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Coalitional Control and the Core with Consumption Externalities

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Abstract
We introduce a family of core notions for pure exchange economies with consumption externalities by separating the coalition whose members benefit from a deviation from the economic domain through which the deviation is implemented. Each nonempty beneficiary coalition S is associated with a control set A_S, which determines both the consumption positions that may be modified and the resources available to the deviation. The framework accommodates self control, Berge type complement control, and intermediate control structures. We study the efficiency and resource properties of the associated cores and provide conditions under which they coincide with the set of Pareto optimal allocations. We also examine how changes in the control correspondence affect blocking power and core inclusion, obtaining implications for existence results. For a class of network economies, we characterize core allocations and show that changing control rights can restore non-emptiness when the self-regarding core is empty. Finally, we introduce a pessimistic version of the core, in which beneficiaries must improve under every feasible redistribution outside the control domain, and establish corresponding results on efficiency, resource characterization, and control expansion.

Keywords: coalitional control, core, exchange economy, consumption externalities, Berge equilibrium, pessimistic blocking.

JEL classification: C71, D51, D62.

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1 Introduction

The core is one of the central solution concepts in cooperative analysis, both in game theory and in exchange economies. It expresses a requirement of coalitional stability: an outcome belongs to the core if no coalition can challenge it through a feasible deviation that improves the welfare of its members. The precise meaning of this requirement depends, however, on how the opportunities available to each coalition are specified.

This dependence on coalitional opportunities becomes particularly transparent in an exchange economy. The blocking mechanism brings together two roles that need not, in principle, coincide. A coalition S identifies the agents whose welfare must improve, while the economic domain over which the deviation can act determines which consumption positions may be changed and which resources may be used to generate that improvement. In the standard core, these two roles are identified: a coalition reallocates its own resources among its own members. With consumption externalities, however, there is no compelling reason to maintain this identification. Self-regarding blocking preserves this coincidence, whereas Berge-type behavior suggests the opposite possibility, in which an agent's welfare is improved through changes in the consumption of others; see, for example, Haller (2024).

The purpose of this note is to separate these two dimensions of coalitional blocking. To every nonempty beneficiary coalition $S \subseteq N$, we associate a nonempty control set $A_S \subseteq N$. The members of S are the agents who must benefit from the deviation, whereas A_S identifies both the consumption coordinates that may be modified and the resources available to the deviation. The correspondence

$$S \longmapsto A_S$$

is taken as a primitive description of coalitional control. The resulting framework generates a family of core notions in which the beneficiary coalition and the domain through which its improvement is implemented need not coincide.

This formulation provides a common framework for several otherwise different blocking mechanisms. At one pole, if

$$A_S = S,$$

beneficiaries control their own consumption positions and resources. Under the baseline conjecture that consumption outside the control domain remains fixed, this gives the usual self-regarding or Nash-type blocking convention for exchange economies with consumption externalities. With selfish preferences, it reduces further to ordinary coalitional blocking. At the other pole, proper beneficiary coalitions have complement control:

$$A_S = N \setminus S \quad \text{for every } \emptyset \neq S \subsetneq N, \qquad A_N = N.$$

For every proper beneficiary coalition, the consumption of its members remains fixed and their improvement can be generated only through changes in the consumption of non-beneficiaries. For singleton beneficiary coalitions, this reproduces the beneficiary-support separation underlying the Berge-Zhukovskii mutual-support principle; for general proper coalitions, it provides its natural coalitional counterpart. The grand coalition, by contrast, has full control and retains its usual role in testing Pareto improvements. Beyond these two polar benchmarks, extended, restricted and mixed structures allow self-control and other-control to coexist. The objective is therefore not to propose a core associated with one particular behavioral interpretation, but to provide a common coalitional control framework within which self-regarding, Berge-type and intermediate forms of blocking can be compared.

The Berge-type pole reflects the mutual support principle underlying Berge equilibrium. In its standard Berge-Zhukovskii formulation, the agents whose actions are chosen and the agent whose welfare is supported are distinct: for each player i, the other players coordinate their

actions so as to support i's payoff, given i's own action[1].

The coalitional formulation introduced by Berge (1957) and studied by Courtois et al. (2017) makes this separation explicit at the group level. Let

$$\mathcal{R} = \{R_m\}_{m \in M}$$

be a partition of the set of players, and let

$$\mathcal{K} = \{K_m\}_{m \in M}$$

be a family of supporting groups. A strategy profile x is a coalitional Berge equilibrium if

$$u_i(x) \geq u_i(x_{-K_m}, y_{K_m})$$

for every $m \in M$, every $i \in R_m$, and every $y_{K_m} \in X_{K_m}$. Thus, R_m identifies the group whose welfare is supported, whereas K_m identifies the group whose strategies provide that support. With singleton beneficiary groups, this formulation includes Nash equilibrium when

$$R_i = \{i\}, \qquad K_i = \{i\},$$

and Berge-Zhukovskii equilibrium when

$$R_i = \{i\}, \qquad K_i = N \setminus \{i\}.$$

The present construction takes this beneficiary-supporter separation as its starting point but extends it in two directions. First, a control domain is associated with every nonempty beneficiary coalition, rather than only with the elements of a fixed partition. Second, its application to a pure exchange economy requires control to be specified over both consumption positions and resources. In this sense, A_S plays a role analogous to that of the supporting group K_m, but it does not identify a second acting coalition. It identifies the economic domain through which the improvement of the members of S can be implemented. Depending on the application, the availability of that domain may reflect control rights, institutional authority, or support available to the beneficiaries.

The beneficiary-control distinction admits several complementary interpretations. One is in terms of property and control rights. Xiong and Zheng (2007) make coalitional opportunities depend on control rights over firms, while Balbuzanov and Kotowski (2019) reinterpret endowments as exclusion rights. Balbuzanov and Kotowski (2024) extend this approach to production networks. Related formulations incorporate tradable rights into coalitional feasibility (Hervés-Beloso and Moreno-García, 2022) or base blocking opportunities on the objects effectively controlled by a coalition (Sun et al., 2026). Although these models differ from the present one, they share the idea that coalition membership need not determine the entire economic domain over which a coalition can exercise effective rights.

At a more abstract level, the control correspondence may be viewed as generating a structured form of the coalitional effectivity studied by Moulin and Peleg (1982). Ray and Vohra (2015) similarly distinguish an active coalition from other agents involved in its opportunities. In the present construction, this general idea acquires a specific exchange economy interpretation: the control set A_S identifies both the consumption positions that may be modified and the resources available to the deviation.

More concrete interpretations arise in households and organizations. Consider a household with three members whose decision rules allow its resources to be pooled and consumption to be redistributed among all three. A plan intended to improve the welfare of two members may then change the third member's consumption and use her endowment, without requiring

[1]Recent work also examines the existence and computation of Berge equilibria in finite games through algebraic methods; see Riedel and Torrente (2026).

that she also benefit. The beneficiaries therefore differ from the household domain through which their improvement is implemented. Collective household models such as Gersbach and Haller (2001) provide a natural background for this interpretation. Similarly, an organization may have authority to use the resources and adjust the consumption allocations of a broader administrative unit to benefit a particular group within it. In both settings, the relevant decision rules or authority determine the domain available to support the beneficiaries.

International agreements provide a further interpretation. Trade policies may affect other countries through market access and terms of trade, creating a rationale for international cooperation; see Bagwell and Staiger (1999). In this setting, an adjustment or compensation mechanism may benefit a group of countries while drawing on resources committed by a broader set of participants. Environmental agreements provide a related example. The conservation or restoration of a shared environmental resource may benefit some countries or communities while relying on the resources or commitments of others, as in the management of a shared water basin, regional fisheries, or a jointly financed conservation program. In all these cases, the beneficiaries need not coincide with the agents whose resources or commitments support the intervention.

Across these different interpretations, the distinction between S and A_S acquires particular importance in the presence of consumption externalities. An agent in $S \setminus A_S$ may benefit even though her own consumption remains fixed, while the consumption of an agent in $A_S \setminus S$ may be modified although that agent is not a beneficiary. Effective blocking power therefore depends jointly on the control domain and on the preferences of the beneficiaries.

The distinction remains relevant under selfish preferences. In this case, effective blocking requires

$$S \subseteq A_S,$$

since every beneficiary's own consumption position must belong to the coalition's control domain. When the inclusion is strict, however, the additional agents in A_S may still contribute resources to the deviation. Preference dependence and resource control are therefore two distinct channels through which the separation between beneficiaries and controllers has economic content.

The benchmark $A_S = S$ connects the construction with the literature on cores of exchange economies with consumption externalities. Under the Nash-type conjecture adopted in our baseline core, a coalition reallocates its own initial resources while consumption outside the coalition remains at the status quo. This is the Walrasian-Nash blocking convention considered by Dufwenberg et al. (2011) and Graziano et al. (2023). The latter refer to the associated core as the γ-core and emphasize that, with consumption externalities, a dominance notion depends both on the resources available to the deviating coalition and on the conjecture about outside consumption.

We also develop a resource formulation of the coalitional control core. This part of the analysis connects the present construction to the literature on resource measures of inefficiency and coalitional instability. The underlying idea goes back to Debreu (1951) and was adapted by Montesano (2002) to obtain a characterization of the core in terms of resource saving. Di Pietro et al. (2022) extend this approach to exchange economies with consumption externalities. The present framework introduces a further distinction: welfare is evaluated for the beneficiaries in S, whereas resource saving is measured on the control domain A_S.

A separate part of the analysis concerns the treatment of consumption outside the control domain. In the baseline core, a Nash-type conjecture keeps consumption on $N \setminus A_S$ fixed at the status quo. Alongside this baseline, we study a pessimistic version in which the members of S must improve under every feasible redistribution on $N \setminus A_S$. When $A_S = S$, this form of pessimistic blocking gives the exchange economy counterpart of the α-dominance introduced by Aumann (1961). For a general control structure, however, the agents outside A_S need not coincide with the agents outside S. The pessimistic condition is therefore a conjecture about

consumption outside the control domain, not a description of the complementary coalition $N \setminus S$ exercising its own control rights. Both the plan on A_S and each redistribution on $N \setminus A_S$ satisfy their respective resource constraints, ensuring feasibility of the resulting social consumption profile.

Taken together, these considerations motivate the family of pure exchange cores studied in this note. Each beneficiary coalition is associated with a possibly different consumption and resource control domain. This separation makes it possible to examine how beneficiary membership, control, consumption externalities, and conjectures about outside consumption jointly affect blocking power, efficiency, resource properties, and core nonemptiness.

Our analysis yields results on efficiency, resource characterization, and comparisons across control structures. The core retains familiar efficiency properties. Core allocations are Pareto optimal whenever some nonempty beneficiary coalition has full control, and the core coincides with the Pareto optimal set under stronger conditions ensuring that only the grand coalition can effectively block. The core also admits a resource formulation in which welfare is evaluated for S and resource saving is measured on A_S. Under an appropriate Control Monotonicity condition, the preference and resource cores coincide.

A further set of results concerns comparisons across control correspondences. Under suitable restrictions on the consumption effects associated with newly controlled positions, expanding the control domain strengthens blocking power and reduces the associated core. Under pessimistic blocking, the same core inclusion holds without a sign restriction on those effects. These comparisons also provide a way of transferring nonemptiness results from benchmark control structures to other control correspondences. Parallel efficiency and resource characterization results are established for the pessimistic core. The network application also illustrates how compatibility between heterogeneous preferences and control rights can support a nonempty core even when selfish and other-regarding agents coexist.

The control correspondence may represent underlying property, authority, or network relations. Individual control sets, for example, may generate coalitional control domains through unions, with family and household structures as natural special cases. Related A-economy models, such as Vasil'ev (2016) and Graziano et al. (2026), show how individual opportunity and budget sets involving the consumption of several agents can enter a competitive framework. These connections point to possible competitive and network foundations for coalitional control.

The remainder of the note is organized as follows. Section 2 introduces the economy, the control correspondence, and the baseline core. Section 3 presents the main benchmark control structures. Section 4 studies efficiency, effective blocking power, and the resource characterization of the core. Section 5 compares control correspondences and derives implications for core non-emptiness. Section 6 introduces and studies the pessimistic version of the core. Section 7 concludes.

2 The economy and the control structure

Consider a finite pure exchange economy

$$E = \left\langle N, \mathbb{R}^{\ell}_{+}, (\succeq_i, \omega_i)_{i \in N} \right\rangle.$$

Here $\omega_i \in \mathbb{R}^{\ell}_{+}$ is agent i's initial endowment, and preferences are defined over social consumption profiles, allowing for consumption externalities.

Throughout, preferences $\succeq_i$ are assumed to be complete, transitive and continuous on the social consumption space

$$\mathbb{R}^{\ell|N|}_{+}.$$

A social consumption profile

$$x = (x_i)_{i \in N}$$

is an element of $\mathbb{R}_+^{\ell|N|}$. For every $B \subseteq N$, we denote by

$$x_B = (x_i)_{i \in B}$$

the restriction of x to the agents in B. Given profiles x_B and $y_{N \setminus B}$, we use

$$(x_B, y_{N \setminus B})$$

to denote the corresponding social consumption profile.

For every agent i, every social consumption profile x, and every nonempty set $B \subseteq N$, define

$$P_i^B(x) := \left\{ z_B \in \mathbb{R}_+^{\ell|B|} : (z_B, x_{N \setminus B}) \succ_i x \right\}.$$

Thus, $P_i^B(x)$ is the strict upper contour set of agent i on the B-section through x: consumption on B may vary, while consumption outside B remains fixed at its value in x. An agent i is said to have selfish preferences if, for any two social consumption profiles x and y,

$$x_i = y_i \quad \Longrightarrow \quad x \sim_i y.$$

A selfish agent cares only about her own consumption bundle and is indifferent to changes in the consumption of other agents.

A social consumption profile x is feasible if

$$\sum_{i \in N} x_i \leq \sum_{i \in N} \omega_i.$$

A feasible social consumption profile will be called an allocation. We denote the set of allocations by $F(E)$. A feasible allocation $x \in F(E)$ is Pareto optimal[2] if there exists no feasible allocation $z \in F(E)$ such that

$$z \succ_i x \qquad \text{for every } i \in N.$$

We denote the set of Pareto optimal allocations by

$$PO(E).$$

For vectors $a, b \in \mathbb{R}^\ell$, we write $a > b$ if $a \geq b$ componentwise and $a \neq b$.

To every nonempty coalition $S \subseteq N$, associate a nonempty set

$$A_S \subseteq N.$$

The set S is the *beneficiary coalition*: every member of S must strictly benefit from the deviation. The set A_S is the *control set*: it identifies the consumption positions that may be changed and the initial resources available to implement the improvement of S.

Definition 1 (Coalitional improvement relative to a control set)**.** *Let x be feasible. A nonempty coalition $S \subseteq N$ improves upon x relative to A_S if there exists*

$$z_{A_S} \in \mathbb{R}_+^{\ell|A_S|}$$

such that

$$\sum_{h \in A_S} z_h \leq \sum_{h \in A_S} \omega_h$$

and

$$(z_{A_S}, x_{N \setminus A_S}) \succ_i x \qquad \textit{for every } i \in S.$$

[2]We use Pareto optimality to mean that no feasible allocation is strictly preferred by every agent. This property is also sometimes called weak Pareto optimality. In economies with consumption externalities, it need not coincide with strong Pareto optimality, defined by weak improvement for every agent and strict improvement for at least one. Sufficient conditions for the two notions to coincide are provided by Graziano et al. (2023, Remark 3.1).

The associated core consists of the feasible allocations that cannot be improved upon by any nonempty coalition S. We denote it by

$$C_{\mathcal{A}}(E),$$

where

$$\mathcal{A} = (A_S)_{\varnothing \neq S \subseteq N}$$

is the underlying control structure.

The following simple result clarifies the role of consumption externalities in turning potential control into effective blocking power. It identifies a necessary preference condition for a control set to support a coalitional improvement and shows what this condition entails under selfish preferences.

Proposition 1 (Preference dependence and blocking)**.** *If a nonempty coalition S improves upon an allocation x relative to A_S, then*

$$P_i^{A_S}(x) \neq \varnothing \qquad \text{for every } i \in S.$$

If preferences are selfish, improvement by S further requires

$$S \subseteq A_S.$$

Proof. Any improving plan on A_S belongs to $P_i^{A_S}(x)$ for every $i \in S$.

If preferences are selfish and $i \in S \setminus A_S$, every change on A_S leaves i's own consumption unchanged. Hence $P_i^{A_S}(x) = \varnothing$, which excludes improvement by S. □

The preference condition in the proposition is necessary but not sufficient for blocking. Each beneficiary may admit an improvement on A_S, yet a block requires a single plan that strictly improves all members of S and satisfies the resource constraint.

Under selfish preferences, the proposition requires every beneficiary to belong to the control set:

$$S \subseteq A_S.$$

The case $A_S = S$ is therefore the natural self-regarding benchmark. Consumption externalities allow improvements beyond this restriction. A beneficiary $i \in S \setminus A_S$ may strictly benefit from changes in consumption on A_S, even though her own bundle remains fixed.

The contribution of a controlled agent need not, however, operate through consumption effects. Even if changes in the consumption of an agent $h \in A_S$ do not affect any beneficiary's preferences, her endowment contributes to the resources available on A_S. Removing h from the control set may therefore change the blocking possibilities.

Remark 1. When A_S contains agents outside S, the definition attributes to S control over resources and consumption opportunities associated with non-members. Thus, A_S should be interpreted as a primitive describing rights, authority, or effective control rather than merely as another coalition.

Remark 2. A related comparison arises with non-bossiness, introduced by Satterthwaite and Sonnenschein (1981) and used in matching models; see, for example, Kojima (2010). Non-bossiness requires that a change in an agent's report leaving her own assignment unchanged cannot alter the assignments of others. The present core is not defined through reports or an allocation rule, so this condition does not directly restrict the admissible coalitional deviations. It may nevertheless become relevant for an implementation of the control structure: a rule allowing members of S to induce changes on A_S while their own assignments remain fixed could fail non-bossiness. Consumption externalities do not remove this possible implementation issue, since standard non-bossiness is formulated in terms of assignments. They do, however, change its economic interpretation: a beneficiary whose own bundle remains fixed need not remain unaffected in welfare terms.

Remark 3. The definition assigns two distinct roles to the same set A_S. It determines both the consumption coordinates that coalition S can modify and the resources available to finance the deviation. A more general control structure could distinguish an *action set* A_S from a *resource set* R_S. The present model corresponds to the benchmark case in which action and resource rights coincide. We keep this simpler specification as the starting point.

The benchmark structures considered below isolate different combinations of preference dependence and resource control.

3 Benchmark cases

The general control structure $S \mapsto A_S$ encompasses a number of benchmark configurations. The main cases considered below are summarized in Table 1.

Table 1: Benchmark control structures

Case	**Control set**	**Interpretation**
Self-regarding control	$A_S = S$	The coalition reallocates only its members' resources and consumption.
Complement control	$A_S = N \setminus S, \quad S \subsetneq N,$ $A_N = N$	Members of every proper coalition keep their own bundles fixed and can improve only through changes in outsiders' consumption, while the grand coalition has full control.
Extended control	$S \subsetneq A_S \subsetneq N$	The coalition controls its members and some outsiders.
Restricted control	$A_S \subsetneq S$	Some beneficiaries have fixed consumption and can improve only through externalities generated inside A_S.
Mixed control	$S \not\subseteq A_S, \quad A_S \not\subseteq S$	Some beneficiaries lie outside the control set, while some outsiders belong to it.
Common control domain	$A_S = A$ for every S	All beneficiary coalitions act on the same consumption and resource domain; $A = N$ gives unrestricted control.
Network-generated control	$A_S = \bigcup_{i \in S} A_i$	Coalitional control is generated by individual control relations; family structures arise as a partition case.

3.1 Self-regarding coalitional control

If

$$A_S = S,$$

a deviation satisfies

$$\sum_{i \in S} z_i \leq \sum_{i \in S} \omega_i, \qquad (z_S, x_{N \setminus S}) \succ_i x \quad \text{for every } i \in S.$$

This is the standard Nash-type blocking convention for an exchange economy with consumption externalities. We denote the core associated with this self-regarding control structure by

$$C^{\mathrm{sr}}(E)$$

where the superscript refers to the control structure and does not impose any restriction on preferences.

If preferences are selfish, outsiders' consumption is irrelevant and the definition reduces to the ordinary blocking notion of a pure exchange economy. Thus, $C^{\mathrm{sr}}(E)$ coincides with the standard Walrasian core, up to the free disposal formulation of coalitional feasibility. With consumption externalities, the same control structure yields instead the Walrasian-Nash core studied, for example, by Dufwenberg et al. (2011) and Graziano et al. (2023).

3.2 Complement control

Consider the control structure

$$A_S = \begin{cases} N \setminus S, & \text{if } \emptyset \neq S \subsetneq N, \\ N, & \text{if } S = N. \end{cases}$$

For every nonempty proper coalition $S \subsetneq N$, the bundles of its members remain fixed, and an improvement satisfies

$$\sum_{h \in N \setminus S} z_h \leq \sum_{h \in N \setminus S} \omega_h, \qquad (x_S, z_{N \setminus S}) \succ_i x \quad \text{for every } i \in S.$$

Thus, improvement by a proper coalition is generated entirely through consumption externalities. When $S = N$, instead, the grand coalition has full control. A blocking plan is then any allocation z_N satisfying

$$z_N \succ_i x \quad \text{for every } i \in N.$$

For singleton beneficiary coalitions, this reproduces the beneficiary-support separation underlying the Berge-Zhukovskii mutual support principle. For general proper coalitions S, it provides its natural coalitional counterpart.

Under selfish preferences, Proposition 1 implies that no proper coalition can block, since its members lie outside the control set. Their own consumption remains fixed, so they cannot be strictly improved. Blocking by proper coalitions in this benchmark therefore has substantive content only through consumption externalities.

The grand coalition, however, has full control and can block precisely those allocations that admit a strict Pareto improvement. Hence, under selfish preferences,

$$C_{\mathcal{A}}(E) = PO(E).$$

Since the self-regarding core is contained in $PO(E)$, it follows that, under selfish preferences,

$$C^{\mathrm{sr}}(E) \subseteq C_{\mathcal{A}}(E).$$

Thus, relative to the standard core, complement control weakens effective blocking power by proper coalitions while preserving the efficiency test performed by the grand coalition.

3.3 Extended control

If

$$S \subsetneq A_S \subsetneq N,$$

the coalition controls its own members and some outsiders. The set

$$A_S \setminus S$$

identifies outsiders whose resources and consumption opportunities belong to the control domain of S. This case combines self-regarding and other-regarding channels of improvement.

With selfish preferences, the other-regarding channel disappears but the case remains non-trivial. The inclusion $S \subseteq A_S$ satisfies the necessary condition in Proposition 1: every beneficiary's own consumption can be modified. Moreover, the endowments of agents in $A_S \setminus S$ enlarge the resources available to the deviation, even though their consumption does not affect the beneficiaries' preferences. Whether a block exists still depends on preferences and resource availability. Thus, beneficiary membership and resource control remain distinct even in the absence of consumption externalities.

This case is related in spirit to approaches in which coalitional blocking power is determined by property, exclusion, or effectivity rights rather than by coalition membership alone. The present formulation specifies the enlarged resource domain A_S directly, without requiring it to coincide with the set of beneficiaries.

3.4 Restricted control

If

$$A_S \subsetneq S,$$

every $i \in S \setminus A_S$ keeps her bundle x_i fixed and can benefit only through externalities generated by changes in A_S. This case separates especially clearly the beneficiary role from the control role.

Under selfish preferences, Proposition 1 excludes blocking by such a coalition, since $S \nsubseteq A_S$. The beneficiaries outside the control set cannot be strictly improved. Restricted control can thus generate blocking power only through consumption externalities.

3.5 Mixed control

If neither set contains the other,

$$S \setminus A_S \neq \varnothing, \qquad A_S \setminus S \neq \varnothing,$$

the control domain excludes some beneficiaries and includes some outsiders. Beneficiaries in $S \cap A_S$, if any, may have their own consumption modified, whereas those in $S \setminus A_S$ keep their own bundles fixed.

As in the restricted control case, Proposition 1 implies that a mixed pair (S, A_S) cannot generate a block under selfish preferences, since $S \nsubseteq A_S$. Its blocking power therefore relies on consumption externalities for the beneficiaries outside the control set, while the outsiders in $A_S \setminus S$ may matter through both their consumption and their resources.

3.6 Common control domain

Suppose that

$$A_S = A \qquad \text{for every nonempty } S \subseteq N,$$

for some fixed nonempty set $A \subseteq N$. All beneficiary coalitions then act on the same consumption and resource domain. This may represent a common institutional or organizational pool available to different beneficiary coalitions.

Under selfish preferences, Proposition 1 implies that only coalitions $S \subseteq A$ can have effective blocking power. A beneficiary outside A keeps her own consumption fixed and cannot be strictly improved. With consumption externalities, this restriction need not hold, because beneficiaries outside A may improve through changes in the consumption positions belonging to A.

The polar case $A = N$ corresponds to unrestricted control. Every coalition then has access to the whole social resource endowment and may propose any allocation. Coalitions differ only in the set of agents whose strict improvement is required. As shown by Proposition 3 below, blocking under a common control domain is already captured by singleton beneficiary coalitions.

3.7 Network-generated control

Individual control relations can be represented by a directed network on N: a link from i to h means that agent i has control over the consumption position and resources of agent h. For each $i \in N$, let the nonempty set

$$A_i \subseteq N$$

collect the agents over whom i has such control. These relations need not be reciprocal, and an agent need not control her own consumption and resources.

Coalitional control is generated by pooling the individual control domains of coalition members:

$$A_S := \bigcup_{i \in S} A_i.$$

Thus, an agent belongs to A_S whenever she belongs to the control domain of at least one member of S. Since each A_i is nonempty, so is A_S for every nonempty coalition S, as required by the general definition of a control structure.

Under selfish preferences, Proposition 1 implies that coalition S can have effective blocking power only if

$$S \subseteq A_S = \bigcup_{i \in S} A_i.$$

Every beneficiary must therefore belong to the control domain of at least one coalition member. In particular, if individual control is reflexive,

$$i \in A_i \qquad \text{for every } i \in N,$$

then every coalition controls at least the consumption positions and resources of its own members. The resulting structure extends the standard self-regarding case by allowing individual control relations to enlarge the domain available to a coalition.

The network-generated construction also provides a link with the coalitional Berge architecture discussed in the Introduction. Let

$$\mathcal{R} = \{R_m\}_{m \in M}$$

be a partition of N, and associate with each R_m a nonempty supporting set $K_m \subseteq N$. If

$$A_i = K_m \qquad \text{for every } i \in R_m,$$

then

$$A_{R_m} = \bigcup_{i \in R_m} A_i = K_m.$$

Thus, for the beneficiary group R_m, the network-generated control domain reproduces the beneficiary-supporter separation underlying a coalitional Berge equilibrium. The connection is structural rather than an equivalence between solution concepts: coalitional Berge equilibrium is defined relative to a fixed partition and strategic deviations, whereas the present core considers all beneficiary coalitions and assigns control over both consumption positions and resources.

The special case $K_m = R_m$, in which each group controls its own consumption and resource domain, generates a family structure based on a partition of the population. To describe this case, let

$$\mathcal{H} = \{H_1, \ldots, H_k\}$$

be a partition of N into families, let $H(i)$ denote the family containing agent i, and set

$$A_i = H(i).$$

Then

$$A_S = \bigcup_{i \in S} H(i)$$

is the union of the families represented in S. Suppose, in addition, that each agent's preferences depend only on consumption within her family:

$$\succeq_i \text{ depends only on } x_{H(i)}.$$

This specification is closely related to the collective household economies studied by Gersbach and Haller (2001), in which individual preferences may exhibit intrahousehold consumption externalities and the household constitutes the relevant consumption and resource domain.

The present blocking notion additionally distinguishes the beneficiary set from this household control domain. If S contains only some members of a family, the entire family nevertheless belongs to A_S, while strict improvement is required only for those family members who belong to S. If, instead, admissible beneficiary coalitions are required to be unions of complete families, then

$$A_S = S,$$

and beneficiary and household control domains coincide. This provides the natural benchmark for comparison with the corresponding core notions in collective household economies.

When the sets A_i overlap without being generated by a partition, the same construction describes more general networks of individual control relations.

4 Core, Pareto optimality and resource characterization

We first examine how the distinction between beneficiaries and control domains affects efficiency and effective blocking power. We then establish a resource characterization of the core.

Proposition 2 (Full control and Pareto optimality)**.** *Suppose that there exists a nonempty coalition $S \subseteq N$ such that*

$$A_S = N.$$

Then

$$C_{\mathcal{A}}(E) \subseteq PO(E).$$

Proof. Let $x \in C_{\mathcal{A}}(E)$. If $x \notin PO(E)$, some allocation $z \in F(E)$ satisfies

$$z \succ_i x \qquad \text{for every } i \in N.$$

Since $A_S = N$, coalition S can implement z, which strictly improves every member of S. This contradicts core membership. □

The condition holds whenever $A_N = N$, as under self-regarding, complement and unrestricted control. For network-generated control, it holds whenever $\bigcup_{i \in N} A_i = N$, in particular when individual control is reflexive. Full control need not, however, belong to the grand coalition: any nonempty beneficiary coalition with control over the whole economy can test strict Pareto improvements.

A second implication concerns coalitions sharing a control domain.

Proposition 3 (Beneficiary monotonicity under common control)**.** *Let $S, T \subseteq N$ be nonempty coalitions such that*

$$S \subseteq T \qquad \text{and} \qquad A_S = A_T.$$

If T improves upon an allocation x, then S improves upon x.

Proof. A blocking plan for T is feasible on the common control domain and strictly improves every member of T. Since $S \subseteq T$, the same plan strictly improves every member of S. □

For a fixed control domain, reducing the beneficiary set weakens the preference requirement for a block. Thus, under common control,

$$A_S = A \qquad \text{for every nonempty } S \subseteq N,$$

all blocking opportunities are captured by singleton beneficiary coalitions. Under unrestricted control, $A = N$, each singleton can propose any feasible allocation, so

$$C_{\mathcal{A}}(E) = \bigcap_{i \in N} \mathrm{Max}_{\succeq_i} F(E),$$

where $\mathrm{Max}_{\succeq_i} F(E)$ is the set of allocations maximal for agent i. The core is therefore nonempty precisely when the agents have a common maximal allocation.

Remark 4. The inclusion $S \subseteq T$ is sufficient, but not necessary, for every T-block to be an S-block. More generally, suppose $A_S = A_T = A$ and, for every social consumption profile x,

$$\bigcap_{i \in T} P_i^A(x) \subseteq \bigcap_{j \in S} P_j^A(x).$$

Then every plan on A that strictly improves all members of T also strictly improves all members of S. Since feasibility is the same for both coalitions, every T-block is an S-block, even if $S \not\subseteq T$. Under additional separability assumptions, such alignment may be related to benevolence or positive preference interdependence; see Borglin (1973).

A natural further question is whether the core coincides with the Pareto optimal set. A proper beneficiary coalition may block a Pareto optimal allocation, since improvement is required only for its members. Equivalence therefore requires stronger assumptions. We consider conditions extending the argument developed for the Walrasian-Nash core in Graziano et al. (2023).

The conditions below ensure that a profitable deviation requires access to the entire resource endowment and that only the grand coalition has such access.

Social Monotonicity. For any two social consumption profiles x and y,

$$x \geq y \quad \Longrightarrow \quad x \succeq_i y \qquad \text{for every } i \in N.$$

Boundary Equivalence. For every social consumption profile x, let

$$Z_x = \{j \in N : x_j \text{ has a zero component}\}.$$

Then

$$x \sim_i (0_{Z_x}, x_{N \setminus Z_x}) \qquad \text{for every } i \in N.$$

Social Monotonicity allows positive consumption externalities and rules out negative ones; related conditions appear in Dufwenberg et al. (2011). Boundary Equivalence makes a profile welfare equivalent to the profile obtained by replacing each boundary bundle with zero.

Glove Market Condition. For every $j \in N$, there exists a commodity k_j such that

$$\omega_h^{k_j} = 0 \qquad \text{for every } h \neq j.$$

Positive Aggregate Endowments.

$$\sum_{i \in N} \omega_i \gg 0.$$

Full Control Reserved to the Grand Coalition.

$$A_N = N, \qquad A_S \neq N \quad \text{for every nonempty } S \subsetneq N.$$

Together, the two endowment conditions imply that every agent is the sole supplier of some commodity with a positive aggregate endowment. The control restriction ensures that only the grand coalition can draw on the entire resource domain.

Proposition 4 (Core-Pareto equivalence)**.** *Suppose that Social Monotonicity, Boundary Equivalence, the Glove Market Condition and Positive Aggregate Endowments hold, and that Full Control is Reserved to the Grand Coalition. Then only the grand coalition can block an allocation, and*

$$C_{\mathcal{A}}(E) = PO(E).$$

Proof. Suppose that S blocks an allocation x through z_{A_S}, and write

$$\widetilde{z} = (z_{A_S}, x_{N \setminus A_S}).$$

If every controlled bundle has a zero component, Boundary Equivalence gives

$$\widetilde{z} \sim_i q \qquad \text{for every } i \in S,$$

where

$$q := (0_{Z_{\widetilde{z}}}, \widetilde{z}_{N \setminus Z_{\widetilde{z}}}).$$

Since all controlled bundles are zero in q and consumption outside A_S is unchanged or reduced, $q \leq x$. Social Monotonicity then contradicts $\widetilde{z} \succ_i x$.

Thus some $z_h \gg 0$, and feasibility implies

$$\sum_{h \in A_S} \omega_h \gg 0.$$

The Glove Market Condition forces $A_S = N$, since omitting any agent j would leave zero resources of commodity k_j. The full control restriction therefore gives $S = N$.

Hence only the grand coalition can block. Since $A_N = N$, its blocks are precisely strict Pareto improvements, proving

$$C_{\mathcal{A}}(E) = PO(E).$$

□

The argument separates two roles: preferences and endowments force any effective blocking domain to be N, while the control restriction forces its beneficiary coalition to be N. The latter restriction holds, in particular, under complement control, where

$$A_N = N, \qquad A_S = N \setminus S \neq N \quad \text{for every nonempty } S \subsetneq N.$$

4.1 A resource characterization of the core

The blocking notion introduced above is formulated in terms of strict preference improvements. An alternative is to ask whether the beneficiaries can attain at least their status quo welfare levels while using strictly less than the initial resources available on the control domain.

This resource interpretation is immediate in a standard exchange economy, but becomes more delicate in the presence of consumption externalities. Whether saved resources can be used to generate a strict welfare improvement depends on how they are redistributed and on the consumption environment faced by the beneficiaries. An appropriate monotonicity condition is therefore needed to connect resource saving with preference improvement. This is the idea

underlying the resource approach introduced by Montesano (2002) and extended to exchange economies with consumption externalities by Di Pietro et al. (2022).

The present framework adds a further distinction. The agents whose welfare is evaluated and the agents whose resources are available need not coincide. Accordingly, welfare continues to be evaluated for the members of S, while resource saving must be measured on the control domain A_S.

The institutional interpretations discussed in the Introduction make this distinction concrete. In adjustment, compensation, or environmental programs, the welfare objective may concern a targeted group of beneficiaries, while the relevant resource constraint is defined by the broader pool made available by the institution.

Definition 2 (Coalitional improvement in resources)**.** *Let $x \in F(E)$. A nonempty coalition $S \subseteq N$ improves upon x in resources relative to A_S if there exists*

$$z_{A_S} \in \mathbb{R}_+^{\ell|A_S|}$$

such that

$$\sum_{h \in A_S} z_h < \sum_{h \in A_S} \omega_h$$

and

$$(z_{A_S}, x_{N \setminus A_S}) \succeq_i x \qquad \text{for every } i \in S.$$

The associated resources core, denoted by

$$C_{\mathcal{A}}^r(E),$$

is the set of allocations that cannot be improved upon in resources by any nonempty coalition.

When $A_S = S$ for every nonempty S, the underlying preference-based blocking notion yields the Walrasian-Nash core studied by Dufwenberg et al. (2011) and referred to as the γ-core in Graziano et al. (2023). The corresponding resource formulation is studied by Di Pietro et al. (2022). More generally, the present definition separates the beneficiaries whose welfare is evaluated from the consumption and resource domain on which saving is assessed.

For the next result, assume that

$$\sum_{h \in A_S} \omega_h \neq 0 \qquad \text{for every nonempty } S \subseteq N.$$

This condition rules out control domains with no resources. Recall that preferences are continuous by the assumptions stated in Section 2.

Proposition 5 (Resources core and core)**.** *Under the preceding condition,*

$$C_{\mathcal{A}}^r(E) \subseteq C_{\mathcal{A}}(E).$$

Proof. Let $x \in C_{\mathcal{A}}^r(E)$ and suppose, by contradiction, that $x \notin C_{\mathcal{A}}(E)$. Then there exist a nonempty coalition S and a plan z_{A_S} such that

$$\sum_{h \in A_S} z_h \leq \sum_{h \in A_S} \omega_h$$

and

$$(z_{A_S}, x_{N \setminus A_S}) \succ_i x \qquad \text{for every } i \in S.$$

If

$$\sum_{h \in A_S} z_h < \sum_{h \in A_S} \omega_h,$$

the same plan constitutes an improvement in resources, contradicting $x \in C^r_{\mathcal{A}}(E)$.

Suppose therefore that

$$\sum_{h \in A_S} z_h = \sum_{h \in A_S} \omega_h.$$

Since the aggregate endowment of A_S is nonzero, at least one component of z_h is positive for some $h \in A_S$. By continuity, this component can be reduced by a sufficiently small positive amount without destroying any beneficiary's strict improvement. Since S is finite, a single reduction can be chosen such that

$$(\widehat{z}_{A_S}, x_{N \setminus A_S}) \succ_i x \qquad \text{for every } i \in S.$$

The modified plan uses strictly fewer resources than the aggregate endowment of A_S. Thus S improves upon x in resources, again contradicting $x \in C^r_{\mathcal{A}}(E)$. □

For the reverse inclusion, resource saving must be convertible into a strict welfare improvement for all beneficiaries. This requires a monotonicity condition adapted to the separation between the beneficiary coalition and its control domain.

Control Monotonicity. For every nonempty coalition S, every social consumption profile

$$(z_{A_S}, x_{N \setminus A_S}),$$

and every vector $r \in \mathbb{R}^\ell_+$ such that

$$r > \sum_{h \in A_S} z_h,$$

there exists

$$\widehat{z}_{A_S} \in \mathbb{R}^{\ell |A_S|}_+$$

such that

$$\sum_{h \in A_S} \widehat{z}_h = r$$

and

$$(\widehat{z}_{A_S}, x_{N \setminus A_S}) \succ_i (z_{A_S}, x_{N \setminus A_S}) \qquad \text{for every } i \in S.$$

Control Monotonicity requires that an increase in the aggregate resources available on A_S can be redistributed over the same control domain so as to make all beneficiaries strictly better off. When $A_S = S$, it reduces to Social Group Monotonicity: additional coalitional resources can be distributed among coalition members to improve each of them strictly. Related forms of this condition are used by Dufwenberg et al. (2011) and, for the resource characterization of the Walrasian-Nash core, by Di Pietro et al. (2022).

Theorem 1 (Resource-core equivalence)**.** *Suppose that*

$$\sum_{h \in A_S} \omega_h \neq 0 \qquad \textit{for every nonempty } S \subseteq N,$$

and that Control Monotonicity holds. Then

$$C_{\mathcal{A}}(E) = C^r_{\mathcal{A}}(E).$$

Proof. By Proposition 5,

$$C^r_{\mathcal{A}}(E) \subseteq C_{\mathcal{A}}(E).$$

For the reverse inclusion, let $x \in C_{\mathcal{A}}(E)$ and suppose that $x \notin C^r_{\mathcal{A}}(E)$. Then some nonempty coalition S has a plan z_{A_S} satisfying

$$\sum_{h \in A_S} z_h < \sum_{h \in A_S} \omega_h$$

and

$$(z_{A_S}, x_{N \setminus A_S}) \succeq_i x \qquad \text{for every } i \in S.$$

Apply Control Monotonicity with

$$r = \sum_{h \in A_S} \omega_h.$$

There exists a plan $\widehat{z}_{A_S}$ satisfying

$$\sum_{h \in A_S} \widehat{z}_h = \sum_{h \in A_S} \omega_h$$

and

$$(\widehat{z}_{A_S}, x_{N \setminus A_S}) \succ_i (z_{A_S}, x_{N \setminus A_S}) \succeq_i x \qquad \text{for every } i \in S.$$

Thus S blocks x, contradicting $x \in C_{\mathcal{A}}(E)$. □

The equivalence shifts strictness from preferences to resources: a resource improvement makes every beneficiary weakly better off while leaving some resources unused on A_S. Control Monotonicity ensures that these resources can be redistributed to obtain a strict improvement for every member of S.

The condition is substantive. Under selfish preferences, whenever $S \setminus A_S \neq \emptyset$, a beneficiary outside the control domain remains indifferent to every change on A_S. Such an agent may satisfy the weak preference requirement for a resource improvement, but cannot be strictly improved through the additional resources. Control Monotonicity therefore fails, and the equivalence theorem does not apply.

The resource characterization thus preserves the distinction central to the model: welfare is evaluated for S, while resource saving and redistribution concern A_S.

5 Control expansion and core existence

The distinction between beneficiaries and control domains raises two different comparative questions. Proposition 3 compares beneficiary coalitions with the same control domain. We now hold the beneficiary coalition fixed and compare its opportunities under different control structures.

The section develops two implications for core existence. First, under suitable preference restrictions, an expansion of control strengthens blocking power and reduces the core. The resulting inclusions allow non-emptiness to be transferred from a benchmark structure. Second, for a class of network economies with one agent having full individual control, we obtain a direct characterization of the core. An example shows that compatibility between heterogeneous preferences and control rights can support a nonempty core even when the self-regarding core of the same economy is empty.

5.1 Control expansion and core inclusion

Let

$$\mathcal{A} = (A_S)_{\emptyset \neq S \subseteq N}, \qquad \mathcal{B} = (B_S)_{\emptyset \neq S \subseteq N}$$

be two control structures satisfying

$$A_S \subseteq B_S \qquad \text{for every nonempty } S \subseteq N.$$

More control gives a coalition access to additional endowments, but does not necessarily preserve its blocking opportunities. Under A_S, consumption on $B_S \setminus A_S$ stays fixed, whatever plan the coalition chooses. Under B_S, the coalition must provide that consumption together with consumption on A_S, using the endowments of B_S. These resources may not suffice to preserve both the additional agents' consumption and the beneficiaries' improvement. If the beneficiaries value those agents' consumption positively, more control may therefore remove a blocking opportunity.

Example 1 (More control need not imply more blocking power)**.** *Let $N = \{1, 2\}$, $\omega_1 = \omega_2 = 1$, and*

$$u_1(x_1, x_2) = x_1 + 2x_2, \qquad u_2(x_1, x_2) = x_2.$$

Consider the feasible allocation $x = (0, 2)$ and control structures satisfying

$$A_{\{1\}} = \{1\}, \qquad B_{\{1\}} = N,$$

with $A_S = B_S = N$ for every other nonempty coalition.

Under $\mathcal{A}$, singleton $\{1\}$ blocks x by choosing $z_1 = 1$, since

$$u_1(1, 2) = 5 > 4 = u_1(0, 2).$$

Under $\mathcal{B}$, its plan must satisfy $z_1 + z_2 \leq 2$. On this set, u_1 is maximized at $x = (0, 2)$. Agent 2 also attains her maximum feasible consumption at x, so neither singleton nor the grand coalition can block. Hence

$$x \in C_{\mathcal{B}}(E) \setminus C_{\mathcal{A}}(E).$$

Thus control expansion alone does not imply $C_{\mathcal{B}}(E) \subseteq C_{\mathcal{A}}(E)$.

The example suggests that expanding control preserves blocking opportunities, and hence reduces the core, if reducing consumption at the newly controlled positions does not harm any beneficiary.

Definition 3 (Non-positive consumption effects from a set)**.** *Agent i has non-positive consumption effects from $J \subseteq N$ if, for every social consumption profile x and every consumption profile $y_J \leq x_J$,*

$$(y_J, x_{N \setminus J}) \succeq_i x.$$

The condition means that reducing consumption on J, while keeping consumption outside J fixed, cannot make agent i worse off. For $J = \{j\}$ with $j \neq i$, it describes a non-positive consumption externality from agent j. For $J = \{i\}$, it requires that reducing i's own consumption cannot make her worse off.

More generally, if $i \in J$, the condition also covers reductions in her own consumption that leave all other agents' consumption unchanged. In the comparison below, we require this property only for $J = B_S \setminus A_S$, the additional positions controlled under $\mathcal{B}$.

Proposition 6 (Core monotonicity under non-positive consumption effects)**.** *Suppose that*

$$A_S \subseteq B_S \qquad \textit{for every nonempty } S \subseteq N.$$

If every $i \in S$ has non-positive consumption effects from $B_S \setminus A_S$, for every nonempty S, then

$$C_{\mathcal{B}}(E) \subseteq C_{\mathcal{A}}(E).$$

Proof. Suppose $x \in F(E) \setminus C_{\mathcal{A}}(E)$, and let z_{A_S} be a blocking plan for some nonempty coalition S. Extend this plan to B_S by setting

$$\widehat{z}_h = \begin{cases} z_h, & h \in A_S, \\ 0, & h \in B_S \setminus A_S. \end{cases}$$

Nonnegative endowments imply

$$\sum_{h \in B_S} \widehat{z}_h = \sum_{h \in A_S} z_h \leq \sum_{h \in A_S} \omega_h \leq \sum_{h \in B_S} \omega_h.$$

For every $i \in S$, non-positive consumption effects give

$$(\widehat{z}_{B_S}, x_{N \setminus B_S}) \succeq_i (z_{A_S}, x_{N \setminus A_S}) \succ_i x.$$

Thus the extended plan blocks x under $\mathcal{B}$. □

Remark 5. The condition of non-positive consumption effects is related to Borglin's notion of nonbenevolence (Borglin, 1973). In his formulation, utilities take the form

$$u_i(x) = U_i(v_1(x_1), \ldots, v_n(x_n)),$$

where v_j represents agent j's ranking of her own consumption bundles, independently of others' consumption. Nonbenevolence towards $j \neq i$ means that U_i is nonincreasing in v_j. If internal utilities are componentwise nondecreasing, nonbenevolence towards every agent in J, with $i \notin J$, implies our non-positive consumption effects condition.

Our condition applies directly to consumption on $B_S \setminus A_S$, without requiring the previous representation. If this set includes the beneficiary herself, it also restricts the effect of her own consumption, which nonbenevolence towards others does not address.

Remark 6. The preference restriction in Proposition 6 requires that reducing consumption on $B_S \setminus A_S$ cannot harm any beneficiary. Social comparisons based on relative consumption provide an economic motivation for this restriction when the additional positions exclude the beneficiaries themselves.

A simple example can be drawn from the discussion in Sobel (2005, Section 3.2), who considers interdependent preferences in which constant negative weights on others' consumption represent spite. In a one-commodity economy, such preferences can be represented by

$$u_i(x) = f_i\left(x_i - \sum_{j \neq i} b_{ij} x_j\right), \qquad b_{ij} \geq 0,$$

where $f_i : \mathbb{R} \to \mathbb{R}$ is strictly increasing. Other agents' consumption acts as a comparison benchmark: an increase in their consumption can lower agent i's welfare even when her own consumption remains unchanged. Conversely, holding her own consumption fixed, reducing other agents' consumption cannot make her worse off. These preferences therefore satisfy the condition in Proposition 6 whenever

$$S \subseteq A_S \subseteq B_S \qquad \text{for every nonempty } S \subseteq N.$$

Indeed, reducing consumption on $B_S \setminus A_S$ leaves each beneficiary's own consumption unchanged and cannot lower her utility.

The role of social comparisons in consumption decisions has recently been explored, for example, by Jappelli and Pistaferri (2026), who study how consumption responses to income shocks depend on their observability and on whether they are shared by coworkers. For a general equilibrium analysis of negative consumption externalities, see Tian and Yang (2009).

Corollary 1 (Comparison with self-regarding control)**.** *Suppose that preferences are selfish.*

1. *If $S \subseteq A_S$ for every nonempty S, then*

$$C_{\mathcal{A}}(E) \subseteq C^{\mathrm{sr}}(E).$$

2. *If $A_S \subseteq S$ for every nonempty S, then*

$$C^{\mathrm{sr}}(E) \subseteq C_{\mathcal{A}}(E).$$

Proof. For the first inclusion, compare self-regarding control with $\mathcal{A}$. The newly controlled positions $A_S \setminus S$ generate no consumption externalities for members of S, so Proposition 6 applies.

For the second inclusion, a coalition with $A_S \subsetneq S$ cannot block under selfish preferences, since some beneficiary's own consumption remains fixed. Thus only coalitions with $A_S = S$ can block, and their deviations coincide with ordinary self-regarding deviations. The control restriction therefore removes some coalitions from the blocking test, which gives

$$C^{\mathrm{sr}}(E) \subseteq C_{\mathcal{A}}(E).$$

□

Under selfish preferences, the two inclusions have a direct interpretation. Expanding control beyond coalition membership preserves ordinary blocking opportunities and may add new ones, so the core can become smaller. Restricting control within coalition membership instead disables coalitions with $A_S \subsetneq S$, so the core can become larger. The expansion case includes reflexive network-generated control, since $i \in A_i$ for every i implies $S \subseteq A_S$.

At the extreme, unrestricted control can make the core empty. Suppose that $A_S = N$ for every nonempty coalition, there are at least two agents, and preferences are selfish and strongly monotone in own consumption. Let

$$\Omega := \sum_{i \in N} \omega_i \neq 0.$$

Every singleton can assign the entire aggregate endowment to its beneficiary. For any allocation x, at least one agent i has $x_i \neq \Omega$. Since $x_i \leq \Omega$, this agent strictly prefers

$$(\Omega, 0_{N \setminus \{i\}})$$

and can block x. Hence

$$C_{\mathcal{A}}(E) = \emptyset.$$

Non-emptiness of the ordinary core therefore does not guarantee non-emptiness after control is expanded. By contrast, restricting control preserves every ordinary core allocation and thus preserves non-emptiness. More generally, the core inclusion established above yields the following transfer of non-emptiness.

Corollary 2 (Transfer of core non-emptiness)**.** *Under the hypotheses of Proposition 6,*

$$C_{\mathcal{B}}(E) \neq \emptyset \quad \Longrightarrow \quad C_{\mathcal{A}}(E) \neq \emptyset.$$

Proof. The conclusion follows from $C_{\mathcal{B}}(E) \subseteq C_{\mathcal{A}}(E)$. □

This principle identifies structures that inherit non-emptiness; it does not establish non-emptiness of the benchmark core. For example, suppose that

$$C^{\mathrm{sr}}(E) \neq \emptyset \qquad \text{and} \qquad A_S \subseteq S \quad \text{for every nonempty } S.$$

Then

$$\emptyset \neq C^{\mathrm{sr}}(E) \subseteq C_{\mathcal{A}}(E)$$

in either of the following cases:

1. preferences are selfish;
2. every $i \in S$ has non-positive consumption effects from $S \setminus A_S$, for every nonempty S.

The first case follows from Corollary 1, and the second from Proposition 6, with $B_S = S$.

The second restriction includes effects on a beneficiary's own consumption whenever $i \in S \setminus A_S$. It ensures that extending a plan to S by assigning zero to these additional positions preserves the beneficiaries' improvement. If, in addition, $A_N = N$, Proposition 2 gives

$$\emptyset \neq C_{\mathcal{A}}(E) \subseteq PO(E).$$

5.2 Full individual control in a network economy

We now describe the core of a network economy in which one agent controls all consumption positions and resources, while the other agents have selfish preferences. This gives a necessary and sufficient condition for core non-emptiness, without assuming that a benchmark core is nonempty.

The agent with full control can block any allocation that does not maximize her utility over the feasible set. The other agents, however, retain their individual control rights. A core allocation must therefore maximize the first agent's utility while giving every other agent at least as much as she can obtain through her own control domain. Under the network structure considered below, these requirements are also sufficient in a one-commodity economy.

Proposition 7 (Core characterization under full individual control)**.** *Consider a one-commodity economy with aggregate endowment*

$$W := \sum_{h \in N} \omega_h > 0$$

and network-generated control

$$A_S = \bigcup_{h \in S} A_h, \qquad h \in A_h \quad \textit{for every } h \in N.$$

Suppose that an agent d has full control,

$$A_d = N,$$

and preferences represented by a continuous utility function u_d. Every other agent $t \in N \setminus \{d\}$ has selfish preferences strictly increasing in her own consumption.

Let

$$M_d := \operatorname{argmax}_{x \in F(E)} u_d(x)$$

and

$$L_t := \sum_{h \in A_t} \omega_h \qquad \textit{for every } t \neq d.$$

Then

$$C_{\mathcal{A}}(E) = \{x \in M_d : x_t \geq L_t \textit{ for every } t \neq d\}.$$

In particular, the core is nonempty if and only if there exists $x \in M_d$ such that

$$x_t \geq L_t \qquad \textit{for every } t \neq d.$$

Proof. Let $x \in C_{\mathcal{A}}(E)$. Since $A_d = N$, singleton $\{d\}$ can choose any feasible allocation. Thus $x \in M_d$.

For every $t \neq d$, reflexivity gives $t \in A_t$. If $x_t < L_t$, singleton $\{t\}$ can block x by choosing the feasible plan

$$z_t = L_t, \qquad z_h = 0 \quad \text{for every } h \in A_t \setminus \{t\}.$$

Her own consumption increases, so strict monotonicity and selfishness imply a strict improvement, regardless of the changes in other agents' consumption. Core membership therefore requires

$$x_t \geq L_t \qquad \text{for every } t \neq d.$$

Conversely, let $x \in M_d$ satisfy $x_t \geq L_t$ for every $t \neq d$. Every coalition containing d has full control and can propose only feasible allocations. It therefore cannot strictly improve d.

For any nonempty coalition $S \subseteq N \setminus \{d\}$,

$$\sum_{t \in S} x_t \geq \sum_{t \in S} L_t = \sum_{t \in S} \sum_{h \in A_t} \omega_h \geq \sum_{h \in A_S} \omega_h.$$

The last inequality holds because the sum over individual control domains counts each endowment in their union at least once.

Since preferences are selfish and strictly increasing, a block by S would require $z_t > x_t$ for every $t \in S$. Reflexivity implies $S \subseteq A_S$, so

$$\sum_{h \in A_S} z_h \geq \sum_{t \in S} z_t > \sum_{t \in S} x_t \geq \sum_{h \in A_S} \omega_h,$$

contradicting feasibility on A_S. Hence no coalition blocks x. □

Since u_d is continuous and $F(E)$ is compact, at least one allocation maximizes agent d's utility. Core non-emptiness depends on whether such an allocation also gives every other agent t at least L_t, the amount she can obtain through her individual control domain. These requirements can be demanding. In particular, when control domains overlap, the same endowments enter the resource claims of several agents.

If an allocation meets these requirements, however, no larger coalition can block it either. Coalitions containing d cannot strictly improve her because her utility is already maximal over the feasible set. Coalitions excluding d cannot finance a strict improvement for all their members: the resources on their control domain do not exceed the sum of the amounts those members can obtain individually. Thus, in this class of economies, excluding singleton blocks is sufficient for core membership.

Example 2 (Preference weights and the self-regarding benchmark)**.** *Under the hypotheses of Proposition 7, suppose that*

$$u_d(x) = \prod_{h \in N} x_h^{\alpha_h}, \qquad \alpha_h > 0, \qquad \sum_{h \in N} \alpha_h = 1.$$

The unique feasible allocation maximizing u_d is

$$x_h^* = \alpha_h W \qquad \textit{for every } h \in N.$$

Thus

$$C_{\mathcal{A}}(E) \neq \emptyset \quad \Longleftrightarrow \quad \alpha_t W \geq L_t \quad \textit{for every } t \neq d,$$

and the core, when nonempty, consists only of x^*.

For an illustrative case with two agents, let

$$\omega_1 = \omega_2 = 1, \qquad A_1 = N, \qquad A_2 = \{2\},$$

and

$$u_1(x) = x_1^{\theta} x_2^{1-\theta}, \qquad u_2(x) = x_2, \qquad 0 < \theta < 1.$$

Agent 1 controls both consumption positions and values both agents' consumption. Agent 2 is selfish and controls only her own consumption and endowment. Here,

$$x^* = (2\theta,\, 2(1-\theta)), \qquad L_2 = 1.$$

Every allocation other than x^* is blocked by agent 1. The allocation x^* is itself blocked by agent 2 precisely when $2(1-\theta) < 1$, since she can consume her own endowment. Hence

$$C_{\mathcal{A}}(E) = \begin{cases} \{x^*\}, & 0 < \theta \le \frac{1}{2}, \\ \emptyset, & \frac{1}{2} < \theta < 1. \end{cases}$$

Existence therefore requires agent 1's preferred allocation to give agent 2 at least what she can obtain independently.

Now keep the same preferences and endowments, but replace network control with self-regarding control. Agent 2's opportunities are unchanged, while agent 1 can modify only her own consumption, using her own endowment.

To avoid singleton blocks, an allocation must satisfy $x_2 \ge 1$ and, since $x_2 > 0$, also $x_1 \ge 1$. The only feasible candidate is therefore $(1,1)$. If $0 < \theta < \frac{1}{2}$, the grand coalition blocks this allocation through x^*: agent 2 receives more than 1, and agent 1 attains her unique feasible maximum. Thus

$$C^{\mathrm{sr}}(E) = \emptyset, \qquad C_{\mathcal{A}}(E) = \{x^*\} \qquad \text{for } 0 < \theta < \frac{1}{2}.$$

For $0 < \theta < \frac{1}{2}$, the difference comes from agent 1's singleton deviations. Under self-regarding control, she can block x^* by increasing her consumption to 1 while agent 2 continues to consume $2(1-\theta)$. The resulting total exceeds the aggregate endowment:

$$1 + 2(1-\theta) > 2.$$

Under network control, agent 1 must finance both consumptions from the aggregate endowment, so this deviation is unavailable.

Stability does not require the agents' interests to coincide. For $\theta = \frac{1}{3}$, agent 1 can block $(\frac{1}{2}, \frac{3}{2})$ by choosing her unique feasible maximizer $(\frac{2}{3}, \frac{4}{3})$, even though agent 2 receives less.

This example shows that changing control rights can restore core non-emptiness in an economy whose self-regarding core is empty. Full control requires agent 1 to finance both the consumption she receives and the consumption of agent 2 that she values. Agent 2, in turn, retains her own blocking opportunity. Existence follows when agent 1's preferred allocation respects that opportunity.

Remark 7. The characterization in Proposition 7 also holds for control structures that are not network-generated. Retain the assumptions on the economy and preferences, with d denoting the agent whose utility is u_d. The network assumptions can be replaced by the following requirements: every coalition containing d has full control,

$$t \in A_{\{t\}} \qquad \text{for every } t \neq d,$$

and

$$\sum_{h \in A_S} \omega_h \le \sum_{t \in S} L_t, \qquad L_t := \sum_{h \in A_{\{t\}}} \omega_h,$$

for every nonempty $S \subseteq N \setminus \{d\}$ with $S \subseteq A_S$.

The inequality limits coalitional resources to the sum available through individual control domains. Coalitions with $S \nsubseteq A_S$ cannot block under selfish preferences. The proof therefore applies without the union construction. Reflexive network-generated control with $A_d = N$ satisfies these conditions. Extensions to several other-regarding agents and multiple commodities remain open.

6 Pessimistic blocking

The analysis so far has adopted a Nash-type conjecture: when coalition S acts on its control set A_S, consumption outside A_S remains fixed at the status quo. An alternative benchmark is obtained by adopting a pessimistic conjecture about consumption outside the control domain. In this case, coalition S blocks only if its members benefit under every feasible redistribution outside A_S.

For a nonempty coalition S, let

$$F(N \setminus A_S) = \left\{ y_{N\setminus A_S} \in \mathbb{R}_+^{\ell|N\setminus A_S|} : \sum_{h\notin A_S} y_h \leq \sum_{h\notin A_S} \omega_h \right\}.$$

When $A_S = N$, this set contains only the empty consumption profile, with the corresponding sum equal to zero.

Definition 4 (Pessimistic coalitional improvement)**.** *Let $x \in F(E)$. A nonempty coalition $S \subseteq N$ pessimistically improves upon x relative to A_S if there exists*

$$z_{A_S} \in \mathbb{R}_+^{\ell|A_S|}$$

such that

$$\sum_{h\in A_S} z_h \leq \sum_{h\in A_S} \omega_h$$

and

$$(z_{A_S}, y_{N\setminus A_S}) \succ_i x \qquad \text{for every } i \in S$$

for every

$$y_{N\setminus A_S} \in F(N \setminus A_S).$$

The associated pessimistic core, denoted by

$$C_{\mathcal{A}}^{p}(E),$$

is the set of allocations that admit no pessimistic block.

Every social consumption profile considered in a pessimistic deviation is feasible for the economy as a whole. Indeed, the plan on A_S and each outside redistribution satisfy their respective resource constraints, so

$$\sum_{h\in A_S} z_h + \sum_{h\notin A_S} y_h \leq \sum_{h\in N} \omega_h.$$

This feature parallels the α-dominance formulation in Graziano et al. (2017), where all profiles considered in the dominance relation are physically feasible for the grand coalition.

When

$$A_S = S \qquad \text{for every nonempty } S \subseteq N,$$

the definition reduces to the α-core with feasible outside redistributions: a coalition redistributes its initial resources and deviates only if all its members are better off under every feasible redistribution of the initial resources of the complementary coalition. The present construction extends this mechanism by allowing the beneficiary coalition and the control domain to differ.

Remark 8. Whenever $A_S = N$, Nash-type and pessimistic blocking coincide for coalition S, since there is no consumption outside its control domain. Consequently, under unrestricted control,

$$A_S = N \qquad \text{for every nonempty } S \subseteq N,$$

the two associated cores coincide. More generally, the blocking notions coincide for a given coalition whenever its members' preferences are independent of consumption outside A_S. This includes selfish preferences when $S \subseteq A_S$.

The difference becomes sharper when $S \setminus A_S \neq \emptyset$. Under Nash-type blocking, these beneficiaries keep their status quo bundles. Under pessimistic blocking, their consumption belongs to the outside redistribution, and improvement must hold even when they receive zero.

For example, if a beneficiary has selfish preferences and strictly prefers her status quo bundle to zero, a coalition excluding her from its control domain cannot pessimistically block. Under complement control, every member of a proper beneficiary coalition lies outside its control domain, since

$$A_S = N \setminus S \qquad \text{for every } \emptyset \neq S \subsetneq N.$$

The same obstruction may arise under restricted or mixed control. For the grand coalition under complement control, by contrast, $A_N = N$, so the two blocking notions coincide.

6.1 Pareto optimality

The efficiency implication established in Proposition 2 extends directly to pessimistic blocking.

Proposition 8 (Full control and Pareto optimality under pessimistic blocking)**.** *Suppose that there exists a nonempty coalition $S \subseteq N$ such that*

$$A_S = N.$$

Then

$$C^p_{\mathcal{A}}(E) \subseteq PO(E).$$

Proof. Let $x \in C^p_{\mathcal{A}}(E)$. If $x \notin PO(E)$, some allocation $z \in F(E)$ strictly improves every agent and hence every member of S. Since $A_S = N$, the outside domain is empty, and z constitutes a pessimistic block by S, contradicting core membership. □

The condition holds, in particular, when $A_N = N$. As in the Nash-type case, however, full control by any nonempty beneficiary coalition suffices for Pareto optimality of core allocations.

Remark 9. Under the assumptions of Proposition 4, the pessimistic core also coincides with the Pareto optimal set.

To see this, suppose that S pessimistically blocks an allocation x. The improvement requirement holds, in particular, when consumption outside A_S is zero. If every controlled bundle has a zero component, Boundary Equivalence makes the resulting profile indifferent to the zero profile. Social Monotonicity then contradicts strict improvement over x. Hence some controlled bundle is strictly positive, and feasibility together with the Glove Market Condition forces $A_S = N$. The full control restriction gives $S = N$.

Only the grand coalition can therefore pessimistically block, and its blocks are precisely strict Pareto improvements. Thus

$$C_{\mathcal{A}}(E) = C^p_{\mathcal{A}}(E) = PO(E).$$

This conclusion relies on the additional preference, endowment and control conditions; it does not follow from pessimism or global feasibility alone.

6.2 Resource characterization

The resource interpretation developed in Section 4.1 extends to pessimistic blocking. Welfare is evaluated for the beneficiary coalition S, while resource saving is measured on A_S. The welfare requirement must now hold for every feasible redistribution outside the control domain.

A nonempty coalition S improves upon an allocation x in resources pessimistically if there exists

$$z_{A_S} \in \mathbb{R}_+^{\ell|A_S|}$$

such that

$$\sum_{h\in A_S} z_h < \sum_{h\in A_S} \omega_h$$

and

$$(z_{A_S}, y_{N\setminus A_S}) \succeq_i x \qquad \text{for every } i \in S$$

for every

$$y_{N\setminus A_S} \in F(N \setminus A_S).$$

Denote the associated resources core by

$$C_{\mathcal{A}}^{p,r}(E).$$

As in the Nash-type case, continuity and the condition

$$\sum_{h\in A_S} \omega_h \neq 0 \qquad \text{for every nonempty } S \subseteq N$$

yield

$$C_{\mathcal{A}}^{p,r}(E) \subseteq C_{\mathcal{A}}^{p}(E).$$

Indeed, a pessimistic blocking plan that already leaves resources unused is also a resource improvement. Otherwise, it exhausts the nonzero resources on A_S, and some component of the plan is positive. Compactness of $F(N \setminus A_S)$, continuity and finiteness of S allow a single sufficiently small reduction of this component that preserves every beneficiary's strict improvement for all admissible outside redistributions. The modified plan therefore generates a strict resource saving.

For the reverse inclusion, we combine Control Monotonicity with a separability condition. This allows an improvement between two plans on A_S, obtained for one outside redistribution, to be preserved for all others.

Control-set Separability. For every nonempty coalition S, every $i \in S$, and every pair of consumption profiles z_{A_S}, z'_{A_S}, if

$$(z_{A_S}, y_{N\setminus A_S}) \succeq_i (z'_{A_S}, y_{N\setminus A_S})$$

for some outside consumption profile $y_{N\setminus A_S}$, then

$$(z_{A_S}, y'_{N\setminus A_S}) \succeq_i (z'_{A_S}, y'_{N\setminus A_S})$$

for every outside consumption profile $y'_{N\setminus A_S}$. The analogous implication is required for strict preference.

When $A_S = S$, this is the Social Group Separability condition used in the resource characterization of the α-core. Here it makes the ranking of alternative plans on the control domain independent of outside consumption.

Proposition 9 (Pessimistic resource-core equivalence)**.** *Suppose that*

$$\sum_{h\in A_S} \omega_h \neq 0 \qquad \textit{for every nonempty } S \subseteq N.$$

Under Control Monotonicity and Control-set Separability,

$$C_{\mathcal{A}}^{p}(E) = C_{\mathcal{A}}^{p,r}(E).$$

Proof. The inclusion

$$C^{p,r}_{\mathcal{A}}(E) \subseteq C^{p}_{\mathcal{A}}(E)$$

follows from the compactness and continuity argument above.

For the reverse inclusion, suppose that

$$x \in F(E) \setminus C^{p,r}_{\mathcal{A}}(E).$$

Some nonempty coalition S has a plan z_{A_S} satisfying

$$\sum_{h \in A_S} z_h < \sum_{h \in A_S} \omega_h$$

and

$$(z_{A_S}, y_{N \setminus A_S}) \succeq_i x \qquad \text{for every } i \in S$$

for every

$$y_{N \setminus A_S} \in F(N \setminus A_S).$$

Fix one such outside redistribution $y_{N \setminus A_S}$. By Control Monotonicity, there exists $\widehat{z}_{A_S}$ such that

$$\sum_{h \in A_S} \widehat{z}_h = \sum_{h \in A_S} \omega_h$$

and

$$(\widehat{z}_{A_S}, y_{N \setminus A_S}) \succ_i (z_{A_S}, y_{N \setminus A_S}) \qquad \text{for every } i \in S.$$

Control-set Separability preserves this strict comparison for every admissible outside redistribution. Therefore

$$(\widehat{z}_{A_S}, y'_{N \setminus A_S}) \succ_i (z_{A_S}, y'_{N \setminus A_S}) \succeq_i x \qquad \text{for every } i \in S$$

for every

$$y'_{N \setminus A_S} \in F(N \setminus A_S).$$

Thus S pessimistically blocks x, proving $x \notin C^{p}_{\mathcal{A}}(E)$. □

The preference-resource interpretation therefore carries over to pessimistic blocking. Control Monotonicity converts resource saving into strict improvement for a fixed outside redistribution; Control-set Separability ensures that the same redistribution of the additional resources works for every admissible outside consumption profile.

6.3 Control expansion

Under pessimistic blocking, expanding control increases blocking power without requiring a sign restriction on consumption effects. Let

$$\mathcal{A} = (A_S)_{\emptyset \neq S \subseteq N}, \qquad \mathcal{B} = (B_S)_{\emptyset \neq S \subseteq N}$$

satisfy

$$A_S \subseteq B_S \qquad \text{for every nonempty } S \subseteq N.$$

Under the Nash-type conjecture, consumption on $B_S \setminus A_S$ is fixed under $\mathcal{A}$ but enters the resource constraint under $\mathcal{B}$. As Section 5 showed, this change can destroy a profitable deviation. Under pessimistic blocking, the improvement requirement already covers every feasible redistribution outside A_S, including those assigning zero to these additional positions.

Proposition 10 (Control expansion under pessimistic blocking)**.** *If*

$$A_S \subseteq B_S \qquad \text{for every nonempty } S \subseteq N,$$

then

$$C^p_{\mathcal{B}}(E) \subseteq C^p_{\mathcal{A}}(E).$$

Proof. Suppose that

$$x \in F(E) \setminus C^p_{\mathcal{A}}(E).$$

Some nonempty coalition S has a plan z_{A_S} feasible on A_S such that

$$(z_{A_S}, y_{N\setminus A_S}) \succ_i x \qquad \text{for every } i \in S$$

for every

$$y_{N\setminus A_S} \in F(N \setminus A_S).$$

Extend the plan to B_S by setting

$$\widehat{z}_h = \begin{cases} z_h, & h \in A_S, \\ 0, & h \in B_S \setminus A_S. \end{cases}$$

Then

$$\sum_{h\in B_S} \widehat{z}_h = \sum_{h\in A_S} z_h \leq \sum_{h\in A_S} \omega_h \leq \sum_{h\in B_S} \omega_h.$$

For every

$$y_{N\setminus B_S} \in F(N \setminus B_S),$$

the profile

$$(0_{B_S\setminus A_S}, y_{N\setminus B_S})$$

belongs to $F(N \setminus A_S)$. Hence

$$(\widehat{z}_{B_S}, y_{N\setminus B_S}) \succ_i x \qquad \text{for every } i \in S.$$

Thus S also pessimistically blocks x under $\mathcal{B}$. □

The universal requirement over outside redistributions therefore allows a blocking plan to be extended to a larger control domain without assumptions on whether beneficiaries value the additional agents' consumption positively or negatively.

As an application, take the self-regarding structure

$$B_S = S \qquad \text{for every nonempty } S \subseteq N.$$

Its pessimistic core is the α-core with feasible outside redistributions. Whenever this benchmark core is nonempty, every control structure satisfying

$$A_S \subseteq S \qquad \text{for every nonempty } S \subseteq N$$

therefore satisfies

$$\emptyset \neq C^p_{\mathcal{B}}(E) \subseteq C^p_{\mathcal{A}}(E).$$

Non-emptiness results for the self-regarding pessimistic core can therefore be transferred to the corresponding restricted-control cores. One source is Asdrubali (1996, Corollary 3.1): with free disposal and monotone preferences, the existence of a distributive Lindahl equilibrium ensures non-emptiness of the α-core with feasible outside redistributions. For other conditions ensuring existence under alternative formulations of the α-core, see Martins-da-Rocha and Yannelis (2026).

Remark 10. The Nash-type and pessimistic formulations are not the only possible conjectures about consumption outside the control domain. One may instead require agents outside A_S to consume their initial endowments, leading to profiles of the form

$$(z_{A_S}, \omega_{N \setminus A_S}),$$

or adopt an optimistic conjecture: a coalition blocks if some feasible redistribution outside A_S makes all its members strictly better off.

These alternatives generate further dominance relations and cores. We leave them for future work and focus here on the Nash-type and pessimistic benchmarks.

7 Concluding remarks

Separating the beneficiary coalition S from its control domain A_S provides a flexible way of describing coalitional blocking in exchange economies with consumption externalities. The distinction affects both preferences and resources: S identifies the agents who must benefit, whereas A_S determines the consumption positions that may be modified and the resources available to finance the deviation.

Consumption externalities are central to this construction. They allow beneficiaries outside the control domain to improve and can therefore give effective blocking power to configurations with $S \not\subseteq A_S$. The distinction remains relevant under selfish preferences. When $S \subsetneq A_S$, the additional agents' consumption does not affect the beneficiaries' welfare, but their endowments enlarge the resources available to the deviation. Preference dependence and resource control are therefore distinct channels through which control affects coalitional opportunities.

The analysis establishes several implications of this separation. Full control by any nonempty beneficiary coalition ensures Pareto optimality of core allocations. Under stronger preference, endowment and control conditions that make only the grand coalition capable of blocking, the core coincides with the Pareto optimal set. Under Control Monotonicity, the core also admits a resource characterization in which welfare is evaluated for S and resource saving is measured on A_S.

Comparisons across control structures show that expanding control need not strengthen blocking power in the presence of consumption externalities. Under suitable restrictions on consumption effects, however, control expansion reduces the core and yields a conditional transfer of non-emptiness. Under pessimistic blocking, the same inclusion holds without a sign restriction on consumption effects. Parallel efficiency and resource results are obtained for the pessimistic core, with an additional separability condition used for the resource equivalence.

The network application provides a direct characterization for a class of one-commodity economies with heterogeneous preferences. One agent has full control, while the remaining agents are selfish and retain their individual control rights. The core is nonempty precisely when some feasible allocation maximizing that agent's utility also gives every other agent at least the amount she can obtain through her individual control domain. The example shows that, with preferences and endowments held fixed, changing the control structure can turn an empty self-regarding core into a nonempty core.

Several issues remain open. A first concerns the origin of the control correspondence. In the present model, $S \mapsto A_S$ is primitive. Property and exclusion rights models suggest that coalitional control may instead be derived from underlying rights or authority structures. The network-generated specification $A_S = \bigcup_{i \in S} A_i$ provides one such construction, but leaves open the economic foundations of the individual domains and their aggregation. Family structures and overlapping networks of control offer natural settings in which to investigate these foundations.

A related question is whether the beneficiary-control distinction has a competitive counterpart. The A-equilibrium of Vasil'ev (2016) and the framework of Graziano, Pesce and Platino

(2026) provide a useful starting point. In those models, an individual may choose consumption on a domain A_i containing several agents, subject to a corresponding group budget. The connection with the present core requires further analysis. Even when coalition members participate fully, the associated A-core allows each participant to propose an individual plan on her domain, with the resource constraint depending on coalition participation. Here, a coalition proposes a single plan on A_S, using the endowments of all agents in that domain. The two blocking notions therefore need not coincide, even when $A_S = \bigcup_{i \in S} A_i$. It remains to determine whether individual control domains and budget rights can be specified to decentralize the coalitional structure studied here.

This question also motivates separating action rights from resource rights. A more general model could distinguish the consumption positions a coalition can modify, A_S, from the agents whose resources it can use, R_S. This would accommodate control over consumption without access to the corresponding endowments, as well as access to resources without control over their owners' consumption. Such a distinction may also help formulate a corresponding equilibrium notion.

Finally, properties such as monotonicity or conditions analogous to superadditivity could be imposed directly on the control correspondence and interpreted in terms of coalitional rights. These structural properties must nevertheless be considered together with preferences: formal control does not by itself determine effective blocking power. The existence result in Section 5.2 suggests studying this interaction through more general conditions of compatibility between preferences and control rights. Extending the analysis to several other-regarding agents, multiple commodities, and structures without an agent having full control would help identify when the flexibility of coalitional control can sustain stability beyond the self-regarding benchmark.

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