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Taxation, Revenue Sharing and Price Discrimination

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Abstract

We study the effects of taxes and fees in markets where sellers practice second-degree price discrimination, offering multiple versions of their product. Sellers distort the quantity (or quality) intended for all types of consumers, except for those with the highest marginal willingness to pay. We show that ad valorem taxes/fees can alleviate this distortion, thereby generating revenue while increasing consumer surplus and welfare, provided the tax rate increases with the size or quality of the version it applies to. We explore the implications of this result for important issues in fiscal policy (taxation of sin goods and of goods affecting labor supply). We also consider applications to the analysis of vertical relations between firms, as well as the strategy of platforms when setting prices for access and when competing with sellers.

JEL Classification: D4, H21, H22, L1

Keywords: Commodity taxation, tax incidence, price discrimination, sin goods.

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1 Introduction

Second-degree price discrimination is a very common marketing strategy. This practice can take various forms, such as offering a product in different package sizes (e.g., soft drinks in 0.5 and 2 liter bottles), or versions with different quality (e.g., first- and second-class tickets) and functionalities (e.g., a car with different engine types). As with most other goods and services, products sold under price discrimination are subject to taxes and/or fees imposed by suppliers or platforms. Besides governments applying taxes such as VAT, firms such as Apple (App Store), Google (Play Store), Microsoft (Microsoft Store), AirBnb and Uber commonly apply fees to transactions between sellers and consumers.[1] In this paper, we explore novel effects of taxes and fees in markets with price discrimination, showing that they can serve efficiency-enhancing purposes that have been ignored thus far.

Our analysis originates from the observation that taxes and fees may vary across different versions of a product. For example, governments apply different tax rates to business- and economy-class flight tickets, to cars of the same model but with different engines, or to versions of food products that contain high levels of unhealthy ingredients.[2] In other cases, the differentiation may be de facto, e.g., because sellers distribute some versions of their product through specific channels subject to different taxes than the standard ones.[3] Moreover, in digital markets, sellers of software and mobile apps typically adopt "freemium" pricing, whereby the basic version is free of charge and a monetary price applies only to the premium version.[4] Hence, only the latter is subject to (ad valorem) fees. The fact that taxes and fees differ across versions is important because, when designing each version of a product, a price-discriminating seller must ensure that consumers self-select on the intended one. The uneven impact of taxes and fees can alter the relative profitability of the different versions. Because the features and prices of such versions interdependent, sellers may

[1]Apple and Google currently charge sellers 30 percent of the price consumers pay to download apps, initial subscriptions or in-app purchases, and 15 percent for repeated subscriptions.

[2]See, e.g., the UK Air Passenger Duty https://www.gov.uk/guidance/rates-and-allowances-for-air-passenger-duty, and https://en.wikipedia.org/wiki/Road_tax for examples of road taxes in different countries. In the UK, a tax on sugary drinks applies at a certain rate for drinks with up to 8g of sugar per 100ml, and a higher tax rate applies to drinks with higher sugar content per ml (https://www.instituteforgovernment.org.uk/explainer/sugar-tax).

[3]For instance, Toyota distributes their high-end luxury vehicles under the Lexus brand, which are available through premium dealerships. On the other hand, Toyota's more affordable and mainstream vehicles are typically sold through their standard dealership network. As another example, Apple sells their high-end, premium smartphones like the iPhone Pro series through their own Apple Stores and upscale electronics retailers. Meanwhile, they also offer more affordable options like the iPhone SE and older models, which are available through a wider range of retail channels including budget-focused stores and online marketplaces.

[4]This strategy is extremely common for mobile apps in some categories including music and video streaming, gaming, and data storage. As reported by ACM (2019), freemium accounts for more than 90% of revenue from the games category of apps in the App Store.

respond to taxation in unexpected ways.

Typically, taxes increase the price and reduce the supply of a product, imposing a burden on consumers and producers. We show that this result does not hold for an ad valorem tax/fee that targets the top version of the product sold by a price-discriminating firm. This tax can alleviate the distortion imposed by the seller, which originates from the classic trade-off between rent extraction and efficiency (Laffont and Martimort, 2002). To understand this claim, consider the classical model where a seller provides a basic and a top version of its product, each intended for different consumer types. The seller must ensure that consumers with the highest willingness to pay do not self-select on the basic version. To relax the incentive compatibility constraint, the seller reduces the quality (or size) of the basic version (Maskin and Riley, 1984), while imposing no distortions at the top. An ad valorem tax targeting the top version makes collecting revenue from the low types relatively more attractive to the seller, so its incentive to distort the basic version diminishes. By the same token, the tax also affects the top version. However, this distortion is of second order. Hence, consumer surplus and (starting from the no tax equilibrium) welfare increase with the tax. Therefore, we show that the tax on the top version produces a "double dividend": it increases surplus while generating revenue. Instead, a tax on the basic version has the opposite effects.

We apply our basic result to several contexts where price discrimination plays an important role, considering first some interesting implications for fiscal policy. We consider "sin goods" like sugary beverages. Consumers tend to underestimate the long-term health costs of these goods, which calls for corrective taxes (O'Donoghue and Rabin, 2003). However, consumers with the strongest preference for sin goods tend to have lower levels of income and education, so taxes raise distributional concerns (Allcott et al., 2019). We note that a common (and controversial) practice by producers of these goods is to provide "supersized" packages that target the most vulnerable consumers.[5] We show that, an ad valorem tax applied on such packages reduces not only their size and harm, but also their price, so the net surplus of vulnerable consumers increases. Although the tax induces an increase in the size of the smaller version, this effect is less socially harmful when this version is intended for less vulnerable consumers.

We also consider goods that directly affect labor supply, such as childcare and transportation, which are also often marketed in multiple versions (e.g., full- vs. part-time nursery subscriptions, first- and second-class tickets). The public finance literature has shown that subsidizing (resp. taxing) complements (substitutes) to labor supply enables the government to make the income tax

[5] See, e.g., the proposed limit on the size of soda drinks by the city of New York (https://en.wikipedia.org/wiki/Sugary_drinks_portion_cap_rule) and similar proposals discussed in the UK (https://www.theguardian.com/society/2015/sep/14/obesity-growing-portion-sizes-overeating-cambridge-university-study).

schedule more efficient, by relaxing the incentive compatibility constraints (Piketty and Saez, 2013). The intuition is that a subsidy to goods that complement labor supply produces a relatively lower benefit on individuals who supply less labor. We show that an ad valorem tax on the top version can relax the incentive constraints even when applied to a complement to labor supply. The tax induces an improvement to the lower version, which benefits low-ability individuals more than high-ability ones who mimick the low ability ones.

In the last part of the analysis, we focus on the implications of our basic mechanism for the analysis of relations between suppliers (including platforms) and sellers. Ad valorem transaction fees are a key component of the "agency model" of vertical relations (Johnson, 2017; Foros et al., 2017), which is typical of digital platforms. Our results contribute interesting insights to the study of this model. We find that separation between the seller and its supplier can be socially preferable to integration, even if both firms are monopolists. If the firms were integrated, they would maximize their joint profit by choosing the same prices and product features as in the "zero fees" equilibrium. Instead, if the supplier is independent and imposes an ad valorem fee that targets the top version sold by the seller, consumer surplus and welfare can increase. This contrasts with the usual prescription that vertical integration increases welfare due to the otherwise imperfect coordination between upstream and downstream firms (Tirole, 1988).

We then study how price discrimination by sellers can affect the way suppliers set their fees, focusing on transaction fees set by platforms. These fees are quite controversial. Many sellers claim that fees unfairly squeeze their profits and force them to raise prices, thereby harming consumers. [6] Our results indicate that, if they target the top version, ad valorem fees can in fact increase welfare and consumer surplus. However, the supplier may set them too high compared to the optimal level.

In part, the controversy regarding transaction fees revolves around their interaction with other sources of revenue for the platforms. To analyze this point, we first let the platform sell a device essential to accessing the marketplace (e.g., a smartphone). By effectively charging consumers for access to the market, the platform internalizes the effect of the transaction fee on their surplus. As established above, this effect is positive if the fee targets the top version, which induces the platform to set a higher transaction fee when it can charge for access. This is in contrast to standard settings (see Oi, 1971), where access and transaction charges are substitutes. We then consider "hybrid" platforms that sell their own products on the marketplace. For example, Apple and Google offer

[6] See the recent lawsuit brought by Epic Games against Apple and Google. Moreover, Apple and Google have been designated as gatekeepers by the European Commission under the Digital Market Act for their app stores (see https://ec.europa.eu/commission/presscorner/detail/en/ip_23_4328). An investigation has been launched on Apple for violating the Digital Market Act in its Apple Store (see https://ec.europa.eu/commission/presscorner/detail/en/IP_24_3433.)

music and video streaming apps that compete with third-party ones. Regulators have raised concerns that transaction fees may be anti-competitive, by putting third-party sellers at a disadvantage.[7] We find that, if the fee targets the top version of the product, the platform prefers a lower transaction fee than if it had no competing products to sell. The intuition is that a marginal increase in the fee induces the third-party seller to adjust its offer in such a way that consumers get more surplus. Hence, the fee increases the competitive pressure on the platform's own product. By contrast, if the fee targets the basic version, or if it applies uniformly to all versions, we find that the platform sets it higher than in the case it did not sell its own product.

The remainder of the paper is organized as follows. Section 2 provides a review of the literature. Section 3 describes the model, and Section 4 characterizes the effects of taxes/fees on quantities, prices, consumer surplus and welfare. Section 5 compares the effects of differentiated ad valorem taxes with other tax instruments. Section 6 applies the basic results to the analysis of fiscal policy, whereas Section 7 considers applications to vertical relations and platforms. Section 8 concludes.

2 Literature

The incidence of indirect taxes is a classic topic in economics (Fullerton and Metcalf, 2002). Many previous studies have looked at imperfectly competitive markets, focusing on firms that supply a single product and adopt uniform pricing (Delipalla and Keen, 1992; Anderson et al., 2001; Auerbach and Hines, 2002; Weyl and Fabinger, 2013; Miklós-Thal and Shaffer, 2021). A fundamental result in the literature is that taxes lower supply and increase prices, hurting consumers as well as producers. Our study uncovers new effects in markets with price discrimination, provided that tax rates can be differentiated.

There is an extensive literature on second-degree price discrimination (Tirole, 1988; Laffont and Martimort, 2002; Stole, 2007), but few studies have investigated the effects of taxes (or fees) when this form of pricing is practiced. Laffont (1987), Cheung (1998) and Jensen and Schjelderup (2011) study taxation of a monopolist that applies nonlinear pricing. They consider tax rates that apply uniformly to all tariffs. D'Annunzio et al. (2020) consider multi-part tariffs allowing for differentiated tax rates on the access and usage parts of a tariff. Our paper focuses instead on forms of price discrimination such as quantity discounts and versioning, and allows for tax rates that vary according to the version of the good they apply to. This differentiation opens the door to novel and counterintuitive results. McCalman (2010) considers second-degree price discrimination when

[7] See, e.g., ACM (2019, chpt. 3 and 4) in the context of the mobile app market. Similar concerns were raised in a recent antitrust lawsuit against Google brought by multiple US States (see https://www.courtlistener.com/docket/60042641/522/state-of-utah-v-google-llc/).

analyzing the effects of unit trade tariffs in presence of a foreign price-discriminating monopolist. Tariffs have conventional effects on equilibrium quantities and prices in his model, although they can increase domestic welfare. We consider ad valorem taxes and uncover different effects on prices and quantities.

A small but growing branch of the literature studies Edgeworth's paradox of taxation, i.e., the fact that multi-product firms may respond to a tax on one good by reducing the prices of all goods (Salinger, 1991; Armstrong and Vickers, 2023; D'Annunzio and Russo, 2024). This result centers on the interdependence between market demands for *different* goods provided by the same firm. In contrast, the key mechanism in the present paper centers on the heterogeneity of individual demands for the same good and on the firm's incentives to elicit the consumer's private information.

We examine the implications of our main mechanism across different settings, thereby contributing to multiple strands of literature.

Within the public finance domain, an established literature considers the taxation of sin goods (e.g., sugary drinks) in presence of behavioral biases (O'Donoghue and Rabin, 2003; Allcott et al., 2019). This literature has recognized the role of market power (O'Connell and Smith, 2024), but has largely ignored price discrimination by sellers. Considering this aspect, we show that sin taxes may at the same time reduce the health damages and the financial burden on more vulnerable consumers. We also apply our mechanism to study the interaction between indirect and income taxation (Piketty and Saez, 2013), focusing on goods that affect labor supply (such as transport and childcare). Again, our contribution is to recognize that these goods are often sold under price discrimination, which allows us to uncover novel effects of taxation.

Our analysis also contributes to the literature studying vertical relations, and in particular the "agency model" (Johnson, 2017; Foros et al., 2017). The literature has found that this model may be more efficient than other vertical arrangements, such as wholesale. However, due to the usual lack of coordination between upstream and downstream firms with market power, welfare and consumer surplus would be higher if they were integrated. We show that this result may be overturned when the seller applies price discrimination and is subject to differentiated fees.

We contribute to the literature analyzing the relation between transaction and access fees, such as the sale of devices by platforms (see, e.g., Etro, 2021; Gaudin and White, 2021). Unlike previous papers, we find that a transaction fee can be complementary to the price of an essential device. Moreover, we study "hybrid" marketplace platforms (Hagiu et al., 2020, 2022). Anderson and Bedre Defolie (2024) consider monopolistic competition among sellers and a platform that provides a range of competing products. They find that, compared to a pure marketplace, a hybrid platform may set higher transaction fees to steer consumers towards its products. Tremblay (2022) shows that, when entering a market, a platform tends to reduce transaction fees applied to other sellers in that market. This result is fairly consistent with our findings, though our setting, and the mechanism behind our results, are different.

Finally, few previous papers studying platforms account for price discrimination. Lin (2020) and Jeon et al. (2022) study second-degree discrimination, focusing on how a platform's incentives and ability to screen participants on one side depend on the externalities generated on the other side. de Cornière et al. (2024) study third-degree price discrimination by a platform hosting different types of sellers. Wang and Wright (2017) show that ad valorem fees allow efficient price discrimination across goods with different costs and values. Differently from these studies, we consider the effects of transaction fees when the sellers on the platform, rather than the platform itself, engage in price discrimination.

3 Model

We consider a monopolist seller providing a single good to two types of consumers, indexed by $i = H, L$, where H stands for "high" and L for "low." We normalize the total number of consumers to one, denoting the share of type-H consumers by $v \in (0,1)$. The utility from the good is

$$u(q, \theta_i) - p,\ i = H, L, \tag{1}$$

where $q \geq 0$ is quantity, p is the price and $\theta_H > \theta_L$ is the parameter determining the marginal willingness to pay, assumed to be private information (Maskin and Riley, 1984). We assume that $\frac{\partial u}{\partial q} > 0$, $\frac{\partial^2 u}{\partial q^2} < 0$, $\frac{\partial u}{\partial \theta} > 0$ and $\frac{\partial^2 u}{\partial q \partial \theta} > 0$. The cost of providing one unit of the good is $c \geq 0$. Although we refer to q as quantity, we could also interpret this variable as quality (Mussa and Rosen, 1978).

The seller offers two "bundles", (q_i, p_i), each intended for one consumer type. These can represent two packages of different size (e.g., "regular" and "supersized") or two versions of the product (e.g., "basic" and "premium"), sold at different prices.[8] For concreteness, we refer to them as "versions" in the following. Each version is subject to an ad valorem tax rate imposed by a government, denoted $t_i \in [0,1]$, $i = H, L$. The latter can also be interpreted as a fee imposed by a platform or supplier. For ease of exposition, in the following we refer to it as a "tax" for concreteness.

[8]The prices p_i can also represent total outlays resulting from nonlinear tariffs applied by the seller, as in the case of network services such as energy or mobile data.

The seller's problem is

$$\max_{q_H, p_H, q_L, p_L} \quad \pi = v\left(\left(1-t_H\right)p_H - cq_H\right) + (1-v)\left(\left(1-t_L\right)p_L - cq_L\right), \tag{2}$$

$$s.t. \quad u\left(q_H, \theta_H\right) - p_H \geq u\left(q_L, \theta_H\right) - p_L, \tag{3}$$

$$u\left(q_L, \theta_L\right) - p_L \geq u\left(q_H, \theta_L\right) - p_H, \tag{4}$$

$$u\left(q_H, \theta_H\right) - p_H \geq 0, \tag{5}$$

$$u\left(q_L, \theta_L\right) - p_L \geq 0, \tag{6}$$

where (3) and (4) are the incentive compatibility constraints, while (5) and (6) are the participation constraints for H and L-type consumers, respectively (we normalize the utility from no consumption to zero).

As expression 2 suggests, the seller provides two versions of its product if and only if the tax rates t_H and t_L are not "too large". We return to this point in Section 4. Throughout the analysis, we concentrate on tax rates such that this condition holds.

To complete the setup, we write the tax revenue as

$$vt_H p_H + (1-v)\,t_L p_L. \tag{7}$$

Social welfare, obtained as the sum of consumer surplus, profit and tax revenue, boils down to total surplus in this market (using the shorthand notation $u_i \equiv u\left(q_i, \theta_i\right),\ i = H, L$)

$$W = v\left(u_H - cq_H\right) + (1-v)\left(u_L - cq_L\right). \tag{8}$$

Discussion. We consider a monopolist firm and two types of consumers for simplicity. In Appendix E.1 and E.2, we show that our results also apply with more than one supplier and more than two consumer types. In Section 7.4, we extend the model to consider endogenous market participation on both sides.

The analysis focuses on ad valorem taxes to concentrate on the most novel results. It is well known that subsidies can correct restrictions on supply in imperfectly competitive markets. We discuss the implications of allowing for subsidies in Section 4.2. Moreover, in Section 5, we consider (possibly differentiated) unit taxes and an undifferentiated ad valorem tax rate, showing that they have conventional effects on prices and quantities.

The differentiation of the tax rates by product version is a key aspect of our analysis. These differentiated tax rates may be applied by a government or a supplier/distributor. For instance, governments often impose car taxes that vary by fuel type or engine displacement, and tax rates health plans where only premium plans are taxed.[9] Governments can also impose higher tax rates on some versions of unhealthy products, such as drinks with a high content of sugar.[10] Differentiation may also apply de facto if the seller provides a low quality version of its product on illegal sales channels, avoiding taxes and regulation.

Differentiated fees can also be applied by suppliers or distributors (including platforms). For instance, the premium version of a product provided by a seller may require specific inputs from a supplier, which may be subject to revenue-sharing arrangements.[11] There are multiple examples of platforms applying differentiated ad valorem fees on different versions of a given service.[12] Another relevant scenario where our model applies is when the seller adopts the "freemium" pricing strategy, whereby the basic version of the product is made available for free. As a result, only the premium version is subject to the fees imposed by a platform.[13] Appendix D.1 makes this point formally by providing two alternative adaptations of the model that include freemium pricing.

4 Analysis

4.1 The effects of taxes on the seller and on consumers

Following standard steps (Laffont and Martimort, 2002), we can show that only (3) and (6) are binding at the allocation that solves the seller's problem. Manipulating the binding constraints, we obtain the standard monotonicity condition, $q_H > q_L$. The equilibrium is such that

$$p_H = u(q_H, \theta_H) - u(q_L, \theta_H) + u(q_L, \theta_L), \quad p_L = u(q_L, \theta_L). \tag{9}$$

[9] https://en.wikipedia.org/wiki/Road_tax

[10] For instance, in the UK a tax on sugary drinks applies at a certain rate for drinks with up to 8g of sugar per 100ml, and a higher tax rate applies to drinks with higher sugar content per ml.

[11] Bang & Olufsen provide high-end audio systems specifically designed for Audi's luxury car models and shares with Audi part of the revenue from the sale of such models. As another example, Marvel TV produces exclusive superhero series such as Daredevil, Jessica Jones, and Luke Cage specifically for Netflix, which agreed to share with Marvel part of the revenue generated by subscriptions. Hermes provides designer bands for limited versions of the Apple watch, sharing some of the revenue with Apple.

[12] For instance, Uber charges different ad valorem commission rates to drivers providing different quality levels, e.g. Uber X and Uber Black. Similarly, AirBnB applies different commission rates for standard listings and premium listings on AirbnbPlus.

[13] This pricing strategy is extremely common in markets such as mobile apps. Many leading apps, including Dropbox, Spotify, Tinder and Duolingo, require users to pay an upfront subscription for the premium version and make the basic one available for free. Other apps (typically videogames) implement freemium by charging heavy users with elective in-app payments. Clearly, if the price p_L is zero, or non-monetary (e.g., nuisance from ads), even a fee that formally applies to both versions would effectively target only the top version.

Substituting 9 in 2, we can rewrite the seller's problem as

$$\max_{q_H, q_L} \quad \pi = v\left((1-t_H)\left(u_H - u_{HL} + u_L\right) - cq_H\right) + (1-v)\left((1-t_L)\,u_L - cq_L\right), \tag{10}$$

where we have used the shorthand notation $u_i \equiv u\left(q_i, \theta_i\right)$, $i = H, L$, and $u_{HL} \equiv u\left(q_L, \theta_H\right)$. We assume this problem is concave in q_H and q_L. In the above expression, $u_{HL} - u_L$ represents the high type's information rent, that the seller must grant to prevent these consumers from choosing the L-version. Therefore, the seller cannot extract the entire surplus from the high types. In contrast, the seller leaves no rent to the low types. We get the following expressions for consumer surplus:

$$CS_H = u_{HL} - u_L, \qquad CS_L = 0. \tag{11}$$

The equilibrium quantities in the two bundles solve the following system of equations

$$\frac{\partial \pi}{\partial q_H} = \frac{\partial u_H}{\partial q_H}\left(1 - t_H\right) - c = 0, \tag{12}$$

$$\frac{\partial \pi}{\partial q_L} = v\left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L}\right)\left(1 - t_H\right) + (1-v)\left(\frac{\partial u_L}{\partial q_L}\left(1 - t_L\right) - c\right) = 0. \tag{13}$$

Setting taxes aside ($t_H = t_L = 0$), these equations indicate that the seller offers the efficient version to the high types, in the sense that their marginal utility equals its marginal cost. The quantity in the L-version, instead, is distorted downwards to reduce the information rent (given $\frac{\partial u_{HL}}{\partial q_L} > \frac{\partial u_L}{\partial q_L}$).

Consider now the effects of the tax rates on q_L and q_H. As we show in Appendix A.1, starting from the above first-order conditions and given the properties of the utility function, we have

$$\tfrac{\partial q_L}{\partial t_L} < 0, \quad \tfrac{\partial q_L}{\partial t_H} > 0, \quad \tfrac{\partial q_H}{\partial t_L} = 0, \quad \tfrac{\partial q_H}{\partial t_H} < 0. \tag{14}$$

The sign of the last derivative is counterintuitive: the quantity in the L-version *increases* with the rate that targets the H-version. To see why, consider that the seller distorts q_L downwards to extract more revenue from the high types, by reducing the information rent. The tax takes part of this revenue away (without affecting the revenue from the other version directly). By the same token, though, t_H introduces a distortion in q_H. However, as we shall see, this latter distortion is of second order.

The effects of taxes on prices mirror those on quantities. We have

$$\tfrac{\partial p_L}{\partial t_L} < 0, \quad \tfrac{\partial p_L}{\partial t_H} > 0, \quad \tfrac{\partial p_H}{\partial t_L} > 0, \quad \tfrac{\partial p_H}{\partial t_H} < 0. \tag{15}$$

Notably, t_H induces a *reduction* in the price of the H-version. This is due to (i) q_H decreasing and (ii)

q_L increasing with the tax, which raises the information rent. On the other hand, this tax increases the price of the L-version, because q_L increases with t_H.

In terms of consumer surplus, quite interestingly, the high types are better off, while the net surplus of the low types remains equal to zero in equilibrium:

$$\frac{\partial CS_H^e}{\partial t_H} = \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)\frac{\partial q_L^e}{\partial t_H} > 0, \quad \frac{\partial CS_H^e}{\partial t_L} = \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)\frac{\partial q_L^e}{\partial t_L} < 0, \quad \frac{\partial CS_L^e}{\partial t_i} = 0, i = H, L. \tag{16}$$

The tax on the L-version has very different effects. It induces a reduction in q_L, as one would expect, but no change in q_H. Thus, p_H increases because the information rent goes down. Consequently, consumers are worse off.

We have so far assumed that the seller provides two versions of the product. However, as we show in Appendix B, whenever one tax rate is exceedingly large, the seller does not provide that version. More specifically, there exists a threshold $\bar{t}_H(t_L)$ such that, for $t_H \geq \bar{t}_H(t_L)$, the seller provides only the L-version, to all consumers. In the following, we concentrate on pairs of tax rates such that the seller provides both versions, because we intend to study the effects of taxation in presence of price discrimination.

Proposition 1. *If the seller provides two versions of its product, the size of the L-version increases (resp. decreases) with an ad valorem tax targeting the H-version (resp. L-version), alleviating (resp. aggravating) the distortion imposed by the seller. Moreover, the ad valorem tax targeting the H-version (resp. L-version) increases (resp. reduces) consumer surplus.*

4.2 Effects of taxes on welfare and socially optimal tax rates

Focus now on social welfare. Differentiating (8) and given the first-order conditions of the seller's problem, we obtain

$$\frac{\partial W}{\partial t_H} = v\frac{\partial q_H}{\partial t_H}\frac{\partial u_H}{\partial t_H}t_H + (1-v)\frac{\partial q_L}{\partial t_H}\left(\frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)(1-t_H) + \frac{\partial u_L}{\partial q_L}t_L\right), \tag{17}$$

$$\frac{\partial W}{\partial t_L} = (1-v)\frac{\partial q_L}{\partial t_L}\left(\frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)(1-t_H) + \frac{\partial u_L}{\partial q_L}t_L\right). \tag{18}$$

As a first step, we consider the above derivatives at the laissez-faire equilibrium, where $t_H = t_L = 0$. Given (14), we obtain

$$\left.\frac{\partial W}{\partial t_H}\right|_{t_H=t_L=0} = v\frac{\partial q_L}{\partial t_H}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) > 0, \qquad \left.\frac{\partial W}{\partial t_l}\right|_{t_H=t_L=0} = v\frac{\partial q_L}{\partial t_L}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) < 0. \tag{19}$$

These expressions show that welfare increases with the introduction of a *tax* on the H-version, which reduces the distortion on q_L. Although the tax also creates a distortion on q_H, this is of second order. This previously unexplored effect has important implications for the design of tax policy and fee structures in vertical relations, which we examine below.

Proposition 2. *Starting from the equilibrium with no taxes, welfare increases (resp. decreases) when introducing a small ad valorem tax that targets the H-version (resp. the L-version).*

Given positive taxes (i.e., $t_i \geq 0$, $i = H, L$), we can show that the set of tax rates that maximizes (8) is

$$t_L^* = 0, \qquad t_H^* = \min\left(\bar{t}_H; 1 + \frac{\frac{\partial q_H}{\partial t_H}\frac{\partial u_H}{\partial q_H}}{\frac{\partial q_L}{\partial t_H}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) - \frac{\partial q_H}{\partial t_H}\frac{\partial u_H}{\partial q_H}}\right) > 0. \tag{20}$$

The tax targeting the top version produces a *double dividend*: it increases total surplus in this market and relaxes the government's budget constraint by generating additional revenue.

The above result is obtained restricting attention to positive tax rates. If we allow for subsidies, the tax rates maximizing (8) are such that $t_L < t_H = 0$ (see Appendix A.2). The intuition is easily grasped by comparing (17) to (18): a subsidy to the L-version would alleviate the distortion on q_L without distorting q_H. This is a standard result. Moreover, it does not account for the fact that subsidies may not be desirable or feasible in practice: even if a government aims to maximise welfare, it typically faces budgetary restrictions. In Appendix A.2, we show that if we account for the cost of public funds (i.e., the opportunity cost of government expenditures), the optimal policy always involves a tax $t_H > 0$ and, if the cost of public funds is large enough, even $t_L \geq 0$.

Proposition 3. *Assuming $t_i \geq 0$, the optimal tax rates are such that a positive tax rate is applied on the high version, while the low version is not taxed (see (20)).*

Our findings suggest that a properly designed tax on goods sold in multiple versions by a price-discriminating seller can be desirable on pure efficiency grounds. More specifically, differentiated tax rates, with higher tax rates on top-end versions or the largest package formats, can alleviate, rather than compound, the typical distortions in these markets.

5 Other tax instruments

To highlight the effects of differentiated ad valorem taxes, we now briefly consider a uniform ad valorem tax rate applied to all versions and (possibly differentiated) unit taxes. We show that none of these instruments can produce the same welfare-enhancing effects described above.

Uniform ad valorem tax. Consider a uniform ad valorem tax, i.e., $t_H = t_L = t$. As we show in Appendix C.1, we get

$$\frac{\partial q_L}{\partial t} < 0, \qquad \frac{\partial q_H}{\partial t} < 0. \tag{21}$$

Therefore, consumer surplus and social welfare both decrease with t. To grasp the intuition, replace $t_H = t_L = t$ in the first-order conditions of the seller's problem, (12) and (13), and divide both expressions by $1-t$, to obtain

$$\frac{\partial \pi}{\partial q_H} = \frac{\partial u_H}{\partial q_H} - \frac{c}{1-t} = 0, \tag{22}$$

$$\frac{\partial \pi}{\partial q_L} = \left(v \left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L} \right) + (1-v) \frac{\partial u_L}{\partial q_L} \right) - (1-v) \frac{c}{1-t} = 0. \tag{23}$$

These expressions show that the tax has the same effect as an increase in the unit cost of production.

Unit taxes. Suppose now that the seller is subject to unit taxes, denoted by τ_i, $i = H, L$. The profit function is therefore

$$\pi = v \left(p_H - (c + \tau_H) q_H \right) + (1-v) \left(p_L - (c + \tau_L) q_L \right). \tag{24}$$

Again, the effect of either tax rate is similar to that of an increase in the cost of production. Therefore, as we show in Appendix C.2, we obtain

$$\frac{\partial q_L}{\partial \tau_L} < 0, \qquad \frac{\partial q_H}{\partial \tau_L} = 0, \qquad \frac{\partial q_H}{\partial \tau_H} < 0, \qquad \frac{\partial q_L}{\partial \tau_H} = 0. \tag{25}$$

6 Applications to fiscal policy

In this section, we extend the above model to consider further applications to government policy. . The analyses we present hinge on the effects of the tax identified above and allow us to revisit some well-established results in the literature. We first consider the case of sin goods. Next, we study the taxation of goods that are strictly related to labor supply decisions.

6.1 Sin goods

Some goods cause "internalities", in the sense that consumers overlook costs that they will sustain in the future caused by their present consumption of the good. The main example are "sin goods", such as sugary beverages and tobacco, which are harmful to health in the long run. These effects call for taxes to control consumption (O'Donoghue and Rabin, 2003). However, taxes on sin goods are controversial because their financial burden hits mainly low-income households. We focus on an often overlooked aspect in this debate: the fact that sellers typically market these goods in multiple package sizes. These include "supersized" packages that target consumers who tend to have lower income and education levels (Conlon et al., 2024). Furthermore, these consumers are possibly more exposed to advertising campaigns, which have been shown to increase taste for larger packages (Dubois et al., 2017).

To study these issues, we assume that consumers have the same utility function as in (1), but suffer an additional cost, $\beta_i q_i$, which they ignore when buying the product. Hence, the equilibrium is the same as in the baseline model. The welfare function is

$$W = v\left(u_H - cq_H - \beta_H q_H\right) + (1-v)\left(u_L - cq_L - \beta_L q_L\right). \tag{26}$$

In this setting, there are additional market distortions that taxes can address. Differentiating (26), using the equilibrium conditions in (12) and (13), and evaluating the resulting expression at $t_H = t_L = 0$, we get

$$\left.\frac{\partial W}{\partial t_i}\right|_{t_H=t_L=0} = v\frac{\partial q_L^e}{\partial t_i}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) - v\frac{\partial q_H^e}{\partial t_i}\beta_H - (1-v)\frac{\partial q_L^e}{\partial t_i}\beta_L, \quad i = H, L. \tag{27}$$

The first term in this expression is the same as in (19). The last two terms capture the marginal effect of introducing t_i on the internalities. We know from (14) that $\frac{\partial q_H^e}{\partial t_H} < 0$ whereas $\frac{\partial q_L^e}{\partial t_H} > 0$. Hence, a tax on version H reduces the overall internality if and only if $-\beta_H \frac{\partial q_H^e}{\partial t_H} > \frac{1-v}{v}\frac{\partial q_L^e}{\partial t_H}\beta_L$. This inequality holds if the behavioral biases are sufficiently more important for consumers with the strongest preference for the good. This is plausible, as empirical evidence suggests that consumers with the strongest preference for sin goods are particularly prone to the internalities (Allcott et al., 2019). In this scenario, (27) shows that t_H produces an additional social benefit by reducing the size and harm of the version intended for these consumers.

In fact, t_H also has interesting effect on consumer surplus. We have

$$CS = vCS_H + (1-v)CS_L = v\left(u_{HL} - u_L - \beta_H q_H\right) - (1-v)\beta_L q_L. \tag{28}$$

The difference $u_{HL} - u_L$ measures the surplus of H-type consumers excluding the internality. As we have shown in Proposition 1, this difference increases with t_H, due to the reduction in the price p_H. This aspect is particularly relevant considering the concerns about the financial incidence of a sin tax. We find that, when applied to the largest version, the incidence of the tax is negative, so the financial burden on disadvantaged consumers decreases. This is a positive effect on disadvantaged consumers, in addition to the reduction in the internality. There is also a negative effect on L-type consumers, as q_L increases and so the internality worsens. Again, if $\frac{\beta_H}{\beta_L} > -\frac{1-v}{v}\frac{\frac{\partial q_L^e}{\partial t_H}}{\frac{\partial q_H^e}{\partial t_H}}$, the net overall effect on consumers is positive.

Finally, the effect of t_L is less clear-cut, because the tax has no effect on q_H, but reduces q_L (see (14)) and consumer surplus. Hence, while the tax can correct the internalities suffered by the L-type consumers, its impact on H-type consumers tends to be negative.

Proposition 4. *Consider a "sin good" that generates internalities affecting primarily the H-type consumers, i.e., such that $\beta_H \frac{\partial q_H^e}{\partial t_H} > -\frac{1-v}{v}\frac{\partial q_L^e}{\partial t_H}\beta_L$ holds. The introduction of an ad valorem tax t_H reduces the social cost of internalities and increases consumer surplus.*

6.2 Complements and substitutes to labor supply

A longstanding literature in public finance studies the relation between income taxes and taxes on goods and services. This relationship is particularly relevant for goods that have a significant impact on labor supply. Goods like childcare, transportation and productivity software tend to complement labor supply, whereas holiday packages, videogames and entertainment tend to be substitutes. Some of these goods are sold in multiple versions under price discrimination (e.g., first- and second-class travel tickets, full- and part-time nursery places). It is therefore interesting to examine how taxes on these goods can interact with income taxation, in light of our previous results.

The standard assumption in the income tax literature is that individuals have heterogeneous and unobservable earning abilities (Piketty and Saez, 2013). Because earning income requires labor supply, the government must design income taxes subject to incentive compatibility constraints. We modify the baseline model to incorporate these aspects. Assume that, in addition to θ_i, individuals differ in their ability to earn income per unit of labor supplied, that we call the "hourly wage" for short and denote by w_j, $j = H, L$. Suppose also that $w_H > w_L$ (we treat the opposite case, $w_H < w_L$, in Appendix A.3). We modify the utility function as follows

$$u(q, \theta_i) - m(q, s_j) + y - T(y) - p,\ i, j = H, L, \tag{29}$$

where $y \equiv w_j s$ is earned income, s is labor supply, and $T(y)$ is the income tax schedule (i.e., the amount of income tax given y). In line with the literature, we assume both w_j and s are unobservable to the government (unlike y). The component $m(.)$ captures the disutility from labor supply.

The seller's good can be either a complement ($\frac{\partial m}{\partial q} < 0$, $\frac{\partial^2 m}{\partial q \partial s} < 0$) or substitute ($\frac{\partial m}{\partial q} > 0$, $\frac{\partial^2 m}{\partial q \partial s} > 0$)

to s.[14] For brevity, we concentrate here on the case where they are complements (the full analysis is available in Appendix A.3).

To justify the existence of an income tax, assume the government needs to generate a minimum amount of revenue, R. Consider an income tax schedule that, given $t_L = t_H = 0$, (i) generates the required amount R and (ii) is such that the incentive compatibility constraint of the H-type individuals binds. This constraint ensures that these individuals prefer to earn the intended after-tax income level, $y_H - T(y_H)$, rather than supply less labor and earn $y_L - T(y_L)$ (in this case, we say that consumers "shirk"). If the introduction of either t_H or t_L relaxes the constraint (keeping the individual and government income levels constant), the government can redesign the income tax schedule more efficiently. For example, it can use the revenue from t_i to reduce the slope of $T(y_L)$, keeping total revenue constant but inducing the L-types to supply more labor.

Consider H-type individuals who shirk and choose to earn y_L rather than the higher income level y_H. They supply less labor, so they benefit relatively less from the complementary good, than if not shirking. As a result, they are not willing to pay enough to acquire the H-version. Moreover, they need to supply less labor than the L-types to earn the same income level, y_L, due to their higher ability. Hence, an increase in q_L has a positive impact on the H-types who shirk, but smaller than the positive impact on the L-type individuals, all else given. Therefore, given Proposition 1, introducing t_H relaxes the incentive constraint, whereas t_L has the opposite effect. Formally, we have

$$\left.\frac{\partial W}{\partial t_i}\right|_{t_H=t_L=0} = \frac{\partial q_L}{\partial t_i} v \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial m\left(q_L, \frac{y_H}{w_H}\right)}{\partial q_L} - \frac{\partial u_L}{\partial q_L} + \frac{\partial m_L}{\partial q_L} \right) + \tag{30}$$

$$+ \frac{\partial q_L}{\partial t_i} (1-v) \left(\frac{\partial m\left(q_L, \frac{y_L}{w_H}\right)}{\partial q_L} - \frac{\partial m\left(q_L, \frac{y_H}{w_H}\right)}{\partial q_L} \right).$$

In the above expression, the first term in brackets is the effect of changes in q_L given the distortion on this quantity imposed by the seller, which is similar to (19) in the baseline model. The second term captures how changes in q_L affect the incentive constraint faced by the government. In the Appendix, we show that a similar result applies when $w_L > w_H$, as well as when the seller's good is a substitute to labor supply, provided that $w_H > w_L$.

Proposition 5. *Consider a good that complements labor supply and is sold in multiple versions. Introducing an ad valorem tax on the top version allows to make the income tax schedule more efficient.*

[14] We assume that overall utility is increasing and concave in q (i.e., $\frac{\partial u}{\partial q} - \frac{\partial m}{\partial q}$ is positive), and that this increases in θ.

The standard result in the literature is that taxes on goods and services improve the efficiency of the income tax schedule if these goods are substitute to labor supply, but not if they are complements. This result is based on the premise that these goods are provided under uniform pricing, so taxes reduce their supply. When taking price discrimination into account, we have shown that ad valorem taxes that target the top version can help to relax the incentive constraints faced by the government even if the good is complementary to labor supply.

7 Applications to vertical relations and platforms

We now focus on the alternative interpretation of the model, where we interpret t_i as revenue-sharing fees applied by either a supplier or a platform that connects the seller to consumers. A notable example of this kind of arrangements is the "agency" model of vertical relations (Johnson, 2017; Foros et al., 2017), which is common in digital markets. We present some interesting implications of our results for the analysis of vertical structures and of the strategy of platforms.

7.1 Vertical mergers

In this section, we re-examine the classical question on vertical mergers (Tirole, 1988) in the scenario where the downstream firm practices second-degree price discrimination, which has been largely overlooked by the literature.

Assume there is a monopolist supplier, S, who applies an ad-valorem fee t_i on the seller and has zero costs. Hence, the profit of the supplier, that we assume to be concave in $t_i,\ i = H, L$, equals the revenue in (7) and are as follows

$$\pi_S = v p_H t_H + (1 - v) p_L t_L. \tag{31}$$

If the seller and the supplier merge, they maximize the sum of (10) and (31), i.e., their gross-of-fees profit. As a result, the prices and quantities for each version, (q_i^e, p_i^e), would be set as in the no-fees equilibrium we characterized in Section 4.1.

Assume now that seller and supplier are independent and, to start, suppose their relationship concerns only the top-end version of the product. For example, the top version may require an essential input and the supplier of this input may demand a revenue-sharing arrangement (see footnote 11). Similarly, the seller may practice "freemium" pricing, which is common for mobile apps distributed on marketplace platforms such as the Apple Store and Google Play. Under this pricing model, the seller provides the basic version for free, so the platform's fee only hits the premium version.

We established in Proposition 1 that consumer surplus increases in t_H (as long as $t_H < \bar{t}_H(t_L)$). This suggests that a merger between the two firms would reduce consumer surplus in this scenario. Moreover, Proposition 2 establishes that a positive and small fee on the H-version increases welfare. Hence, the socially optimal level of t_H must be strictly positive and thus above the level chosen by the merging firm.

However, the supplier will generally select a fee different from the optimal one. The equilibrium fee, t_H^e, is such that

$$\frac{\partial \pi_S}{\partial t_H} = v\left(u_H^e - u_{HL}^e + u_L^e\right) - vt_H\left(\frac{\partial q_L}{\partial t_H}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) - \frac{\partial q_H}{\partial t_H}\frac{\partial u_H}{\partial q_H}\right) = 0. \tag{32}$$

The welfare-maximizing fee, t_i^*, are characterized in (20). Assuming concavity of the welfare function, we can compare t_H^e and t_H^* by evaluating $\frac{\partial W}{\partial t_H}$ in $t_H = t_H^e$ (using (32)). We get that $t_H^e < t_H^*$ if and only if the following derivative is strictly positive

$$\left.\frac{\partial W}{\partial t_H}\right|_{t_H = t_H^e} = -v\left(u_H^e - u_{HL}^e + u_L^e\right) + v\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)\frac{\partial q_L}{\partial t}. \tag{33}$$

The terms on the right hand side of this expression capture two effects that the supplier does not internalize when setting t_H: the per-consumer profit loss of the seller and the increase in consumer surplus. Therefore, expression (33) tells us that the supplier may set the fee too high compared to the socially optimal level if and only if the second term dominates. It follows that a sufficient (but not necessary) condition for welfare to be higher when the two firms are separate is that $t_H^e \leq t_H^*$.

Finally, consider the case where the supply relationship concerns only the L-version. In this scenario, it is easily established that a merger between the two firms can only have a positive effect on welfare. This is because the effect of t_L on consumer surplus and welfare is negative. Hence, separation between the supplier and the seller must reduce welfare compared to integration. We reach a similar conclusion when the supply relationship applies to all versions and the supplier applies a single, uniform fee t. This follows from Section 5.

Proposition 6. *Assume a monopolist supplier applies an ad valorem fee to the H-version of the product. Vertical integration between the seller and the supplier reduces consumer surplus. Moreover, it reduces welfare if $t_H^e \in [0, t_H^*]$. Instead, if the supply relationship concerns only the L-version or both versions with a uniform fee, the effect of integration on consumer surplus and welfare is positive.*

This finding contrasts with the common presumption that, by removing the frictions caused by the lack of coordination between suppliers and sellers, vertical integration improves market performance (Tirole, 1988). In our setting, the fact that the supplier does not internalize the effect of its fee on the profit of the downstream seller is beneficial if the relationship concerns the top version of the product.

7.2 Essential devices and access fees

Suppose now that, in addition to having a revenue sharing agreement with the seller, the supplier sells an essential good for consumers to access the seller's product. For instance, the supplier may be a platform that connects sellers to consumers. Many such platforms charge consumers for accessing the marketplace, in addition to imposing fees on sellers. In the case of platforms hosting mobile apps, the platform itself provides an essential device, such as a smartphone.[15] Another example are console makers (as Sony, Microsoft, Nintendo), that provide the hardware/platform needed to access the games and have revenue sharing agreements with game developers. Similarly, streaming TV providers (e.g., Roku, Amazon Fire TV, Apple TV) supply the hardware necessary to access streaming services sold by streaming services (e.g., Netflix, Disney+, HBO Max), and they negotiate revenue-sharing agreements for subscriptions sold via their devices.

We now study the relation between the price of essential devices and the transaction fees applied by the same supplier. Let p_D be the price of the essential device. Suppose also that consumers' outside option and the marginal cost of the device are zero. Consumers get an intrinsic utility d from the device. Given they cannot access the marketplace without the device, we assume that consumers do not observe their own θ beforehand. The timing is as follows: at stage 1, the supplier sets its own fees. At stage 2, consumers decide whether to buy the device and then observe their type. The seller sets the price and quality of its products. At stage 3, consumers who bought the device observe the seller's product and decide which version to buy, if any.

We solve this model in Appendix A.5. The assumption that consumers have identical valuation for the device simplifies the exposition, but it is not crucial: in Appendix E.3, we show that the main result in this section hold when demand for the device is downward sloping.

At stage 3, the solution to the seller's problem is the same as in Section 3. Consumers do not observe the products until they have bought the device, so the seller takes the size of the market as given when designing and pricing such products. Given (11), the expected consumer surplus from the seller's product at stage 3 is $E(CS) = v(u^e_{HL} - u^e_L)$, where $u^e_i \equiv (q^e_i, \theta_i)$ and $u_{HL} \equiv u(q^e_H, \theta_L)$. At

[15] Anderson and Bedre-Defolie (2024) study the market for apps, focusing on cross-market externalities between consumers and sellers, rather than on price discrimination.

stage 1, the supplier can recover this expected surplus through the price of the device, setting

$$p_D = d + E(CS) = d + v\left(u^e_{HL} - u^e_L\right).$$

This is the highest price such that consumers buy the device. When choosing its transaction fees, t_i, therefore, the supplier maximizes the following

$$\pi_P = d + v\left(u^e_{HL} - u^e_L\right) + v p^e_H t_H + (1-v)\, p^e_L t_L. \tag{34}$$

For the sake of exposition, we consider first the case where the supplier only imposes a fee on the top version, t_H. As discussed in Section 7.1, this could be because the seller adopts freemium pricing and provides the base version for free or because the relation among the firms only pertain the high quality version of the product. By Proposition 2, we know that consumer surplus increases in t_H. Hence, p_D increases with this fee as well. Therefore, the equilibrium fee set by the supplier to maximize (34) is *larger* than if the supplier does not sell the essential device. Hence, from the supplier's perspective, the transaction fee on the top version is *complementary* to the access charge, rather than substitute. The reason is that, through the access/device price, the supplier internalizes the positive effect of the transaction fee on consumer surplus. This result contrasts with previous literature, which suggests that access and transaction charges should be substitutes from the supplier's perspective (see, e.g., Etro, 2021; Gaudin and White, 2021).

The relationship between the access charge and a transaction fee that targets the L-version, t_L, is completely opposite. As Proposition 1 suggests, t_L reduces consumer surplus, and thus causes a decrease in p_D. We find a similar result when both versions are subject to the same fee, t (given the findings of Section 5).

Proposition 7. *A supplier charging consumers for access implements a transaction fee on the top version of the seller's product above the level chosen in the absence of this access fee. However, if the fee targets the base version or both versions uniformly, the supplier sets it below the level chosen in the absence of this access fee.*

7.3 Hybrid suppliers

Suppliers and platforms hosting marketplaces not rarely compete in downstream markets with the sellers. For instance, console makers like Sony (PlayStation) and Microsoft (Xbox) produce and sell first-party games (e.g., The Last of Us by Sony or Halo by Microsoft) that compete with third-party ones distributed on their platforms. Moreover, Apple and Google provide apps that compete with (often well established) third-party ones in, e.g., video and music streaming, office utilities and cloud storage, available on their app stores. The literature has referred to these platforms as "hybrids" between marketplace and seller (see, e.g., Anderson and Bedre Defolie, 2024; Hagiu et al., 2020, 2022).

The above observations raise concerns about potential anti-competitive behavior by the supplier, for two reasons. First, third-party products are subject to transaction fees, which potentially puts them at a disadvantage with respect to the supplier's own products. Furthermore, the suppliers tend to make their own products prominent on their marketplaces.[16] We contribute to this debate by studying how a hybrid marketplace would set the transaction fees, as opposed to a "pure marketplace", in the case where products are sold under price discrimination.

Consider the setting presented in Section 3, and assume the supplier also provides a product that competes with the seller's. We assume that a share $\lambda \in [0,1]$ of "loyal" consumers only buys the seller's product, if any. This is consistent with the third-party seller having an established user base. The other consumers obtain the same utility as (1) from either product. The distribution of θ is independent of whether consumers are loyal. We assume that all consumers observe the supplier's product at no cost, because it is prominent. By contrast, non-captive consumers incur a small search cost to observe the seller's product (but they have rational expectations). For simplicity, the supplier and seller have the same production cost.

The timing is as follows. At stage 1, the supplier sets its fees, t_i, and the characteristics (p_i^P and q_i^P) of its product. At stage 2, the seller sets the features (p_i and q_i) of its own product. At stage 3, consumers land on the marketplace and observe the supplier's product. Non-loyal consumers decide whether and which version to buy from the supplier, or search the third-party one. Finally, at stage 4, non-loyal consumers who searched and loyal ones observe the third-party product and decide which version of this product to buy, if any.

We describe the main findings here and relegate the analysis to Appendix A.6. Conditional on t_i, the values of p_i^e and q_i^e chosen by the seller are the same as in Section 3.[17] In equilibrium, the supplier sets its prices in such a way that only loyal consumers buy the third-party product. These consumers obtain the surplus given in (11). The non-loyal consumers buy the supplier's product, so its profit is

$$\pi_S = (1-\lambda)\left[v\left(u_H^P - CS_H^e - cq_H^P\right) + (1-v)\left(u_L^P - cq_L^P\right)\right] + t_H \lambda v p_H^e + t_L \lambda (1-v) p_L^e. \tag{35}$$

[16] For instance, Apple and Google pre-install some of their own apps on smartphones and tablets running the respective operating systems.

[17] The seller is effectively a monopolist for all the loyal consumers and it treats the share of consumers that search as given. Therefore, its problem is identical to that in the baseline model.

The last term is the revenue from the transaction fees, whereas the first term captures the profit from selling its own product. The revenue from the H-types is constrained by the expected surplus, CS_H^e,that they would get from the seller's product: to attract the non-loyal consumers, the supplier must ensure they get the same surplus as with the third-party product.

Suppose first that the supplier implements a fee targeting the H-version of the seller's product (see above for examples of this relationship). The key observation is that CS_H^e increases with t_H, as shown in Proposition 1. Hence, the fee t_H makes the seller's product *more* competitive. Therefore, the equilibrium fee is *smaller* than the fee the supplier would choose if it did not sell its own product.

Suppose now the fee applies to the L-version. From Proposition 1, we know that this fee has the opposite effect on CS_H^e, and thus we can conclude by the same reasoning as above that the supplier has an incentive to set this fee above the level it would choose if it did not supply the product. The same applies in the case of a uniform fee t applying to both versions.

Proposition 8. *A hybrid supplier sets its transaction fee below the level that a pure marketplace supplier would choose, provided the fee targets the top version of the seller' product. By contrast, if the fee targets the base version or applies uniformly to all versions, a hybrid supplier sets it higher than a pure marketplace supplier.*

This result speaks to the debate on the fees applied by hybrid digital platforms. We find that a hybrid marketplace platform may increase its transaction fees to relax price competition with third-party products, but not if the fees target the top-end version of such products. Quite surprisingly, we find that price competition is relaxed by reducing the fee that targets the top version, in contrast with previous literature (see Anderson and Bedre Defolie, 2024).

7.4 Endogenous market participation

We now focus on the scenario where the supplier is a marketplace platform that brings consumers and sellers together. In this context, it is interesting to relax the assumption of exogenous participation by sellers and consumers, to study the effect of fees in presence of network effects.

Let the number of consumers and sellers that join the marketplace be n_c and n_s, respectively. For simplicity, assume that each seller is a monopolist in its product category, and that each consumer interacts with all sellers in the marketplace.[18] Suppose also that sellers and consumers are symmetric, conditional on joining the marketplace. A seller's profit when joining is given by

$$\pi = n_c \left(v \left((1 - t_H) \, p_H - c q_H \right) + (1 - v) \left((1 - t_L) \, p_L - c q_L \right) \right). \tag{36}$$

[18]These assumptions are not crucial. For instance, we would obtain qualitatively similar results assuming that each consumer interacts with only one seller.

We assume that $n_s = \phi_s(\pi)$ and $n_c = \phi_c(n_s E(CS))$, where $E(CS)$ is the surplus that each consumer expects to get when interacting with a seller. The functions $\phi_s(.)$ and $\phi_c(.)$ are increasing and continuously differentiable.

We consider the following timing: at stage 1, the platform sets t. At stage 2, sellers and consumers decide whether to join and each seller sets p_i, q_i and x_i, for $i = H, L$. At stage 3, consumers observe the features and prices of the products available and decide which to buy, if any. We assume each seller takes n_c as given when deciding whether to join the platform and choosing the values of p_i, q_i and x_i, $i = H, L$. This is because consumers do not observe the features of the products available on the platform prior to joining, but have rational expectations. Consumers derive the same utility from consuming each good of each active seller and observe their type only after joining.

Given the above setting, we can show the following results (see Appendix A.7). Given the number of sellers, the fee t_H increases $E(CS)$ (Proposition 1), which increases the number of consumers willing to join. Hence, although the fee reduces the profit per consumer, its overall effect on sellers can be positive if this effect is compensated by the expansion in the number of consumers. On the other hand, even if consumer surplus per seller increases with the fee, the net effect on consumers can be negative if the fee induces too many sellers to abandon the marketplace. Interestingly, the fee can create a virtuous circle in which participation by *both* sides increases. We obtain that

$$\frac{dn_s}{dt_H} > 0 \iff -v\frac{\partial CS_H^e}{\partial t} n_s \frac{\pi^e}{\frac{\partial \pi^e}{\partial t}} > \frac{n_c}{\phi_c'}, \qquad \frac{dn_c}{dt_H} > 0 \iff \frac{n_s}{\phi_s'} > -n_c \frac{\partial \pi^e}{\partial t} \frac{CS_H^e}{\frac{\partial CS_H^e}{\partial t}}. \tag{37}$$

The above expressions suggest that the a fee targeting the top version will increase participation on both sides whenever consumers are significantly more responsive than sellers, i.e., $\frac{n_c}{\phi_c'}$ is small relative to $\frac{n_s}{\phi_s'}$.

Consider now the effects of t_L. As we have shown in Proposition 1, this fee reduces consumer surplus, as well as profit for the sellers. Hence, the effect on market participation is negative on both sides. The same conclusion applies to a uniform fee on both versions, t, given the results of Section 5.

Proposition 9. *If the transaction fee targets the top version, it determines an increase in the number of both consumers and sellers joining the marketplace if and only if the conditions in expression (37) hold. By contrast, a fee on the base version and a uniform fee reduce participation on both sides of the market.*

7.5 Allowing the seller to bypass the supplier

In this last extension, we consider the implications of allowing the seller to transact with consumers by alternative channels, thereby avoiding the fees imposed by the supplier. This scenario is particularly relevant for digital goods, such as mobile apps, in light of recent policy developments related to the Digital Market Act (DMA). Traditionally, platforms hosting app marketplaces (e.g., the App Store) did not allow sellers to distribute their apps via alternative channels and to collect payments without using the platform's proprietary system. The European Commission has recently forced the platforms to relax these restraints.Taking stock of these recent developments, in this section we briefly consider whether allowing the app seller to transact with consumers outside the platform may change the effects of the transaction fee considered above. Specifically, we assume the platform allows the seller to distribute its product independently, thereby avoiding the transaction fee. For concreteness, focus on the case where the fee only applies to version H.[19]

Let there be a share of consumers, $(1-b) \in [0,1]$, who acquires the seller's product outside the platform and sustain a finite cost $\gamma \geq 0$ when doing so. This cost captures the fact that using an alternative system may require, for instance, to install additional software, set up a new password and re-enter payment data. The other consumers are unwilling to use the alternative distribution channel, so they only transact on the platform. These may be consumers who are less tech savvy or more time constrained. For simplicity, we assume that b and θ are independently distributed. We assume that the seller can charge different prices to consumers inside and outside the platform, but the (quality of) the products sold must be the same.

The seller has nothing to gain from distributing the low version of its product outside the platform, because it is not subject to the transaction fee. However, if t_H is high enough, the seller wants consumers to use its alternative channel to acquire the H-version, despite the transaction cost. As we show in Appendix A.4, the seller sets the same prices as in the baseline for the products purchased on the platform, but discounts the price charged outside the platform, p_H^o, in order to compensate consumers for the transaction cost, i.e., $p_H^o = p_H - \gamma$. The seller's problem can thus be written as

$$\begin{aligned} \max_{q_H, q_L} \quad \pi = {} & v\left(b\left(1-t_H\right)\left(u_H - u_{HL} + u_L\right) + (1-b)\left(u_H - u_{HL} + u_L - \gamma\right) - cq_H\right) + \\ & + (1-v)\left(u_L - cq_L\right) = \\ & v\left(\left(1-bt_H\right)\left(u_H - u_{HL} + u_L\right) - (1-b)\gamma - cq_H\right) + (1-v)\left(u_L - cq_L\right). \end{aligned} \tag{38}$$

[19] The model presented in this section can also be used to study taxation when the seller provides goods on the legal and the black market, thereby avoiding taxes and safety regulations for the latter. In this case, good H should be interpreted as the good traded on the legal market, subject to the tax, while good L is the good traded on the black market.

This expression suggest that the effects of t_H on the profits of the seller, and on the choice of price and quality, are qualitatively similar to the baseline model. By the same mechanism as in Proposition 1 and 2, q_H decreases with t_H, while q_L increases. As a result, consumer surplus and welfare increase.

8 Conclusions

This paper studied the effects of taxation in markets with second-degree price discrimination. We have shown that, by imposing a well-designed ad valorem tax on the bundle intended for the high types, the government can obtain a double dividend, collecting revenues while concurrently raising consumer surplus and, possibly, welfare. A subsidy could increase consumer surplus and welfare, but would impose a cost on the government that may not be viable. Uniform ad valorem and unit taxes (both uniform and differentiated) decrease quantities and welfare. In the baseline model we consider a monopolist seller and two types, but the results are robust to considering more than two types and competing sellers. In practical terms, our results call for (ad valorem) taxes targeting, e.g., larger packages of beverages, and first-class travel tickets.

The ad valorem tax we model may be charged by a government or represent a revenue sharing agreement the seller have with a supplier or a distributor. For both these scenarios, we extended the model to deal with some interesting applications of the results.

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Appendix

A Proofs of results in the text

A.1 Establishing the signs of the derivatives in (14) and (15)

By totally differentiating the first-order conditions of the monopolist's problem in(12) and (13), we find that

$$\frac{\partial q_i}{\partial t_i} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial t_i} - \frac{\partial^2 \pi}{\partial q_j \partial t_i}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{Z}, \quad \frac{\partial q_j}{\partial t_i} = -\frac{\frac{\partial^2 \pi}{\partial q_i^2}\frac{\partial^2 \pi}{\partial q_j \partial t_i} - \frac{\partial^2 \pi}{\partial q_i \partial t_i}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{Z}, \; i,j = H,L, \; j \neq i.$$

where $Z \equiv \frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H^2} - \left(\frac{\partial^2 \pi}{\partial q_H \partial q_L}\right)^2 > 0$, $\frac{\partial^2 \pi}{\partial q_j^2} < 0$, $\frac{\partial^2 \pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial t_L} = 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial t_H} = \frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) > 0$, $\frac{\partial^2 \pi}{\partial q_H \partial t_H} = \frac{\partial u_H}{\partial q_H} > 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial t_L} = -\frac{\partial u_L}{\partial q_H} < 0$. Hence, we have

$$\text{sgn}\left(\frac{\partial q_H}{\partial t_H}\right) = \text{sgn}\left(\frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial u_H}{\partial q_H}\right) < 0$$

$$\text{sgn}\left(\frac{\partial q_L}{\partial t_H}\right) = \text{sgn}\left(-\frac{\partial^2 \pi}{\partial q_H^2}\frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)\right) > 0$$

$$\frac{\partial q_H}{\partial t_L} = 0$$

$$\text{sgn}\left(\frac{\partial q_L}{\partial t_L}\right) = \text{sgn}\left(\frac{\partial^2 \pi}{\partial q_H^2}\frac{\partial u_L}{\partial q_L}\right) < 0.$$

Let us now compute the derivatives of the equilibrium prices $p_H = u_H - u_{HL} + u_L$ and $p_L = u_L$ with respect to t_i, $i = H,L$. Taking into account that $\frac{\partial u}{\partial q} > 0$ and $\frac{\partial^2 u}{\partial q \partial \theta} > 0$, we have

$$\frac{\partial p_H}{\partial t_H} = \frac{\partial u_H}{\partial q_H}\frac{\partial q_H}{\partial t_H} < 0, \qquad \frac{\partial p_L}{\partial t_H} = \frac{\partial u_L}{\partial q_L}\frac{\partial q_L}{\partial t_H} > 0.$$

$$\frac{\partial p_H}{\partial t_L} = -\frac{\partial q_L}{\partial t_L}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) > 0, \qquad \frac{\partial p_L}{\partial t_L} = \frac{\partial u_L}{\partial q_L}\frac{\partial q_L}{\partial t_L} < 0.$$

A.2 Subsidies and the cost of public funds

Consider the system of first order conditions $\frac{\partial W}{\partial t_L} = \frac{\partial W}{\partial t_H} = 0$ in (17) and (18). If we relax the assumption that $t_i \geq 0$, the unique solution to this system is such that

$$t_L^{FB} = -\frac{v}{1-v}\frac{\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)}{\frac{\partial u_L}{\partial q_L}} < 0, \qquad t_H^{FB} = 0.$$

Let us now introduce a cost of public funds, $\lambda \geq 1$. Given this cost, the welfare function is modified as

$$W = v\left(u_H - cq_H\right) + (1-v)\left(u_L - cq_L\right) + (\lambda - 1)\left(vt_H p_H + (1-v)\, t_L p_L\right).$$

The solution to the system of first order conditions $\frac{\partial W}{\partial t_L} = \frac{\partial W}{\partial t_H} = 0$ is such that

$$t_L = -\frac{v}{1-v}\frac{\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)}{\lambda \frac{\partial u_L}{\partial q_L}} + \frac{\lambda - 1}{\lambda} u_L \left(\frac{1}{\frac{\partial u_L}{\partial q_L}\frac{\partial q_L}{\partial t_L}} + \frac{\frac{\partial q_L}{\partial t_H}}{\frac{\partial q_L}{\partial t_L}} \frac{1}{\frac{\partial u_H}{\partial q_H}\frac{\partial q_H}{\partial t_H}} \frac{\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}}{\frac{\partial u_L}{\partial q_L}}\right),$$

$$t_H = \frac{\lambda - 1}{\lambda}\frac{1-v}{v} u_L \frac{\frac{\partial q_L}{\partial t_H}}{\frac{\partial q_L}{\partial t_L}} \frac{1}{\frac{\partial u_H}{\partial q_H}\frac{\partial q_H}{\partial t_H}} \geq 0.$$

The expression for t_H is zero only if $\lambda = 1$ and strictly positive otherwise. As for t_L, we see two components in the formula. The first is negative, whereas the second is strictly positive whenever $\lambda > 1$, and zero if $\lambda = 1$. Moreover, the second term is increasing in λ, whereas the first term decreases in magnitude. It follows that as λ increases away from one, we have $t_H > 0$, when λ is large enough, $t_L > 0$.

A.3 Income taxes

We consider four scenarios, given by the intersection of $w_H \gtreqless w_L$ and whether the seller's good is either a complement ($\frac{\partial m}{\partial q} < 0$, $\frac{\partial^2 m}{\partial q \partial s} < 0$) or substitute ($\frac{\partial m}{\partial q} > 0$, $\frac{\partial^2 m}{\partial q \partial s} > 0$) to s. The seller's problem is

$$\begin{aligned}
\max_{q_H, p_H, q_L, p_L} \quad & \pi = v\left(\left(1-t_H\right)p_H - cq_H\right) + (1-v)\left(\left(1-t_L\right)p_L - cq_L\right), && (39)\\
s.t. \quad & u\left(q_H, \theta_H\right) - m\left(q_H, s_H\right) - p_H \geq u\left(q_L, \theta_H\right) - m\left(q_L, s_H\right) - p_H - p_L, && (40)\\
& u\left(q_L, \theta_L\right) - m\left(q_L, s_L\right) - p_L \geq u\left(q_H, \theta_L\right) - m\left(q_H, s_L\right) - p_H, && (41)\\
& u\left(q_H, \theta_H\right) - m\left(q_H, s_H\right) - p_H \geq -m\left(0, s_H\right), && (42)\\
& u\left(q_L, \theta_L\right) - m\left(q_L, s_L\right) - p_L \geq -m\left(0, s_L\right). && (43)
\end{aligned}$$

In the last two constraints above, $m(0,s_i)$ is the disutility from effort given no consumption of the good. The solution to this problem follows the same steps as in the baseline model and we obtain

$$p_H = u_H - m_H - u_{HL} + m\left(q_L, s_H\right) + u_L - m_L + m\left(0, s_L\right), \tag{44}$$

$$p_L = u_L - m_L + m\left(0, s_L\right), \tag{45}$$

where we have used similar shorthand notation to the baseline model. Observe that, in expression (44), the terms $u_{HL} - m(q_L, s_H) + u_L - m_L + m(0, s_L)$ capture the high type's information rent, which accounts for the effect of choosing the L-version on the disutility from effort, in addition to the effect on the utility from consumption of the good. The equilibrium quantities satisfy the following first-order conditions:

$$\frac{\partial \pi}{\partial q_H} = \left(\frac{\partial u_H}{\partial q_H} - \frac{\partial m_H}{\partial q_H}\right)\left(1-t_H\right) - c = 0, \tag{46}$$

$$\frac{\partial \pi}{\partial q_L} = v\left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L} - \frac{\partial m_L}{\partial q_L} + \frac{\partial m\left(q_L, s_H\right)}{\partial q_L}\right)\left(1-t_H\right) + (1-v)\left(\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial m_L}{\partial q_L}\right)\left(1-t_L\right) - c\right) = 0. \tag{47}$$

These expressions have a similar interpretation to (12) and (13). In particular, the first term in (47) captures the effect of q_L on the information rent of the high types. Under our assumptions, this rent increases with q_L. Hence, this quantity is distorted downwards compared to the efficient level. Furthermore, starting from these FOCs we can show that

$$\tfrac{\partial q_L}{\partial t_L} < 0, \quad \tfrac{\partial q_L}{\partial t_H} > 0, \quad \tfrac{\partial q_H}{\partial t_L} = 0, \quad \tfrac{\partial q_H}{\partial t_H} < 0,$$

so that the effect of the tax rates on the price and versions of the goods provided by the price-discriminating seller is similar to our baseline model.

To justify the existence of income taxes, suppose the government must generate a minimum amount of revenue R to cover its expenditures. That is, the combination of income and indirect taxes satisfies the following constraint

$$v\left(T\left(y_H\right)+t_H p_H\right)+(1-v)\left(T\left(y_L\right)+t_L p_L\right)\geq R. \tag{48}$$

To streamline the analysis, we assume the seller takes individual income as given. Let welfare be the sum of utility and profits, net of taxes (whose revenue finances R). Thus, assuming (48) is binding in the design of optimal policy, we can write welfare as

$$W=v\left(u_H-m_H+y_H-cq_H\right)+(1-v)\left(u_L-m_L+y_L-cq_L\right)-R. \tag{49}$$

Case 1: $w_H > w_L$ and $\frac{\partial m}{\partial q} < 0, \frac{\partial^2 m}{\partial q \partial s} < 0$

The incentive compatibility constraint faced by the government is

$$u_H-m_H-p_H+y_H-T(y_H)\geq$$

$$\max\left(u_{HL}-m\left(q_L,\frac{y_L}{w_H}\right)-p_L+y_L-T(y_L);u_H-m\left(q_H,\frac{y_L}{w_H}\right)-p_H+y_L-T(y_L)\right)$$

To understand the expressions on the right hand side, observe that, in principle, an individual with wage w_H that decides to "shirk" could consume either the H-version of the good or the L-version. Under the assumption that $\frac{\partial m}{\partial q}<0, \frac{\partial^2 m}{\partial q \partial s}<0$ and given (44), it can be shown that

$$u_{HL}-m\left(q_L,\frac{y_L}{w_H}\right)-p_L+y_L-T(y_L)\geq u_H-m\left(q_H,\frac{y_L}{w_H}\right)-p_H+y_L-T(y_L),$$

so the constraint writes as

$$u_H-m_H-p_H+y_H-T(y_H)\geq u_{HL}-m\left(q_L,\frac{y_L}{w_H}\right)-p_L+y_L-T(y_L).$$

Rearranging the constraint and replacing the prices p_H and p_L from (44) and (45) we obtain

$$y_L\leq T(y_L)-m\left(q_L,\frac{y_H}{w_H}\right)+y_H-T(y_H)+m\left(q_L,\frac{y_L}{w_H}\right).$$

Setting the above at equality and replacing for $y_L-T(y_L)$ in (49) we get

$$W=v\left(u_H-m_H+y_H-cq_H\right)+$$

$$+(1-v)\left(u_L-m_L+T(y_L)-m\left(q_L,\frac{y_H}{w_H}\right)+y_H-T(y_H)+m\left(q_L,\frac{y_L}{w_H}\right)-cq_L\right)-R.$$

Taking the derivative of this expression with respect to t_H, evaluated at $t_L=t_H=0$, and using the equilibrium conditions in (46) and (47), we get

$$\frac{\partial W}{\partial t_H}=\frac{\partial q_L}{\partial t_H}\left(v\left(\frac{\partial u_{HL}}{\partial q_L}-\frac{\partial m\left(q_L,\frac{y_H}{w_H}\right)}{\partial q_L}-\frac{\partial u_L}{\partial q_L}+\frac{\partial m_L}{\partial q_L}\right)+(1-v)\left(\frac{\partial m\left(q_L,\frac{y_L}{w_H}\right)}{\partial q_L}-\frac{\partial m\left(q_L,\frac{y_H}{w_H}\right)}{\partial q_L}\right)\right)>0. \tag{50}$$

The derivative with respect to t_L is instead

$$\frac{\partial W}{\partial t_L}=\frac{\partial q_L}{\partial t_L}\left(v\left(\frac{\partial u_{HL}}{\partial q_L}-\frac{\partial m\left(q_L,\frac{y_H}{w_H}\right)}{\partial q_L}-\frac{\partial u_L}{\partial q_L}+\frac{\partial m_L}{\partial q_L}\right)+(1-v)\left(\frac{\partial m\left(q_L,\frac{y_L}{w_H}\right)}{\partial q_L}-\frac{\partial m\left(q_L,\frac{y_H}{w_H}\right)}{\partial q_L}\right)\right)<0. \tag{51}$$

Case 2: $w_H > w_L$ and $\frac{\partial m}{\partial q} > 0, \frac{\partial^2 m}{\partial q \partial s} > 0$

The government faces the same incentive compatibility constraint as in Case 1:

$$u_H-m_H-p_H+y_H-T(y_H)\geq$$
$$\max\left(u_{HL}-m\left(q_L,\frac{y_L}{w_H}\right)-p_L+y_L-T(y_L);u_H-m\left(q_H,\frac{y_L}{w_H}\right)-p_H+y_L-T(y_L)\right).$$

This constraint ensures that the additional income tax imposed on an individual with wage w_H, $T(y_H)-T(y_L)$, is small enough that the individual prefers to earn the “intended” income level, y_H, rather than make less effort and earn y_L. To understand the expressions on the right hand side, observe that, in principle, an individual with wage w_H that decides to “shirk” could decide to consume either the H-version of the good or the L-version. Under the assumption that $\frac{\partial m}{\partial q}<0, \frac{\partial^2 m}{\partial q\partial s}<0$ and given (44), it can be shown that

$$u_{HL}-m\left(q_L,\frac{y_L}{w_H}\right)-p_L+y_L-T(y_L)\leq u_H-m\left(q_H,\frac{y_L}{w_H}\right)-p_H+y_L-T(y_L),$$

so the incentive constraint writes as

$$u_H-m_H-p_H+y_H-T(y_H)\geq u_H-m\left(q_H,\frac{y_L}{w_H}\right)-p_H+y_L-T(y_L).$$

Rearranging the constraint and replacing the prices p_H and p_L from (44) and (45) we obtain

$$y_L \leq T(y_L) - m_H + m\left(q_H, \frac{y_L}{w_H}\right) + y_H - T(y_H).$$

Setting the above at equality and replacing for y_L in (49) we get

$$W = v(u_H - m_H + y_H - cq_H) + (1-v)\left(u_L - m_L + T(y_L) + m\left(q_H, \frac{y_L}{w_H}\right) + y_H - T(y_H) - m_H - cq_L\right) - R.$$

Taking the derivative of this expression with respect to t_H, evaluated at $t_L = t_H = 0$, and using the equilibrium conditions in (46) and (47), we get

$$\frac{\partial W}{\partial t_H} = \frac{\partial q_L}{\partial t_H} v\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial m\left(q_L, \frac{y_H}{w_H}\right)}{\partial q_L} - \frac{\partial u_L}{\partial q_L} + \frac{\partial m_L}{\partial q_L}\right) + (1-v)\frac{\partial q_H}{\partial t_H}\left(\frac{\partial m\left(q_H, \frac{y_L}{w_H}\right)}{\partial q_H} - \frac{\partial m_H}{\partial q_H}\right) > 0. \tag{52}$$

The derivative with respect to t_L is instead

$$\frac{\partial W}{\partial t_L} = \frac{\partial q_L}{\partial t_L} v\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial m\left(q_L, \frac{y_H}{w_H}\right)}{\partial q_L} - \frac{\partial u_L}{\partial q_L} + \frac{\partial m_L}{\partial q_L}\right) < 0. \tag{53}$$

Case 3: $w_H < w_L$ and $\frac{\partial m}{\partial q} < 0, \frac{\partial^2 m}{\partial q \partial s} < 0$

Given $w_H < w_L$, the incentive compatibility constraint the government faces is

$$u_L - m_L - p_L + y_L - T(y_L) \geq$$

$$\max\left(u_{LH} - m\left(q_H, \frac{y_H}{w_L}\right) - p_H + y_H - T(y_H); u_L - m\left(q_L, \frac{y_H}{w_L}\right) - p_L + y_H - T(y_H)\right).$$

Note that, unlike in Cases 1 and 2, it is now the "low" type individual that has the highest earning ability and, hence, may want to choose the lower level of income, y_H, to save on effort. Given (44), it can be shown that

$$u_{LH} - m\left(q_H, \frac{y_H}{w_L}\right) - p_H + y_H - T(y_H) \leq u_L - m\left(q_L, \frac{y_H}{w_L}\right) - p_L + y_H - T(y_H),$$

by the assumption that $\frac{\partial u}{\partial q} - \frac{\partial m}{\partial q}$ is positive and increases in θ. Hence, the constraint writes as

$$u_L - m_L - p_L + y_L - T(y_L) \geq u_L - m\left(q_L, \frac{y_H}{w_L}\right) - p_L + y_H - T(y_H).$$

Rearranging the constraint and replacing the prices p_H and p_L from (44) and (45) we obtain

$$y_H \leq T(y_H) + m\left(q_L, \frac{y_H}{w_L}\right) + y_L - T(y_L) - m_L.$$

Setting the above at equality and replacing for y_H in (49) we get

$$W = v\left(u_H - m_H + T(y_H) + m\left(q_L, \frac{y_H}{w_L}\right) + y_L - T(y_L) - m_L - cq_H\right) + (1-v)(u_L - m_L + y_L - cq_L).$$

Taking the derivative of this expression with respect to t_H, evaluated at $t_L = t_H = 0$, and using the equilibrium conditions in (46) and (47), we get

$$\frac{\partial W}{\partial t_H} = \frac{\partial q_L}{\partial t_H} v \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial m\left(q_L, \frac{y_H}{w_H}\right)}{\partial q_L} - \frac{\partial u_L}{\partial q_L} + \frac{\partial m_L}{\partial q_L} + \frac{\partial m\left(q_L, \frac{y_H}{w_L}\right)}{\partial q_L} - \frac{\partial m_L}{\partial q_L}\right) > 0. \tag{54}$$

The derivative with respect to t_L is instead

$$\frac{\partial W}{\partial t_L} = \frac{\partial q_L}{\partial t_L} v \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial m\left(q_L, \frac{y_H}{w_H}\right)}{\partial q_L} - \frac{\partial u_L}{\partial q_L} + \frac{\partial m_L}{\partial q_L} + \frac{\partial m\left(q_L, \frac{y_H}{w_L}\right)}{\partial q_L} - \frac{\partial m_L}{\partial q_L}\right) < 0. \tag{55}$$

Case 4: $w_H < w_L$ and $\frac{\partial m}{\partial q} > 0, \frac{\partial^2 m}{\partial q \partial s} > 0$

Consider now the problem of the government. Given $w_H < w_L$, the incentive compatibility constraint it faces is

$$u_L - m_L - p_L + y_L - T(y_L) \geq$$
$$\max\left(u_{LH} - m\left(q_H, \frac{y_H}{w_L}\right) - p_H + y_H - T(y_H); u_L - m\left(q_L, \frac{y_H}{w_L}\right) - p_L + y_H - T(y_H)\right).$$

Given (44), it can be shown that

$$u_{LH} - m\left(q_H, \frac{y_H}{w_L}\right) - p_H + y_H - T(y_H) \leq u_L - m\left(q_L, \frac{y_H}{w_L}\right) - p_L + y_H - T(y_H),$$

by the assumption that $\frac{\partial u}{\partial q}-\frac{\partial m}{\partial q}$ is positive and increases in θ. Hence, the constraint writes as

$$u_L-m_L-p_L+y_L-T(y_L)\geq u_L-m\left(q_L,\frac{y_H}{w_L}\right)-p_L+y_H-T(y_H).$$

Rearranging the constraint and replacing the prices p_H and p_L from (44) and (45) we obtain

$$y_H\leq T(y_H)+m\left(q_L,\frac{y_H}{w_L}\right)+y_L-T(y_L)-m_L.$$

Setting the above at equality and replacing for y_H in (49) we get

$$W=v\left(u_H-m_H+T(y_H)+m\left(q_L,\frac{y_H}{w_L}\right)+y_L-T(y_L)-m_L-cq_H\right)+(1-v)\left(u_L-m_L+y_L-cq_L\right).$$

Taking the derivative of this expression with respect to t_H, evaluated at $t_L=t_H=0$, and using the equilibrium conditions in (46) and (47), we get

$$\frac{\partial W}{\partial t_H}=\frac{\partial q_L}{\partial t_H}v\left(\left(\frac{\partial u_{HL}}{\partial q_L}-\frac{\partial m\left(q_L,\frac{y_H}{w_H}\right)}{\partial q_L}-\frac{\partial u_L}{\partial q_L}+\frac{\partial m_L}{\partial q_L}\right)+\frac{\partial m\left(q_L,\frac{y_H}{w_L}\right)}{\partial q_L}-\frac{\partial m_L}{\partial q_L}\right). \tag{56}$$

The sign of the first term in this expression is positive, but the second term has negative sign given $0<\frac{\partial m\left(q_L,\frac{y_H}{w_L}\right)}{\partial q_L}<\frac{\partial m_L}{\partial q_L}$ by the assumption that $\frac{\partial^2 m}{\partial q\partial s}>0$, given $\frac{y_H}{w_L}<\frac{y_L}{w_L}$. Hence, the overall sign is ambiguous. It is however significant to note that the second term is negative due to fact that the introduction of t_H increases q_L. This tightens the incentive constraint because the good is substitute to labor supply. When choosing to exert less effort, the increase in the marginal disutility from effort caused by the increase in q_L is smaller than when the individual exerts the intended level of effort, $\frac{y_L}{w_L}$.

The derivative with respect to t_L is instead

$$\frac{\partial W}{\partial t_L}=\frac{\partial q_L}{\partial t_L}v\left(\left(\frac{\partial u_{HL}}{\partial q_L}-\frac{\partial m\left(q_L,\frac{y_H}{w_H}\right)}{\partial q_L}-\frac{\partial u_L}{\partial q_L}+\frac{\partial m_L}{\partial q_L}\right)+\frac{\partial m\left(q_L,\frac{y_H}{w_L}\right)}{\partial q_L}-\frac{\partial m_L}{\partial q_L}\right). \tag{57}$$

Again, the sign is ambiguous.

A.4 Allowing for alternative payment systems

Given our assumptions, we set $p_L = 0$ and can write the seller's problem as follows

$$
\begin{aligned}
\max_{q_H, p_H, x_H, q_L, x_L, x_L, p_H^0, x_L} \quad & \pi = b\left(v\left((1-t_H)\,p_H - cq_H\right) + (1-v)\,(p_L - cq_L)\right) + \\
& + (1-b)\left(v\left(p_H^o - cq_H\right) + (1-v)\,(p_L - cq_L)\right), \\
s.t. \quad & u(q_H, \theta_H) - p_H \geq u(q_L, \theta_H) - p_L, && (58)\\
& u(q_H, \theta_H) - p_H^o - \gamma \geq u(q_H, \theta_H) - p_H, && (59)\\
& u(q_L, \theta_L) - p_L \geq u(q_H, \theta_L) - p_H, && (60)\\
& u(q_L, \theta_L) - p_L \geq u(q_H, \theta_L) - p_H^0 - \gamma, && (61)\\
& u(q_H, \theta_H) - p_H \geq 0, && (62)\\
& u(q_H, \theta_H) - p_H^o - \gamma, \geq 0, && (63)\\
& u(q_L, \theta_L) - p_L \geq 0. && (64)
\end{aligned}
$$

In the above problem, in addition to the usual incentive compatibility and participation constraints for the consumers who would transact with the seller only inside the platform, we include incentive constraints for those consumers that would be willing to transact with the seller outside of it, sustaining the cost γ. Specifically, constraint 59 requires that high type consumers who are willing to transact outside the platform prefer to do so rather than go through the platform. Constraint (61) requires that low type consumers prefer to choose the version of the app intended for them than the high version (accessed outside the platform). Finally, constraint (63) requires that high type consumers that transact outside the platform get non-negative utility. Clearly, (59) is satisfied optimally by setting $p_H^o = p_H - \gamma$, so constraints (61) and (63) are redundant. As a result, we can use the same arguments as in the baseline model to show that the solution to the seller's problem is such that $p_H = u_H - u_{HL} + u_L$ and $p_L = u_L$. Hence, we can write the objective as (38). The remainder of the analysis follows from the main text.

A.5 Device sales

We solve the model by backward induction. Consider Stage 3. Let $n \in [0,1]$ be the number of consumers who bought the device at Stage 2. At Stage 3, these consumers observe θ and select the version of the product based on the same utility function as in (1). Consider now Stage 2. Each consumer gets an expected payoff equal to $d + E(CS) - p_D$ when buying the device, where $E(CS)$ is the ex-ante expected surplus a consumer gets from the seller's products. It follows that all consumers buy the device, i.e., $n = 1$, if and only if $d + E(CS) \geq p_D$, whereas $n = 0$ otherwise. The latter scenario cannot be optimal to the platform, so we restrict attention to $p_D \leq d + E(CS)$ and $n = 1$.

Given that the share of high types, v, is the same as in the baseline model, the profit of the seller is isomorphic to (2), except that it is multiplied by n. Note that the seller takes n as given, because consumers make their decision whether to buy the device prior to observing the values of the variables p_i, q_i and x_i, for $i = H, L$. It follows that the solution to the seller's problem, given p_D and t, is the same as in the model of Section 3. Hence, at stage 3, H-type consumers get the same surplus as in (11) and their expected surplus at stage 2 is $E(CS) = v(u^e_{HL} - u^e_L)$, where $u^e_i \equiv (q^e_i, \theta_i)$ and $u_{HL} \equiv u(q^e_H, \theta_L)$ and the superscript e denotes the values chosen by the seller in equilibrium (given t).

Finally, consider Stage 1. The solution to the platform's problem must be such that $p_D = d + E(CS)$. When choosing t_H, therefore, the platform maximizes (34). Compare this expression to (10), and notice that $u^e_{HL} - u^e_L$ increases with t_H. Hence the derivative of (34) with respect to t_H is everywhere greater than the derivative of (10) (we focus on the $0 \leq t_H < \bar{t_H}(t_L = 0)$ interval, where $\bar{t_H}(t_L)$ is defined in Appendix B). Thus, one can apply the results of Milgrom and Shannon (1994) to conclude that the equilibrium level of the fee must be higher when the platform sells the device than when it does not.

A.6 Analysis with hybrid platform

The game is described in Section 7.3. At stage 3, consumers can buy the platform's product or search the seller's. Consumers of type i who search expect to get the surplus $CS^e_i - \sigma$, where σ is the search cost and CS^e_i is the surplus conditional on the equilibrium values of p_i and q_i (that the seller chooses at stage 2, given t_H), that we shall denote with the superscript e. Recall that consumers have a rational expectation about this surplus, but they need to search to observe the characteristics of the seller's product. The search cost is small, i.e., $\sigma \to 0$, and thus omitted in the expressions that follow. In equilibrium, no loyal consumers buy the platform's product, while the non-loyal search it if and only if $CS^e_i \geq CS^P_i$. Therefore, all consumers of type i are available to the seller if $CS^e_i \geq CS^P_i$, while only a share s is available otherwise.

Consider now stage 2. The seller chooses p_i and q_i, given t_H, p^P_i, and q^P_i, and the share of consumers that is available. Non-loyal consumers do not observe the equilibrium values of p_i and q_i prior to searching, but only have a rational expectation about such values. Hence, the seller treats the shares of consumers that are available as given when choosing these variables (since the loyal consumers only buy the seller's product by definition). Let these shares be S_H and S_L among, respectively, high- and low-type consumers. We have $S_H = vs$ if $CS_H^e < CS_H^P$, and $S_H = v$ otherwise. Similarly, $S_L = s(1-v)$ if $CS_L^e < CS_L^P$, and $S_L = 1-v$ otherwise. The constraints faced by the seller are the same as in the baseline model (i.e., (3)-(6)). The seller adopts price discrimination, with prices set as in expression (9). Specifically, $p_H = u_H - u_{HL} + u_L$ and $p_L = u_L$ must hold, where $u_i \equiv (q_i, \theta_i)$ and $u_{HL} \equiv u(q_H, \theta_L)$. Hence, the seller's problem reduces to

$$\max_{q_H, q_L} \quad \pi = S_H \left((1-t_H)(u_H - u_{HL} + u_L) - cq_H \right) + S_L (u_L - cq_L). \tag{65}$$

Note that if $S_H = vs$ and $S_L = (1-v)s$, or if $S_H = v$ and $S_L = (1-v)$, the objective is isomorphic (up to a multiplicative constant) to (2), so the two problems must have the same solution. In words, the seller faces the same problem as in the baseline model when either all consumers or only the captive ones search.

Whenever $t_H < \bar{t_H}(t_L = 0)$ (where $\bar{t_H}(t_L)$ is defined in Appendix B), the seller serves both consumer types, high-type consumers get a surplus $CS_H^e = u_{HL}^e - u_L^e$ in equilibrium, where $u_i^e \equiv (q_i^e, \theta_i)$ and so on, whereas low type consumers get $CS_L^e = 0$. If $t_H \geq \bar{t_H}(t_L)$, the seller only sells a single version of its product, targeting the high-types and sets $p_H = u_H$, so that $CS_H^e = CS_L^e = 0$. Consumers would of course obtain the same levels of expected surplus if t_H was so large that the seller simply dropped out of the market. Observe that the solution to the seller's problem only depends on the platform's decisions at stage 1 through t_H and the surpluses CS_i^P (which affect the shares S_H and S_L).

Focus now on stage 1. We assume that the platform wants to serve all consumer types with its product. The platform's problem is therefore

$$\max_{t, q_H^P, p_H^P, x_H^P, q_L^P, x_L, x_L^P} \quad \pi_P = (1-S_H)\left(p_H^P - cq_H^P\right) + (1-v)(1-S_L)\left(p_L^P - cq_L^P\right) + t_H S_H p_H.$$

with $S_H = vs$ and $S_L = (1-v)s$ (i.e., only the loyal consumers buy from the seller). The platform must satisfy the following constraints

$$u\left(q_H^P, \theta_H\right) - p_H^P \geq u\left(q_L^P, \theta_H\right) - p_L^P, \tag{66}$$

$$u\left(q_L^P, \theta_L\right) - p_L^P \geq u\left(q_H^P, \theta_L\right) - p_H^P, \tag{67}$$

$$u\left(q_H^P, \theta_H\right) - p_H^P \geq \max\left(0, CS_H^e\right), \tag{68}$$

$$u\left(q_L, \theta_L\right) - p_L^P \geq \max\left(0, CS_L^e\right). \tag{69}$$

The first two constraints are incentive compatibility constraints. The last two constraints are participation constraints: each (non-loyal) consumer type must receive at least the surplus it can expect to get by searching the third-party seller's product. Following standard procedures, and noting that $\max(0, CS_H^e) = CS_H^e$, while $\max(0, CS_L^e) = 0$, we have

$$\begin{aligned} p_H^P &= \min\left(u_H^P - u_{HL}^P + u_L^P, CS_H^e\right), \\ p_L^P &= u_L^P, \end{aligned}$$

where $u_i^P \equiv \left(q_i^P, \theta_i\right)$ and $u_{HL}^P \equiv u\left(q_H^P, \theta_L\right)$. Note that in this setting the incentive compatibility constraint for the H-type is not necessarily binding in equilibrium, because the third-party seller's product tightens the participation constraints. Assuming that $p_L^P = u_L^P$, as in the baseline model, we have

$$\begin{aligned} p_H^P &= \min\left(u_H^P - u_{HL}^P + u_L^P, u_H^P - CS_H^e\right), \\ p_L^P &= u_L^P. \end{aligned} \tag{70}$$

Suppose that the H-type's incentive compatibility constraint binds, that is, that $u_H^P - u_{HL}^P + u_L^P \leq u_H^P - CS_H^e$, so that $p_H^P = u_H^P - u_{HL}^P + u_L^P$. The platform's problem would then reduce to

$$\max_{t, q_H^P, q_L^P} \quad (1-s)\left(v\left(u_H^P - u_{HL}^P + u_L^P - cq_H^P\right) + (1-v)\left(u_L^P - cq_L^P\right)\right) + st_H v p_H^e.$$

Since p_H^e does not depend on q_i^P, the pair $\left(q_{H0}^P, q_{L0}^P\right)$ that solves this problem must be the same as the pair solving (65) when $t_H = 0$ (and $S_H = vs$ and $S_L = (1-v)s$ hold). Hence, conditional on $t_H = 0$, the surplus of the high types, $u_{HL0}^P - u_{L0}^P$ equals CS_H^e. However, by Proposition 1, CS_H^e increases with t_H, for any $0 \leq t_H < \bar{t}$. Hence, for any $0 < t_H < \bar{t}$, we have $u_{HL0}^P - u_{L0}^P < CS_H^e$, so $u_H^P - u_{HL}^P + u_L^P > u_H^P - CS_H^e$ must hold. That is, the H-type's participation constraint binds. Finally, when $t_H \geq \bar{t}$, $CS_H^e = 0$, so $u_H^P - u_{HL}^P + u_L^P \leq u_H^P - CS_H^e$ must hold. This is because the seller only serves the H-types in this case, and extract all their surplus.

Summing up, we can write the platform's problem as

$$\max_{q_H^P, q_L^P, t_H} \quad \pi_P = \begin{cases} (1-s)\left(v\left(u_H^P - CS_H^e - cq_H^P\right) + (1-v)\left(u_L^P - cq_L^P\right)\right) + st_H v p_H & \text{if } 0 \leq t_H < \bar{t_H}(t_L), \\ (1-s)\left(v\left(u_H^P - u_{HL}^P + u_L^P - cq_H^P\right) + (1-v)\left(u_L^P - cq_L^P\right)\right) + st_H v p_H & \text{if } t_H \geq \bar{t_H}(t_L). \end{cases} \tag{71}$$

Let us first focus on the case where $0 \leq t_H < \bar{t_H}(t_L)$. We compare the platform's profit in expression (71) to (31), and note that CS_H^e increases with t_H, while $\left(q_H^P, q_L^P\right)$ do not depend on it. Note also that the term $t_H v p_H^e$ is identical in the two expressions, for any t_H. Hence the derivative of (71) with respect to t_H is everywhere smaller than the derivative of (31). Thus, one can apply the results by Milgrom and Shannon (1994) to conclude that the equilibrium level of the fee must be smaller when the platform sells its own product than when it does not (again, conditional on the solution being such that $0 \leq t_H < \bar{t_H}(t_L = 0)$).

However, we cannot exclude the possibility that the platform prefers a fee such that $t_H \geq \bar{t_H}(t_L = 0)$ when selling its own product, and a fee such that $0 \leq t_H < \bar{t_H}(t_L = 0)$ when it is a pure marketplace. This is because there is a discrete increase in the revenue from selling the product when t_H reaches the level $\bar{t_H}(t_L = 0)$, compared to when $0 < t_H < \bar{t_H}(t_L = 0)$, as established above. Nevertheless, by setting $t_H \geq \bar{t_H}(t_L = 0)$ the platform already ensures that no consumer gets a positive surplus when buying from the seller, because the latter only serves the high types and captures all their surplus (setting $p_H = u_H$), so that $CS_H^e = 0$. Therefore setting t_H to a level such that the seller makes zero net profit, and exits the market, cannot be optimal: the platform would then earn the same profit from the sale of its product as in the second row of (71), but forgo the fee revenue $t_H v s p_H^e$.

A.7 Endogenous number of consumers and sellers

We solve the model backwards. At stage 3, consumers obtain the same surplus from each seller as in the baseline model, i.e., $CS_H^e = u_{HL}^e - u_L^e$ and $CS_L^e = 0$. To see why, consider that at stage 2, each seller faces the same problem as in (88), except that the total number of consumers is n_c. Since each seller takes this number as given, the solution is identical to (88) and we get the same values of q_i^e, p_i^e and x_i^e. Prior to joining the platform, each consumer expects to obtain the surplus $n_s E(CS) = n_s \left(v CS_H^e + (1-v) CS_L^e\right) = n_s v CS_H^e$, whereas each seller gets the profit in (36), that we denote by π^e after replacing for the equilibrium values q_H^e and q_L^e (given t). Hence, we have $n_s = \phi_s(n_c \pi^e)$ and $n_c = \phi_c(n_s v CS_H^e)$. Starting from these expressions, we can write the following derivatives

$$\frac{\partial n_s}{\partial t} = \phi_s' n_c \frac{\partial \pi^e}{\partial t} < 0, \qquad \frac{\partial n_c}{\partial t} = \phi_c' n_s v \frac{\partial CS_H^e}{\partial t} > 0,$$

$$\frac{dn_s}{dt} = \frac{\partial n_s}{\partial t} + \phi_s' \frac{dn_c}{dt} \pi^e, \qquad \frac{dn_c}{dt} = \frac{\partial n_c}{\partial t} + \phi_c' v CS_H^e \frac{dn_s}{dt}.$$

Combining the above derivatives and rearranging, we obtain

$$\frac{dn_s}{dt} = \frac{\phi_s' \left(n_c \frac{\partial \pi^e}{\partial t} + \phi_c' n_s v \frac{\partial CS_H^e}{\partial t} \pi^e\right)}{1 - \phi_c' \phi_s' v CS_H^e \pi^e}, \qquad \frac{dn_c}{dt} = \frac{\phi_c' \left(n_s v \frac{\partial CS_H^e}{\partial t} + \phi_s' n_c \frac{\partial \pi^e}{\partial t} v CS_H^e\right)}{1 - \phi_c' \phi_s' v CS_H^e \pi^e}.$$

We assume the denominator in the above expressions is positive, i.e. $1 > \phi_c' \phi_s' v CS_H^e \pi^e$. These derivatives show that, if the number of consumers is much less responsive than the number of sellers (i.e., $\phi_s' \to 0$), then $\frac{dn_s}{dt} \to 0$, whereas $\frac{dn_c}{dt} \to \phi_c' n_s v \frac{\partial CS_H^e}{\partial t} > 0$. By contrast, if the number of sellers is much less responsive than the number of consumers (i.e., $\phi_c' \to 0$), we have $\frac{dn_c}{dt} \to 0$, whereas $\frac{dn_s}{dt} \to \phi_s' n_c \frac{\partial \pi^e}{\partial t} < 0$. Furthermore, both total derivatives can be positive, provided the condition stated in the Propositon holds.

ONLINE APPENDIX

B Provision of differentiated versions (Online)

Suppose that the pair of tax rates is such that $q_L^e > 0$, where q_L^e is the unique value of q_L such that 13 holds. For this to be the case, it must be that t_L is not too large, given t_H. Indeed, we can argue that there exists a unique $\bar{t_L}$ such that $q_L^e = 0$ whenever $t_L > \bar{t_L}$. Note from expressions 12 and 13 that q_L^e decreases with t_L, whereas q_H^e does not depend on this tax rate. Hence, there exists a threshold $\bar{t_L}$ such that $q_L^e = 0$ whenever $t_L > \bar{t_L}$.

We now show that, for any t_L such that $q_L^e > 0$, whenever $0 \leq t_H < \bar{t}_H(t_L)$ the seller's optimal strategy is to provide two differentiated products. To prove the existence of the threshold $\bar{t}_H(t_L)$, observe from expressions 12 and 13 that the necessary condition $q_H^e > q_L^e$ holds when $t_H = 0$, but $q_H^e = 0 < q_L^e$ when $t_H = 1$, for any $1 \geq t_L \geq 0$.[20] Given that q_H^e and q_L^e respectively decrease and increase monotonically with t_H (see (14)), there exists a unique value of t_H, that we denote by $\bar{t}_H(t_L)$, such that $q_H^e = q_L^e$ holds. Whenever $t_H \geq \bar{t}_H(t_L)$, the seller cannot charge a premium for the H-bundle without violating the constraint in (3).

Given $t_H \geq \bar{t}_H(t_L)$, the seller has two options in principle. The first is to serve only the high types (Option H). Under this option, it is a dominant strategy for the seller to set $\overline{p}_H = u_H$, violating the low-type's participation constraint. The optimal q for the seller in this case, that we denote by $\bar{q}_H$, maximizes $u(\theta_H, q)(1 - t_H) - cq$ and is thus equal to the value q_H^e that satisfies 12 (note that this is independent of t_L). The seller would therefore earn $\pi_H = v((1 - t_H)u(\bar{q}_H, \theta_H) - c\bar{q}_H)$. Observe that, by definition, when $t_H = \bar{t}_H(t_L)$, $\bar{q}_H = q_L^e$ holds and, since $\bar{q}_H$ is decreasing in t_H, we must have $\bar{q}_H < q_L^e$ for $t_H > \bar{t}_H(t_L)$.

The second option is to serve the the L-version to both types (Option L). Under this option, it is a dominant strategy for the seller to set $p_L = u(\theta_L, q)$, as any higher price level would violate the low type's participation constraint. Therefore, the optimal q for the seller in this case, that we denote by $\bar{q}_L$, would maximize $u(\theta_L, q)(1 - t_L) - cq$. The seller's profit would thus be equal to $\pi_L = \bar{u}_L(1 - t_L) - c\bar{q}_L$, where $\bar{u}_L = u(\theta_L, \bar{q}_L)$. Observe that $\bar{q}_L > q_L^e$ when the latter value is evaluated at $\bar{t}_H(t_L)$.

By assumption, the seller prefers to supply two differentiated versions when $t_H \leq \bar{t}_H(t_L)$, the

[20] If $c = 0$ when $t_H = 1$ setting $q_H = 0$ is only weakly optimal for the seller. However, it seems reasonable to assume that if faced with an extreme 100% fee rate, the seller would have no interest in providing the product.

following inequality must hold for any $t_H \in [0, \bar{t}_H(t_L)]$:

$$v\left(\left(1-t_H\right)\left(u_H^e - u_{HL}^e + u_L^e\right) - cq_H^e\right) + \left(1-v\right)\left(\left(1-t_L\right)u_L^e - cq_L^e\right) > v\left(\left(1-t_H\right)u\left(\bar{q}_H, \theta_H\right) - c\bar{q}_H\right).$$

Since, as established above, $\bar{q}_H = q_H^e$, we have $u_H^e = u\left(\bar{q}_H, \theta_H\right)$, so we can rearrange the above inequality as

$$\left(1-v\right)\left(u_L^e\left(1-t_L\right) - cq_L^e\right) > v\left(1-t_H\right)\left(u_{HL}^e - u_L^e\right). \tag{72}$$

We now establish that Option L dominates Option H for any $t_H \geq \bar{t}_H(t_L)$.

$$\bar{u}_L\left(1-t_L\right) - c\bar{q}_L > v\left(\left(1-t_H\right)u\left(\bar{q}_H, \theta_H\right) - c\bar{q}_H\right), \tag{73}$$

The right hand side of this inequality is decreasing in t_H, whereas the left hand side does not depend on it. Therefore, it is sufficient to establish that

$$\bar{u}_L\left(1-t_L\right) - c\bar{q}_L > v\left(\left(1-t_H\right)u\left(\bar{q}_H, \theta_H\right) - c\bar{q}_H\right)\big|_{t_H = \bar{t}_H(t_L)},$$

which given $\bar{q}_H = q_L^e$ when $t_H = \bar{t}_H(t_L)$, we can also write as

$$\bar{u}_L\left(1-t_L\right) - c\bar{q}_L > v\left(\left(1-t_H\right)u\left(q_L^e, \theta_H\right) - cq_L^e\right)\big|_{t_H = \bar{t}_H(t_L)}.$$

Adding and subtracting $v\left(1-t\right)u\left(q_L^e, \theta_L\right)\big|_{t_H = \bar{t}_H(t_L)}$ from the right hand side, we can rewrite the inequality as

$$\bar{u}_L\left(1-t_L\right) - c\bar{q}_L > v\left(\left(1-t\right)u\left(q_L^e, \theta_H\right) - u\left(q_L^e, \theta_L\right) + u\left(q_L^e, \theta_L\right) - cq_L^e\right)\big|_{t_H = \bar{t}_H(t_L)}.$$

Given (72), the right hand side of this inequality is bounded from above by $u\left(q_L^e, \theta_L\right) - cq_L^e\big|_{t_H = \bar{t}_H(t_L)}$. Hence, to establish (73), it is sufficient to check that

$$\bar{u}_L - c\bar{q}_L \geq u\left(q_L^e, \theta_L\right) - cq_L^e\big|_{t_H = \bar{t}_H}.$$

This inequality must be satisfied given that, by definition, $\bar{q}_L$ maximizes $u\left(q, \theta_L\right) - cq$, whereas q_L^e does not.

C Other tax instruments (Online)

C.1 Uniform ad valorem tax

Consider profits in (2) with $t_L = t_H = t$. The first-order conditions of the monopolist's problem are

$$\frac{\partial \pi}{\partial q_H} := \frac{\partial u_H}{\partial q_H}(1-t_H) - c = 0, \tag{74}$$

$$\frac{\partial \pi}{\partial q_L} := v\left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L}\right)(1-t_H) + (1-v)\left(\frac{\partial u_L}{\partial q_L}(1-t_L) - c\right) = 0. \tag{75}$$

By totally deriving the above first-order conditions of the monopolist's problem with respect to a uniform tax t, we find that

$$\frac{\partial q_i}{\partial t} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial t} - \frac{\partial^2 \pi}{\partial q_i \partial t}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H},$$

where $H \equiv \frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H^2} - \left(\frac{\partial^2 \pi}{\partial q_H \partial q_L}\right)^2 > 0$, $\frac{\partial^2 \pi}{\partial q_j^2} < 0$, $\frac{\partial^2 \pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial t} = -v\frac{\partial u_H}{\partial q_H} < 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial t} = -\left(\frac{\partial u_L}{\partial q_L} - v\frac{\partial u_{HL}}{\partial q_L}\right) < 0$. Hence,

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial t}\right) = \operatorname{sgn}\left(-\frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H \partial t}\right) = \operatorname{sgn}\left(-v\frac{\partial u_H}{\partial q_H}\right) < 0,$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial t}\right) = \operatorname{sgn}\left(-\frac{\partial^2 \pi}{\partial q_H^2}\frac{\partial^2 \pi}{\partial q_L \partial t}\right) = \operatorname{sgn}\left(-\left(\frac{\partial u_L}{\partial q_L} - v\frac{\partial u_{HL}}{\partial q_L}\right)\right) < 0.$$

Furthermore, the introduction of a small ad valorem tax has negative effects on welfare

$$\left.\frac{\partial W}{\partial t}\right|_{t=0} = \frac{\partial q_H}{\partial t}v\left(\frac{\partial u_H}{\partial q_H} - c\right) + \frac{\partial q_L}{\partial t}(1-v)\left(\frac{\partial u_L}{\partial q_L} - c\right) < 0.$$

C.2 Unit taxes

We solve this problem following the same steps as in Section 4: constraints (3) and (6) are binding, meaning that equilibrium prices are as given in (9). Replacing these prices in (24), and using again the shorthand notation $u_i \equiv u(q_i, \theta_i)$, $i = H, L$, and $u_{HL} \equiv u(q_L, \theta_H)$, we get

$$\pi = v(u_H + u_{HL} - u_L - (c + \tau_H)q_H) + (1-v)(u_L - (c + \tau_L)q_L). \tag{76}$$

Given the constraints (3)-(6), and using similar steps, we obtain that the equilibrium quantities solve the following system of equations

$$\frac{\partial \pi}{\partial q_H} := v\left(\frac{\partial u_H}{\partial q_H} - c - \tau_H\right) = 0, \tag{77}$$

$$\frac{\partial \pi}{\partial q_L} := v\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{HL}}{\partial q_L}\right) + (1-v)\left(\frac{\partial u_L}{\partial q_L} - c - \tau_L\right) = 0. \tag{78}$$

By totally differentiating the first-order conditions of the monopolist's problem in(77) and (78), we find that

$$\frac{\partial q_i}{\partial \tau_i} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial \tau_i} - \frac{\partial^2 \pi}{\partial q_i \partial \tau_j}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \quad \frac{\partial q_j}{\partial \tau_i} = -\frac{\frac{\partial^2 \pi}{\partial q_i^2}\frac{\partial^2 \pi}{\partial q_j \partial \tau_i} - \frac{\partial^2 \pi}{\partial q_i \partial \tau_i}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \; i,j = H,L, \; j \neq i.$$

where $H \equiv \frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H^2} - \left(\frac{\partial^2 \pi}{\partial q_H \partial q_L}\right)^2 > 0, \quad \frac{\partial^2 \pi}{\partial q_j^2} < 0, \quad \frac{\partial^2 \pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau_L} = 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau_H} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau_H} = -v$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau_L} = -(1-v) < 0$. Hence, we have

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial \tau_H}\right) = \operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_L^2} v\right) < 0$$

$$\frac{\partial q_L}{\partial \tau_H} = 0, \frac{\partial q_H}{\partial \tau_L} = 0$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial \tau_L}\right) = \operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_H^2}(1-v)\right) < 0.$$

Consider now a uniform unit tax $\tau_L = \tau_H = \tau$. By totally differentiating the first-order conditions of the monopolist's problem in(77) and (78), we find that

$$\frac{\partial q_i}{\partial \tau} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial \tau} - \frac{\partial^2 \pi}{\partial q_i \partial \tau}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \quad \frac{\partial q_j}{\partial \tau} = -\frac{\frac{\partial^2 \pi}{\partial q_i^2}\frac{\partial^2 \pi}{\partial q_j \partial \tau} - \frac{\partial^2 \pi}{\partial q_i \partial \tau}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \; i,j = H,L, \; j \neq i.$$

where $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau} = -v$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau} = -(1-v)$. Hence, we have

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial \tau}\right) = \operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_L^2} v\right) < 0,$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial \tau}\right) = \operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_H^2}(1-v)\right) < 0.$$

D Freemium pricing (Online)

D.1 Freemium pricing with ads as a nonmonetary price

We now provide an extended version of the model of Section 3, where we allow the seller to choose between monetary and nonmonetary prices for each version of its good. The first objective of this part is to provide sufficient conditions for the case of full monetary and freemium pricing. Secondly, we aim to establish that the effects of a transaction fee that we have shown in Propositions 1 carry through to this more general setting.

Assume a consumer of type i sustain a disutility $\alpha_i > 0$ for every non-monetary unit paid. Thus, the utility of a type-i consumer is

$$U_i(p,q,x) = u(q,\theta_i) - p - \alpha_i x,\ i = H, L. \tag{79}$$

For simplicity, we assume perfect correlation (either positive or negative) between the parameters θ and α. The seller earns a revenue r_i for every unit of non-monetary price on version i. Given these assumptions, and using the same notation for utility as in the baseline model, the seller's problem is

$$\max_{q_H, p_H, x_H, q_L, p_L, x_L} \quad \pi = v((1-t)p_H + r_H x_H - c q_H) + (1-v)((1-t)p_L + r_L x_L - c q_L), \tag{80}$$

$$s.t. \quad u_H - p_H - \alpha_H x_H \geq u_{HL} - p_L - \alpha_H x_L, \tag{81}$$

$$u_L - p_L - \alpha_L x_L \geq u_{LH} - p_H - \alpha_L x_H, \tag{82}$$

$$u_H - p_H - \alpha_H x_H \geq 0, \tag{83}$$

$$u_L - p_L - \alpha_L x_L \geq 0. \tag{84}$$

We assume that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L}$ holds, i.e., the difference between the disutility from ads of the high and low types is small relative to the difference in their marginal utility from product quality. This assumption guarantees that the incentive compatibility constraint of the high type is satisfied. In Appendix D.1.1 we show that under this assumption, the usual constraints (81) and (84) bind at equilibrium, so we have

$$p_H + \alpha_H x_H = u_H - u_{HL} + \alpha_H x_L + u_L - \alpha_L x_L, \quad p_L + \alpha_L x_L = u_L. \tag{85}$$

Consequently, we can rewrite the seller's problem as

$$\max_{q_H,x_H,q_L,x_L} \quad \pi = v\left((1-t)\left(u_H-\alpha_H x_H-u_{HL}+\alpha_H x_L+u_L-\alpha_L x_L\right)+r_H x_H-cq_H\right)+ \\ +(1-v)\left((1-t)\left(u_L-\alpha_L x_L\right)+r_L x_L-cq_L\right). \tag{86}$$

Given the linearity of the objective in x_H and x_L, the solution is such that

$$\begin{cases} x_L=\frac{u_L}{\alpha_L},\, x_H=\frac{1}{\alpha_H}\left(u_H-u_{HL}+\frac{\alpha_H}{\alpha_L}u_L\right) & \text{if} \quad \alpha_L-\frac{v}{1-v}\left(\alpha_H-\alpha_L\right)\leq\frac{r_L}{(1-t)}, \text{ and } \alpha_H\leq\frac{r_H}{(1-t)} \\ x_L=0,\, x_H=\frac{1}{\alpha_H}\left(u_H-u_{HL}+u_L\right) & \text{if} \quad \alpha_L-\frac{v}{1-v}\left(\alpha_H-\alpha_L\right)>\frac{r_L}{(1-t)}, \text{ and } \alpha_H\leq\frac{r_H}{(1-t)} \\ x_L=\frac{u_L}{\alpha_L},\, x_H=0 & \text{if} \quad \alpha_L-\frac{v}{1-v}\left(\alpha_H-\alpha_L\right)\leq\frac{r_L}{(1-t)}, \text{ and } \alpha_H>\frac{r_H}{(1-t)}, \\ x_L=0,\, x_H=0 & \text{if} \quad \alpha_L-\frac{v}{1-v}\left(\alpha_H-\alpha_L\right)>\frac{r_L}{(1-t)}, \text{ and } \alpha_H>\frac{r_H}{(1-t)}. \end{cases}$$

In words, the seller offers version i for free if and only if the revenue r_i is large enough compared to the disutility α_i. Although the model contemplates many possible cases, in the main text we concentrate on the case of freemium pricing (which applies if $\frac{r_H}{1-t}<\alpha_H$ and $\alpha_L-\frac{v}{1-v}\left(\alpha_H-\alpha_L\right)\leq\frac{r_L}{1-t}$). The case of full monetary pricing can be analysed in a way that is very similar to the baseline model, as the fee applies to all versions. Note that the other two cases are less empirically relevant, as they entail a greater amount of ads shown on the H-version (premium) than on the L-version (base), i.e. $x_H>x_L$.

Commenting briefly on the conditions for freemium pricing to emerge, we note that α_L must be small, relative to r_L, while α_H must be larger than $\frac{r_H}{1-t}$. For the latter condition to be satisfied when t=0, $\alpha_H>r_H$ is necessary. Moreover, the transaction fee must not exceed $1-\frac{r_H}{\alpha_H}$. In words, in this setting freemium emerges whenever low-type consumers' disutility from ads is relatively small, contrary to that of the high-types. In addition, the transaction fee must not be too large.

We now on the case of freemium pricing. In this case, the equilibrium is such that

$$\begin{aligned} p_L=0, &\quad p_H=u_H-u_{HL}+\frac{\alpha_H}{\alpha_L}u_L, \\ x_L=\frac{u_L}{\alpha_L}, &\quad\quad x_H=0. \end{aligned}$$

We can therefore write the expressions for consumer surplus in this setting as

$$CS=vCS_H+(1-v)CS_L=v\left(u_{HL}-\frac{\alpha_H}{\alpha_L}u_L\right), \tag{87}$$

where $CS_H=u_{HL}-\frac{\alpha_H}{\alpha_L}u_L$ and $CS_L=0$. Note that the condition $\frac{u_{HL}}{u_L}>\frac{\alpha_H}{\alpha_L}$ guarantees that CS_H is strictly positive.

Given these assumptions, we can rewrite the seller's problem as

$$\max_{q_H,q_L} \quad \pi=v\left((1-t)\left(u_H-u_{HL}+\frac{\alpha_H}{\alpha_L}u_L\right)-cq_H\right)+(1-v)\left(\frac{u_L r_L}{\alpha_L}-cq_L\right). \tag{88}$$

The above expression is fundamentally identical to (10). Hence, the analysis follows along the same lines as in Section 3.

D.1.1 Binding constraints in problem (80)

As a first step, we show that (81) and (84) imply that (83) holds. Constraint (84) can be rewritten as $x_L \leq u(q_L, \theta_L) - \alpha_L x_L$. Setting x_L at the upper bound of this constraint gets the right hand side of (81) as close as possible to zero. Hence, if $u(q_L, \theta_H) - u(q_L, \theta_L) + \alpha_L x_L - \alpha_H x_L \geq 0$, constraint (83) must be implied by (81). Given the linearity of the problem in x_L, we can anticipate that either $x_L = 0$ or $x_L = u(q_L, \theta_L) / \alpha_L$ holds at the solution. In the former case, $u(q_L, \theta_H) - u(q_L, \theta_L) + \alpha_L x_L - \alpha_H x_L \geq 0$ is satisfied because $u(q_L, \theta_H) > u(q_L, \theta_L)$ by assumption. In the latter case, the constraint boils down to $u(q_L, \theta_H) - \alpha_H u(q_L, \theta_L) / \alpha_L \geq 0$, which is satisfied given the assumption that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L}$. Summing up, we can ignore constraint (83) and anticipate that (81) must be binding at the solution of (80).

In the second step, we show that (81) being binding implies that (82) is slack and can be ignored. Given the linearity of the problem, we can anticipate that if (81) binds, either $x_H = 0$ or $x_H = \frac{u(q_H, \theta_H) - u(q_L, \theta_H) + x_L + \alpha_H x_L}{\alpha_H}$ hold. Suppose first that $x_H = 0$, so that $p_H = u(q_H, \theta_H) - u(q_L, \theta_H) + u(q_L, \theta_L)$. Plugging these expressions in the right hand side of (82) we get after some rearrangements: $u(q_H, \theta_L) - u(q_L, \theta_L) - (u(q_H, \theta_H) - u(q_L, \theta_H))$. This expression is strictly negative by assumption, which implies that (82) is slack. Suppose now that $x_H = \frac{u(q_H, \theta_H) - u(q_L, \theta_H) + x_L + \alpha_H x_L}{\alpha_H}$ and $p_H = 0$. Plugging these expressions in (82) we get

$$u(q_L, \theta_L) - x_L - \alpha_L x_L \geq u(q_H, \theta_L) - \frac{\alpha_L}{\alpha_H} (u(q_H, \theta_H) - u(q_L, \theta_H) + x_L + \alpha_H x_L).$$

Suppose the solution is such that $x_L = 0$ and $x_L = u(q_L, \theta_L)$. The above constraint can then be written after some rearrangements as

$$0 \geq u(q_H, \theta_L) - \frac{\alpha_L}{\alpha_H} (u(q_H, \theta_H) - u(q_L, \theta_H) + u(q_L, \theta_L)).$$

The last term in brackets on the right hand side is positive. Hence, given the assumption that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L} \Longleftrightarrow \frac{\alpha_L}{\alpha_H} > \frac{u_L}{u_{HL}}$, the constraint must hold if it holds when $\frac{\alpha_L}{\alpha_H} = \frac{u_L}{u_{HL}}$. Plugging this expression in the constraint, we have after some rearrangements that

$$\frac{u(q_H, \theta_H)}{u(q_L, \theta_H)} \geq \frac{u(q_H, \theta_L)}{u(q_L, \theta_L)},$$

which holds strictly by our assumptions on utility. Finally, suppose that the solution is such that

$x_L = u(q_L, \theta_L)/\alpha_L$ and $x_L = 0$. The constraint (82) can then be written as

$$0 \geq u(q_H, \theta_L) - u(q_L, \theta_L) - \frac{\alpha_L}{\alpha_H}(u(q_H, \theta_H) - u(q_L, \theta_H)).$$

The last term in brackets on the right hand side is positive. Hence, given the assumption that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L} \iff \frac{\alpha_L}{\alpha_H} > \frac{u_L}{u_{HL}}$, the constraint must hold if it holds when $\frac{\alpha_L}{\alpha_H} = \frac{u_L}{u_{HL}}$. Plugging this expression in the constraint, we have after some rearrangements that

$$\frac{u(q_H, \theta_H)}{u(q_L, \theta_H)} \geq \frac{u(q_H, \theta_L)}{u(q_L, \theta_L)},$$

which holds strictly by our assumptions on utility.

D.2 Freemium with ads as quality reduction

We provide an alternative formulation of the model where q captures (the reduction in) the quantity of ads consumers are exposed to. In order to incorporate freemium pricing in this version of the model, we shall assume that the seller faces a transaction cost when collecting payments from consumers directly, and that the willingness to pay of low-type consumers for the basic version of the product (given the volume of ads) is not sufficient to justify incurring such cost. As we shall see, given these modifications, the key effects of the transaction tax on quality and consumer surplus are as in the baseline model.

Let $Q - q_i$ be the quantity of ads shown to a consumer choosing version i of the product, where Q is the maximum level of ads that can be shown. Assume the seller derives some revenue $r(Q - q_i)$ per consumer from such ads, which is such that

$$\begin{aligned}
r'(.) > 0 \quad if \quad r(Q - q_i) < \tilde{q}, \\
r'(.) = 0 \quad if \quad r(Q - q_i) = \tilde{q}, \\
r'(.) < 0 \quad if \quad r(Q - q_i) > \tilde{q},
\end{aligned}$$

and $r'' < 0$. That is, there are diminishing returns to showing ads to a given consumer and a level of ad intensity, $\tilde{q} < Q$, beyond which the revenue from the marginal ad is negative. This can be due, for instance, to the ability of consumers to register ads being diminishing in the quantity of ads they receive and, more generally, to advertising clutter (Anderson and de Palma, 2009; Anderson and Peitz, 2023).

We assume there is a transaction cost, z, the seller faces when collecting a monetary payment from each consumer. For simplicity, suppose that the price charged for the high version of the product, p_H, is large enough that $p_H > z$ holds, so the seller does indeed charge a positive monetary price for that version. The seller's profit is therefore

$$\pi = v((1-t)\,p_H - z + r(Q-q_H)) + (1-v)\,((1-t)\,p_L - z + r(Q-q_L)) \quad if \quad p_L > 0$$

$$\pi = v((1-t)\,p_H - z) + (1-v)\,(r(Q-q_L)) \quad if \quad p_L = 0$$

Note that we assume for simplicity (and without loss) that $c = 0$ in this version of the model. It follows from the above expressions that, quite intuitively, whenever $p_L(1-t) \geq z$, the seller will choose not to apply any monetary price to the L-version of the product, generating revenue only from the sale of ads.

As in the baseline model, given t, the seller maximises π subject to the constraints (3)-(6). Note that $x_L = 0$ in this version. Given the same utility function as in the baseline, one can again show that constraints (4) and (5) can be ignored. The upper bound on p_L faced by the seller to satisfy (6) is u_L. Let us assume that $u_L < z$, so that the seller chooses $p_L = 0$ due to the transaction cost z. This implies that (6) is slack, i.e. low-type consumers get some positive surplus, equal to u_L. Regarding p_H, conditional on $p_L = 0$, it is a dominant strategy for the seller to set $p_H = u_H - u_{HL}$, satisfying constraint (3). Thus, the seller's problem reduces to

$$\max_{q_H, q_L} \pi = v((1-t)\,(u_H - u_{HL}) + r(Q-q_H)) + (1-v)\,(r(Q-q_L)).$$

The equilibrium quality levels, q_H^e and q_L^e, satisfy the following equations, respectively:

$$\frac{\partial \pi}{\partial q_H} = v\left(\frac{\partial u_H}{\partial q_H}(1-t) - r'(Q-q_H)\right) = 0, \tag{89}$$

$$\frac{\partial \pi}{\partial q_L} = -v\frac{\partial u_{HL}}{\partial q_L}(1-t) - (1-v)\,r'(Q-q_L) = 0. \tag{90}$$

The above expressions indicate that, for the high version, the level of reduction in the quantity of ads, q_H^e, is such that the marginal utility gain for the high-type consumer equals the marginal ad revenue loss for the seller. Consider now equation (90) and note that, if the first term was absent, the value of q_L^e satisfying the equation would simply be such that $r'(Q-q_L) = 0$. That is, given its unwillingness to extract any monetary revenue from the consumer, the seller would just maximize the ad revenue from the low version. However, the first term in (90) indicates that the seller has an incentive to expose consumers to even more ads on the low version, despite $r'(Q-q_L^e) < 0$, because this relaxes constraint (3) and reduces the high-types' information rent.

Let us now consider the effects of t. As in the baseline model, starting from (89) and (90), we can show that $\frac{\partial q_H^e}{\partial t} < 0$ and $\frac{\partial q_L^e}{\partial t} > 0$. The intuition for the latter is that the fee reduces the incentive for the seller to extract monetary revenue from the high-type consumers and reduce their information rent. By the same token, we obtain that both u_{HL} and u_L increase with the fee, so that all consumers are strictly better off. Similarly, welfare increases when starting from t=0. Hence, Proposition 1 and 2 would still hold.

E Robustness checks (online)

E.1 More than two types

We assume there are three types of consumers, characterized by the preference parameter $\theta \in \{\theta_H, \theta_M, \theta_L\}$, with $\theta_H > \theta_M > \theta_L$. Let v_H, v_M and v_L be the shares of consumers of type H, M and L, respectively, with $v_H + v_M + v_L = 1$. Furthermore, to avoid "bunching" of types we assume that $\frac{v_L}{v_M} < \frac{v_L + v_M}{v_H}$, i.e. that the distribution of types satisfies the monotone hazard rate property (Laffont and Martimort, 2002, p.90). The model is otherwise identical to our baseline setup.

The seller offers to consumers three bundles, (q_i, p_i), each intended for one type. These bundles must satisfy six incentive constraints (two for each type)

$$u(q_i, \theta_i) - p_i \geq u(q_j, \theta_i) - p_j, \quad i, j = L, M, H \quad i \neq j,$$

and three participation constraints (one per each type)

$$u(q_i, \theta_i) - p_i \geq 0, \quad i = L, M, H.$$

Following standard steps (Laffont and Martimort, 2002), one can show that, in equilibrium, there are two binding incentives constraints (the ones such that a higher type want to mimic a lower type) and one binding participation constraint (the one of low types). From these binding constraints we derive the equilibrium prices. Hence, the seller maximizes the following problem

$$
\begin{aligned}
\max_{(q_i,p_i)} \quad & \pi = \sum_{i=L,M,H} v_i \left((1-t_i)\, p_i - c q_i \right), && (91)\\
s.t. \quad & p_H = u_H + u_M + u_L - u_{ML} - u_{HM}, && (92)\\
& p_M = u_M + u_L - u_{ML}, && (93)\\
& p_L = u_L, && (94)
\end{aligned}
$$

where $u_i \equiv u(q_i,\theta_i)$ for each $i = L,M,H$, and $u_{ij} \equiv u\left(q_j,\theta_i\right)$ for each $i,j = L,M,H$ with $i \neq j$. Hence, we derive the following first-order conditions

$$
\frac{\partial \pi}{\partial q_H} := v_H \left(\frac{\partial u_H}{\partial q_H} (1-t_H) - c \right) = 0, \tag{95}
$$

$$
\frac{\partial \pi}{\partial q_M} := v_H \left(\frac{\partial u_M}{\partial q_M} - \frac{\partial u_{HM}}{\partial q_M} \right) (1-t_H) + v_M \left(\frac{\partial u_M}{\partial q_M} (1-t_M) - c \right) = 0, \tag{96}
$$

$$
\frac{\partial \pi}{\partial q_L} := v_H \left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L} \right) (1-t_H) + v_M \left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L} \right) (1-t_M) + v_L \left(\frac{\partial u_L}{\partial q_L} (1-t_L) - c \right) = 0. \tag{97}
$$

Totally differentiating the above equations and taking into account that cross-profits derivatives are zero ($\frac{\partial^2 \pi}{\partial q_i \partial q_j} = 0$ for $i,j = L,M,H$ with $i \neq j$), we find that

$$
\frac{\partial q_H}{\partial t_H} = - \frac{\begin{vmatrix} \frac{\partial^2 \pi}{\partial q_L^2} & \frac{\partial^2 \pi}{\partial q_L \partial q_M} & \frac{\partial^2 \pi}{\partial q_L \partial t_H} \\ \frac{\partial^2 \pi}{\partial q_M \partial q_L} & \frac{\partial^2 \pi}{\partial q_M^2} & \frac{\partial^2 \pi}{\partial q_M \partial t_H} \\ \frac{\partial^2 \pi}{\partial q_H \partial q_L} & \frac{\partial^2 \pi}{\partial q_H \partial q_M} & \frac{\partial^2 \pi}{\partial q_H \partial t_H} \end{vmatrix}}{H} = - \frac{\frac{\partial^2 \pi}{\partial q_L^2} \frac{\partial^2 \pi}{\partial q_M^2} \frac{\partial^2 \pi}{\partial q_H \partial t_H}}{H} \leq 0,
$$

where H is the determinant of the Hessian matrix, which is negative by second order conditions, $\frac{\partial^2 \pi}{\partial q_i^2} < 0$ for $i = L,M,H$ also by second order conditions, and $\frac{\partial^2 \pi}{\partial q_H \partial t_H} = -v_H \frac{\partial u_H}{\partial q_H} < 0$. Following similar steps, we find that the derivatives of q_M and q_L with respect to t_H are, respectively, such that

$$
\operatorname{sgn}\left(\frac{\partial q_M}{\partial t_H} \right) = \operatorname{sgn}\left(-v_H \left(\frac{\partial u_M}{\partial q_M} - \frac{\partial u_{HM}}{\partial q_M} \right) \right) \geq 0, \qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial t_H} \right) = \operatorname{sgn}\left(-v_H \left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L} \right) \right) \geq 0 \ .
$$

These signs follow from the assumption that $\frac{\partial^2 u}{\partial q \partial \theta} > 0$. This establishes that the effect of the ad valorem tax applied to the H-bundle is such that the quantity of the other two bundles increases, reducing the distortion applied by the seller.

Similarly, the derivatives of the equilibrium quantities with respect to t_M and t_L are such that

$$\frac{\partial q_H}{\partial t_M}=0, \qquad \operatorname{sgn}\left(\frac{\partial q_M}{\partial t_M}\right)=\operatorname{sgn}\left(-v_M\frac{\partial u_M}{\partial q_M}\right)\leq 0, \qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial t_M}\right)=\operatorname{sgn}\left(-v_M\left(\frac{\partial u_L}{\partial q_L}-\frac{\partial u_{ML}}{\partial q_L}\right)\right)\geq 0,$$

$$\frac{\partial q_H}{\partial t_L}=0, \qquad \frac{\partial q_M}{\partial t_L}=0, \qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial t_L}\right)=\operatorname{sgn}\left(-v_L\frac{\partial u_L}{\partial q_L}\right)\leq 0.$$

E.2 Duopoly

We consider two symmetric sellers, indexed by $s\in\{1,2\}$ and four consumer types, indexed by $i\in\{H_1,L_1,H_2,L_2\}$, differing in (i) their intensity of preferences for the good and (ii) their preference for the two sellers. The utility when buying from seller s is $u_s(q,\theta_i)-p$, where p is the price and θ_i is the preference parameter. Let v_i be the share of consumers of type i, with $\sum_{i=H_1,L_1,H_2,L_2} v_i=1$, and assume that each consumer buys from at most one seller. We assume the utility function satisfies the following conditions:

$$u_1(q,\theta_{H_1})>u_1(q,\theta_{L_1})>u_1(q,\theta_{L_2})>u_1(q,\theta_{H_2})=0, \quad \forall q>0,$$

$$u_2(q,\theta_{H_2})>u_2(q,\theta_{L_2})>u_1(q,\theta_{L_1})>u_1(q,\theta_{H_1})=0, \quad \forall q>0,$$

$$\frac{\partial u_1}{\partial q}(q,\theta_{H_1})>\frac{\partial u_1}{\partial q}(q,\theta_{L_1})>\frac{\partial u_1}{\partial q}(q,\theta_{L_2})>\frac{\partial u_1}{\partial q}(q,\theta_{H_2})=0, \quad \forall q>0,$$

$$\frac{\partial u_2}{\partial q}(q,\theta_{H_2})>\frac{\partial u_2}{\partial q}(q,\theta_{L_2})>\frac{\partial u_2}{\partial q}(q,\theta_{L_1})>\frac{\partial u_2}{\partial q}(q,\theta_{H_1})=0, \quad \forall q>0.$$

These conditions imply a perfect correlation between the preference for one seller and the intensity of preference for the good it supplies (Spulber, 1989). For simplicity, we assume only the "low" types are willing to buy from either seller, whereas the "high" types do not get any utility from buying from their least preferred seller.

Let (q_i,p_i) denote the bundle that a seller proposes to consumers of type i. Given the condition that consumers self-select on the intended bundle, there is no loss in proceeding under the assumption that seller 1 only offers bundles intended for the couple of consumer types that prefer its product, i.e. H_1 and L_1, whereas seller 2 only serves H_2 and L_2. We are now going to state the constraints that the sellers face regarding each type of consumer. Considering a seller s, we have the following incentives and participation constraints that apply to the H_s-bundle:

$$u_s(q_{H_s},\theta_{H_s}) - p_{H_s} \geq u_s(q_{L_s},\theta_{H_s}) - p_{L_s},\ s = 1,2, \tag{98}$$

$$u_s(q_{H_s},\theta_{H_s}) - p_{H_s} \geq u_{s'}\left(q_{L_{s'}},\theta_{H_s}\right) - p_{L_{s'}},\ s,s' = 1,2,\ s' \neq s, \tag{99}$$

$$u_s(q_{H_s},\theta_{H_s}) - p_{H_s} \geq u_{s'}\left(q_{H_{s'}},\theta_{H_s}\right) - p_{H_{s'}},\ s,s' = 1,2,\ s' \neq s, \tag{100}$$

$$u_s(q_{H_s},\theta_{H_s}) - p_{H_s} \geq 0,\ s = 1,2. \tag{101}$$

Constraint (98) must hold in order for H_s types not to choose the bundle offered to L_s consumers by the same seller . The next two constraints, (99) and (100), must hold to avoid that H_s types buy any of the bundles offered by the other seller, s'. Finally, (101) must hold for H_s types to prefer the bundle intended for them to not participating in the market at all.

Symmetrically, the constraints that apply to the L_s-bundle are as follows

$$u_s(q_{L_s},\theta_{L_s}) - p_{L_s} \geq u_s(q_{H_s},\theta_{L_s}) - p_{H_s},\ s = 1,2, \tag{102}$$

$$u_s(q_{L_s},\theta_{L_s}) - p_{L_s} \geq u_{s'}\left(q_{L_{s'}},\theta_{L_s}\right) - p_{L_{s'}},\ s,s' = 1,2,\ s' \neq s, \tag{103}$$

$$u_s(q_{L_s},\theta_{L_s}) - p_{L_s} \geq u_{s'}\left(q_{H_{s'}},\theta_{L_s}\right) - p_{H_{s'}},\ s,s' = 1,2,\ s' \neq s, \tag{104}$$

$$u_s(q_{L_s},\theta_{L_s}) - p_{L_s} \geq 0,\ s = 1,2. \tag{105}$$

Constraint (102) must hold in order for L_s types not to choose the bundle offered to H_s consumers by seller s. The next two constraints, (99) and (100), must hold to avoid that L_s types buy from the other seller. Finally, (101) must hold for L_s types to prefer the bundle intended for them to not participating in the market at all.

Given the differentiated ad valorem tax rates we consider in the baseline setting, the problem of seller s is

$$\max_{q_{H_s},p_{H_s},q_{L_s},p_{L_s}} \quad \pi = v_{H_s}\left[(1-t_H)\,p_{H_s} - cq_{H_s}\right] + v_{L_s}\left[(1-t_L)\,p_{L_s} - cq_{L_s}\right],\ s = 1,2, \tag{106}$$

subject to constraints (98)-(105).

We are now going to solve seller s's problem characterized above, focusing on symmetric equilibria. Our first step is to establish which constraints are going to be binding in equilibrium to determine equilibrium prices. Given $u_{s'}\left(q_{L_{s'}},\theta_{H_s}\right) = u_{s'}\left(q_{H_{s'}},\theta_{H_s}\right) = 0$, constraints (99) and (100) cannot be binding, because of the participation constraints in (101). Furthermore, given (105), and that $u_s(q_{L_s}, \theta_{H_s}) > u_s(q_{L_s}, \theta_{L_s})$, constraint (101) cannot be binding either. Hence, the equilibrium must be such that (98) is binding. We have

$$p_{H_s} = p_{L_s} + u_s(q_{H_s}, \theta_{H_s}) - u_s(q_{L_s}, \theta_{H_s}), \; s = 1, 2. \tag{107}$$

Given (107), we can write the constraints (102), after some rearrangements, as

$$u_s(q_{H_s}, \theta_{H_s}) - u_s(q_{L_s}, \theta_{H_s}) \geq u_s(q_{H_s}, \theta_{L_s}) - u_s(q_{L_s}, \theta_{L_s}), \; s = 1, 2,$$

which must hold strictly by the assumption that $\frac{\partial u_s}{\partial q}(q, \theta_{H_s}) > \frac{\partial u_s}{\partial q}(q, \theta_{L_s})$. Hence, these constraints cannot be binding. Consider now the constraints (104). These can be rewritten, using (107) and after a few rearrangements as

$$u_{s'}\left(q_{H_{s'}}, \theta_{H_{s'}}\right) - u_{s'}\left(q_{L_{s'}}, \theta_{H_{s'}}\right) - p_{L_s} \geq u_{s'}\left(q_{H_{s'}}, \theta_{L_s}\right) - u_s(q_{L_s}, \theta_{L_s}) - p_{L_{s'}}, \; s = 1, 2.$$

In a symmetric equilibrium (where $p_{L_s} = p_{L_{s'}}$ and $q_{L_s} = q_{L_{s'}}$), this inequality must hold strictly by the assumption that $\frac{\partial u_{s'}}{\partial q}\left(q, \theta_{H_{s'}}\right) > \frac{\partial u_s}{\partial q}(q, \theta_{L_s}) > \frac{\partial u_{s'}}{\partial q}(q, \theta_{L_s})$. Therefore, the only constraints that can be binding are (103) and (105). We have

$$p_{L_s} = u_s(q_{L_s}, \theta_{L_s}) - max\left(0, u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - p_{L_{s'}}\right) \; s, s' = 1, 2, s' \neq s. \tag{108}$$

Given (107) and (108), we can therefore write the the problem of seller s as

$$\begin{aligned} \max_{q_{H_s}, q_{L_s}} \quad \pi_s = v_{H_s} \left[(1 - t_H)\left(u_s(q_{L_s}, \theta_{L_s}) - max\left(0, u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - p_{L_{s'}}\right) + u_s(q_{H_s}, \theta_{H_s}) - (q_{L_s}, \theta_{H_s})\right) - cq_{H_s}\right] \\ + v_{L_s}\left[(1 - t_L)\left(u_s(q_{L_s}, \theta_{L_s}) - max\left(0, u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - p_{L_{s'}}\right)\right) - cq_{L_s}\right], \quad s, s' = 1, 2, s' \neq s. \end{aligned} \tag{109}$$

Observe that $u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - p_{L_{s'}}$ does not depend on q_{H_s} nor on q_{L_s}. The first-order conditions of this problem are

$$\frac{\partial \pi}{\partial q_{H_s}} := \frac{\partial u_s(q_{H_s}, \theta_{H_s})}{\partial q_{H_s}}(1 - t_H) - c = 0 \; s = 1, 2, \tag{110}$$

$$\frac{\partial \pi}{\partial q_{L_s}} := v_{H_s}\left(-\frac{\partial u_s(q_{L_s}, \theta_{H_s})}{\partial q_{L_s}} + \frac{\partial u_s(q_{L_s}, \theta_{L_s})}{\partial q_{L_s}}\right)(1 - t_H) + v_{L_s}\left(\frac{\partial u_s(q_{L_s}, \theta_{L_s})}{\partial q_{L_s}}(1 - t_L) - c\right) = 0 \; s = 1, 2. \tag{111}$$

The key observation is that these equations have the same form as (12) and (13), which implies that the effects of taxation must be also be the same, and so are the implications for optimal policy.

E.3 Device: downward sloping demand

Suppose that consumers have a downward sloping demand for the device. Under our assumptions, this demand only depends on the net expected surplus from accessing the market, i.e., $d+E(CS)-p_D$. Let the quantity of consumers that acquire the device be $Q(d+E(CS)-p_D)$, assumed increasing in its argument, i.e., $Q'>0$. We can therefore write the profit of the platform as

$$\pi_P = Q\cdot\left(vp_H^e t_H + (1-v)\,p_L^e t_L + p_D\right). \tag{112}$$

The first-order condition characterizing the profit maximising level of t_H is as follows

$$\frac{\partial\pi_P}{\partial t_H} = Q\cdot v\cdot\left(\frac{\partial p_H}{\partial t_H}t_H + p_H\right) + Q'\cdot\frac{\partial E(CS)}{\partial t_H}\cdot\left(vp_H^e t_H + (1-v)\,p_L^e t_L + p_D\right) = 0. \tag{113}$$

Suppose now the platform does not sell the device, so that $p_D=0$. The first-order condition above is now

$$\frac{\partial\pi_P}{\partial t_H} = Q\cdot v\cdot\left(\frac{\partial p_H}{\partial t_H}t_H + p_H\right) + Q'\cdot\frac{\partial E(CS)}{\partial t_H}\cdot\left(vp_H^e t_H + (1-v)\,p_L^e t_L\right) = 0. \tag{114}$$

Noting that $\frac{\partial E(CS)}{\partial t_H}>0$ and comparing the two first-order conditions, it is clear that for any t_L, t_H and $p_D>0$ the derivative $\frac{\partial\pi_P}{\partial t_H}$ referring to the case with the device sale is larger than when this device is not present. It follows that the equilibrium value of t_H is larger, as stated in Proposition 7. Given $\frac{\partial E(CS)}{\partial t_L}<0$ and $\frac{\partial E(CS)}{\partial t}<0$, we obtain the opposite result for t_L and the uniform tax t.