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**Matteo Bizzarri, Marco Pangallo, and Francisco Queirós**

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University of Naples Federico II

University of Salerno

Bocconi University, Milan

CSEF - Centre for Studies in Economics and Finance
DEPARTMENT OF ECONOMICS AND STATISTICS – UNIVERSITY OF NAPLES FEDERICO II
80126 NAPLES - ITALY
Tel. and fax +39 081 675372 – e-mail: csef@unina.it
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# *Production Networks, Time to Build and Endogenous Oscillations*

**Matteo Bizzarri[*], Marco Pangallo[†], and Francisco Queirós[‡]**

**Abstract**
We study how sector-specific shocks propagate in a production economy with input-output linkages and heterogeneous time to build. We show that, depending on the sector and network characteristics, one-time idiosyncratic shocks can induce a non-monotonic response of aggregate output as it converges back to steady state – a phenomenon we term 'endogenous oscillations' – and get amplified over time. We study the conditions on the network structure that generate this behavior. We introduce a measure to quantify the magnitude of such endogenous oscillations generated by a single small productivity shock. We quantify the model on US input-output data, showing that for some sectors a single shock can generate aggregate fluctuations. In particular, the magnitude of oscillations is twice as large as it would be if convergence to the steady state were always monotonic.

**JEL Classification:** C67, D57, D85, E23, E32.

**Keywords:** Endogenous Oscillations, Business Cycles, Production Networks.

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[*] University of Naples Federico II and CSEF: E-mail: matteo.bizzarri@unina.it
[†] CENTAI Institute. E-mail: marco.pangallo@centai.eu
[‡] ISEG - Lisbon School of Economics and Management. E-mail: queiros.francisco@gmail.com

of shocks of alternating sign. We consider a multi-industry RBC model with input-output linkages across sectors and heterogeneous time to build, in the spirit of Long Jr and Plosser (1983). We show that productivity shocks to individual sectors can generate endogenous oscillations in aggregate output, which eventually converge back to the steady state. We study the conditions on the network structure and time to build that make this possible, and we develop an operational measure to quantify the size of such oscillations. Finally, we calibrate the model on the US input-output network.

We can illustrate our mechanism with a simple circular network with two sectors, L and S. Suppose that both sectors sell to final consumers, but $L$ has a larger share of consumer demand; $L$ is 'large' and $S$ 'small'. Suppose further that i) each sector uses the output of the other as intermediate input ($L$ sells to S and S sells to $L$); and that ii) there is one period time-to-build, namely that the final output obtained from inputs bought today is ready with one time period delay. What happens when the $L$ sector is hit by a positive, one-period shock at time zero? Since inputs are invested with a one-period lag, sectors alternate in the moments of their expansion; that is, after period zero, a sector expands only if its supplier expanded in the previous period. This implies that sector $L$ will expand at even periods (time zero, two, four...), while sector $S$ expands at odd periods (one, three, five...). However, since sector $L$ has a larger share in final consumption, aggregate output will be larger in even periods (when sector L expands) than in odd periods (when $S$ expands). This can lead to a sequence of (damped) endogenous oscillations, where expansions and contractions alternate, even if the economy was hit by just a one-period shock. The same intuition can be extended to a vertical economy where 'small' and 'large industries' alternate.

More generally, endogenous oscillations can happen when the diffusion of the shock hits groups of sectors with very different size as it travels through the network. Formally, the size of the group of sectors reached is a network statistic: the weighted $k$-degree, measuring the number of sectors in the network that are at a distance $k$ from the sector that receives the shock, appropriately weighted. It is weigthed in two senses: first, the importance of each link is measured by the input intensity of that input for the buyer; second, each path is weighted by the Domar weight of the end node. The model has a unique steady state so, after a transitory shock, in the long run GDP converges back to steady state GDP: the weighted $k$-degree does too, because longer paths are weighted less and less. However, in the short run, for some sectors, the weighted $k$-degree increases, amplifying the effect of the initial shock on GDP: this amplification is the key mechanism leading to the endogenous oscillations we study. In particular, for some sectors the weighted $k$-degrees undergoes nonmonotonic changes as $k$ grows: these are the sectors that generate more oscillations. While one-period time to build is enough to generate endogenous oscillations, our model allows for the possibility of longer and heterogenous production lags across sectors. When time to build is heterogeneous, the $k$ distance must be generalized to a measure of the sectors reached with $k$ time steps: if time to build is large, $k$ steps may mean a lower network distance. Thus, heterogeneity in time to build has a crucial role in determining which and where endogenous oscillations can arise.

The detailed contributions are as follows. In Theorem 1, we characterize the response of aggregate GDP to a small transitory productivity shock to a sector, in terms of the $k$-degree of the shocked sector. We use this characterization to derive conditions under which the combination network+time-to-build generates amplification, and endogenous oscillations. We illustrate the insight using two simple networks: the vertical and circular networks. Then, we define a measure to quantify the variability generated by a small productivity shock. The measure is the impact of a shock on the total absolute variation of GDP over time, and we think of it as a measure of oscillatory behavior. In Theorem 2, we show that this is equal to the (normalized) Domar weight of the shocked sector if and only if the dynamic is monotonic. We illustrate how the measure sheds light on the circular economy example.

To quantify the importance of our mechanism, we calibrate our model to the U.S. economy. Following Liu and Tsyvinski (2024), we use sector-specific estimates of time to build from the backlog computed from 'Manufacturers Shipments, Inventories, and Orders', which is a monthly survey containing information on manufacturers' shipments, inventories, and unfilled orders. We simulate our model at a monthly frequency to take maximal advantage of the heterogeneity of time to build across sectors. Regarding the input-output network, we use the B.L.S. inter-industry relationships data, ending up with 172 connected sectors.

We are interested in the possibility of endogenous oscillations arising from temporary idiosyncratic shocks. To investigate this, we conduct the following experiment: we shock each one of our industries (with a one-period productivity shock) and study the reaction of aggregate output in a 30-month period. First, qualitatively, we show some impulse response functions to build intuition into how these shocks cause endogenous oscillations while propagating through the input-output network with heterogeneous time to build. Taking the example of the semiconductors sector, we show that a positive one-off productivity shock leads to sizable oscillations even 10 months later, due to the long time to build of the motor vehicles parts sector. We also show that our oscillation measure is positively correlated with upstreamness, but it is uncorrelated with time to build and sector size. We find that out of our 172 idiosyncratic shocks, 154 can lead to endogenous oscillations – that is, when we shock these industries, aggregate output converges to the steady-state in a non-monotonic fashion. We also show that, on average, these oscillations are large. In particular, we measure the volatility of the growth rate of aggregate output arising from these individual impulse responses; we find that it is twice as large as it would be if convergence to the steady state were monotonic.

**Related Literature** Our paper can be related to two main literatures: the literature on business cycles, and the literature on production networks.

Standard one-sector models of real business cycles rely on exogenous productivity shocks or wedges to generate aggregate fluctuations (Kydland and Prescott, 1982; King and Rebelo, 1999; Chari et al., 2007). We construct a model of real business cycles, where aggregate fluctuations are driven by productivity shocks (Kydland and Prescott, 1982; King and Rebelo, 1999). However, we depart from the one-sector model and show that the combination of time to build and a production network can generate endogenous oscillations, in the sense clarified in the Introduction.

We also characterize the conditions for the existence of endogenous oscillations. We contribute to this literature by showing that the combination of time to build and a production network can generate endogenous oscillations, in the sense clarified in the Introduction. We also characterize the conditions for the existence of endogenous oscillations. Endogenous fluctuations that are not driven by productivity shocks can also be driven by changes in agents' beliefs (self-fulfilling prophecies) in economies subject to multiple equilibria (Azariadis, 1981; Benhabib and Farmer, 1994; Farmer and Guo, 1994; Galí, 1994; Schmitt-Grohe, 2000). Instead, the endogenous oscillations we highlight do not depend on the multiplicity of equilibria: our model has a unique competitive equilibrium, and a unique steady state in the absence of shocks.

Other papers have studied cyclical behavior in deterministic models in the form of limit cycles. Benhabib and Nishimura (1979) consider a neoclassical growth model and show that limit cycles can arise in a deterministic economy that features multiple capital goods, which do not fully depreciate after one period. Asea and Zak (1999) show that time to build in a neoclassical growth model can also generate limit cycles. Recently, Beaudry et al. (2020) finds limit cycles in a model with financial frictions, and Beaudry et al. (2024) study the role of complementarity and substitutability in generating limit cycles in an abstract setting. Pangallo (2025) study how the trade network contributes to comovement through synchronization of limit cycles.[1] Our results are complementary to these papers; even if our economy can be seen as a neoclassical growth model with full depreciation, we show that non-monotonic behavior can appear and be quantitatively relevant even without long-run cyclical dynamics. Moreover, with imperfect depreciation the model allows very limited analytical solutions. We exploit the tractability of the Long Jr and Plosser (1983) model to derive analytical measures for the oscillatory behavior.

The literature on production networks has focused on the amplifying role of the network as a tool to explain aggregate fluctuations: the seminal paper by Acemoglu et al. (2012) shows that network asymmetries in a static economy can generate aggregate fluctuations even in the limit of large $N$. The literature has since studied many other amplifying forces, such as market power (Grassi et al., 2017), endogenous linkages (Taschereau-Dumouchel, 2020; Kopytov et al., 2024). Instead, our contribution is to show that the input-output connections, beyond amplifying fluctuations, can also endogenously generate oscillatory behavior characteristic of business cycles.

We build upon the seminal model of Long Jr and Plosser (1983), who proposed a multisector model with input-output linkages and one-period time-to-build. Foerster et al. (2011) show that the network component in their model has a quantitatively important role in explaining business cycle volatility, in a standard long-run DSGE analysis, with recurrent shocks. We extend this framework by considering heterogeneous time-to-build across sectors. The main difference is that, instead of studying how the network amplifies a shock process, we derive conditions under which shocks to one sector can generate endogenous oscillations. The dynamic diffusion of shocks through production networks has recently gained interest. Liu and Tsyvinski (2024) show that, under

[1] Endogenous oscillations or cycles have been shown to arise in overlapping generations models (Gale, 1973; Grandmont, 1985; Reichlin, 1986; Dos Santos Ferreira and Lloyd-Braga, 2005).

convex adjustment costs, temporary productivity shocks can have long-lasting effects; this is especially true for shocks hitting upstream sectors, as they travel through longer supply chains. Even if our focus is different (we are interested in the volatility of output and the possibility of endogenous oscillations), our results corroborate some of the findings of Liu and Tsyvinski (2024). In particular, we show that shocks to upstream sectors can have a larger welfare impact than what is simply predicted by their Domar weight. The closest paper is Leng et al. (2024), which shows that time to build and the input-output network structure can rationalize the 'bullwhip' effect, a well-known phenomenon in supply chain management, where upstream sectors are more volatile than downstream sectors. As in their model, we consider a dynamic network economy with heterogeneous time-to-build. The main difference is that we do not study amplification, but the ability of the model to generate endogenous oscillations. Moreover, we focus on productivity shocks, rather than demand shocks.

# 2 Model

## 2.1 Setup

Time is infinite and discrete: $t = 0, 1, \ldots$. There are $M$ sectors and one representative household. In each sector $i$ there is a representative firm with a Cobb-Douglas production function:

$$Y_{i,t} = \frac{1}{\mathcal{A}_i} A_{i,t} \, L_{i,t-d_{iL}}^{\alpha_i} \prod_{j=1}^{M} M_{ij,t-d_{ij}}^{\omega_{ij}} \tag{1}$$

where $M_{ij,t-d_{ij}}$ is the quantity of intermediate input $j$, $L_{i,t-d_{iL}}$ is labor, $A_{i,t}$ is the total factor productivity of $i$. We denote the vectors that collect these quantities by omitting the sector index $i$, so that $\log A_t = (\log A_{1,t}, \ldots, \log A_{M,t})$, $\log Y_t = (\log Y_{1,t}, \ldots, \log Y_{M,t})$. In this section, we keep $A_t$ generic: in the next sections we add more structure when we compute the impulse responses.

A crucial element of (1) is time to build: $d_{ij}$ and $d_{iL}$ denote the number of periods that input $j$ or labor need to become active in the production of $i$. We assume constant returns to scale, implying that $1 = \alpha_i + \sum_{j=1}^{M} \omega_{ij}$. We define the input-output matrix $\Omega$ as the matrix with entries $\omega_{ij}$. Note that, with this definition, the i.o. matrix $\Omega$ is row-stochastic. Following Leng et al. (2024), we also define the input-output matrix with time to build $d$: $\Omega_d$ is the matrix with entries $\omega_{ij} 1(d_{ij} = d)$. With this definition, we have $\Omega = \sum_d \Omega_d$. The value $\mathcal{A}_i = \alpha_i^{\alpha_i} \prod_j \omega_{ij}^{\omega_{ij}}$ is a constant that depends purely on technological parameters, that we include to simplify further expressions.

The consumer has a lifetime utility given by

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \sum_{i=1}^{M} \gamma_i \, \log \left( C_{i,t} \right) \right)$$

The consumer is endowed with one unit of labor, which is supplied inelastically.

We look for the competitive equilibrium of this economy, namely a set of prices, intermediate inputs, labor allocations, consumption, and output sequences $(P_{j,t}, M_{ij,t}, L_{i,t}, Y_{it}, C_{it})$ such that:

1. The consumer optimizes their intertemporal utility:

$$\max_{(C_t)_t} \mathbb{E}_0 \left( \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{M} \gamma_i \log\left(C_{i,t}\right) \right)$$

subject to:

$$\sum_i P_{i,t} C_{i,t} + a_{t+1}/R_t = W_t + a_t + \Pi_t$$

where savings $a_t$ are in terms of the numeraire (that we choose below); $\Pi_t$ are the firms profits (or cash flow); $R_t$ is the interest rate.

2. Firms optimize their expected profits, so for all $t$ they solve:

$$\max_{M_{ij,t}} \mathbb{E}_t \left( \sum_s \prod_{k=1}^{s} \frac{1}{R_{t+k}} P_{i,t+s} Y_{i,t+s} - \sum_j P_{j,t} M_{ij,t} - w_t L_{i,t} \right)$$

subject to (1)

3. Markets clear:

$$\sum_i L_{it} = 1$$

$$Y_{i,t} = \sum_j M_{ji,t} + C_{i,t}$$

$$a_t = 0$$

## 2.2 Equilibrium

The following proposition characterizes the competitive equilibrium of the model.

**Proposition 1.** *Choose the marginal utility of income in each period as a numeraire. Then:*

1. *The sectoral revenues $\lambda_{i,t} = P_{i,t} Y_{i,t}$ are constant in time, and satisfy:*

$$\lambda = (I - \sum_d \beta^d \Omega_d')^{-1} \gamma.$$

2. *The labor allocation is constant:*

$$L_i = \frac{\alpha_i \beta^{d_{iL}} \lambda_i}{\sum_j \alpha_j \beta^{d_{L,j}} \lambda_j}$$

3. *The dynamics of sectoral output is:*

$$\log Y_t = \eta + \log A_t + \sum_d \Omega_d \log Y_{t-d} \tag{2}$$

*where $\eta$ is the vector with components:*

$$\eta_i = \left(\alpha_i d_{iL} + \sum_j \omega_{ij} d_{ij}\right) \log \beta - \alpha_i \log \sum_i \alpha_i \beta^{d_{iL}} \lambda_i + \log \lambda_i - \sum_j \omega_{ij} \log \lambda_j \tag{3}$$

4. *If the productivity sequence is constant $A_{i,t} = \overline{A}_i$ (e.g in a steady-state):*

$$\log \overline{P} = (I - \Omega)^{-1} \left(\overline{\eta} - \log \overline{A}\right) \tag{4}$$

$$\log \overline{Y} = (I - \Omega)^{-1} \left(\eta + \log \overline{A}\right) \tag{5}$$

*where $\overline{\eta}_i = \log \beta - \alpha_i \log \sum_i \alpha_i \beta^{d_{iL}} \lambda_i$.*

The proof follows calculations analogous to Leng et al. (2024) (that focuses on demand, rather than productivity shocks).

The key difference of our model with Long Jr and Plosser (1983) is that we allow for heterogeneous time to build. This affects the equilibrium Domar weights. In static models, the Domar weight of a sector reflects the number of weighted direct and indirect connections of the sector to the consumer throughout the network. In dynamic models, such as Long Jr and Plosser (1983) and the Liu and Tsyvinski (2024) models, these connections have to be weighted by the discount factor, because each further step in the network represents demand that will be realized in the future. With heterogeneous time to build, network connections have the additional heterogeneity implied by time to build: so, in the Domar weights, the links with time to build $d$ must be weighted with the corresponding higher discount factor $\beta^d$. Note that, instead, the time to build for the labor input does not affect the Domar weights (but it affects the dynamics) as shown by (2) and (3).

The Long Jr and Plosser (1983) model is the special case of this model when time to build is homogeneous and equal to 1. In this case the dynamic of output is described in the following corollary.

**Corollary 1.** *If $d_{iL} = d_{ij} = 1$ for all $j$, sector-level outputs evolve according to:*

$$\log Y_{t+1} \;=\; \eta \;+\; \log A_{t+1} \;+\; \Omega \, \log Y_t.$$

The following examples will be useful for illustrating the theoretical results throughout the paper.

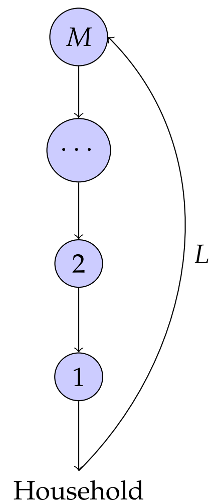


Figure 1: The vertical economy

**Example 1** (Vertical economy with homogeneous time to build of one period)**.** Consider a line network with $M$ industries/firms, where $i = M$ is the most upstream firm and $i = 1$ is the most downstream firm, as in Figure 1. Suppose that the consumer only consumes good 1 ($\gamma_1 = 1$) and all the sectors $i \neq M$ use only the intermediate input in production, while sector $M$ uses only labor, so that $\omega_{i,i-1} = 1$ and $\alpha_M = 1$. Assume that time to build is homogeneous and equal to $d$. Then the Domar weights are equal to

$$\lambda_i \;=\; \beta^{d(i-1)}.$$

The Domar weights (and revenues) are increasing as sectors are closer to the consumers, because their output is consumed in a nearer future.

**Example 2** (Vertical economy with heterogeneous time to build)**.** Now consider the example with $M = 2$ sectors, and suppose that the downstream sector has a time to build of 2 periods. Then:

$$\lambda_2 = \beta^2, \quad \lambda_1 = 1$$

so that the distance in the Domar weights (and revenues) increases further: a longer time to build means that sector 2 is now at three steps from the final consumer, not two (because of its own time to build, and because of the time to build of the downstream sector 1). So, the sales have to be weighted by $\beta^2$, decreasing the Domar weight. In general, longer time to build translates in smaller Domar weights.

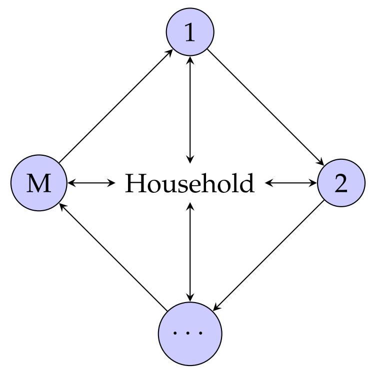


Figure 2: The circle economy

**Example 3** (Circle economy)**.** Consider a circular network as in Figure 2, in which firm $i$ only uses inputs from firm $i-1$ and possibly labor. Suppose that time to build is homogeneous and equal to 1. Consider first the case $M=2$. The Domar weights are

$$\boldsymbol{\lambda} = \frac{1}{1-\beta^2\omega_{12}\omega_{21}} \begin{bmatrix} \gamma_1 + \beta\gamma_2\omega_{21} \\ \\ \gamma_2 + \beta\gamma_1\omega_{12} \end{bmatrix}$$

So, the Domar weight is larger for sector 1 if and only if

$$\gamma_1(1-\beta\omega_{12}) > \gamma_2(1-\beta\omega_{21}).$$

So, either sector 1 has a larger consumer demand ($\gamma_1 > \gamma_2$), or relies less on the intermediate input than sector 2 ($\omega_{12} < \omega_{21}$).[2] The relative importance of the intermediate input channel is weighted by $\beta$ – if $\beta$ is small, longer connections matter less. In general, the Domar weights are

$$\lambda = \left(I - \beta\Omega'\right)^{-1} = \sum_{s=0}^{+\infty}(\Omega')^s\gamma = \frac{1}{1-\beta^N\prod_{i=1}^N \omega_{i+1,i}}\sum_{s=0}^{M-1}(\Omega')^s\gamma$$

and similar intuitions apply.

### 2.3 Aggregate Variables

In this paper, we are interested in how an idiosyncratic shock affects the dynamics of some aggregate variables, such as real GDP or consumption. We follow Long Jr and Plosser (1983) and define GDP at constant prices, which

[2]In the limit in which a sector does not rely at all on the intermediate input we fall in the case of the vertical economy of Example 1, in which the upstream sector has smaller Domar weight.

we choose to be the steady-state ones (given by (5))

$$GDP_t := \sum_{i=1}^{M} \overline{P}_i Y_{i,t}.$$

Consistent with Long Jr and Plosser (1983), we treat intermediate inputs as gross capital formation. For this reason, we do not subtract input depreciation when computing GDP: $\overline{P}_i Y_{i,t}$ will correspond to the (real) gross value added of sector $i$. We also define real aggregate consumption as

$$C_t := \sum_{i=1}^{M} \overline{P}_i C_{i,t}.$$

Even though these two measures are different, we will show that they exhibit similar dynamics. In particular, both can feature endogenous oscillations.

# 3 Theoretical results

## 3.1 Theoretical impulse responses

The goal of this section is to compute the impulse response of GDP for a small productivity shock. Specifically, we assume that the economy is in the steady state $A_{h,s} = \overline{A}_h$ for all periods $s$, and we consider a small one-period shock to GDP that hits a specific sector $h$ at time $t$.

Before stating the Theorem, we need some additional notation. Define $\tilde{\lambda}_i = \dfrac{\lambda_i}{\sum_j \lambda_j}$ the *normalized* Domar weights. Moreover, define the *propagation matrix* $\overline{\Omega}_s = \sum_{\phi \in \Phi_s} \prod_j \Omega_{\phi_j}$, where $\Phi_s$ is the set of all sequences $\phi = (\phi_1, \dots, \phi_n) \in \mathbb{N}_+^n$ such that $\sum_j \phi_j = s$, with the convention that the product over the empty sequence $\phi = \emptyset$ is the identity matrix: $\overline{\Omega}_0 = I$. The entries of the matrix $\overline{\Omega}_s$ represent weighted counts of all the direct and indirect paths that goods travel in the network in $s$ time units: since production happens with time to build, the count must acknowledge the fact that some steps take longer time than others. If time to build is homogeneous equal to 1, we have $\overline{\Omega}_s = \Omega^s$: in $s$ periods the goods travel exactly $s$ steps in the network.

**Example 4** (Circle economy)**.** To illustrate the calculation of the $\overline{\Omega}$ matrices, consider the circular economy of Example 3. Suppose sector 1 has time to build 1 and sector 2 has time to build 2. We have:

$$\Omega_1 = \begin{pmatrix} 0 & \omega_{12} \\ 0 & 0 \end{pmatrix} \qquad \Omega_2 = \begin{pmatrix} 0 & 0 \\ \omega_{21} & 0 \end{pmatrix}$$

We have: $\Phi_1 = \{(1)\}$, $\Phi_2 = \{(1,1),(2)\}$, $\Phi_3 = \{(1,1,1),(1,2),(2,1)\}$,

$\Phi_4 = \{(1,1,1,1),(1,1,2),(2,1,1),(1,2,1),(2,2)\}$ and so on. So, taking advantage from the property that both $\Omega_1$ and $\Omega_2$ are nilpotent, $\Omega_1^2 = 0$, $\Omega_2^2 = 0$, we get:

$$\overline{\Omega}_1 = \sum_{\phi \in \Phi_1} \prod_j \Omega_{\phi_j} = \Omega_1$$

$$\overline{\Omega}_2 = \sum_{\phi \in \Phi_2} \prod_j \Omega_{\phi_j} = \Omega_1\Omega_1 + \Omega_2 = \Omega_2$$

$$\overline{\Omega}_3 = \sum_{\phi \in \Phi_3} \prod_j \Omega_{\phi_j} = \Omega_2\Omega_1 + \Omega_1\Omega_2 + \Omega_1^3 = \begin{pmatrix} 0 & 0 \\ 0 & \omega_{12}\omega_{21} \end{pmatrix} + \begin{pmatrix} \omega_{12}\omega_{21} & 0 \\ 0 & 0 \end{pmatrix} = \omega_{12}\omega_{21} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and, in general, we get that, if $x, r \in \mathbb{N}$ are the values such that $k = 3x + r$, we have:

$$\overline{\Omega}_k = \begin{cases} (\omega_{12}\omega_{21})^x \Omega_1 & r = 1 \\ (\omega_{12}\omega_{21})^x \Omega_2 & r = 2 \\ (\omega_{12}\omega_{21})^x I & r = 0 \end{cases}$$

The next Theorem is the main result of this section, and characterizes the impact on GDP of a small productivity shock, as well as the consequences for amplification and oscillations.

**Theorem 1.** *Suppose the economy is in the steady state with $A_t = A$ for all $t$.*

1. *The impact of a productivity shock affecting sector $h$ at time $t$ on GDP $s$ steps later is:*

$$\frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}} = \sum_{i=1}^{M} \tilde{\lambda}_i \, (\overline{\Omega}_s)_{ih} \tag{6}$$

   *If time to build is 1 in all sectors, we have:*

$$\frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}} = \sum \tilde{\lambda}_i (\Omega^s)_{ih}$$

2. *As a consequence, a small shock generates:*

   *(a)* Amplification *from $t+n$ to $t+n+1$ if and only if:*

$$\sum_{i=1}^{M} \lambda_i \, \left( (\overline{\Omega}_{n+1})_{ih} - (\overline{\Omega}_n)_{ih} \right) > 0 \tag{7}$$

   *(b)* Oscillations, *if the dynamic moves towards the steady state for some $n$, and there is amplification for some $s > n$.*

*Formally, we have* perpetual damped oscillations *if there is a sequence of time indices $n_t$ such that:*

$$\sum_{i=1}^{M} \lambda_i \left( (\overline{\Omega}_{n_t+1})_{ih} - (\overline{\Omega}_{n_t})_{ih} \right) < 0$$

$$\sum_{i=1}^{M} \lambda_i \left( (\overline{\Omega}_{n_t+s+1})_{ih} - (\overline{\Omega}_{n_t+s})_{ih} \right) > 0 \quad \text{for some } s > 0 \tag{8}$$

The proof is in Appendix A.1. Point 1) characterizes the effect of a shock to sector $h$ in terms of the network position of $h$. The immediate effect has an impact on GDP of the magnitude (in percentage) of the normalized Domar weight of $h$ (recall $\overline{\Omega}_0 = I$). The normalized Domar weight also matters for diffusion, as it affects the response of other sectors, too. The impact in $s$ periods depends on the amount of steps the shock travels in the network. If time to build is equal to 1, at $s = 1$ the immediate neighbors are affected, and so the size of the effect on GDP is the weighted sum of the Domar weights of the neighbors, that is a *weighted degree* of node $h$. After $s$ steps, the neighbors at distance $s$ are reached, and so the impact depends on the weighted $s$-degree. If time to build is higher than 1, or heterogeneous, the amount of steps accounts for time to build, and so the computation of the weighted $s$-degree changes accordingly.

To illustrate point 2), first suppose that time to build is homogeneous and equal to 1. In that case the conditions become:

a) There is amplification from $t + n$ to $t + n + 1$ if and only if: $\sum_{i=1}^{M} \lambda_i \left( (\Omega^{n+1})_{ih} - (\Omega^{n})_{ih} \right) > 0$.

b) After a positive shock, the dynamic displays perpetual damped oscillations every period if

$$\sum_{i=1}^{M} \lambda_i \left( (\Omega^{n+1})_{ih} - (\Omega^{n})_{ih} \right) < 0$$

for $n$ odd and

$$\sum_{i=1}^{M} \lambda_i \left( (\Omega^{n+2})_{ih} - (\Omega^{n+1})_{ih} \right) > 0$$

for $n$ even.

Notice that both conditions are expressed in terms of the (non normalized) Domar weights, because for the inequalities to be satisfied the normalization is irrelevant.

The next two examples illustrate two cases that generate amplification and oscillations.

**Example 5** (Amplification in a vertical economy)**.** Suppose that time to build is homogeneous and equal to 1. To fix ideas, consider $M = 3$ sectors. Let us consider a small shock to the most upstream sector $i = 3$. There is amplification from $t$ to $t + 1$ if

$$\sum_{i=1}^{M} \lambda_i \, \omega_{i3} > \lambda_3 \Leftrightarrow \lambda_2 > \lambda_3$$

There is amplification from $t+1$ to $t+2$ if

$$\sum_{i=1}^{M} \lambda_i \, (\Omega^2)_{i3} > \sum_{i=1}^{M} \lambda_i \, \omega_{i3}$$

$$\Leftrightarrow \quad \lambda_1 \, \omega_{12}\omega_{23} > \lambda_2 \, \omega_{23} \Leftrightarrow \lambda_1 \, > \lambda_2$$

The dynamic impact from period $t+3$ onwards is zero, since $\Omega^3 = 0$. So, given that we know that the Domar weights are increasing moving downstream (towards the consumer) from Example 1, we conclude that there is amplification for all periods $s$ with $s < 3$. In general, in a vertical economy of length $M$ and for a shock to sector $i$, we have amplification for $i-1$ periods, and then a monotonic decrease to the steady state for $s > i-1$.

**Example 6** (Damped oscillations in a circle economy)**.** We need to check whether: $\Delta_{n+1,h} = \sum_{i=1}^{M} \lambda_i \, \left( (\Omega^{n+1})_{ih} - (\Omega^n)_{ih} \right)$ is positive for some $n$ and negative for some other. In the circle economy with 2 firms, $\Delta_{n+1,h} = (\Omega^{n+1})_{ii}\tilde{\lambda}_i + (\Omega^{n+1})_{ji}\tilde{\lambda}_j - \left( (\Omega^n)_{ii}\tilde{\lambda}_i + (\Omega^n)_{ji}\tilde{\lambda}_j \right)$.

Suppose, without loss of generality, that the shock hits firm 1 at time 0. We have:

$$\Omega^{2t} = (\omega_{21}\omega_{12})^{2t} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \Omega^{2t+1} = (\omega_{21}\omega_{12})^{2t} \begin{pmatrix} 0 & \omega_{12} \\ \omega_{21} & 0 \end{pmatrix}$$

So, if $t$ even:

$$\begin{aligned}
\Delta_{t+1,1} &= -\left(\omega_{12}\omega_{21}\right)^t \tilde{\lambda}_1 + \left(\omega_{12}\omega_{21}\right)^t \omega_{21}\tilde{\lambda}_2 = \left(\omega_{12}\omega_{21}\right)^t \left(-\tilde{\lambda}_1 + \omega_{21}\tilde{\lambda}_2\right) \\
\Delta_{t+2,1} &= \left(\omega_{12}\omega_{21}\right)^{t+1} \tilde{\lambda}_1 - \left(\omega_{12}\omega_{21}\right)^t \omega_{21}\tilde{\lambda}_2 = \left(\omega_{12}\omega_{21}\right)^t \omega_{21} \left(\omega_{12}\tilde{\lambda}_1 - \tilde{\lambda}_2\right) \\
\Delta_{t+3,1} &= \left(\omega_{12}\omega_{21}\right)^{t+2} \left(-\tilde{\lambda}_1 + \omega_{21}\tilde{\lambda}_2\right) \\
&\vdots
\end{aligned}$$

So we can see that $\Delta_{t,1}$ is proportional to $-\tilde{\lambda}_1 + \omega_{21}\tilde{\lambda}_2$ or $\omega_{12}\tilde{\lambda}_1 - \tilde{\lambda}_2$, at alternating periods. Since the dynamic converges to the steady state, there cannot be amplification in every period, and so at least one of them has to be negative.[3] There are oscillations if one of the two is positive. So, we have oscillations if either $\tilde{\lambda}_2\omega_{21} < \tilde{\lambda}_1$ and $\tilde{\lambda}_1\omega_{12} > \tilde{\lambda}_2$ (recessions at odd periods), or vice versa (recessions at even periods). In either case, if GDP expands at a period $t$ then it also does every other period: so there are perpetual damped oscillations. For a shock to firm 2, we would get the same conditions (implying expansion and recession in alternating periods). So, in this economy GDP

[3] Formally, if they were both positive, we would conclude $\tilde{\lambda}_2\omega_{21}\omega_{12} > \omega_{12}\tilde{\lambda}_1 > \tilde{\lambda}_2$ that is impossible.

either converges monotonically, or there are perpetual damped oscillations. Note that, by the formula above we have that GDP goes up in period 1 if $\tilde{\lambda}_2 \omega_{21} < \tilde{\lambda}_1$.

The condition to have recessions at odd periods is satisfied if and only if:

$$
\begin{aligned}
&(\gamma_2 + \beta\gamma_1\omega_{12})\omega_{21} > \gamma_1 + \beta\gamma_2\omega_{21} \iff \\
&\frac{\omega_{21}(1-\beta)}{1-\beta\omega_{12}\omega_{21}} > \frac{\gamma_1}{\gamma_2}
\end{aligned} \tag{9}
$$

The left-hand side of (9) is smaller than 1, because $1-\beta \leq 1-\beta\omega_{12}\omega_{21}$, and the expression for even periods is analogous. This implies that when $\gamma_2$ and $\gamma_1$ are equal (or close), the conditions cannot be satisfied: when the consumption shares are symmetric, there are no oscillations. To have oscillations, we need sufficiently asymmetric consumption shares. To understand this, consider a positive shock to the largest sector, say 1. Since the sector is large, the immediate effect on GDP is also large. However, in the next period the output of sector 1 will not be affected by the shock, since it uses as input only labor, and the contemporaneous production of sector 2. So, at time $t+1$ the effect on GDP is only due to an increase in production of sector 2, that being small, has a small effect on GDP. So, GDP increases less than at time $t+1$. At time $t+2$, the increased amount of good 2 implies that again sector 1 must increase the output, but by a smaller amount than before. If the sectors are sufficiently similar, this increase is not sufficient to offset the damping effect of the propagation, and GDP keeps decreasing monotonically to approach the steady state. If the sectors are sufficiently asymmetric, instead, the increase can offset the progagation, and GDP increases at $t+2$: this generates oscillations.

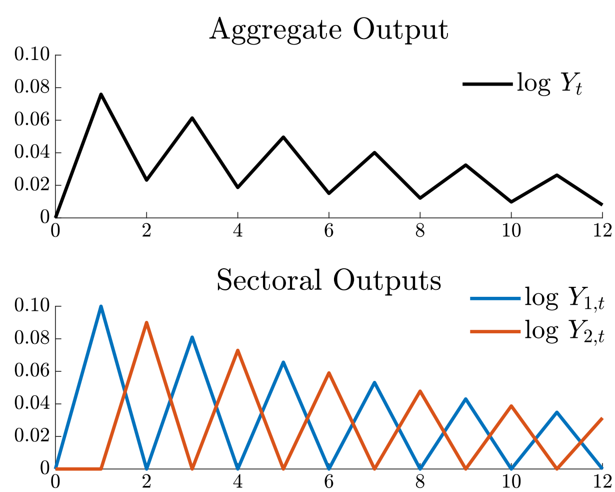


Figure 3: **Endogenous oscillations in a 2-sector circle network**
This figure shows the reaction of aggregate and sectoral outputs to a one-period shock to $A_{1,t}$. We consider a 2-sector economy with $\beta = 0.1$, $\gamma_1 = 0.8$, $\gamma_2 = 0.2$, $\alpha_1 = \alpha_2 = 0.1$, $\omega_{11} = \omega_{22} = 0$, $\omega_{12} = \omega_{21} = 1$ and $d_{12} = d_{21} = 1$.

Figure 3 represents the evolution of real aggregate and sectoral output to a one-time shock to industry one. This shock takes place at time 1. In this period, the output of sector 1 expands, while sector 2 remains unchanged (given that inputs are predetermined); as a result, aggregate output expands. At time 2, sector 2 expands, but sector 1 contracts. Since sector 2 represents a small fraction of aggregate output, GDP is above steady-state but below its time 1 level. However, at time 3, sector 1 expands again. Since sector 1 represents a large share of GDP, GDP can end up expanding again. Thus, GDP is higher when sector 1 expands, and lower when sector 2 expands. This can lead to a sequence of damped oscillations, as shown in Figure 3.

In general, in a larger circle network of size $M$, we can have also amplification for many periods. For example, building on the intuitions of the example above for the vertical economy, if the sector $i$ supplies $i-1$ and the Domar weights are in decreasing order, so that $\lambda_i < \lambda_{i-1}$, then we can have amplification for $M$ periods, and then either a monotonic return to the steady state, or cyclic behavior with period $M$.

The next example shows that, with heterogeneous time to build, oscillations arise even more often.

**Example 7** (Circle economy - Oscillations with heterogeneous time to build)**.** In the circle network with $M = 2$ sectors, now suppose that sector 2 has time to build of 2 periods. If the shock hits sector 1 we have: $\sum_{i=1}^{2} \lambda_i \left(\overline{\Omega}\right)_{i1} - \lambda_1 = -\lambda_1 < 0$. Since sector 2 needs two periods to react, at $t = 1$ GDP can only go down. But then: $\sum_{i=1}^{2} \lambda_i \left( (\overline{\Omega}^2)_{i1} - (\overline{\Omega})_{i1} \right) = \omega_{21}\lambda_2 > 0$: GDP goes up. So we have oscillations without assumptions on $\gamma$, contrary to Example 6. $\sum_{i=1}^{2} \lambda_i \left( (\overline{\Omega}^3)_{i1} - (\overline{\Omega}^2)_{i1} \right) = \omega_{12}\omega_{21}\lambda_1 - \omega_{21}\lambda_2 > 0 \iff \omega_{12}\lambda_1 > \lambda_2$. So, we get an oscillatory behavior that at periods 0, 3, 6 and so on has a peak, then a decrease, due to the fact that sector 2 does not react, then one or two periods of expansion, depending on whether the condition $\omega_{12}\lambda_1 > \lambda_2$ is satisfied. The intuition is that a longer time to build shifts the reaction of the corresponding sector by some periods, while the rest of the dynamic is drifting towards the steady state: when the "right" period arrives, the production of the lagged sector can cause another spike in GDP. This example is represented in Figure 4.

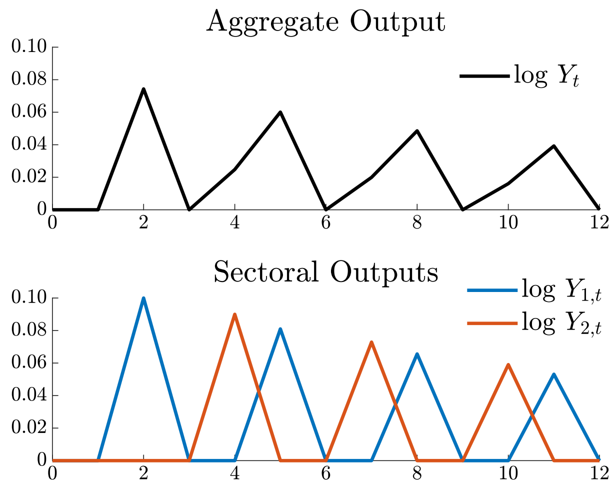


Figure 4: **Endogenous oscillations in a 2-sector circle network**

This figure shows the reaction of aggregate and sectoral outputs to a one-period shock to $A_{1,t}$. We consider a 2-sector economy with $\beta = 0.1$, $\gamma_1 = \gamma_2 = 0.5$, $\alpha_1 = \alpha_2 = 0.1$, $\omega_{11} = \omega_{22} = 0$, $\omega_{12} = \omega_{21} = 1$, $d_{12} = 1$ and $d_{21} = 2$.

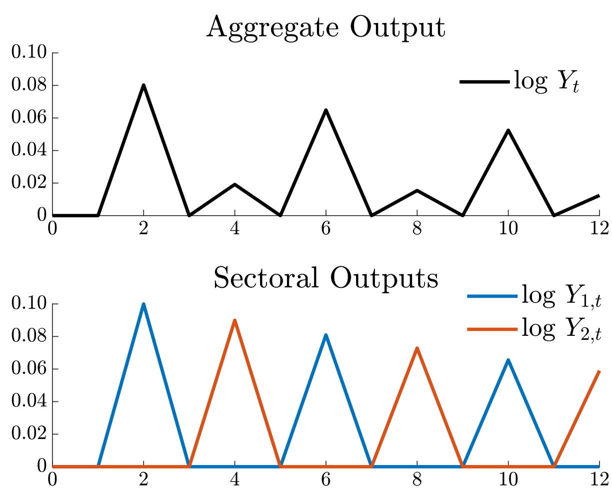


Figure 5: **Endogenous oscillations in a 2-sector circle network**

This figure shows the reaction of aggregate and sectoral outputs to a one-period shock to $A_{1,t}$. We consider a 2-sector economy with $\beta = 0.1$, $\gamma_1 = 0.8$, $\gamma_2 = 0.2$, $\alpha_1 = \alpha_2 = 0.1$, $\omega_{11} = \omega_{22} = 0$, $\omega_{12} = \omega_{21} = 1$ and $d_{12} = d_{21} = 2$.

In this figure we have heterogeneous $\gamma$, but homogeneous time to build equal to 2.

## 3.2 A measure of shock-implied volatility

We are interested in formally studying how time to build and the network structure affect volatility. One may also be interested in quantifying their magnitude: if these oscillations exist, are they large or small? How much additional volatility do they introduce compared to a scenario where GDP converges monotonically to the steady state? We introduce a measure of the magnitude of oscillations triggered by a one-period, idiosyncratic productivity shock. The measure has a twofold purpose. In Theorem 2 , we show that the value of the measure provides a simple indicator for the existence of endogenous oscillations. In addition, the measure is also useful to quantify the magnitude of such oscillations.

To keep analytic tractability, we define it using the *absolute variation* instead of the quadratic variation (as in the standard variance): this will allow to obtain a sharp characterization when the dynamic is monotonic, in Theorem 2.[4]

**Definition 1.** Define the *total variation* of GDP as:

$$T_V := \frac{1}{GDP_t} \lim_{T\to\infty} \sum_{s=1}^{T} |GDP_{t+s+1} - GDP_{t+s}| \tag{10}$$

The total variation is a classic way to measure the amount of oscillations in a time series. It aims to quantify the total deviation from an initial value $GDP_t$ (e.g., triggered by a one-off shock), assuming that dynamics asymptotically converge back to $GDP_t$ (otherwise the measure would diverge for $T \to \infty$).

**Definition 2.** Define the *oscillations measure* implied by a positive productivity shock at sector $h$ at time $t$ as:

$$\text{osc}_h := \left(\frac{\partial \log GDP_t}{\partial_+ \log A_{h,t}}\right)^{-1} \frac{\partial}{\partial_+ \log A_{h,t}} T_V \mid_{ss} \tag{11}$$

where $\dfrac{\partial}{\partial_+ \log A_{h,t}} f = \lim_{\Delta\to 0^+} \dfrac{f(\log A_{h,t} + \Delta) - f(\log A_{h,t})}{\Delta}$ denotes the *right* derivative with respect to the size of the shock, and the subscript $ss$ denotes the fact that the derivative is computed for the steady state values of productivity: $A_{i,t} = \overline{A}_i$ for all $t$ and $i$.

The measure is normalized by the initial size of the shock $\dfrac{\partial \log GDP_t}{\partial_+ \log A_{h,t}}$: this normalization ensures comparability of the oscillations across different sectors. It is important that in the definition we specify that the derivative is taken from the right, because the absolute value is not differentiable in zero. Moreover, the derivative in (11) may fail to exist if $GDP_t = GDP_{t+1}$ for some $t$: Appendix A.3 shows that this can happen at most for a non-generic set of technology and preference parameters. Outside of this non-generic set, the oscillation measure is equal to:

$$\begin{aligned}\text{osc}_h &= \left(\frac{\partial \log GDP_t}{\partial_+ \log A_{h,t}}\right)^{-1} \sum_{s=0}^{+\infty} \left| \frac{\partial \log GDP_{t+s+1}}{\partial_+ \log A_{h,t}} - \frac{\partial \log GDP_{t+s}}{\partial_+ \log A_{h,t}} \right| \\ &= \frac{1}{\tilde{\lambda}_h} \sum_{s=0}^{+\infty} \left| \sum_i \tilde{\lambda}_i \left( (\overline{\Omega}^{t+s+1})_{ih} - (\overline{\Omega}^{t+s})_{ih} \right) \right| \end{aligned} \tag{12}$$

The next result is our main result on the volatility measure, and it clarifies the role of the normalization.

[4] As a drawback, we have to be careful in measuring the impact of a shock as the *right* derivative, since the absolute value is not differentiable in zero. Moreover, the increments may not be differentiable in the knife-edge case in which GDP is constant for some periods even after the shock. The case can arise, but for generic values of the parameters it does not.

**Theorem 2.** *The oscillations measure is always larger than 1:*

$$osc_h \geq 1$$

*Furthermore, the GDP is monotonically decreasing after the (positive) shock if and only if: $osc_h = 1$.*

The proof is in Appendix A.2. The theorem above shows that a value of $\text{osc}_h$ larger than 1 represents oscillations. This measure, as shown by Equation (12), is the ratio of two numbers. The numerator is the cumulative sum of the growth rates of GDP (in absolute terms), starting one period after the shock. The denominator is the contemporaneous impact of the shock on GDP, that is the normalized Domar weight of the sector that was shocked, which coincides with the growth rate of GDP in the period when the shock occurs. When there are no oscillations, the two numbers coincide – the cumulative sum of all GDP growth rates in absolute terms as GDP converges back to the steady state (the numerator) must equal the GDP growth rate in the period when the shock occurred (the denominator). If the former is greater than the latter, this indicates the presence of oscillations.[5] The value of $\text{osc}_h$ therefore has a simple interpretation. For example, a value of $\text{osc}_h = 1.3$ implies that the cumulative sum of GDP growth rates is 30% larger than it would be if, in response to a small shock to sector $h$, convergence to the steady state were monotonic.

**Examples**

**Example 8** (Perpetual damped oscillations)**.** If the sign of the deviation from the steady state follows a simple pattern, it is possible to further simplify the expression for $\text{osc}_h$. To see an example, suppose that time to build is equal to 1 and there are oscillations at every period (not including the moment when the shock hits), for example if: $\sum_{i=1}^{M} \lambda_i \left((\Omega^{n+1})_{ih} - (\Omega^n)_{ih}\right) > 0$ for $n$ even and $\sum_{i=1}^{M} \lambda_i \left((\Omega^{n+1})_{ih} - (\Omega^n)_{ih}\right) < 0$ for $n$ odd. Then we can show that the formula becomes:

$$\begin{aligned}
\text{osc}_h &= \frac{1}{\tilde{\lambda}_h} \sum_{s=0} \left| \frac{\partial \log GDP_{t+s+1}}{\partial \log A_{h,t}} - \frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}} \right| = \\
&= 1 - 2\frac{1}{\tilde{\lambda}_h} \sum_i ((I + \Omega')^{-1})_{ih} \tilde{\lambda}_i
\end{aligned}$$

**Example 9** (Circle economy)**.** An example in which we have oscillations at every period is the case of the circle network of Example 3, in which condition (9) is satisfied, and the shock hits the smaller sector. Let us suppose the smaller sector is $h = 1$, and $\omega_{12} = \omega_{21} = \omega$. In such a case, for we know that for $k$ even $\frac{\partial \log GDP_{t+k+1}}{\partial \log A_{1,t}} - \frac{\partial \log GDP_{t+k}}{\partial \log A_{1,t}} > 0$, and for $k$ odd: $\frac{\partial \log GDP_{t+k}}{\partial \log A_{1,t}} - \frac{\partial \log GDP_{t+k-1}}{\partial \log A_{1,t}} < 0$, and so, by the example above we can

[5] In fact, it immediately follows from Theorem 2 that $\text{osc}_h \geq 1$ and that $\text{osc}_h = 1$ if and only if there are no oscillations.

explicitly compute the measure:

$$\text{osc}_1 = 1 - 2\sum_i ((I+\Omega')^{-1})_{i1}\tilde{\lambda}_i/\tilde{\lambda}_1 = 1 - \frac{2}{1-\omega^2}\frac{\tilde{\lambda}_1 - \omega\tilde{\lambda}_2}{\tilde{\lambda}_1}$$

that, indeed, satisfies $\text{osc}_1 > 1$ if and only if $\omega\tilde{\lambda}_2 > \tilde{\lambda}_1$, that is exactly the condition (9).

Instead, if $h = 1$ is the *larger* sector, there are *downward* oscillations at even periods, and so:

$$\text{osc}_1 = -1 + 2\frac{1}{1-\omega^2}\frac{\tilde{\lambda}_1 - \omega\tilde{\lambda}_2}{\tilde{\lambda}_1}$$

that satisfies $\text{osc}_1 > 1$ if and only if $\frac{1}{1-\omega^2}\left(\tilde{\lambda}_1 - \omega\tilde{\lambda}_2\right) > \tilde{\lambda}_1$, that is equivalent to $\omega\tilde{\lambda}_1 > \tilde{\lambda}_2$.

# 4 Quantitative Results

## 4.1 Data

To check if the theoretical insights described in Section 3 also hold in real production networks, we calibrate our model to the U.S. economy. We need two main data sources. One is the U.S. input-output network, the other is a proxy for time-to-build across sectors.

For time-to-build, we rely on the Manufacturers' Shipments, Inventories, and Orders (M3) survey by the U.S. census (Liu and Tsyvinski, 2024). The M3 survey provides timely and detailed data on economic conditions in the manufacturing sector. It tracks monthly changes in manufacturers' shipments, inventories, and unfilled orders, offering insights into production trends and demand for goods across sectors. We construct a backlog measure by dividing unfilled orders by the value of shipments. This is a reasonable measure because it normalizes the backlog relative to the rate at which manufacturers fulfill orders, providing a measure of how many months of demand remain outstanding at the current pace of shipments. Assuming no prioritization, the time-to-build for all sectors using input $j$ is the same, that is $d_{ij} = d_j$ for all sectors $i$ using input $j$.[6] We consider seasonally-adjusted measures and compute the mean of time-to-build across all months from 2010 to 2019 (Liu and Tsyvinski, 2024). Finally, we convert the backlog measures to a quarterly frequency, rounding to the nearest quarter. While the median time-to-build is one quarter, some sectors have longer time-to-build. The maximum time-to-build is four quarters (Ship and boat building), several other sectors have a three quarters time-to-build (e.g. Railroad rolling stock manufacturing, Aerospace product and parts manufacturing).

[6]The M3 survey uses its own classification of manufacturing sectors that does not necessarily align with more common classifications, such as NAICS. We address this issue following Liu and Tsyvinski (2024): (i) for the NAICS sectors that we can match to M3 sectors we use the M3 backlog measure; (ii) for the NAICS sectors that we can match to manufacturing but not to any specific M3 sector, we use the aggregate backlog of manufacturing (that is, we divide the total unfilled orders by the manufacturing sector by total shipments); (iii) for the NAICS sectors that are not manufacturing, lacking better data we use the simple mean of the backlog across all manufacturing sectors.

The second dataset that we use is the 2023 B.L.S. Input-Output matrix. We estimate the input-output coefficients $\omega_{ij}$ from the Use matrix, normalizing interindustry sales and value added so that $1 = \alpha_i + \sum_{j=1}^{M} \omega_{ij}$. We also use private consumption to estimate the consumption shares $\gamma_i$. We remove sectors that either do not sell to any other sector or do not buy from any sector (Liu and Tsyvinski, 2024), ending up with 172 connected sectors.

## 4.2 Impulse responses

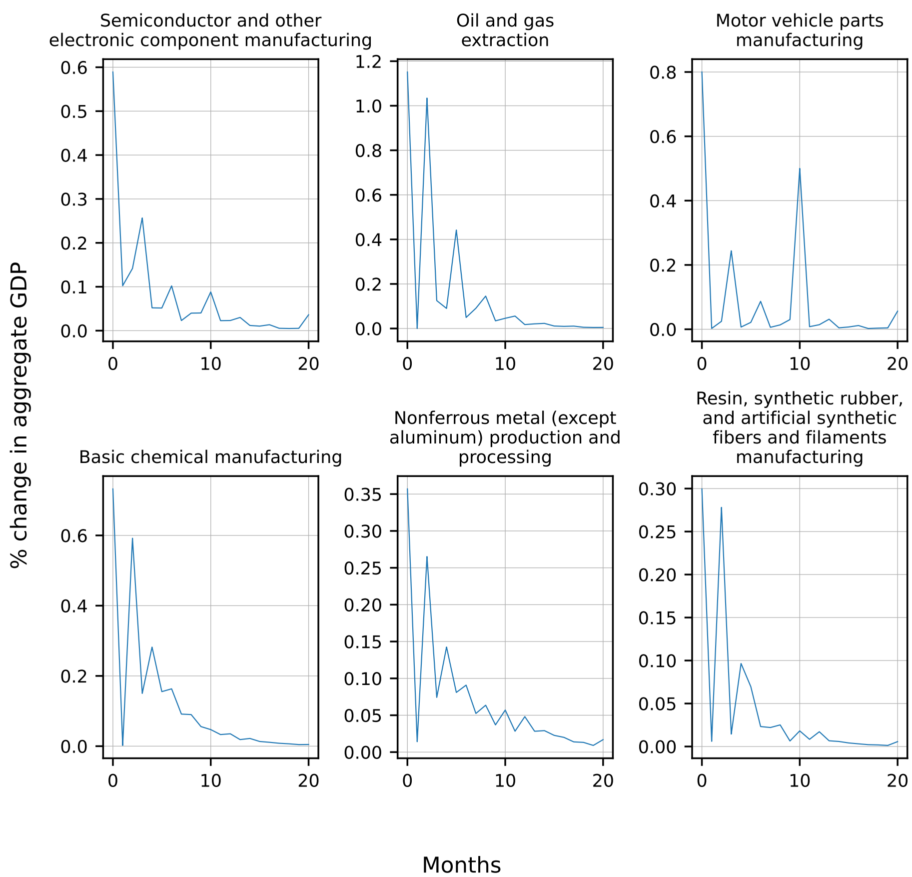


Figure 6: Deviation in aggregate GDP in the 20 steps after a 100% positive productivity shock hits each of the named sectors. The sectors are chosen for illustrative purposes among the ones that imply the largest total variation $TV$.

We now simulate the model with productivity shocks hitting one of the 172 sectors at time $t$, and disappearing thereafter. We systematically check the impact on both aggregate and sectoral output by shocking all sectors, one at a time, with a 100% positive productivity shock. Figure 6 shows the impact on GDP of hitting a few sectors, selected to illustrate the type of oscillations that the model produces among the ones with largest oscillation measure $CV_h(g_Y)$.

To understand the oscillations in more detail, Figure 7 zooms in on the case of a productivity shock hitting the "Semiconductor and other electronic component manufacturing" sector. The positive shock leads this sector to double its output, and so, due to time to build equal to one, immediately increase aggregate GDP by a factor equal

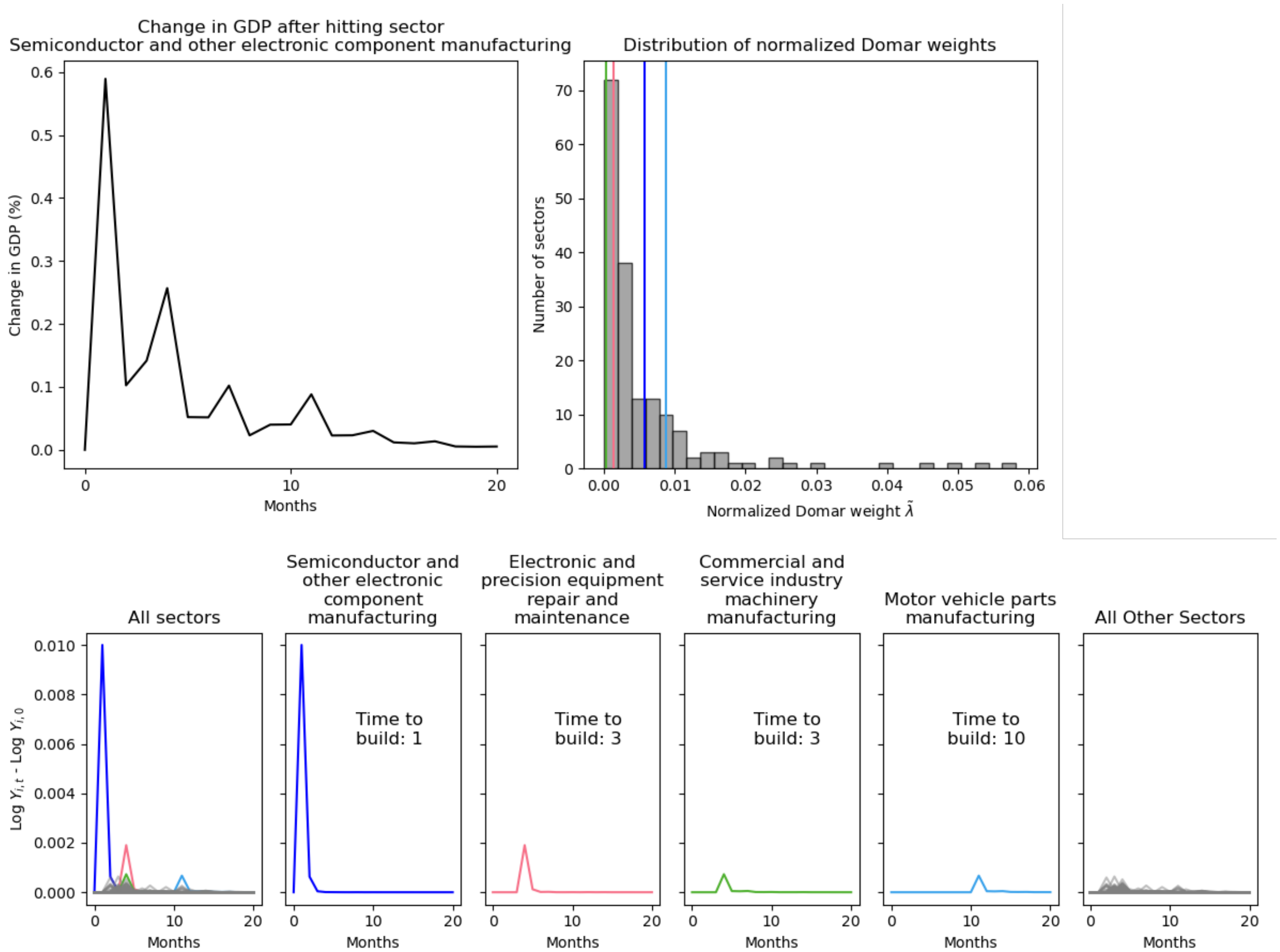


Figure 7: Illustration of a positive productivity shock hitting the "Semiconductor and other electronic component manufacturing" sector. Top left: Change in aggregate GDP (corresponds exactly to the top left panel of Figure 6). Top right: Distribution of normalized Domar weights $\tilde{\lambda}$. The colored vertical lines show the normalized Domar weights of the sectors highlighted in the bottom row. Bottom row: Log-deviations from the steady state for a few selected sectors. The second panel from the left shows the log deviation of the hit sector, the next three panels show the log deviation of the three biggest customers of the hit sector. The panel on the right shows the dynamics of all remaining sectors, and the panel on the left overlays the five plots on its right. This visualization makes it possible to understand which sectors contribute to the dynamics at which time, and how substantial their contribution to aggregate oscillations is, depending both on how the shock propagates in the network and on their Domar weight.

to its normalized Domar weight. At this point, its two main customers "'Electronic and precision equipment repair and maintenance" and "Commercial and service industry machinery manufacturing" get cheaper inputs thanks to the positive productivity shock, but because they have time to build equal to three months cannot immediately increase their production. Thus, even though other sectors with shorter time to build do increase their production, they do not generate a sizable increase in GDP that compensates from the fall in GDP due to the end of the positive productivity shock that hit the semiconductor sector. Three months after the shock, the two main customers of the semiconductor sector increase their production, and this leads to a spike in GDP. This peak is then followed by another trough, and then by another peak that is caused by the contribution of other sectors that are farther away from semiconductors. The peak after 10 months is finally caused by the very long time to build of the 'Motor vehicle parts manufacturing' sector, which is the third largest customer of semiconductors. Although the production increase is relatively small (0.1%) and isolated, this sector is large (its Domar weight is larger than that of the semiconductor sector), and so gives rise to a sizable peak.

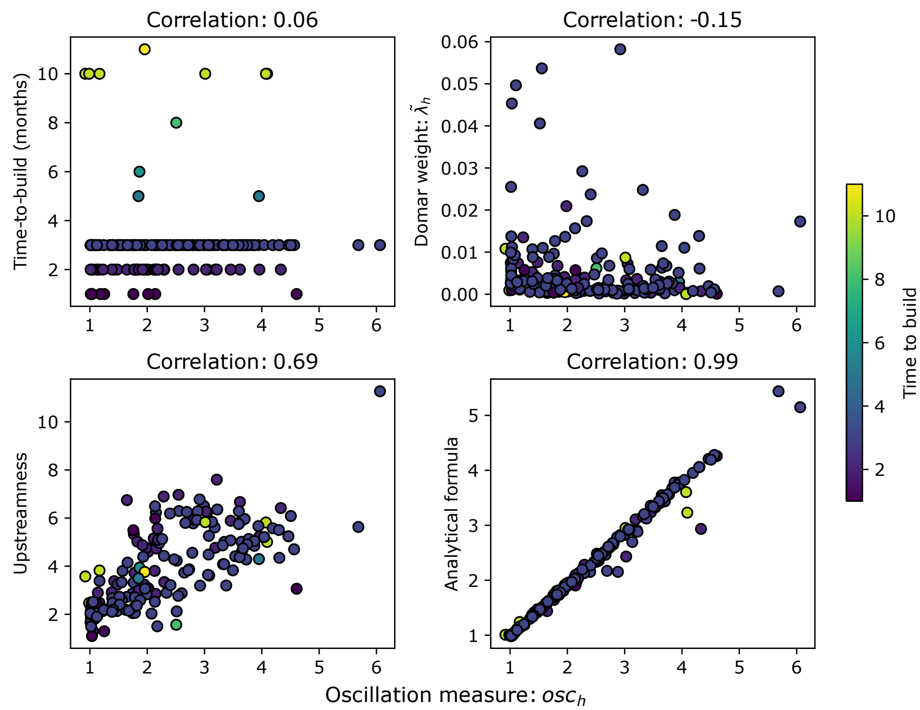


Figure 8: Correlation between the oscillation measure $\text{osc}_h$ and four metrics of the sector hit by the productivity shock.

Next, we try to understand how much simple network metrics can explain the oscillation measure $\text{osc}_h$. To do so, we compute this measure by running simulations hitting each sector at a time, and computing Eq. (12) for 100 steps of simulated data. First, we compare this measure to time to build, showing that the time to build of the hit sector, by itself, cannot explain the oscillation measure. Thus, hitting a sector with long time to build is not enough to generate endogenous oscillations. Second, we compare our oscillation measure with the Domar weights. Again, we do not find any correlation. This suggests that, after normalizing our oscillation measure by the effect of size $\tilde{\lambda}_h$, size is no longer important to explain oscillations. Third, we correlate our oscillation measure with upstreamness (Antràs et al., 2012). In this case, the correlation is quite high, suggesting that hitting sectors that are very upstream with productivity shocks is likely to generate substantial oscillations in economic activity. This can be expected, as the effect of productivity shocks propagates downstream across many sectors, with potentially long time to build. However, upstreamness by itself cannot fully account for our oscillation metric. Thus, in the fourth panel we show the oscillation metric computed analytically replacing the theoretical impulse response functions (Eq. (6)) in Eq. (12). We see that the correlation is almost perfect, apart from small deviations mostly due to computational limits.[7]

**Quantifying oscillations in impulse responses** The results shown above suggest that sectoral shocks can trigger oscillations in aggregate output. To conclude the discussion of this section, we quantify the prevalence and magnitude of these oscillations. First, we note that, out of the 172 impulse responses that we consider, we observe endogenous oscillations in 154 of them. That is, approximately 90% of all sector-level productivity shocks generate a non-monotonic convergence of GDP to its steady state. Second, we apply our measure $\text{osc}_h$ to each one of the 172 impulse responses. Figure 9 shows the distribution of this measure.

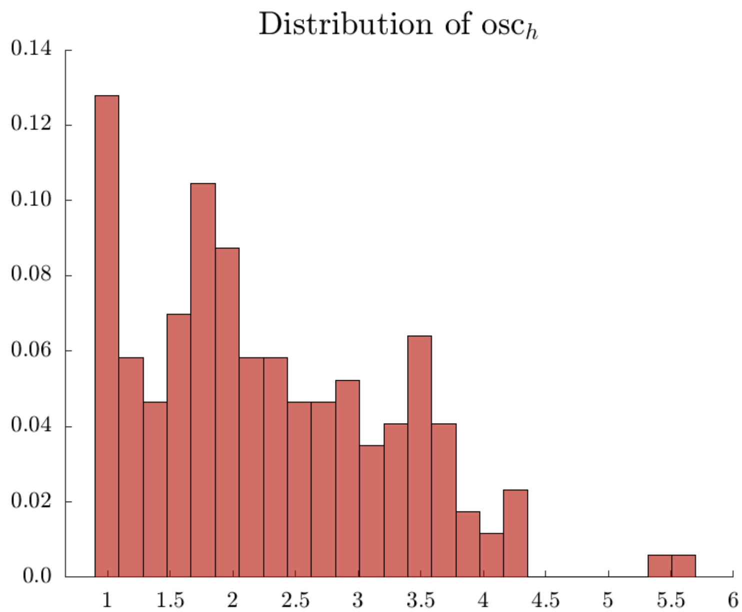


Figure 9: This figure shows the distribution of the measure $\text{osc}_h$.

Figure 9 suggests that the endogenous oscillations generated by sector-level shocks are significant. The average value of our oscillations measure is $\overline{\text{osc}_h} = 2.28$. This means that, for the average shock, the volatility of the growth rate of output is 128% higher than what it would be if GDP were to converge monotonically to steady-state. In Table 1 we report the three industries with the lowest and highest values of $\text{osc}_h$. While in the first group we see mostly service industries, in the second we see industries in both the primary sector and in manufacturing.

[7]To compute the theoretical impulse response functions in Eq. (6), one needs to consider all possible sequences $\phi = (\phi_1, \ldots, \phi_n) \in \mathbb{N}_+^n$, which increase exponentially with the number of steps $s$ after the shock. Thus, we could compute the analytical measure up to 15 steps after the shock.

| Industry name | $\text{osc}_h$ |
|---|---|
| Offices of physicians | 1.0 |
| Nursing and residential care facilities | 1.0 |
| Child day care services | 1.0 |
| Manufacturing and reproducing magnetic and optical media | 4.28 |
| Support activities for mining | 5.35 |
| Agencies, brokerages, and other insurance related activities | 5.68 |

Table 1: Smallest and largest values of the oscillations measure $\text{osc}_h$.

# Conclusion

We show that a very standard economy with an input-output network and time to build can generate endogeous oscillations out of a single productivity shock, and we show that these can be quantitatively sizable. To assess the contribution of this channel when considering other forces for amplification and cyclical behavior, such as financial frictions, multiple equilibria or market power is an interesting avenue for further research.

# Appendix

# A Proofs

## A.1 Proof of Theorem 1

We prove a more general statement, without the assumption that the productivity process is in the steady state. In general, we have:

$$\frac{\partial \log Y_{i,t+s}}{\partial \log A_{h,t}} = (\overline{\Omega}_s)_{ih} \tag{13}$$

The statement above implies:

$$\frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}} = \frac{1}{GDP_t} \sum_{i=1}^{M} (\overline{\Omega}_s)_{ih} \overline{P}_i Y_{i,t+s} \tag{14}$$

Now assuming that $A_{h,t} = \overline{A}_h$ for all $t$, since in the steady state $\overline{P}_i \overline{Y}_i = \lambda_i$, we get:

$$\frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}} = \frac{1}{\sum_j \lambda_j} \sum_{i=1}^{M} (\overline{\Omega}_s)_{ih} \lambda_i = \sum_{i=1}^{M} (\overline{\Omega}_s)_{ih} \tilde{\lambda}_i$$

which is what we wanted to show.

We prove the statement in Equation (13) by induction on $s$.

If $s = 0$:

$$\frac{\partial \log Y_{i,t}}{\partial \log A_{h,t}} = \delta_{ih} = (\overline{\Omega}_0)_{ih}$$

where $\delta_{ij} = 1$ is the usual notation for the identity matrix, such that $\delta_{ij} = 1$ if and only if $i = j$. Now, suppose the statement (13) is true for $s$ periods after the shock. We compute $\frac{\partial \log Y_{i,t+s+1}}{\partial \log A_{h,t}}$.

We have that:

$$\begin{aligned} \frac{\partial \log Y_{i,t+s+1}}{\partial \log A_{h,t}} &= \sum_d \sum_{j=1}^{M} (\Omega_d)_{ij} \frac{\partial \log Y_{j,t+s+1-d}}{\partial \log A_{h,t}} \\ &= \sum_d \sum_{j=1}^{M} (\Omega_d)_{ij} \frac{\partial \log Y_{j,t+s+1-d}}{\partial \log A_{h,t}} \end{aligned}$$

Now, since $d \geq 1$, we have that $s + 1 - d \leq s$, so by inductive hypothesis we get:

$$\begin{aligned} \frac{\partial \log Y_{i,t+s+1}}{\partial \log A_{h,t}} &= \sum_d \sum_{j=1}^{M} (\Omega_d)_{ij} (\overline{\Omega}_s)_{jh} \\ &= \sum_d \sum_{j=1}^{M} (\Omega_d)_{ij} \, (\overline{\Omega}_s)_{jh} \\ &= (\overline{\Omega}_{s+1})_{ih} \end{aligned}$$

which proves the statement. ■

## A.2 Proof of Theorem 2

In the proof, we are going to need the following Lemma. The proof is in Appendix A.3.

**Lemma 1.** 1. *The total variation $T_V$ is right-(and left-)differentiable, except possibly for a non-generic set of technology and preference parameters.*

2. *Outside of this non-generic set, the oscillation measure is equal to (as in Equation (12)):*

$$osc_h = \left(\frac{\partial \log GDP_t}{\partial_+ \log A_{h,t}}\right)^{-1} \sum_{s=0}^{+\infty} \left| \frac{\partial \log GDP_{t+s+1}}{\partial_+ \log A_{h,t}} - \frac{\partial \log GDP_{t+s}}{\partial_+ \log A_{h,t}} \right| \\ = \frac{1}{\tilde{\lambda}_h} \sum_{s=0}^{+\infty} \left| \sum_i \tilde{\lambda}_i \left( (\overline{\Omega}^{t+s+1})_{ih} - (\overline{\Omega}^{t+s})_{ih} \right) \right| \tag{15}$$

First of all, by Theorem 1 we have that $\frac{\partial \log GDP_t}{\partial \log A_{h,t}} = \tilde{\lambda}_h$. Now suppose that $\frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}}$ is monotonically decreasing in $s$. In this case, calculating:

$$\frac{\partial}{\partial_+ \log A_{h,t}} T_V = \sum_{s=0} \left| \frac{\partial \log GDP_{t+s+1}}{\partial \log A_{h,t}} - \frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}} \right| = \lim_{T\to\infty} \sum_{s=0}^{T} \left( \frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}} - \frac{\partial \log GDP_{t+s+1}}{\partial \log A_{h,t}} \right) \\ = \lim_{T\to\infty} \frac{\partial \log GDP_t}{\partial \log A_{h,t}} - \frac{\partial \log GDP_{t+T+1}}{\partial \log A_{h,t}} = \frac{\lambda_h}{\sum_j \lambda_j} = \tilde{\lambda}_h$$

and so $\mathrm{osc}_h = 1$.

Now consider an arbitrary sequence of $\frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}}$, and define:

$$m_s^h = \min_{1\le k\le s} \frac{\partial \log GDP_{t+k}}{\partial \log A_{h,t}}$$

It clearly is monotonically decreasing (no oscillations), so that $\frac{\partial}{\partial_+ \log A_{h,t}} T_V(m) = \tilde{\lambda}_h$ and $m_s^h \le \frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}}$. We have that $m_s^h - m_{s+1}^h$ is either zero, or the impact at time $t+s+1$ is the new minimum: $m_{t+s+1}^h = \frac{\partial \log GDP_{t+s+1}}{\partial \log A_{h,t}} < m_{t+s}^h \le \frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}}$. In both cases, we have that: $\left| \frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}} - \frac{\partial \log GDP_{t+s+1}}{\partial \log A_{h,t}} \right| \ge m_s^h - m_{s+1}^h$. So:

$$\frac{\partial}{\partial_+ \log A_{h,t}} T_V = \sum_{s=0} \left| \frac{\partial \log GDP_{t+s+1}}{\partial \log A_{h,t}} - \frac{\partial \log GDP_{t+s}}{\partial \log A_{h,t}} \right| = \\ \ge \sum \left| m_{s+1}^h - m_s^h \right| = \tilde{\lambda}_h$$

and so, $\text{osc}_h \geq 1$, with strict inequality if and only if the dynamic is not monotonic. ∎

### A.3 Proof of Lemma 1

**1. Right-differentiability** Consider all the productivities fixed at the steady state values $\overline{A}_i$, except for $A_{h,t}$, that receives a shock. So, all the dyamic variables are considered functions of one variable, $A_{h,t}$. Using a Taylor approximation around the steady state value $\overline{A}_h$, we find that:

$$GDP_{t+s+1} - GDP_{t+s} = \sum_i \lambda_i \left( (\overline{\Omega}^{t+s+1})_{ih} - (\overline{\Omega}^{t+s})_{ih} \right) (\log A_{h,t} - \log \overline{A}_h) + o(\log A_{h,t} - \log \overline{A}_h)$$

So, if $\sum_i \lambda_i \left( (\overline{\Omega}^{t+s+1})_{ih} - (\overline{\Omega}^{t+s})_{ih} \right) \neq 0$, then there is a right neighborhood of the steady state $(\log \overline{A}_h, \log \overline{A}_h + \varepsilon)$ in which $GDP_{t+s+1} - GDP_{t+s} \neq 0$ and does not change sign. In this case, we have that the limit:

$$\lim_{\log A_{h,t} \to \log \overline{A}_h^+} \frac{|GDP_{t+s+1} - GDP_{t+s}|}{\log A_{h,t} - o \log \overline{A}_h} = \sum_i \lambda_i \left( (\overline{\Omega}^{t+s+1})_{ih} - (\overline{\Omega}^{t+s})_{ih} \right)$$

exists and so the right derivative is well defined. Since this is true for all $s$, we can conclude that, except for a non-generic set of parameters, the function $\sum_{s=1}^{T} |GDP_{t+s+1} - GDP_{t+s}|$ is right-differentiable in $\log A_{h,t} = \log \overline{A}_h$.

**2. Expression of the measure** If in Equation (11) we can exchange derivative and limit, we can conclude:

$$\begin{aligned}
\text{osc}_h &= \left( \frac{\partial \log GDP_t}{\partial_+ \log A_{h,t}} \right)^{-1} \frac{\partial}{\partial_+ \log A_{h,t}} T_V \,|_{ss} && (16) \\
&= \left( \frac{\partial \log GDP_t}{\partial_+ \log A_{h,t}} \right)^{-1} \left( \frac{1}{GDP_{ss}} \frac{\partial}{\partial_+ \log A_{h,t}} \lim_{T \to \infty} \sum_{s=1}^{T} |GDP_{t+s+1} - GDP_{t+s}| - \frac{T_V \,|_{ss}}{GDP_{ss}^2} \right) && (17) \\
&= \left( \frac{\partial \log GDP_t}{\partial_+ \log A_{h,t}} \right)^{-1} \left( \frac{1}{GDP_{ss}} \frac{\partial}{\partial_+ \log A_{h,t}} \lim_{T \to \infty} \sum_{s=1}^{T} |GDP_{t+s+1} - GDP_{t+s}| \right) && (18) \\
&= \sum_{s=1}^{\infty} \left| \frac{\partial \log GDP_{t+s+1}}{\partial_+ \log A_{h,t}} - \frac{\partial \log GDP_{t+s}}{\partial_+ \log A_{h,t}} \right| && (19)
\end{aligned}$$

provided the series converge. This is the thesis. In the next paragraph, we show that we can indeed do that.

**Exchanging derivative and limit** We want to exchange the derivative and the limit. Formally, define the sequence of functions $f_T = \sum_{s=1}^{T} |GDP_{t+s+1} - GDP_{t+s}|$.

We are going to show that $\frac{\partial}{\partial_+ \log A_{h,t}} \lim_{T \to \infty} f_T = \lim_{T \to \infty} \frac{\partial}{\partial_+ \log A_{h,t}} f_T$. We show this by showing that the sequence of derivatives $\frac{\partial}{\partial_+ \log A_{h,t}} f_T$ converges uniformly in a neighborhood of the steady state (Theorem 7.17 in Rudin (1976)).

Fix an $\varepsilon$, and consider a right-neighborhood of the steady state $(\log \overline{A}_h, \log \overline{A}_h + \varepsilon)$. First of all, the sequence $f_T$

converges. By the mean value theorem, there is a point $\zeta_s \in (\log \overline{A}_h, \log \overline{A}_h + \varepsilon)$ such that:

$$GDP_{t+s+1} - GDP_{t+s} = \left( \sum_{i=1}^{M} \overline{P}_i \, Y_{i,t+s}(\zeta_s)(\overline{\Omega}_s)_{ih} - \sum_{i=1}^{M} \overline{P}_i \, Y_{i,t+s}(\zeta_s)(\overline{\Omega}_s)_{ih} \right) (\zeta_s - \log \overline{A}_h)$$

Moreover, we know that for any initial shock $\log A_{h,t}$, each $Y_{i,t}$ converges to $\overline{Y}_i$ as $t \to \infty$. So, $Y_{i,t}$ must have a finite maximum over $t$. We also know that $Y_{i,t}$ is monotonic in the shock, so it reaches a maximum in $\log \overline{A}_h + \varepsilon$. Define:

$$\overline{Y} = \max_{i,s} \frac{Y_{i,t+s}(\overline{A}_h + \varepsilon)}{\overline{Y}_i}$$

So, for all points in $(\log \overline{A}_h, \log \overline{A}_h + \varepsilon)$:

$$\begin{aligned}
|GDP_{t+s+1} - GDP_{t+s}| &= \left| \sum_{i=1}^{M} \overline{P}_i \, Y_{i,t+s}(\zeta_s)(\overline{\Omega}_s)_{ih} - \sum_{i=1}^{M} \overline{P}_i \, Y_{i,t+s}(\zeta_s)(\overline{\Omega}_s)_{ih} \right| |(\zeta_s - \log \overline{A}_h)| \\
&\leq \overline{Y} \left| \sum_{i=1}^{M} \overline{P}_i \overline{Y}_i \, (\overline{\Omega}_s)_{ih} - \sum_{i=1}^{M} \overline{P}_i \overline{Y}_i \, (\overline{\Omega}_s)_{ih} \right| \varepsilon \\
&= \overline{Y} \left| \sum_{i} \lambda_i \, (\overline{\omega}_{ih,s+1} - \overline{\omega}_{ih,s}) \right| \varepsilon \\
&\leq \overline{Y} \left| \sum_{i} \lambda_i \overline{\omega}_{ih,s+1} \right| \varepsilon
\end{aligned}$$

So, we can conclude that:

$$\sum_{s=1}^{\infty} |GDP_{t+s+1} - GDP_{t+s}| \leq \overline{Y} \sum_{s=1}^{\infty} \sum_{i} \lambda_i \overline{\omega}_{ih,s+1} | \varepsilon$$

so, the series on the left-hand side converges, because the right-hand side does.

Now, we need to show that the sequence of derivatives $\frac{\partial}{\partial_+ \log A_{h,t}} f_T$ converges uniformly in $(\log \overline{A}_h, \log \overline{A}_h + \varepsilon)$. To do that, we show that, except for a non generic set of parameters, the derivatives $\frac{\partial}{\partial_+ \log A_{h,t}} f_T$ are Lipschitz with a common constant. This, by Ascoli-Arzelá's Theorem (Theorem 7.25 in Rudin (1976)) implies that the sequence converges uniformly.

The sequence of the right derivatives is, using Theorem 1:

$$\left| \frac{\partial}{\partial_+ \log A_{h,t}} f_T \right| \tag{20}$$

$$= \sum_{s=1}^{T} \left| \frac{\partial GDP_{t+s+1}}{\partial_+ \log A_{h,t}} - \frac{\partial GDP_{t+s}}{\partial_+ \log A_{h,t}} \right| \tag{21}$$

$$= \sum_{s=1}^{T} \left| \sum_{i=1}^{M} \overline{P}_i \, Y_{i,t+s+1}(\overline{\Omega}_{s+1})_{ih} - \sum_{i=1}^{M} \overline{P}_i \, Y_{i,t+s}(\overline{\Omega}_s)_{ih} \right| \tag{22}$$

$$= \sum_{s=1}^{T} \left| \sum_{i=1}^{M} \overline{P}_i \, Y_{i,t+s+1}(\overline{\Omega}_{s+1})_{ih} - \sum_{i=1}^{M} \overline{P}_i \, Y_{i,t+s}(\overline{\Omega}_s)_{ih} \right| \tag{23}$$

Now in $\log A_{h,t}$ we have:

$$\left(\frac{\partial GDP_{t+s+1}}{\partial_+ \log A_{h,t}} - \frac{\partial GDP_{t+s}}{\partial_+ \log A_{h,t}}\right) |_{\log \overline{A}_h} = \sum_{i=1}^{M} \lambda_i \left((\overline{\Omega}_{s+1})_{ih} - (\overline{\Omega}_s)_{ih}\right)$$

So, whenever the above expression is different from zero, there is a neighborhood of the steady state value in which the expression does not change sign. This, is true for every $s$, so everywhere except possibly for a non-generic set of parameters, the function: $\sum_{s=1}^{T} \left| \frac{\partial GDP_{t+s+1}}{\partial_+ \log A_{h,t}} - \frac{\partial GDP_{t+s}}{\partial_+ \log A_{h,t}} \right|$ is differentiable.

Define $\delta_s = sgn\left(\sum_{i=1}^{M} \lambda_{i,ss} \left((\overline{\Omega}_{s+1})_{ih} - (\overline{\Omega}_s)_{ih}\right)\right)$ the sign of the $s$-th term of the sum. The derivative is:

$$\left| \frac{\partial}{\partial_+ \log A_{h,t}} f_T \right| = \tag{24}$$

$$\left| \sum_{s=1}^{T} \delta_s \left( \sum_{i=1}^{M} \overline{P}_i \frac{\partial}{\partial \log A_{h,t}} Y_{i,t+s+1} (\overline{\Omega}_s)_{ih} - \sum_{i=1}^{M} \overline{P}_i \frac{\partial}{\partial \log A_{h,t}} Y_{i,t+s} (\overline{\Omega}_s)_{ih} \right) \right| = \tag{25}$$

$$\left| \sum_{s=1}^{T} \delta_s \left( \sum_{i=1}^{M} \overline{P}_i \, Y_{i,t+s+1} \left((\overline{\Omega}_s)_{ih}\right)^2 - \sum_{i=1}^{M} \overline{P}_i \, Y_{i,t+s} \left((\overline{\Omega}_s)_{ih}\right)^2 \right) \right| \leq \tag{26}$$

$$\overline{Y} \left| \sum_{s=1}^{T} \delta_s \left( \sum_{i=1}^{M} \overline{P}_i \overline{Y}_i \, \left( \left((\overline{\Omega}_s)_{ih}\right)^2 - \left((\overline{\Omega}_s)_{ih}\right)^2 \right) \right) \right| \tag{27}$$

Now the last term converges for $T \to \infty$, to a finite limit $K$. So, each function of the sequence is Lipschitz with common constant $K$. It follows that the sequence is equicontinuous, as we wanted to show. ■

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