CENTRO STUDI IN ECONOMIA E FINANZA

CENTRE FOR STUDIES IN ECONOMICS AND FINANCE

WORKING PAPER NO. 73

Sequential Choice and R&D Racing

Hans W. Gottinger (University of Maastricht)

November 2001

DIPARTIMENTO DI SCIENZE ECONOMICHE - UNIVERSITÀ DEGLI STUDI DI SALERNO
Via Ponte Don Melillo - 84084 FISCIANO (SA)
Tel. 089-96 3167/3168 - Fax 089-96 3169 – e-mail: csef@unisa.it

CSEF WORKING PAPER NO. 73

Sequential Choice and R&D Racing

Hans W. Gottinger (University of Maastricht)

Abstract

This paper develops a framework to analyze how choices are made when R&D competition occurs between two firms, and the aggressiveness-time tradeoffs have to be resolved in multiple stages. At issue is the way in which resources are used at each stage, i.e. are aggressiveness problems undertaken and solved (slowly) or are quick solutions adopted in an effort to get the product to market faster? We first analyze why differently positioned firms choose different targets. We focus on this translation between ex ante asymmetries between firms and ex post asymmetries in the equilibrium outcomes. Our second focus is on understanding the implications of the tradeoff between the level of aggressiveness and time spent on each stage in a multi-stage process.

Keywords. Innovation, Race, Competition, Strategy, Industrial Economics

Table of contents

1. Introduction

Most of the existing work on sequential R&D is of decision theoretic nature. The idea is usually to study different aspects of the resource allocation process as the stages of R&D unfold. Grossman and Shapiro (1986) analyse the optimal profile of R&D effort over multiple stages and distinguish between situations where there is a stochastic relationship. Payoffs are received only upon completion of all stages. In both cases, they show that optimal resource allocation increases as more stages elapse. In contrast, Dutta (1997) shows that, in the case where the total available budget is given exogenously and flow payoffs result from completion of the stages, the optimal allocation involves greater expenditures in the early stages. However, if the payoffs are only received upon completion of all stages, then the optimal allocation is to spread the budget evenly across all stages. Gallini and Kotowitz (1985) model R& D as drawing experiments, without replacement, from a series of known potential experiments. The characteristics, including the length, of each experiment are fixed, but the length of the entire program is determined endogenously. They derive time profiles of R&D, involving both parallel consideration of multiple experiments, and sequential consideration of separate experiments. Granot and Zuckerman (1991) are also concerned with the sequencing of known activities. In earlier work, Roberts and Weitzman (1981) have derived operational rules for investing criteria in sequential R&D projects. They distinguish between processes for which all stages must be completed before payoffs result, and those where the stages are optional and the project may be terminated at any stage and also offer a taxonomy of sequential processes along other dimensions.

Analyses of sequential R&D that take into account the strategic interaction between firms are fewer and far between. Grossman & Shapiro (1987) analyse a two firm, two stage game, with benefits accruing to the first firm to finish both stages, with the intention of studying how the investment level of a firm depends on its relative position in an R&D race. They show that typically the firm that gets in the lead steps up its effort level, while the firm that falls behind lowers its effort level. Park (1987) carries out a similar analysis. As we shall see below, our analysis of the aggressiveness-speed tradeoff results in quite different resource utilization patterns in similar two stage games. In their study of multi-stage races, Harris and Vickers (1987) reach the principal conclusion that the-race leader invests more than the follower. This conclusion too is modified in our analysis of multi-stage games. Two stage strategic interactions have been used in other situations as well. eg. Fershtman and Kamien (1990) study the strategic interaction between two firms when the product requires development of two complementary technologies. Their focus is on investigating the circumstances under which cross licensing results at the intermediate stage where each firm has developed only one technology. In a different vein, Vickers (1986) analyses the response of a duopoly to a sequence of process innovations and contrasts situations of “increasing dominance” where the incumbent firm remains dominant with those of “action-reaction” where the firm with the current best technology alternates. Beath et al. (1987) extend this analysis to consider a sequence of product innovations.

In this paper, we model the R&D process as one that is composed of a series of stages, at each of which some set of product characteristics are determined . At each stage, the firm exercises discretion as to the extent to which the particular stage characteristics will be developed. Further, when the firm's R&D resource endowment is fixed and stage specific, the firm faces a tradeoff between the extent to which it will develop the stage characteristics and the time that it will spend in doing so.We will refer to the degree to which the stage characteristic is developed as the characteristic's (targeted) level of aggressiveness. In this sense, the more aggressive are the product characteristics, the greater the profits the firm can expect to get from the product but the longer is the expected time to completion of R&D. We will speak of this as the aggressiveness-time tradeoff in each stage. When the completion of R&D requires many such tradeoffs to be resolved sequentially, firms will, at the beginning of each stage, want to make their choices in light of the choices made by all firms in previous stages and their stage completion records to that time.

We investigate how firms choose the aggressiveness of their product characteristics when there is a tradeoff between the aggressiveness of product characteristics and the speed with which the stages of R&D are completed, and when strategic interactions between firms are accounted for. While products with more aggressive characteristics are "better" in the sense that they earn the firm greater profits, the required R&D takes more time. This paper thus incorporates strategic considerations into the analysis of a problem, that of the way in which given resources are used in a sequential R&D context, that has hitherto been dealt with only in a decision theoretic fashion, lately in a stochastic version (Gottinger, 2001) We offer two sets of conclusions to build on the existing literature. First, in the two stage game (as, for example, in Grossman and Shapiro (1987) we demonstrate the importance of explicitly accounting for differences in product quality and cost. It is in the assumption of scarce resources for each stage (and the implied tradeoff between product aggressiveness and time to completion of R&D) that this analysis differs from previous analyses. Further, in situations of strategic rivalry, relative R&D resource endowment influences the pattern of resource utilisation, as does incumbency (Gilbert, 1989). These analyses are motivated by the empirical work described in Cebon et al.(2001). Secondly, this paper analyses the changing way in which R&D resources are used in a multi-stage game as one firm gets further ahead of the other, and as the firms' proximity to the last stage increases. Throughout, we abstract from "dropout behavior" (Lippman & McCardle (1987)).

It should be stressed that it is not our purpose to analyse how firms vary the amount they spend on R&D as stages in the sequential game evolve (against nature or against a specific opponent). Rather, the issue is: how do firms use given resources in the face of the existence of a very natural tradeoff between the aggressiveness of product characteristics and the speed of execution.

There are two sets of reasons why the resources available for R&D might be limited. The first interpretation has to do with the assumption that it is the budget available to R&D management that is fixed, and consequently limited use can be made of the services of engineers and scientists to do the R&D(even though there is a well developed market for such services). The second interpretation is that the resources available to do R&D (in technically complex, knowledge based industries)are firm specific and therefore subject to factor market imperfections. As Rubin(1973) points out, "...the value of a resource typically exceeds the market value of the individual parts due to the cohesiveness of the human part of the resource developed through mutual experience within the firm". In a similar vein, Camerer (1991) suggests that workers might be tied to a firm if they are productive together but cannot collectively agree to move to a better-paying firm. Thus scientists function best within (and are worth more in) the milieu in which much of their work on firm specific R&D has proceeded. The notion of resources being tied to a firm also implies that a firm cannot augment its resources easily. This 'resource based' view within the strategy literature (see Connor (1991) for a review) analyzes the effects of product market competition taking as given initial differences in firms resource endowments. Game theoretic advances (Fudenberg and Tirole, 1989, Tirole, 1988) also suggest foundations for the resource based view. For example, information asymmetries in the 'new industrial organization' (Jacquemin, 1987; Reinganum, 1989) might preclude the use of an asset outside the boundaries of the firm that owns the asset. Kreps (1990) suggests that in many situations where unforeseen contingencies arise, certain transactions are facilitated by the reputation of a trusted party, i.e. a firm. Thus a firm's resources might not be able to operate independently of the firm. The obvious question that the resource-based view leaves unanswered is that of how the firms came to acquire different endowments in the first place. Here again Rubin (1973) suggests that when resources are firm-specific, then existing resources will have to be used to augment the stock available for future use. Given the opportunity costs of engaging in such augmentation , different firms might opt for different augmentation rates. As such, at any point in time, different firms would have access to different resource endowments.

R&D resources are assumed to be firm specific. Further, each stage is assumed to have its own resource endowment (non-transferable across stages). This reflects our belief that the nature of work done by, say, other scientists in basic research, is considerably different from that done by, say, other scientists and engineers in development. Just as firms would incur significant costs in augmenting their R&D resources, so also deploying existing resources across the stages is also assumed to be very costly.

The paper is organized as follows. Section 2 introduces a simple model that will allow us to think about multiple stage racing and analyzes a two-stage interaction between symmetric firms. Section 3 elaborates on this analysis to consider some of the asymmetries that empirical work suggests are important (Cohen and Levin, 1989, Lerner, 1997). We show that an incumbent firm will choose more aggressively in each stage of the race than an entrant firm. We analyze factors that cause relatively resource rich firms to choose differently from resource poor ones. Finally , Section 4 is devoted to understanding how getting ahead (falling behind) alters the choices that a firm makes. We identify the importance of two factors : the magnitude of the lead that one firm has over the other, and the distance between the firms and the finish line and suggest that the patterns of resource allocation, in a situation where there is an aggressiveness-speed tradeoff, can be quite different from that suggested in the existing literature. Section 5 draws some conclusions and identifies some new research issues.

2. A Model for Symmetric Firms

Product development proceeds in a sequence of stages, each responsible for determining a particular characteristic of the product. Each stage involves a number of problem solving activities, split into two parts, 'research' and 'development', each of which improves the product characteristic being determined in that stage. The firm exercises discretion as to the extent to which the stage characteristic will be developed. At each stage, the firm has access to an exogeneously determined resource endowment (which cannot be transferred across stages) and has to target the set of activities over which it will allocate these resources. The larger the number of activities in the targeted set, the greater is the extent to which the stage characteristic is developed. However, the larger the number of targeted activities, the smaller will be the resource amount allocated to each activity, and the greater will be the expected time to completion of the activities in that stage. We will speak of a firm's target being more aggressive if its target set contains more activities.

System complexity considerations are incorporated here by thinking of each problem solving activity in a stage as linked to every other problem solving activity in that stage. We assume that the stage specific resource is divided equally among the activities in the targeted set and that the problems cannot be solved independent of each other. Thus, at any point in time, the targeted set of problems has either been completely solved or not at all. Let the hazard rate for solving the targeted set of problems be denoted 'h'.[1] Then a higher hazard rate is obtained by allocating more resources to each activity, which implies consideration of fewer activities. Thus a higher hazard rate implies a less aggressive target set.

In this section, we model a two-firm, two-stage interaction. Each firm has two choices for the degree of aggressiveness of its product characteristics in each stage. The choice of the higher rate 'x' in a stage gives the firm a less aggressive stage target, while the choice of the lower hazard rate 'y' corresponds to a more aggressive stage target. While there is uncertainty in the time taken to achieve a particular target, the target itself is chosen at the beginning of each stage. Let p_R, q_R be the choices of hazard rate by firms 1 and 2 respectively for the first stage (hereafter, RES stage). Similarly, let firms 1 and 2 choose hazard rates p_D, q_D respectively for the second stage (hereafter, DEV stage). At time 0, firm 1 chooses $p_R = x$ or $p_R = y$, while firm 2 simultaneously chooses $q_R = x$ or $q_R = y$. Assume, without loss of generality, that the first firm to finish the RES stage is firm 2. Then firm 2 will choose $q_D = x$ or $q_D = y$ as its hazard rate for the DEV stage. Finally, when firm 1 is done with stage RES (which may or may not be before firm 2 has finished stage DEV) it will choose $p_D = x$ or $p_D = y$.

For most of the paper (except in parts of Section 3), we will employ the following reduced form expression for the payoffs received by firms as a function of p_R. p_D, q_R, q_D . Payoffs will depend on the “points” accumulated by a firm. Firm 1’s points are as follows

$$p_R < q_R \Rightarrow 2 \text{ points},\ p_R = q_R \Rightarrow 1 \text{ point},\ p_R > q_R \Rightarrow 0 \text{ points}$$

$$p_D < q_D \Rightarrow 2 \text{ points},\ p_D = q_D \Rightarrow 1 \text{ point},\ p_D > q_D \Rightarrow 0 \text{ points}$$

Firm 2’s points are determined analogously. Thus, a firm has more points the more aggressive is its target relative to that of its rival. The notion that more aggressive targets lead to higher profits is reflected in the assumption that a higher point score leads to greater profits. Monopoly profits do not depend on the relative merits of the products, since consumers do not have a choice between products when only one firm has finished R&D. Accordingly, monopoly profits are denoted $M(p_R, p_D)$ or $M(q_R, q_D)$ and not as a function of points. We abstract from 'dropout behaviour' and assume that both firms will eventually finish the multiple stages. Since we are concerned with the utilisation of given R&D resources, flow

[1] The hazard rate function should be interpreted thus. h(t) denotes the instantaneous conditional probability of completion of a stage that has lasted until time t. Formally, h(t)/f(t)/(1-F(t)) for any continuous random variable having distribution F(t) and density f(t). For an exponentially distributed random variable with parameter m>0,

$$f(t)=me^{-mt},$$

and the hazard rate, h(t)=m, is a constant for all t.

costs of R&D are fixed regardless of the level of aggressiveness chosen. We do not explicitly include this flow cost term in our expressions. Firms are risk neutral and maximise discounted expected profits, with r denoting the discount rate. We will look for a subgame-perfect equilibrium in this game (Fudenberg and Tirole, 1989). The solution proceeds by backwards induction in three steps, which we turn to after discussing the notation that we will employ.

Notation: For algebraic convenience, we will employ the notation described here. Let W(i) be the expected payoff for a firm when it has finished stage DEV with i points while its rival is still in its DEV stage (having completed its RES stage). Then, for firm 1, we would write

$$rW(i) = M(p_R, p_D) + q_D\,(D(i) - W(i)) \Rightarrow W(i) = \frac{M(p_D, P_D) + q_D D(i)}{(r + q_D)}$$

where $M(p_R, p_D)$, D(i) denote monopoly, duopoly profits

Analogously, when W(i) refers to a payoff for firm 2, it is given by

$$[M(q_R, q_D) + p_D D(i)]/(r + p_D).$$

Similarly, let L(i) be the expected payoff for a firm when it is still in stage DEV, which it will finish with i points, when its rival has already finished its DEV stage. Then, for firm 1, we would write

$$rL(i) = p_D(D(i)\text{-}L(i)) \Rightarrow L(i) = \frac{p_D D(i)}{(r + p_D)}$$

Analogously, when L(i) refers to a payoff for firm 2, it is given by $[q_D(D(i)]/(r + q_D)$. We assume W'(i) > 0, L'(i) > 0. Occasionally, we will write the monopoly profit term without its arguments for convenience. The reduced form expressions that we analyse are in terms of W(i) and L(i). However, we will reinterpret the propositions in terms of monopoly and duopoly profit terms. This translation between the W(i), L(i) notation and the monopoly, duopoly notation will be accomplished using the following lemmas.

For convenience, define the quantity T(q) = q/(r + q-1).

Lemma 1: For any value of points i , if monopoly profits with less aggressive actions in each of the RES and DEV stages exceeds T(y)D(i), then (a) M > W(i) for all x, y combinations that map into i. (b) W(i) > L(i). Further, the greater is M(x,x)/D(i), the greater are the ratios M(x,x)/W(i) and W(i)/L(i).

Proof: see appendix 1.

***Lemma 2*:** The following conditions

$$yD\,(i+1) > xD(i), \qquad yM(y,v) > xM(x, v) \quad \text{where } v = x,y$$

are sufficient to ensure that

$$yL(i + 1) > xL(i), \qquad yW(i + 1) > xW(i)$$

Proof: follows directly from the expressions

***Lemma 3*:** Even when M is greater than W(i), L(i) in the sense of Lemma 1, there exist ranges of values of x and y such that the following conditions are compatible

$$xW(i) < yW(i + 1), \qquad xL(i) < yL(i + 1), \qquad xM(.,x) > yM(. , y)$$

Proof: see appendix 1

We turn now to the backwards induction. A firm's strategy consists of its choices; of hazard rate (and hence level of aggressiveness) in each of the two stages. Without loss of generality, we assume that firm 2 is the first firm to finish the RES stage. Then the game can be solved in three parts. Part 3, which we analyse first, corresponds to the situation where both firms have had their RES stage choices, firm 2 has made its DEV stage choice, and firm 1 has to make its DEV stage choice. Part 2 corresponds to the part of the game where firm 1 is still in its RES stage, while firm 2 has just finished its RES stage and has to make its DEV stage choice. Part 2 is solved taking as given the results of the Part 3 solution by backwards induction. Finally, Part 1 refers to the part of the game where both firms simultaneously make their RES stage choices. We solve Part 1 for specific cases that we identify from the analysis of Parts 2 and 3.

Part 3

Here firm 1 chooses its DEV stage strategy, p_D, given its own previous choice of p_R, firm 2's choice of q_R and q_D , and information regarding whether firm 2 has completed its DEV stage or not. (Recall that we assumed that firm 2 was the first to finish the RES stage. Consequently, when firm 1 has to choose p_D, we know that firm 2 has already finished its RES stage and has chosen q_D.) There are two cases to consider depending on whether firm 2 has completed its DEV stage or not.

First consider the case where firm 2 has not yet competed its DEV stage. Then firm 1's expected payoff $S_{1,a}(i)$ is given by

$$rS_{1,a}(i) = p_D\,(W(i) - S_{1,a}(i)) + q_D(L(i) - S_{1,a}(i))$$

$$\Rightarrow S_{1,a}(i) = \frac{p_D W(i) + q_D L(i)}{(r + p_D + q_D)}$$

Now, firm 1 chooses p_D = x or p_D= y to maximize this expression for various realizations of p_R, q_R, q_D (i.e. all the previous decisions made by the firms). As an example, consider firm 1's best response to the triple (p_R, q_R, q_D) = (y, y, x). Then for p_D = y to be preferred ($\succ$) to p_D= x, we need $S_{1,a}(i+1)$ when p_D = y to be greater than $S_{1,a}(i)$ when p_D= x. So

$$y \succ x \Rightarrow \frac{yW(3) + yL(3)}{(r + 2y)} > \frac{xW(2) + yL(2)}{(r + x + y)}$$

Here, on the LHS, $p_R = q_R$, $p_D < q_D$ implies that firm 1 has 3 points. On the RHS, $p_R = q_R$ **,** $p_D = q_D$ implies that firm 1 has 2 points. A sufficient condition for this inequality to hold is that $xW(2) \le yW(3)$. More generally,

$$xW(i) \le yW(i+1) \ \forall\, i \Rightarrow y \succ x \ \ \forall\, (p_R, q_R, q_D)$$

Now consider the case where firm 2 has completed its DEV stage. Then firm 1's expected payoff is given by $S_{1,b}(i)$ where

$$rS_{1,b}(i) = p_D\,(L(i) - S_{1,b}(i)) \Rightarrow S_{1,b}(i) = \frac{p_D L(i)}{(r + p_D)}$$

So for p_D = y to be preferred to p_D = x, we need

$$y \succ x \Rightarrow \frac{yL(i+1)}{(r+y)} > \frac{xL(i)}{(r+x)}$$

So

$$xL(i) \le yL(i+1) \ \forall\, i \Rightarrow y \succ x \ \ \forall\, (p_R, q_R).$$

In summary, when $xW(i) \le yW(i + 1)$, $xL(i) \le yL(I + 1)$ for all i, firm 1 will choose p_D = y regardless of whether firm 2 has completed its DEV stage or not.

Part 2

Prior to firm 1 choosing its DEV stage strategy , firm 2 is the first firm to finish the RES stage and has to choose q_D given (p_R, q_R) and given firm 1's actions as determined by conditions in Part 1 of the backward induction.

Let us define W_2 as firm 2's expected payoff if it finishes stage DEV before firm 1 finishes stage RES.

$$rW_2(i) = p_R(W(i)–W_2(i)) + M(q_R, q_D)$$

$$\Rightarrow W_2(i) = [M(q_R, q_D) + p_R\, W(i)]/(r + p_R)$$

Similarly, let L_2 be firm 2's expected payoff if it finishes stage DEV after firm 1 finishes stage RES. Then

$$rL_2(i) = q_D(W(i) - L_2(i) + p_D(L(i) - L_2(i))$$

$$\Rightarrow L_2(i) = \frac{q_D W(i) + p_D L(i)}{(r + p_D + q_D)}$$

Here W(i) and L(i) are the quantities from Part 3 of the backward induction and refer to payoffs received by firm 2. Recall from Part 3 that $xW(i) \leq yW(i+1)$, $xL(i) \leq yL(i+1)$ for all i implies that firm 1 chooses $p_D = y$ regardless of whether firm 1 chooses its DEV stage strategy before or after firm 2 finishes its DEV stage. So firm 1's action is the same in the two cases whose payoffs (for firm 2) are represented by $W_2(i)$ and $L_2(i)$. Thus both $W_2(i)$ and $L_2(i)$ have the same index as argument.

Now, firm 2 chooses q_D to maximize $S_2(i)$ where

$$rS_2(i) = q_D(W_2(i) - S_2(i)) + p_D(L_2(i) - S_2(i))$$

$$\Rightarrow S_2(i) = \frac{q_D W_2(i) + p_R L_2(i)}{(r + q_D + p_R)}$$

Substituting $W_2(i)$ and $L_2(i)$ into S_2, we have

$$S_2(i) = q_D = [\frac{M(q_R, q_D) + p_R W(i)}{(r + p_R)}] + p_R[\frac{q_D W(i) + p_D L(i)}{(r + p_D + q_D)}] \Big/ (r + q_D + p_R)$$

Let $k_1 = 2r + p_R + p_D$ and $k_2 = r + p_D$. Then $S_2(i)$ can be rewritten as

$$\frac{p_R k_1 W(i) q_D + p_R W(i) q_D^2 + k_2 M(q_R, q_D) q_D + M(q_R, q_D) q_D^2 + L(i) p_D p_R (r + p_R)}{(r + p_R)(r + p_D + q_D)(r + p_R + q_D)}$$

Let us compare the magnitudes that the various terms in this expression for $S_2(i)$ take on for firm 2' two possible choices of q_D. We see that the denominator term increases if $q_D = x$ is chosen instead of $q_D = y$. The L term in the numerator takes on the value $p_D p_R (r+p_R) L(i)$ when $q_D = x$ and $p_D p_R (r + p_R) L(i+1)$ when $q_D = y$, and is therefore larger when $q_D = y$. Thus both the L term in the numerator and the reciprocal of the denominator are larger when $q_D = y$ than when $q_D = x$. We need the following assumptions to ensure that the other numerator terms are at least as large when $q_D = y$ than when $q_D = x$.

$$x^2 W(i) \leq y^2 W(i+1)\ \forall\ i.\ x^2 M(q_R, x) \leq y^2 M(q_R, y) \text{ for } q_R = x, y \qquad (*)$$

Recall from the Part 3 analysis above that $xW(i) \leq yW(i+1)$, $xL(i) \leq yL(i+1)$ for all i implied that firm 1 would set $p_D = y$ regardless of firm 2's choice of q_D. When these conditions hold, the inequalities in (*) above are sufficient to ensure that $q_D = y$ is a dominant strategy for firm 2.

We now want to establish sufficient conditions for firm 2 to choose $q_D = x$. For x to be chosen, S_2 with $q_D = x$ should be greater than S_2 when $q_D = y$. If T_{c1} is a term that includes all non-monopoly terms in the inequality that expresses this comparison, then the relevant condition for x to be chosen is

$$\frac{xM(.,x)}{(r+p_R+x)} + T_{c1} > \frac{yM(.,y)}{(r+p_R+y)}$$

This can be re-expressed as

Condition C1:

$$\theta < \frac{x}{y}\frac{(r+p_R+y)}{(r+p_R+x)} + \frac{T_{c1}}{M(.,x)}(1+\frac{r+p_R}{y}), \text{where } \theta = \frac{M(.,y)}{M(.,x)}$$

This condition is more likely to hold the closer θ is to l. Note that T_{c1} may be positive or negative. However, the second term on the right hand side of the inequality has less of an effect the greater is the monopoly profit term, M(., x), relative to the terms in T_{c1},. Note also that the first term on the right hand side is greater than 1.

The above analysis of Parts 3 and 2 of the backwards induction can be summarised as follows:

Part 3: (Action when a firm is the last firm to finish RES stage and is the last to choose its DEV stage strategy)

- Sufficient conditions to choose y:

$$xW(i) \leq yW(i + l), xL(i) \leq yL\,(i + 1)\ \forall\ i$$

Part 2: (Action when a firm is the first firm to finish the RES stage and is the first to choose its DEV stage strategy)

- Sufficient conditions to choose y:

$$x^2W(i) \leq y^2W(i + 1)\ \forall\ i,\ x^2M(v, x) \leq y^2M(v, y)\ v = x,y$$

- Sufficient conditions to choose x:

Condition C1 holds.

In what follows, let us distinguish two cases as follows.

<u>Case I.</u>

$$x^2W(i) \leq y^2W(I + 1)\ xL(i) \leq yL(I + 1)\ \forall\ i,\ x^2\,M(v, x) \leq y^2M(v, y)\ ,v = x, y$$

In Case 1, the conditions specified are sufficient to ensure that firms always choose y when they complete the RES stage. Lemma 2 suggests that these Case I conditions will hold when both duopoly and monopoly profit terms are sufficiently larger under the more aggressive strategy choice (than under the less aggressive choice) to overcome the effect of the longer expected time to completion under the more aggressive choice.

Case II:

$$xW(i) \leq yW(i + 1),\ xL(i) \leq yL(i + 1),\ xM(v, x) > yM(v, y),\ v = x, y$$

and condition C1 holds.

In Case II, the conditions specified are sufficient to ensure that the first firm to finish the RES stage chooses x, while the second firm to finish the RES stage chooses y. Lemma 3 shows that ranges of x and y exist such that these conditions are satisfied.

Part 1

In the game's initial stage, both firms simultaneously choose hazard rates for the RES stage. After introducing some preliminaries, we shall solve separately for the two cases identified by the backwards induction thus far.

Let V_1 denote firm 1's expected payoff if it finishes the RES stage first and subsequent play is optimal, and let $\overline{V_1}$, denote its expected payoff if it finishes the RES stage second and subsequent play is optimal. Then firm 1's overall expected payoff, P_1, is given by

$$rP_1 = p_R(V_1 - P_1) + q_R(\overline{V_1} - P_1) \Rightarrow P_1 = \frac{p_R V_1 + q_R \overline{V_1}}{(r + p_R + q_R)}$$

Since we have already derived an expression for firm 2's expected payoff if it finished RES first (called S_2, in Part 2 of the backwards induction), we can interchange p's and q's to get an expression for 1's expected payoff if it finishes RES first.

$$V_1 = p_D\left[\frac{M(p_R p_D) + q_R W(i)}{(r + q_R)}\right] + q_R\left[\frac{p_D W(j) + q_D L(j)}{(r + p_D + q_D)}\right] \Big/ (r + p_D + q_R)$$

Firm 1 will maximise P_1 with p_D and q_D set by backwards induction. Thus the index i associated with the p_D term in the numerator of V_1 should assume that firm 1 finished the RES stage first, while the corresponding index j for the q_R term should assume that firm 1 finished the RES stage second. However, for exactly the same reason that W_2 and L_2 in Part 2 of the backwards induction shared the same index as their argument, we have i = j here. In all subsequent references to V_1, the common index will be denoted by i. Also

$$r\overline{V}_1 = p_R(A - \overline{V}_1) + q_D(\zeta - \overline{V}_1)$$

where

A = firm 1's payoff when it has finished stage RES but rival hasn't finished stage DEV

ζ = firm 1's payoff when it hasn't finished stage RES but rival has finished stage DEV

Then the terms in the expression can be calculated as follows

$$r\zeta = p_R(B-\zeta) \Rightarrow \zeta = \frac{p_R B}{(r + p_R)}$$

where B = firm 1's payoff when it has finished stage RES ,rival has finished stage DEV

Also

$$A = \frac{p_D W + q_D L}{(r + p_D + q_D)}, B = \frac{p_D L}{(r + p_D)}.$$

This leads to the following expression

$$\overline{V_1} = \frac{p_R A + q_D [\frac{p_R B}{(r+p_R)}]}{(r + p_R + q_D)}$$

where A, B are as above

Case I:

Recall that this is the case where the first firm to finish the RES stage chooses y, as does the second. We want to look for Firm 1's best response to the choices of q_R by firm 2. First suppose that q_R = y. When p_R = x, P_1 is given by

$$\frac{xV_1(i) + y\overline{V_1}(j)}{(r + x + y)}$$

Here $V_1(i)$ is what firm 1 gets if it finishes the RES stage first. Then p_R = x, q_R = y, p_D = y, q_D = y, and the index i = 1. Similarly, $V_1(j)$ is what firm 1 gets if it finishes the RES stage last. Here also p_R = x, q_R = y, q_D = y, q_D = y, and the index j = 1. Similarly, when p_R = y, P_1 is given by

$$\frac{yV_1(k) + y\overline{V_1}(1)}{(r + 2y)}$$

and the indices are k = 2 and 1= 2. So, when q_R = y,

$$\frac{yV_1(2) + y\overline{V_1}(2)}{(r + 2y)} > \frac{xV_1(1) + y\overline{V_1}(1)}{(r + x + y)} \Rightarrow y \Rightarrow y \succ x$$

So, firm 1 chooses p_R = y when q_R = y if $yV_1(2) \geq xV_1(1)$.

Now, consider the case where q_R = x. By proceeding as above, we have

$$\frac{yV_1(3) + y\overline{V}_1(3)}{(r + 2y)} > \frac{xV_1(2) + y\overline{V}_1(2)}{(r + x + y)} \Rightarrow y \succ x$$

So firm 1 chooses p_R = y when q_R = x if $yV_1(3)\,) \geq xV_1(2)$.

Combining these results, we can say that $xV_1(i) \leq yV_1(i + 1)$ for i=1,2 is a sufficient condition for y to be a dominant strategy for firm 1 in stage RES.

Let us check how this condition relates to the Case I conditions. When $p_D = q_D = y$ (as in Case I), for the case where $q_R = y$, we can rewrite $yV_1(i + 1) \geq xV_1(i)$, after some manipulation, as

$$y(r + 2y) [yM(y, y) - xM(x, y)] + y^2 (2r + 3y) [yW(i + 1) - xW(i)]$$

$$+ y^2 (r + y) [yL(i + 1) - xL(i)] \geq 0 \qquad (**)$$

Now, recall that under Case I assumptions, $yW(I + 1) \geq xW(i)$, $yL(I + 1) \geq xL(i)$ for all i. So the only additional assumption required to make (**) hold is that $yM(y, y) \geq xM(x, y)$. We can also verify that no additional assumptions are required to make $yV_1(i + 1) \geq xV_1(i)$ hold when $q_R = x$. So we have the following proposition.

Proposition 1: When the following conditions hold

$$x^2W(i) \leq y^2W(i + 1),\ xL(i)\ ) \leq yL(i + 1)\ \forall i,$$

$$x^2M(v, x) \leq y^2M(v, y),\ v = x, y\ ,\ xM(x, y) \leq yM(y, y)$$

both firms choosing aggressive strategies (y) in each of the RES and DEV stages constitutes a subgame perfect equilibrium.

The conditions in this proposition can be reinterpreted using Lemma 2. They are the same as saying that both monopoly and duopoly profits are sufficiently larger when the aggressive[2] option is chosen to outweigh the cost of achieving these higher profits at a later (expected) date. So this proposition simply establishes that if the advantages to introducing a better product are high enough to outweigh the benefits of introducing the product sooner, then both firms will opt for the aggressive targets in both stages.

Case II:

Recall that this is the case where the first firm to finish the RES stage chooses its DEV stage strategy to be x, while the second chooses y. We can now rule out an equilibrium where one firm does x in the RES stage, and the other does y in the RES stage, when the condition C1 (under which Case II arises) holds. Let us develop a condition as we did condition C1 in Part 2. Consider firm 1's response to $q_R = x$. We have to compare the payoff P_1 when $p_R = y$ with that when $p_R = x$. Let T_{c2} be the term that includes all non-monopoly terms in the inequality that expresses this comparison. Then the relevant condition for $p_R = y$ to be chosen in response to $q_R = x$ is

[2] To see this, note that if T_{C1} and T_{C2} are small enough so that their terms can be ignored in conditions C1 and C3, then condition C1 implies condition C3.

$$\frac{yM(y,x)}{(r+x+y)} > \frac{xM(x,x)}{(r+2x)} + T_{C2}$$

This can be restated as

Condition C2:

$$\theta > \frac{x}{y}\left(\frac{r+x+y}{r+2x}\right) + \frac{T_{C2}}{M(x,x)}\left(1+\frac{r+x}{y}\right), where\, \theta = \frac{M(y,x)}{M(x,x)}$$

While T_{C2} may be positive or negative, the T_{C2} term in the condition C2 inequality has less of an effect the greater is the monopoly profit term, M(x,x), relative to the terms in T_{C2}. Note also that the first term (FT) on the right hand side is greater than 1. Then the condition C2 inequality can be approximated by $\theta >$ FT. This is less likely to hold the closer θ is to 1. But θ was required to be close to 1 to make condition C1 hold (which is the condition under which we are in Case II). So conditions C1 and C2 cannot hold simultaneously.

Now define Condition C3 to be condition C2 with a reversal in the inequality. Then condition C3 is likely to hold under the same conditions that condition C1 held.[2] When condition C3 holds, firm 1 will choose $p_R = x$ in response to firm 2's choice of $q_R = x$. We can now state the following proposition.

Proposition 2: When Conditions C1 and C3 hold and

$$xW(i) \leq yW(i+1),\ xL(i) \leq yL(i+1)\ \forall\ i$$

then both firms choosing x in the RES stage, the first firm to finish RES choosing x in the DEV stage, and the second to finish RES choosing y in the DEV stage, constitute a subgame perfect equilibrium.

Lemma 3 established that the conditions under which this result holds are indeed met for some parameter values[3]. The intuition behind this proposition is as follows. When the monopoly profits are large enough and do not vary much with the product characteristics, both firms choose the strategy that will enable them to introduce the product quickly. However, following the introduction of the product by one firm (or even the getting ahead of one firm), the rival finds it advantageous to choose the more aggressive (& slower) option in the latter stage since the duopoly profits do vary enough with the quality of the product to make the delayed introduction worthwhile. An example of such a situation is as follows. Demand inelasticity implies that aggressive cost reduction (and associated long product development times) is not optimal. Bertrand competition implies that the only way to earn any profits when there is a rival is to have a product that is superior on some stage characteristic. So both firms will start by trying to become the monopolist with the less aggressive strategy, but once one firm has introduced the product, the laggard will prefer the more aggressive strategy.[3]

[3] Both Lemma 1 and Condition C1 hold if M(. ,x) and M(.,y) are close enough such that xM(.,x)>yM(.,y)

In these two results, symmetric firms choose identically in the RES stage. Depending on whether monopoly profits are large enough or not, they will either choose the quicker strategy or the more aggressive (slower) strategy. Whereas in the situation analyzed in Proposition 1, firms are aggressive in the DEV stage regardless of the order in which they finished the RES stage, in the Proposition 2 situation, whether a firm is aggressive or not in the DEV stage depends on whether it finishes the RES stage last or first[4]. The underlying notion is that a firm that gets ahead and closer to the finish line makes different choices from a firm that falls behind. We shall return to develop this notion further in the context of multi-stage races.

3. Models for Asymmetric Firms

In this section, we modify the analysis of the two-firm two-stage game in Section 2 to consider two kinds of asymmetries between firms that our empirical work has suggested is important. The notation used here parallels that used above.

Of the two cases considered in Section 2 ,Case II is more appropriate to the industry setting in which empirical work was focussed on because of two characteristics of the mainframe (server)computer industry. Historically most mainframe sales have been to large institutions whose sole criterion is typically to buy the mainframe with the highest available performance. New generation mainframes have typically surpassed previous generations' performance. A firm that finds itself the first to introduce a new generation mainframe will be able to sell its product regardless of the exact performance that its new generation mainframe attains. In other words, monopoly profits (where a monopolist should be thought of as the sole producer of a new generation mainframe) with less aggressive characteristics are not very different from monopoly profits with more aggressive characteristics. In our model, this is a factor that increases the likelihood of condition C1 holding (part of the Case II assumptions). The second industry characteristic is that of intense competition when more than one producer has a product on the market. This, together with the fact that the mainframe customer base has remained fairly stable, translates into monopoly profits being considerably greater than duopoly profits. Lemma 1 suggests that this is another factor that increases the likelihood of condition C1 holding. The remaining Case II assumptions simply say, by Lemma 2, that when there is duopoly competition, it pays to choose the more aggressive targets.

For the analysis of the incumbent-entrant interaction, we shall make the assumptions under which Case II holds. For the analysis of the interaction between firms with differing endowments, we shall make suitably modified versions of the assumptions under which Case II holds.

[4] The result depends upon homogeneity of consumer preferences. We can construct an example of the following form to show this.Suppose some consumers prefer high performance products while others put greater value on lower price. Then, when there is strong product market competition, a segmented equilibrium can result in that one firm chooses an aggressive option in the first research stage, while the rival chooses aggressively in the second stage but not in the first stage.In equilibrium, one firm serves the high performance preferring consumers, while the other serves te consumers who value lower price.

3. 1. Incumbent-Entrant Asymmetry

Let firm 1 be an incumbent firm that chooses hazard rates p_R, p_D, in the RES and DEV stages respectively, and let firm 2 be an entrant that chooses hazard rates q_R, q_D similarly. The incumbent earns flow rents I from its existing product, which cease to accrue to it once the new product is introduced by either firm. We will analyse this incumbent-entrant interaction by backwards induction in the same manner as in Section 2.

The conditions governing the actions of the entrant firm are the same as those discussed in the symmetric case in Section 2. Following the discussion at the beginning of this section, let us assume that the sufficient conditions hold under which the entrant chooses $q_D = y$ if it is the last to finish the RES stage and $q_D = x$ if it is the first firm to finish the RES stage (Case II of Section 2). Recall that these were derived in Parts 3 and 2 of the backwards induction in Section 2. We need $xW(i) \leq yW(i+1), xL(i) \leq yL(i+1)$ for all i, and Condition C1 should hold. Now we turn to analyzing the incumbent's choice of actions.

Part 3

Suppose the incumbent is the last to finish the RES stage. If, at the time of its choosing p_D, the entrant has not finished the DEV stage, then the incumbent earns $S^I_{1,a}(i)$ where

$$S_{1,a}^{I}(i) = \frac{p_D W(i) + q_D L(i) + I}{(r + p_D + q_D)}$$

For $p_D = y$ to be preferred to $p_D = x$, the following inequality must hold

$$\frac{yW(i+1) + q_D L(i+1) + I}{(r + y + q_D)} > \frac{xW(i) + q_D L(i) + I}{(r + x + q_D)}$$

This can be rewritten as

$$I(x - y) + (r + q_D)\,[yW(i + 1) - xW(i)] + xy\,[W(i + 1) - W(i)] +$$

$$q_D(r + q_D)[L(i + 1) - L(i)] + q_D\,[xL(i + 1) - yL(i)] > 0$$

When $yW(I + 1) \geq xW(i)$ for all i, this inequality bolds and the incumbent chooses $p_D = y$ regardless of the entrant's choice of q_D.

If, at the time of the incumbent's choice of p_D, the entrant has finished the DEV stage, then the incumbent earns $S_{1,b}^{I}(i)$

$$S_{1,b}^{I}(i) = \frac{p_D L(i)}{(r + p_D)}$$

Here $p_D = y$ is preferred to $p_D = x$ if $yL(I + 1) \geq xL(i)$ for all i.

Part 2

If the incumbent is the first firm to finish the RES stage, calculations similar to those in the symmetric firm analysis (where we developed an expression for the term $S_2(i)$) show that it gets $S^I_2(i)$ where

$$S^I_2(i) = S_2^1(i) = p_D[\frac{M + q_R W(i)}{(r + q_R)}] + q_R[\frac{p_D W(i) + q_D L(i) + I}{(r + p_D + q_D)}] + I \Big/ (r + p_D + q_R)$$

We can establish conditions under which the incumbent chooses $p_D = y$ as follows. Let T_{C4} include all non flow-profit (I) terms in the inequality that compares S^I_2 when $p_D = y$ with S^I_2 when $p_D = x$. Then the relevant condition for $p_D = y$ to be chosen is

Condition C4:

$$E + \frac{T_{c4}}{I} > F$$

where

$$E = \frac{(r + q_R + q_D + y)}{(r + y + q_R)(r + y + q_D)}, F = \frac{(r + q_R + q_D + x)}{(r + x + q_R)(r + x + q_D)}$$

Here the q_D term is the same in E and F because the entrant's DEV stage action, when it is the second to finish the RES stage, does not depend on the incumbent's choice of p_D. This follows from the assumptions made at the beginning of the backwards induction.

In condition C4, the non T_{C4} terms are of the form

$$T^I = \frac{k + p_D}{(\alpha + p_D)(\beta + p_D)}, where \;\; k = r + q_R + q_D, \alpha = r + q_D, \beta = r + q_R$$

Then

$$\text{sign } (\frac{\partial T^I}{\partial p_D}) = sign((\alpha + p_D)(\beta + p_D) - (k + p_D)(\alpha + \beta + 2p_D)) < 0, \Rightarrow E > F$$

It follows that condition C4 will hold when I is large enough relative to the terms in T_{C4}, regardless of the sign of T_{C4}. Then the incumbent chooses $p_D = y$ when it is the first to finish the RES stage regardless of the entrant's choice of q_D.

Part 1

Now we discuss the incumbent's choice of p_R in the RES stage. Let V^I denote the incumbent's payoff if it finishes the RES stage before the entrant, and subsequent play is optimal, and let 6^I denote the incumbent's payoff if it finishes the RES stage after the entrant, and subsequent play is optimal. Let P^I denote the incumbent's payoff when it has to choose its strategy p_R for the RES stage. Then proceeding as in Part 1 of the backwards induction in Section 2, we have

$$p^I(i) = \frac{p_R V^I(i) + q_R \bar{V}^I(i) + I}{(r + p_R + q_R)}$$

Note that $V^I(i)$ and $6^I(i)$ are the analogues of $V_I(i)$ and $6_I(i)$ in Part 1 of the earlier backwards induction. We can show that, when the I term in the expressions for $V^I(i)$ and $6^I(i)$ is large relative to the other terms, then we can approximate for $V^I(i)$ and $6^I(i)$ by the following

$$V^I(i) = I\,\frac{(r + p_D + q_R + q_D)}{(r + p_D + q_D)(r + p_D + q_R)}\,\forall i, \bar{V}^I(i) = I\,\frac{(r + p_R + p_D + q_D)}{(r + p_D + q_D)(r + p_R + q_D)}\,\forall i$$

Since $V^I(i)$ is what the incumbent gets when it finishes the RES stage first, plug in $p_D = y$, $q_D = y$ into its expression. Similarly, since $6^I(i)$ is what the incumbent gets when it finishes the RES stage second, plug in $p_D = y$, $q_D = x$ into its expression. Then we have

$$\mathbf{V^I(i) = I}\,\frac{(r + 2y + q_R)}{(r + 2y)(r + y + q_R)}\,\forall i, \bar{V}^I(i) = I\,\frac{(r + x + y + p_R)}{(r + x + y)(r + x + p_R)}\,\forall i$$

Substitute these terms into the expression for P^I. Substitute also $q_R = x$ to see what the incumbent's best response is to the entrant choosing the less aggressive strategy in the RES stage. Then we have

$$P^I = I = \frac{\left(\frac{p_R(r + 2y + x)}{(r + 2y)(r + y + x)} + \frac{x(r + x + y + p_R)}{(r + x + y)(r + x + p_R)} + 1\right)}{(r + x + p_R)}$$

Numerical simulations show that the incumbent's best response to the entrant's choice of $q_R = x$ is "usually" to choose $p_R = y$. Further, by exactly the same reasoning as in Case II of the symmetric firm interaction in Section 2, Condítion C3 implies that $q_R = x$ is a dominant strategy for the entrant in the RES stage. The conditions that we have accumulated in establishing this equilibrium include $xW(i) \leq yW(i + I)$, $xL(i) \leq yL(i + 1)$ for all i, and that Conditions C1 and C3 should hold. But these are exactly the conditions in Proposition 2. So we can state the following proposition.

Proposition 3: *When Condition C4 holds and the conditions in Proposition 2 hold, then the following constitutes a subgame perfect equilibrium. The incumbent chooses y in each of the RES and DEV stages. The entrant chooses x in the RES stage, x in the DEV stage if it is the first to finish the RES stage, and y in the DEV stage if it is the second to finish the RES stage.*

The entrant behaves exactly as it did in the symmetric firm interaction.

When monopoly profits are large enough and do not vary much with the product characteristics, the entrant chooses the faster, less aggressive option in each stage as long as it has not fallen behind in the race. The incumbent's behavior is influenced by what the literature identifies as the “replacement effect" (sec, eg, Tirole (1988), Chapter 10). The conventional “replacement effect” says that, in an effort to maximize the discounted value of its existing profit stream, an incumbent monopolist invests less in R&D than an entrant, and thus expects to be replaced by the entrant (in the case where the innovation is drastic enough that the firm with the older technology would not find it profitable to compete with the newer technology). In our model, when the incumbent's flow profits are large enough to make condition C4 hold, this same replacement effect causes the incumbent to be replaced only temporarily (if the innovation is drastic). Subsequently it regains a dominant position in the market since it has a superior version of the new technology.

In the usual models, investing more simply shortens your expected time to discovery; what you discover is always the same (i.e. the patent value is not a function of the amount invested). The difference in our model arises because trying to accelerate discovery implies that the innovation will be inferior in some sense (i.e. the analog to the patent, the value of the innovation, is a function of the firm’s investment strategy). This result flows from the more realistic assumption that in developing products as complex as mainframe computers, the R&D resources at the firm's disposal are fixed. *The relevant question then is not how much you invest, but how you allocate what you do have at your disposal.*

Recall that our empirical analyses had shown a strong correlation between our measure of incumbency and the degree of aggressiveness of firms’ targets. This is in accord with what

Proposition 3 suggests. The incumbent always chooses the more aggressive targets, while the entrant chooses the less aggressive target in the RES stage. Further, the probability that the entrant chooses the less aggressive target in the DEV stage as well is $x/(x + y) > 0.5$.

3. 2. Resource Endowment Asymmetries

In the model setup in Section 2, we suggested that each stage involves a number of interdependent problem solving activities. We incorporated the idea of system complexity by thinking of each activity as being linked to every other activity in the stage. So a choice of a target set is simultaneously a choice of the number of links that are included within the target set. A target of *n* activities involves a maximum of $C_2 = n(n-1)/2$ links, in standard combinatorial notation. Let us assume that the extent to which a stage characteristic is developed is an increasing function of the maximum number of links in the target set.

Recall that the larger the number of activities in the targeted set, the smaller will be the resource amount allocated to each activity. Since it is the resources allocated per activity that determines the hazard rate, a larger number of activities in the target set also implies a greater expected time to completion of the activities in that stage. Note also that, for a given choice of hazard rate, the target set is larger the greater is the firm's resource endowment.

Formally, let $s(B, h)$, $s_B > 0$, $s_h < 0$, denote the extent to which a stage characteristic is developed when B is the stage specific resource endowment and h the choice of the stage hazard rate. In this section, we will sometimes refer to monopoly profits as being a function of s(B,h). This simply makes explicit the earlier implicit assumption that monopoly profit is a function of the resource endowment B, in addition to being a function of the chosen hazard rate.

Our model of stage specific problem solving activities leads us to the system complexity assumption (SCA) below. A given resource increment allows the larger target set to expand more than the smaller one The maximum number of links in the target set also increascs as the target set gets larger. The SCA says that this increase in the number of links, as a proportion of the number of links in an existing target set, is greater the larger is the existing target set.

System Complexity Assumption (SCA):

$$\frac{d}{dh}\left[\frac{s_B(B,h)}{s(B,h)}\right] \langle \; 0$$

For ease of exposition, the following lemma specializes to particular reduced forms for monopoly profit. Later we suggest that this restriction to specific forms is more severe than necessary. Also, for notational convenience, we will sometimes refer to the monopoly profit function as though it depends only on the choice of hazard rate in one stage. This is not crucial to the analysis in any way.

Lemma 4: When the SCA holds, a range of reduced form monopoly payoff functions of the form M = Ks (B, h), K = constant, q is any power, satisfy

$$\frac{\partial}{\partial B}\left[\frac{M(s,(B,y)}{M(s(B,x)}\right] > 0$$

Proof: see appendix 1.

The SCA, together with the fact that $s_B > 0$, $s_h < 0$, also implies that $s_{Bh} < 0$. Therefore,

$$s_B(B, y) > s_B(B, x), \text{ or}$$

$$\frac{d}{dB}[s,(B,y)-s(B,x)] \; > \; 0$$

So, the increase in the extent to which a stage characteristic is developed, as we go from a higher to a lower hazard rate, is greater for a firm that has a greater resource endowment. Furthermore, for any given choice of hazard rate, the resource rich firm can undertake more activities in any stage. To capture these two effects of resource asymmetry, the "points" system used in the symmetric interaction is revised as follows.

	Resource Poor	**Resource Rich**
$p_R < q_R$	2	5
$p_R = q_R$	1	3
$p_R > p_R$	0	1

The points allocated to the resource poor firm are the same as in the symmetric interaction. Proceeding by backwards induction as before, we can summarize sufficient conditions for the resource rich firm's choices in Parts 3 and 2 as below (the conditions for the resource poor firm are as in Section 2).

Part 3: (Action when the resource rich firm is the last firm to finish the RES stage and is the last to choose its DEV stage strategy)

- Sufficient conditions to choose y:

$$xW(i) \leq yW(i+2),\ xL(i) \leq yL(i+2)\ \forall\ i$$

Part 2: (Action when the resource rich firm is the first firm to finish the RES stage and is the first to choose its DEV stage strategy)

- Sufficient conditions to choose y:

$$x^2W(i) \leq y^2W(i+2)\ \forall\ i,\ x^2M(v,x) \leq y^2M(v,y),\ v = x,y$$

- Sufficient conditions to choose x:

Condition C1 holds.

We can consider cases analogous to Cases I and II in Section 2. Proceeding as in the previous section, we can show that the results in Part 1 of the backward induction are also the same as in the symmetric firm game. Thus, in the analogue to Case I, both the resource rich firm and the resource poor firm choose the aggressive strategies in each of the RES and DEV stages. In the analogue to Case II, the first firm to finish the RES stage chooses the faster, less aggressive strategy, while the second to finish the RES stage chooses the more aggressive strategy. Thus, these two cases correspond to situations where the sufficient conditions specified are strong enough to obscure the effect of the resource endowment asymmetry.

A more interesting situation arises when the effect of the asymmetry is not obscured. We will now analyze this case by backwards induction in our usual three parts.

Part 3

If $xW(i) \leq yW(i + 2)$, $xL(i) \leq yL(i + 2)$ for all i, then the resource rich firm, will choose y as its DEV stage strategy if it is the second to finish the RES stage. Now consider conditions under which the resource poor firm chooses x in Part 3. Let us suppose that the resource poor firm is firm 1 and has to choose p_D for firm 2's choice of q_D. Then, paralleling the Section 2 discussion, there are two cases to consider depending on whether firm 2 has completed its DEV stage at the time of firm 1's choice of p_D or not. If firm 2 has not completed its DEV stage, then firm 1 will choose $p_D = x$ if

$$\frac{xW(i)+yL(i)}{(r+x+q_D)} \rangle \frac{yW(i+1)+yL(i+1)}{(r+y+q_D)}, \forall i$$

This can be rewritten as

$$\text{xW(i)} \rangle \left(\frac{r+x+q_D}{r+y+q_D}\right) yW(i+1) + T_{cs,}\ \forall\ i$$

where Tc_5 denotes the terms in L. If firm 2 bas completed its DEV stage, then firm 1 will choose $p_D = x$ if

$$\frac{xL(i)}{(r+x)} \rangle \frac{yL(i+1)}{(r+y)},\ \forall\ i$$

Now let us collect all the conditions on the W and L terms to get

Condition C5:

$$\text{yW(I + 2)} \geq \text{xW(i)} > \left(\frac{r+x+q_D}{r+y+q_D}\right) yW(i+1)\ \ + T_{cs,}\ \forall\ i$$

$$\text{yL(i+2)} \geq \text{xL(i)} > \quad \left(\frac{r+x}{r+y}\right) yL(i+1), \quad \forall \quad i$$

If Tc_5 is small, this amounts to making the following assumption on the ranges in which xW(i) and xL(i) lie for all i

$$xW(i)\varepsilon[\sigma_w yW(i+1), yW(i+2)] xL(i)\varepsilon[\sigma_1 yL(i+1), yL(i+2)]\ \forall\ i$$
$$where\ \ \sigma_w = \left(\frac{r+k+x}{r+k+y}\right),\ \sigma_1 = \left(\frac{r+x}{r+y}\right)$$

So condition C5 simply collects assumptions under which the "points" assumption to choosing x or to choosing y are not extreme enough to mask the effects of the resource endowment asymmetries.

Part 2

In Part 2 of the backwards induction in Section 2, we showed that condition C1 was sufficient to ensure that firm 2 choose x to be its DEV stage strategy if it found that it was the first firm to finish the RES stage. We can similarly argue that if the inequality in condition C1 is reversed, then the firm finishing the RES stage first will choose y as its DEV stage strategy. To write this more formally, let us adopt the convention that $M^{rich}(p_R, p_D)$ and $M^{poor}(p_R, p_D)$ refer to the monopoly profits of the resource rich and the resource poor firm respectively for given choices of hazard rates p_R and p_D. Then we can write a modified condition $C1_{RA}$ (where the subscript “RA” is intended to denote the case of “resource asymmetries”)

Condition $C1_{RA}$

$$\theta^{poor} \langle \; \frac{x}{y}(\frac{r+p_R+y}{r+p_R+x}) + \frac{T_{c1}}{M^{poor}(.,x)}(1+\frac{r+p_R}{y}), \qquad \theta^{poor} = \frac{M^{poor.}(.,y)}{M^{poor.}(.,x)}$$

$$\frac{x}{y}(\frac{r+p_R+y}{r+p_R+x}) + \frac{T_{c1}}{M^{rich}(.,x)}(1+\frac{r+p_R}{y}), \;\langle \theta^{rich}, \quad \theta^{rich} =$$

$$\theta^{rich} = \frac{M^{rich(.}(.,y}{M^{rich(.}(.,x)})$$

By lemma 4, a small enough DEV stage resource endowment for the resource poor firm and a large enough DEV stage resource endowment for the rescource rich firm will ensure that this condition holds. Also, as in condition C1, T_{Cl} may be positive or negative. However, the larger are monopoly profits relative to T, the less is the effect of the Tc1 term. When condition $C1_{RA}$ holds, there exist resource endowment asymmetries such that the resource poor firm would choose x as its DEV stage strategy if it was the first to finish the RES stage, while the resource rich firm would choose y as its DEV stage strategy if it was the first to finish the RES stage (i.e. in Part 2 of the backwards induction). Note that the resource endowment asymmetry being discussed here refers to the endowment available to each firm in the DEV stage.

Summarizing Parts 3 and 2, when conditions $C1_{RA}$ and C5 hold, the resource rich firm will choose y as its DEV stage strategy, while the resource poor firm will choose x as its DEV stage strategy.

Part 1

Now we analyze the choices that each firm makes for its RES stage strategy (Part 1 of the backwards induction). As in the analysis of Part 1 in Seetion 2, we shall examine firm 1's decision, alternately adopting the perspective that the resource rich firm or the resource poor firm is firm 1. Let $P1^{rich}$ and $P1^{poor}$ refer to the expected payoffs of the resource rich and the resource poor firm respectively at the beginning of the RES stage (analogous to P_1 in Part 1 in Section 2).

First suppose that firm 1 is the resource poor firm. Then

$$P_1^{poor} = \frac{p_R p_D M^{poor}(p_R, p_D)}{(r+q_R)(r+p_R+q_R)(r+p_D+q_R)} + \; T$$

where T denotes the non monopoly terms in the expression. Since this is the resource poor firm and condition C5 holds, set p_D = x. Look at the best response to the resource rich firm’s choice of q_D = y. Then we can rewrite

$$P_1^{poor} = \frac{x}{(r+y)(r+x+y)} \frac{M^{poor}(p_R, x)}{(1+\frac{r+y}{p_R})} + T$$

We see that the resource poor firm will choose $p_R = x$ in stage RES, when the rich firm chooses $q_R = z$, if

$$\frac{M^{poor}(x,x)}{1+\frac{r+y}{x}} > \frac{M^{poor}(y,x)}{1+\frac{r+y}{y}} + T_{c6}$$

where T_{C6} collects the non-monopoly terms in the inequality. We can state this as

Condition C6:

$$\frac{M^{poor}(y,x)}{M^{poor}(x,x)} < \frac{1+\frac{r+y}{y}}{1+\frac{r+y}{x}} - \frac{T_{c6}}{M^{poor}(x,x)}\left(1+\frac{r+y}{y}\right)$$

By lemma 4, the left hand side of condition C6 will be small enough to make condition C6 hold if the poor firm has a small enough RES stage resource endowment and if the monopoly terms M^{poor} (x, x) is large enough relative to the terms in T_{C6}.

Now suppose that firm 1 is the resource rich firm. Proceeding exactly as above, we can show that it will choose $p_R = y$ in response to the resource poor firm's choice of $q_R = x$ if the following condition holds.

Condition C7:

$$\frac{M^{rich}(y,y)}{M^{rich}(x,x)} > \frac{1+\frac{r+x}{y}}{1+\frac{r+x}{x}} + \frac{T_{c7}}{M^{rich}(x,y)}\left(1+\frac{r+x}{y}\right)$$

By lemma 4, the left hand side of condition C7 will be large enough to make condition C7 hold if the rich firm has a large enough RES stage resource endowment and if the monopoly term M^{rich}(x, y) is large enough relative to the terms in T_{C7}.

Now we can state the following proposition.

Proposition 4. *If monopoly profits are of a form that satisfy lemma 4, and if conditions $C1_{RA}$, C5, C6 and C7 hold, then the resource rich firm chooses y in each of the RES and DEV stages, while the resource poor firm chooses x in each of the RES and DEV stages.*

Recall that conditions $C1_{RA}$, C6 and C7 are satisfied when there is sufficient resource asymmetry for the resource endowments in each of the RES and DEV stages. These conditions are all more likely to hold when the monopoly terms are larger than the non-monopoly terms in the relevant expressions. Lemma 3 allows us to say that this is the same as monopoly profits being significantly larger than duopoly profits. This condition is consistent with the environment in modern network industries (Gottinger,2002) . Condition C5 simply collects conditions under which the "points" obtained by choosing either x or y are not too extreme to obscure the effects of the resource asymmetries.

The restriction to monopoly functions of the type in Lemma 4 is more severe than necessary for the result. To see this consider the following. As a firm's stage resource endowment increases, it could use the additional resources to either choose more aggressive targets or to attempt to finish the stage quicker, or both. Proposition 4 seems to suggest that the rich firm will choose the more aggressive targets. Consider the case when a particular stage of the R&D process is directed toward cost reduction. In situations where demand is inelastic enough, there is less of an incentive for aggressive cost reduction, and we would expect firms to choose to finish the stages faster. In this regard, note that Lemma 4 holds for more general monopoly functions that satisfy

$$\frac{M_s(s(B,y))/M(s(B,y))}{M_s(s(B,x)/M(s(B,x))} \rangle \frac{s(B,x)}{s(B,y)}$$
$$\Rightarrow \frac{\partial}{\partial h}[s(B,h)\frac{M_s(s(B,h))}{M/(s(B,h))}] \ \langle \ 0$$

Now, s(B, h) falls as h rises (becomes less aggressive) and we can show that $M_s(s(B, h)) / M(s(B, h))$ has less effect as demand becomes more inelastic. Consequently, Lemma 4 will hold for reduced form monopoly expressions as demand becomes more inelastic. In situations of sufficient demand elasticity, Lemma 4 (and, consequently, Proposition 4) may not hold. Then resource rich firms may not choose to direct their additional resources toward more aggressive targets.

Empirical results show a weak correlation between aggressiveness and resource-richness (Gilbert, 1989). The theoretical discussion suggests two possible interpretations. (a) If, for example, the demand for personal computers displays different elasticities in different local markets (US, Japan, Europe)[5], then we might expect there to be only imperfect correlation between aggressiveness and resource-richness when projects from the different markets are grouped together as here. (b) If demand for personal computers is not inelastic enough, then we would expect resource rich firms to aim for both higher speed in R&D and greater aggressiveness. This could once again cause imperfect correlation of the kind we observe.

4. Multi-Stage Races

In section 2, we saw that, under some conditions, identical firms behave differently depending on whether they are the first or the second firm to finish the first stage of a two-stage race. The underlying notion is that getting ahead and closer to the finish line results

[5] This could be due to differences in the availability of substitute computing resources eg distributed computer networks.

in different choices (aggressive behaviour or quick behaviour) than falling behind. These results flowed from our introduction of system complexity and from our departure from the conventional patent-race framework which ignores the competition that occurs when two or more firms have access to a new technology. Then, in section 3, we demonstrated the relevance of this framework by showing how it casts some light on the data that we gathered from our study of product development in the mainframe computer industry. Now, we investigate the implications of this framework for multistage races and suggest that some of the existing results in the literature hold only under very specific assumptions when we move away from the patent-race framework.

All the existing literature on multi-stage races is in the patent race framework. Harris and Vickers (1987) show that the leader invests more than the follower in a multistage patent race scenario. Their result generalizes a similar result due to Grossman and Shapiro (1987) for two-stage games. In contrast, instead of analyzing aggregate resource allocation, we discuss how given resources are allocated. As in earlier parts of this paper, there is a tradeoff between being aggressive or being fast in each stage of a multi-stage race. Our focus will be on characterizing the differences in the expected payoff function of firms as they get ahead of their rivals (or fall behind) and closer to the finish line. We will speak of the monopoly (respectively, duopoly) term becoming more important in a payoff expression as the ratio of its coefficient to that of the duopoly (respectively, monopoly) term rises. In particular, we show the following.

Proposition 5*: The monopoly term in the expected payoff expression of the leading firm in a two-firm multi-stage race becomes progressively more important as it gets further ahead of its rival, providing the lead meets a minimum threshold. This threshold lead is smaller the closer is the lead firm to the finish fine. Conversely, the duopoly term in the expected payoff expression of the 1agging firm becomes more important as it falls further behind, subject to the same threshold lead considerations as the leading firm.*

Proof:

The detailed analytics are in appendix 2. Here we sketch the method used.

First, we derive an expression for the leading firm's payoff, when it bas finished all stages and is reaping monopoly profits, as a function of the lead it has over its rival. Let W_{n-1} be the payoff of the lead firm when it has finished all n stages, and the rival is in stage 1. Let $q_1, \dots , q_n$. be the lagging firm's choices of hazard rates for the n stages. Then

$$W_{n-1} = \frac{M}{(r+q_1)}[1+\frac{q_1}{(r+q_{1+1})}+\frac{q_1 q_{1+1}}{(r+q_{1+1})(r+q_{1+2})}+\dots+\frac{q_1 \dots q_{n-1}}{(r+q_{1+1})..(r+\mathrm{qn})}$$

$$+\frac{q_1 q_{1+1} \dots q_n}{(r+q_1)(r+q_{1+1})\dots(r+q_n)}D \quad for\ n-1 \rangle\ 0$$

$$W_o = \frac{M}{(r+q_n)}+\frac{q_n D}{(r+q_n)}, \quad for\ n-1\ =\ 0$$

We show that the coefficient on the monopoly term rises faster than that on the duopoly term as the lead increases.

Then, using this property of the coefficients, we consider the expression for the leading firm's payoff, $V_{n,n-j}$, as a function of its lead j, when it is in the last stage of the n stage race.

$$V_{n,n-j} = \frac{p_n W_j + q_{n-j} V_{n.n(j-1)}}{r + p_n + q_{n-j}}$$

Once again, we show that as long as the lead exceeds a threshold lead (which may be 0), the coefficient on the monopoly term rises faster than that on the duopoly term as the lead increases. This method of recursion, where the relationships on the coefficients when the lead firm is at stage S of the race is used to derive similar relationships when the lead firm is at stage S-1, yields our result.

The procedure is similar far the lagging firm. First we derive an expression far its payoff, as a function of how much it lags the rival, when the rival has finished all stages and the lagging firm is at stage *f*. Denote this by L_{n-f}. Then

$$L_{n-f} = \frac{p_f \cdots p_n}{(r + p_f)\ldots(r + p_n)} D$$

So the payoff when the firm is j + 1 stages bebind the leading firm at stage n is given by

$$V_{n-(j+1).n} = \frac{p_{n-(j+1)} V_{n-j+} q_n L_{j+1}}{r + p_{n(j+1)} + q_n}$$

Then we show that the duopoly coefficients rise faster than the monopoly ones as the lead increases, subject to the threshold lead considerations. The same is shown to be true recursively when the lead firm is at position n-1, n-2, etc.

Hence the result.

This characterization highlights two forces that influence a firm's choices in the various stages: proximity to the finish line and distance between the firms. The probability of reaping monopoly profits is higher the farther ahead a firm is of its rival, and even more so the closer the firm is to the finish line. If the lead firm is far from the finish line, even a sizeable lead may not translate into the dominance of the monopoly profit term, since there is plenty of time far the lead situation to be reversed and failure to finish first remains a probable outcome. In contrast, the probability that the lagging firm will get to be a monopolist becomes smaller as it falls behind the lead firm. This raises the following question. What kinds of actions cause a firm to get ahead? Intuitively, one would expect that a firm that is ahead of its rival at any time t, in the sense of having completed more stages by time t, is likely to bave chosen the faster, less aggressive strategy more often. We can construct numerical estimates of the probability that a leading firm is more likely to have chosen less aggressively (faster) to verify this intuition[6].

We have shown that the monopoly term is increasingly important to a firm as it gets ahead of its rival, and that the duopoly term is increasingly important to a firm that falls behind. Further simple calculations suggest that the firm that is ahead is likely to have made less aggressive choices than the firm that is behind in the race.

One question of interest is whether chance leads result in greater likelihood of increasing lead, or in more catchup behavior. The existing literature (Grossman and Shapiro (1987), Harris and Vickers (1987)) has suggested that a firm that surges ahead of its rival increases its investment in R&D and speeds up while a lagging firm reduces its investment in R&D and slows down. Consequently these papers suggest that the lead continues to increase. However, when duopoly competition and system complexity are accounted for, the speeding up of a leading firm occurs only under special circumstances. We suggest that the computer industry is one in which monopoly profits do not change substantially with increased aggressiveness, but duopoly profits do not change substantially with increased aggressiveness(Shapiro,1989) Then a firm getting far enough ahead such that the monopoly term dominates its payoff expression will always choose the fast strategy, while a firm that gets far enough behind will always choose the slow and aggressive approach. Then the lead is likely to continue to increase. If, on the other hand, both monopoly and duopoly profits increase substantially with increased aggressiveness then even large leads can vanish with significant probability.

[6] Suppose that the lagging firm has completed m stages of the race at some time t. *If we* assume that each firm chooses either the more aggressive action in each stage or the less aggressive option at each stage, then the number of stages that have been completed by a given time can be treated as the number of arrivals generated by a random variable with a Poisson distribution. Without this assumption, the stage completion times are exponentially distributed with different parameters, and the sum of the m stage completion times does not follow a standard distribution.
Let E be the event that the leading fim chose less aggressively than the lagging firm in each of the first m stages, and let E- be the complement event that the leading firm chose more aggressively than the lagging firm in each of the first m stages. Let A be the event lbat the lagging fim is at stage m at time t. Then we want to calculate

$$\text{Prob}(E|A) = \text{Pr ob}(A|E) \,.\, \text{Prob}(E) / [\, \text{Prob}(A|E) \,.\, \text{Prob}(E) + \text{Prob}(A|E^c) \,.\, \text{Prob}(E^c)$$

In this expression, if x denotes the less aggressive strategy and y the more aggressive one, then P(A |E) can be shown to be given by

$$P(A|E) = \sum_{1-m}^{2m-1} \binom{2m-1}{i} \left(\frac{x}{x+y}\right)^{i} \left(\frac{y}{x+y}\right)^{2m-1-i}$$

This is so because P(A |E) is the probability that the leading firm's Poisson process has completed more than the m stages than the lagging firm's Poisson process at time t, given that the leading firm chose less aggressively. Think of an event being the completion of a stage by either firm i.e. an arrival by either of the Poisson process then the probability that an event is due to the leading firm's process is x/(x + y). Then P(A | E), the probability that the leading firm’s process completes m stages before the lagging firm’s process, is the probability that, of the first 2m-1 events, at last m events are due to the leading firm’s process.

5. Conclusion

The main contribution of this paper has been to formalize the implications of system complexity for firms' R&D strategies. Product development typically proceeds in a sequence of stages, each of which determines the degree to which some subset of product characteristics is developed. In contrast to the existing literature's focus on how firms choose their aggregate level of investment in R&D, we discuss how *given* R&D rcsources are utilized in the different stages of the R&D process. Our model makes explicit how scarce R&D resources result in a tradeoff between the aggressiveness of the targeted objectives at each tage and the speed with which that stage is completed. Ex-ante differences between firms influence the way in which firms exercise their discretion in each stage.

Observations of R&D processes in the mainframe computer industry were used to motivate our framework. We found strong agreement between the theoretical prediction and the empirical results regarding the correlation between market incumbency and the aggressiveness of firms' targets. We showed that the fact that higher market share firms appear to choose more aggressive targets is an instance of the “replacement effect”[7]. Further, we found a weak correlation between a firm's R&D resource endowment and its targeted level of aggressiveness. Theory allows us to suggest some reasons why this might be so. For example, theory suggests that we could expect a strong correlation if the degree of demand elasticity was determined by the availability of substitute computing resources.

The second contribution of this paper was to make explicit the two forces that influence a firm's choices in the various stages of a multi-stage race: proximity to the finish line and distance between the firms. In doing so, we suggested two things. First, the existing literature, by focusing on the issue of aggregate resources devoted to R&D as the sequential game progresses, ignores the factors that influence the way in which these resources are utilized. Second, while conclusions in the existing literature continue to hold when we depart from the conventional patent-race framework, they do so only under special circumstances.

[7] The conventional “replacement effect” suggests that an incumbent monopolist will invest less in R&D than an entrant in order to delay cannibalization of the profit stream from its existing product.

Bibliography

Beath, J., Katsoulacos, Y., & Ulph, D. (1987): Sequential Product Innovation and Industry Evolution, *Economic Journal,* 97 (Conference 87), 32-43

Camerer, C. (1991): Does Strategy Research Need Game Theory?, *Strategic Management Journal, Vol.* 12, 137-152

Cebon, P. B., Hauptman, O. and Shekar, C.(2001): Industries in the Making: Product Modularity, Technological Innovation and the Product Lifecycle, Melbourne Business School, mimeo

Cohen, W. M. and Levin, R. C. (1989): Empirical Studies of Innovation and Market Structure, in: R. Schmalensee and R. D. Willig (eds.), *Handbook of Industrial Organization,Vol. II,* Elsevier Science Publ., New York, 1061-1249

Connor, K. (1991): A Historical Comparison of Resource-Based Theory and Five Schools of Thought within Industrial Organization Economics: Do we have a new Theory of the Firm?, *Journal of Management, Vol.* 17, No. 1, 121-154

Clark, K. (1985): The Interaction of Design Hierarchies and Market Concepts in Technological Evolution, *Research Policy,* 14, 235-251

Dutta, P. (1997): Optimal Management of an R&D Budget, *Journal of Economic Dynamics and Control* 21, 575-602

Fershtman, C. and Kamien, M. (1990): Cross Licensing of Complementary Technologies, Center for Mathematical Studies in Economics & Management Science Discussion Paper 866, Northwestern University

Fudenberg,D. and Tirole, J. (1989): Noncooperative Game Theory for Industrial Organization: An Introduction and Overview, in: R. Schmalensee and R.D. Willig (eds.), *Handbook of Industrial Organization,Vol. I,* Elsevier Science Publ., New York, 260-327

Gallini, N. and Kotowitz, Y. (1985): Optimal R&D Processes & Competition, *Economica,* 52:321-334

Gilbert, R.J. (1989): Mobility Barriers and the Value of Incumbency, in: R. Schmalensee and R.D. Willig (eds.), *Handbook of Industrial Organzation, Vol.I,* Elsevier Science Publishers, New York, 475-535

Gottinger, H. W. (2001): Modeling Stochastic Innovation Races, *Technological Forecasting and Social Change ,Vol.68, 1-18*

Gottinger, H. W. (2002): Economics of Network Industries, (Book in Progress, Rouledge.London)

Granot, D. and Zuckerman, D. (1991): Optimal Sequencing and Resource Allocation in R&D Projects, *Management Science, Vol.* 37, No. 2, February 1991

Grossman, G. and Shapiro, C. (1986): Optimal Dynamic: R&D Programs, *Rand Journal of Economics,* 17, 581-596

Grossman, G. and Shapiro, C. (1987): Dynamic R&D Competition, *Economic Journal,* 97, June, 372-387

Harris, C. and Vickers, J. (1987): Racing with Uncertainty, *Review of Economic Studies*, LIV, 1-21

Henderson, R. and Clark, K. (1990): The Reconfiguration of Existing Product Technologies & the Failure of Established Firms, *Administrative Science Quarterly,* 35

Iansiti. M. (1991): Technology Integration: Exploring the Interface between Applied Science and Product Development, Harvard Business School Working Paper #92-026

Iansiti, M. (1992): System Focused Technology Development: An Empirical Study *of* the Development of Novel Science Based Capabilities in the Computer Industry, Harvard Business School Working Paper

Jacquemin, A. (1987): *The New Industrial Organization, Market Forces and Strategic Behavior,* MIT Press, Cambridge, MA

Kreps, D. (1990): Corporate Culture and Economic: Theory, in J. Alt & K. Shepsle (eds.) *Perspectives on Positive Political Economy,* Cambridge University Press, Cambridge, MA

Lerner,J. (1997): An Empirical Exploration of a Technology Race, *The Rand* Journal of *Economics* 28(2), 228-247

Lippman, S. & McCardIe, K. (1987): Dropout Behavior in R&D Races with Learning, *The Rand Journal of Economics,* 18, 287-295

Lucas, R. (1971): Optimal Management *of* an R&D Project, *Management Science,* 17, 679-697

Park, J. (1987): Dynamic Patent Races with Risky Choices, *Management Science,* 33, 1563-1571

Reinganum,J.F.(1989): The Timing of Innovation: Research, Development, and Diffusion, in: R. Schmalensee and R.D. Willig (eds.), *Handbook of Industrial Organzation, Vol.I,* Elsevier Science Publishers, New York, 849-908

Roberts, K. And Weitzman, M. (1981): Funding Criteria for Research, Development and Exploration Projects, *Econometrica,* 49, 1261-1288

Rosenberg, N. (1982): *"Inside the Black Box: Technology and Economics",* Cambridge University Press

Rubin, P. (1973): The Expansion *of* Firms, *Journal of Political Economy, Vol.* 81, No. 4, (July-August) 936-949

Saviotti, P. and Metcalfe, J. (1984): A Theoretic Approach to the Construction *of* Technology Output Indicators, *Research Policy, 13,* 141-151

Shapiro,C., Theories of Oligopoly Behavior, in: R. Schmalensee and R.D. Willig (eds.), *Handbook of Industrial Organzation, Vol.I,* Elsevier Science Publishers, New York, 329-411

Tirole, J. (1988): "The *Theory of Industrial Organization",* MIT Press

Vickers, J. (1986): The Evolution *of* Market Structure when there is a Sequence of Innovations, *Journal of Industrial Economics,* September

Appendix 1

Proof of Lemmas

Define $T(q) = q/(r + q - 1)$.

Proof of Lemma 1

The terms can be expressed as follows.

$$rW = M + q(D-W) \Rightarrow W = (M + qD)/(r+q)$$

where

q = rival's D stage strategy

M, D = monopoly, duopoly profits

$$rL -= p(D-L) \Rightarrow L = pD/(r+p)$$

where p = own D stage strategy

$$M > W(i) \quad \Leftrightarrow \quad M > qD(i) / (r + q - 1)$$

There are two cases to consider. If $r + q < 1$, then lemma is true trivially. So consider $r + q > 1$. Since $T'(q) < 0$, so $T(y) > T(x)$. Also $M(x,x)$ is the lowest value that $M(p_r,p_d)$ can assume. So if $M(x, x) > T(y)D(i)$, then $M > T(q)D(i)$ will hold for all realizations of q, p_r, p_d. Note finally that $M(p,p.) > T(q)D(i)$ suffices to ensure that $M > W(i)$ for all x, y combinations that map into i. Part (a) follows. For part (b),

$$W > L \quad \Leftrightarrow \quad M > r(p - q)\, D / (r + p)$$

Here p and q are the two firms' D stage strategies. If $p < q$, or $p = q$, then result follows immediately.If $p = x$, $q = y$ so that $p > q$, then define

$$\gamma = \frac{r(x-y)}{(x+y)}$$

Then

$$\gamma > T(y) \Rightarrow \; r > xy + y^2 /[xy - y^2 - (1-r)(x-y)] > 1$$

So

$$\gamma < T(y). \text{ Then } M(x,x) > T(z)D(i) \Rightarrow M(x,x) > \gamma D(i)$$

So part (b) holds.

The fact that M/W and W/L are increasing functions of M/D can be seen by writing the following expressions

$$M/W = [1 + q/(M/D)]^{-1} \ , \ W/L = [(r + p)/(r + q)]\ [1M/pD + q/p]$$

Hence the result.

Proof of Lemma 3

We want to show that the following conditions are consistent.

As in the proof of lemma 1, M larger than W, L follows if M is larger than D.

As in lemma 2, the condition

$$xD(i) < yD(i + 1) \Rightarrow xL(i) < yL(i + 1)$$

The only thing that remains to be checked is that it is possible to have xM(.,x)>yM(.,y) and xW(i) < yW(i + 1) together. To see this, note that

$$xW(i) < yW(I + 1) \Rightarrow q(yD(I + 1) - xD(i)) > (xM(., x) - yM(., y) - c(x - y)$$

Rewrite this as

$$\frac{q(yD(i+1) - xD(i))}{(x\dfrac{M(.,x)}{M(.,y)} - y)} \rangle \ M(.,y)$$

Let q = x (since we just want to show possibility) and let

$$\text{x=ky, k > 1} \ \alpha = \frac{D(i+1)}{d(i)}, \beta = \frac{M(.,y)}{D(i)}, \tau = \frac{M(.,y)}{M(.,x)}$$

Then, the condition can be reexpressed as

$$\frac{(\alpha - k)}{(\dfrac{1}{\tau} - \dfrac{1}{k})} > \frac{\beta}{y}$$

This is satisfied for any realization of the RHS if k is small enough.

So, if x and y are sufficiently close, x > y, x,y in (0, 1), then the conditions stated in the lemma can hold concurrently.

Hence the result.

Proof of Lemma 4

Let M, denote the derivative of M with respect to s(B,h). Then $M_s = Kqs^{q-1}$, and M_s/M = Kq/s. So

$$\frac{s(B,x)}{s(B,y)} = \frac{\frac{M_s(s(B,y)}{M(s(B,y)}}{\frac{M_s(s(B,x)}{M(s(B,x)}} > \frac{s_B(B,x)}{s_B(B,y)}$$

where the inequality follows from the SCA. Rearranging, we have

$$M(s(B, x)M_s(s(B, y)s_B(B, y) > M(s(B, y)M_s(s(B, x)s_B(B, x)$$

$$\Rightarrow M(s(B, x))\frac{\partial}{\partial B}[M(s(B, y)] > M(s(B, y)\frac{\partial}{\partial B}[\frac{M(s(B,y))}{M(s(B,x))}] > 0$$

$$\Rightarrow \frac{\partial}{\partial B}[\frac{M(s(B,y))}{M(s(B,x))}] > 0$$

Hence the result.

Appendix 2

Proof of Proposition 5

We will speak of the monopoly (respectively, duopoly) term becoming more important in a payoff expression as the ratio of its coefficient to that of the duopoly (respectively, monopoly) term rises. We show that the monopoly term in the payoff expression of a leading firm becomes progressively more important than the duopoly term as it gets further ahead of its rival in a multi stage race (as long as the lead is great enough), while the duopoly term assumes greater significance for the lagging firm as it gets further behind.

Let $p_{1,\ldots}$.,p_n indicate the choices of hazard rate by the leading firm in each of the n stages, and let $q_{1,\ldots}$.,q_n, indicate the analogous choices by the lagging firm. For convenience, we will denote a firm's flow monopoly profits, which are a function of the its hazard rates, by M and flow duopoly profits, which are a function of the "points" system introduced in the paper, by D.

First consider the leading firm. We will derive an expression for its payoff when it has finished all stagges while its rival has not and will show that the coefficient of M rises faster than the coefficient of D as the rival gets further behind, as long as the rival is far enough bebind. We will then show that this property of the coefficients holds whatever the position of the leading firm (i.e. it need not already have finished all the stages of the race).

Let W_{n-1} be the expected payoff of the leading firm when it has finished all stages, and its rival is still engaged in stage 1. Then

$$rW_{n-1} = M+q_l(W_{n-(l+1)} - W_{n-1})$$

where $M = M(p_{l,\ldots}p_n)$ = leaders monopoly profits

q_1 = laggard’s stage l strategy

$$\text{So } W_{n-1}= \frac{M}{(r+q_l)}(1+\frac{q_l}{(r+q_{l+1})}+\frac{q_l}{(r+q_l)}\frac{q_{l-1}}{(r+q_{l+1})}W_{n-l-2}$$

By repeated substitution as above, and by using the fact that

$$W_o = \frac{M}{(r+q_n)}+\frac{q_nD}{(r+q_n)}$$

we can show that

$$W_{n-1} = \frac{M}{(r+q_l)}[1+\frac{q_l}{(r+q_{1+1})}+\frac{q_{l+1}}{(r+q_{l+1})(r+q_{l+2})}+\ldots+\frac{q_{l.}q_{n-1}}{(r+q_{l+1})..(r+q_n)}$$

$$+\frac{q_{l+1}\ldots q_n}{(r+q)(r+q_{l-1}\ldots(r+q_n)}D \quad for\ n-l>\ 0$$

$$W_0 = \frac{M}{(r+q_n}) + \frac{q_n D}{(r+q_n)}, \quad for\ n-l=0$$

So W_o is the expected payoff of the leading firm when it has finished all stages and the laggard is in the last stage.

Using the following notation

$$\eta_j = W_{n(n-j)}\text{'s M term coefficient}$$

$$\gamma_j = W_{n(n-j)}\text{'s D term coefficient}$$

the following recurrence relations hold

$$n_{j+1} = \frac{1}{(r+q_{n-(j+1)})}+\frac{q_{n-lj-1}}{(r+q_{n-(j+1)})}\eta_j \quad \gamma_{j+1} = \frac{q_{n-(j+1)}}{(r+q_{n-(j+1)})}\gamma_j$$

It follows that, when the leading firm has finished all n stages, as the lead widens, the coefficient of M rises relative to that of D since

$$\frac{n_{j+1}}{n_j} > \frac{\gamma_{j+1}}{\gamma_j}, \forall j \geq\ 0$$

Now consider the situation when the lead firm is in the final stage n. If $V_{n,n-j}$, is its payoff when it is in stage n and the laggard is j stages behind, then

$$V_{n,n-j} = \frac{p_n W_j + q_{n-j}V_{n,n(j-1)}}{r+p_n+q_{n-j}}$$

If the rival's actions are the same (i.e. q_{n-j}, is the same for all j)[8], then we can write this as

$$V_{n,n-j} = AW_j + BV_{n,n-(j-1)} - C$$

where A,B,C are constants.

Using the following notation

8 It is not necessary to use this. The critical fact used in the recursive relations below is that both A and B, defined below, are always less than 1, regardless of the rival’s strategic choice.

$$\mu_j = V_{n,nj}\text{'s M term coefficient}$$

$$\lambda_j = V_{n,nj}\text{'s D term coefficient}$$

the following recurrence relations hold

$$\mu_{j+1} = A\eta_{j+1}\text{'s} + B\mu_j \quad \text{(i)}$$

$$\lambda_{j+1} = A\gamma_{j+1} + B\lambda_j$$

where η_j, γ_j are as before

Then

$$\mu_j = A\eta_j + B\mu_{j-1} = A\eta j + B(A\eta_{j-1} + B\mu_{j-2}) = A(\eta_j + B\eta_{j-1}) + B^2\mu_{j-1}$$

Repeating this recursion gives us

$$\mu_j = (A\eta_j + B\eta_{j-1} + B^2\eta_{j-2} + \ldots B^{j-1}\eta_j) + B^j\mu_0$$

where $\mu_0 = V_{n,n}$'s M term coefficient

Similarly,

$$\lambda_j = A(\gamma_j + B\gamma_{j-1} + B^2\gamma_{j-2} + \ldots + B^{j-1}) + B^j\lambda_0$$

where $\lambda_0 = V_{n,n}$'s D term coefficient

From (i) we see that

$$\frac{\mu_{j+1}}{\mu_j} > \frac{\lambda_{j+1}}{\lambda_j} \Leftrightarrow \frac{\mu_j}{\eta_{j+1}} < \frac{\lambda_j}{\gamma_{j+1}}$$

The inequality on the right hand side can be expanded to read as

$$\frac{A(\eta_j + B\eta_{j-1} + .. + B^{j-1}\eta_1)}{\eta_{j+1}} + \frac{B^j\mu_0}{\eta_{j+1}} < \frac{A(\gamma_j + B\gamma_{j-1} + .. + B^{j-1}\gamma_1)}{\gamma_{j+1}} + \frac{B^j\lambda_0}{\gamma_{j+1}}, \quad (ii)$$

Now, recall from the analysis of the coefficients when the leading firm had finished all the stages that

$$\frac{\eta_j}{\eta_{j+1}} < \frac{\gamma_j}{\gamma_{j+1}}$$

$$\text{Further, } \frac{\eta_{j-1}}{\eta_{j+1}} = \frac{\eta_{j-1}}{\eta_j}\frac{\eta_j}{\eta_{j+1}} < \frac{\gamma_{j-1}}{\gamma_j}\frac{\gamma_j}{\gamma_{j+1}} = \frac{\gamma_{j-1}}{\gamma_{j+1}}$$

$$\text{Similarly, } \frac{\eta_{j-i}}{\eta_{j+1}} < \frac{\gamma_{j-i}}{\gamma_{j+1}}, \forall\ i\ \in (0, j)$$

Inequality (ii) will be satisfied for j large enough regardless of the relative magnitude of μ_o, and λ_o (since B < 1). It follows that the coefficients of the monopoly term rise faster than those of the duopoly term once the lead (j) gets large enough and the leader is in the final stage.

Thus far we have used our knowledge of the fact that the monopoly coefficients rise faster than the duopoly ones when the lead firm has finished all stages to derive a similar result for the corresponding coefficients when the lead firm is in the final stage and the lead is large enough. A similar recursive analysis (i.e. using the pattern of coefficients when the leader is at stage n-m to derive the pattern of coefficients when the leader is at stage n-m-l) establishes that the monopoly coefficients will rise faster than the duopoly ones regardless of the lead firm's position as long as the lead is large enough.

Further, it is possible to show that the further the lead firm gets from the finishing line, the greater is the lead needed before the property about this monopoly and duopoly coefficients holds. This can be seen if we reinterpret the notation from above as follows. Let the η and γ terms refer to the monopoly and duopoly coefficients when the leader is at stage n-m, and let g and X refer to the coefficients when the leader is at n-m-1[9]. Let j refer to the lead as before. Call the following inequalities *condition (iii)*

$$\text{j = 0 For } \frac{\mu_1}{\mu_o} > \frac{\lambda_1}{\lambda_0}, need\, \frac{\mu_0}{\eta_1} < \frac{\lambda_0}{\gamma_1}$$

$$\text{j = 1: For } \frac{\mu_2}{\mu_1} > \frac{\lambda_2}{\lambda_1} need\, \frac{\mu_1}{\eta_2} < \frac{\lambda_1}{\gamma_2} \Rightarrow \frac{A\eta_1 + B\mu_0}{\eta_2} < \frac{A\gamma_1 + B\lambda_0}{\gamma_2}$$

$$\text{j=2: For } \frac{\mu_3}{\mu_2} > \frac{\lambda_3}{\lambda_2}, need\, \frac{\mu_2}{\eta_3} < \frac{\lambda_2}{\gamma_3} \Rightarrow \frac{A(\eta_2 + B\eta_1) + B^2\mu_0}{\eta_3} < \frac{A(\gamma_2 + B\gamma_1) + B^2\lambda_0}{\gamma_3}$$

Similarly for j = 3,4......

For any m, the condition (iii) inequality for a particular lead j must hold true for the monopoly coefficient to rise faster than the duopoly coefficient when the leader is at stage n-m-1.

Now consider the conditions (iii) when m = S. If μ_o is larger than λ_o, this is more likely to cause a problem for the conditions for low j than for higher j. Suppose the conditions above

9 Then m varies from -1 to n, and m = -1 is the situation when the lead firm has finished the race. Then the above deduction of the properties of the coefficients when the lead firm is at stage n from the properties when the lead firm has finished all n stages corresponds to m = -1.

do not hold for j = 0 and j = 1, but do hold for j > 1 (in stage m = S). Then, in the next recursion when m = S + 1, we have

$$\frac{\eta_1}{\eta_0} < \frac{\gamma_1}{\gamma_0}, \frac{\eta_2}{\eta_1} < \frac{\gamma_2}{\gamma_1}, \frac{\eta_{j+1}}{\eta_j} > \frac{\gamma_{j+1}}{\gamma_j} \forall j \geq \ 2$$

Then we see that in conditions (iii) for m = S + 1, it is even more likely that the low j inequalities will not hold, and we will need j to be higher than for the m = S situation for the inequalities to hold. Thus the higher is m, i.e. the greater is the distance of the lead firm from the end of the race, the greater has to be the lead before the monopoly coefficient starts rising faster than the duopoly coefficient.

Now we turn our attention to the lagging firm. The analysis procceds in a manner similar to that for the leading firm. Let L, denote the lagging firm's payoff when the leader has finished all n stages. Let $p_1 .., p_n$ denote the choices of hazard rate for each of the n stages by the lagging firm. Then

$$L_{n-f} = \frac{p_{f..}p_n}{(r+p_f)...(r+p_n)} D$$

As above, we are interested in understanding how the coefficients of the M and D terms behave. Consider the lagging firm's payoff when il is at stage n-(j + 1) and the leading firm is at stage n. Then

$$V_{n-(j+1)n} = \frac{p_{n(j+1)} V_{n-j} + q_n L_{j+1}}{r + p_{n-(j+1)}) + q_n}$$

Adopting notation similar to that used above, we have

γ_j = L_{n-j}'s D term coefficient

μ_j = V_{n-jn}'s M term coefficient

λ_j = V_{n-jj}'s D term coefficient

Then

$$\mu_{j+1} = \frac{p_{n-(j+1)}}{r + p_{n-(j+1)} + q_n} \mu_{j+1}, \lambda_{j+1} = \frac{p_{n-(j+1)}}{r + p_{n-(j+1)} + q_n} \lambda_j + \frac{q_n}{r + p_{n-(j+1)} + q_n} \gamma_{j+1}$$

$$\Rightarrow \frac{\lambda_{j+1}}{\lambda_j} > \frac{\mu_{j+1}}{\mu_J} \forall_J$$

i.e. duopoly term coefficients rise faster than the monopoly term coefficients as the lagging firm gets further behind the leading firm when the latter is at stage n.

Then we carry out the recursion. So when the leader is at stage n-1, and the laggard is j + 1 stages behind, the latter's payoff is

$$V_{n-1(j+1)n-1} = \frac{p_{n-j-2}V_{n-1-jn-1} + q_{n-1}V_{n-1(j+1)n} - c}{r + p_{n-j-2} + q_{n-1}}$$

Then we let η_j and γ_j be the M and D term coefficients when the leader is at stage n, and μ_j, and λ_j be those when the leader is at stage n-1, and we derive recurrence relations for the μ and λ coefficients in much the same way as the analysis for the leading firm. The conclusions are similar. The coefficients of the duopoly term in the lagging firm's payoff expression rise faster than those of the monopoly term when the lead is large enough.