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On Guilt Aversion in Symmetric 2×2 Anti-Coordination Games

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Abstract
This paper examines how guilt aversion affects the equilibria of symmetric 2×2 games with the same Nash equilibrium structure as the Hawk–Dove game: two asymmetric strict pure equilibria and one completely mixed-strategy equilibrium. We classify these generalized Hawk–Dove games into two subclasses, Type 1 and Type 2, based on players' preferences over deviations toward symmetric profiles. We characterize best-reply correspondences and equilibria under guilt aversion, showing that outcomes are highly sensitive to guilt parameters. In Type 1 games, when guilt sensitivity exceeds a threshold, a new symmetric equilibrium emerges while the mixed-strategy equilibrium disappears. In Type 2 games, guilt aversion affects only the mixed equilibrium, leaving the two asymmetric equilibria unchanged.

JEL Classification: C72; D81.

Keywords: Hawk-Dove games, equilibria, guilt aversion, psychological games, ambiguous beliefs.

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1 Introduction

This paper revisits symmetric 2×2 anti-coordination games by introducing guilt aversion as a key psychological factor influencing players' strategic behavior. Unlike traditional models that assume purely self-interested agents, we account for emotional responses in decision-making. In particular, guilt aversion captures the disutility players experience when they believe their actions have caused disappointment or harm to others. This trait has been widely recognized in the literature as a crucial determinant of strategic choices and equilibrium selection [Battigalli, Dufwenberg, 2007, 2009; Attanasi, Nagel, 2008].

Our approach builds on psychological game theory, an extension of classical game theory introduced by Geanakoplos et al. [1989], which allows utilities to depend not only on players' actions but also on their beliefs about others' beliefs, and on higher-order beliefs. In this framework, the classical Nash equilibrium is replaced by the psychological Nash equilibrium [Battigalli, Siniscalchi, 1999; Battigalli et al., 2019; Battigalli, Dufwenberg, 2020]. This perspective has proven effective in capturing a broad range of phenomena involving emotional and motivational drivers such as trust, reciprocity, and guilt [Rabin, 1993; Guerra, Zizzo, 2004; Dufwenberg, Kirchsteiger, 2019].

Within this framework, we focus on a class of symmetric 2×2 games, which we term *generalized Hawk–Dove games*. These games share the same equilibrium structure as the classical Hawk–Dove game: two asymmetric strict pure Nash equilibria and one completely mixed-strategy equilibrium. We further classify them into two subclasses based on players' preferences over deviations toward symmetric profiles. Type 1 games, which include the classical Hawk–Dove game, represent situations where players would prefer their opponent's unilateral deviation toward a specific symmetric outcome. Type 2 games, in contrast, describe situations where symmetric profiles are Pareto-dominated by the asymmetric equilibria.

The main goal of this paper is to characterize how guilt aversion modifies the best-reply correspondences and the resulting equilibria in these games. Our analysis shows that the sensitivity of players to guilt has a significant impact on equilibrium outcomes. In Type 1 games, when guilt sensitivity exceeds a threshold, the equilibrium structure changes substantially: a new symmetric equilibrium emerges while the mixed-strategy equilibrium disappears, although the asymmetric equilibria persist. In Type 2 games, guilt aversion affects only the probabilities of the mixed-strategy equilibrium, leaving the two asymmetric pure equilibria unchanged.

Overall, our findings illustrate how incorporating psychological motives such as guilt aversion enriches the analysis of coordination problems, providing insights into how emotional factors influence equilibrium selection and conflict resolution in strategic interactions.

2 The general game model

The model in its standard form consists in a classical 2×2 symmetric game: Row player Ann and Column player Bob have to choose among two possible actions H and L, so that the strategy sets are $S_A = S_B = \{\text{H, L}\}$. The matrix form is the following:

Ann \ Bob	H	L
H	a / a	c / b
L	b / c	d / d

The model we consider is characterized by the structure of the set of its Nash equilibria: two pure asymmetric strict Nash equilibria with asymmetric payoffs. Therefore the model we consider is characterized by the following conditions:

$$a < c, \ d < b, \ b \neq c \qquad \text{(Generalized Hawk-Dove)}$$

We call this class of games *generalized Hawk-Dove games*. Denote a generic mixed strategy of Ann with $p \in [0, 1]$, where, as usual (with an abuse of notation), $p = \text{prob}(H)$ and $1 - p = \text{prob}(L)$. Similarly, a generic mixed strategy of Bob is $q \in [0, 1]$, where (with an abuse of notation), $q = \text{prob}(H)$ and $1 - q = \text{prob}(L)$. Therefore, (p, q) denotes a generic strategy profile. Note that the set of mixed strategies reduces respectively to $\Sigma_A = [0, 1]$ and $\Sigma_B = [0, 1]$. The term $\pi_A(p, q)$ (resp. $\pi_B(p, q)$) denotes the standard expected payoff of Ann (resp. Bob) for every $(p, q) \in \Sigma_A \times \Sigma_B$, whose functional forms are given by:

$$\pi_A(p, q) = p\left[q\left((a + d) - (c + b)\right) + b - d\right] + (c - d)q + d, \tag{1}$$

and

$$\pi_B(p, q) = q\left[p\left((a + d) - (c + b)\right) + b - d\right] + (c - d)p + d. \tag{2}$$

For the sake of simplicity, we denote with s_A the generic pure strategy of Ann where $s_A = 1$ means that Ann plays H and $s_A = 0$ means that Ann plays L. Similarly, we denote with s_B the generic pure strategy of Bob where $s_B = 1$ means that Bob plays H and $s_B = 0$ means that Bob plays L.

Equilibria

Under condition (Generalized Hawk-Dove), the structure of the set of Nash equilibria is always the same. Denoting with

$$\Psi(G) = \frac{b - d}{(b + c) - (a + d)}, \tag{3}$$

then the best reply correspondences for Ann and Bob are:

$$BR_A(q) = \begin{cases} 1 & \text{if } 0 \leqslant q < \Psi(G), \\ [0,1] & \text{if } q = \Psi(G), \\ 0 & \text{if } \Psi(G) < q \leqslant 1, \end{cases} \qquad BR_B(p) = \begin{cases} 1 & \text{if } 0 \leqslant p < \Psi(G), \\ [0,1] & \text{if } p = \Psi(G), \\ 0 & \text{if } \Psi(G) < p \leqslant 1. \end{cases} \tag{4}$$

Therefore, the game has three Nash equilibria:

- $(p, q) = (1, 0)$;
- $(p, q) = (0, 1)$;
- $(p, q) = (\Psi(G), \Psi(G))$.

REMARK 2.1: In the literature, the Hawk-Dove game refers to a particular subclass of (Generalized Hawk-Dove) given by:

$$a < c < d < b, \tag{5}$$

where the conditions

$$b = 2d \quad \text{and} \quad c = 0$$

are usually added.

Note also that

$$d < b < a < c$$

is substantially equivalent to (5) as it corresponds to the case where the pure strategy H plays the role of pure strategy L and vice versa.

3 Modeling guilt aversion

The mainly considered perspective in the theoretical papers devoted to this issue looks at guilt aversion as a consequence of letting others down. In particular, we refer to the formal model of guilt aversion by [Battigalli, Dufwenberg, 2007], [Battigalli, Dufwenberg, 2009], [Attanasi, Nagel, 2008], in which a guilt-averse agent (say Ann) has a disutility if she believes that her opponent (in our case Bob) is disappointed by her play, as he receives a lower payoff than the one he originally expected given his beliefs. More precisely, it is said that player j is let down if his actual material payoff $\hat{\pi}_j$, received after the play, is lower than the payoff he initially expected to get, $E_j[\pi_j^e]$. Therefore, player j disappointment is given by:

$$\max\{0, E_j[\pi_j^e] - \hat{\pi}_j\}.$$

Given the strategy profile σ, player i's beliefs b_i and player i's guilt-sensitivity parameter $\theta_i > 0$, the guilt-dependent utility of player i can be constructed as follows:

$$u_i(b_i, \theta_i, \sigma) = \hat{\pi}_i(\sigma) - \theta_i \max\{0, E_j[\pi_j^e(\sigma), b_i] - \hat{\pi}_j(\sigma)\}, \tag{6}$$

where $\max\{0, E_j[\pi_j^e(\sigma), b_i] - \hat{\pi}_j(\sigma)\}$ represents player i's expectation of player j disappointment. In particular, $E_j[\pi_j^e(\sigma), b_i]$ represents what player i believes is the payoff that player j initially expects to get and $\hat{\pi}_j(\sigma)$ is what player j actually gets.

Ann's guilt

Suppose that Ann is a guilt-averse agent. In order to construct Ann's guilt-dependent utility function, denote with $\hat{q}$ the expectation of Bob's first-order beliefs about Ann's strategy choice p, and with $\tilde{p}$ the expectation of Ann's second-order beliefs about the first-order belief of Bob $\hat{q}$. Let θ_A be Ann's sensitivity to guilt. If a pure strategy profile is actually played, say (s_A, s_B) with $s_A, s_B \in \{0,1\}$, then Ann believes that Bob's initially expected payoff is $E_B[\pi_B^e(s_A, s_B), \tilde{p}]$, which corresponds to Bob choosing s_B and Ann randomizing with probabilities $\tilde{p}$ and $1-\tilde{p}$. More precisely,

$$E_B[\pi_B^e(s_A, s_B), \tilde{p}] = \pi_B(\tilde{p}, s_B) = \tilde{p}\pi_B(1, s_B) + (1-\tilde{p})\pi_B(0, s_B).$$

Then, for every pure strategy profile $(s_A, s_B) \in \{0,1\} \times \{0,1\}$, Ann's guilt-dependent utility is:

$$u_A(\tilde{p}, \theta_A, (s_A, s_B)) = \pi_A(s_A, s_B) - \theta_A \max\{0, E_B[\pi_B^e(s_A, s_B), \tilde{p}] - \pi_B(s_A, s_B))\}.$$

Since

$$\pi_B(\tilde{p}, 1) = \tilde{p}a + (1-\tilde{p})b, \quad \pi_B(\tilde{p}, 0) = \tilde{p}c + (1-\tilde{p})d,$$

it follows that:

$$\begin{cases} u_A(\tilde{p}, \theta_A, (1,1)) = a - \theta_A \max\{0, \tilde{p}a + (1-\tilde{p})b - a\} = a - \theta_A \max\{0, (1-\tilde{p})(b-a)\}, \\ u_A(\tilde{p}, \theta_A, (1,0)) = b - \theta_A \max\{0, \tilde{p}c + (1-\tilde{p})d - c\} = b - \theta_A \max\{0, (1-\tilde{p})(d-c)\}, \\ u_A(\tilde{p}, \theta_A, (0,1)) = c - \theta_A \max\{0, \tilde{p}a + (1-\tilde{p})b - b\} = c - \theta_A \max\{0, \tilde{p}(a-b)\}, \\ u_A(\tilde{p}, \theta_A, (0,0)) = d - \theta_A \max\{0, \tilde{p}c + (1-\tilde{p})d - d\} = d - \theta_A \max\{0, \tilde{p}(c-d)\}. \end{cases} \tag{7}$$

Bob's guilt

Suppose now that Bob is guilt-averse. In this case, let $\hat{p}$ be the expectation of Ann's first-order beliefs about Bob's strategy choice q and $\tilde{q}$ the expectation of Bob's second-order beliefs about the first-order belief of Ann $\hat{p}$. Let θ_B denote Bob's sensitivity to guilt.

As in the Ann's case, if the pure strategy profile (s_A, s_B) is actually played, Bob believes that Ann initially expected a payoff $E_A[\pi_A^e(s_A, s_B), \tilde{q}]$ which corresponds to Ann choosing s_A and Bob randomizing with probabilities $\tilde{q}$ and $1-\tilde{q}$. More precisely:

$$E_A[\pi_A^e(s_A, s_B), \tilde{q}] = \pi_A(s_A, \tilde{q}) = \tilde{q}\pi_A(s_A, 1) + (1-\tilde{q})\pi_A(s_A, 0).$$

Then, for every pure strategy profile $(s_A, s_B) \in \{0,1\} \times \{0,1\}$, Bob's guilt-dependent utility is:

$$u_B(\tilde{q}, \theta_B, (s_A, s_B)) = \pi_B(s_A, s_B) - \theta_B \max\{0, E_A[\pi_1^e(s_A, s_B), \tilde{q}] - \pi_A(s_A, s_B))\}.$$

Since

$$\pi_A(1, \tilde{q}) = a\tilde{q} + b(1 - \tilde{q}), \quad \pi_A(0, \tilde{q}) = c\tilde{q} + d(1 - \tilde{q}),$$

we get:

$$\begin{cases} u_B(\tilde{q}, \theta_B, (1,1)) = a - \theta_B \max\{0, \tilde{q}a + (1-\tilde{q})b - a\} = a - \theta_B \max\{0, (1-\tilde{q})(b-a)\}, \\ u_B(\tilde{q}, \theta_B, (1,0)) = c - \theta_B \max\{0, \tilde{q}a + (1-\tilde{q})b - b\} = c - \theta_B \max\{0, \tilde{q}(a-b)\}, \\ u_B(\tilde{q}, \theta_B, (0,1)) = b - \theta_B \max\{0, \tilde{q}c + (1-\tilde{q})d - c\} = b - \theta_B \max\{0, (1-\tilde{q})(d-c)\}, \\ u_B(\tilde{q}, \theta_B, (0,0)) = d - \theta_B \max\{0, \tilde{q}c + (1-\tilde{q})d - d\} = d - \theta_B \max\{0, \tilde{q}(c-d)\}. \end{cases} \tag{8}$$

The Psychological Game

From the previous arguments, it follows that in the psychological game with guilt averse players, Ann's utility matrix is:

Ann	H	L
H	$a - \theta_A \max\{0, (1-\tilde{p})(b-a)\}$	$b - \theta_A \max\{0, (1-\tilde{p})(d-c)\}$
L	$c - \theta_A \max\{0, \tilde{p}(a-b)\}$	$d - \theta_A \max\{0, \tilde{p}(c-d)\}$

Bob's utility matrix is:

Bob	H	L
H	$a - \theta_B \max\{0, (1-\tilde{q})(b-a)\}$	$c - \theta_B \max\{0, \tilde{q}(a-b)\}$
L	$b - \theta_B \max\{0, (1-\tilde{q})(d-c)\}$	$d - \theta_B \max\{0, \tilde{q}(c-d)\}$

It can be easily observed that the psychological terms in the utility functions depend on whether

$$a \gtreqless b, \; c \gtreqless d.$$

First of all, note that condition

$$a > b \tag{9}$$

represents the situation in which Ann (resp. Bob) would prefer the unilateral deviation of Bob (resp. Ann) from the asymmetric equilibrium (H, L), i.e. $(1, 0)$ (resp. (L, H), i.e $(0, 1)$), to the symmetric (non-equilibrium) profile (H, H), that is $(1, 1)$. Similarly,

$$d > c \tag{10}$$

represents the situation in which Ann (resp. Bob) would prefer the unilateral deviation of Bob (resp. Ann) from the asymmetric equilibrium (L, H), that is $(0, 1)$ (resp. (H, L), i.e. $(1, 0)$), to the symmetric (non-equilibrium) profile (L, L), that is $(0, 0)$.
Note also that, under conditions (Generalized Hawk-Dove), inequalities (9) and (10) cannot hold simultaneously since they would imply

$$b < a < c < d < b,$$

which is impossible.

Finally, observe that

$$a = b, \ c = d$$

cannot hold simultaneously for the same reasons, since we would get

$$c > a = b > d = c,$$

which is impossible.

Therefore, we can identify three different types of the Hawk-Dove game that give rise to three different kind of psychological games:

	Conditions	Cases
Hawk-Dove Game	(Generalized Hawk-Dove): $a < c$, $d < b$, $b \neq c$	
Type 1	(Generalized Hawk-Dove) (+) $a < b$, $c < d$	$a < c < d < b$
Type 2	(Generalized Hawk-Dove) (+) $a < b$, $d < c$	$d < a \leqslant b < c$ $d < a < c < b$ $a < d \leqslant c < b$ $a < d < b < c$ $a = d < c < b$ $a = d < b < c$
Type 3	(Generalized Hawk-Dove) (+) $b < a$, $d < c$	$d < b < a < c$

Type 1: As explained above, this family of games corresponds to the situation in which both players would prefer the unilateral deviation of their opponent, from the pure strategy equilibrium towards the symmetric profile (L, L) (where the identity of the deviant depends on which of the two equilibria is played). Note also that in Type 1 games, the other symmetric profile (H, H) is Pareto dominated by the two pure Nash equilibria. Finally, as explained in Section 2, this case includes the classical Hawk-Dove game.

Type 2: The nature of this family of games is substantially different from Type 1. In fact, in Type 2 games, players would never prefer a deviation of their opponents from a Nash equilibrium to a symmetric profile. Equivalently, both the symmetric profiles are Pareto dominated by the two Nash equilibria.

Type 3: This family of games is equivalent to Type 1 games. The unique difference is that the pure strategy H plays the role of the strategy L (for both players) and vice versa. Consequently, both players would prefer the unilateral deviation of their opponent, from the pure strategy equilibrium towards the symmetric profile (H, H) so that the other symmetric profile (L, L) is Pareto dominated by the two pure Nash equilibria.

4 Psychological Games and Equilibria

In order to include psychological aspects in the Hawk-Dove game, we take into account the model for static psychological games introduced in [Geanakoplos et al., 1989].

In the general model, we consider a finite set of players $I = \{1, \dots n\}$ and, for each player $i \in I$, we denote with $A_i = \{a_i^1, \dots, a_i^{k(i)}\}$ the finite set of pure strategies of player i. Following the standard notation, $A = A_1 \times \cdots \times A_n = \prod_{i \in I} A_i$ represents the set of strategy profiles and $A_{-i} = A_1 \times \cdots \times A_{i-1} \times A_{i+1} \times \cdots \times A_n = \prod_{j \neq i} A_j$ represents the set of i's opponents strategy profiles. The set Σ_i denotes the set of mixed strategies of player i and $\Sigma = \prod_{i \in I} \Sigma_i$, $\Sigma_{-i} = \prod_{j \neq i} \Sigma_j$ denote respectively the set of mixed strategies profiles and the set of i's opponents mixed strategies with the classical notation that (σ_i, σ_{-i}) with $\sigma_i \in \Sigma_i$ and $\sigma_{-i} \in \Sigma_{-i}$ to indicate the mixed strategies profile $\sigma \in \Sigma$.
We denote with $b_i = (b_i^1, b_i^2, \cdots, b_i^k, \cdots)$ a generic (infinite) hierarchy of beliefs of player i that, roughly speaking, represents what player i believes the others will play, what player i thinks the others believe their opponents will play, and so on. We will restrict our attention to the subset of collectively coherent hierarchies of beliefs $\overline{B}_i$, which is the set of hierarchies of beliefs of player i in which he is sure (i.e. with probability equal to 1) that it is common knowledge that beliefs are coherent. We relegate to Appendix A an exhaustive and precise description of (collective coherent) hierarchy of beliefs.

A *standard psychological game* is described by $G^{GPS} = \{A_1, \cdots, A_n, u_1, \cdots, u_n\}$ where, for every $i \in I$, the utility functions u_i have the form $u_i : \overline{B}_i \times \Sigma \to \mathbb{R}$ ([Geanakoplos et al., 1989]).

Equilibrium notion

The notion of psychological Nash equilibrium introduced in [Geanakoplos et al., 1989] is based on the idea that the entire hierarchy of beliefs must be correct in equilibrium. More precisely, each player is equipped with a function $\beta_i : \Sigma \to \overline{B}_i$ which selects, for every $\sigma \in \Sigma$, the hierarchy of beliefs $\beta_i(\sigma) = (b_i^1, b_i^2, \cdots, b_i^k, \cdots)$ in which player i believes (with probability 1) that his opponents follow the mixed strategy profile σ_{-i}, that each opponent $j \neq i$ believes that his opponents follow σ_{-j}, that each opponent $j \neq i$ believes that his opponents believe that the others follow the mixed strategy profile σ and so on. Then, a *psychological Nash equilibrium* is defined as a pair (b^*, σ^*) where $b^* = (b_1^*, \cdots, b_n^*)$ with $b_i^* \in \overline{B}_i$ and $\sigma^* \in \Sigma$ such that, for every player i:

i) $b_i^* = \beta_i(\sigma^*)$;

ii) $u_i(b_i^*, \sigma^*) \geqslant u_i(b_i^*, (\sigma_i, \sigma_{-i}^*))$ for every $\sigma_i \in \Sigma_i$.

We can also say that $(\beta(\sigma^*), \sigma^*)$ is a psychological Nash equilibrium of the game G^{GPS}.

Summary utility functions and best replies

In Geanakoplos et al. [1989], a characterization for psychological Nash equilibria has been given. The summary utility functions are defined as follows:

$$w_i^{GPS}(\sigma, \tau) = u_i(\beta_i(\sigma), \tau), \qquad \forall (\sigma, \tau) \in \Sigma \times \Sigma. \tag{11}$$

Then $(\beta(\sigma^*), \sigma^*)$ is a psychological Nash equilibrium if and only if, for every $i \in I$,

$$w_i^{GPS}((\sigma_i^*, \sigma_{-i}^*), (\sigma_i^*, \sigma_{-i}^*)) \geqslant w_i^{GPS}((\sigma_i^*, \sigma_{-i}^*), (y_i, \sigma_{-i}^*)), \qquad \forall y_i \in \Sigma_i, \tag{12}$$

or equivalently $\sigma_i^* \in BR_i^{GPS}(\sigma_{-i}^*)$ with

$$BR_i^{GPS}(\sigma_{-i}^*) = \left\{ \sigma_i \in \Sigma_i \,|\, w_i^{GPS}((\sigma_i, \sigma_{-i}^*), (\sigma_i, \sigma_{-i}^*)) \geqslant w_i^{GPS}((\sigma_i, \sigma_{-i}^*), (y_i, \sigma_{-i}^*)),\ \forall y_i \in \Sigma_i \right\}. \tag{13}$$

5 Equilibria under Guilt Aversion

In this section, we characterize the psychological equilibria of the Hawk-Dove game in presence of guilt aversion. Building on the utilities for pure strategies described in Section 3 and illustrated in (7) and (8), the expected utilities are constructed in the classical way. Ann's expected utility from playing the mixed strategy y, assuming Bob plays the mixed strategy q and holding a second-order belief $\tilde{p}$, is given by:

$$\begin{aligned} u_A(\tilde{p}, \theta_A, y, q) = {} & yq\, u_A(\tilde{p}, \theta_A, 1, 1) + y(1-q)\, u_A(\tilde{p}, \theta_A, 1, 0) + \\ & (1-y)q\, u_A(\tilde{p}, \theta_A, 0, 1) + (1-y)(1-q)\, u_A(\tilde{p}, \theta_A, 0, 0). \end{aligned} \tag{14}$$

Similarly, Bob's expected utility from playing the mixed strategy y, assuming Ann plays the mixed strategy p and holding a second-order belief $\tilde{q}$, is:

$$\begin{aligned} u_B(\tilde{q}, \theta_B, p, y) = {} & py\, u_B(\tilde{q}, \theta_B, 1, 1) + p(1-y)\, u_B(\tilde{q}, \theta_B, 1, 0) + \\ & (1-p)y\, u_B(\tilde{q}, \theta_B, 0, 1) + (1-p)(1-y)\, u_B(\tilde{q}, \theta_B, 0, 0). \end{aligned} \tag{15}$$

Recall that $p \in BR_A^{GPS}(q)$ if and only if p maximizes u_A, given that Bob plays q and that $\tilde{p}$ is correct (i.e., $\tilde{p} = p$). More precisely, $p \in BR_A^{GPS}(q)$ if and only if

$$u_A(p, \theta_A, (p,q)) = w_A^{GPS}((p,q),(p,q)) \geqslant w_A^{GPS}((p,q),(y,q)) = u_A(p, \theta_A, (p,y)) \quad \forall y \in [0,1].$$

Consequently, computing $BR_A^{GPS}(q)$ requires maximizing $u_A(\tilde{p}, \theta_A, (y,q))$ with respect to y, under the condition that $\tilde{p}$ must be consistent with the maximizer.

Similarly, $q \in BR_B^{GPS}(p)$ if and only if q maximizes u_B, given that Ann plays p and that $\tilde{q} = q$. More precisely, $q \in BR_B^{GPS}(p)$ if and only if

$$u_B(q, \theta_B, (p,q)) = w_B^{GPS}((p,q),(p,q)) \geqslant w_B^{GPS}((p,q),(p,y)) = u_B(q, \theta_B, (p,y)) \quad \forall y \in [0,1].$$

From the previous section, it follows that a psychological Nash equilibrium in this setting is any pair (p^*, q^*) such that

$$\begin{cases} p^* \in BR_A^{GPS}(q^*) \\ q^* \in BR_B^{GPS}(p^*). \end{cases}$$

In particular, we will focus only on Type 1 and Type 2 games. As clearly shown in Section 2, Type 3 is essentially equivalent to Type 1 when strategy L plays the role of strategy H, and viceversa. This symmetry is clearly reflected in the characterization of the psychological Nash equilibria. However, the computations differ slightly[2].

Main results

In the next subsections, we characterize the best-reply correspondences and the resulting equilibria for both *Type 1* and *Type 2* games in the model with guilt. The results presented below show that the scenario is rather diversified.

Regarding Type 1 games, we observe that the shape of the best-reply correspondences depends on whether the sensitivity parameters θ_A and θ_B exceed the threshold ratio $\frac{b-d}{d-c}$, which measures Ann's losses (respectively gains) with respect to Bob's gains (respectively losses). Whenever the sensitivity parameters do not exceed this ratio, the set of equilibria is only weakly affected by guilt aversion: the two pure-strategy equilibria persist, and there exists a unique equilibrium in completely mixed strategies that depends explicitly on the sensitivity parameters and converges to the mixed equilibrium of the standard game as the sensitivity to guilt converges to zero.

In contrast, when θ_A and θ_B are larger than this ratio, the scenario changes significantly: a new pure strategy equilibrium emerges, corresponding to (L, L), i.e., the symmetric strategy profile that is not Pareto dominated by the two Nash equilibria. The two asymmetric equilibria of the material game are not destroyed by guilt aversion in this case either. Finally, there are no longer equilibria in completely mixed strategies; instead, two new equilibria emerge in which one player chooses L while the other randomizes.

[2]They are available upon request.

In Type 2 games, players' guilt aversion affects only the completely mixed-strategy equilibrium (due to the perturbation of probabilities), while the two pure asymmetric equilibria survive. No other equilibria emerge.

5.1 Type 1: $a < b,\ c < d$

In this case, the strategic form game is the following:

Type 1	$a < b,\ c < d$	
Ann \ Bob	H	L
H	$a - \theta_A(1-\tilde{p})(b-a)$, $a - \theta_B(1-\tilde{q})(b-a)$	$b - \theta_A(1-\tilde{p})(d-c)$, c
L	c , $b - \theta_B(1-\tilde{q})(d-c)$	d , d

During the next calculations, it will be useful the notation:

$$\begin{cases} (a+d)-(b+c) = \gamma < 0, \\ b-d = \delta > 0, \\ d-c = \lambda, \\ b-a = \mu. \end{cases} \tag{16}$$

Ann's best reply correspondence

In this case, Ann's expected utility from playing the mixed strategy y, expecting Bob playing the mixed strategy q and having second order belief $\tilde{p}$ is

$$u_A(\tilde{p}, \theta_A, y, q) =$$

$$y\left[q\Big(\big(1+(1-\tilde{p})\theta_A\big)\big((a+d)-(b+c)\big)\Big) + (b-d) - (1-\tilde{p})\theta_A(d-c)\right] + cq + d(1-q). \tag{17}$$

Now, we can prove the subsequent proposition:

Proposition 5.1: *Let G be an Hawk-Dove game ($a < c$, $d < b$). Assume that $a < b$, $c < d$ and denote with:*

$$\eta_1(\theta_A) = \frac{d - b + \theta_A(d - c)}{\big(1 + \theta_A\big)\big((a + d) - (b + c)\big)}, \tag{18}$$

$$P_1(q, \theta_A) = 1 - \frac{b - d + q[(a + d) - (b + c)]}{\theta_A(d - c - q[(a + d) - (b + c)])}. \tag{19}$$

Then, Ann's best reply correspondence is given by the following:

i) If $\theta_A \leqslant \frac{b-d}{d-c}$, then:

$$BR_A^{GPS}(q) = \begin{cases} 1 & \text{if } 0 \leqslant q < \eta_1(\theta_A), \\ \{0, 1\} & \text{if } q = \eta_1(\theta_A), \\ \{0, 1, P_1(q, \theta_A)\} & \text{if } \eta_1(\theta_A) < q < \Psi(G), \\ \{0, 1\} & \text{if } q = \Psi(G), \\ 0 & \text{if } \Psi(G) < q \leqslant 1, \end{cases} \tag{20}$$

where

- $\eta_1 : \left]0, \frac{b-d}{d-c}\right] \to \mathbb{R}$ *is strictly decreasing with*

$$\lim_{\theta_A \to 0^+} \eta_1(\theta_A) = \Psi(G) \quad \text{and} \quad \eta_1\left(\frac{b - d}{d - c}\right) = 0;$$

- $P_1(\cdot, \theta_A) : [\eta_1(\theta_A), \Psi(G)] \to \mathbb{R}$ *is strictly increasing for every θ_A and*

$$P_1\left(\eta_1(\theta_A), \theta_A\right) = 0 \quad \text{and} \quad P_1\left(\Psi(G), \theta_A\right) = 1. \tag{21}$$

ii) If $\theta_A > \frac{b-d}{d-c}$, then:

$$BR_A^{GPS}(q) = \begin{cases} \{0, 1, P_1(q, \theta_A)\} & \text{if } 0 \leqslant q < \Psi(G), \\ \{0, 1\} & \text{if } q = \Psi(G), \\ \{0\} & \text{if } \Psi(G) < q \leqslant 1, \end{cases} \tag{22}$$

where the function $P_1(\cdot, \theta_A) : [0, \Psi(G)] \to \mathbb{R}$ is strictly increasing for every θ_A and

$$P_1\left(0, \theta_A\right) = 1 - \frac{b - d}{\theta_A(d - c)} \quad \text{and} \quad P_1\left(\Psi(G), \theta_A\right) = 1. \tag{23}$$

Proof. Consider the expected utility function $u_A(\tilde{p}, \theta_A, y, q)$ as defined in (17). Then

(1) If

$$q\Big(\big(1+(1-\tilde{p})\theta_A\big)\big((a+d)-(b+c)\big)\Big)+(b-d)-(1-\tilde{p})\theta_A(d-c)>0, \tag{24}$$

then $u_A(\tilde{p}, \theta_A, (\cdot, q))$ is strictly increasing, therefore it is maximized only by $y = 1$. The consistency condition with the maximum for correct beliefs implies that $\tilde{p} = 1$; it follows that (24) becomes:

$$q\Big((a+d)-(b+c)\Big)+(b-d)>0. \tag{25}$$

Since $(a+d)-(b+c) < 0$ and $d-b < 0$, (25) is equivalent to

$$q < \frac{d-b}{(a+d)-(b+c)} = \Psi(G) < 1.$$

Hence, for $q \in [0, \Psi(G)[$,

$$w_A^{GPS}((1,q),(1,q)) \geqslant w_A^{GPS}((1,q),(y,q)) \qquad \forall y \in [0,1],$$

and $1 \in BR_A^{GPS}(q)$.

There are no other maximizers in this case.

(2) If

$$q\Big(\big(1+(1-\tilde{p})\theta_A\big)\big((a+d)-(b+c)\big)\Big)+(b-d)-(1-\tilde{p})\theta_A(d-c)<0, \tag{26}$$

then $u_A(\tilde{p}, \theta_A, (\cdot, q))$ is strictly decreasing and it is maximized only by $y = 0$. In this case, the consistency condition with the maximum for correct beliefs implies that $\tilde{p} = 0$; it follows that (26) becomes:

$$q\Big(\big(1+\theta_A\big)\big((a+d)-(b+c)\big)\Big)+(b-d)-\theta_A(d-c)<0 \tag{27}$$

or, equivalently, bearing in mind that $(a+d)-(b+c) < 0$,

$$q > \frac{d-b+\theta_A(d-c)}{\big(1+\theta_A\big)\big((a+d)-(b+c)\big)} = \eta_1(\theta_A). \tag{28}$$

Now we have to find the values of $q \in [0,1]$ that satisfy (28). For this purpose firstly observe that:

$$\lim_{\theta_A \to 0^+} \eta_1(\theta_A) = \frac{d-b}{(a+d)-(b+c)} = \Psi(G),$$

and

$$\lim_{\theta_A \to +\infty} \eta_1(\theta_A) = \frac{d-c}{(a+d)-(b+c)} < 0 < \Psi(G).$$

Using the notation in (16), we have that:

$$\eta_1(\theta_A) = \frac{-\delta + \lambda\theta_A}{\gamma(1+\theta_A)}, \qquad \frac{\partial \eta_1(\theta_A)}{\partial \theta_A} = \frac{\lambda+\delta}{\gamma(1+\theta_A)^2} < 0 \quad \forall \theta_A > 0,$$

since $\lambda + \delta = d - c + b - d = b - c > 0$ and $\gamma < 0$. So $\eta_1(\theta_A)$ is strictly decreasing and there exists a unique point $\theta'_A > 0$ such that $\eta_1(\theta'_A) = 0$, that corresponds to

$$\theta'_A = \frac{\delta}{\lambda} = \frac{b-d}{d-c}. \tag{29}$$

Therefore, we get that for $\theta_A \leqslant \theta'_A$, $\eta_1(\theta_A) \geqslant 0$ and (28) is satisfied for all $q \in]\eta_1(\theta_A), 1]$. If $\theta_A > \theta'_A$, $\eta_1(\theta_A) < 0$ and (28) is satisfied for all $q \in [0,1]$.
This finally implies that

i) If $\theta_A \leqslant \frac{b-d}{d-c}$, then, for all $q \in]\eta_1(\theta_A), 1]$,

$$w_A^{GPS}((0,q),(0,q)) \geqslant w_A^{GPS}((0,q),(y,q)) \qquad \forall y \in [0,1],$$

and $0 \in BR_A^{GPS}(q)$.

ii) If $\theta_A > \frac{b-d}{d-c}$, then, for all $q \in [0,1]$,

$$w_A^{GPS}((0,q),(0,q)) \geqslant w_A^{GPS}((0,q),(y,q)) \qquad \forall y \in [0,1],$$

and $0 \in BR_A^{GPS}(q)$.

There are no other maximizers in this case.

(3) If

$$q\Big(\big(1+(1-\tilde{p})\theta_A\big)\big((a+d)-(b+c)\big)\Big) + (b-d) - (1-\tilde{p})\theta_A(d-c) = 0, \tag{30}$$

then $u_A(\tilde{p}, \theta_A, (y,q))$ is constant with respect to y, therefore every $y \in [0,1]$ maximizes $u_A(\tilde{p}, \theta_A, (\cdot, q))$. Solving for $\tilde{p}$ in (30) we get:

$$\tilde{p} = 1 - \frac{b-d+q[(a+d)-(b+c)]}{\theta_A(d-c-q[(a+d)-(b+c)])} := P_1(q, \theta_A).$$

So, in this case, $p = P_1(q,\theta_A)$ is the unique Ann's best reply, provided that $P_1(q,\theta_A) \in [0,1]$. Now, from (16), $P_1(q,\theta_A)$ can be rewritten as

$$P_1(q,\theta_A) = 1 - \frac{\delta + q\gamma}{\theta_A(\lambda - q\gamma)}.$$

Then:

$$\frac{\partial P_1(q,\theta_A)}{\partial q} = -\frac{\gamma(\lambda+\delta)}{\theta_A(\lambda-q\gamma)^2} > 0 \quad \forall \theta_A > 0,$$

as $\gamma < 0$ and $\lambda + \delta = b - c > 0$.
So $P_1(\cdot, \theta_A)$ is strictly increasing for every $\theta_A > 0$. Now,

$$P_1(q, \theta_A) = 1 \iff q = -\frac{\delta}{\gamma} = \frac{d-b}{(a+d)-(b+c)} = \Psi(G) < 1.$$

Moreover,

$$P_1(0, \theta_A) = 1 - \frac{\delta}{\lambda \theta_A} > 0 \iff \theta_A > \frac{\delta}{\lambda} = \frac{b-d}{d-c}.$$

So, if $\theta_A > \frac{b-d}{d-c}$, then

$$P_1(q, \theta_A) \in [0,1] \quad \forall q \in [0, \Psi(G)]$$

and

$$P_1(0, \theta_A) = 1 - \frac{b-d}{\theta_A(d-c)} > 0. \tag{31}$$

If $\theta_A \leqslant \frac{b-d}{d-c}$, there exists a unique point $\overline{q}(\theta_A)$ such that:

$$P_1(\overline{q}(\theta_A), \theta_A) = 1 - \frac{b-d+\overline{q}(\theta_A)[(a+d)-(b+c)]}{\theta_A(d-c-\overline{q}(\theta_A)[(a+d)-(b+c)])} = 0.$$

It can be easily computed that

$$\overline{q}(\theta_A) = \frac{\theta_A(d-c)+(d-b)}{(1+\theta_A)[(a+d)-(b+c)]} = \eta_1(\theta_A)$$

and that $\eta_1(\theta_A) \geqslant 0$ for every $\theta_A \leqslant \frac{b-d}{d-c}$. Hence, in this case

$$P_1(q, \theta_A) \in [0,1] \quad \forall q \in [\eta_1(\theta_A), \Psi(G)]\,.$$

Summarizing:

i) If $\theta_A \leqslant \frac{b-d}{d-c}$, then, for all $q \in [\eta_1(\theta_A), \Psi(G)]$,

$$w_A^{GPS}((P_1(q,\theta_A), q), (P_1(q,\theta_A), q)) \geqslant w_A^{GPS}((P_1(q,\theta_A), q), (y, q)) \qquad \forall y \in [0,1],$$

and $P_1(q, \theta_A) \in BR_A^{GPS}(q)$. Moreover

$$P_1(\eta_1(\theta_A), \theta_A) = 0, \quad P_1(\Psi(G), \theta_A) = 1.$$

ii) If $\theta_A > \frac{b-d}{d-c}$, then for all $q \in [0, \Psi(G)]$,

$$w_A^{GPS}((P_1(q,\theta_A), q), (P_1(q,\theta_A), q)) \geqslant w_A^{GPS}((P_1(q,\theta_A), q), (y, q)) \qquad \forall y \in [0,1],$$

and $P_1(q, \theta_A) \in BR_A^{GPS}(q)$. Moreover

$$P_1(0, \theta_A) = 1 - \frac{b-d}{\theta_A(d-c)} > 0, \quad P_1(\Psi(G), \theta_A) = 1.$$

□

Bob's best reply correspondence

Bob's expected utility from playing the mixed strategy y, expecting Ann playing the mixed strategy p and having second order belief $\tilde{q}$ is

$$u_B(\tilde{q}, \theta_A, p, y) =$$

$$y\left[p\Big(\big(1+(1-\tilde{q})\theta_B\big)\big((a+d)-(b+c)\big)\Big)+(b-d)-(1-\tilde{q})\theta_B(d-c)\right]+cp+d(1-p). \quad (32)$$

It follows that Bob's expected utility is substantially the same of Ann's one when we replace q with p, $\tilde{p}$ with $\tilde{q}$ and θ_A with θ_B. Using the same arguments we can easily deduce Bob's best reply correspondence. For the sake of completeness we report a complete characterization of the Bob's best reply correspondence in the proposition below.

PROPOSITION 5.2: *Let G be an Hawk-Dove game ($a < c$, $d < b$). Assume that $a < b$, $c < d$ and denote with:*

$$Q_1(p, \theta_B) = 1 - \frac{b-d+p[(a+d)-(b+c)]}{\theta_B(d-c-p[(a+d)-(b+c)])}. \quad (33)$$

Let η_1 be defined as in Proposition 5.1. Then, Bob's best reply correspondence is given by the following:
i) If $\theta_B \leqslant \frac{b-d}{d-c}$, then:

$$BR_B^{GPS}(p) = \begin{cases} 1 & \text{if } 0 \leqslant p < \eta_1(\theta_B), \\ \{0,1\} & \text{if } p = \eta_1(\theta_B), \\ \{0,1,Q_1(p,\theta_B)\} & \text{if } \eta_1(\theta_B) < p < \Psi(G), \\ \{0,1\} & \text{if } p = \Psi(G), \\ 0 & \text{if } \Psi(G) < p \leqslant 1, \end{cases} \quad (34)$$

where

- $\eta_1 : \left]0, \frac{b-d}{d-c}\right] \to \mathbb{R}$ *is strictly decreasing with*

$$\lim_{\theta_B \to 0^+} \eta_1(\theta_B) = \Psi(G) \quad \text{and} \quad \eta_1\left(\frac{b-d}{d-c}\right) = 0;$$

- $Q_1(\cdot, \theta_B) : [\eta_1(\theta_B), \Psi(G)] \to \mathbb{R}$ *is strictly increasing and*

$$Q_1\left(\eta_1(\theta_B), \theta_B\right) = 0 \quad \text{and} \quad Q_1\left(\Psi(G), \theta_B\right) = 1. \quad (35)$$

ii) If $\theta_B > \frac{b-d}{d-c}$, then:

$$BR_B^{GPS}(p) = \begin{cases} \{0,1,Q_1(p,\theta_B)\} & \text{if } 0 \leqslant p < \Psi(G), \\ \{0,1\} & \text{if } p = \Psi(G), \\ \{0\} & \text{if } \Psi(G) < p \leqslant 1, \end{cases} \quad (36)$$

where $Q_1(\cdot, \theta_B) : [0, \Psi(G)] \to \mathbb{R}$ *is strictly increasing and*

$$Q_1(0, \theta_B) = 1 - \frac{b-d}{\theta_B(d-c)} \quad \text{and} \quad Q_1(\Psi(G), \theta_B) = 1. \tag{37}$$

Equilibrium analysis

Making use of the best reply correspondences computed in the previous subsection, in this section we analyze the set of psychological Nash equilibria in the different cases. Firstly, we will provide a characterization of equilibria in mixed strategies that will be useful.

Characterization of equilibria in mixed strategies

From the structure of the best reply correspondences, we get that a completely mixed strategy profile (p^*, q^*) is an equilibrium if and only if it is a solution of the following system:

$$\begin{cases} p = P_1(q, \theta_A) = 1 - \dfrac{\delta + q\gamma}{\theta_A(\lambda - q\gamma)}, & (i) \\ \\ q = Q_1(p, \theta_B) = 1 - \dfrac{\delta + p\gamma}{\theta_B(\lambda - p\gamma)}. & (ii) \end{cases} \tag{38}$$

From equation (ii), we obtain

$$p = \frac{\lambda\theta_B(1-q) - \delta}{\gamma(1 + \theta_B(1-q))} := I_1(q, \theta_B).$$

Let $D_1 : E \to \mathbb{R}$ be the function defined by:

$$D_1(q) = P_1(q, \theta_A) - I_1(q, \theta_B),$$

where E is the intersection of the domain of $P_1(q, \theta_A)$ with the image set through $Q_1(p, \theta_A)$ of its domain.

It follows that a completely mixed strategy profile (p^*, q^*) is equilibrium if and only if q^* is a zero for D_1, that is, $D_1(q^*) = 0$. We have:

$$\frac{\partial P_1(q, \theta_A)}{\partial q} = -\frac{\gamma(\lambda + \delta)}{\theta_A(\lambda - q\gamma)^2}; \qquad \frac{\partial I_1(q, \theta_A)}{\partial q} = -\frac{\theta_B(\lambda + \delta)}{\gamma(1 + \theta_B(1-q))^2};$$

So,

$$\frac{\partial^2 P_1(q, \theta_A)}{\partial q^2} = -\frac{2\gamma^2(\lambda + \delta)}{\theta_A(\lambda - q\gamma)^3} < 0 \quad \text{and} \quad \frac{\partial^2 I_1(q, \theta_A)}{\partial q^2} = -\frac{2\theta_B^2(\lambda + \delta)}{\gamma(1 + \theta_B(1-q))^3} > 0 \quad \forall q \in [0, 1]$$

being $\theta_A, \theta_B, \lambda, \delta > 0$, $\gamma < 0$, $q \in [0, 1]$. So D_1 is twice differentiable with

$$\frac{\partial^2 D_1(q, \theta_A)}{\partial q^2} = \frac{\partial^2 P_1(q, \theta_A)}{\partial q^2} - \frac{\partial^2 I_1(q, \theta_A)}{\partial q^2} < 0 \quad \forall q \in [0, 1]$$

implying that D_1 is strictly concave.

Threshold parameters

The previous propositions show that the structure of the best reply correspondences depends on

$$\theta_A, \theta_B \gtrless \frac{b-d}{d-c}.$$

Note that:

- $b-d$ represents the absolute value of the loss incurred by Ann (resp. Bob) when she (resp. he) unilaterally deviates from the pure Nash equilibrium to the Pareto undominated symmetric profile (L, L).
- $d-c$ represents the absolute value of the gain obtained by Bob (resp. Ann) when Ann (resp. Bob) unilaterally deviates from the pure Nash equilibrium to the Pareto undominated symmetric profile (L, L).

Therefore, from Ann's point of view, the ratio $\frac{b-d}{d-c}$ captures her losses (resp. gains) relative to Bob's gains (resp. losses) when Ann chooses H (resp. L) as the best response to Bob's choice of L. The same reasoning applies symmetrically to Bob.

Equilibria in case $0 < \theta_A,\, \theta_B \leqslant \dfrac{b-d}{d-c}$

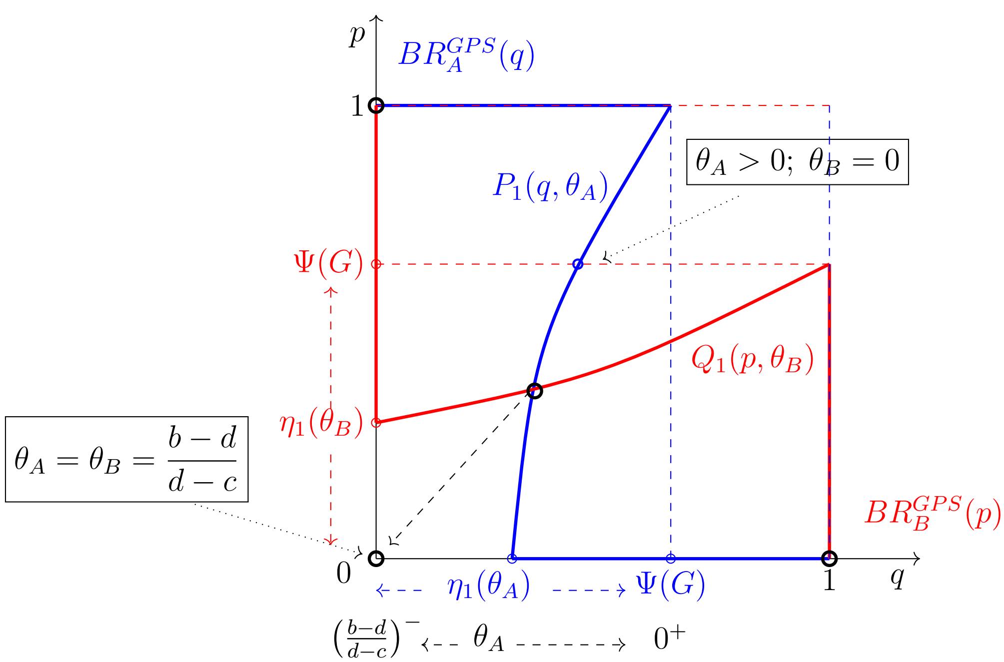


Figure 1: Case $0 < \theta_A,\, \theta_B \leqslant \dfrac{b-d}{d-c}$

In Figure 1, the best reply correspondences are illustrated. It is clear that the two pure strategy equilibria $(1,0) = (H, L)$ and $(0,1) = (L, H)$ persist under guilt aversion. It is also evident that there are no other equilibria in which one of the strategies is pure. When $\theta_A, \theta_B < \frac{b-d}{d-c}$, there exists a unique equilibrium in completely mixed strategies. This follows from the fact that the function D_1 has a unique zero in the interval $]\eta_1(\theta_A), \Psi(G)[$. This conclusion is based on the following arguments:

1) $D_1(\eta_1(\theta_A)) < 0$ (since $P_1(\eta_1(\theta_A), \theta_A) = 0$ and $I_1(\eta_1(\theta_A), \theta_B) > 0$), and $D_1(\Psi(G)) > 0$ (since $P_1(\Psi(G), \theta_A) = 1$ and $I_1(\Psi(G), \theta_B) < 1$); by the continuity of D_1, this implies the existence of at least one zero in $]\eta_1(\theta_A), \Psi(G)[$.

2) D_1 cannot have more than one zero in this interval, otherwise it would have a local minimum between them, contradicting the fact that D_1 is twice differentiable and its second derivative is always negative, as shown above.

The figure demonstrates the effect of guilt aversion on the completely mixed strategy equilibrium: both components are lower than $\Psi(G)$, implying that both players assign a higher probability to the pure strategy L. As θ_A, θ_B approach $\frac{b-d}{d-c}$, the equilibrium converges to $(0,0)$, which corresponds to (L, L). In fact, when $\theta_A, \theta_B = \frac{b-d}{d-c}$, it holds that $\eta_1(\theta_A) = \eta_1(\theta_B) = 0$, so D_1 has a unique zero at $q = 0$, making (L, L) a new pure strategy equilibrium, with no other mixed strategy equilibria.

As a final remark, guilt aversion affects only the opponent's equilibrium strategy. This is clearly seen in the case where one player, say Bob, is not affected by guilt aversion ($\theta_B = 0$), while Ann is ($\theta_A > 0$). In this scenario, the equilibrium is identified in Figure 1 as the intersection between the graph of $P_1(q, \theta_A)$ (in blue) and the horizontal dotted line (in red) at level $\Psi(G)$. In this equilibrium, Ann's strategy is exactly $p^* = \Psi(G)$, while Bob's equilibrium strategy is $q^* < \Psi(G)$. It follows that Bob has to play L with a larger probability to compensate Ann's disutility from guilt.

Equilibria in case $\theta_A, \theta_B > \dfrac{b-d}{d-c}$

The scenario is now illustrated in Figure 2. First, observe that a completely mixed strategy equilibrium may occur if and only if the function D_1 admits a zero in the open interval $]Q_1(0, \theta_B), \Psi(g)[$. However, this condition cannot be satisfied because:
1) D_1 attains a positive value at the extreme points of the interval; in fact, $D_1(Q_1(0, \theta_B)) > 0$ (since $P_1(Q_1(0, \theta_B), \theta_A) > 0$ and $I_1(Q_1(0, \theta_B), \theta_B) = 0$) and $D_1(Q_1(0, \theta_B)) > 0$ also holds as $P_1(Q_1(0, \theta_B), \theta_A) = 1$ and $I_1(Q_1(0, \theta_B), \theta_B) < 1$. Since D_1 attains a positive value at the extreme points of the interval, then D_1 cannot have a unique zero in the interval.
2) If D_1 had more than one zero, it would require the existence of a local minimum. However, this is impossible (as previously explained), because the second derivative is strictly negative.

So, there are five equilibria in which at least one player chooses a pure strategy. In fact, in addition to the three pure strategy equilibria from the previous case ((H, L), (L, H), and (L, L)), two new equilibria appear: $(p, q) = (0, Q_1(0, \theta_B))$ (resp. $(p, q) = (P_1(0, \theta_A), 0)$), in which Ann

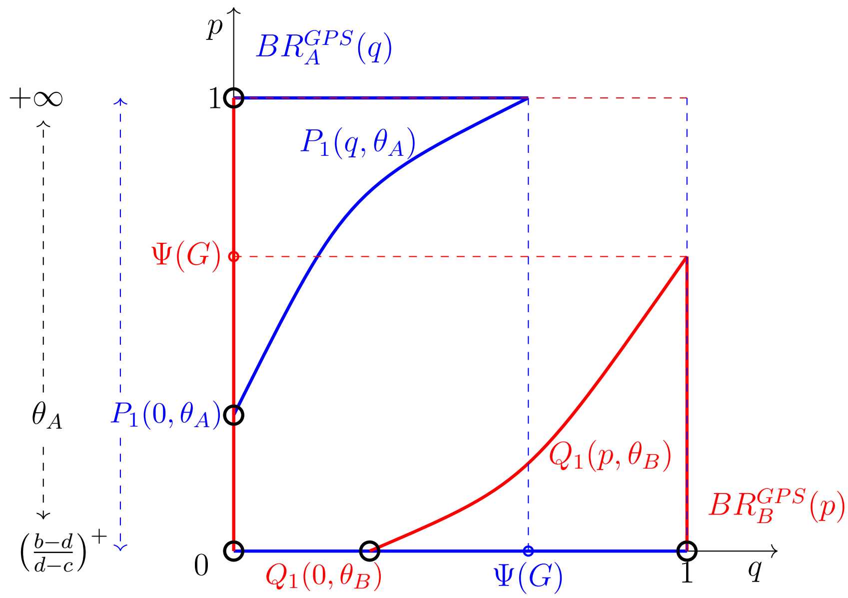


Figure 2: Case $\theta_A, \theta_B > \dfrac{b-d}{d-c}$

(resp. Bob) plays L and Bob (resp. Ann) randomizes with probability $Q_1(0,\theta_B)$ (resp. $P_1(0,\theta_A)$). Note also that $Q_1(0,\theta_B)$ (resp. $P_1(0,\theta_A)$) tends to 1 as θ_B (resp. θ_A) diverges to infinity.

Equilibria in case $0 < \theta_B \leqslant \dfrac{b-d}{d-c} < \theta_A$

This scenario corresponds to the asymmetric situation in which one player (Bob in this case) has a low sensitivity to guilt aversion, i.e. $\theta_B \leqslant \dfrac{b-d}{d-c}$ while Ann has high sensitivity, i.e. $\dfrac{b-d}{d-c} < \theta_A$.

We first observe that, in general, $P_1(0,\theta_A)$ can be greater, smaller or equal to $\eta_1(\theta_B)$ as the following example shows.

Example 5.3: Consider the Type 1 game $a = 1$, $c = 2$, $d = 3$, $b = 4$ so that

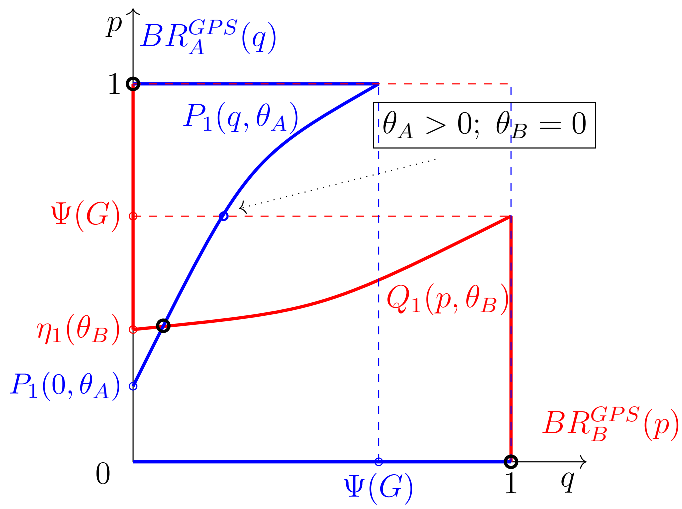
$$\theta_B < 1 = \frac{b-d}{d-c} < \theta_A.$$

We get:

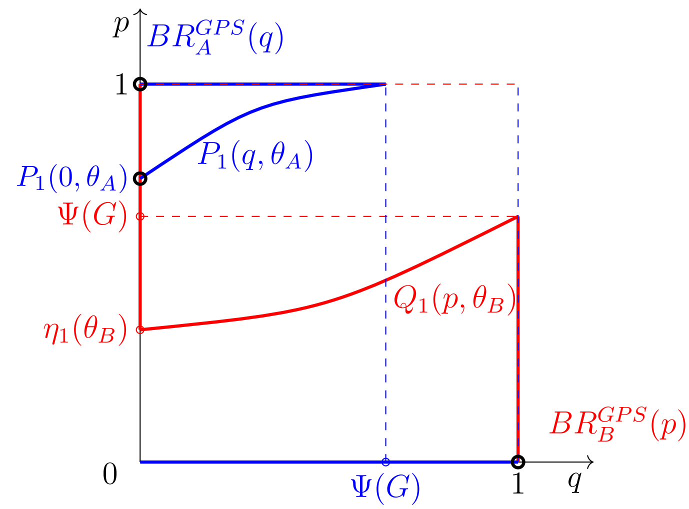
$$P_1(0,\theta_A) = 1 - \frac{1}{\theta_A} \quad \text{and} \quad \eta_1(\theta_B) = -\frac{\theta_B - 1}{2(1+\theta_B)}$$

So, $P_1(0,2) = 1/2 < \eta_1(1/6) = 5/14 < P_1(0,5) = 4/5$.

Therefore, we can have both the situations described in Figures (3a) and (3b). Observe first that (H,H) and (L,L) are equilibria in both cases. In case $P_1(0,\theta_A) < \eta_1(\theta_B)$, following the same steps of the case in Figure 1, we get that D_1 has a unique zero in $]0,\psi(G)[$ which gives the unique equilibrium in completely mixed strategies. In case $P_1(0, \theta_A) > \eta_1(\theta_B)$, following the same steps of the case in Figure 2, we observe that D_1 does not have zeros, so there are no equilibria in completely mixed strategies, but there is an equilibrium in which Bob plays L and Ann randomizes with probability $P_1(0, \theta_A)$. When $P_1(0, \theta_A) = \eta_1(\theta_B)$, this latter equilibrium persist providing a zero for D_1, as, in this case, we have $D_1(0) = 0$. Finally, note that the case $\theta_B = 0$ affects only the case $P_1(0, \theta_A) < \eta_1(\theta_B)$; in this case we have a completely mixed strategy equilibrium $(p, q) = (\Psi(G), q)$ with $q < \Psi(G)$)



(a) Subcase $P_1(0, \theta_A) < \eta_1(\theta_B)$



(b) Subcase $P_1(0, \theta_A) > \eta_1(\theta_B)$

Figure 3: Case $0 < \theta_B \leqslant \dfrac{b-d}{d-c} < \theta_A$

5.2 Type 2: $a < b$; $d < c$

In this case, the strategic form game is the following:

Type 2:	$a<b,\ d<c$	
Bob / Ann	H	L
H	$a-\theta_A(1-\tilde{p})(b-a)$, $a-\theta_B(1-\tilde{q})(b-a)$	b, c
L	c, b	$d-\theta_A\tilde{p}(c-d)$, $d-\theta_B\tilde{q}(c-d)$

Ann's best reply correspondence

Ann's expected utility, from playing the mixed strategy y, expecting Bob playing the mixed strategy q and having second order belief $\tilde{p}$, in this case is

$$u_A(\tilde{p},\theta_A,y,q)=$$

$$y\left[q\Big((a+d)-(b+c)+\theta_A(a-b)+\theta_A\tilde{p}\big(b+d-a-c\big)\Big)+b-d+\tilde{p}\theta_A\big(c-d\big)\right]+$$

$$(1-q)\big(d-\theta_A\tilde{p}(c-d)\big)+cq \tag{39}$$

PROPOSITION 5.4: *Let G be an Hawk-Dove game ($a<c,\ d<b$). Assume that $a<b,\ d<c$ and denote with*

$$\eta_2(\theta_A)=\frac{d-b+\theta_A\big(d-c\big)}{(a+d)-(b+c)+\theta_A\big(d-c\big)}, \tag{40}$$

$$\tag{41}$$

$$\eta_3(\theta_A)=\frac{d-b}{(a+d)-(b+c)+\theta_A(a-b)}, \tag{42}$$

$$\tag{43}$$

$$P_2(q,\theta_A)=-\frac{q[(a+d)-(b+c)+\theta_A(a-b)]+(b-d)}{\theta_A\Big(q(b+d-a-c)+c-d\Big)}. \tag{44}$$

Then, for every $\theta_A>0$, Ann's best reply correspondence is given by the following:

$$BR_A^{GPS}(q) = \begin{cases} 1 & \textit{if } 0 \leqslant q < \eta_3(\theta_A), \\ \{0,1\} & \textit{if } q = \eta_3(\theta_A), \\ \{0,1,P_2(q,\theta_A)\} & \textit{if } \eta_3(\theta_A) < q < \eta_2(\theta_A), \\ \{0,1\} & \textit{if } q = \eta_2(\theta_A), \\ 0 & \textit{if } \eta_2(\theta_A) < q \leqslant 1, \end{cases} \tag{45}$$

where

- $\eta_2 :]0, +\infty[\to \mathbb{R}$ *is strictly increasing and*

$$\lim_{\theta_A \to 0^+} \eta_2(\theta_A) = \Psi(G); \qquad \lim_{\theta_A \to +\infty} \eta_2(\theta_A) = 1.$$

- $\eta_3 :]0, +\infty[\to \mathbb{R}$ *is strictly decreasing and*

$$\lim_{\theta_A \to 0^+} \eta_3(\theta_A) = \Psi(G); \qquad \lim_{\theta_A \to +\infty} \eta_3(\theta_A) = 0.$$

- $P_2(\cdot, \theta_A) : [\eta_3(\theta_A), \eta_2(\theta_A)] \to \mathbb{R}$ *is strictly increasing and*

$$P_2\left(\eta_3(\theta_A), \theta_A\right) = 0 \quad \text{and} \quad P_2\left(\eta_2(\theta_A), \theta_A\right) = 1. \tag{46}$$

Proof. First note that from (16) we get:

$$\lambda < 0, \quad \mu > 0.$$

Consider the utility defined in (39). Then:

(1) If

$$q\Big((a+d) - (b+c) + \theta_A(a-b) + \theta_A\tilde{p}\big(b+d-a-c\big)\Big) + b - d + \tilde{p}\theta_A\big(c-d\big) > 0, \tag{47}$$

then $u_A(\tilde{p}, \theta_A, (\cdot, q))$ is strictly increasing, therefore it is maximized only by $y = 1$. The consistency condition with the maximum for correct beliefs implies that $\tilde{p} = 1$; it follows that (47) becomes:

$$q\Big((a+d) - (b+c) + \theta_A\big(d-c\big)\Big) + b - d + \theta_A\big(c-d\big) > 0 \tag{48}$$

Since $(a+d) - (b+c) < 0$, $d - c < 0$ and $d - b < 0$, (48) is equivalent to

$$q < \frac{d - b + \theta_A\big(d-c\big)}{(a+d) - (b+c) + \theta_A\big(d-c\big)} = \eta_2(\theta_A).$$

It can be easily checked that

$$\eta_2(\theta_A) \in]0,1[\quad \forall \theta_A > 0.$$

Moreover

$$\lim_{\theta_A \to 0^+} \eta_2(\theta_A) = \Psi(G); \qquad \lim_{\theta_A \to +\infty} \eta_2(\theta_A) = 1.$$

Since $\eta_2(\theta_A)$ can be rewritten as

$$\eta_2(\theta_A) = \frac{-\delta + \theta_A \lambda}{\gamma + \theta_A \lambda},$$

we get

$$\frac{\partial \eta_2(\theta_A)}{\partial \theta_A} = \frac{\lambda(\gamma+\delta)}{(\gamma+\theta_A\lambda)^2} > 0 \iff \lambda(\gamma+\delta) = (d-c)(a-c) < 0,$$

where the latter inequality is satisfied because $d - c < 0$, and $(a - c) < 0$. It follows that $\eta_2(\theta_A)$ is strictly increasing in $]0, +\infty[$. Hence, for $q \in [0, \eta_2(\theta_A)[$,

$$w_A^{GPS}((1,q),(1,q)) \geqslant w_A^{GPS}((1,q),(y,q)) \qquad \forall y \in [0,1],$$

and $1 \in BR_A^{GPS}(q)$.

There are no other maximizers in this case.

(2) If

$$q\Big((a+d)-(b+c)+\theta_A(a-b)+\theta_A\tilde{p}\big(b+d-a-c\big)\Big)+b-d+\tilde{p}\theta_A\big(c-d\big) < 0, \tag{49}$$

then $u_A(\tilde{p}, \theta_A, (\cdot, q))$ is strictly decreasing and attains a maximum point only in $y = 0$. The consistency condition with the maximum for correct beliefs implies that $\tilde{p} = 0$. Then, (49) becomes:

$$q\Big((a+d)-(b+c)+\theta_A(a-b)\Big)+b-d < 0.$$

Since $(a + d) - (b + c) + \theta_A(a - b) < 0$ for all $\theta_A > 0$ and $d - b < 0$, it can be easily checked that (49) is equivalent to

$$q > \frac{d-b}{(a+d)-(b+c)+\theta_A(a-b)} = \eta_3(\theta_A)$$

with

$$\eta_3(\theta_A) \in]0,1[\quad \forall \theta_A \in]0,+\infty[, \qquad \lim_{\theta_A \to 0^+} \eta_3(\theta_A) = \Psi(G), \qquad \lim_{\theta_A \to +\infty} \eta_3(\theta_A) = 0.$$

Finally, it immediately follows that $\eta_3(\theta_A)$ is strictly decreasing in $]0, +\infty[$. Therefore, for $q \in]\eta_3(\theta_A), 1]$,

$$w_A^{GPS}((0,q),(0,q)) \geqslant w_A^{GPS}((0,q),(y,q)) \qquad \forall y \in [0,1],$$

and $0 \in BR_A^{GPS}(q)$.

(3) If

$$q\Big((a+d)-(b+c)+\theta_A(a-b)+\theta_A\tilde{p}\big(b+d-a-c\big)\Big)+b-d+\tilde{p}\theta_A\big(c-d\big)=0, \qquad (50)$$

then $u_A(\tilde{p}, \theta_A, (y, q)$ is constant with respect to y, therefore every $y \in [0, 1]$ maximizes $u_A(\tilde{p}, \theta_A, (\cdot, q))$. Solving for $\tilde{p}$ in (50) we get:

$$\tilde{p} = -\frac{q[(a+d)-(b+c)+\theta_A(a-b)]+(b-d)}{\theta_A\Big(q(b+d-a-c)+c-d\Big)} := P_2(q, \theta_A).$$

Now:

$$P_2(q,\theta_A)=0 \iff q[(a+d)-(b+c)+\theta_A(a-b)]+(b-d)=0 \iff$$

$$q = \frac{d-b}{(a+d)-(b+c)+\theta_A(a-b)} = \eta_3(\theta_A),$$

and

$$P_2(q,\theta_A)=1 \iff q[(a+d)-(b+c)+\theta_A(a-b)]+(b-d)+\theta_A\Big(q(b+d-a-c)+c-d\Big)=0 \iff$$

$$q = \frac{d-b+\theta_A(d-c)}{(a+d)-(b+c)+\theta_A(d-c)} = \eta_2(\theta_A).$$

Moreover, using notation in (16), $P_2(q, \theta_A)$ can be rewritten as:

$$P_2(q,\theta_A) = -\frac{q(\gamma-\mu\theta_A)+\delta}{\theta_A\Big(q(\mu+\lambda)-\lambda\Big)}.$$

We have that:

$$\frac{\partial P_2(q,\theta_A)}{\partial q} = -\frac{1}{\theta_A}\frac{\partial}{\partial q}\left[\frac{q(\gamma-\mu\theta_A)+\delta}{\Big(q(\mu+\lambda)-\lambda\Big)}\right] = -\frac{1}{\theta_A}\left[\frac{-\gamma\lambda+\theta_A\mu\lambda-\delta(\mu+\lambda))}{\Big(q(\mu+\lambda)-\lambda\Big)^2}\right]$$

From the assumptions we know that $\delta > 0$, $\mu > 0$, $\lambda, \gamma < 0$, therefore it follows that $\mu\lambda < 0$ and

$$\frac{\partial P_2(q,\theta_A)}{\partial q} > 0 \iff \theta_A > \frac{\gamma\lambda+\delta(\mu+\lambda)}{\mu\lambda}.$$

Now observe that $\delta\mu > 0$ and $\gamma + \delta = a - c < 0$. It follows that $\lambda(\gamma + \delta) > 0$ and

$$\frac{\gamma\lambda+\delta(\mu+\lambda)}{\mu\lambda} = \frac{\lambda(\gamma+\delta)+\delta\mu}{\mu\lambda} < 0;$$

so, $P_2(\cdot, \theta_A)$ is strictly increasing in the interval $[\eta_3(\theta_A), \eta_2(\theta_A)]$ for every $\theta_A > 0$.

□

Bob's best reply correspondence

As already noticed in the previous Type 1 section, Bob's expected utility is substantially the same of Ann's one when we replace q with p, $\tilde{p}$ with $\tilde{q}$ and θ_A with θ_B, so we can easily deduce Bob's best reply correspondence. For the sake of completeness, we report it in the proposition below.

Proposition 5.5: *Let G be an Hawk-Dove game ($a < c$, $d < b$). Assume that $a < b$, $d < c$ and let η_2 and η_3 defined as in Proposition 5.4. Let*

$$Q_2(p, \theta_B) = -\frac{p[(a+d)-(b+c)+\theta_B(a-b)]+(b-d)}{\theta_B\Big(p(b+d-a-c)+c-d\Big)} \tag{51}$$

Then, for every $\theta_B > 0$, Bob's best reply correspondence is given by the following:

$$BR_B^{GPS}(p) = \begin{cases} 1 & \text{if } 0 \leqslant p < \eta_3(\theta_B) \\ \{0,1\} & \text{if } p = \eta_3(\theta_B) \\ \{0,1,Q_2(p,\theta_B)\} & \text{if } \eta_3(\theta_B) < p < \eta_2(\theta_B) \\ \{0,1\} & \text{if } p = \eta_2(\theta_B), \\ 0 & \text{if } \eta_2(\theta_B) < p \leqslant 1, \end{cases} \tag{52}$$

where

- $\eta_2 :]0, +\infty[\to \mathbb{R}$ *is strictly increasing and*

$$\lim_{\theta_B \to 0^+} \eta_2(\theta_B) = \Psi(G); \qquad \lim_{\theta_B \to +\infty} \eta_2(\theta_B) = 1.$$

- $\eta_3 :]0, +\infty[\to \mathbb{R}$ *is strictly decreasing and*

$$\lim_{\theta_B \to 0^+} \eta_3(\theta_B) = \Psi(G); \qquad \lim_{\theta_B \to +\infty} \eta_3(\theta_B) = 0.$$

- $Q_2(\cdot, \theta_B) : [\eta_3(\theta_B), \eta_2(\theta_B)] \to \mathbb{R}$ *is strictly increasing and*

$$Q_2\left(\eta_3(\theta_B), \theta_B\right) = 0 \quad \text{and} \quad Q_2\left(\eta_2(\theta_B), \theta_B\right) = 1. \tag{53}$$

Equilibrium analysis

In this section, following the steps provided for Type 1 games, we give a characterization of the equilibria in completely mixed strategies in Type 2 games. Then, we analyze the equilibria of the game. In particular, we will show that, regardless of the sensitivity parameters θ_A, θ_B, the two pure strategy equilibria of the game (H, L) and (L, H) survive to guilt aversion. Moreover, one equilibrium in completely mixed strategies, which depends on θ_A, θ_B, always exists.

Characterization of equilibria in mixed strategies

From the structure of the best reply correspondences, we get that a completely mixed strategy profile (p^*, q^*) is an equilibrium if and only if it is a solution of the following system:

$$\begin{cases} p = P_2(q, \theta_A) = -\dfrac{q(\gamma - \mu\theta_A) + \delta}{\theta_A\Big(q(\mu+\lambda) - \lambda\Big)}, & (i) \\ q = Q_2(p, \theta_B) = -\dfrac{p(\gamma - \mu\theta_B) + \delta}{\theta_B\Big(p(\mu+\lambda) - \lambda\Big)}. & (ii) \end{cases} \tag{54}$$

From equation (ii), we obtain

$$p = \frac{q\lambda\theta_B - \delta}{\gamma - \mu\theta_B + q\theta_B(\mu+\lambda)} := I_2(q, \theta_B).$$

Let $D_2 : [\eta_3(\theta_A), \eta_2(\theta_A)] \to \mathbb{R}$ be the function defined by:

$$D_2(q) = P_2(q, \theta_A) - I_2(q, \theta_B),$$

It follows that a completely mixed strategy profile (p^*, q^*) is equilibrium if and only if q^* is a zero for D_2, that is, $D_2(q^*) = 0$.

Lemma 5.6: *For every $\theta_A, \theta_B > 0$, there exists a unique point $q^* \in]\eta_3(\theta_A), \eta_2(\theta_A)[$ such that $D_2(q^*) = 0$. Moreover:*

$$\begin{cases} \mu + \lambda > 0 \implies \frac{\partial^2 P_2(q,\theta_A)}{\partial q^2} < 0,\ \frac{\partial^2 I_2(q,\theta_A)}{\partial q^2} > 0,\ \frac{\partial^2 D_2(q,)}{\partial q^2} < 0 & \forall q \in]\eta_3(\theta_A), \eta_2(\theta_A)[, \\ \mu + \lambda < 0 \implies \frac{\partial^2 P_2(q,\theta_A)}{\partial q^2} > 0,\ \frac{\partial^2 I_2(q,\theta_A)}{\partial q^2} < 0,\ \frac{\partial^2 D_2(q)}{\partial q^2} > 0 & \forall q \in]\eta_3(\theta_A), \eta_2(\theta_A)[, \\ \mu + \lambda = 0 \implies \frac{\partial^2 P_2(q,\theta_A)}{\partial q^2} = \frac{\partial^2 I_2(q,\theta_A)}{\partial q^2} = \frac{\partial^2 D_2(q)}{\partial q^2} = 0 & \forall q \in]\eta_3(\theta_A), \eta_2(\theta_A)[. \end{cases} \tag{55}$$

Proof. It can be immediately checked that D_2 is twice differentiable with:

$$\frac{\partial P_2(q, \theta_A)}{\partial q} = \frac{\lambda(\gamma - \theta_A\mu) + \delta(\mu+\lambda)}{\theta_A\Big(q(\mu+\lambda) - \lambda\Big)^2}, \qquad \frac{\partial I_2(q, \theta_A)}{\partial q} = \frac{\theta_B(\delta\mu + (\gamma+\delta)\lambda - \mu\lambda\theta_B)}{(\gamma - \mu\theta_B + q\theta_B(\mu+\lambda))^2},$$

and

$$\frac{\partial^2 P_2(q, \theta_A)}{\partial q^2} = \frac{2(\mu+\lambda)\left(\lambda\theta_A\mu - \lambda(\gamma+\delta) - \delta\mu\right)}{\theta_A\Big(q\mu + (q-1)\lambda\Big)^3}, \tag{56}$$

$$\frac{\partial^2 I_2(q, \theta_A)}{\partial q^2} = \frac{2(\mu+\lambda)\theta_B^2(\lambda\mu\theta_B - \lambda(\gamma+\delta) - \mu\delta)}{(\gamma + q\theta_B\lambda + (q-1)\mu\theta_B)^3}. \tag{57}$$

Now, the denominator in (56) is positive for every q as $q\mu \geqslant 0$ and $(q-1)\lambda \geqslant 0$ and they cannot be equal to 0 simultaneously. Moreover,

$$\lambda\mu\theta_A - \lambda(\gamma+\delta) - \mu\delta < 0 \tag{58}$$

as $\delta, \mu > 0$, $\lambda, \gamma < 0$ and $\gamma + \delta = a - c < 0$, so that $\mu\lambda < 0$, $\delta\mu > 0$ and $\lambda(\gamma+\delta) > 0$. Therefore

$$\frac{\partial^2 P_2(q,\theta_A)}{\partial q^2} \gtreqless 0 \iff \mu + \lambda \lesseqgtr 0.$$

Similarly, the denominator in (57) is negative for every q as $\gamma < 0$, $q\lambda \leqslant 0$ and $(q-1)\mu \leqslant 0$. Then, from (58), it follows that

$$\frac{\partial^2 I_2(q,\theta_A)}{\partial q^2} \gtreqless 0 \iff \mu + \lambda \gtreqless 0.$$

Hence, (55) follows. Moreover

$$P_2(\eta_3(\theta_A),\theta_A) = 0;\ I_2(\eta_3(\theta_A),\theta_A) > 0 \implies D_2(\eta_3(\theta_A)) < 0$$

and

$$P_2(\eta_2(\theta_A),\theta_A) = 1;\ I_2(\eta_3(\theta_A),\theta_A) < 1 \implies D_2(\eta_2(\theta_A)) > 0;$$

Consequently, there exists at least a zero for D_2 in $]\eta_3(\theta_A), \eta_2(\theta_A)[$. If there exists more than one zero, the function D_2 should have at least a local maximum point and a local minimum point, but this is not possible whatever the value of $\mu + \lambda$ is, as the second order derivative of D_2 has constant sign. □

Equilibria

From the structure of the best reply correspondences (see Figures 4 and 5), we immediately see that the two pure strategy equilibria of the game (H, L) and (L, H) persist in case of guilt aversion regardless of the sensitivity parameters θ_A, θ_B. The previous Lemma 5.6 shows that there exists a unique equilibrium in completely mixed strategies that is the solution of system (54) and therefore depends on θ_A, θ_B. The sign of the term $\lambda + \mu$ affects only the concavity/convexity of the function D_2 and the asymptotic behavior of the completely mixed strategy equilibrium.

Note that, by the definition in (16),

$$\lambda + \mu > 0 \iff b + d > a + c,$$

where $b+d$ is the sum of Bob's payoffs when Ann plays L, and $a+c$ is the sum of his payoffs when she plays H. Thus, the condition $\lambda + \mu > 0$ indicates that Bob is better off, on average, when Ann plays L rather than H.

Another interpretation can be provided: since

$$\lambda + \mu > 0 \iff b - a > c - d,$$

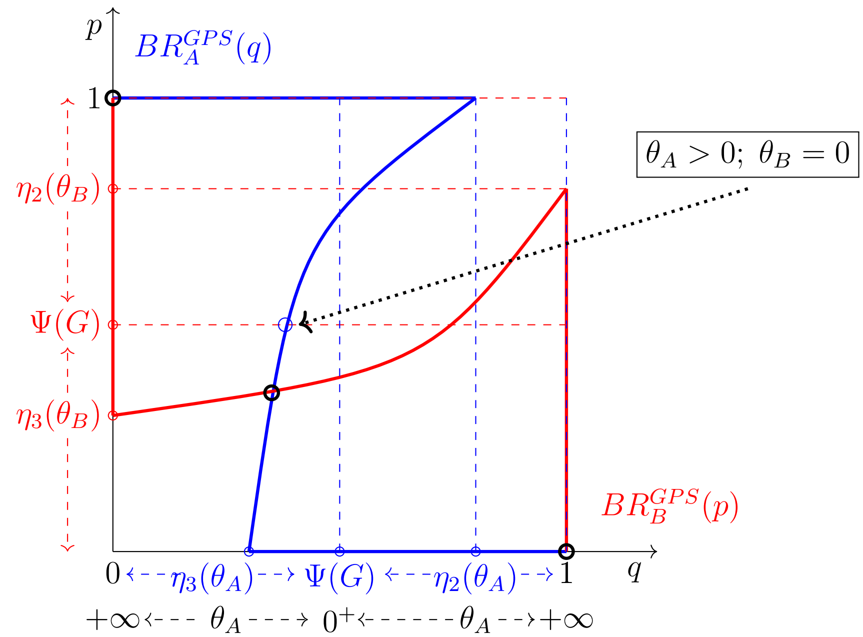


Figure 4: Type 2: Subcase $\mu + \lambda > 0$.

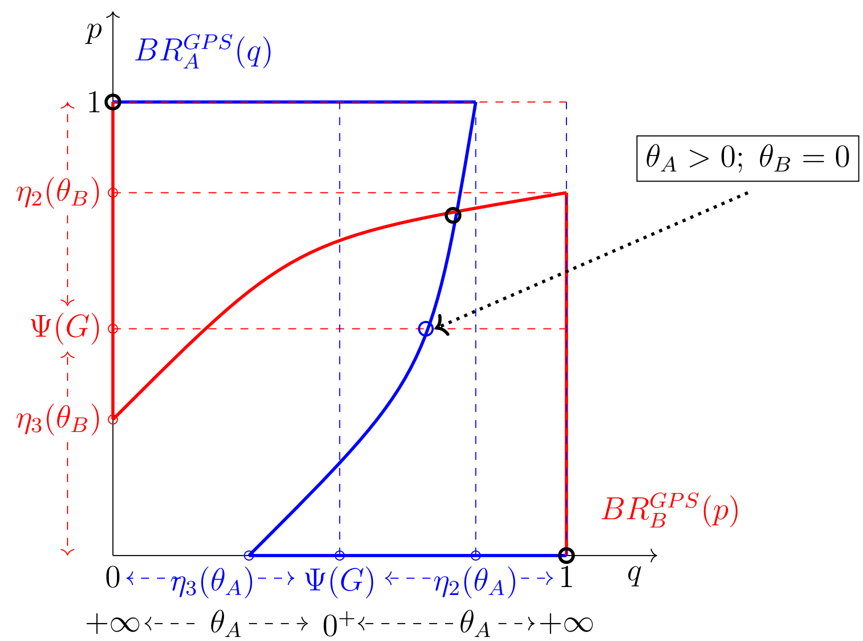


Figure 5: Type 2: Subcase $\mu + \lambda < 0$.

the condition $\lambda + \mu > 0$ also means that Ann prefers Bob's deviation from the symmetric strategy profile (H, H) towards the asymmetric equilibrium (H, L) rather than his deviation from the symmetric profile (L, L) towards the asymmetric equilibrium (L, H).

Finally, consider the case in which only Ann is affected by guilt aversion, i.e., $\theta_A > 0$ and

$\theta_B = 0$. We immediately observe that, as for Type 1 games, only Bob's mixed equilibrium strategy is perturbed. When $\lambda + \mu > 0$ (Figure 4), in equilibrium Bob plays H with a lower probability compared to the case without guilt. Conversely, when $\lambda + \mu < 0$ (Figure 5), Bob plays H with a higher probability.

Remark 5.7: Some insight into the asymptotic behavior of the equilibria in completely mixed strategies can be given. To this aim, denote with $H = \lambda + \mu$. Solving the system (54) we get that the second component $q^*(\theta_A, \theta_B)$ of the completely mixed strategy equilibrium is one (and only one) of the two solutions

$$q_1(\theta_A, \theta_B) = \frac{-b - \sqrt{\Delta}}{2a}, \quad q_2(\theta_A, \theta_B) = \frac{-b + \sqrt{\Delta}}{2a}$$

where

$$-b = \gamma^2 - \mu\gamma(\theta_A + \theta_B) - \delta H(\theta_A - \theta_B) + (\mu - \lambda)H\theta_A\theta_B,$$
$$a = H\left(-\theta_B\gamma + (\mu - \lambda)\theta_A\theta_B\right)$$

and

$$\Delta = \left(-\gamma^2 + \mu\gamma(\theta_A + \theta_B) + \delta H(\theta_A - \theta_B) + (\lambda - \mu)H\theta_A\theta_B\right)^2 - 4H\left(-\theta_B\gamma + (\mu - \lambda)\theta_A\theta_B\right)\delta\left(-\gamma - \lambda\theta_A + \mu\theta_B\right).$$

In order to understand the asymptotic behavior of the equilibrium points, we focus on the case $\theta_A = \theta_B = \theta$. We get

$$\lim_{\theta \to +\infty} q_1(\theta) = \lim_{\theta \to +\infty} \frac{(\mu^2 - \lambda^2)\theta^2 - |\lambda^2 - \mu^2|\theta^2}{2(\mu^2 - \lambda^2)\theta^2}, \quad \lim_{\theta \to +\infty} q_2(\theta) = \lim_{\theta \to +\infty} \frac{(\mu^2 - \lambda^2)\theta^2 + |\lambda^2 - \mu^2|\theta^2}{2(\mu^2 - \lambda^2)\theta^2}$$

where the previous limits can attain the values 0 or 1 depending on the sign of $\lambda + \mu$. As the equilibrium component $q^*(\theta)$ is equal to $q_1(\theta)$ or $q_2(\theta)$, we get that a converging subsequence, denoted (with an abuse of notation) with $\{q^*(\theta_n)\}_{n\in\mathbb{N}}$ converges to 0 or 1 depending on the data of the game. Therefore, the corresponding sequence of equilibria $\{(p^*(\theta_n), q^*(\theta_n))\}_{n\in\mathbb{N}}$ converges respectively to (L, L) or to (H, H).

6 Conclusions

Our analysis shows the extent to which guilt aversion influences the equilibrium structure of generalized Hawk–Dove games. In Type 1 games, guilt aversion can drive the system toward the Pareto-undominated symmetric profile, especially when sensitivity parameters exceed a specific threshold, thereby providing a mechanism for conflict resolution. In Type 2 games, guilt aversion only modifies the mixed strategy equilibrium. These findings highlight the role of guilt aversion in shaping strategic interactions, offering insights into how psychological and behavioral factors can resolve or exacerbate conflicts in coordination problems.

Finally, this paper represents the first contribution to a broader project aimed at characterizing equilibria under guilt aversion across all classes of symmetric 2×2 games, which will be the focus of future research.

7 Appendix A: Beliefs Structure

The hierarchical structure of beliefs is constructed below.
Denote with $\Delta(X)$ the set of probability measures on a given set X, then

$$B_i^1 := \Delta(\Sigma_{-i}) \text{ is the set of first order beliefs of player } i,$$
$$B_i^2 := \Delta(\Sigma_{-i} \times B_{-i}^1) \text{ is the set of second order beliefs of player } i,$$
$$\vdots$$
$$B_i^k(\Sigma_{-i} \times B_{-i}^1 \times \cdots \times B_{-i}^{k-1}) \text{ is the set of } k\text{-th order beliefs of player } i,$$

and so on, where

$$B_{-i}^k := \prod_{j \neq i} B_j^k \text{ is the set of } k\text{-th order beliefs of } i\text{'s opponents},$$

for every player i and for every $k \in \mathbb{N}$.

Therefore, k-th order beliefs of player i is represented by a probability measure over others' mixed strategies and other's beliefs up to the $(k-1)$-th order. In the end, the set of hierarchies of beliefs of player i is

$$B_i = \prod_{k=1}^{\infty} B_i^k,$$

whose elements are infinite hierarchies of beliefs $b_i = (b_i^1, b_i^2, \cdots, b_i^k, \cdots)$. Hence, player i's beliefs represent (via probability measures) what player i believes the others will play, what player i thinks the others believe their opponents will play, and so on. Since, for every $k \in \mathbb{N}$, B_i^k is compact and can be metrized as a separable metric space, the set B_i is, in turn, metrizable and separable. Moreover, it results to be compact under the topology induced by this metric (see [De Marco et al., 2022] for further details).

We will restrict our attention to the subset of collectively coherent beliefs $\overline{B}_i \subset B_i$, which is the set of beliefs of player i in which he is sure (i.e. with probability equal to 1) that it is common knowledge that beliefs are coherent. Specifically, a belief $b_i \in B_i$ is said to be coherent if, for every $k \in \mathbb{N}$, the following holds:

$$\text{marg}(b_i^{k+1}, \Sigma_{-i} \times B_{-i}^1, \times \cdots \times B_{-i}^{k-1}) = b_i^k. \tag{59}$$

The set $\overline{B}_i$ is compact as well (the detailed construction of the set of collectively coherent beliefs can be found in [Geanakoplos et al., 1989] and the proof of its compactness in [De Marco et al., 2022].

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