and on the monitoring investor.

2 The Second Best Contract

2.1 The No Commitment Contract With One Investor

This case has been analysed by K&P [13]; there is a single investor who has monitoring rights. If the low state occurs the firm declares it and pays R_L to the investor. If the high state occurs the firm can either declare it and pay R_H to the investor or can falsely declare the low state, in which case it is monitored with probability m. If monitored and caught cheating the firm pays $R_{HL} = R_H + \delta$ to the investors, i.e. what she had a right to if a truthful report had been made, plus the penalty for misreporting; if not monitored, then again the firm pays R_L.[1]

Distinguishing between two stages of the game, the time line is the following:

- Ex-ante stage. At time zero a financial contract is offered by the entrepreneur specifying the repayments due in each monitored or non moni-

[1] K&P allow for the low state repayments to be conditioned on the act of monitoring. However they show that optimally there is no gain to be had from using this information to affect repayments.

tored state. At time one production takes place and at time two the state of nature is realised and observed by the entrepreneur.

- Ex-post stage. At time three the entrepreneur makes a report and the monitor decides whether to verify the report. These strategies are chosen as mutual best responses. At time 4, conditional on the reported state and on the result of monitoring, if any, the relevant transfers are made.

At the ex-post stage the decision variable for the firm is the probability l with which it falsely declares the low state when the high state has occurred. For the monitoring investor, the decision variable is the probability m with which she will audit the firm following a low state report. As standard in models of this type (e.g. in K&P [13]), we assume that the firm never cheats when the low state occurs; and the investor never monitors when a high state report is received:

$$E\pi_E|_H = (1-l)(f_H - R_H) + l[m(f_H - R_H - \delta) + (1-m)(f_H - R_L)], \quad (1)$$

$$E\pi_R|_{\hat{L}} = m[\frac{pl}{1-p+pl}(R_H + \delta) + \frac{1-p}{1-p+pl}R_L - \phi] + (1-m)R_L. \quad (2)$$

l, m are selected as mutual best responses. K&P restrict attention to the case in which the Nash equilibrium is in mixed strategies and show that this is defined by:

$$l = \frac{(1-p)\phi}{p(R_H + \delta - R_L - \phi)};$$

$$m = \frac{R_H - R_L}{R_H + \delta - R_L}.$$

Then $0 < m < 1$ requires $R_H + \delta > R_H$ and $R_H > R_L$; $0 < l < 1$ requires $R_H + \delta - R_L - \phi/p > 0$. If the investor-firm contract does not satisfy these conditions, then no interior mixed strategy Nash equilibrium is possible.

Given these Nash equilibrium values for l and m, K&P solve the following contract problem: choose R_s, D to

$$\max\ p(f_H - R_H) + (1-p)(f_L - R_L) \quad (3)$$

$$\text{st } rD \leq p(1-l)R_H + (1-p+pl)R_L \quad (4)$$

$$f_H - R_H - \delta \geq 0 \quad (5)$$

$$f_L - R_L \geq 0 \quad (6)$$

$$l = \frac{(1-p)\phi}{p(R_H + \delta - R_L - \phi)} \quad (7)$$

$$m = \frac{R_H - R_L}{R_H + \delta - R_L} \quad (8)$$

where (4) is the individual rationality constraint for investors, (5) and (6) are the feasibility conditions, (7) and (8) are respectively the probability of lying and the probability of monitoring determined at the ex-post stage.

Under the assumption that $pf_H + (1-p)f_H - rD - \phi > 0$, it is found that the optimal contract has the following properties:

- maximum punishment: $R_{HL} = R_H + \delta = f_H$;
- binding low state feasibility: $R_L = f_L$;
- rent to the firm in the high state:

$$R_H = \frac{rD(f_H - f_L - \phi) - f_L(f_H - f_L)(1-p)}{p(f_H - f_L) - \phi} < f_H;$$

- positive probability of lying by the firm:

$$l = \frac{(1-p)\phi}{p(f_H - f_L - \phi)};$$

  and positive probability of monitoring by the investor:

$$m = \frac{(rD - f_L)(f_H - f_L - \phi)}{[p(f_H - f_L) - \phi](f_H - f_L)};$$

- the expected payoff to the firm is:

$$Ef - rD - \frac{(1-p)\phi(rD - f_L)}{p(f_H - f_L) - \phi}$$

As expected, the firm's payoff is below the first best level due to the asymmetric information. The loss depends on $rD - f_L$, on the level of observation costs ϕ, and on the spread of revenues of the firm between the two states, $f_H - f_L$. We can measure it by the optimal expected observation cost:

$$m(1 - p + pl)\phi = \frac{\phi(1-p)(rD - f_L)}{p(f_H - f_L) - \phi}$$

We take this contract as a benchmark to measure the impact of the introduction of the third party into the venture.

2.2 The No Commitment Contract With Two Investors

We assume here that the amount of finance required by the entrepreneur can be provided by two investors in shares to be determined. Distinguishing again between two stages of the game, the time line follows the single investor contract and is only modified by the entrepreneur offering a contract to both investors and corresponding repayments contingent on the reported monitored or non-monitored states.

If the low state occurs the firm declares it and pays R_L, P_L respectively to the investors. If the high state occurs the firm can either declare it and pay R_H, P_H to the investors or can falsely declare the low state. A low state report can be monitored with probability m. If monitored and caught cheating the firm pays $R_{HL} = R_H + \delta, P_H$ respectively to the monitoring and non monitoring investors; if not monitored, then again the firm pays R_L, P_L.

At the ex-post stage the decision variable for the firm is the probability l with which it falsely declares the low state when the high state has occurred. For the monitoring investor the decision variable is the probability m with which the investor will audit the firm following a low state report. Introducing the third party will now change the firm's expected profits conditional on a high state having occurred (1) in the following way:

$$\begin{aligned} E\pi_{E|H} &= (1-l)(f_H - R_H - P_H) + l[m(f_H - R_{HL} - P_H) \\ &\quad +(1-m)(f_H - R_L - P_L)] \end{aligned} \tag{9}$$

In line with K&P, we restrict attention to the case in which the Nash equilibrium is in mixed strategies and show that these are defined by:

$$\begin{aligned} l &= \frac{(1-p)\phi}{p(R_{HL} - R_L - \phi)}; \\ m &= \frac{R_H + P_H - R_L - P_L}{R_H + \delta + P_H - R_L - P_L}. \end{aligned}$$

Then $0 < m < 1$ requires $R_H + \delta > R_H$ and $R_H + P_H > R_L + P_L$; $0 < l < 1$ requires $R_H + \delta - R_L - \phi/p > 0$. If the investor-firm contract does not satisfy these conditions, then no interior mixed strategy Nash equilibrium is possible.

Thus, given a subsequent mixed strategy equilibrium, we can write the contract problem as one of choosing R_s, P_s, α to:

$$\max\ p(f_H - P_H - R_H) + (1-p)(f_L - P_L - R_L) \tag{10}$$

$$\text{st } \alpha r D \leq p(1-l)R_H + (1-p+pl)R_L \tag{11}$$

$$(1-\alpha)rD \leq (p - pl + plm)P_H + (1-p+pl-plm)P_L \tag{12}$$

$$f_H - P_H - R_H - \delta \geq 0 \tag{13}$$

$$f_L - R_L - P_L \geq 0 \tag{14}$$

$$l = \frac{(1-p)\phi}{p(R_H + \delta - R_L - \phi)} \tag{15}$$

$$m = \frac{R_H + P_H - R_L - P_L}{R_H + \delta + P_H - R_L - P_L} \tag{16}$$

where the constraints have the usual meaning: (11) and (12) are the individual rationality constraints for the monitor and for the non monitoring investor respectively, (13) and (14) are the feasibility conditions, (15) and (16) are respectively the probability of lying and the probability of monitoring determined at the ex-post stage.

The case in which $\alpha = 1$ is of interest: here the second principal plays the role of a pure insurance company to the firm; it allows the firm to alter the distribution of its net revenues across states. We could then think of the problem as one of optimal information revelation by the firm when it faces auditing investors and an insurance company. It can defraud both by cheating but may be punished for this. Similarly we can interpret the case of $\alpha = 0$ as one where parties specialise: one acts as supervisor and one as investor.

We can derive various properties of the optimal contract (the technical details are in the appendix).

Proposition 1 *With a subsequent mixed strategy, the optimal contract under no commitment has:*

i. *investors getting their reservation utility level;*

ii. *maximum punishment:* $f_H - R_H - \delta - P_H = 0$;

iii. *zero rent to the firm in the low state however it reports* ($f_L = R_L - P_L$)*, but positive rent in the high state with truthful reports* ($f_H - R_H - P_H > 0$)*.*

Result (i) is not surprising: since the firm is writing the contract it has no gain from leaving any rent to either investor: if it did, it could reduce either P or R in a way that leaves the other constraints preserved.

Maximum punishment reduces the incentive of the firm to cheat and so the frequency of low state reports and, for given m, the amount of monitoring that will be undertaken, thus saving on monitoring costs. Put slightly differently, given that there is a mixed strategy equilibrium, δ only enters the contract problem via the participation constraints of the investors through l. Raising the punishment reduces l and reduces the impact of the participation constraint.

Binding low state feasibility is a common result in agency problems that assists in ensuring incentive compatibility.

Given maximum punishment and binding low state feasibility, we can set $R_H + P_H = f_H - \delta$, and $R_L = f_L - P_L$. Using the monitoring investor participation constraint to eliminate R_H, the problem reduces to choose P_H, P_L and α to

$$\max \ p\delta \tag{17}$$

$$\text{st } \mu P_H + (1-\mu)P_L = (1-\alpha)rD \tag{18}$$

where $\mu = p - pl + plm$ and

$$\delta = f_H - P_H - f_L + P_L + \frac{(f_L - P_L - \alpha rD)}{p(1-l)}. \tag{19}$$

We then have:

Theorem 1 *Any contract which supports a mixed strategy, that has maximum punishment and binding low state feasibility has* $P_H < P_L$*.*

At first sight this seems surprising; it arises because the firm wishes to maximise the net return it gets in state H, given that in state L it gets no rent: the contract requires $R_H \geq R_L$ in order to preserve the incentive for monitoring; by itself this then gives an incentive to cheat but this incentive can be reduced by raising P_L relative to P_H. Negative correlation between the returns of the two investors thus reduces the variance in the firms' resources. It arises because the non-monitoring investors get caught up in the incentive problem: partly the repayments they get affects the incentive compatibility of monitoring; partly they affect the probability with which the high transfer will be made. The monitoring investor receives a high transfer in the high state, but actually may make a payment to the firm in the low state. The non monitoring investor receives a low state return higher than that received in the high state. By making the payments negatively correlated with the profits of the firm and with the repayments to monitoring investors, the spread in the retained surplus of the firm between truthfully reported high states and non-monitored false reports of low states is reduced. By reducing the variability of the net profit received across states, the firm reduces its incentive to cheat when the high state occurs and mitigates the investor's incentive to monitor. Theorem 1 allows us to derive the following Corollary.

Corollary 1 *The equilibrium probabilities of monitoring and lying are lower in the contract with two investors than in the contract with a single investor.*

The proof is in the appendix. There we show that the greater the spread $P_L - P_H$ the lower is m, the closer are the marginal rates of substitution of the firm and the non monitoring investors and the higher is the objective of the firm. This raises the question of what are the optimal P_s and R_s combinations. As the spread $P_L - P_H$ continually rises, m approaches zero but as it does, of course the mixed strategy form of the second period equilibrium is lost. Also as P_L gets high, then since low state feasibility is binding, R_L becomes negative. This serves to reduce l and the incentive of the firm to cheat. With risk neutrality and no limit on the signs or sizes of payments R_s, P_s, the feasible set is unbounded due to the strict inequality $m > 0$ and there is no optimum: each further increase in $P_L - P_H$ raises the firms objective. Thus there is no optimal solution.

Alternatively we can impose limited liability on investors, e.g. that all repayments should be nonnegative. Then, given the negative correlation result established in Theorem 1, the upper bound on P_L is f_L with $R_L = 0$. Similarly the lower bound on P_H is 0 with $R_{HL} = f_H$. Hence the optimal contract has $R_L = P_H = 0$ with a subsequent Nash equilibrium in mixed strategies defined by: $l = \dfrac{(1-p)\phi}{p(f_H - \phi)}$; $m = \dfrac{f_H - f_L - \delta}{f_H - f_L}$; with $\delta = f_H - \dfrac{\alpha r D}{p(1-l)}$

We use the limited liability solutions to derive the share of investment provided by each investor and the welfare analysis of the three-party contract.

3 The share α

Suppose that we allow the firm to choose the shares of investment in the contract by allowing it to set α. We can show that $0 < \alpha < 1$.

First suppose $\alpha = 0$. Then with limited liability so that optimally $R_L = 0$, (11) can be written as:

$$p(1-l)R_H \geq 0.$$

Since the monitoring investor just needs an expected nonnegative payoff, in fact the firm can set payoffs equal to zero in each state. This would lead to a pooling contract where the entrepreneur always lies and there is zero monitoring. As $P_H = 0$ and $P_L = f_L$, P's participation becomes $(1-p+pl-plm)f_L = rD$, i.e. $f_L = rD$. However this is infeasible as it implies that the firm can afford a single repayment contract, which we have ruled out by assumption.

An analogous argument applies if $\alpha = 1$. In this case it is the non monitoring investor who exits the contract. With limited liability so that optimally $P_H = 0$ and $P_L = f_L$, (12) can be written as:

$$(1-p+pl-plm)f_L \geq 0.$$

The contract involves then a strictly positive transfer to the third party, which the firm can avoid by setting a payoff equal to zero in each state and thus turning the problem into a K&P type of problem. We will however see in the next section that such a contract is dominated by any other contract which has α strictly positive. Hence $0 < \alpha < 1$.

4 Is one or two investors best?

To get a direct comparison with the firm's value function with one investor, we eliminate α as follows. The reaction curve of the firm, obtained using (9), tells us that:

$$-(f_H - R_H - P_H) + m(f_H - R_H - \delta - P_H) + (1-m)(f_H - R_L - P_L) = 0.$$

With maximum punishment, binding low state feasibility and $P_H = 0$,

$$R_H = f_L + m(f_H - f_L).$$

On the other hand from maximum punishment and $P_H = 0$, $p\delta = p(f_H - R_H) = p(1-m)(f_H - f_L)$. If we sum the reservation constraints for the two investors we get:

$$p(1-l)(f_L + mf_H - mf_L) - rD + (1-p+pl-plm)f_L = 0.$$

Solving this for m, given that $l = \dfrac{(1-p)\phi}{p(f_H-\phi)}$, gives:

$$m = \frac{(rD - f_L)(f_H - \phi)}{pf_H^2 - f_L p f_H - \phi f_H + f_L \phi p}.$$

Putting this in the expected payoff for the firm yields:

$$\begin{aligned} E\pi_2 &= p\delta = p(f_H - R_H) \\ &= p(f_H - f_L - \frac{(rD - f_L)(f_H - \phi)(f_H - f_L)}{pf_H^2 - pf_Lf_H - \phi f_H + f_L\phi p}) \end{aligned} \tag{20}$$

Similarly, with a single investor we can write the maximum payoff of the firm as:

$$E\pi_1 = p(f_H - f_L - \frac{(rD - f_L)(f_H - f_L - \phi)}{(p(f_H - f_L) - \phi)}. \tag{21}$$

Comparing (20) and (21) yields:

$$E\pi_2 - E\pi_1 = \frac{pf_L\phi^2(rD - f_L)(1 - p)}{(pf_H^2 - f_Lpf_H - \phi f_H + f_L\phi p)(p(f_H - f_L) - \phi)} > 0 \tag{22}$$

thus showing that multiple provision of funding is preferable to provision by a single investors. It also follows that if the contract with limited liability dominates the K&P contract, so does the unlimited liability contract where repayments are such that $m \to 0$.

5 Collusion and renegotiation

So far, we have not analysed collusion problems arising in this setting. Generally the literature has focused on those arising in a principal-supervisor-agent hierarchy (Tirole [22], Khalil and Lawarrée [12], Kofman and Lawarrée [14], Strausz [21]).

In our framework a collusive agreement can occur between the firm and the monitor, but not between the non monitoring investors and the firm and nor between the two investors (the monitoring and the non monitoring one). A collusive agreement between the non monitoring investors and the firm would be harmless as it would not alter the risk of the firm of being audited. Similarly an agreement between the two investors would not affect the agent's incentives to misreport.

The only possibility is then a collusive agreement occurring between the firm and the monitor. We can show that the non monitoring investors are not damaged by such an agreement. Suppose that, having falsely claimed the low state to have occurred, the entrepreneur endeavours to prevent the monitor from verifying, by bribing her. Because of the result established in Theorem 1 of negative correlation of repayments, the non monitoring investor will be better off accepting the low report than querying it as she gets a payoff higher than the one received under truthful reporting. So there is no possibility for the monitor and the firm to collude against the passive investors.

Similarly the firm could report the high state while in fact the low state has occurred, thus making the lower repayment to the non monitoring investor associated with a high state report, and offer the monitor a side contract so as to induce her not to monitor. For the monitor to accept it, the bribe offer would have to be such that:[2]

$$\alpha rD \leq B_0 < R_H, \tag{23}$$

whereas for the firm to make it

$$f_L - P_H - B_0 > f_L - P_L - R_L. \tag{24}$$

Using (23) and (24), we deduce that the contract is collusion proof iff:

$$f_L - P_H - \alpha rD \leq 0,$$

which certainly holds.

It remains to see whether the entrepreneur and the monitor gain any increase in utility by engaging in a form of renegotiation between themselves. Suppose that after a low state report, the firm offers a bribe to the investor to induce her not to monitor. The bribe offer will convey some information to the monitor, which will now compare the payoff from monitoring with the payoff from accepting the bribe. A bribe B_1 will thus be accepted iff:

$$R_L + B_1 > R_H + \delta - \phi.$$

From the form of the optimal contract we know that there is binding low state feasibility and maximum punishment, so this simplifies to:

$$B_1 > f_H - P_H - f_L + P_L - \phi \tag{25}$$

On the other hand the entrepreneur will only offer bribes that satisfy

$$f_H - R_L - P_L - B_1 > (1-m)(f_H - R_L - P_L) + m(f_H - R_H - \delta - P_H),$$

which simplifies to:

$$B_1 < m(f_H - f_L). \tag{26}$$

Using (25) and (26), we can deduce that the contract is renegotiation-proof iff:

$$\delta - \phi + P_L - P_H > 0,$$

which certainly holds under the K&P assumption that $\delta - \phi = Ef - rD - \phi > 0$.[3]

Another renegotiation possibility could still arise between the firm and the monitor. Suppose the high state occurs and suppose the firm announces that it

[2] In such circumstances the firm anticipates that the the monitor could first accept and then, after the high report is made, ask the enforcement of the original contract paying R_H, thus undoing the side contract. However, we suppose that ex post she will have no possibility of doing so.

[3] Using (18) and (19), $\delta - \phi$ can be written as $Ef - rD - \phi - plm(P_H - P_L)$, i.e., under limited liability $Ef - rD - \phi + plmf_L$.

will make a high state report if the payoff B_2, with $\alpha rD \leq B_2 < R_H$, is accepted, triggering to report low in case the renegotiation offer were not accepted.[4] In deciding whether to accept the bribe offer $B_2 \geq \alpha rD$, the monitor will compare it with the one she would get if a low report was made and she had to monitor to enforce her rights, $R_H + \delta - \phi$. She will thus accept the bribe iff:[5]

$$B_2 \geq \alpha rD > R_H + \delta - \phi. \tag{27}$$

On the other hand the entrepreneur will only offer bribes that satisfy

$$f_H - P_H - B_2 > (1-m)(f_H - R_L - P_L) + m(f_H - R_H - \delta - P_H),$$

which, by binding low state feasibility and maximum punishment, simplifies to:

$$B_2 < f_H - P_H - (1-m)(f_H - f_L). \tag{28}$$

Using (27) and (28), we deduce that the contract is renegotiation-proof iff:

$$\delta - \phi > 0,$$

which also holds under the K&P assumption. Thus even in this case there is no gain to be had from a renegotiation between the monitor and the firm.

Conclusions

In this paper we have explored the optimal financial contract, together with incentive compatible and optimal reporting and monitoring probabilities, when the revenues of a production process are private information to the firm and are revealed only after the funding necessary for the project has been raised. The amount of finance can be provided by two investors, say a big shareholder and a group of small dispersed shareholders. Costly monitoring can be carried out by one group of investors with the other group of investors learning the results of it for free. The amount of monitoring is not a variable that can be written into the contract.

Given that there is a subsequent mixed strategy Nash equilibrium in the probabilities of monitoring and cheating, we find that the non monitoring investors have a role in creating correct reporting incentives for the firm, which leads to a negative correlation between the true revenues of the firm and their return. This allows the firm to better self-police its incentive to cheat and hence the amount of state observation cost that it must ultimately bear from its residual profit. It also discourages non monitoring investors from ever wishing to

[4] Also in such circumstances the firm anticipates that the the monitor could first accept and then, after the high state report is received, ask the enforcement of the original contract paying R_H, thus undoing the side contract. Also for this case we suppose that ex post she will have no possibility of doing so.

[5] Note that such private agreement between the firm and the monitor will not damage the non monitoring investor who gets the payoff she had a right to (P_H).

monitor: in the event of false report by the firm, they would have no incentive to overturn a low state report since they would have received get a higher return from this than from a high state report. As a consequence of the smoothing out of repayments, we also find that the two investor scenario is Pareto superior to a single investor model à la Khalil and Parigi [13], with the extent of the gain in welfare depending on the investors' degree of limited liability. If we allow for post-contract offers being made only by the entrepreneur, we also find that this type of contract is collusion-proof. One implication of this result is that, since because of negative correlation the non monitoring investor cannot suffer any loss in utility, the contract is also renegotiation-proof.

One remark concerns the choice of the party who writes the contract. This is in fact a crucial issue in the problem of giving the right incentives to the firm when the uninformed party cannot commit to an audit policy. Intuitively, if the uninformed party writes the contract, she will try and extract all the surplus from the informed one thus maximising his incentive to cheat. Conversely, by designing the contract, the informed party can set the repayments so as to keep a rent for correct reporting (i.e. impose a sure loss for misreporting), thus making cheating less attractive.

Appendix

Proof of Proposition 1. The proof proceeds as follows: we first prove that there is maximum punishment; then that the firm gets a positive rent in high state and zero rent in low state. Instead of working with R_{HL} as a variable, we use the punishment $\delta = R_{HL} - R_H$.

To derive properties of the optimal contract we form the Lagrangean

$$\begin{aligned}
&p(f_H - P_H - R_H) + (1-p)(f_L - P_L - R_L) + \\
&+\lambda_1[p(1-l)R_H + (1-p+pl)R_L - \alpha r D] + \\
&+\lambda_2[\mu P_H + (1-\mu)P_L - (1-\alpha)rD] + \\
&+\lambda_3(f_H - P_H - R_H - \delta) + \lambda_4(f_L - R_L - P_L)
\end{aligned}$$

where $\mu = p(1-l)+plm$, $l = \dfrac{(1-p)\phi}{p(R_H + \delta - R_L - \phi)}$, $m = \dfrac{R_H + P_H - R_L - P_L}{R_H + P_H + \delta - R_L - P_L}$ and α is the share of capital provided by the monitoring investor. We then use the FOC's:

$$\begin{aligned}
\frac{\partial L}{\partial R_H} &= -p + \lambda_1\left(p - pl + \frac{pl(R_H - R_L)}{R_H + \delta - R_L - \phi}\right) + \\
&+\lambda_2(P_H - P_L)\left[\frac{pl(1-m)}{R_H + \delta - R_L - \phi} + \frac{pl(1-m)}{R_H + \delta - R_L + P_H - P_L}\right] - \lambda_3 = 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial R_L} &= -(1-p) + \lambda_1\left(1 - p + pl - \frac{pl(R_H - R_L)}{R_H + \delta - R_L - \phi}\right) - \\
&-\lambda_2(P_H - P_L)\left[\frac{pl(1-m)}{R_H + \delta - R_L - \phi} + \frac{pl(1-m)}{R_H + \delta - R_L + P_H - P_L}\right] - \lambda_4 = 0
\end{aligned}$$

$$\frac{\partial L}{\partial \delta} = \lambda_1 \frac{pl(R_H - R_L)}{R_H + \delta - R_L - \phi} + \lambda_2 (P_H - P_L)[\frac{pl(1-m)}{R_H + \delta - R_L - \phi} - $$
$$ - \frac{plm}{R_H + \delta - R_L + P_H - P_L}] - \lambda_3 \leq 0$$

$$\frac{\partial L}{\partial P_H} = -p + \lambda_2[\mu + \frac{pl(1-m)(P_H - P_L)}{R_H + \delta - R_L + P_H - P_L}] - \lambda_3 = 0$$

$$\frac{\partial L}{\partial P_L} = -(1-p) + \lambda_2[(1-\mu) - \frac{pl(1-m)(P_H - P_L)}{R_H + \delta - R_L + P_H - P_L}] - \lambda_4 = 0$$

- From $\frac{\partial L}{\partial R_H} + \frac{\partial L}{\partial R_L}$ and $\frac{\partial L}{\partial P_H} + \frac{\partial L}{\partial P_L}$:

$$1 + \lambda_3 + \lambda_4 = \lambda_1 \quad (29)$$
$$= \lambda_2 \quad (30)$$

  whence we deduce that $\lambda_1 = \lambda_2 > 1$.

- For $m < 1$, $\delta > 0$ and $\frac{\partial L}{\partial \delta} = 0$.

- To prove maximum punishment ($\lambda_3 > 0$), we use $\lambda_1 = \lambda_2$ from (29) and (30) in the *FOC* on δ, and get:

$$\lambda_3 = \lambda_1 \{ \frac{pl(R_H - R_L)}{R_H + \delta - R_L - \phi} + (P_H - P_L)[\frac{pl(1-m)}{R_H + \delta - R_L - \phi} - $$
$$ - \frac{plm}{R_H + \delta - R_L + P_H - P_L}]\}$$

$$\lambda_3 = \lambda_1 pl \{ \frac{R_H - R_L + P_H - P_L}{R_H + \delta - R_L - \phi} - m(P_H - P_L)[\frac{1}{R_H + \delta - R_L - \phi} + $$
$$ + \frac{1}{R_H + \delta - R_L + P_H - P_L}]\}$$

$$\lambda_3 = \lambda_1 \frac{pl}{R_H + \delta - R_L - \phi} \{ R_H - R_L + P_H - P_L$$
$$ - m(P_H - P_L)(1 + \frac{R_H + \delta - R_L - \phi}{R_H + \delta - R_L + P_H - P_L})\}$$

$$\lambda_3 = \lambda_1 \frac{plm(R_H + \delta - R_L)}{R_H + \delta - R_L - \phi}[1 + \frac{R_H + \delta - R_L - \phi}{R_H + \delta - R_L + P_H - P_L}]$$

  which is certainly positive. Hence $\delta = f_H - R_H - P_H$.

- The next step is to prove that the low state feasibility constraint binds: $R_L + P_L = f_L$ ($\lambda_4 > 0$). Using the FOC on P_L :

$$\lambda_4 = -(1-p) + \lambda_2[(1-p+pl-plm) - \frac{pl(1-m)(P_H - P_L)}{R_H + \delta - R_L + P_H - P_L}].$$

  This can also be rearranged to:

$$\lambda_4 = (\lambda_2 - 1)(1-p) + \lambda_2 \frac{pl(1-m)(R_H + \delta - R_L)}{R_H + \delta - R_L + P_H - P_L},$$

  which, using (29), is certainly positive.

■

Proof of Theorem 1. Given maximum punishment and binding low state feasibility, we can set $R_H + \delta = f_H - P_H$, and $R_L = f_L - P_L$. The problem hence reduces to:

$$\max_{\delta,\alpha,P_L,P_H} p\delta$$

$$\text{st } p(1-l)(f_H - P_H - \delta) + (1-p+pl)(f_L - P_L) = \alpha r D \quad (31)$$

$$\mu P_H + (1-\mu)P_L = (1-\alpha)rD \quad (32)$$

From (31)

$$\begin{aligned}
p(1-l)\delta &= p(1-l)(f_H - P_H) + \frac{(1-p+pl)(f_L - P_L) - \alpha r D}{1-l} \\
p\delta &= p(f_H - P_H - f_L + P_L) + \frac{f_L - P_L - \alpha r D}{1-l} \\
&= p(f_H - P_H - f_L + P_L) + \frac{p(f_H - P_H - f_L + P_L - \phi)}{p(f_H - P_H - f_L + P_L) - \phi}(f_L - P_L - \alpha r D) \\
&= p(f_H - P_H - f_L + P_L) + \frac{f_L - P_L - \alpha r D}{1-l}
\end{aligned}$$

Hence

$$\max_{\alpha,P_L,P_H} p\delta \quad (33)$$

$$\text{st } \mu P_H + (1-\mu)P_L = (1-\alpha)rD \quad (34)$$

where

$$\delta = f_H - P_H - f_L + P_L + \frac{f_L - P_L - \alpha r D}{p(1-l)}. \quad (35)$$

Remark 1 *Note here that $f_L - P_L - \alpha r D < 0$. Suppose not, then the entrepreneur could offer a single repayment contract to the monitoring investor, and a state contingent contract to the non monitoring one (in particular with*

$P_L < P_H$). The monitoring investor will hence give up her audit rights ($m = 0$) thus maximising the entrepreneur's incentive to lie ($l = 1$). Using $m = 0$ and $l = 1$ into (34) we deduce that $P_L = (1-\alpha)rD$, the non monitoring investor is also offered a single repayment contract. But this implies that $f_L - rD > 0$, thus making a single repayment contract implementable even under asymmetric information, a contradiction.

Using a similar argument, it can be shown that this holds for any $0 \leq \alpha \leq 1$. Suppose $\alpha = 0$. In this case the two uninformed parties specialise: one as the investor and one as the supervisor. If $f_L - P_L \geq 0$, then the entrepreneur could offer a single repayment contract to the only investor left (say $P_{SR} = rD$) and a state contingent contract to the supervisor with $R_H > 0$ and $R_L < 0$ (so as to hold her down to her reservation utility, while still inducing her to monitor). However, $R_L < 0$ and binding low state feasibility ($f_L - R_L - P_{SR} = 0$) imply that $f_L - P_{SR} < 0$, a contradiction.

Working out the FOC's

$$\frac{\partial L}{\partial P_H} \quad : \quad p\frac{\partial \delta}{\partial P_H} = -\lambda\left\{\mu - \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi} - \frac{pl(P_H - P_L)}{f_H - f_L}\frac{\partial \delta}{\partial P_H}\right\} \tag{36}$$

$$\frac{\partial L}{\partial P_L} \quad : \quad p\frac{\partial \delta}{\partial P_L} = -\lambda\left\{1 - \mu + \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi} - \frac{pl(P_H - P_L)}{f_H - f_L}\frac{\partial \delta}{\partial P_L}\right\} \tag{37}$$

$$\frac{\partial L}{\partial \alpha} \quad : \quad \frac{rD}{1-l} = \lambda rD\left\{1 + \frac{l(P_H - P_L)}{(1-l)(f_H - f_L)}\right\} \tag{38}$$

$$\frac{\partial L}{\partial \lambda} \quad : \quad p(1-l)P_H + (1-p+pl)P_L + \frac{pl(P_H - P_L)}{f_H - f_L}[P_H - P_L - \frac{f_L - P_L - \alpha rD}{p(1-l)}] = $$

$$= \quad (1-\alpha)rD \tag{39}$$

where $\frac{\partial \delta}{\partial P_H} = -1 + \frac{l(f_L - P_L - \alpha rD)}{(1-l)(p(f_H - P_H - f_L + P_L) - \phi)}$, $\frac{\partial \delta}{\partial P_L} = 1 - \frac{1}{p(1-l)} - \frac{l(f_L - P_L - \alpha rD)}{(1-l)(p(f_H - P_H - f_L + P_L) - \phi)}$.

- For a mixed strategy we must have $m = \frac{f_H - \delta - f_L}{f_H - f_L} > 0$ implying we need $f_H - \delta - f_L > 0$, i.e. $\delta < f_H - f_L$. Substituting out $\delta = f_H - P_H - f_L + P_L + \frac{f_L - P_L - \alpha rD}{p(1-l)}$ in the inequality, we find that:

$$P_L - P_H + \frac{f_L - P_L - \alpha rD}{p(1-l)} < 0. \tag{40}$$

This holds at any P_L, P_H, α which subsequently has $m > 0$. Using (32)

$$P_L = -(p - pl + plm)(P_H - P_L) + (1-\alpha)rD.$$

Substituting out in (40), gives

$$plm(P_L - P_H) > f_L - rD. \tag{41}$$

Lemma 1 *$MRS_E < MRS_P$ iff (40) holds. So at any P_H, P_L, α satisfying the conditions of the theorem we have $MRS_E < MRS_P$.*

Proof. *Suppose not. Then, looking at the marginal rates of substitution of the entrepreneur (MRS_E) and the non monitoring investor (MRS_P):*

$$MRS_E = -\frac{\frac{\partial\delta}{\partial P_H}}{\frac{\partial\delta}{\partial P_L}}, \tag{42}$$

$$MRS_P = -\frac{\mu - \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi} - \frac{pl(P_H - P_L)}{f_H - f_L}\frac{\partial\delta}{\partial P_H}}{1 - \mu + \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi} - \frac{pl(P_H - P_L)}{f_H - f_L}\frac{\partial\delta}{\partial P_L}}; \tag{43}$$

where

$$\begin{aligned}
\frac{\partial\delta}{\partial P_H} &= -1 + \frac{l(f_L - P_L - \alpha rD)}{(1-l)(p(f_H - P_H - f_L + P_L) - \phi)}, \\
\frac{\partial\delta}{\partial P_L} &= 1 - \frac{1}{p(1-l)} - \frac{l(f_L - P_L - \alpha rD)}{(1-l)(p(f_H - P_H - f_L + P_L) - \phi)}.
\end{aligned}$$

Assuming an interior solution to exist ($MRS_E = MRS_P$) :

$$-\frac{\frac{\partial\delta}{\partial P_H}}{\frac{\partial\delta}{\partial P_L}} = -\frac{\mu - \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi} - \frac{pl(P_H - P_L)}{f_H - f_L}\frac{\partial\delta}{\partial P_H}}{1 - \mu + \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi} - \frac{pl(P_H - P_L)}{f_H - f_L}\frac{\partial\delta}{\partial P_L}}$$

$$-\frac{\partial\delta}{\partial P_H}\left\{1 - \mu + \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi} - \frac{pl(P_H - P_L)}{f_H - f_L}\frac{\partial\delta}{\partial P_L}\right\} = -\frac{\partial\delta}{\partial P_L}\left\{\mu - \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi} - \frac{pl(P_H - P_L)}{f_H - f_L}\frac{\partial\delta}{\partial P_H}\right\}$$

$$-\frac{\partial\delta}{\partial P_H}\left\{1 - \mu + \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi}\right\} = -\frac{\partial\delta}{\partial P_L}\left\{\mu - \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi}\right\}$$

$$-\frac{\partial\delta}{\partial P_H} = -\left\{\mu - \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi}\right\}\left(\frac{\partial\delta}{\partial P_H} + \frac{\partial\delta}{\partial P_L}\right).$$

$$\frac{\partial\delta}{\partial P_H} + \frac{\partial\delta}{\partial P_L} = -\frac{1}{p(1-l)}$$

$$1 - \frac{l(f_L - P_L - \alpha rD)}{(1-l)(p(f_H - P_H - f_L + P_L) - \phi)} = \frac{1}{p(1-l)}\left\{\mu - \frac{pl(1-m)(P_H - P_L)}{f_H - P_H - f_L + P_L - \phi}\right\}$$

$$-p(1-l)+\frac{pl(f_L-P_L-\alpha rD)}{p(f_H-P_H-f_L+P_L)-\phi}=-\mu+\frac{pl(1-m)(P_H-P_L)}{f_H-P_H-f_L+P_L-\phi}$$

$$\frac{(f_L-P_L-\alpha rD)}{p(f_H-P_H-f_L+P_L)-\phi}=-m+\frac{(1-m)(P_H-P_L)}{f_H-P_H-f_L+P_L-\phi}. \tag{44}$$

Using $\frac{f_H-P_H-f_L+P_L-\phi}{p(f_H-P_H-f_L+P_L)-\phi}=\frac{1}{p(1-l)}$ *in (44) gives*

$$\begin{aligned}\frac{f_L-P_L-\alpha rD}{p(1-l)} &= -m(f_H-P_H-f_L+P_L-\phi)+(1-m)(P_H-P_L)\\ &= -m(f_H-f_L-\phi)+(P_H-P_L)\\ &= -\frac{f_H-f_L-\delta}{f_H-f_L}(f_H-f_L-\phi)+(P_H-P_L). \end{aligned}\tag{45}$$

Using δ *from (35) in (45) gives:*

$$\begin{aligned}\frac{(f_L-P_L-\alpha rD)(f_H-f_L)}{p(1-l)} &= -(P_H-P_L-\frac{1}{p(1-l)}(f_L-P_L-\alpha rD))*\\ &\quad *(f_H-f_L-\phi)+(f_H-f_L)(P_H-P_L)\end{aligned}$$

$$\begin{aligned}\frac{(f_L-P_L-\alpha rD)(f_H-f_L)}{p(1-l)} &= \frac{(f_L-P_L-\alpha rD))(f_H-f_L-\phi)}{p(1-l)}+(P_H-P_L)\phi\\ 0 &= -\frac{(f_L-P_L-\alpha rD)}{p(1-l)}\phi+(P_H-P_L)\phi\\ P_L-P_H &= -\frac{(f_L-P_L-\alpha rD)}{p(1-l)},\end{aligned}\tag{46}$$

which is a contradiction. ■

Lemma 2 *For any fixed* P_H, P_L, α *which give the firm a nonnegative expected utility in a contract leading to a subsequent mixed strategy where the repayments have been optimally chosen,* $\frac{\partial E\pi_E}{\partial P_L}<0$.

Proof. *We can write* $\frac{\partial E\pi_E}{\partial P_L}$ *as:*

$$-\frac{(1-p)[(x_H-x_L-\phi)(p(x_H-x_L)-\phi)+\phi(px_H+(1-p)x_L-\alpha rD-\phi))]}{(1-l)(x_H-x_L-\phi)(p(x_H-x_L)-\phi)}.$$

which is non positive if

$$C=p(x_H-x_L-\phi)(p(x_H-x_L)-\phi)+p\phi(px_H+(1-p)x_L-\alpha rD-\phi)>0, \tag{47}$$

where $x_H = f_H - P_H$ *and* $x_L = f_L - P_L$.

We can show that so long as the firm only offers contracts yielding nonnegative expected utility to it then this condition is satisfied. The objective function of the firm is:

$$p(f_H - P_H - R_H) = p(x_H - R_H). \tag{48}$$

The investors participation constraint is:

$$\begin{aligned} \alpha r D &= p(1-l)R_H + (1-p+pl)x_L \\ &= pR_H + (1-p)x_L - (R_H - x_L)\frac{(1-p)\phi}{(x_H - x_L - \phi)} \\ &= pR_H + (1-p)x_L - (R_H - x_L)\frac{(1-p)\phi}{(x_H - x_L - \phi)} \\ &= pR_H + (1-p)x_L - (R_H - x_L)\frac{(1-p)\phi}{(x_H - x_L - \phi)}, \end{aligned} \tag{49}$$

where $R_L = x_L - P_L$ *because of binding low state feasibility. Solve (49) for* R_H *and substitute into (48):*

$$\begin{aligned} E\pi &= px_H + (1-p)x_L - \alpha r D - \frac{(1-p)\phi}{p(x_H - x_L) - \phi}(\alpha r D - x_L) \\ &= \frac{p(px_H^2 - 2x_H p x_L - x_H\phi + x_H x_L - x_L^2 + px_L^2 - \alpha r D(x_H - x_L - \phi)}{p(x_H - x_L) - \phi} \end{aligned} \tag{50}$$

Subtract (47) from (50) to get

$$E\pi - C = -p(x_H - x_L - 2\phi)(\alpha r D - x_L) < 0 \tag{51}$$

Hence if $E\pi > 0$ *then* $C > 0$.

Suppose that we take any fixed values of P_H, P_L *and* α *that satisfy the non monitoring investor participation constraint:*

$$(p - pl + plm)P_H + (1 - p + pl - plm)P_L = (1-\alpha)rD.$$

Given these fixed values, compute the Nash equilibrium l, m *and then solve the contract problem for optimal values of* R_s *conditional on these fixed values of* P_s*. The resulting maximal utility level of the firm is given by (50). However this contract with a subsequent mixed strategy will be offered by the firm only if* $E\pi$ *given by (50) is positive and hence only if* $C > 0$ *and so will involve* $\frac{\partial E\pi_E}{\partial P_L} < 0$.
■

Lemma 3 *From any point* P_H, P_L *satisfying the conditions of the theorem the firm is better off reducing* P_H *and raising* P_L *until a mixed strategy fails to hold.*

Proof. *To see this vary the repayments to* P *according to:*

$$dP_L = MRS_P dP_H.$$

The non monitoring investor's utility is then constant:

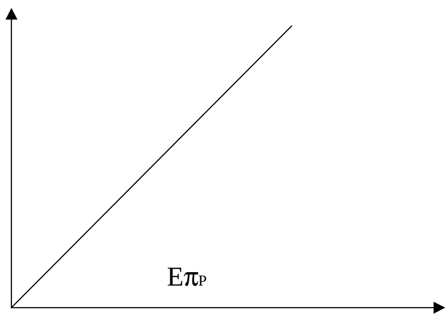
$$d\pi_P = \frac{\partial \pi_P}{\partial P_H} dP_H + \frac{\partial \pi_P}{\partial P_L} dP_L = \frac{\partial \pi_P}{\partial P_L}[-MRS_P dP_H + dP_L] = 0; \tag{52}$$

the entrepreneur's change in utility is

$$\begin{aligned} dE\pi_E &= \frac{\partial E\pi_E}{\partial P_L} dP_L + \frac{\partial E\pi_E}{\partial P_H} dP_H \\ &= [\frac{\partial E\pi_E}{\partial P_L} MRS_P + \frac{\partial E\pi_E}{\partial P_H}] dP_H \\ &= \frac{\partial E\pi_E}{\partial P_L}[MRS_P - MRS_E] dP_H, \end{aligned}$$

which is positive if $dP_H < 0$ *since* $MRS_P - MRS_E > 0, \frac{\partial E\pi_E}{\partial P_L} < 0.$ ■

We can illustrate the negative correlation result using a diagram where we have the repayments to P on the axis for any given α. We see that the firm's isoprofit contour is steeper that the investor's. By holding the investor's utility constant, it is then possible to vary the repayments so as to increase the firm's profits by shifting its profit function to the left. It is then optimal to set $P_H < P_L$.



In order to determine the share of investment provided by each investor, we bound the repayments to be non negative. The firm's profit function will thus shift to the left until it hits the vertical axis where $P_L = f_L$ and $P_H = 0$. Using the limited liability repayments into the first order condition for λ (39) and solving this for α gives:

$$\alpha = \frac{[rD - (1-p)f_L](f_H - f_L) - plf_H f_L}{rD[(f_H - f_L)(pf_H - \phi) - (1-p)\phi f_L]}(pf_H - \phi).$$

■

Proof of Corollary 1. Under the conditions of the Theorem the numerator of m can be written as: $f_H - f_L - \delta = p(f_H - f_L) - E\pi_E$. For Lemma

3 this is increasing in P_H and decreasing in P_L.[6] Analogously, under the same conditions, $l = \dfrac{(1-p)\phi}{p(f_H - f_L + P_L - P_H - \phi)}$, which is decreasing in the spread between P_L and P_H. This then implies that the equilibrium probabilities of monitoring and lying are lower in the contract with two investors than in the contact with a single investor. ■

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