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Price Stability and Financial Stability: Designing the Central Bank Mandate

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Emmanuel Caiazzo* **and Alberto Zazzaro**†

Abstract

In this paper, we model a novel trade-off between price stability and financial stability in central banking. This trade-off arises from the interaction between the monetary policy interest rate, the central bank rescue interventions, and the degree of bank illiquidity. We characterize and compare the equilibrium outcomes, in terms of monetary policy, rescue policy, and bank investment decisions, that arise under a strict inflation-targeting mandate with those that instead emerge under a dual mandate, in which the central bank is required to account for both the costs of inflation and the costs associated with financial instability. Our analysis suggests that an 3inflation-targeting mandate may be advisable when the economy is subject to frequent and severe inflationary shocks that would require substantial policy rate adjustments, or when liquidity risks in the banking system are neither too high nor too low. Otherwise, a mandate that explicitly requires the central bank to take financial stability into account, even at the cost of relaxing strict inflation control, may be preferable.

JEL classification: G01, G21, G28.

Keywords: Central banking; Inflation targeting; Financial stability; Rescue policies

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1 Introduction

The potential tension between price stability and financial stability as monetary policy targets has been explored extensively in the literature. Attention has been paid to the impact of interest rate changes – the primary monetary policy tool for maintaining inflation at desired levels – on the soundness of financial institutions and the probability of financial crises. A common argument is that a sharp increase in policy interest rates aimed at countering inflationary pressures leads, both directly and indirectly, to a devaluation of fixed-income assets and, particularly when it follows a prolonged period of low interest rates, to heightened moral hazard among firms. The resulting increase in credit risk and nonperforming loans renders banks more vulnerable, exposed to liquidity crises and fearful of default, while firms become more financially constrained and the banking system, as a whole, more fragile (Demirgüç-Kunt and Detragiache, 1998; Goodfriend, 2002; Maddaloni and Peydró, 2011; Borio, 2014; Gomez *et al.*, 2021; Grimm *et al.*, 2023; Boissay *et al.*, 2024).

In this paper, we highlight a distinct and novel trade-off in central banking between maintaining inflation at its target level and safeguarding the stability of the banking system. This trade-off arises from the interplay between the policy interest rate set by the central bank, the intervention policies aimed at rescuing illiquid banks, and the optimal liquidity positions chosen by banks.

We consider a three-period banking system populated by commercial banks and a central bank. Commercial banks collect demand deposits and decide whether to invest in a liquid short-term asset or in an illiquid long-term asset. Short-term assets yield returns in each period in line with the policy rate set by the central bank and can be immediately liquidated at face value. Long-term assets also yield a certain return, but only in the final period; however, in the interim period, they can be sold on the financial market to other banks to meet unexpectedly large deposit withdrawals *à la* Diamond and Dybvig (1983), at a price that depends on the liquidity available in the banking system. The central bank plays a dual role: (i) it sets the policy interest rate, and (ii) intervenes to rescue illiquid banks.

In the interim period, the economy may be hit by an inflationary shock. By increasing the nominal interest rate to the level prescribed by a fixed inflation targeting rule, the central bank can counteract the inflationary surge. In contrast, if the interest rate set by the central bank deviates from the inflation targeting rule, the latter incurs losses proportional to the magnitude of the imbalance. At the same time, in the interim period, commercial banks can face a liquidity shock triggered by massive deposit withdrawals. If the market price of long-term assets is insufficient to allow banks hit by a run to recover the liquidity needed to meet depositors' withdrawal requests, they will experience financial distress and, in the absence of central bank intervention, eventual failure. The central bank may intervene by implementing two costly, non-mutually exclusive rescue policies. First, it can act as a lender of last resort by providing emergency liquidity directly to financially distressed banks. Second, the central bank may inject liquidity into the financial market through healthy banks to support demand and maintain the price of long-term assets held by distressed banks at a sufficiently high level to allow for their orderly liquidation and ensure the banks' solvency. The losses incurred by the central bank as a result of the rescue policy implemented to prevent the failure of distressed banks are proportional to the amount of liquidity injected into the financial market.

The monetary policy interest rate rule followed by the central bank affects the stability of the banking system and the costs of rescue interventions through two interdependent channels: (i) the optimal rescue policy announced and implemented by the central bank, and (ii) the returns banks earn from investing in short-term and long-term assets, which determine the overall liquidity (or illiquidity) of the banking system.

The first channel arises from the fact that the policy rate implicitly sets the maximum price that liquid banks are willing to pay for the long-term assets of the distressed bank. When the policy rate in the interim period is high enough, the willingness to pay for long-term assets becomes so low that the price at which illiquid banks can sell their assets does not allow them to recover the liquidity needed to meet depositors' withdrawals. In this scenario, injecting liquidity into the market by extending credit to healthy banks is ineffective in preventing the default of distressed banks. Therefore, the only possible rescue policy for the central bank is to act as a lender of last resort, providing distressed banks with the liquidity to repay depositors and avoid failure.

When the policy interest rate set by the central bank is below this threshold, the price that liquid banks are willing to pay for long-term assets may be high enough to allow for the liquidation of distressed banks' assets and prevent their insolvency without the need for emergency liquidity injections by the central bank. However, for this to occur, the overall liquidity of the banking system and therefore the total demand in the financial market must be sufficiently high (Shleifer and Vishny, 1992; Allen and Gale, 1994, 1998). Otherwise, central bank intervention is still required. However, if the policy interest rate remains low, the central bank is not obliged to act as a lender of last resort; it also has the option to prevent the default of distressed banks by lending to healthy banks, providing the additional liquidity necessary to absorb the assets of distressed banks at a price that allows them to raise the resources needed to meet depositor withdrawals. These two alternatives, and any combination thereof, are neutral in terms of intervention costs as both require a liquidity injection equal to the difference between the total liquidity needs and the total liquidity available in the market. In any case, however, since the central bank is required to cover only a portion of the liquidity that distressed banks must raise to avoid failure, the intervention costs remain lower compared to those under a high-interest rate regime.

A second channel through which the central bank's interest rate rule affects the stability of the banking system and the costs of rescue interventions operates indirectly via the effects that the rescue policy exerts on banks' incentives to hold short- and long-term assets. In fact, the central bank's rescue policy is not neutral with respect to banks' investment decisions. Lender-of-last-resort interventions enable illiquid banks facing unexpectedly large deposit withdrawals to obtain from the central bank the full liquidity required to meet depositors' demands without having to liquidate their assets on the market at fire-sale prices. In contrast, when the central bank intervenes by injecting liquidity through healthy banks, the financial market remains the sole source of liquidity for illiquid banks in financial distress, which are forced to sell long-term assets to other banks at a discount. Accordingly, and under the reasonable assumption that the fundamental returns on long-term assets exceed those on short-term assets, if commercial banks anticipate that the central bank will intervene either by lending directly to distressed banks, the incentive to hold illiquid long-term assets increases relative to that of holding liquid assets. As a result, with less liquidity available in the market, the fragility of the banking system increases, and the costs of intervention are, on average, higher.

Therefore, it would be optimal for the central bank to relinquish its role as lender of last resort whenever possible. However, in order to do so, the central bank must keep the policy rate below the threshold beyond which the market price of long-term assets would become so low that lending to the market to rescue illiquid banks is ineffective. If the central bank's mandate is to maintain inflation at its target, then, in the presence of strong inflationary pressures, it may be forced to raise interest rates beyond this threshold, leaving it with no alternative but to intervene as a lender of last resort to support banks in financial distress. However, in order to implement bailout policies that incentivize banks to hold liquidity, the central bank must credibly commit to keeping the policy interest rate below the target rate in situations where inflationary pressures are severe and the target rate required to counteract them would exceed the threshold beyond which the prices of long-term assets in financial markets fall so low as to discourage demand from liquid banks. However, this would imply that the central bank fails to uphold its price stability mandate and leaves the economy to bear higher inflation costs.

If, instead, the central bank is formally assigned a dual mandate to ensure both monetary and financial stability, it faces a trade-off between inflation costs and rescue costs when setting its monetary policy rule. On the one hand, it may maintain the policy rate at the target level, minimizing inflation costs, but at the expense of implementing rescue policies that, if anticipated, reduce the incentives of banks to hold liquidity and increase intervention costs. On the other hand, to enable the implementation of rescue policies that incentivize banks to invest in liquid assets and reduce the costs of the expected bailout interventions, the central bank may set the policy rate below the inflation-targeting interest rate rule required to prevent an increase in the price level, thus accepting higher inflation costs.

Our model examines this trade-off and the optimal central bank mandate. We determine the optimal weight to assign to the objectives of monetary stability and financial stability under a dual mandate and identify the conditions under which such a mandate reduces the social costs of monetary policy relative to an exclusive inflation-targeting mandate. Intuitively, this occurs in economies where inflationary pressures are relatively infrequent and mild, such that the general price level is highly responsive to monetary tightening, and increases in the policy rate can therefore remain limited. Alternatively, a dual mandate can be optimal when the incidence of deposit withdrawal shocks in the banking system is neither so rare as to make the risk of lender-of-last-resort interventions negligible, nor so common as to discourage banks from holding portfolios of illiquid assets and make the potential welfare gains from deviating from the inflation-targeting rule very small.

The recent turmoil in the US banking sector provides anecdotal evidence of how central banks' anti-inflationary measures compel them to implement costly lender-of-last-resort interventions to mitigate potential destabilizing waves of banking crises. During the COVID-19 outbreak, many banks allocated a significant portion of their asset portfolios to Treasury securities and other long-term low-yield assets. Despite this, the low interest rates paid on deposits allowed banks to realize profitable interest margins (Zhou and Meng, 2023). However, when the Fed markedly increased interest rates to counter inflationary shocks and growing pressures on consumer prices, the market value of long-term assets plummeted sharply. This raised concerns about the financial stability of numerous banks, prompting withdrawals from uninsured depositors that forced banks to liquidate their securities and recognize losses (Rajan and Acharya, 2023). The bank turmoil eventually led to the bankruptcy of Silicon Valley Bank (SVB), marking the second largest default in the US banking system. To prevent the SVB default from escalating into a systemic crisis, the Federal Reserve Bank introduced the "Bank Term Funding Program" (BTFP), a lender-of-last-resort facility designed to provide emergency liquidity to banks with substantial unrealized losses on long-term Treasury securities. The program allows banks to exchange long-term assets (such as US Treasury securities) for Federal Reserve funds at par value, irrespective of their current market valuation (Acharya *et al.*, 2023).

The BTFP has attracted criticism from those who have highlighted the effects of moral hazard on bank behavior and the potential fiscal costs to taxpayers. Buiter (2023), for example, argued that the BTFP offers too many advantages compared to market conditions. Consequently, financing through the BTFP becomes the first option for all banks, including those that could raise the liquidity they need at market conditions.[1] In the face of such criticism, our model interprets the Fed's actions as a consequence of its high interest rate policy, which necessitated the adoption of a bailout program such as the BTFP, under which loans are extended to all illiquid banks against collateralizable securities valued above their market price. Indeed, had the Fed marked to market the assets pledged as collateral by the borrowing banks, these institutions would have been unable to meet their liquidity needs either by borrowing from the central bank or by selling their securities at prevailing market prices. The resulting liquidity shortfall would have prevented these banks from meeting their immediate obligations, thus exposing them to a substantial risk of default. Ultimately, by choosing to counter inflationary pressures through a sharp hike in the policy interest rate, the Federal Reserve has constrained the tools available to preserve financial stability and has increased the costs of its rescue interventions. Our model suggests that, had the rate hike been less pronounced, the Fed might have supported financial stability by injecting liquidity into the banking system, rather than acting as a lender of last resort and by allowing illiquid banks to be disciplined by market forces. However, this approach would have entailed a weaker response to the inflationary shock.

Our study relates to two relevant strands of literature. First, we contribute to the literature on the conflicts between price and financial stability and the proper weight of financial stability in the central bank's mandate (Ferguson, 2002). A commonly accepted point of view is that if central banks are charged with financial and price stability duties, there is "*the risk of financial dominance, i.e., the risk that financial stability considerations undermine the credibility of the central bank's price stability*

[1] Precisely, in Buiter (2023)'s words, "*[...] this prudential response was not optimal, because the new Bank Term Funding Program created by the Fed, which offers one-year loans to banks with the collateral valued at par, should have been made available only on penalty terms. With market value well below par for many eligible debt instruments, the lender of last resort has become the lender of first resort – offering materially subsidized loans.*"

mandate" (Smets, 2018, p. 267). Therefore, financial and price stability duties should be separated, and monetary authorities should consider financial stability only insofar as it affects price stability (Bernanke and Gertler, 2000; Bernanke, 2012). However, the global financial crisis demonstrated that keeping the inflation rate under control is insufficient to ensure financial stability (Rajan, 2006; Stiglitz, 2010; Bernanke, 2012), and, consequently, that central banks need to be endowed with powers and responsibilities directly aimed at safeguarding financial stability (Borio, 2014). The underlying idea is that "*a central bank can significantly reduce the incidence of financial crises in the medium term by tolerating higher price volatility in the short–term*" (Boissay *et al.*, 2024, p. 5). The objectives of financial and price stability should not be considered in isolation, given the role of banks in money creation and the effects of interest rates on output fluctuations and refinancing costs of distressed banks (Schwartz, 1998; Bordo and Wheelock, 1998; Mishkin, 2009; Fahri and Tirole, 2012; Brunnermeier and Sannikov, 2014; Boissay *et al.*, 2024). Rather, close coordination between monetary policy and prudential regulation is necessary to limit the impact of policy rate changes on banks' default risk (Rajan, 2006; Paries *et al.*, 2011; Whelan, 2013; Cecchetti and Kohler, 2014). We contribute to this literature by introducing a novel rationale for why it may be optimal for the central bank to follow an interest rate rule that allows deviations from the inflation-target interest rate, referring to the potential costs arising from bailout interventions in favor of distressed banks, rather than to the risks of increasing the fragility of the banking system.

Another related strand of literature examines the effects of bank rescue policies on bank incentives and investment behavior. The traditional approach focuses on lender-of-last-resort interventions, emphasizing the distorting effects on banks' risk-taking (Goodfriend and King, 1988; Freixas, 1999; Calomiris and Haber, 2015). More recently, new resolution mechanisms for distressed banks have been examined. In particular, a policy of intervention through lending to liquid banks, similar to the one considered in this article, has been analyzed by Acharya and Yorulmazer (2007) and Acharya and Yorulmazer (2008). Their starting point is the "too-many-to-fail" problem: banks, anticipating that the regulator will find it efficient to bail out troubled institutions when the number of failures is high, have an incentive to invest in common assets to increase the probability of joint distress, but in this way making the banking system more fragile. To resolve the conflict between crisis prevention and resolution, central banks can credibly commit to subsidizing surviving banks that have not invested in common assets. However, while assisting surviving banks in acquiring distressed banks creates an incentive for each bank to differentiate itself from others (Perotti and Suarez, 2002), it also induces banks to hold excessive liquidity during crises and insufficient liquidity during booms (Acharya *et al.*, 2011).[2] We contribute to this literature by explicitly analyzing the links between resolution mechanisms and monetary policy, and how they mutually influence each other.

The remainder of the paper is organized as follows. In Section 2, we present the model setup. In Sections 3 and 4, we characterize the equilibrium under an inflation-targeting mandate and a dual mandate, respectively. In Section 5, we compare the two mandates and analyze the conditions under which the dual mandate leads to welfare losses that are smaller or larger than those under inflation targeting. All proofs and extensions are reported in Appendix A and Appendix B, respectively.

2 Model set-up

The economy lasts for three periods, initial ($t = 0$), interim ($t = 1$) and final ($t = 2$), and consists of a continuum of anonymous and atomless banks indexed by $j \in [0, 1]$, with total mass 1, and a central bank.

2.1 Banks

In the initial period, banks collect one unit of money in the form of demand deposits d and equities $e = 1 - d$ and invest it in financial assets. There are two investment opportunities: (i) liquid reserves, i.e., short-term securities that are safely remunerated in each period at the policy interest rate i_t and can be immediately liquidated at par, and (ii) illiquid long-term assets that yield with certainty $R > 1$ units of money in the final period ($t = 2$). Following Acharya (2009), we assume that each bank can invest in only one of the two types of assets, but not both. In other words, banks have two possible actions: choosing to hold a portfolio of illiquid long-term assets $P_{\mathcal{A}}$ or a portfolio of short-term liquid securities $P_{\mathcal{R}}$. Each bank j allocates its money to $P_{\mathcal{A}}$ or $P_{\mathcal{R}}$, depending on whether the expected payoff from long-term assets exceeds or falls short of that from short-term securities, that is, $u_j(P_{\mathcal{A}}) - u_j(P_{\mathcal{R}}) = \Delta_j \gtreqless 0$. For brevity, we refer to banks that opt for long-term assets as *illiquid banks*, and to those that opt for short-term securities as *liquid banks*. Therefore, letting $\mathbb{1}_{\Delta_j > 0}$ denote

[2] Other studies have considered bail-in policies consisting of a debt-equity swap imposed by the regulator. The use of bail-in resolution mechanisms allows banks to be recapitalized by imposing losses on a small fraction of unsecured bank debt holders, thereby avoiding the burden on taxpayers from distortionary taxes, and eliminates the distortions introduced by bailout expectations. However, bail-ins increase the cost of debt for banks and introduce time-inconsistency problems for both the regulator and the banks, thus once again distorting banks' risk-taking behavior (Chari and Kehoe, 2016; Walther and White, 2020; Pandolfi, 2022).

an indicator variable that takes the value 1 if $\Delta_j > 0$, the share of illiquid banks (or, equivalently, the degree of illiquidity) of the banking system is given by

$$\rho = \int_0^1 \mathbb{1}_{\Delta_j > 0} \, \mathrm{d}j, \tag{1}$$

while $1 - \rho$ denotes the share of liquid banks (the degree of liquidity) of the banking system.

In the interim period, with probability ω a bank experiences a run and all its depositors withdraw deposits, while with probability $1-\omega$ there are no deposit withdrawals. Assuming that the probability of bank run is independent between banks, ω is also the fraction of banks that suffer a run at $t = 1$. Liquid banks that experience a run are able to satisfy depositors' requests by operating with the central bank and liquidating their short-term assets at nominal value. Illiquid banks experiencing a deposit run can sell long-term assets in the financial market to other banks that have not faced withdrawals and possess liquidity. If the liquidation proceeds are short of the value of demand deposits, banks are financially distressed and unable to repay depositors.

2.2 Monetary policy interest rate

The central bank operates under a binding and publicly announced mandate that defines its objectives in terms of price stability and financial stability. It pursues these goals by setting the monetary policy interest rate and designing the rescue intervention strategy for banks facing financial distress.

At $t = 0$, the policy interest rate is set at a level consistent with the inflation target, which, for simplicity, we normalize to zero: $i_0^\star = i_{0,T} = 0$. In the interim period, an inflationary shock occurs with probability π. In this case, the central bank can counteract inflationary pressures and preserve price stability by raising the interest rate to the new target level $i_\pi > 0$. With probability $1 - \pi$, pressure on the general price level does not materialize, and the target interest rate remains at zero:

$$i_{1,T} = \begin{cases} 0 & \text{with prob. } 1 - \pi \\ i_\pi & \text{with prob. } \pi \end{cases} \tag{2}$$

If the policy rate deviates from the target rate, the central bank incurs losses proportional to the absolute value of the deviation:

$$\Gamma = \gamma \big| i_t - i_{t,T} \big| \tag{3}$$

with $\gamma > 0$. In what follows, we will refer to Γ as the inflation costs.

2.3 Financial market and distressed banks

At $t = 1$, a financial market opens in which banks can trade long-term assets. Following Wagner (2011), we assume that illiquid banks can sell the portfolio of long-term assets only as a whole. Since the policy interest rate represents the opportunity cost of forgoing liquidity, the return for banks that demand long-term assets on the market, R/p, must be at least equal to the return on liquid reserves $1 + i_1$. This means that the market cannot clear at a price higher than the present value of the fundamental return of the asset, that is, $p \leq \overline{p} = R/(1+i_1)$. When aggregate liquidity is not sufficient to absorb the total supply of illiquid bank assets at $R/(1+i_1)$, cash-in-the-market pricing prevails, and the financial market clears at a fire-sales price (Allen and Gale, 1994, 1998; Wagner, 2011).

The total liquidity in the financial market is given by the total amount of short-term securities held by the fraction $1-\omega$ of the mass of liquid banks $1-\rho$ that do not suffer a run, while the long-term asset supply comes from the group illiquid banks that face deposit withdrawal $\omega\rho$. The market liquidity and the supply of assets in the interim period are, respectively:

$$L = (1-\omega)(1-\rho) \tag{4}$$

$$S = \omega\rho \tag{5}$$

If $L/S \geq R/(1+i_1)$, illiquid banks can sell their long-term assets at the fundamental value $R/(1+i_1)$, otherwise, the market clearing price is such that $L = pS$. Therefore:

$$p^\star = \min\left[\frac{R}{1+i_1}, \frac{(1-\omega)(1-\rho)}{\omega\rho}\right] \tag{6}$$

An illiquid bank experiencing a run is financially distressed if the total liquidity it can raise by selling long-term assets at the market-clearing price is lower than its total liabilities. Since illiquid banks invest all their funds in long-term assets, they are distressed if $p^\star < d$. This can happen in two scenarios. First, when the central bank policy rate is high enough, such that $i_1 > \tilde{i} = R/d - 1$. In this case, the discounted fundamental value of the long-term asset is lower than d and any illiquid banks that experience a run on deposits would be in financial distress. Second, when the market liquidity is low. In this case, even if the central bank's policy rate is below $\tilde{i}$, illiquid banks can be forced to liquidate long-term assets at a fire sale price below d, at which they might not be able to absorb a potential bank run. From equations (4) and (5), it is straightforward to verify that this second scenario arises when $\rho > \overline{\rho} = [1-\omega]/[1-\omega(1-d)]$, that is, when the overall illiquidity (liquidity) of the banking system is sufficiently high (low).

2.4 Rescue interventions

In the absence of emergency liquidity injections by the central bank, commercial banks experiencing financial distress will have no choice but to default. We assume that a bank default generates prohibitive (not explicitly modeled) costs for the economy as a whole, such that central bank interventions aimed at preventing the default of distressed banks are always welfare-enhancing.

When the policy interest rate i_1 remains below the threshold $\tilde{i}$, but the liquidity available in the banking system is also low, the central bank can intervene in two non-mutually exclusive ways to support the price of long-term assets at $p^\star = d$ and avoid bank defaults. First, it can lend d units of money to a fraction q of the number of distressed banks $\omega\rho$ at a zero interest rate. In this way, the supply of long-term assets on the market decreases to $\widetilde{S} = S(1-q)$, allowing the remaining fraction $1-q$ of banks experiencing a run to sell their assets on the financial market at a price sufficiently high to repay depositors. Second, the central bank can lend an amount of liquidity Θ at a zero interest rate to all or some banks not affected by a run, thus increasing aggregate liquidity to $\widetilde{L} = L + \Theta$ and maintaining the asset price above the survival threshold. Any combination $\mathcal{B} = (\Theta, q)$ such that $\widetilde{L} = d\widetilde{S}$ allows the supply of long-term assets in the financial market to be absorbed by the available liquidity at a market price equal to d.

When $i_1 > \tilde{i}$, the fundamental value of the long-term assets of liquid banks in the interim period is lower than d. In this scenario, solvent banks that are not affected by runs do not find it optimal to acquire long-term securities, irrespective of the amount of liquidity the central bank may be willing to extend. The only way to prevent the default of distressed banks is for the central bank to act as a lender of last resort by lending the necessary liquidity to all illiquid banks that experience a run $d\omega\rho$.

We assume that the injection of liquidity by the central bank during the interim period generates social costs (hereafter, rescue costs) that depend on the total amount of liquidity injected, regardless of the type of rescue intervention adopted.[3] In particular, for simplicity, we will assume that rescue

[3]Rescue costs can be interpreted as the shadow price of the additional liquidity injected into the banking system to prevent the failure of illiquid banks in financial distress, and they encompass various types of costs for the central bank and the economy, such as the operational expenses associated with initiating and managing

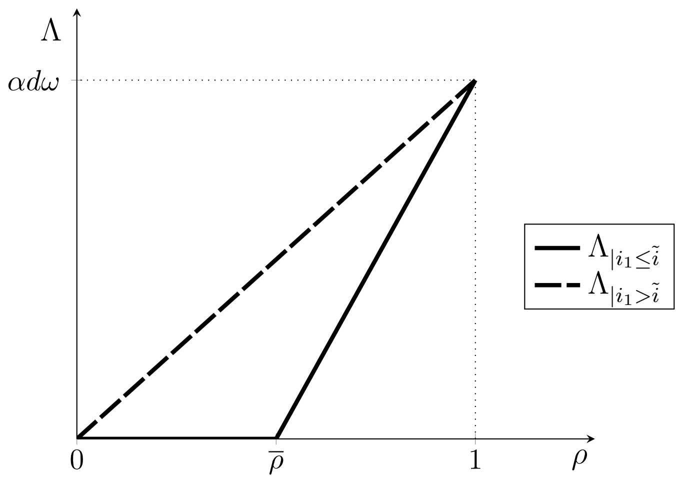
costs increase linearly with liquidity injected by the central bank $\Lambda = \alpha(dq\omega\rho + \Theta)$, where $\alpha > 0$ is the unitary cost of emergency liquidity. By condition $\tilde{L} = d\tilde{S}$, it follows that the amount of liquidity injected through the financial market when $i_1 < \tilde{i}$ is inversely related to the amount of liquidity injected through lender of last resort interventions, and precisely that $\Theta = d\omega\rho(1-q) - (1-\omega)(1-\rho)$. By substituting this expression into Λ, we have that rescue costs are:

$$\Lambda = \begin{cases} \max\big[0, \lambda(\rho - \overline{\rho})\big] & \text{if } i_1^\star \leq \tilde{i} \\ \alpha d\omega\rho & \text{if } i_1^\star > \tilde{i} \end{cases} \tag{7}$$

where $\lambda = \alpha\big[1 - \omega(1-d)\big]$.

Figure 1 illustrates the rescue costs borne by the central bank. It is interesting to note that rescue

Figure 1: Rescue costs



costs increase with the share of illiquid banks in the banking system, regardless of the specific design of the rescue intervention (that is, irrespective of the particular combination of Θ and q chosen by the central bank). However, for any $\rho < 1$, rescue costs are strictly higher when the policy interest rate exceeds $\tilde{i}$ (the dashed line) than when it lies below that threshold (the solid line). This occurs because, when $i_1 < \tilde{i}$, the central bank can rely, at least partially, on the liquidity of other banks available to purchase the assets of distressed institutions and, as a result, needs to inject a smaller amount of additional liquidity into the market.

From now on, to make the trade-off between price-level stability and financial stability relevant, we assume that the inflationary shock, when it occurs, is sufficiently strong to require the central bank

lending facilities, the fiscal costs arising from potential distortionary taxation, the credit risk, and the reputation costs for the central bank (Lucas, 2019; Fahri and Tirole, 2023).

to set an interest rate above $\tilde{i}$ to counteract it.

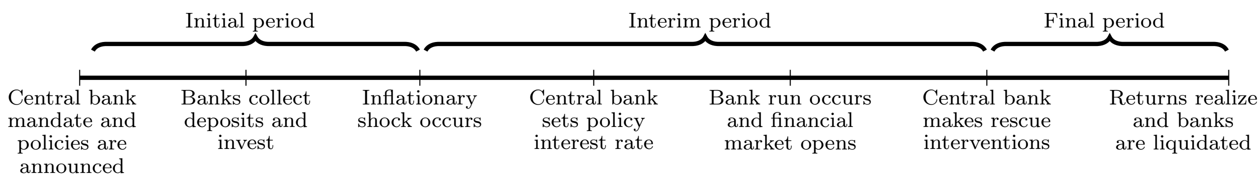
Assumption 1. $i_\pi > \tilde{i}$.

In this way, in the presence of an inflationary shock, a trade-off arises between inflation costs and rescue costs. If the central bank adjusts the policy interest rate at the inflation target i_π the rescue cost function is the dashed line in figure 1, but inflation costs are zero. Otherwise, if the central bank wants to reduce rescue costs along the solid line in the figure 1 it must keep the policy rate below $\tilde{i}$ and bear inflation costs.

2.5 Model timeline and equilibrium

Figure 2 summarizes the sequence of model events. During the initial period, after that the mandate of the central bank is set and the monetary and rescue policies are announced, banks collect deposits and allocate them to one of the two investment opportunities, either liquid short-term securities or illiquid long-term assets. At the beginning of the interim period, a random inflationary shock occurs and the central bank sets the policy interest rate i_1 that will prevail between the interim and final periods. Then, possible bank runs materialize and, if necessary, the central bank implements rescue interventions. In the final period, banks are liquidated and the realized value is distributed to the depositors and equity holders for consumption.

Figure 2: Model timeline



In the next section, we derive the time-consistent Nash equilibrium under two possible mandates for the central bank. The single inflation-targeting mandate, or $\mathcal{I}$-mandate, which establishes that the central bank prioritizes inflation costs over financial stability and rescue costs. In this case, the equilibrium is characterized by the triplet $\mathcal{I}^\star = \{i^\star_\mathcal{I}, \mathcal{B}^\star_\mathcal{I}, \rho^\star_\mathcal{I}\}$ for which:

E1. each bank maximizes its expected payoff, conditional on anticipated monetary and rescue policies and taking the investment choices of all other banks as given (i.e., treating the share of liquid and illiquid banks in the system as fixed, assuming its individual decision has no influence on aggregate market liquidity);

E2. the central bank sets the policy rate to ensure price stability;

E3. the central bank determines the rescue policy with the aim of minimizing rescue costs conditional on the optimal policy rate;

E4. banks' expectations are confirmed in equilibrium.

Under the second mandate, referred to as the dual mandate or $\mathcal{D}$-mandate, the central bank is tasked with ensuring both price stability and banking system stability. In this setting, equilibrium is characterized by the triplet $\mathcal{D}^{\star} = \{i^{\star}_{\mathcal{D}}, \mathcal{B}^{\star}\mathcal{D}, \rho^{\star}_{\mathcal{D}}\}$, where, since optimal monetary policy influences and is influenced by the degree of illiquidity in the banking system, equilibrium conditions **E2**-**E3** and condition **E4** are replaced by the following:

E2′. the central bank sets the policy rate and the rescue policy to minimize a weighted sum of inflation and rescue costs;

E3′. banks' expectations are confirmed in equilibrium and consistent with the central bank's policy rate.

3 Inflation-targeting ($\mathcal{I}$-)mandate

When operating under an inflation-targeting mandate, the central bank is required to maintain inflation at its target, and the announced optimal monetary policy rule stipulates that the policy rate coincide with the target rate so that the inflation costs in equation (3) are equal to zero:

$$i^{\star}_{1\mathcal{I}} = \begin{cases} 0 & \text{if } i_{1T} = 0 \\ i_{\pi} & \text{if } i_{1T} = i_{\pi}. \end{cases} \tag{8}$$

Accordingly, if an inflationary shock materializes during the interim period, the policy interest rate is set at $i^{\star}_{1,\mathcal{I}} = i_{\pi}$. In this case, given Assumption 1, the only rescue policy for the central bank is to act as the lender of last resort by lending to all illiquid banks experiencing a bank run. In the absence of an inflationary shock, the policy interest rate remains unchanged at its initial level, normalized to zero. In this case, when the share of banks investing in illiquid assets is less than $\overline{\rho}$, liquid banks will be able to absorb the long-term assets of illiquid banks hit by a bank run at a price equal to or greater than d, and the central bank will not need to intervene to inject additional liquidity into the banking system. When $\rho > \overline{\rho}$, aggregate liquidity in the banking system is insufficient to ensure the liquidation of long-term assets at a price at least equal to d, which is required to prevent the failure of distressed illiquid banks. Under these circumstances, the central bank is *ex post* indifferent between all possible rescue policy designs such that $\tilde{L} = d\tilde{S}$. All of these policies ensure that the market price of long-term assets is fixed at d and entail the same level of rescue costs, represented by the point along the solid line in Figure 1 corresponding to the equilibrium share of illiquid banks on the x axis

Banks know the central bank's mandate and rationally anticipate its optimal monetary and rescue policies in the different possible states of the world. Therefore, they understand that, under the inflation-targeting rule (8), the central bank, if faced with a wave of inflation, will have no choice but to act as a lender of last resort, supplying liquidity directly to banks in financial distress. Likewise, banks are aware that in periods when the economy is not subject to inflationary pressures, the central bank will refrain from intervening to rescue banks if market liquidity $1-\rho$ exceeds $1-\overline{\rho}$, or will find it indifferent to supply emergency liquidity by lending to solvent banks or to illiquid distressed banks, as long as the market price of long-term assets is maintained at d. Hence, banks must formulate conjectures about the specific policy mix that the central bank will adopt to support banks during the phases of monetary stability when $i^{\star}_{1,T} = 0$. In summary, bank j's expectations about rescue interventions under the $\mathcal{I}$-mandate are as follows:

$$\mathcal{B}^{e}_{\mathcal{I}j} = \begin{cases} (0,0) & \text{if } \; i_{1T} = 0 \; \text{ and } \; \rho \leq \overline{\rho} \\ (d\omega\rho(1-q^{e}_{j}) - (1-\omega)(1-\rho), q^{e}_{j}) & \text{if } \; i_{1T} = 0 \; \text{ and } \; \rho > \overline{\rho} \\ (0,1) & \text{if } \; i_{1T} = i_{\pi} \end{cases} \tag{9}$$

where q^{e}_{j} denotes the expectation of banks regarding the fraction of distressed banks that will receive a capital injection from the central bank during the interim period

Each bank treats ρ as parametrically given, recognizing that its individual investment decision does not affect the investment choices of other banks and has no impact on aggregate market liquidity. Therefore, based on expectations regarding the policy rate (8), the central bank's rescue interventions in the event of a bank run (9), and the overall degree of illiquidity in the banking system, banks decide their investments by comparing the expected payoff from holding an illiquid portfolio composed of long-term assets with that from holding a liquid portfolio composed of short-term securities, $u_j(P_{\mathcal{A}}, \mathcal{B}^{e}_{j}, i^{e}_{t,T}, \rho)$ and $u_j(P_{\mathcal{R}}, \mathcal{B}^{e}_{j}, i^{e}_{t,T}, \rho)$. From now on, for simplicity of notation, we denote the expected payoffs by $\mathcal{A}_{\mathcal{I}j}$ and $\mathcal{R}_{\mathcal{I}j}$, respectively. Given anonymity in beliefs and payoff functions, we can henceforth omit the subscript j for simplicity and without risk of ambiguity.

Since the optimal rescue policy depends on whether the share of illiquid banks in the economy is above or below the threshold $\overline{\rho}$, the expected payoffs of banks also vary according to whether $\rho \lesseqgtr \overline{\rho}$. When $\rho \leq \overline{\rho}$, that is, when bank j assumes that the banking system is sufficiently liquid, the expected payoff functions from investing in long-term and short-term assets are given by:

$$\mathcal{A}_{\mathcal{I}}^{-} = \omega\left[\pi R + (1-\pi)p^{\star}(\rho)\right] + (1-\omega)R \tag{10}$$

$$\mathcal{R}_{\mathcal{I}}^{-} = \omega + (1-\omega)\left[\pi(1+i_{\pi}) + (1-\pi)\frac{R}{p^{\star}(\rho)}\right] \tag{11}$$

where the superscript $-$ indicates that ρ is below the threshold $\overline{\rho}$.

With probability ω an illiquid bank suffers a run and its value depends on whether the economy as a whole is experiencing an inflationary wave or not. In the first case, with probability π, the illiquid bank is rescued by the central bank by receiving a loan equal to d that allows it to pay the depositors and not liquidate the long-term assets that yield R in the final period; with probability $1-\pi$ there is no inflationary pressure in the economy and the bank can sell the long-term assets on the market at a price $p^{\star}$ which depends on the share of banks expected to have invested in liquid reserves. In the first case, with probability π, the illiquid bank is rescued by the central bank through a direct loan of amount d, enabling it to repay depositors without liquidating its long-term assets, which yield R in the final period; with probability $1-\pi$ no inflationary pressures materialize, and the bank can sell its long-term assets on the market at a price $p^{\star}$, which depends on the share of banks expected to have invested in liquid reserves. With probability $1-\omega$, the bank does not face deposits withdrawals and earns R on its long-term assets in the final period. If bank j invests in liquid reserves, the returns are equal to 1 if a bank run occurs with probability ω. In contrast, if a bank run does not occur, the returns are equal to $1+i_{\pi}$ during an inflationary wave, and to the expected return from the purchase of long-term assets in the financial market, $R/p^{\star}$, if there are no inflationary pressures.

When $\rho > \overline{\rho}$, that is, when a great proportion of banks is assumed to be illiquid, the expected payoff functions are as follows:

$$\mathcal{A}_{\mathcal{I}}^{+} = \omega\left[\pi R + (1-\pi)(q^{e}R + (1-q^{e})d)\right] + (1-\omega)\left[R + (1-\pi)\left(\frac{R}{d} - 1\right)\theta_{\rho}^{e}\right] \tag{12}$$

$$\mathcal{R}_{\mathcal{I}}^{+} = \omega + (1-\omega)\left[\pi(1+i_{\pi}) + (1-\pi)\frac{R}{d} + (1-\pi)\left(\frac{R}{d} - 1\right)\theta_{1-\rho}^{e}\right] \tag{13}$$

where the superscript $+$ indicates that ρ is above the threshold $\overline{\rho}$, while θ_{ρ}^{e} and $\theta_{1-\rho}^{e}$ denote the amount of loans that each bank j expects to receive from the central bank in the case it holds illiquid or liquid assets.

For the bank that chooses to invest in long-term assets, the expected payoff in (12) differs in two respects from those reported in equation (10). First, when the bank experiences a run and the economy is in a period without inflationary pressures, it can benefit from central bank lending only with probability q^e, whereas with probability $1 - q^e$ it will sell its long-term assets to non-distressed banks at a price d, supported by the liquidity provided to the latter by the central bank. Second, when, with probability $(1 - \omega)(1 - \pi)$, the bank does not face deposit withdrawals and no inflationary pressures materialize, it can use the liquidity borrowed from the central bank, θ^e_ρ, to purchase the asset from other banks at a price d, thus earning an additional return equal to $(R/d - 1)\theta^e_\rho$. If bank j chooses to hold a portfolio of short-term liquid securities, it will always be able to meet any withdrawals of deposits without experiencing financial distress, regardless of the degree of illiquidity in the banking system. Therefore, with probability ω, their payoff remains equal to 1. However, when $\rho > \overline{\rho}$, liquid banks rationally anticipate that the central bank will be compelled to intervene and support illiquid financial institutions experiencing distress, even in the absence of inflationary pressures. In this case, the expected payoff from investing in reserves includes the possibility that, with probability $(1-\omega)(1-\pi)$, the central bank will offer liquid banks the opportunity to borrow resources in the amount of $\theta^e_{1-\rho}$ to purchase long-term assets from distressed banks, thus earning a return of $(R/d - 1)\theta^e_{1-\rho}$.

To rule out trivial cases, we assume that the return on long-term assets exceeds the expected returns on short-term reserves when the latter are remunerated at the policy rate consistent with the inflation-targeting rule. Otherwise, investment in reserves would always dominate investment in long-term assets, and the only possible equilibrium distribution of banks between the two investment opportunities would lead to $\rho = 0$.

Assumption 2. $R - \pi(1 + i_\pi) - (1 - \pi) > 0$.

By comparing equations (12) and (13), it is straightforward to verify that investing in long-term assets becomes increasingly profitable for banks as the values of q^e and θ^e_ρ rise and the value of $\theta^e_{1-\rho}$ decreases. In other words, the wider the access to central bank credit available to illiquid banks, whether or not they are experiencing financial distress, and the more restricted it is for liquid banks, the greater the incentive for banks to invest in long-term assets rather than in liquid securities. From (7), rescue costs increase with the number of illiquid banks in the economy, while they are unaffected by the mix of rescue interventions and how the injected liquidity is distributed among banks. Therefore, when $\rho > \overline{\rho}$, the optimal design of the central bank's rescue policy is to fully abstain from lender-of-last-resort interventions and to restrict access to credit facilities to liquid banks only, that is, to set $q^\star = 0$ and $\theta^\star_\rho = 0$. Since this policy design is time-consistent, each bank anticipates it correctly and forms their expectations consistently as follows:

$$q^e = 0; \qquad \theta^e_\rho = 0; \qquad \theta^e_{1-\rho} = \frac{\Theta}{(1-\omega)(1-\rho)} = \frac{d\omega\rho - (1-\omega)(1-\rho)}{(1-\omega)(1-\rho)}, \tag{14}$$

where we assume that the $(1-\omega)(1-\rho)$ banks holding liquid assets and accessing central bank lending receive an equal share of the total amount Θ of emergency liquidity injected into the market. Thus, substituting (14) into (12), we have that, for any $\rho > \overline{\rho}$, the expected payoff from investing in long-term assets is:

$$\mathcal{A}^+_{\mathcal{I}} = \omega\big[\pi R + (1-\pi)d\big] + (1-\omega)R = \mathcal{A}^-_{\mathcal{I}}(\overline{\rho}) \tag{12$'$}$$

where $\mathcal{A}^-_{\mathcal{I}}(\overline{\rho})$ is the value of the payoff function (10) when $\rho = \overline{\rho}$ and $p^\star = d$. Similarly, the payoff from investing in reserves is:

$$\mathcal{R}^+_{\mathcal{I}} = \mathcal{R}^-_{\mathcal{I}}(\overline{\rho}) + (1-\omega)(1-\pi)\left(\frac{R}{d} - 1\right)\left[\frac{\omega d\rho}{(1-\omega)(1-\rho)} - 1\right] \tag{13$'$}$$

where, once again, $\mathcal{R}^-_{\mathcal{I}}(\overline{\rho})$ corresponds to the payoff derived from the function in (11) at $\rho = \overline{\rho}$ and $p^\star = d$.

Let $\Delta^-_{\mathcal{I}}$ and $\Delta^+_{\mathcal{I}}$ denote the difference between the expected payoffs from investing in long-term assets and liquid securities when the central bank is subject to an inflation-targeting mandate and the overall state of illiquidity in the banking system is, respectively, lower or higher than $\overline{\rho}$:

$$\Delta_{\mathcal{I}} = \begin{cases} \Delta^-_{\mathcal{I}} = \mathcal{A}^-_{\mathcal{I}} - \mathcal{R}^-_{\mathcal{I}} & \text{if } \rho \leq \overline{\rho} \\ \Delta^+_{\mathcal{I}} = \mathcal{A}^+_{\mathcal{I}} - \mathcal{R}^+_{\mathcal{I}} & \text{if } \rho > \overline{\rho} \end{cases} \tag{15}$$

Lemma 1. *$\Delta_{\mathcal{I}}$ is continuous and strictly decreasing in $\rho \in [0,1]$. Moreover, there exists a unique value $\rho^-_{\mathcal{I}} \in (0, \overline{\rho}]$ or $\rho^+_{\mathcal{I}} \in (\overline{\rho}, 1)$ such that $\Delta_{\mathcal{I}}$ is equal to zero.*

Proof. See Appendix A1. □

Intuition is straightforward. When $\rho = 0$, Assumption 2 ensures that long-term assets are more profitable than liquid assets and that $\Delta_{\mathcal{I}} > 0$. As the overall illiquidity of the banking system increases, the prices at which a bank j expects to sell its long-term assets, if it chooses to invest in them, decline. Therefore, as ρ increases, the expected payoff for the bank from long-term investments decreases, while the payoff from investing in liquid securities increases. Moreover, when $\rho > \overline{\rho}$, banks anticipate that, in the absence of inflationary pressures in the economy, any rescue interventions by the central bank will take the form of liquidity injections into the market through subsidized loans extended to liquid banks. In these cases, the amount of liquidity that the central bank must provide to liquid banks in order to maintain the market price of long-term assets at a level that enables illiquid banks to meet withdrawal demands increases with the share of illiquid banks. This raises the expected payoff from investing in liquid securities, while leaving unchanged the return from investing in long-term assets, ultimately driving $\Delta_{\mathcal{I}}$ into negative territory.

Given that banks are anonymous, the time-consistent Nash equilibrium can be fully characterized by the optimal monetary and rescue policies when the central bank operates under a single inflation-targeting mandate, and the share of illiquid banks consistent with these policies and with banks maximizing their expected returns.

Proposition 1. Under $\mathcal{I}$-mandate, the Nash equilibrium is unique and is as follows:

(i) if $i_\pi \geq \max(\tilde{i}, \iota)$, the equilibrium triplet is

$$\mathcal{I}_1^\star = \begin{cases} i_{\mathcal{I}}^\star = i_T \\ \mathcal{B}_{\mathcal{I}}^\star = \begin{cases} (0,0) & \text{if } i_T = 0 \\ (0,1) & \text{if } i_T = i_\pi \end{cases} \\ \rho_{\mathcal{I}}^\star = \rho_{\mathcal{I}}^- \in \ (0, \overline{\rho}] \ : \Delta_{\mathcal{I}}(\rho_{\mathcal{I}}^-) = 0 \end{cases} \tag{16}$$

(ii) if $i_\pi \in (\tilde{i}, \max(\tilde{i}, \iota))$, the optimal central bank's policy rate and rescue policy and the equilibrium share of illiquid banks are

$$\mathcal{I}_2^\star = \begin{cases} i_{\mathcal{I}}^\star = i_T \\ \mathcal{B}_{\mathcal{I}}^\star = \begin{cases} (d\omega\rho_{\mathcal{I}}^\star - (1-\omega)(1-\rho_{\mathcal{I}}^\star), 0) & \text{if } i_T = 0 \\ (0,1) & \text{if } i_T = i_\pi \end{cases} \\ \rho_{\mathcal{I}}^\star = \rho_{\mathcal{I}}^+ \in \ (\overline{\rho}, 1) \ : \Delta_{\mathcal{I}}(\rho_{\mathcal{I}}^+) = 0 \end{cases} \tag{17}$$

where $\iota = \tilde{i}\left[\frac{1}{\overline{\rho}} - \frac{(1-d)}{\pi}\right] - \frac{(1-d)}{\pi(1-\omega)} \gtrless \tilde{i}$ if and only if $\pi \gtrless \frac{1-d}{\omega}\left(\frac{1-\omega}{d} + \frac{1}{R-d}\right)$. Both equilibria $\mathcal{I}_1^\star$ and $\mathcal{I}_2^\star$ are globally stable.

Proof. See Appendix A2. □

The prevailing equilibrium depends on the relative magnitude of the inflation risk and the risk of liquidity shock. If the former is sufficiently low and the latter sufficiently high, so that $\iota < \tilde{i}$, then the resulting equilibrium corresponds to $\mathcal{I}_1^{\star}$. In this scenario, inflation exerts a disciplinary effect that indirectly helps contain the expected rescue costs borne by the central bank. The probability that the central bank, in response to inflationary pressures in the economy, will be forced to raise the policy interest rate increases the expected return on liquid securities, while the relatively high likelihood of encountering liquidity problems reduces the expected benefits of investing in long-term, illiquid assets. The combined effect of these two mechanisms is to push aggregate liquidity in the banking system to the point that, in equilibrium, $1 - \rho_{\mathcal{I}}^{-} > 1 - \overline{\rho}$. As a result, when inflationary pressures are absent, the central bank can refrain from injecting additional liquidity into financial markets to support illiquid banks affected by deposit shocks, thus incurring zero rescue costs.

If, instead, the probability of the economy experiencing inflationary surges is high, while the risk of banks facing sudden deposit withdrawals is low, the resulting equilibrium depends on the severity of the inflationary shock and the responsiveness of the economy and inflation to policy rates; in other words, the equilibrium is determined by the magnitude of the policy rate adjustment required for the central bank to achieve its inflation target. When the central bank is forced to substantially raise interest rates, bringing them to $i_\pi > \iota$, the share of illiquid banks remains $\rho_{\mathcal{I}}^{-} < \overline{\rho}$, as does the optimal strategy for rescuing distressed banks, which corresponds to that specified in $\mathcal{I}_1^{\star}$.

Instead, when the central bank can maintain control over inflation through moderately tight adjustments of the policy interest rate (that is, when $i_\pi < \iota$), investing in liquid assets becomes relatively less attractive to banks compared with long-term assets. This is due to both the low expected return on liquid assets and the high probability that the central bank will intervene as a lender of last resort to support illiquid distressed banks, thus reducing the opportunities to acquire long-term assets on the secondary market at a discount. This leads to an increase in the share of illiquid banks that, in equilibrium, exceeds $\overline{\rho}$, along with a deterioration in the general liquidity of the banking system. As a consequence, the system becomes unable to support the market value of long-term assets in the event of forced liquidation triggered by deposit withdrawals. In this second scenario, even in the absence of inflationary pressures, illiquid banks that experience a deposit shock fall into financial distress, thereby compelling the central bank to intervene with additional liquidity injections and to bear the corresponding rescue costs. Therefore, in order to reduce incentives to invest in illiquid assets and to minimize rescue costs, the central bank's intervention is structured to allow liquid banks to earn excess returns by using central bank funding to purchase long-term assets of financially distressed banks at a discounted minimum price equal to d.

In summary, irrespective of the policy interest rate consistent with keeping inflation at its target, the equilibrium that arises under the $\mathcal{I}$-mandate is characterized by zero inflation costs and positive expected rescue costs, which rise with the share of illiquid banks in the economy. For this reason, it is important to examine how liquidity and inflation risks affect the overall degree of illiquidity in the banking system and, consequently, the expected cost of potential bank rescues.

Proposition 2. Under $\mathcal{I}$-mandate, the equilibrium share of banks investing in the long-term asset $\rho^\star_{\mathcal{I}}$: (i) strictly decreases with i_π; (ii) strictly decreases with ω; (iii) increases (decreases) with π if $\rho^\star_{\mathcal{I}}$ is greater (less) than $1-\omega$.

Proof. See Appendix A3 □

The comparative statics results in Proposition 2 merely confirm the underlying economic intuition driving the equilibrium determination. An increase in the likelihood of facing deposit withdrawals and liquidity shortages discourages banks from investing in long-term illiquid assets, while expanding the opportunities for liquid banks to acquire long-term assets from distressed banks forced to liquidate them at a discount. The same effect arises when the central bank needs to implement a more contractionary monetary policy to counteract upward pressures on the price level. In fact, a higher policy rate i_π does not have effects on central bank rescue policies but increases the return on liquid assets, thus reducing the equilibrium share of banks that find it optimal to invest in long-term assets.

In contrast, an increase in the probability that the economy enters a phase of inflationary pressures has mixed effects on the benefits for banks of maintaining a liquid position. On the one hand, this makes investing in liquid securities more attractive, as banks are more likely to benefit from the high returns at i_π induced by the central bank's anti-inflationary response. On the other hand, an increase in π increases the likelihood that the central bank will be forced to intervene as a lender of last resort in support of illiquid banks facing a run. As a result, opportunities for liquid banks to gain from purchasing long-term assets from banks facing liquidity shortages on the secondary market become less frequent. If the overall liquidity in the banking system is such that, absent inflation and central bank lender-of-last-resort interventions, the market price of long-term assets is sufficiently high (that is, when $\rho^\star_{\mathcal{I}} > 1-\omega$), the second effect dominates the first. Consequently, in equilibrium, a larger number of banks find it more profitable to maintain an illiquid portfolio of long-term assets. Otherwise, if $\rho^{\star}_{\mathcal{I}} < 1 - \omega$, an increase in π leads to a further reduction in the equilibrium share of illiquid banks.

4 Dual ($\mathcal{D}$-)mandate

In a dual mandate, the central bank's objective is to minimize a weighted sum of inflation and rescue costs:

$$\mathcal{C} = \beta\Lambda + \Gamma \tag{18}$$

where $\beta \geq 0$ is the relative importance of financial stability versus price stability.

As Λ is a step function that shifts upwards when the policy rate is above $\tilde{i}$, in the absence of inflationary pressures, price stability and financial stability are not in conflict, and the central bank has no reason to set a policy rate different from the target rate. However, when the central bank faces an inflationary shock, a trade-off between inflation costs and rescue costs emerges, and the policy interest rate that minimizes the loss function (18) may be different from the interest rate implied by the inflation-targeting rule.

During periods of inflationary pressure, if the policy rate is fixed at its target value i_π, the only intervention tool available to the central bank to rescue financially distressed illiquid banks is to act as a lender of last resort for all of them, thus incurring a total loss equal to:

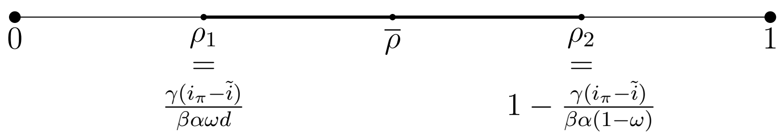
$$\mathcal{C}(i_\pi) = \beta\alpha d\omega\rho \tag{19}$$

Suppose, instead, that the central bank chooses to deviate from the inflation target. In this case, the policy interest rate is set at $\tilde{i}$. In fact, any policy rate lower than $\tilde{i}$ would increase inflation costs without affecting rescue costs, while rates higher than $\tilde{i}$ but different from the target i_π would still entail positive inflation costs without generating savings in expected rescue costs. Therefore, in the event of a deviation from the inflation target, the total loss incurred by the central bank amounts to:

$$\mathcal{C}(\tilde{i}) = \beta \max\left[0, \lambda(\rho - \overline{\rho})\right] + \gamma\big(i_\pi - \tilde{i}\big) \tag{20}$$

By solving inequality $\mathcal{C}(i_\pi) > \mathcal{C}(\tilde{i})$ for ρ, we can characterize situations in which, given the general degree of illiquidity that prevails in the banking system, it is optimal for the central bank to deviate from the target rate during periods of inflationary pressure. Specifically, it is possible to identify two thresholds, $\rho_1 = \frac{\gamma(i_\pi - \tilde{i})}{\beta\alpha\omega d}$ and $\rho_2 = 1 - \frac{\gamma(i_\pi - \tilde{i})}{\beta\alpha(1-\omega)}$, such that if the illiquidity of the banking system takes values in the interval $\mathcal{P} = [\rho_1, \rho_2]$, the total losses of the central bank when the policy rate is set according to the inflation-targeting rule are greater than those incurred when $i_1^\star = \tilde{i}$ (see Figure 3).

Figure 3: The deviation set $\mathcal{P}$



Note: In bold the values of ρ for which $\mathcal{C}(i_\pi) > \mathcal{C}(\tilde{i})$ and the central bank, in the face of an inflationary shock, prefers to deviate from the inflation-target interest rate.

The economic intuition underlying this threshold pattern is straightforward. If the share of illiquid banks is very high or very low, the benefits of setting a policy rate lower than the one that minimizes inflation costs are limited. This occurs because the savings in rescue costs allowed by the deviation from the target rate (that is, the distance between the solid and dashed lines in Figure 1) remain low when the mass of banks investing in long-term assets is small or large enough. In the first case, the number of illiquid banks that could potentially require lender-of-last-resort interventions when the reference rate is set at i_π is so low that it does not justify the central bank deviating from the inflation target. On the other hand, when $\rho > \rho_2$, the number of banks requiring a central bank bailout would remain very high, even if the policy rate were kept at $\tilde{i}$, once again making the abandonment of the inflation target an unnecessarily costly option. In the other cases, where $\rho \in [\rho_1, \rho_2]$, the benefits in terms of lower rescue costs can justify the costs of deviating from the inflation-target rule.

Obviously, the existence of a non-empty deviation set $\mathcal{P}$ depends on the model parameters. In particular, deviations of the policy rate from the inflation target can be valuable if and only if the weight attached to financial stability and rescue costs in the $\mathcal{D}-$mandate is sufficiently large:[4]

$$\beta > \underline{\beta} = \frac{\gamma\big(i_\pi - \tilde{i}\big)}{\alpha\omega d\overline{\rho}} \tag{21}$$

From (21), it is straightforward to show that if there exists a set of levels of banking system illiquidity for which the central bank finds it optimal to deviate from the inflation-targeting rule, then this set necessarily includes $\overline{\rho}$:

[4]Inequality (21) follows from the necessary conditions $\rho_1 \leq 1$ and $\rho_2 \geq \rho_1$ in order for $\mathcal{P}$ to not be an empty set.

Lemma 2. *If $\mathcal{P} \neq \emptyset$, then $\overline{\rho} \in \mathcal{P}$.*

Proof. See Appendix A4 □

The rationale behind this lemma can be easily illustrated by again referring to Figure 1, which depicts the rescue costs. When the share of illiquid banks is equal to $\overline{\rho}$, the savings in rescue costs that can be achieved by setting the policy rate below the target i_π are at their maximum value. Therefore, if in this case the central bank does not consider it worth deviating from the target rate, then $\mathcal{P} = \emptyset$ and, as under the $\mathcal{I}$-mandate, setting a policy rate that minimizes the costs of inflation would always be the optimal choice, regardless of the degree of liquidity of the banking system.

Then, restricting the analysis to the case in which the set $\mathcal{P}$ exists,[5] under the $\mathcal{D}$-mandate, the optimal policy rate for the central bank is

$$i^\star_{1\mathcal{D}} = \begin{cases} 0 & \text{if } \ i_{1T} = 0 \\ \tilde{i} & \text{if } \ i_{1T} = i_\pi \ \text{ and } \ \rho \in \mathcal{P} \\ i_\pi & \text{if } \ i_{1T} = i_\pi \ \text{ and } \ \rho \notin \mathcal{P}. \end{cases} \tag{22}$$

Moving on to rescue policy, by departing from the inflation-targeting rule, the central bank expands the set of conditions under which it may refrain from injecting emergency liquidity into the banking system to rescue illiquid banks affected by a deposit shock or, if compelled to intervene, may abstain from acting as a lender of last resort in favor of illiquid banks. In particular, let $\mathcal{P}^-$ be the subset of $\mathcal{P}$ that contains values of ρ less than or equal to $\overline{\rho}$, and $\mathcal{P}^+$ be the subset of those greater than $\overline{\rho}$; similarly let $\mathcal{P}^{C^-}$ be the subset of the complement set $\mathcal{P}^C$ that contains values of ρ less than or equal to $\overline{\rho}$, and $\mathcal{P}^{C^+}$ be the subset of $\mathcal{P}^C$ including values of ρ greater than $\overline{\rho}$:

$$\begin{aligned} \mathcal{P}^- &= \big\{\rho \in \mathcal{P} : \rho \leq \overline{\rho}\big\}; \quad \mathcal{P}^{C^-} = \big\{\rho \notin \mathcal{P} : \rho \leq \overline{\rho}\big\} \\ \mathcal{P}^+ &= \big\{\rho \in \mathcal{P} : \rho > \overline{\rho}\big\}; \quad \mathcal{P}^{C^+} = \big\{\rho \notin \mathcal{P} : \rho > \overline{\rho}\big\} \end{aligned} \tag{23}$$

When the state of illiquidity of the banking system falls within the set $\mathcal{P}^-$, setting the policy interest rate at $\tilde{i}$ allows the central bank to prevent a collapse in the price of long-term assets and enhances the ability of illiquid banks to raise in the market the liquidity required to meet deposit withdrawals. On the other hand, when $\rho \in \mathcal{P}^+$, by maintaining the policy rate at x, the central bank is relieved

[5] If $\mathcal{P} = \emptyset$, the case of the mandate $\mathcal{D}$ reduces to that of the inflation-targeting mandate analyzed in Section 3.

of the obligation to act as a lender of last resort and can limit its intervention to rescuing distressed institutions, injecting into the market the emergency liquidity necessary for liquid banks to absorb long-term assets at a price d.

Since, as in the case of the inflation-targeting mandate, the central bank has an interest in making investments in long-term assets relatively less attractive compared to investments in liquid assets, a rescue policy design that limits access to credit to only liquid banks, setting $q = 0$ and $\theta_{1-\rho} = 0$, is optimal and intertemporally consistent. Assuming that banks correctly anticipate central bank's policy actions, the rescue strategy expected by each bank can therefore be described as follows:

$$\mathcal{B}^e_{\mathcal{D}} = \begin{cases} (0,0) & \text{if } i_{1T} = 0 \text{ and } \rho \leq \overline{\rho} \text{ or } i_{1T} = i_\pi \text{ and } \rho \in \mathcal{P}^- \\ (d\omega\rho - (1-\omega)(1-\rho), 0) & \text{if } i_{1T} = 0 \text{ and } \rho > \overline{\rho} \text{ or } i_{1T} = i_\pi \text{ and } \rho \in \mathcal{P}^+ \\ (0,1) & \text{if } i_{1T} = i_\pi \text{ and } \rho \notin \mathcal{P} \end{cases} \tag{24}$$

Therefore, even when the central bank operates under a dual mandate, the expected payoff of bank j still depends on whether $\rho \lesseqgtr \overline{\rho}$. If $\rho \in \mathcal{P}^-$, banks' beliefs about overall liquidity in the banking system are such that they expect the market to be able to absorb the sales of illiquid banks facing a run at a price sufficient to repay deposits, without the need for the central bank to intervene with additional liquidity injections. In this case, the expected payoffs from investing in long-term assets and short-term securities are, respectively

$$\mathcal{A}^-_{\mathcal{P}} = \omega\big[\pi d + (1-\pi)p^\star(\rho)\big] + (1-\omega)R \tag{25}$$

and

$$\mathcal{R}^-_{\mathcal{P}} = \omega + (1-\omega)\left[\pi\frac{R}{d} + (1-\pi)\frac{R}{p^\star(\rho)}\right]. \tag{26}$$

With probability ω banks investing in the long-term asset experience a run. If there are no inflationary pressures in the economy, with probability $(1-\pi)$, and the policy rate in the intermediate period is set at $i^\star_{1\mathcal{D}} = 0$, banks will be able to sell their assets on the financial market at $p^\star > d$. If inflationary pressures materialize, the central bank will set the interest rate at $\tilde{i}$; however, even at this rate, liquid banks are sufficiently numerous to allow illiquid banks to recover the funds needed to repay depositors by selling their assets at price d. Finally, with probability $1-\omega$ there are no deposit withdrawals and banks investing in long-term assets earn R.

For banks that invest in liquid assets, in the event of deposit withdrawals, with probability ω, the returns are equal to 1. However, when they do not experience a deposit shock, with probability $1-\omega$, they have the opportunity to buy long-term illiquid assets at a price of $p^\star$ or d, depending on whether the policy rate $i^\star_{1\mathcal{D}}$ is 0 or $\tilde{i}$ with probability $1-\pi$ and π, respectively, earning a return R.

When $\rho \in \mathcal{P}^+$, regardless of the state of inflationary pressure and the policy rate prevailing in the interim period, whether 0 or $\tilde{i}$, banks expect the central bank to intervene to prevent bank failures by providing additional liquidity to liquid banks, enabling them to absorb the long-term assets of distressed illiquid banks at the price d. Therefore, the expected payoff from investment in long-term assets is the weighted average of their fundamental return if held to maturity and the market price d at which they can be liquidated to meet deposit withdrawals,

$$\mathcal{A}^+_{\mathcal{P}} = \omega d + (1-\omega)R. \tag{27}$$

On the other hand, banks that invest in reserves can leverage the capital lent by the central bank and obtain a payoff

$$\mathcal{R}^+_{\mathcal{P}} = \omega + (1-\omega)\left[\frac{R}{d} + \left(\frac{R}{d} - 1\right)\frac{d\omega\rho - (1-\omega)(1-\rho)}{(1-\omega)(1-\rho)}\right] \tag{28}$$

Let $\Delta_{\mathcal{P}}$ be the difference between the expected payoffs from investing in the two types of assets when the central bank deviates from their inflation target mandate,

$$\Delta_{\mathcal{P}} = \begin{cases} \Delta^-_{\mathcal{P}} = \mathcal{A}^-_{\mathcal{P}} - \mathcal{R}^-_{\mathcal{P}} & \text{if } \rho \leq \overline{\rho} \\ \Delta^+_{\mathcal{P}} = \mathcal{A}^+_{\mathcal{P}} - \mathcal{R}^+_{\mathcal{P}} & \text{if } \rho > \overline{\rho} \end{cases} \tag{29}$$

We have the following lemma:

Lemma 3. *$\Delta_{\mathcal{P}}$ is a continuous and strictly decreasing function of $\rho \in \mathcal{P}$, with $\Delta_{\mathcal{P}}(\rho_2) < 0$ and a zero at $\rho^-_{\mathcal{P}} < \overline{\rho}$.*

Proof. See Appendix A5 □

From Lemma 3, it follows that if the function $\Delta_{\mathcal{P}}$ takes a positive value at ρ_1, then there exists a level of banking system illiquidity $\rho^-_{\mathcal{P}} < \overline{\rho}$ that constitutes a potential equilibrium in which no bank can improve its expected payoff by reallocating its portfolio from long-term assets to short-term liquid securities, or conversely from short-term to long-term assets.

When banks' expectations about the level of illiquidity in the banking system lead them to believe that the central bank will not find it optimal to deviate from its inflation target, that is, when $\rho \in \mathcal{P}^C$, the expected benefits of investing in long-term assets and liquid reserves under a dual mandate regime are the same as those obtained under the $\mathcal{I}$-mandate. Therefore, under the dual mandate, the difference between the expected payoffs from investing in long-term assets and liquid reserves when the central bank, $\Delta_{\mathcal{D}}$, coincides piecewise with $\Delta_{\mathcal{I}}$ and $\Delta_{\mathcal{P}}$; specifically,

$$\Delta_{\mathcal{D}} = \begin{cases} \Delta_{\mathcal{I}}^{-} & \text{if } \rho \in \mathcal{P}^{C^-} \\ \Delta_{\mathcal{P}} & \text{if } \rho \in \mathcal{P} \\ \Delta_{\mathcal{I}}^{+} & \text{if } \rho \in \mathcal{P}^{C^+} \end{cases} \tag{30}$$

Lemma 4. *$\Delta_{\mathcal{D}}$ is piecewise continuous in ρ, with discontinuity of first kind in ρ_1 and ρ_2. Moreover, $\Delta_{\mathcal{D}}$ is strictly decreasing on all sub-intervals of its domain.*

Proof. See Appendix A6. □

Unlike what occurs under the $\mathcal{I}$-mandate (see Lemma 1), the flexibility granted to the central bank under the dual mandate to deviate from the inflation target causes the differential between the payoffs of the two types of investment to vary discontinuously with expectations about the overall degree of illiquidity in the banking system. The reason is that the policy interest rate in the interim period depends not only on the realization of the inflationary shock, but also on the relative weight that the central bank's mandate assigns to financial stability and on the current degree of illiquidity in the banking system. These factors make a deviation from the inflation target desirable or undesirable, and this occurs discontinuously at the threshold values ρ_1 and ρ_2.

The discontinuity of the function Δ_D implies that there may be no value of ρ such that $\Delta_{\mathcal{D}} = 0$, or that banks may not find it profitable to carry out portfolio reallocation that alter the illiquidity of the banking system and induce a shift from one branch of $\Delta_{\mathcal{D}}$ to the other. On the other hand, it is possible that there exist two values, $\rho_{\mathcal{P}}^{-}$ and $\rho_{\mathcal{I}}^{-}$ (or $\rho_{\mathcal{I}}^{+}$), at which the expected payoff of holding a portfolio of long-term assets is equal to that of investing in short-term liquid assets. This outcome depends on whether the deviation set $\mathcal{P}$ includes the values of $\rho_{\mathcal{I}}^{\star}$ associated with the equilibrium under the inflation-targeting mandate.

For the sake of presentation, and without loss of generality, we focus on the case in which the model parameters imply that the equilibrium under $\mathcal{I}$-mandate is given by $\mathcal{I}_1^{\star}$, that is, on the case in which $\Delta_{\mathcal{I}} = \Delta_{\mathcal{I}}^{-}$. The alternative case $\Delta_{\mathcal{I}} = \Delta_{\mathcal{I}}^{+}$, in which the equilibrium triplet corresponds to $\mathcal{I}_2^{\star}$, is discussed in the Supplementary appendix B available online. In order to characterize the equilibrium under the dual mandate, it is useful to first establish the following two Lemmas.

Lemma 5. *$\rho_{\mathcal{P}}^{-} \gtrless \rho_{\mathcal{I}}^{-}$ according as $i_\pi \gtrless \tilde{i}/\bar{\rho}$. The value of $\rho_{\mathcal{P}}^{-}$ is (i) strictly decreasing in ω, and (ii) increasing in π if and only if $\rho_{\mathcal{P}}^{-} > 1 - \omega$.*

Proof. See Appendix A7 □

Lemma 6. *Let ρ_1^{-} denote the value of $\rho < \bar{\rho}$ at which $\Delta_{\mathcal{I}}^{-} = \omega\pi(R - d)$. If $\rho_1^{-} \in \mathcal{P}^{C-}$, then $\rho_1^{-} < \min\left\{\rho_{\mathcal{I}}^{-}, \rho_{\mathcal{P}}^{-}\right\}$. Moreover, ρ_1^{-}: (i) strictly decreases with i_π; (ii) strictly decreases with ω; (iii) increases or decreases with π depending on whether $\rho_1^{-} \gtrless 1 - \omega$.*

Proof. See Appendix A8. □

We can now prove the following proposition.

Proposition 3. Let
$$\beta_{\mathcal{I}_1} = \frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d \rho_{\mathcal{I}}^{-}}; \quad \beta_{\mathcal{P}} = \frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d \rho_{\mathcal{P}}^{-}}; \quad \beta_1 = \frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d \rho_1^{-}}$$
denote the relative weight assigned to financial stability in the dual mandate for which ρ_1 is equal to, respectively, $\rho_{\mathcal{I}}^{-}$, $\rho_{\mathcal{P}}^{-}$ and $\tilde{\rho}_1$. Under $\mathcal{D}$-mandate, the time-consistent Nash equilibrium is as follows:

(i) when $\beta \geq \beta_1$, the equilibrium is unique and globally stable, given by the triplet

$$\mathcal{D}_1^{\star} = \begin{cases} i_{1\mathcal{D}}^{\star} = \begin{cases} 0 & \text{if } i_T = 0 \\ \tilde{i} & \text{if } i_T = i_\pi \end{cases} \\ \mathcal{B}_{\mathcal{D}}^{\star} = (0, 0) \\ \rho_{\mathcal{D}}^{\star} = \rho_{\mathcal{P}}^{-} \in (\rho_1, \bar{\rho}] : \Delta_{\mathcal{P}}(\rho_{\mathcal{P}}^{-}) = 0 \end{cases} ; \tag{31}$$

(ii) when $\beta_1 > \beta \geq \max\left(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\right)$: the $\mathcal{D}$-mandate admits two possible equilibrium triplets: the deviation equilibrium $\mathcal{D}_1^{\star}$ and the threat equilibrium

$$\mathcal{D}_2^{\star} = \begin{cases} i_{1\mathcal{D}}^{\star} = i_T \\ \mathcal{B}_{\mathcal{D}}^{\star} = \begin{cases} (0, 0) & \text{if } i_T = 0 \\ (0, 1) & \text{if } i_T = i_\pi \end{cases} \\ \rho_{\mathcal{D}}^{\star} = \rho_1 = \frac{\gamma(i_\pi - \tilde{i})}{\beta\alpha\omega d} \end{cases} \tag{32}$$

which are, respectively, locally stable and locally stable from the left;

(iii) when $\max\left(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\right) > \beta \geq \min\left(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\right)$: **(iii.a)** if $i_\pi \geq \tilde{i}/\overline{\rho}$, the $\mathcal{D}$-mandate admits two possible locally stable equilibria: the deviation equilibrium $\mathcal{D}_1^\star$ and the equilibrium $\mathcal{D}_3^\star \equiv \mathcal{I}_1^\star$ that coincides with the inflation-targeting equilibrium triplet $\mathcal{I}_1^\star$ in (16); **(iii.b)** if $\max(\tilde{i}, \iota) < i_\pi < \tilde{i}/\overline{\rho}$ the equilibrium triplet is unique, globally stable, and equal to $\mathcal{D}_2^{*\prime}$, with $\rho_1' > \rho_1^-$.

(iv) when $\min\left(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\right) > \beta \geq \underline{\beta}$, the $\mathcal{D}$-mandate equilibrium is unique, globally stable, and coincides with the inflation-targeting equilibrium, $\mathcal{D}_3^\star \equiv \mathcal{I}_1^\star$.

Proof. See Appendix A9. □

Under the dual mandate, the equilibrium depends on the relative weight assigned to financial stability and price stability or, more formally, on the measure of the set $\mathcal{P}$. If β is sufficiently large so that the deviation interval $\mathcal{P}$ includes both the values $\rho_{\mathcal{P}}^-$ and $\rho_{\mathcal{I}}^-$, the economy exhibit a time-consistent equilibrium characterized by a policy interest rate that remains low enough (at $\tilde{i}$) to prevent the market value of banks' long-term assets from falling below d. As a result, there are no rescue interventions or associated social costs, but the economy incurs positive inflation costs during periods of inflationary pressure. It is worth noting that, by Lemma 5, the equilibrium degree of illiquidity in the banking system under the dual mandate may be higher or lower than that which would prevail if the central bank operated under a single inflation-targeting mandate. This is due to two opposing effects on expected payoffs stemming from the value the central bank assigns to financial stability under the dual mandate. On the one hand, by deviating from the inflation target, the central bank can avoid injecting additional liquidity into the banking system, thereby making investments in illiquid long-term assets relatively less attractive. On the other hand, by departing from the inflation objective, the central bank prevents policy interest rates from rising above $\tilde{i}$, which reduces the expected profitability of short-term liquid assets, whose returns follow the policy rate. The effect that prevails depends on the severity of potential inflation waves and the extent to which the central bank has to increase the policy rate to neutralize the rise in the general price level. If i_π is sufficiently low, that is, if $i_\pi < \tilde{i}/\overline{\rho}$, the discouragement effect on long-term asset investments predominates, and, in equilibrium, the banking system is more liquid under the dual mandate than under an inflation-targeting mandate. In contrast, if $i_\pi > \tilde{i}/\overline{\rho}$, the banking system under the $\mathcal{D}$-mandate is, in aggregate, more illiquid than it would be under an inflation-targeting mandate.

If β is very large (i.e., if $\beta \geq \beta_1$), the deviation equilibrium $D_1^\star$ is unique and globally stable, in the sense that for any value of ρ different from $\rho_{\mathcal{P}}^-$, banks have an incentive to adjust their portfolios until the share of illiquid banks exactly equals $\rho_{\mathcal{P}}^-$. However, if β is not large enough to include ρ_1^- in the deviation set $\mathcal{P}$, an additional "threat" equilibrium $\mathcal{D}_2^\star$ coexists with the deviation equilibrium $\mathcal{D}_1^\star$, in which the share of illiquid banks is exactly equal to the threshold value ρ_1. In this case, although the payoffs of illiquid banks exceed those of banks holding liquid assets, no arbitrarily small subset of banks with positive mass holding short-term liquid securities finds it profitable to reallocate their portfolios toward long-term assets. This is because such a reallocation would increase the aggregate degree of illiquidity in the banking system to a level $\rho > \rho_1$, at which it would be optimal for the central bank to deviate from its objective of price stability, set the policy rate below the inflation-targeting rule, and refrain from acting as lender of last resort by directly extending emergency loans to distressed illiquid banks. If the difference between the expected payoffs from liquid and illiquid assets at $\rho = \rho_1$, under inflation-targeting monetary and rescue policies, is relatively small compared to the probability of being hit by a shock and the benefit of accessing central bank emergency liquidity – formally, when $\Delta_{\mathcal{I}}(\rho_1) \leq \omega\pi(R - d))$ –, then liquid banks would reduce, rather than increase, their expected payoff by reallocating their portfolios toward illiquid assets. In this case, assigning the central bank a dual mandate would represent a credible threat of not rescuing distressed banks, which discourages banks from holding illiquid assets and allows the central bank to fully achieve its price-stability objective. Since $\Delta_{\mathcal{D}}$ is strictly decreasing in ρ, the threat equilibrium is locally stable, but only on the left, in the sense that when the share of illiquid banks is below ρ_1, banks have an incentive to increase their holdings of illiquid assets; however, if the share of illiquid banks exceeds ρ_1, banks still find it profitable to further increase the overall illiquidity of the system to $\rho_{\mathcal{P}}^-$. The deviation equilibrium $\mathcal{D}_1^\star$, on the contrary, is locally stable to both upward and downward perturbations in the share of illiquid banks, or globally stable if $\Delta_{\mathcal{I}}(\rho_1) > \omega\pi(R - d)$.

At the opposite extreme, when the weight assigned by the central bank's mandate to financial stability is so small that the deviation set $\mathcal{P}$ includes neither $\rho_{\mathcal{I}}^-$ nor $\rho_{\mathcal{P}}^-$, deviating from the inflation target is never optimal for the central bank. In this case, the equilibrium under dual mandate $D_3^\star$ coincides with that under inflation targeting and converges globally to it.

Finally, when β takes intermediate values, the equilibria that emerge depend on whether the deviation set contains $\rho_{\mathcal{P}}^-$ or $\rho_{\mathcal{I}}^-$. In the first case, when the policy rate required to counteract potential inflationary pressures is sufficiently high such that $\rho_{\mathcal{P}}^- \geq \rho_1 > \rho_{\mathcal{I}}^-$, the dual mandate admits two equilibria, the deviation equilibrium $\mathcal{D}_1^\star$ and the inflation-targeting equilibrium $\mathcal{D}_3^\star$. Otherwise, when $i_\pi < \tilde{i}/\overline{\rho}$ and $\rho_{\mathcal{I}}^- \geq \rho_1 > \rho_{\mathcal{P}}^-$, the equilibrium is unique and corresponds to the threat equilibrium $D_2^\star$.

5 Optimal mandate

In the previous section, we showed that when the central bank's mandate requires it to consider both inflation costs and financial instability costs, the equilibrium that emerges depends on the relative weight assigned to the objectives of monetary and financial stability. Precisely, depending on the value of β, three outcomes are possible, which may emerge as a unique equilibrium or coexist as multiple equilibria: the deviation triplet $\mathcal{D}_1^\star$, the threat triplet $\mathcal{D}_2^\star$, and the inflation-targeting triplet $\mathcal{D}_3^\star$. Associated with these equilibrium triplets and with the monetary policy and bailout rules consistent with them are the costs that the central bank must bear in order to comply with its mandate. In this section, we examine which of the equilibria characterized in Proposition 3 entails the lowest expected costs or losses for the central bank. This analysis provides guidance not only on the relative weight that should be assigned to the objectives of inflation and financial stability within the dual mandate, but also, since $\mathcal{D}_3^\star$ coincides with the unique equilibrium $\mathcal{I}_1^\star$ arising under the inflation mandate, on which mandate should ultimately be assigned to the central bank.

5.1 Weighting financial and monetary stability

When the equilibrium triplet corresponds to the deviation triplet, the central bank sets the policy rate at $\tilde{i}$ below the inflation target level and refrains from intervening in support of distressed banks. Consequently, the costs of its monetary and financial policy actions coincide with the inflation losses arising from the failure to adhere to a strict inflation-targeting rule:

$$\mathcal{C}(\mathcal{D}_1^\star) = \pi\gamma\big(i_\pi - \tilde{i}\big) \tag{33}$$

By contrast, the triplet characterizing the inflation-targeting equilibrium implies that the central bank sets the interest rate at the inflation target i_π, thereby reducing inflation losses to zero. However, during periods of inflationary pressure, the central bank commits to act as a lender of last resort by providing emergency liquidity assistance to banks in distress. Consequently, the costs of its policy actions correspond to the rescue costs:

$$\mathcal{C}(\mathcal{D}_3^\star) = \pi\omega d\alpha\rho_{\mathcal{I}}^- \tag{34}$$

Finally, as in the case of the inflation-targeting equilibrium, the threat equilibrium involves only rescue costs for the central bank. However, unlike in equilibrium $\mathcal{D}_3^\star$ (as well as $\mathcal{D}_1^\star$), in the threat equilibrium the central bank's costs depend on β. In particular, since the threshold ρ_1 at which the equilibrium threat is established decreases as β increases, the costs are lower the greater the weight that the mandate assigns to financial stability:

$$\mathcal{C}(\mathcal{D}_2^\star) = \pi\omega d\alpha\rho_1(\beta) \tag{35}$$

with $\partial\mathcal{C}(\mathcal{D}_2^\star)/\partial\beta < 0$.

In cases where the dual mandate admits a unique equilibrium, the expected costs faced by the central bank under the $\mathcal{D}$-mandate correspond to those associated with that equilibrium. For certain values of β, however, Proposition 3 indicates that, under the dual mandate, two equilibria may alternatively prevail. In this case, the expected costs associated with the dual mandate are a weighted average of the costs of the two possible equilibria. Therefore:

$$E(\mathcal{C}\,|\beta) = \begin{cases} \mathcal{C}(\mathcal{D}_1^\star) & \text{if } \ \beta > \beta_1 \\ \varepsilon\mathcal{C}(\mathcal{D}_1^\star) + (1-\varepsilon)\mathcal{C}(\mathcal{D}_2^\star) & \text{if } \ \beta \in \big[\max\big(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\big), \beta_1\big] \\ \varepsilon'\mathcal{C}(\mathcal{D}_1^\star) + (1-\varepsilon')\mathcal{C}(\mathcal{D}_3^\star) & \text{if } \ \beta \in \big[\min\big(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\big), \max\big(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\big)\big] \ \text{ and } \ i_\pi \geq \tilde{i}/\overline{\rho} \\ \mathcal{C}(\mathcal{D}_2^{\star\prime}) & \text{if } \ \beta \in \big[\min\big(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\big), \max\big(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\big)\big] \ \text{ and } \ i_\pi < \tilde{i}/\overline{\rho} \\ \mathcal{C}(\mathcal{D}_3^\star) & \text{if } \ \beta \in \big[\underline{\beta}, \min\big(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\big)\big] \end{cases} \tag{36}$$

where ϵ and $1-\epsilon$ denote the exogenously given probabilities that the central bank assigns to the occurrence of the deviation equilibrium and the threat equilibrium when the value of β falls within the range consistent with case (ii) of Proposition 3, whereas ϵ' and $1-\epsilon'$ denote the probabilities that the central bank assigns to the occurrence of the deviation equilibrium and the inflation-targeting equilibrium when the value of β falls within the range consistent with case (iiia).

We can now characterize the optimal weight to be assigned, within the dual mandate, to the objectives of price stability and financial stability.

Proposition 4. We distinguish between two cases depending on the level i_π of the policy rate consistent with the inflation-targeting rule.

(i). Let i_π be greater than or equal to $\tilde{i}/\overline{\rho}$. In this case, the value of β that minimizes the expected costs of the dual mandate for the economy is:

$$\beta^{\star}_{|i_\pi \geq \tilde{i}/\overline{\rho}} = \begin{cases} \big(\beta_1, \infty\big) & \text{if } \ \alpha > \alpha_1 = \frac{\gamma(i_\pi - \tilde{i})}{\omega d \rho_1^-} \\ \beta_1 & \text{if } \ \alpha_1 \geq \alpha \geq \alpha_2 = \frac{\varepsilon\gamma(i_\pi - \tilde{i})}{\omega d \left[\rho_{\mathcal{I}}^- - (1-\varepsilon)\rho_1^-\right]} \\ \big[0, \beta_{\mathcal{I}_1}\big) & \text{if } \ \alpha < \alpha_2 \end{cases} \tag{37}$$

(ii). Let $i_\pi < \tilde{i}/\overline{\rho}$. In this case, the optimal value of β is:

$$\beta^{\star}_{|i_\pi < \tilde{i}/\overline{\rho}} = \begin{cases} \big(\beta_1, \infty\big) & \text{if } \ \alpha > \alpha_1 \\ \beta_1 & \text{if } \ \alpha_1 \geq \alpha \geq \alpha_2' = \frac{\varepsilon\gamma(i_\pi - \tilde{i})}{\omega d \left[\rho_1^{-\prime} - (1-\varepsilon)\rho_1^-\right]} \\ \beta_{\mathcal{P}} - \eta & \text{if } \ \alpha < \alpha_2' \end{cases} \tag{38}$$

where η is an arbitrarily small positive number and $\rho_1^{-\prime} = \gamma(i_\pi - \tilde{i})/(\beta_{\mathcal{P}} - \eta)\alpha\omega d$.

Proof. See Appendix A10. □

Proposition 4 indicates that the optimal weight assigned to the financial stability objective in a dual mandate can be expressed as a decreasing step function of the marginal social cost of emergency liquidity α. The underlying intuition is straightforward. All else being equal, as α decreases, the social cost of maintaining inflation at the target – that is, the cost of supplying the liquidity required to support banks that, at the prevailing interest rate, would otherwise be unable to raise funds in the financial market – declines.

When α is very high, exceeding a threshold value α_1, aligning the policy rate with the inflation-targeting rule, while having to provide the emergency liquidity necessary to rescue distressed banks, becomes extremely costly. In this case, the deviation equilibrium, where the central bank keeps the policy rate sufficiently low to avoid the need for bank bailouts and the associated liquidity costs, constitutes the least socially costly outcome. Consequently, since Proposition 3 implies that for any $\beta > \beta_1$ the deviation equilibrium emerges as the unique equilibrium under the dual mandate, and given that the corresponding fraction of illiquid banks prone to distress is independent of the relative importance associated with financial stability and price inflation, any value of β exceeding β_1 minimizes the expected social costs generated by a dual mandate.

When α falls below α_1, the threat of not deviating from the inflation target becomes credible, making the threat equilibrium feasible and less costly than the deviation equilibrium. In this case, since the social cost of the threat equilibrium decreases with β, it is optimal to set the weight assigned to the financial stability objective at the maximum value, β_1, consistent with this equilibrium. However, since the threat equilibrium at ρ_1^- is not globally stable, at $\beta = \beta_1$ the dual mandate also admits a second more costly equilibrium at the deviation triplet $\mathcal{D}_1^\star$.

In cases where the marginal cost of emergency liquidity provision falls further below a second threshold α_2, the inflation-targeting equilibrium $\mathcal{D}_3^\star$ becomes less costly than the deviation equilibrium $\mathcal{D}_1^\star$. However, since the threat equilibrium remains the least socially costly outcome, the value of the second threshold for α, and more importantly, the optimal β characterizing the dual mandate, depend on the probability assigned to the occurrence of the deviation equilibrium and on the expected severity of inflationary pressures. If the latter is high, so that the policy rate required to keep inflation under control is sufficiently elevated ($i_\pi \geq \tilde{i}/\overline{\rho}$), then, provided that the probability ε' of a deviation equilibrium prevailing over a threat equilibrium is non-negligible, the inflation-targeting equilibrium constitutes the least socially costly outcome. Accordingly, it is optimal to reduce the relative weight assigned to the financial stability objective to a value lower than $\beta_{\mathcal{I}_1}$. On the other hand, when the expected severity of inflationary waves is moderate, such that $i_\pi < \tilde{i}/\overline{\rho}$, the inflation-targeting equilibrium can never constitute the least socially costly outcome, as the share of illiquid banks is always higher than that which would prevail under a threat equilibrium. In this case, therefore, the optimal weight to assign to the financial stability objective is $\beta_{\mathcal{P}} - \eta$, with η arbitrarily small. This setting steers the banking system toward a 'milder' threat equilibrium $\mathcal{D}_2^{*\prime}$, characterized by a lower share of illiquid banks than under the inflation-targeting equilibrium, yet sufficiently high to prevent the emergence of a deviation equilibrium that would be excessively costly.

5.2 $\mathcal{D}$- versus $\mathcal{I}$-mandate

Since the share of illiquid banks and the monetary and rescue policies implemented in equilibrium by the central bank under a strict inflation-targeting mandate are exactly the same as those prevailing in the $\mathcal{D}_3^\star$ equilibrium under a dual mandate, Proposition 4 allows for an immediate assessment of the relative optimality of the two mandates. More precisely, to the extent that the central bank's mandate is calibrated to minimize total social costs, the following result is implied by Proposition 4.

Proposition 5. If $i_\pi \geq \tilde{i}/\overline{\rho}$, when $\alpha < \alpha_2$, the inflation-targeting mandate can be socially preferable to the dual mandate; otherwise, if $\alpha \geq \alpha_2$, it is optimal to assign the central bank a dual mandate with β set according to (37). If $i_\pi < \tilde{i}/\overline{\rho}$, the $\mathcal{I}$-mandate is never optimal.

Intuitively, the inflation-targeting mandate corresponds to the case where $\beta = 0$. When $i \geq \tilde{i}/\overline{\rho}$, either because the economy is subject to strong inflationary pressures or because of severe frictions in the transmission of monetary policy, the costs of deviating from the inflation target become particularly high. If the marginal cost of emergency liquidity falls below the threshold α_2, according to Proposition 4, $\beta = 0$ belongs to the set of optimal solutions. In this case, adopting a single mandate that assigns monetary policy the sole objective of maintaining price stability may be preferable to granting the central bank the flexibility to pursue a dual mandate. Conversely, when the costs of deviating from the inflation target are not particularly high, it is never optimal to assign the central bank a strict inflation-targeting mandate. This is because a dual mandate that also entrusts the central bank with a financial stability objective, albeit with a relatively small weight, can serve as a credible threat to commercial banks, guiding them toward a stable equilibrium characterized by more liquid investment choices, without compromising the control of inflation.

The higher the value of α_2, the wider the set of circumstances under which it is optimal to assign the central bank the pursuit of price stability as the sole objective of monetary policy, as in the $\mathcal{I}$-mandate. It is therefore of interest to examine how the threshold value of α_2 responds to variations in parameters that capture the credibility of the central bank and the likelihood of a threat equilibrium, as well as the average degree of monetary and financial fragility of the economy.

Proposition 6. (i) The threshold α_2 increases strictly in ε. Moreover, as ε approaches 1, the threshold α_2 is: (ii) strictly increasing in i_π; (iii) increasing or decreasing in π depending on whether $\rho_{\mathcal{I}}^{-} \lesseqgtr 1-\omega$; (iv) non-monotonic in ω, with $\lim_{\omega \to 0} \alpha_2 = \lim_{\omega \to 1} \alpha_2 = \infty$.

Proof. See Appendix A11 □

First, the threshold α_2 depends negatively on the probability that the threat equilibrium materializes and that the banking system remains relatively liquid, rather than converging toward a deviation equilibrium characterized by a relatively large share of illiquid banks. In fact, the threat equilibrium always dominates the inflation-targeting equilibrium in terms of lower rescue costs. Therefore, the more likely it is that the central bank's threat to maintain the policy rate at a level consistent with the inflation target effectively shapes bank investment decisions (that is, the larger $1-\varepsilon$), the narrower the set of circumstances in which it is optimal to assign the central bank the sole objective of maintaining price stability. In other words, a strict inflation-targeting mandate emerges as optimal only when the social cost of emergency liquidity provision is particularly low.

As for the impact of the parameters of monetary and financial fragility, π, i_π, and ω, on the choice of the optimal mandate, general closed-form results cannot be derived. Since the degree of illiquidity of the banking system in both the inflation-targeting and the threat equilibria responds in the same way to changes in these parameters (see Proposition 2 and Lemma 5), the effect of such variations on the threshold α_2 depends on the relative velocity with which ρ_I^- and ρ_1^- adjust, as well as on the likelihood that banks coordinate on a threat rather than on a deviation equilibrium.

General results can, however, be obtained for the case in which ε approaches 1, that is, when the probability that the banking system settles on a threat equilibrium (which, it is should be recalled, is unstable from the right) is negligible. In this case, the choice of optimal mandate reduces to a comparison between the inflation-targeting equilibrium and the deviation equilibrium. Hence, greater severity of inflationary waves and a higher policy rate required to counter them, by increasing inflation costs and reducing the share of banks that, under an inflation-targeting equilibrium, would invest in illiquid assets, result in higher costs associated with the dual mandate and lower costs under a pure inflation-targeting regime. Accordingly, the threshold value of α_2 increases, thus expanding the set of circumstances under which a single inflation-targeting objective constitutes the optimal policy mandate.

An increase in the likelihood of inflationary waves, instead, affects only the cost of the inflation-targeting equilibrium. The threshold α_2 increases or decreases with π through its effect on the share of illiquid banks, which, in turn, depends on the degree of illiquidity of the banking system (see Proposition 2). When the latter is relatively low ($\rho_{\mathcal{I}}^- < 1-\omega$), a higher probability of inflation induces banks to shift toward more liquid investments, thus reducing the potential need for rescue interventions. In this case, the inflation-targeting equilibrium, and hence the $\mathcal{I}$-mandate, is less costly than the deviation equilibrium even for relatively higher values of the emergency liquidity cost. Conversely, when $\rho_{\mathcal{I}}^- > 1-\omega$, an increase in π raises the degree of illiquidity in the banking system, thus restricting the range of circumstances in which the $\mathcal{I}$-mandate is optimal to those characterized by relatively low emergency liquidity costs.

Finally, the probability ω that banks experience liquidity shocks has no bearing on inflation costs, but it has non-univocal effects on rescue costs and, hence, on α_2. On the one hand, greater financial fragility results in a larger number of banks being exposed, on average, to unexpectedly high deposit withdrawals, which raises rescue costs. On the other hand, a higher probability of experiencing a bank run reduces rescue costs, as it lowers the share of illiquid banks that find it optimal to invest in long-term assets at the risk of facing financial distress (see Proposition 2). The net effect is therefore ambiguous. However, the threshold α_2 tends to infinity as ω approaches zero or one. Therefore, it does not possess a global maximum and attains its minimum for intermediate values of ω. This implies that maintaining price stability is more likely to be the optimal mandate when liquidity risks in the banking system are either very high or very low. In the former case, the number of banks that, in equilibrium, find it optimal to invest in a portfolio of long-term illiquid assets is very small, and hence rescue costs are minimal. The same holds when the probability of bank run is very low, regardless of the share of illiquid banks. In all other cases, when the probability of a liquidity crisis takes intermediate values, the costs of emergency liquidity may be sufficiently high to make the deviation equilibrium less costly and justify a trade-off with inflation control.

6 Conclusion

In this paper, we argue that price stability and financial stability objectives can conflict as a result of interactions among the policy interest rate, the market value of long-term assets, the interventions required to rescue banks hit by funding shocks, and the overall degree of liquidity in the banking system following the investment decisions of banks. The policy interest rate helps determine the maximum price the market is willing to pay for the assets of illiquid banks. When the central bank is forced to raise the policy rate sharply to counter an inflationary surge, this price may fall below the minimum level that would allow distressed banks to obtain in the market the liquidity required to avert failure. In such cases, if the central bank seeks to safeguard the stability of the banking system, it must intervene by providing emergency liquidity to distressed illiquid banks through lender-of-last-resort operations. This is anticipated by banks, triggering moral hazard behavior that induces them to increase their investments in illiquid assets, thereby inflating the social costs of maintaining financial stability.

Therefore, in establishing the central bank's mandate, parliament and policy makers face a trade-off between a monetary policy focused exclusively on combating inflation, which entails more costly bailout measures and greater emergency liquidity injections, and a policy that accounts for these costs, allowing for a more flexible pursuit of inflation targets. Our model suggests that an inflation-targeting mandate may be advisable when the economy is subject to frequent and severe inflationary shocks that would require substantial policy rate adjustments, or when liquidity risks in the banking system are neither too high nor too low. In such cases, unless the shadow cost of banking liquidity is particularly elevated, the welfare losses associated with potential lender-of-last-resort interventions do not outweigh the welfare losses that would arise from refraining from increasing interest rates to the inflation target. Otherwise, when inflationary episodes are neither extremely frequent nor severe, or when the likelihood that banks experience unexpected liquidity shocks on the funding side is high, a mandate that explicitly requires the central bank to take financial stability into account, even at the cost of relaxing strict inflation control, may be preferable.

Our results are relevant to the debate on the value of granting central banks flexibility in pursuing price stability in order to take the objective of financial stability into account. Although the pros and cons of central bank flexibility in decision-making are widely discussed and recognized in the literature (Schwartz, 1998; Bordo and Wheelock, 1998; Borio, 2014), the channel we highlight has been largely overlooked. The argument commonly used to justify the central bank's flexibility in achieving the inflation target is that–during periods of financial stress–flexibility allows the central bank to tolerate some inflation variability in order to provide liquidity support to banks or to prevent credit crunches. In this wisdom, the flexibility of the monetary policy can be interpreted as a crisis resolution mechanism (Farhi and Tirole, 2012). Our results, instead, can be interpreted in a macroprudential sense. That is, flexibility enables the central bank to threaten banks with deviations from the inflation rule in response to liquidity reductions in the banking system. When the mandate and the parameters of the economy are such that the threat is credible, the equilibrium liquidity in the banking system will be higher than what would emerge in the case of a rigid inflation-targeting mandate. To the extent that a liquid banking system is more resilient to shocks than an illiquid one, a dual mandate, if properly designed, could promote overall financial stability.

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Appendix A: Proofs

A1 Proof of Lemma 1

From equations (10)–(13), it follows that the payoffs $\mathcal{A}_{\mathcal{I}}^-$, $\mathcal{R}_{\mathcal{I}}^-$, $\mathcal{A}_{\mathcal{I}}^+$ and $\mathcal{R}_{\mathcal{I}}^+$ are continuous in ρ. The continuity of $\mathcal{A}_{\mathcal{I}}$ and $\mathcal{R}_{\mathcal{I}}$, and hence the continuity of $\Delta_{\mathcal{I}}$, follows from the fact that at $\rho = \overline{\rho}$ we have $\mathcal{A}_{\mathcal{I}}^-(\overline{\rho}) = \mathcal{A}_{\mathcal{I}}^+(\overline{\rho})$ and $\mathcal{R}_{\mathcal{I}}^-(\overline{\rho}) = \mathcal{R}_{\mathcal{I}}^+(\overline{\rho})$. Moreover, from (10) and (12′), $\mathcal{A}_{\mathcal{I}}$ strictly decreases in ρ for $\rho \in [0, \overline{\rho}]$ and is constant in ρ for $\overline{\rho} < \rho \leq 1$; while from (11) and (13′), $\mathcal{R}_{\mathcal{I}}$ strictly increases in ρ. Thus, $\Delta_{\mathcal{I}}$ strictly decreases with ρ. Finally, assumption 2 ensures that $\Delta_{\mathcal{I}}(0) > 0$, while from (12′) and (13′) it is straightforward to verify that $\Delta_{\mathcal{I}}$ tends to minus infinity as ρ approaches 1. This establishes that the function $\Delta_{\mathcal{I}}(\rho)$ has a unique zero at $\rho_{\mathcal{I}}^-$ or $\rho_{\mathcal{I}}^+$, respectively below or above $\overline{\rho}$.

A2 Proof of Proposition 1

In equilibrium, maximization of expected bank payoffs and consistency of expectations (see conditions E1 and E4 in Section 2.5) require that the shares of illiquid and liquid banks in system be such that no bank, given its expectations about the central bank's monetary and rescue policies, can increase its payoff by choosing a portfolio of short- or long-term assets different from the one it already holds, and that the central bank finds it optimal to implement the policies anticipated by the banks. From Lemma 1, under the $\mathcal{I}$-mandate, an equilibrium value $\rho_{\mathcal{I}}^{\star}$ exists, is unique, and is stable, and corresponds to the value $\tilde{\rho}$ such that $\Delta_{\mathcal{I}}(\tilde{\rho}) = 0$. In fact, for any $\rho < \tilde{\rho}$ we have $\Delta_{\mathcal{I}}(\rho) > 0$, and liquid banks expect to achieve higher returns by shifting their investments from liquid securities to illiquid long-term assets; conversely, when $\rho > \tilde{\rho}$ and $\Delta_{\mathcal{I}}(\rho) < 0$, it is the illiquid banks that find it optimal to reallocate their investments to liquid securities. However, depending on the values assumed by the model parameters, the equilibrium share of illiquid banks for which $\Delta_{\mathcal{I}} = 0$ can take a value $\rho_{\mathcal{I}}^-$ below $\overline{\rho}$ or a value $\rho_{\mathcal{I}}^+$ above it, thereby affecting the time-consistent rescue policy. Since $\Delta_{\mathcal{I}}$ is strictly decreasing in ρ, a necessary and sufficient condition for $\rho_{\mathcal{I}}^{\star} = \rho_{\mathcal{I}}^-$ is that:

$$\Delta_{\mathcal{I}}^-(\overline{\rho}) < 0 \tag{A1}$$

which, using equations (10) and (11) and solving for i_π, can be expressed as:

$$i_\pi \geq \iota = \tilde{i}\left[\frac{1}{\overline{\rho}} - \frac{(1-d)}{\pi}\right] - \frac{(1-d)}{\pi(1-\omega)} \tag{A2}$$

From inequality (A2), it is immediate to verify that if $\pi < \frac{1-d}{\omega}\left(\frac{1-\omega}{d} + \frac{1}{R-d}\right)$, the threshold value ι is lower than $\tilde{i}$, and hence, by Assumption 1, the condition $i_\pi > \iota$ is necessarily satisfied. In this case, $\rho^\star_{\mathcal{I}} = \rho^-_{\mathcal{I}} < \overline{\rho}$, and the time-consistent rescue policy reduces, as specified by triplet $\mathcal{I}^\star_1$, to possible lender-of-last-resort interventions during periods characterized by inflationary pressures and high policy rates. Conversely, if $\pi > \frac{1-d}{\omega}\left(\frac{1-\omega}{d} + \frac{1}{R-d}\right)$, then, whenever $\tilde{i} < i_\pi < \iota$, we have $\Delta^-_{\mathcal{I}}(\overline{\rho}) > 0$, and in equilibrium the share of illiquid banks is $\rho^\star_{\mathcal{I}} = \rho^+_{\mathcal{I}} > \overline{\rho}$. In this case, as indicated by triplet $\mathcal{I}^\star_2$, given the low liquidity available in the banking system, the optimal rescue policy requires that, in addition to any lender-of-last-resort interventions during inflationary periods, the central bank must intervene during periods of price stability by injecting emergency liquidity through the financial market.

A3 Proof of Proposition 2

From Lemma 1, $\partial\Delta_{\mathcal{I}}/\partial\rho^\star_{\mathcal{I}} < 0$. Therefore, using the implicit function theorem, we have that

$$\text{sign}\ \frac{\partial\rho^\star_{\mathcal{I}}}{\partial x} = \text{sign}\ \frac{\partial\Delta_{\mathcal{I}}(\rho^\star, x)}{\partial x} \tag{A3}$$

where $x = \{\omega, i_\pi, \pi\}$. According to Proposition 1, depending on the model parameters, $\rho^\star_{\mathcal{I}}$ may take the value $\rho^-_{\mathcal{I}}$ or $\rho^+_{\mathcal{I}}$, and the relevant segment of the function $\Delta_{\mathcal{I}}$ will be $\Delta^-_{\mathcal{I}} = \mathcal{A}^-_{\mathcal{I}} - \mathcal{R}^-_{\mathcal{I}}$ or $\Delta^+_{\mathcal{I}} = \mathcal{A}^+_{\mathcal{I}} - \mathcal{R}^+_{\mathcal{I}}$, respectively.

(i). Since i_π enters only in $\mathcal{R}^-_{\mathcal{I}}$ and $\mathcal{R}^+_{\mathcal{I}}$, it follows directly that $\Delta_{\mathcal{I}}$ always decreases with i_π, which implies that:

$$\frac{\partial\rho^\star_{\mathcal{I}}}{\partial i_\pi} < 0. \tag{A4}$$

(ii). Substituting the expressions (10), (11), and (6) for $\mathcal{A}^-_{\mathcal{I}}$, $\mathcal{R}^-_{\mathcal{I}}$, and $p^\star$ into $\Delta^-_{\mathcal{I}}$, and (12′) and (13′) for $A^+_{\mathcal{I}}$ and $R^+_{\mathcal{I}}$ into $\Delta^+_{\mathcal{I}}$, both functions can be expressed in the linear form $\Delta_{\mathcal{I}} = c_0 + c_1\omega$, as follows:

$$\Delta^-_{\mathcal{I}} = \left[\frac{(1-\pi)(1-\rho^-_{\mathcal{I}})}{\rho^-_{\mathcal{I}}} + R - \pi(1+i_\pi)\right] + \left[\pi i_\pi - (1-\pi)\left(\frac{R}{1-\rho^-_{\mathcal{I}}} + \frac{1}{\rho^-_{\mathcal{I}}}\right)\right]\omega = 0 \tag{A5}$$

$$\Delta^+_{\mathcal{I}} = \Big[R - \pi(1+i_\pi) - (1-\pi)\Big] + \left[\pi i_\pi - (1-\pi)\left(\frac{R-d}{1-\rho^+_{\mathcal{I}}}\right)\right]\omega = 0 \tag{A6}$$

Given Assumption 2, it is straightforward to verify that in both expressions the constant term c_0 — namely, the sum of the elements appearing in the first square bracket — is positive. It follows that, in both (A5) and (A6), for any $\omega > 0$, the sum of the factors multiplying ω, and hence the partial derivatives of $\Delta_{\mathcal{I}}^{-}$ and $\Delta_{\mathcal{I}}^{+}$ with respect to ω, is negative. Hence:

$$\frac{\partial \rho_{\mathcal{I}}^{\star}}{\partial \omega} < 0. \tag{A7}$$

(iii). To determine how the equilibrium share of illiquid banks changes with the probability of inflationary pressures, we proceed as in case (ii) by rewriting $\Delta_{\mathcal{I}}^{-}$ and $\Delta_{\mathcal{I}}^{+}$ as linear functions of π. Specifically, rearranging the terms appearing in (10) and (11):

$$\Delta_{\mathcal{I}}^{-} = \left[\omega\big(p^{\star}-1\big) + \big(1-\omega\big)\frac{R}{p^{\star}}\big(p^{\star}-1\big)\right] + \left[\omega\big(R-p^{\star}\big) - (1-\omega)\left(1+i_{\pi}-\frac{R}{p*}\right)\right]\pi = 0 \tag{A8}$$

and using (A6):

$$\Delta_{\mathcal{I}}^{+} = \left[R-1-\left(\frac{R-d}{1-\rho_{\mathcal{I}}^{+}}\right)\omega\right] + \left[\frac{R-d}{1-\rho_{\mathcal{I}}^{+}}\right]\pi = 0 \tag{A9}$$

The constant term in (A8), given by the sum of the terms in the first square bracket, is positive or negative depending on whether $p^{\star} \gtrless 1$ or, substituting for $p^{\star}$, whether $\rho_{\mathcal{I}}^{-} \lesseqgtr 1-\omega$. Therefore, for any $\pi > 0$, the sign of $\partial \Delta_{\mathcal{I}}^{-}/\partial \pi$ (the sum of the factors multiplying π appearing in the second square bracket) is negative or positive according to whether $\rho_{\mathcal{I}}^{-} \lesseqgtr 1-\omega$. From (A9), instead, we have $\partial \Delta_{\mathcal{I}}^{+}/\partial \pi$ for any $\rho_{\mathcal{I}}^{+}$. Therefore, since $\rho_{\mathcal{I}}^{+} > \rho_{\mathcal{I}}^{-} > 0$, it follows that:

$$\frac{\partial \rho_{\mathcal{I}}^{\star}}{\partial \pi} \lesseqgtr 0 \iff \rho_{\mathcal{I}}^{\star} \lesseqgtr 1-\omega. \tag{A10}$$

A4 Proof of Lemma 2

Let inequality (21) hold and let

$$\beta = b\frac{\gamma\big(i_{\pi}-\tilde{i}\big)\big[1-\omega(1-d)\big]}{\alpha d\omega(1-\omega)} \tag{A11}$$

with $b \geq 1$. As $\overline{\rho} = (1-\omega)/[1-\omega(1-d)]$, it is straightforward to verify that:

$$\frac{\gamma\big(i_{\pi}-\tilde{i}\big)}{\omega\alpha d\beta} \leq \overline{\rho} \leq 1 - \frac{\gamma\big(i_{\pi}-\tilde{i}\big)}{\big(1-\omega\big)\alpha\beta}. \tag{A12}$$

A5 Proof of Lemma 3

From equations (25)–(28), it follows that both $\Delta^-_{\mathcal{P}}$ and $\Delta^+_{\mathcal{P}}$ are continuous in ρ. The continuity of $\Delta_{\mathcal{P}}$ follows from the fact that at $\rho = \overline{\rho}$ the functions $\Delta^-_{\mathcal{P}}$ and $\Delta^+_{\mathcal{P}}$ take the same value. Moreover, since $p^\star$ decreases and $\theta^e_{1-\rho}$ increases as ρ increases, it also follows that $\Delta_{\mathcal{P}}$ strictly decreases with ρ. From the comparison of (27) and (28), it is straightforward to verify that $A^+_{\mathcal{P}} < R^+_{\mathcal{P}}$ for any $\rho \geq \overline{\rho}$. Since $\Delta^-_{\mathcal{P}}(\overline{\rho}) = \Delta^+_{\mathcal{P}}(\overline{\rho}) < 0$ and $\Delta^-_{\mathcal{P}}$ strictly decreases with ρ, there must exist a $\rho^-_{\mathcal{P}}$ such that $\Delta^-_{\mathcal{P}}(\rho^-_{\mathcal{P}}) = 0$.

A6 Proof of Lemma 4

For $\rho \in (0, \rho_1)$, we have $\Delta_{\mathcal{D}} = \Delta_{\mathcal{I}}$. Therefore, Lemma 1 ensures that $\Delta_{\mathcal{D}}$ is decreasing and continuous over the interval $\rho \in (0, \rho_1)$. The same argument applies for the interval $\rho \in (\rho_2, 1)$. The discontinuity of $\Delta_{\mathcal{D}}$ at ρ_1 and ρ_2 clearly emerges by comparing (10) with (25) and (12′) with (27). Finally, Lemma 3 proves that $\Delta_{\mathcal{D}}$ is strictly decreasing at any $\rho \in \mathcal{P}$.

A7 Proof of Lemma 5

From Lemma 3, we have that $\rho^-_{\mathcal{P}}$ is characterized by the condition $\Delta^-_{\mathcal{P}} = 0$ and that $\Delta^-_{\mathcal{P}}$ is strictly decreasing in ρ. Therefore, $\rho^-_{\mathcal{P}} \gtreqless \rho^-_{\mathcal{I}}$ if and only if $\Delta^-_{\mathcal{P}}(\rho^-_{\mathcal{I}}) \gtreqless 0$, or, by using expressions (25) and (26) for $\mathcal{A}^-_{\mathcal{P}}$ and $\mathcal{R}^-_{\mathcal{P}}$ and rearranging terms, if and only:

$$(1-\pi)\left[\omega p^\star(\rho^-_{\mathcal{I}}) - (1-\omega)\frac{R}{p^\star(\rho^-_{\mathcal{I}})}\right] + (1-\omega)R - \omega \gtreqless (1-\omega)\pi\frac{R}{d} - \omega\pi d \tag{A13}$$

In turn, for Proposition 1, $\rho^-_{\mathcal{I}}$ is characterized by the condition $\Delta^-_{\mathcal{I}} = 0$. That is, substituting from (10) and (11) and rearranging terms, $\rho^-_{\mathcal{I}}$ is such that:

$$(1-\pi)\left[\omega p^\star(\rho^-_{\mathcal{I}}) - (1-\omega)\frac{R}{p^\star(\rho^-_{\mathcal{I}})}\right] + (1-\omega)R - \omega = \pi\big[(1-\omega)(1+i_\pi) - \omega R\big] \tag{A14}$$

By substituting the left-hand side of (A14) for the right-hand side of inequality (A13) and solving the latter for i_π, it follows that $\Delta^-_{\mathcal{P}}(\rho^-_{\mathcal{I}}) \gtreqless 0$ if:

$$i_\pi \gtreqless \left(\frac{R}{d} - 1\right)\frac{1-\omega(1-d)}{1-\omega} = \frac{\tilde{i}}{\overline{\rho}}. \tag{A15}$$

To derive the comparative static effects of a change in liquidity risk and inflation risk on $\rho^-_{\mathcal{P}}$, we proceed as in Appendix A3. Since $\Delta_{\mathcal{P}}$ decreases with ρ, sign $\partial\rho^-_{\mathcal{P}}/\partial x$ = sign $\partial\Delta_{\mathcal{P}}(\rho^-_{\mathcal{P}}, x)/\partial x$, where $x = \{\omega, \pi\}$. Because $\rho^-_{\mathcal{P}} \in \mathcal{P}^-$, it follows that the relevant segment of $\Delta_{\mathcal{P}}$ $\Delta^-_{\mathcal{P}}$. Now, the function $\Delta^-_{\mathcal{P}}$ can be expressed in linear form as $c_0 + c_1 x$. Hence, the sign of $\partial\Delta_{\mathcal{P}}/\partial x$ will coincide with minus the sign of the constant term c_0. By using (25) and (26):

$$\text{sign}\,\frac{\partial\Delta_{\mathcal{P}}}{\partial\omega} = -\text{sign}\left[(1-\pi)\frac{1-\rho^-_{\mathcal{P}}}{\rho^-_{\mathcal{P}}} + R - \pi\frac{R}{d}\right] \tag{A16}$$

and

$$\text{sign}\,\frac{\partial\Delta_{\mathcal{P}}}{\partial\pi} = -\text{sign}\left[\omega\Big(p^\star(\rho^-_{\mathcal{P}}) - 1\Big) + \frac{(1-\omega)R}{p^\star(\rho^-_{\mathcal{P}})}\Big(p^\star(\rho^-_{\mathcal{P}}) - 1\Big)\right] \tag{A17}$$

Since $R/d = 1 + \tilde{i} < 1 + i_\pi$, Assumption 2 ensures that the sum of terms in (A16) is strictly positive. On the other hand, by substituting equation (6) for $p^\star(\rho^-_{\mathcal{P}})$ into (A17) it follows that:

$$\frac{\partial\Delta^-_{\mathcal{P}}}{\partial\pi} \gtrless 0 \iff \rho^\star \geq 1 - \omega \tag{A18}$$

Therefore:

(i).

$$\frac{\partial\rho^-_{\mathcal{P}}}{\partial\omega} > 0 \tag{A19}$$

(ii).

$$\frac{\partial\rho^-_{\mathcal{P}}}{\partial\pi} > 0 \iff \rho^\star \geq 1 - \omega \tag{A20}$$

.

A8 Proof of Lemma 6

The first part of the proof follows immediately from Lemma 4, according to which $\Delta_{\mathcal{D}}$ is strictly decreasing in ρ. The second part of the proof follows steps similar to what we stated in Appendix A3. In fact, the term ρ^-_1 is implicitly defined by the condition:

$$\Delta^-_{\mathcal{I}} - \psi(\pi, \omega) = 0 \tag{A21}$$

where $\psi = \omega\pi(R-d)$. From Lemma 1, $\partial\Delta_{\mathcal{I}}/\partial\rho < 0$. Therefore, using the implicit function theorem, we have that

$$\text{sign}\,\frac{\partial\rho_1^-}{\partial x} = \text{sign}\left[\frac{\partial\Delta_{\mathcal{I}}}{\partial x} - \frac{\partial\psi}{\partial x}\right] \tag{A22}$$

where $x = \{i_\pi, \omega, \pi\}$.

(i). Since i_π enters only in $\mathcal{R}_{\mathcal{I}}^-$, it follows that $\Delta_{\mathcal{I}}$ always decreases with i_π, which implies that:

$$\frac{\partial\rho_1^-}{\partial i_\pi} < 0 \tag{A23}$$

(ii). In Appendix A3, we noted that $\Delta_{\mathcal{I}}$ is linear in ω. As ψ is also linear, we can state $\Delta_{\mathcal{I}}^- - \psi = a_0 + a_1\omega$. Therefore:

$$\left[\frac{\partial\Delta_{\mathcal{I}}}{\partial\omega} - \frac{\partial\psi}{\partial\omega}\right] = a_1 \tag{A24}$$

Moreover, since ρ_1^- must satisfy the condition $a_0 + a_1\omega = 0$, at $\rho = \rho_1^-$, we have that:

$$\left[\frac{\partial\Delta_{\mathcal{I}}}{\partial\omega} - \frac{\partial\psi}{\partial\omega}\right] = a_1 = -\frac{a_0}{\omega} \tag{A25}$$

Note that ψ does not involve a constant term. Therefore, the term a_0 coincides with the terms of $\Delta_{\mathcal{I}}^-$ that do not depend on ω. Equation (A5) states that this term is positive. Hence

$$\left[\frac{\partial\Delta_{\mathcal{I}}}{\partial\omega} - \frac{\partial\psi}{\partial\omega}\right] < 0 \tag{A26}$$

(iii) In Appendix A3, we noted that $\Delta_{\mathcal{I}}$ is linear in π. As ψ is also linear, we can state $\Delta_{\mathcal{I}}^- - \psi = b_0 + b_1\pi$. Therefore:

$$\left[\frac{\partial\Delta_{\mathcal{I}}}{\partial\pi} - \frac{\partial\psi(\omega,\pi)}{\partial\pi}\right] = b_1 \tag{A27}$$

Moreover, since ρ_1^- must satisfy the condition $b_0 + b_1\pi = 0$, we have that, at $\rho = \rho_1^-$:

$$\left[\frac{\partial\Delta_{\mathcal{I}}}{\partial\pi} - \frac{\partial\psi}{\partial\pi}\right] = b_1 = -\frac{b_0}{\pi} \tag{A28}$$

Note that ψ does not involve a constant term. Therefore, the term b_0 coincides with the terms of $\Delta_{\mathcal{I}}^-$ that do not depend on π. In Appendix A3, we showed that this term is positive if and only if $p > 1$.

Hence, if and only if the term $\rho_1^- < 1-\omega$. Therefore:

$$\frac{\partial \rho_1^-}{\partial \pi} \lesseqgtr 0 \iff \rho_1^- \lesseqgtr 1-\omega. \tag{A29}$$

A9 Proof of Proposition 3

(i). By Lemma 6, $\beta_1 \geq \max\left(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\right)$. Hence, when $\beta \geq \beta_1$, it follows that $\rho_1 < \min\left(\rho_{\mathcal{I}}^-, \rho_{\mathcal{P}}^-\right)$ and, using Lemmas 2 and 3, both $\rho_{\mathcal{I}}^-$ and $\rho_{\mathcal{P}}^-$ belong to the deviation set $\mathcal{P}$. Applying Lemma 3, $\Delta_{\mathcal{D}}$ admits a unique zero at $\rho_{\mathcal{P}}^-$. Therefore, a share $\rho_{\mathcal{P}}^-$ of illiquid banks is consistent with the equilibrium requirements E1, E2′, E3 and E4′, under which no bank, given its expectations, can increase its expected payoff by reallocating its investments between long-term and short-term assets, and the central bank minimizes the cost function (18) by deviating from the inflation-targeting rule, setting the policy rate at $\tilde{\imath}$, and refraining from injecting emergency liquidity to rescue banks hit by liquidity shocks.

(ii). When $\beta \in \left[\max\left(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\right), \beta_1\right]$, the triplet $\mathcal{D}_1^\star$ continues to constitute a possible equilibrium. However, another equilibrium candidate is the discontinuity point ρ_1. In this case, although the expected payoff from a portfolio of illiquid assets is not equal to that of a portfolio of liquid assets, banks with the lowest payoffs may not have any interest in adjusting their portfolios, as such a reallocation would trigger a credible shift in the central bank's monetary policy rule and in the design of rescue policies, leading to a reduction rather than an increase in the realized payoffs. Specifically, since the function $\Delta_{\mathcal{D}}$ is decreasing in ρ, the left-hand limit of $\Delta_{\mathcal{D}}$ as ρ approaches ρ_1 always exceeds its right-hand limit, and moreover, since in the case under consideration $\rho_1 < \rho_{\mathcal{I}}^-$, we have $\lim_{\rho\to\rho_1^-} \Delta_{\mathcal{D}} = \Delta_{\mathcal{I}}^-(\rho_1) > 0$. Therefore, the discontinuity point ρ_1 constitutes an equilibrium if one or more of the following inequalities hold: (a) $\mathcal{R}_{\mathcal{I}}^-(\rho_1) \geq \mathcal{A}_{\mathcal{P}}^-(\rho_1)$; (b) $\mathcal{R}_{\mathcal{P}}^-(\rho_1) > \mathcal{A}_{\mathcal{I}}^-(\rho_1)$, when $\lim_{\rho\to\rho_1^+} \Delta_{\mathcal{D}} = \Delta_{\mathcal{P}}(\rho_1) > 0$; (c) $\mathcal{A}_{\mathcal{P}}^-(\rho_1) > \mathcal{R}_{\mathcal{I}}^-(\rho_1)$, when $\lim_{\rho\to\rho_1^+} \Delta_{\mathcal{D}} = \Delta_{\mathcal{P}}(\rho_1) < 0$. Now, it is easy to show that inequalities (b) and (c) cannot hold. In fact, by comparing equation (10) with (25) and equation (11) with (26), it is immediate to verify that banks' expected payoffs are always higher when monetary policy and rescue policy follow the inflation-targeting rule than when the central bank deviates from these policies, that is, $\mathcal{R}_{\mathcal{I}}^- > \mathcal{R}_{\mathcal{P}}^-$ and $\mathcal{A}_{\mathcal{I}}^- > \mathcal{A}_{\mathcal{P}}^-$ for any ρ. Therefore, if $\mathcal{R}_{\mathcal{P}}^-(\rho_1) < \mathcal{A}_{\mathcal{P}}^-(\rho_1)$, then it necessarily follows that $\mathcal{R}_{\mathcal{P}}^-(\rho_1) < \mathcal{A}_{\mathcal{I}}^-(\rho_1)$; likewise, if $\mathcal{A}_{\mathcal{P}}^-(\rho_1) < \mathcal{R}_{\mathcal{P}}^-(\rho_1)$, it follows that $\mathcal{A}_{\mathcal{P}}^-(\rho_1) < \mathcal{R}_{\mathcal{I}}^-(\rho_1)$. In both cases, this rules out the possibility that ρ_1 constitutes an equilibrium when the central bank adopts policies deviating from the inflation-targeting regime. However, if $\mathcal{A}_{\mathcal{I}}^-(\rho_1) - \mathcal{A}_{\mathcal{P}}^-(\rho_1) > \mathcal{A}_{\mathcal{I}}^-(\rho_1) - \mathcal{R}_{\mathcal{I}}^-(\rho_1)$, that is, if $\omega\pi(R-d) > \Delta_{\mathcal{I}}^-(\rho_1)$, then $\mathcal{R}_{\mathcal{I}}^-(\rho_1) > \mathcal{A}_{\mathcal{P}}^-(\rho_1)$.

This implies that, when the share of illiquid banks is equal to ρ_1 and the central bank follows an inflation-targeting policy, no bank finds it profitable to deviate from its chosen portfolio, due to the credible threat by the central bank to depart from the inflation-targeting rules and to refrain from providing emergency liquidity — either directly or indirectly — to banks in distress. As a result, in this case the dual mandate admits two locally stable equilibria, $\mathcal{D}_1^\star$ and $\mathcal{D}_2^\star$ (the latter being locally stable only from the left). Otherwise, $\omega\pi(R-d) \leq \Delta_{\mathcal{I}}^-(\rho_1)$, $\mathcal{D}_1^\star$ is the unique equilibrium and is globally stable.

(iv). When $\beta \in \left[\underline{\beta}, \min\left(\beta_{1\mathcal{I}}, \beta_{1\mathcal{P}}\right)\right]$, neither $\rho_{\mathcal{I}}^-$ nor $\rho_{\mathcal{P}}^-$ belong to the deviation set $\mathcal{P}$ and, by using Lemma 3, $\Delta_{\mathcal{D}}$ has a unique zero at $\rho_{\mathcal{I}}^-$. In this case, the equilibrium under the dual mandate coincides with the inflation-targeting equilibrium, with the central bank setting the policy rate at the level consistent with price stability and acting as a lender of last resort in support of all banks affected by liquidity shocks. $\mathcal{D}_3^\star$ is also the unique globally stable equilibrium. In fact, when the share of illiquid banks coincides with the discontinuity point ρ_1, banks always find it optimal to adjust their portfolios, regardless of the monetary and bailout policies adopted by the central bank. Since $\rho_{\mathcal{P}}^- < \rho_1$, it follows that $\Delta_{\mathcal{P}}(\rho_1)$ is always strictly negative and, thus, by applying the same reasoning as in case **(ii)**, we obtain $\mathcal{A}_{\mathcal{P}}^-(\rho_1) < \mathcal{R}_{\mathcal{I}}^-(\rho_1)$. As a result, illiquid banks find it profitable to reallocate their investments to liquid assets, thereby reducing the value of ρ and making deviations from inflation-targeting policies no longer optimal for the central bank. However, when β is very small, it also holds that $\rho_1 > \rho_{\mathcal{I}}^-$. Hence, when the central bank finds it optimal to maintain the policy rate at a level consistent with price stability, we have $\mathcal{A}_{\mathcal{I}}^-(\rho_1) < \mathcal{R}_{\mathcal{I}}^-(\rho_1)$, and banks continue to find it more rewarding to hold liquid assets until the share of illiquid banks declines to $\rho_{\mathcal{I}}^-$.

(iii). When $\beta \in \left[\min\left(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\right), \max\left(\beta_{\mathcal{I}_1}, \beta_{\mathcal{P}}\right)\right[$, from Lemma 5 it follows that depending on whether $i_\pi \gtrless \tilde{i}/\overline{\rho}$, either $\rho_{\mathcal{P}}^-$ or $\rho_{\mathcal{I}}^-$ belong to $\mathcal{P}$, while the other belongs to $\mathcal{P}^{C-}$. In the first case, **(iii.a)**, the function $\Delta_{\mathcal{D}}$ has a zero both in the $\Delta_{\mathcal{I}}^-$ segment in $\rho_{\mathcal{I}}^-$ and in the $\Delta_{\mathcal{P}}^-$ segment at $\rho_{\mathcal{P}}^-$. Therefore, the dual mandate admits two possible locally stable equilibrium triplets, $\mathcal{D}_1^\star$ and $\mathcal{D}_3^\star$, whereas, applying the reasoning followed above for the case **(iv)**, $\mathcal{D}_2^\star$ cannot constitute an equilibrium. If instead $i_\pi < \tilde{i}/\overline{\rho}$ and $\rho_{\mathcal{P}}^- < \rho_{\mathcal{I}}^-$, then the function $\Delta_{\mathcal{D}}$ has no zeros. However, by Lemma 3, it follows that $\Delta_{\mathcal{I}}^-(\rho_1) > 0$ and $\Delta_{\mathcal{P}}^-(\rho_1) < 0$ and, applying the same reasoning as for case **(i)**, the triplet $\mathcal{D}_2^\star$ is an equilibrium, locally stable from the left.

A10 Proof of Proposition 4

To determine the optimal relative weight, β, to be assigned in the dual mandate to the objectives of financial and monetary stability, it is necessary to compare the costs associated with the different possible equilibria identified in Proposition 3, whose prevalence depends precisely on β. However, before proceeding to verify the results stated in the proposition, it is useful to make two general preliminary observations explicit. The first point to note is that while $\mathcal{C}(\mathcal{D}_1^\star)$ and $\mathcal{C}(\mathcal{D}_3^\star)$ do not depend on β, $\mathcal{C}(\mathcal{D}_2^\star)$ is decreasing in β. Consequently, the threat equilibrium that arises when β takes the maximum value consistent with its formation, β_1, is the one that dominates all other possible threat equilibria in terms of social cost. From (32), at this threat equilibrium, the share of illiquid banks is $\rho_1(\beta_1) = \rho_1^-$, and the associated social cost is given by

$$\min_{\rho_1} \mathcal{C}(\mathcal{D}_2^\star) = \underline{\mathcal{C}}(\mathcal{D}_2^\star) = \pi\omega d\rho_1^-. \tag{A30}$$

It is therefore this value that must be compared with the social cost associated with the deviation and inflation-targeting equilibrium. The second point to note is that, by Lemma 3, since $\rho_{\mathcal{I}}^- > \rho_1^-$ always holds, the threat equilibrium invariably entails lower costs than the inflation-targeting equilibrium. Therefore, if $\mathcal{C}(\mathcal{D}_1^\star) < \underline{\mathcal{C}}(\mathcal{D}_2^\star)$, then also $\mathcal{C}(\mathcal{D}_1^\star) < \mathcal{C}(\mathcal{D}_3^\star)$ certainly holds, and the deviation equilibrium $\mathcal{D}_1^\star$ is the equilibrium that generates the lowest social costs. We can now turn to the proof of the proposition.

(i). Let $i_\pi \geq \tilde{i}/\overline{\rho}$. In this case, $\rho_{\mathcal{P}}^- > \rho_{\mathcal{I}}^-$, and $\beta_{\mathcal{P}} < \beta_{\mathcal{I}_1}$. By comparing equations (33) and (A30), it is straightforward to verify that if *alpha* is greater than a threshold α_1, the deviation equilibrium is the one that minimizes the social costs associated with the dual mandate.

$$\mathcal{C}(\mathcal{D}_1^\star) < \underline{\mathcal{C}}(\mathcal{D}_2^\star) \iff \alpha \geq \alpha_1 = \frac{\varepsilon\gamma(i_\pi - \tilde{i})}{\omega d\rho_1^-}. \tag{A31}$$

Therefore, it is optimal to assign to the central bank a mandate that places a high weight on financial stability, i.e., $\beta > \beta_1$, in order to ensure the realization of this equilibrium.

If $\alpha \leq \alpha_1$, then the threat equilibrium with a share ρ_1^- of illiquid banks represents the outcome with the lowest social costs. However, this equilibrium is not unique, but it always coexists with the deviation equilibrium. Hence, the expected cost associated with this situation of multiple equilibria is equal to the weighted average of the social costs associated with the deviation and threat equilibria, where the weights are given by the probabilities ε and $1-\varepsilon$ that the system settles, respectively, on the former or the latter equilibrium. To determine the optimal value of β, this average cost must be compared with the social cost of the inflation-targeting equilibrium, regardless of whether it is lower or higher than the cost associated with the deviation equilibrium. Indeed, suppose that $\mathcal{C}(\mathcal{D}_3^\star) < \mathcal{C}(\mathcal{D}_1^\star)$ and, in particular, that $\mathcal{C}(\mathcal{D}_1^\star) - \mathcal{C}(\mathcal{D}_3^\star) = x$. Then, recalling that $\underline{\mathcal{C}}(\mathcal{D}_2^{*\prime}) < \mathcal{C}(\mathcal{D}_1^\star)$, it is straightforward to verify that $\mathcal{C}(\mathcal{D}_3^\star) < \varepsilon'\mathcal{C}(\mathcal{D}_1^\star) + (1-\varepsilon')\mathcal{C}(\mathcal{D}_3^\star) < \varepsilon\mathcal{C}(\mathcal{D}_1^\star) + (1-\varepsilon)\underline{\mathcal{C}}(\mathcal{D}_2^{*\prime})$. On the other hand, if $C_3 - C_1 = x > 0$, we have that $\mathcal{C}(\mathcal{D}_3^\star) > \varepsilon'\mathcal{C}(\mathcal{D}_1^\star) + (1-\varepsilon')\mathcal{C}(\mathcal{D}_3^\star) > \varepsilon\mathcal{C}(\mathcal{D}_1^\star) + (1-\varepsilon)\underline{\mathcal{C}}(\mathcal{D}_2^{*\prime})$. Now, by using expressions (33), (A30) and (34), it is immediate to verify that:

$$\varepsilon\mathcal{C}(\mathcal{D}_1^\star) + (1-\varepsilon)\underline{\mathcal{C}}(\mathcal{D}_2^\star) < \mathcal{C}(\mathcal{D}_3^\star) \iff \alpha \geq \alpha_2 = \frac{\varepsilon\gamma(i_\pi - \tilde{i})}{\omega d\left[\rho_{\mathcal{I}}^- - (1-\varepsilon)\rho_1^-\right]}. \tag{A32}$$

In this case, since the deviation equilibrium does not depend on β, the relative weight on the financial stability objective that minimizes the costs of central bank action is equal to β_1, which allows the banking system to potentially converge to the least costly threat equilibrium.

Finally, if $\alpha < \alpha_2$, it is optimal to reduce the relative weight assigned to financial stability to $\beta_{\mathcal{I}_1}$, thereby steering the economy toward the inflation-targeting equilibrium, which constitutes the socially least costly outcome.

(ii). Let $i_\pi < \tilde{i}/\overline{\rho}$. In this case, $\rho_{\mathcal{P}}^- < \rho_{\mathcal{I}}^-$, and $\beta_{\mathcal{P}} > \beta_{\mathcal{I}_1}$. As Proposition 3 shows, regardless of the level of the anti-inflationary policy rate, the equilibrium that emerges when $\beta > \beta_1$ is the deviation equilibrium. Therefore, even when i_π is low, if $\alpha > \alpha_1$, the optimal choice is for the dual mandate to assign to the financial stability objective a relative weight that exceeds β_1, thus minimizing the social costs of central bank monetary and rescue policies.

However, when $\alpha < \alpha_1$ and $i_\pi < \tilde{i}/\overline{\rho}$, the alternative to the multiple equilibrium formed by $\mathcal{D}_1^\star$ and $\mathcal{D}_2^\star$ is a second deviation equilibrium, $\mathcal{D}_2^{*\prime}$, which is characterized by a higher share of illiquid banks than $\mathcal{D}_2^\star$, yet lower than the share prevailing in the inflation-targeting equilibrium. Once again, since the costs associated with the threat equilibrium decrease as β increases, the minimum costs arising from this equilibrium are attained when β takes the maximum value consistent with its formation, which, in this case according to Proposition 3, is $\beta_1' = \beta_{\mathcal{P}} - \eta$, with η arbitrarily small. At this threat equilibrium, the share of illiquid banks is $\rho_1(\beta_{\mathcal{P}} - \eta) = \rho_1^{-\prime}$, and the social cost is

$$\min_{\rho_1} \mathcal{C}(\mathcal{D}_2^{*\prime}) = \underline{\mathcal{C}}(\mathcal{D}_2^{*\prime}) = \pi\omega d\rho_1^{-\prime}. \tag{A33}$$

Therefore, by using the expressions (33), (A30) and (A33), we have:

$$\varepsilon\mathcal{C}(\mathcal{D}_1^\star) + (1-\varepsilon)\underline{\mathcal{C}}(\mathcal{D}_2^\star) < \underline{\mathcal{C}}(\mathcal{D}_2^{*\prime}) \iff \alpha \geq \alpha_2' = \frac{\varepsilon\gamma(i_\pi - \tilde{i})}{\omega d\big[\rho_1^{-\prime} - (1-\varepsilon)\rho_1^-\big]}. \tag{A34}$$

Since the deviation equilibrium does not depend on β, the optimal relative weight between the objectives of financial and monetary stability is $\beta_{\mathcal{P}} - \eta$, for which the banking system converges to the threat equilibrium $\mathcal{D}_2^{*\prime}$.

A11 Proof of Proposition 6

From (37):

$$\alpha_2 = \frac{\varepsilon\gamma(i_\pi - \tilde{i})}{\omega d\big[\rho_{\mathcal{I}}^- - (1-\varepsilon)\rho_1^-\big]} \tag{A35}$$

(i). By Lemma 6, $\rho_{\mathcal{I}}^- > \rho_1^-$. Therefore:

$$\frac{\partial\alpha_2}{\partial\varepsilon} = \frac{\gamma\big(i_\pi - \tilde{i}\big)\big(\rho_{\mathcal{I}}^- - \rho_1^-\big)}{\omega d\big[\rho_{\mathcal{I}}^- - (1-\varepsilon)\rho_1^-\big]^2} > 0. \tag{A36}$$

Let now $\varepsilon \to 1$, and $\alpha_2 \to \gamma(i_\pi - \tilde{i})/\omega d\rho_{\mathcal{I}}^-$.

(ii) From Proposition 2, $\partial\rho_{\mathcal{I}}^-/\partial i_\pi < 0$. Therefore:

$$\frac{\partial\alpha_2}{\partial i_\pi}_{|\varepsilon\to1} = \frac{\gamma}{\omega d\rho_{\mathcal{I}}^-}\left(1 - \frac{\partial\rho_{\mathcal{I}}^-/\partial i_\pi}{\rho_{\mathcal{I}}^-}\right) > 0. \tag{A37}$$

(iii) Similarly, using Proposition 2, it is immediate to verify that

$$\frac{\partial\alpha_2}{\partial\pi}_{|\varepsilon\to1} = -\frac{\gamma(i_\pi - \tilde{i})}{\omega d(\rho_{\mathcal{I}}^-)^2}\frac{\partial\rho_{\mathcal{I}}^-}{\partial\pi} \gtrless 0 \iff \rho_{\mathcal{I}}^- \lesseqgtr 1-\omega. \tag{A38}$$

(iv) Differentiating expression (A35) with respect to ω, by Proposition 2 we have:

$$\frac{\partial\alpha_2}{\partial\omega}_{|\varepsilon\to1} = -\frac{\gamma(i_\pi - \tilde{i})}{\omega^2 d(\rho_{\mathcal{I}}^-)^2}\left(\rho_{\mathcal{I}}^- + \omega\frac{\partial\rho_{\mathcal{I}}^-}{\partial\omega}\right) \gtrless 0 \iff \rho_{\mathcal{I}}^- \lesseqgtr -\omega\frac{\partial\rho_{\mathcal{I}}^-}{\partial\omega}. \tag{A39}$$

By using condition $\mathcal{A}_{\mathcal{I}}^- = \mathcal{R}_{\mathcal{I}}^-$, we can express $\rho_{\mathcal{I}}^-$ as an implicit function of ω:

$$\omega\big[\pi R + (1-\pi)p^\star(\rho_{\mathcal{I}}^-)\big] + (1-\omega)R = \omega + (1-\omega)\left[\pi(1+i_\pi) + (1-\pi)\frac{R}{p^\star(\rho_{\mathcal{I}}^-)}\right]. \tag{A40}$$

By substituting expression (6) for the market clearing price, we obtain:

$$\omega\pi R + (1-\pi)(1-\omega)\frac{1-\rho_{\mathcal{I}}^{-}}{\rho_{\mathcal{I}}^{-}} + (1-\omega)R = \omega + (1-\omega)\pi(1+i_\pi) + (1-\pi)\omega R\frac{\rho_{\mathcal{I}}^{-}}{1-\rho_{\mathcal{I}}^{-}}. \tag{A41}$$

Let us set $z = (1-\rho_{\mathcal{I}}^{-})/\rho_{\mathcal{I}}^{-}$. We can then rewrite equation (A40) as a quadratic equation in z as follows.

$$(1-\pi)(1-\omega)z^2 + \big[(1-\omega)(R-\pi(1+i_\pi)) - \omega(1-\pi R)\big]z - (1-\pi)\omega R = 0 \tag{A42}$$

Since the constant term, $-(1-\pi)\omega R$, is negative, and the second degree term, $(1-\pi)(1-\omega)$, is positive, there exists a unique positive root of z that solves equation (A42), given by

$$\begin{aligned}\hat{z} = &\frac{\sqrt{\big[(1-\omega)(R-\pi(1+i_\pi)) - \omega(1-\pi R)\big]^2 + 4(1-\pi)^2\omega R(1-\omega)}}{2(1-\pi)(1-\omega)} \\ &- \frac{\big[(1-\omega)(R-\pi(1+i_\pi)) - \omega(1-\pi R)\big]}{2(1-\pi)(1-\omega)}\end{aligned} \tag{A43}$$

or, more compactly, by collecting the terms in ω,

$$\hat{z} = \frac{\sqrt{A\omega^2 + B\omega + C} + D\omega + E}{F(1-\omega)} \tag{A44}$$

where A, B, C, D, E and F are collections of model parameters independent of ω. Reverting to $\rho_{\mathcal{I}}^{-} = 1/(1+\hat{z})$ and differentiating with respect to ω, we obtain:

$$\frac{\partial\rho_{\mathcal{I}}^{-}}{\partial\omega} = \frac{-\partial\hat{z}/\partial\omega}{(1+\hat{z})^2} = -\frac{(1-\omega)\big(D + \frac{2A\omega+B}{2\sqrt{A\omega^2+b\omega+C}}\big) + (1-\omega)\hat{z}}{\big[(1-\omega)+(1-\omega)\hat{z}\big]^2}. \tag{A45}$$

By substituting for $(1-\omega)\hat{z} = (\sqrt{A\omega^2 + B\omega + C} + D\omega + E)/F$, we have that the limit of the derivative (A45) as ω approaches 1 exists, is finite, and, by Lemma 5, is strictly negative:

$$\lim_{\omega\to 1}\frac{\partial\rho_{\mathcal{I}}^{-}}{\partial\omega} = -\frac{F}{\sqrt{A+B+C}+D+E} < 0. \tag{A46}$$

Moreover, from (A44), $\lim_{\omega\to 1}\hat{z} = \infty$, and hence:

$$\lim_{\omega\to 1}\rho_{\mathcal{I}}^{-} = 0 \tag{A47}$$

As a result, by using (A46) and (A47), we have that:

$$\frac{\partial \alpha_2}{\partial \omega}_{|\varepsilon \to 1 \text{ and } \omega = 1} > 0 \tag{A48}$$

Let us now consider the case in which ω approaches zero. First, by using (A44),

$$\lim_{\omega \to 0} \rho_{\mathcal{I}}^{-} = \frac{F}{F + \sqrt{C} + E} > 0 \tag{A49}$$

Second, from A44, it is readily verified that the limit of $\hat{z}$ for $\omega \to 0$ exists and is finite and, therefore, that:

$$\lim_{\omega \to 0} \omega \frac{\partial \rho_{\mathcal{I}}^{-}}{\partial \omega} = 0 \tag{A50}$$

Thus, by using (A49) and (A50), we obtain:

$$\frac{\partial \alpha_2}{\partial \omega}_{|\varepsilon \to 1 \text{ and } \omega = 0} < 0 \tag{A51}$$

thereby demonstrating that α_2 is not monotone. Finally, from the expression (A35) for α_2, and by using (A49) and (A47), it immediately follows that

$$\lim_{w \to 0} \alpha_2 = \lim_{w \to 1} \alpha_2 = \infty. \tag{A52}$$

Supplementary Appendix B (intended online): Case $\mathcal{I}_2^\star$

B1 Equilibrium

In this section, we discuss the case where the model's parameters are such that the equilibrium triplet in the inflation targeting mandate corresponds to $\mathcal{I}_2^\star$.

Lemma B1. *Let ρ_2^- denote the value of ρ at which $\mathcal{A}_\mathcal{I}^+ = \mathcal{R}_\mathcal{P}^+$. If $\rho_2^- \in \mathcal{P}^{\mathcal{C}+}$, then $\rho_2^- > \rho_\mathcal{I}^+$.*

Proof. By definition, at $\rho_\mathcal{I}^+$, we have $\mathcal{A}_\mathcal{I}^+ = \mathcal{R}_\mathcal{I}^+$. By comparing equations (13′) and (28) follow that $\mathcal{R}_\mathcal{I}^+ > \mathcal{R}_\mathcal{P}^+$, for all ρ. Finally, as the expected return of the short-term asset increases as ρ increases, it must be that $\rho_2^- > \rho_\mathcal{I}^+$. □

The equivalent of Proposition 3 can be stated as follows:

Proposition B1. Let

$$\beta_{\mathcal{I}_2} = \frac{\gamma(i_\pi - \tilde{i})}{\alpha(1-\omega)(1-\rho_\mathcal{I}^+)} \quad \beta_2 = \frac{\gamma(i_\pi - \tilde{i})}{\alpha(1-\omega)(1-\rho_2^+)} \tag{B1}$$

Under $\mathcal{D}$-mandate, the subgame perfect Nash equilibrium is as follows:

(i) when $\beta < \min\left(\beta_\mathcal{P}, \beta_{\mathcal{I}_2}\right)$, the equilibrium is unique and globally stable, and coincides with the inflation target equilibrium $\mathcal{I}_2^\star$

(ii) If $\beta \in \left(\min\left(\beta_\mathcal{P}, \beta_{\mathcal{I}_2}\right), \min\left(\beta_2, \max\left(\beta_\mathcal{P}, \beta_{\mathcal{I}_2}\right)\right)\right.$: **(ii.a)** If $\min\left(\beta_\mathcal{P}, \beta_{\mathcal{I}_2}\right) = \beta_\mathcal{P}$, the $\mathcal{D}-$mandate admits two possible locally stable equilibrium: the deviation equilibrium $\mathcal{D}_1^\star$, and the inflation target equilibrium; If $\min\left(\beta_\mathcal{P}, \beta_{\mathcal{I}_2}\right) = \beta_{\mathcal{I}_2}$, the equilibrium is unique, locally stable from the right and described by the triplet

$$\mathcal{D}_4^\star = \begin{cases} i_{1\mathcal{D}}^\star = i_T \\ \mathcal{B}_\mathcal{D}^\star = \begin{cases} \left(\frac{\omega d\rho_2}{(1-\omega)(1-\rho_2)} - 1, 0\right) & \text{if } i_T = 0 \\ (0,1) & \text{if } i_T = i_\pi \end{cases} \\ \rho_\mathcal{D}^\star = \rho_2 = 1 - \frac{\gamma(i_\pi - \tilde{i})}{\beta\alpha(1-\omega)} \end{cases} \tag{B2}$$

(iii) If $\beta \in \left(\min\left(\beta_2, \max\left(\beta_\mathcal{P}, \beta_{\mathcal{I}_2}\right)\right), \max\left(\beta_2, \max\left(\beta_\mathcal{P}, \beta_{\mathcal{I}_2}\right)\right)\right)$, if the model admits equilibria, it admits two possible equilibria: the deviation equilibrium $\mathcal{D}_2^\star$, and the triplet $\mathcal{D}_4^\star$.

(iv) If $\beta \geq \max\left(\beta_2, \max\left(\beta_\mathcal{P}, \beta_{\mathcal{I}+}\right)\right)$, the equilibrium is unique and globally stable, and coincides with the deviation equilibrium $\mathcal{D}_1^\star$

Proof. **(i)** When $\beta < \min\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right)$, neither $\rho_{\mathcal{I}}^{+}$ nor $\rho_{\mathcal{P}}^{-}$ belong to the deviation set $\mathcal{P}$ and, by using Lemma 3, $\Delta_{\mathcal{D}}$ has a unique zero at $\rho_{\mathcal{I}}^{+}$. In this case, the equilibrium under the dual mandate coincides with the inflation target equilibrium, with the central bank setting the policy rate at the level consistent with price stability, acting as a lender of last resort in support of all banks affected by liquidity shocks during inflation waves, and lending to liquid banks when the realized target rate is null.

The inflation target equilibrium is also the unique globally stable equilibrium. In fact, when neither $\rho_{\mathcal{I}}^{+}$ nor $\rho_{\mathcal{P}}^{-}$ belong to the deviation set $\mathcal{P}$, at $\rho = \rho_2$, we have $\Delta_{\mathcal{P}} > 0$ meaning that banks always find it optimal to adjust their portfolios, regardless of the monetary and bailout policies adopted by the central bank.

(ii) For $\beta \geq \min\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right)$, one of $\rho_{\mathcal{I}}^{+}$ or $\rho_{\mathcal{P}}^{-}$ belongs to the set $\mathcal{P}$. In cases where $\min\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right) = \beta_{\mathcal{P}}$, there exist values of β such that $\rho_{\mathcal{D}}^{-} \in \mathcal{P}$ and $\rho_{\mathcal{I}}^{+} \notin \mathcal{P}$. In this case, by using Lemma 3, $\Delta_{\mathcal{D}}$ takes on the value zero at two different points: $\rho_{\mathcal{D}}^{-}$ and $\rho_{\mathcal{I}}^{+}$. Lemma B1 implies that $\beta_2 > \beta_{\mathcal{I}_2}$. Therefore, as long as $\beta \in [\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2})$, the dual mandate has two equilibria: the deviation equilibrium $\mathcal{D}_1^{\star}$ and the inflation target equilibrium $\mathcal{I}_2^{\star}$.

In cases where $\min\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right) = \beta_{\mathcal{I}_2}$, there exist values of β such that $\rho_{\mathcal{D}}^{-} \notin \mathcal{P}$ and $\rho_{\mathcal{I}}^{+} \in \mathcal{P}$. In this case, by using Lemma 3, there are no values of β for which $\Delta_{\mathcal{D}}$ is zero. Lemma B1 implies that, for $\beta_2 > \beta > \beta_{\mathcal{I}_2}$, at $\rho = \rho_2$ there are no banks that will increase expected return by switching investment decision. Therefore, as long as $\beta \in [\beta_{\mathcal{I}_2}, \min(\beta_2, \beta_{\mathcal{P}})]$, the model admits a unique equilibrium described by the triplet $\mathcal{D}_4^{\star}$.

(iii) In cases where $\beta > \min\left(\beta_2, \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right)\right)$, both $\rho_{\mathcal{I}}^{+}$ and $\rho_{\mathcal{P}}^{-}$ belong to the deviation set $\mathcal{P}$. When $\min\left(\beta_2, \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right)\right) = \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right)$, values for β such both $\rho_{\mathcal{I}}^{+}$ and $\rho_{\mathcal{P}}^{-}$ belong to the deviation set $\mathcal{P}$ while ρ_2^{-} does not exist. In accordance to the bullet (ii.b), ρ_2 constitutes an equilibrium. However, as $\rho_{\mathcal{P}}^{-} \in \mathcal{P}$, the deviation equilibrium is also an equilibrium of the dual mandate. When $\min\left(\beta_2, \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right)\right) = \beta_2$, Lemma B1 implies that at $\rho = \rho_2$, the discontinuity point ρ_2 cannot constitute an equilibrium. In fact, at ρ_2, banks investing in long-term asset will prefer to hold reserves. Moreover, as $\beta_2 > \beta_{\mathcal{I}_2}$, $\min\left(\beta_2, \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right)\right) = \beta_2$ implies $\max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right) = \beta_{\mathcal{P}}$. Therefore, in this case, for $\beta \in \left(\beta_2, \beta_{\mathcal{P}}\right)$ the model does not admit equilibria.

(iv) For $\beta \geq \max\left(\beta_2, \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}_2}\right)\right)$, $\rho_{D}^{-}, \rho_{\mathcal{I}}^{+}, \rho_2^{-}$ all belong to $\mathcal{P}$. Hence, Lemma 4 and B1 implies that the deviation equilibrium is the unique equilibrium of the dual mandate.

□

B2 Mandate costs

When the equilibrium in the inflation target mandate is $\mathcal{I}_2^\star$, total costs are:

$$\mathcal{C}(\mathcal{D}_4^\star) = \pi\omega d\alpha\rho_{\mathcal{I}}^{+} + (1-\pi)\alpha(1-\omega)(1-\rho_{\mathcal{I}}^{+})\theta_{1-\rho}^\star \tag{B3}$$

The difference from the equilibrium triplet $\mathcal{D}_3^\star$ is that, in this case, rescue costs are also positive if inflation waves do not materialize. We keep the assumption that, when two alternative possible equilibria emerge, the expected costs associated with the dual mandate are a weighted average of the costs of the two possible equilibria. Therefore, we can state the expected costs of the dual mandate as:

$$E(\mathcal{C}\,|\beta) = \begin{cases} \mathcal{C}(\mathcal{D}_1^\star) & \text{if } \ \beta > \max\left(\beta_2, \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}+}\right)\right) \\ \psi\mathcal{C}(\mathcal{D}_1^\star) + (1-\psi)\mathcal{C}(\mathcal{D}_4^\star) & \text{if } \ \beta \in \left(\min\left(\beta_2, \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}+}\right)\right), \max\left(\beta_2, \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}+}\right)\right)\right) \\ \psi'\mathcal{C}(\mathcal{D}_1^\star) + (1-\psi')\mathcal{C}(\mathcal{I}_2^\star) & \text{if } \ \beta \in \left(\min\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}+}\right), \min\left(\beta_2, \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}+}\right)\right)\right) \\ & \text{and } \ \min\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}+}\right) = \beta_{\mathcal{P}} \\ \mathcal{C}(\mathcal{D}_4^\star) & \text{if } \ \beta \in \left(\min\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}+}\right), \min\left(\beta_2, \max\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}+}\right)\right)\right) \\ & \text{and } \ \min\left(\beta_{\mathcal{P}}, \beta_{\mathcal{I}+}\right) = \beta_{\mathcal{I}+} \\ \mathcal{C}(\mathcal{I}_2^\star) & \text{if } \ \beta \in \left[\underline{\beta}, \min\left(\beta_{\mathcal{I}+}, \beta_{\mathcal{P}}\right)\right] \end{cases} \tag{B4}$$

From Lemma B1 follows that the equilibrium triplet $\mathcal{D}_4^\star$ always provides more costs than the inflation target equilibrium. Therefore, to determine the cheapest mandate of the economy we need the comparison between the deviation equilibrium and the inflation target equilibrium. From equations (B3) and (33), we get that the deviation equilibrium provides costs lower than the inflation target equilibrium if and only if α is larger than a critical threshold:

$$\alpha \leq \hat{\alpha}_2 = \frac{\gamma\pi(i_\pi - \tilde{i})}{\left[\pi d\omega\rho_{\mathcal{I}}^{+} + (1-\pi)(1-\omega)(1-\rho_{\mathcal{I}}^{+})\theta_{1-\rho}^\star\right]} \tag{B5}$$

Economic intuitions and comparative static exercise on the threshold $\hat{\alpha}_2$ are specular to those we derive in the paper. Therefore, we can conclude that, in cases where $i_\pi < \iota$, the inflation target mandate involves costs lower than the dual mandate if and only if $\alpha < \hat{\alpha}_2$.