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A Comparison of Stock Market Mechanisms

Giovanni Cespa

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DIPARTIMENTO DI SCIENZE ECONOMICHE - UNIVERSITÀ DEGLI STUDI DI SALERNO
Via Ponte Don Melillo - 84084 FISCIANO (SA)
Tel. 089-96 3167/3168 - Fax 089-96 3169 – e-mail: csef@unisa.it

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ABSTRACT

I analyze a static, noisy rational expectations equilibrium model where traders exchange vectors of assets accessing multi-dimensional information under two alternative market structures. In the first (the unrestricted system), informed speculators condition their demands for each asset on all equilibrium prices and market makers set prices observing all order flows; in the second (the restricted system), speculators are restricted to condition their demand on the price of the asset they want to trade and market makers only observe the order flow of the asset they price. I show that informed traders' incentives to collect and exploit multi-dimensional private information depend on the number of prices they can condition upon when submitting their demand schedules, and on the specific price formation process one considers. Building on this insight, I then give conditions under which the restricted system is more efficient than the unrestricted system.

Keywords: Financial Economics, Asset Pricing, Information and Market Efficiency, Market Mechanisms

JEL Classification: G100, G120, G140

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* CSEF, Università di Salerno, and Departament d'Economia i Empresa, Universitat Pompeu Fabra, Ramon Trias Fargas, 25–27, E-08005 Barcelona. E-mail: giovanni.cespa@econ.upf.es.

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1 Introduction

The literature on stock market design has recently devoted attention to mechanisms allowing traders to exchange *portfolios of assets*. The idea behind these contributions is that the impossibility of operating in more than one market at the same time, a feature that characterizes virtually all of the existing stock markets, may either affect traders' capability to rebalance their portfolios (Bossaerts, Fine, and Ledyard (2002)) or seriously hamper their ability to exploit trade relevant information, and trigger program trades that cause price *oscillations* (Amihud and Mendelson (1991a, 1991b)). A mechanism allowing the trade of asset portfolios would thus mitigate price volatility and permit better portfolio re-balancing.

From the perspective of market design it is then important to understand how to concretely implement such a trading system. Consider, for instance, a trader submitting an order to buy a given vector of assets. She may want to condition her demand not only on the price of the asset she is trading, but also to take advantage of *cross-conditioning* possibilities. In particular, she may want to condition her decision to buy say a hundred shares of company A both on the price of company A and on that of company B, to the extent that information flows about the two companies are somewhat *related*. This type of cross-conditioning has been advocated by many authors on grounds of improved efficiency and reduced volatility (Beja and Hakansson (1979), Amihud and Mendelson (1991a), Economides and Schwartz (1995)). Surprisingly, little theoretical analysis has assessed the desirability of its introduction.

Aside from theoretical considerations, this analysis is motivated by the deep changes in trading procedures spurred by recent advances in information technology. ITG, the technology company running the POSIT network, has recently started allowing its clients the submission of multi price contingent orders. [1] Optimark, a trading system directed to institutional traders, allowed the specification of different parameters upon which to condition trade execution. [2] Archipelago, an open limit

[1] The electronic equity-matching system ITG started operating 14 years ago. Its trading platform QuantEX permits an order submission strategy ("Pairs") that automatically executes orders "when the spread differential between two stocks reaches a specified level." *QuantEX, Electronic Trading Made Intelligent*, available at `http://www.itginc.com`. I thank Ekkehart Boehmer for pointing this evidence to me.

[2] Besides submitting traditional limit and market orders, traders could condition their demand on a number of contingencies. For instance, a trader could specify her willingness to pay more for a larger order size in a confidential way, so that the actual transaction price would not be affected. See Clemons and Weber (1998).

order book system, allows participants to submit *non standard* types of orders. [3]

Motivated by these considerations, I analyze the properties of two call-auction trading mechanisms in which a vector of assets is traded among risk averse, informed competitive speculators and liquidity traders with the intermediation of a competitive, risk neutral market-making sector. In the *unrestricted* mechanism, informed agents submit *multi-price* contingent orders. In the *restricted* mechanism they submit *standard* limit orders. As far as market makers: in the unrestricted mechanism they are able to observe all assets' order flows, whereas in the restricted mechanism they only observe the order flow of the asset they price. Equilibrium behavior is analyzed and implications for price informativeness and traders' welfare are addressed.

Contrary to common intuition, I challenge the view that a multi-price contingent system should *always* render the market more efficient. Amihud and Mendelson (1991b) argue that "a mechanism which enables *simultaneous conditioning* of orders for different assets (...) would increase the information available to traders, improve value discovery and reduce volatility." This assertion points at the positive effect that observing multiple sources of correlated information has. By contrast, my paper unveils the negative side of a multi-price contingent system, by analyzing its *feedback* effect on traders' incentives to exploit private information. Indeed, a fundamental insight of the paper is that *different* trading mechanisms generate *different* incentives to exploit private information. The unrestricted system spurs traders to use *non* payoff-relevant multidimensional information. Conversely, the restricted system enhances incentives to exploit payoff-related multidimensional information.

In the unrestricted system, a trader conditions her order on multidimensional private information if and only if this allows her to separate noise from fundamental information. To see this, suppose a trader receives *good news* about two assets. This can either be a signal that both assets are valuable or the consequence of a bias in her private signals. However, if signal error terms are, say, positively correlated, observing that both prices are, for example, lower than her private signals reinforces the trader's suspicion that her private information is biased. As a consequence, she revises downwards her estimation of payoff values and scales down her position in

[3]For instance, traders can post "discretionary orders," where they specify both a limit price and the price difference they are willing to accept to get the order executed (for instance, a trader may want to buy 1000 shares at 10$ but may be willing to pay 10 1/4$ at most. The order is posted at 10 and if a sell at 10 1/4 enters the book, it is executed). Also, they can post tracking orders that are automatically adjusted to the National Best Bid and Offer (NBBO) changes. See *Wall Street Letter*, December 4, 2000. For a survey of recent trading platforms' innovations see the *Economist*, May, 18th 2000.

both assets. But what if error terms are uncorrelated? In this case, knowing that both prices are lower than private signals does not help the trader learn whether signals are upwardly biased or indeed asset values are actually high. She thus refrains from exploiting all private information and submits *single* signal contingent orders. Summarizing, traders exploit the *whole* vector of private signals *provided* the error terms affecting their information are correlated.

Things are different in a restricted system where, as traders cannot offset the content of *all* private signals with the market's opinion, they rely *more* on private information. As a consequence, traders exploit the whole vector of private signals *not only* when these are correlated because of error terms but *also* when signal correlation is due to payoffs.

Comparing incentives across mechanisms, it turns out that speculators in the unrestricted system have more of an incentive to collect private information whenever private signals' correlation is only due to error terms. Conversely, when private signals are correlated only through payoffs, speculators' incentives to gather private information in the restricted system top those they have in the unrestricted system. As the flip side of the incentives' coin is traders' speculative aggressiveness, it thus follows that speculators trade more (less) aggressively on private information in the unrestricted system whenever private signals' correlation is only due to error terms (payoffs).

Based on this effect, I find conditions under which the unrestricted mechanism is less efficient than the restricted one. Indeed, as trading aggressiveness impacts market efficiency, one expects prices to be *more informative* in the system where speculators trade more aggressively on fundamental-related private information. This intuition is correct, provided the information structure is symmetric. [4] In this case, as argued above, traders speculate *more aggressively* on their signals in the restricted system, embed *more* payoff-relevant information in the order flows, and thus render prices *better* estimators of the payoff values. This, however, comes at the cost of making the price impact of trades harsher and, thus, noise traders' expected losses higher.

The paper also analyzes a market where, at the opening call, market makers set prices observing more than one order flow while traders bear single price restrictions in their orders. An example of such an *intermediate* market is given by the opening

[4] In other words, when the random variables representing the information structure of the market have equal variances.

call auction carried out in the NYSE. [5] Numerical simulations show that insofar as noise traders' demand is not very dispersed, when the information structure is symmetric and only correlation across asset payoffs affects order flows, the *restricted* system delivers more informative prices than the *intermediate* system and this, in turn, is more informative than the *unrestricted* system. The effect at work is the same as outlined above: restricting the amount of *public* information that informed speculators observe forces them to exploit their *private* information more aggressively, enhancing the informativeness of order flows.

Related Literature

This paper contributes to the literature studying the effects of different trading mechanisms on agents' behavior and market patterns. Most of this literature has concentrated on the analysis of single (risky) asset markets. Madhavan (1992) compares the properties of quote driven systems with those of order driven systems. Biais (1993) contrasts centralized and fragmented markets. Pagano and Röell (1996) assess the effects of market transparency on uninformed traders' losses. Grossman (1992), in a closely related paper, justifies the coexistence of upstairs and downstairs markets. He argues that, contrary to what economic theory usually assumes, technical limitations prevent investors from expressing their demands as a function of a price vector, and from continuously updating them as new information arrives. This precludes investors' preferences from being accurately represented on organized markets, and gives upstairs dealers, acting as *repositories of information* about unexpressed demands, a transaction costs' advantage vis-à-vis downstairs dealers. In view of this paper's results, and insofar as a major function stock markets perform is to signal firms' *true* assets' payoffs, overcoming technological limitations may not always be a good idea, as it can impair price efficiency. [6]

To summarize, little is known about the properties of markets where traders' private information is multi-dimensional. A notable exception is the paper by Manzano (1997) where the author, in a multi-dimensional Kyle model, compares multi-price and single-price contingent systems. Also related are the analyses of Wohl and Kandel (1997) and Brown and Holden (2002). These papers study a trading mechanism where agents condition their demand for a given asset on a market index. Hence,

[5] See Lindsay and Schaede (1990) and O'Hara (1995).

[6] See Fishman and Hagerty (1992) for a discussion of the importance of stock price efficiency for production decisions within *and* outside the firm.

their focus is rather on the advantages of avoiding *mispricing risk*. [7] However, none of the above papers has assessed the effect that observing multiple sources of *endogenous* public information (i.e. equilibrium prices) has on traders' incentives to exploit multidimensional private information and on price efficiency.

The paper is organized as follows. In the next section, I compare a one-asset market where traders submit limit orders to one where they submit market orders. This provides a useful benchmark on which to build the comparison of market mechanisms in the multi-asset setup. In the third section, I characterize the unique equilibria of the two mechanisms. In the fourth section, I compare their properties and in the fifth section I introduce the *intermediate* mechanism and numerically compare its properties to those of the mechanisms studied in the previous sections. The sixth section concludes the paper. A final appendix collects most of the proofs.

2 The benchmark: limit orders vs. market orders

In this section I compare the properties of two markets where informed speculators submit either *limit orders* or *market orders*. As will become clear later, insofar as traders in the restricted system fail to condition their demand on all the sources of information related to the asset payoff (as in a market order market), this provides a useful benchmark on which to build the comparison of market mechanisms in the multi-asset setup.

In both markets a single risky asset with liquidation value $v \sim N(\bar{v}, \tau_v^{-1})$ and a riskless asset with unitary return, are traded among risk averse informed agents and noise traders with the intermediation of a competitive, risk neutral market making sector. There is a continuum of informed agents in the interval $[0, 1]$. Each informed agent k receives a private signal $s_k = v + \epsilon_k$ about the unknown v, where $\epsilon_k \sim N(0, \tau_\epsilon^{-1})$, and ϵ_k, ϵ_h are independent for $k \neq h$. Assume that her preferences are represented by a CARA utility $U(\pi_k) = -\exp\{-\pi_k/\gamma\}$ where $\gamma > 0$ is the coefficient of constant absolute risk tolerance and $\pi_k = x_k(v - p)$ is the profit of buying x_k units of the asset at price p. Normalize the informed traders' initial wealth to zero and let noise traders submit a random demand $u \sim N(0, \tau_u^{-1})$. Finally, assume that the random variables v, u, ϵ_k are independent $\forall k$ and that the collective private

[7] Mispricing risk is the risk that a limit order is executed at a mispriced limit price (as is the case, e.g. when some relevant information is revealed to the market and the limit price is not updated to take it into account).

information of informed agents reveals v: $\int_0^1 s_k dk = v$, a.s..

2.1 The limit-order market

Suppose that informed agents submit limit orders, i.e. an agent k submits a schedule $X_{Lk}(s_k, p)$ indicating her desired position in the risky asset contingent on her private signal and on the price and restrict attention to linear equilibria where $X_{Lk}(s_k, p) = a_L s_k + \psi(p)$, and $\psi(\cdot)$ is a linear function. Competitive, risk neutral market makers set a semi-strong efficient equilibrium price conditional on the observation of the order book $L = \int_0^1 x_k dk + u = a_L v + u + \psi(p)$. Let $z_L = a_L v + u$ be the informational content of the order book. Then, $p = E[v|z_L]$ and the following result applies

Proposition 1 *In the limit-order market there is a unique linear equilibrium. It is symmetric and given by* $X_L(s_k, p) = a_L(s_k - p)$ *and* $p = \lambda_L z_L + (1 - \lambda_L a_L)\bar{v}$, *where* $a_L = \gamma\tau_\epsilon$, $\lambda_L = a_L \tau_u / \tau_L$ *and* $\tau_L = (Var[v|z_L])^{-1} = \tau_v + a_L^2 \tau_u$.

Proof. See Vives (1995a). QED

According to intuition, informed speculators' trading aggressiveness in the limit-order market a_L increases in the (conditional) precision of their private signal and in the risk tolerance coefficient. Market makers' reaction to the presence of informed speculators $\lambda_L = a_L \tau_u / \tau_L$ is captured by the OLS regression coefficient of the unknown payoff value on the order-book. As common in this literature λ_L measures the reciprocal of market depth (see e.g. Kyle (1985) and Vives (1995a, 1995b)). The informativeness of the equilibrium price is measured by the reciprocal of the payoff conditional variance given the order flow: $(\mathrm{Var}[v|z_L])^{-1} = \tau_L$. The higher τ_L, the smaller the residual uncertainty on the *true* payoff value once the order-flow has been observed.

2.2 The market-order market

Suppose instead informed agents submit market orders, i.e. each agent k submits a schedule $X_{Mk}(s_k)$ contingent on the private signal she receives and restrict attention to linear equilibria where $X_{Mk}(s_k) = a_M s_k + b_M$. Competitive, risk neutral market makers set a semi-strong efficient equilibrium price conditional on the observation of the order book $L = \int_0^1 x_k dk + u = a_M v + u$. Let $z_M = a_M v + u$ be the informational content of the order book. Then, $p = E[v|z_M]$ and the following result applies

Proposition 2 *In the market-order market there is a unique linear equilibrium. It is symmetric and given by* $X_M(s_k) = a_M(s_k - \bar{v})$ *and* $p = \lambda_M z_M + (1 - \lambda_M a_M)\bar{v}$*, where* $a_M = \gamma(\tau_\epsilon^{-1} + \mathrm{Var}[p])^{-1}$ *is the unique positive root of the cubic equation* $F(a_M) = ((\tau_\epsilon/\gamma)a_M - 1)\tau_v + (\lambda_M/\gamma)a_M^2 = 0$*, with* $\lambda_M = a_M\tau_u/\tau_M$ *and* $\tau_M = (\mathrm{Var}[v|z_M])^{-1} = \tau_v + a_M^2\tau_u$*.*

Proof. See Vives (1995b). QED

Informed speculators' trading aggressiveness in the market-order market a_M is inversely related to price volatility $\mathrm{Var}[p]$. Indeed, while traders condition on private information, they *do not* anticipate the equilibrium price. Thus, the larger the equilibrium price variance, the higher the *execution* risk, i.e. the risk of having their order executed at a price different from the one prevailing when they submitted it, and the smaller a_M.

2.3 Comparing limit orders with market orders

Given the previous results, we can now compare traders' behavior, market performance and traders' wealth in the two markets. Indicate with $\mathrm{Var}[p; a_L]$ and $\mathrm{Var}[p; a_M]$ respectively the *ex ante* volatility in the limit-order market and in the market-order market.

Proposition 3

1. *Informed traders in the market-order market trade less aggressively than in the limit-order market:* $a_M < a_L$*; as a result prices in the market-order market are less informative and ex-ante less volatile than in the limit-order market:* $\tau_M < \tau_L$ *and* $\mathrm{Var}[p; a_M] < \mathrm{Var}[p; a_L]$*;*

2. *the market-order market is deeper than the limit-order market if and only if* $a_M/a_L < \tau_M/\tau_L$*.*

Proof. The first part follows immediately from the definitions of a_M and a_L, since $a_M \equiv \gamma(\tau_\epsilon^{-1} + \mathrm{Var}[p])^{-1} < \gamma\tau_\epsilon \equiv a_L$. Given this, $\tau_M < \tau_L$ and owing to price efficiency $\mathrm{Var}[p; a_M] = \tau_v^{-1} - \tau_M^{-1} < \tau_v^{-1} - \tau_L^{-1} = \mathrm{Var}[p; a_L]$. Part 2 follows from the definition of λ_M and λ_L. It is immediate to see that there are values of the parameters for which $\lambda_M < \lambda_L$ as rearranging this inequality leads to $(a_M - a_L)(\tau_v - a_M a_L \tau_u) < 0$. As $a_M < a_L$, for this condition to hold it must be that $\tau_v > a_M a_L \tau_u$. Suppose this is never possible, i.e. $\tau_v/a_L\tau_u \leq a_M$, then as $a_M < a_L$, this implies $\tau_v/\tau_u < a_L^2$ which is clearly not always true (e.g. choose $\gamma\tau_\epsilon > \tau_v = \tau_u$). QED

The intuition for the above results is straightforward: risk averse informed speculators in the market-order market suffer from execution risk. As a consequence, they scale back their aggressiveness w.r.t. speculators in the limit-order market. Therefore, they embed *less* information in the order flow, lowering the market-order market price informativeness and making it less volatile.

Comparing depth across the two markets, two effects are at play: first, as $a_M < a_L$, market makers' adverse selection problem is less important in the market-order market; second, since $\tau_M < \tau_L$, market makers in the market-order market are less able to disentangle noise from information. If the positive effect coming from the reduction in traders' aggressiveness is stronger than the negative effect due to the reduction in transparency, the market-order market is deeper.

Remark 1 The result on price informativeness contrasts with Rochet and Vila (1994), who in their analysis of Kyle (1985) show that price informativeness does not depend on the type of order the insider submits. The reason is that in their limit order model strategic behavior leads the insider to scale down her aggressiveness; this equalizes the amount of information flowing to the market in the limit order model to the one of the market order model. In the present context, no strategic effects arise while risk aversion translates execution risk in a trading aggressiveness reduction.

Remark 2 In a semi-strong efficient market, price volatility is due to the arrival of information. Thus, the more volatile is the market, the more information is being embedded into the price by traders' speculative activity and the more informative is the price about the asset payoff.

Proposition 4

1. *An informed agent k prefers to trade in the limit-order market rather than in the market-order market if and only if $(a_L/\lambda_L^{-1})/(a_M/\lambda_M^{-1}) \leq (\tau_\epsilon + \tau_L)/\tau_L$;*

2. *noise traders' expected losses are larger in the limit-order market if and only if $a_M/a_L \leq \tau_M/\tau_L$.*

Proof. Applying lemma 3, $E[U(x_M(v-p))] = -((\tau_M\tau_v + a_M^2\tau_u\tau_\epsilon)/(\tau_M(\tau_v+\tau_\epsilon)))^{1/2}$ and $E[U(x_L(v-p))] = -(\tau_L/(\tau_L+\tau_\epsilon))^{1/2}$. Simplifying, $E[U(x_L(v-p))] \geq E[U(x_M(v-p))] \Leftrightarrow (a_L/\lambda_L^{-1})/(a_M/\lambda_M^{-1}) \leq (\tau_\epsilon+\tau_L)/\tau_L$. To see that there are parameter values for which this condition can be satisfied, choose $\gamma = \tau_\epsilon = \tau_v = \tau_u = 1$. For part 2, $E[u(v-p)] = -\lambda_M\tau_u^{-1}$ in the market-order market and $E[u(v-p)] = -\lambda_L\tau_u^{-1}$ in the limit-order market. The result follows. QED

Two factors influence a trader's choice between the limit-order market and the market-order market: the relative impact that traders' aggressiveness has on market depth *and* the informational advantage traders retain over market makers. The smaller the ratio $(a_L/\lambda_L^{-1})/(a_M/\lambda_M^{-1})$, the lower the relative impact on the market that limit orders have with respect to market orders. The higher the ratio $(\tau_\epsilon+\tau_L)/\tau_L$, the larger is the informational advantage that limit order traders retain over market makers. Thus, if (and only if) $(a_L/\lambda_L^{-1})/(a_M/\lambda_M^{-1}) \leq (\tau_\epsilon+\tau_L)/\tau_L$ a trader finds that the advantages of trading in the limit-order market overcome those of trading in the market-order market.

As noise traders' expected losses are inversely proportional to market depth, whenever the market-order market is deeper than the limit-order market, noise traders' experience lower expected losses in that market.

Concluding, owing to execution risk, risk averse informed speculators trade *less* aggressively in a market-order market than in a limit-order market. As a consequence, the equilibrium price in the former market is *less* informative than in the latter. As markets cannot be unambiguously ranked according to depth, noise traders' expected losses can either be larger or smaller in the limit-order market. Finally, an informed trader's choice to submit a limit order instead of a market order depends on the relative impact his strategy has on market depth *and* on the informational advantage she retains over market makers.

3 Multi-price vs. single price contingent trading mechanisms

In this section I extend the assumptions of section 2 to a multi-asset setup. For the notation let us indicate with $\mathbf{\Pi}_{\boldsymbol{x}}$ the precision matrix of the two-dimensional random vector $\boldsymbol{x}$; with τ_{x_i} the precision of the random variable x_i and with ρ_x the correlation coefficient of the random vector (x_1, x_2). Suppose that informed and noise traders exchange a vector of *two* risky assets with random liquidation value $\boldsymbol{v} = (v_1, v_2) \sim N(\bar{\boldsymbol{v}}, \boldsymbol{\Pi}_{\boldsymbol{v}}^{-1})$ and a riskless one with unitary return with the intermediation of a competitive, risk neutral market making sector. There is a continuum of informed agents in the interval $[0, 1]$. Each informed agent k receives a vector of private signals $\boldsymbol{s}_k = \boldsymbol{v} + \boldsymbol{\epsilon}_k$ about the unknown $\boldsymbol{v}$, where $\boldsymbol{\epsilon}_k = (\epsilon_{k1}, \epsilon_{k2}) \sim N(\mathbf{0}, \boldsymbol{\Pi}_{\boldsymbol{\epsilon}}^{-1})$, and $\boldsymbol{\epsilon}_k$ and $\boldsymbol{\epsilon}_h$ are independent for $k \neq h$. Assume that her preferences are represented by a CARA utility $U(\pi_k) = -\exp\{-\pi_k/\gamma\}$ where $\gamma > 0$ is the coefficient of constant absolute risk tolerance and $\pi_k = \boldsymbol{x}_k'(\boldsymbol{v} - \boldsymbol{p})$ is the profit of buying (x_{k1}, x_{k2}) units of each asset at price $\boldsymbol{p}$. Normalize the informed traders' initial wealth to zero and let noise traders submit a random demand $\boldsymbol{u} = (u_1, u_2) \sim N(\mathbf{0}, \boldsymbol{\Pi}_{\boldsymbol{u}}^{-1})$. Finally, assume that the random vectors $\boldsymbol{v}, \boldsymbol{u}, \boldsymbol{\epsilon}_k$ are independent $\forall k$ and that the collective private information of informed agents reveals $\boldsymbol{v}$: $\int_0^1 \boldsymbol{s}_k dk = \boldsymbol{v}$, a.s..

With the above assumptions, I consider two market mechanisms:

1. the *unrestricted mechanism* where *(a)* speculators condition their demand for each asset j on the vector of private signals $\boldsymbol{s}_k$ and on the price of assets $j = 1, 2$, and *(b)* market makers set the price of asset j conditionally on the observation of the order flow of both assets $j = 1, 2$;

2. the *restricted mechanism* where *(a)* speculators condition their demand for an asset j on the vector of private signals $\boldsymbol{s}_k$ and on the price of asset j only and *(b)* market makers set the price of asset j conditionally on the observation of the order flow j.

3.1 The unrestricted system

The unrestricted model is a version of the multi-asset model of Admati (1985) with the addition of a risk-neutral, competitive, market-making sector as in Vives (1995a). [8]

Suppose informed traders submit multi-price contingent orders. Thus, each trader k submits a vector of demand schedules $\mathbf{X}_k(\boldsymbol{s}_k, \boldsymbol{p})$, indicating the position desired in each asset j at every price vector $\boldsymbol{p}$, contingent on the available private information; I restrict attention to linear equilibria where $\mathbf{X}_k(\boldsymbol{s}_k, \boldsymbol{p}) = \mathbf{A}\boldsymbol{s}_k + \phi(\boldsymbol{p})$, and $\mathbf{A}$, $\phi(\cdot)$ are, respectively, the matrix of trading intensities and a linear function of current prices.

Market makers observe the vector of aggregate order flows $\mathbf{L}(\cdot) = \int_0^1 \boldsymbol{x}_k dk + \boldsymbol{u}$. Therefore, in pricing asset j each market maker uses *both* the information contained

[8] For noisy rational expectations equilibrium models with a single risky asset see Hellwig (1980), Diamond and Verrecchia (1981) and Grossman and Stiglitz (1980).

in order flow j and that contained in order flow $i \neq j$. Due to traders' linear strategies, the aggregate order flow is then $\mathbf{L}(\cdot) = \boldsymbol{z} + \phi(\boldsymbol{p})$, where $\boldsymbol{z} = \mathbf{A}\boldsymbol{v} + \boldsymbol{u}$, is the vector of order flows' informational contents. Owing to competition for each order flow and risk neutrality, market makers set a semi-strong efficient price vector $\boldsymbol{p} = E[\boldsymbol{v}|\boldsymbol{z}] = \mathbf{\Pi}^{-1}\left(\mathbf{\Pi}_{\boldsymbol{v}}\bar{\boldsymbol{v}} + \mathbf{A}'\mathbf{\Pi}_{\boldsymbol{u}}\boldsymbol{z}\right)$, where $\mathbf{\Pi} = \mathbf{\Pi}_{\boldsymbol{v}} + \mathbf{A}'\mathbf{\Pi}_{\boldsymbol{u}}\mathbf{A}$, and the following result holds:

Proposition 5 *In the unrestricted system there exists a unique equilibrium in linear strategies. It is symmetric and given by*

$$\mathbf{X}_k(\boldsymbol{s}_k, \boldsymbol{p}) = \mathbf{A}(\boldsymbol{s}_k - \boldsymbol{p}), \tag{3.1}$$

and $\boldsymbol{p} = \mathbf{\Lambda}\boldsymbol{z} + (I - \mathbf{\Lambda}\mathbf{A})\,\bar{\boldsymbol{v}}$, *where* $\mathbf{A} = \gamma\mathbf{\Pi}_{\boldsymbol{\epsilon}}$ *and* $\mathbf{\Lambda} = \mathbf{\Pi}^{-1}\mathbf{A}'\mathbf{\Pi}_{\boldsymbol{u}}$.

Remark 3 The matrix $\mathbf{\Lambda}$ maps order flows into prices. For the equilibrium to be well-defined, $\mathbf{\Lambda}$ must be invertible and, given the model's assumptions, this is always the case. Notice also that, owing to multicollinearity effects, the diagonal elements of this matrix can be negative (see Admati 1985).

The next corollary characterizes how speculators exploit public and private information in equilibrium.

Corollary 1 *In the unrestricted system, an informed speculator's demand for each asset* $j = 1, 2$, *depends on the whole private signal vector* $\boldsymbol{s}_k$ *and on the whole price vector* $\boldsymbol{p}$ *if and only if* $\rho_\epsilon \neq 0$.

Proof. Follows from the fact that $\mathbf{A} = \gamma\mathbf{\Pi}_{\boldsymbol{\epsilon}}$. QED

According to corollary 1, informed multi-price conditioning is optimal if and only if the conditional precision matrix of the speculators' private signals is *not* diagonal. The intuition is as follows. As prices are set contingently on the observation of *all* the order flows, cross-asset public information is already fully exploited. Hence, informed traders cannot improve upon market makers in their estimation of $\boldsymbol{v}$ by combining public information. However, market makers cannot observe the signals informed traders receive. Therefore, to the extent that error terms are correlated, multi-price conditioning allows informed agents to disentangle price realizations due to fundamental information from those due to liquidity traders' demands. Consider the following example.

Example 1 Writing in scalar form a trader's strategy one can see that the trading intensity in an asset j is the composition of two effects: a *direct* one stemming from the informational advantage the speculator has over the rest of the market in asset j, and an *indirect* one coming from the informational advantage she has on the remaining asset, to the extent that the received signals are correlated. To see this, indicate with τ_{ϵ_j}, $j = 1, 2$ the (conditional) signal precision in asset j. Then, the strategy of a speculator in asset j can be written as follows:

$$X_{kj}(\boldsymbol{s}_k, \boldsymbol{p}) = \frac{\gamma\tau_{\epsilon_j}}{(1-\rho_\epsilon^2)}(s_{kj} - p_j) - \frac{\gamma\rho_\epsilon\sqrt{\tau_{\epsilon_j}\tau_{\epsilon_i}}}{(1-\rho_\epsilon^2)}(s_{ki} - p_i).$$

Assume that $\rho_\epsilon > 0$ and that speculator k receives two signals s_{kj}, s_{ki} such that $s_{kj} > p_j$ and $s_{ki} > p_i$. This can happen for two reasons: either both assets are worth more than what the market thinks (i.e. asset prices are biased downwards e.g. by noise traders' selling pressure); or both signals are biased upwards. A downward bias in equilibrium prices is good news since it gives the trader the possibility of taking advantage of the market's forecast error. Her demand in each asset is larger, the more precise are the signals she has received. However, the existence of positive correlation across signal-error terms strengthens the hypothesis of a contemporaneous, upward bias in the speculator's signals. [9] Given this, the speculator reinforces her belief that the good news she received about both assets is due to the effect of error terms and reduces her demand in both asset j and asset i. [10]

When no correlation across error terms exists ($\rho_\epsilon = 0$), speculators have no way to reduce the bias in their strategies by pooling together private signals and find it optimal to submit single-signal *and* single-price contingent orders.

Notice, however, that even if $\rho_\epsilon = 0$ market makers still use the information contained in all the order flows when pricing an asset. Indeed, their demand can be written as

$$\mathbf{X}_k^{MM}(\boldsymbol{p}) = (\boldsymbol{\Lambda}^{-1} - \mathbf{A})(\bar{\boldsymbol{v}} - \boldsymbol{p}),$$

and it is easy to see that the diagonality of $\boldsymbol{\Pi}_{\boldsymbol{\epsilon}}$ does not imply the diagonality of $(\boldsymbol{\Lambda}^{-1} - \mathbf{A})$.

I now turn the attention to the characterization of the restricted system.

[9] This is the case because an error that biases upward the information contained in s_{ki} is more likely to happen together with an error biasing upwards the information about asset one as well.

[10] The correction that correlated information induces is stronger (weaker) the higher (lower) is the correlation across error terms. Indeed, for a bivariate normal distribution, the value of $F_{\rho_\epsilon}(\epsilon_{k1}, \epsilon_{k2})$ is increasing in ρ_ϵ for all $\rho_\epsilon \in [-1, 1]$ and all fixed $(\epsilon_{k1}, \epsilon_{k2})$: a higher correlation across error terms increases the probability that a joint bias in private signals occurs (see e.g. Tong 1990).

3.2 The restricted system

In the restricted system, a speculator k can condition her demand for an asset j on the whole vector of private signals $\boldsymbol{s}_k$ and on the price of asset j only. In this case, we can interpret market makers as uninformed speculators. Therefore, the model captures the features of the opening auction of those markets where all traders are allowed to condition their demand of an asset j on its price only. [11]

In any linear equilibrium, private and public information are conditionally independent, so the speculator's strategy depends both on her signal and on the price. In particular, assume that a speculator k submits a demand schedule $X_{Rkj}(\boldsymbol{s}_k, p_{Rj})$, $j = 1, 2$ indicating the desired position in asset j at every price p_{Rj}, contingent on the available information, and let us restrict attention to linear equilibria.

The market makers of asset j, observe the asset order flow (that carries information about all the assets) but *do not* observe the order flow of the other asset. Formally, they thus observe $L_{Rj}(\cdot) = \int_0^1 x_{Rkj} dk + u_j$, $j = 1, 2$. Consider a candidate symmetric equilibrium $X_{Rkj}(\boldsymbol{s}_k, p_{Rj}) = \boldsymbol{a}'_{Rj}\boldsymbol{s}_k + \phi_{Rj}(p_{Rj})$, where $\boldsymbol{a}_{Rj}$ is the 2×1 vector of trading intensities and $\phi_{Rj}(\cdot)$ is a linear function of the j-th price. The aggregate order flow of asset j is then $L_{Rj}(\cdot) = z_{Rj} + \phi_{Rj}(p_{Rj})$, where $z_{Rj} = \boldsymbol{j}'(\mathbf{A}_R\boldsymbol{v} + \boldsymbol{u})$, is the informational content of order flow j, $\boldsymbol{j}$ is a column vector containing a 1 in the j-th position and a zero elsewhere and $\mathbf{A}_R$ is the matrix of trading intensities in the restricted system.

Given competition and market makers' risk neutrality, the equilibrium price of asset j is $p_{Rj} = \bar{v}_j + \lambda_{Rj}\boldsymbol{j}'(\mathbf{A}_R(\boldsymbol{v} - \bar{\boldsymbol{v}}) + \boldsymbol{u})$, where

$$\lambda_{Rj} = \frac{\tau_{u_j}\tau_{v_i}\left(\gamma\tau_{\epsilon_j} + a_{Rji}\left(\rho_v\sqrt{\tau_{v_j}/\tau_{v_i}} - \rho_\epsilon\sqrt{\tau_{\epsilon_j}/\tau_{\epsilon_i}}\right)\right)}{\tau_{u_j}\left((a_{Rjj})^2\tau_{v_i} + (a_{Rji})^2\tau_{v_j} + 2\rho_v a_{Rjj}a_{Rji}\sqrt{\tau_{v_j}\tau_{v_i}}\right) + \tau_{v_j}\tau_{v_i}}, \tag{3.2}$$

$j \neq i = 1, 2$, is the regression coefficient of v_j on z_{Rj} i.e. the usual measure of market depth and a_{Rji} is the (j, i)-th element of $\mathbf{A}_R$. Consequently, we have the following

Lemma 1 *In every linear equilibrium of the restricted system, the vector of equilibrium prices is given by*

$$\boldsymbol{p}_R = \boldsymbol{\Lambda}_R\boldsymbol{z}_R + (I - \boldsymbol{\Lambda}_R\mathbf{A}_R)\,\bar{\boldsymbol{v}}, \tag{3.3}$$

where $\boldsymbol{\Lambda}_R = \operatorname{diag}(\lambda_{R1}, \lambda_{R2})$ *and* $\boldsymbol{z}_R = \mathbf{A}_R\boldsymbol{v} + \boldsymbol{u}$ *are respectively the matrix of market depths and the vector of order flows' informational contents in the restricted model.*

[11] To the best of my knowledge, this is the first attempt to characterize the equilibrium in a multi-asset framework where competitive, *risk averse* traders receive different signals and bear restrictions in the number of asset prices they can condition upon. Manzano (1997) studies the case where traders are risk neutral and act strategically.

In the restricted system market makers can exploit cross-asset information in estimating an asset value *if and only* if speculators use both their signals when trading the asset. Conversely, in the unrestricted system even if $\mathbf{A}$ is diagonal, the price of an asset j depends on the order flow of asset $i \neq j$ (to the extent that either $\mathbf{\Pi}_{\boldsymbol{v}}$ or $\mathbf{\Pi}_{\boldsymbol{u}}$ are not diagonal).

Informed speculators' equilibrium demand parameters are characterized in the following lemma.

Lemma 2 *In every linear equilibrium of the restricted system, an informed speculator k's demand for asset $j = 1, 2$ is given by $X_{Rkj}(\boldsymbol{s}_k, p_{Rj}) = \boldsymbol{j}'\mathbf{A}_R(\boldsymbol{s}_k - \bar{\boldsymbol{v}}) + b_{Rj}(\bar{v}_j - p_{Rj})$, where,*

$$\boldsymbol{j}'\mathbf{A}_R = \gamma\left(Var[v_j|\boldsymbol{s}_k, p_{Rj}]\right)^{-1}\mathbf{c}_{2j}, \text{ and } b_{Rj} = \gamma\left(Var[v_j|\boldsymbol{s}_k, p_{Rj}]\right)^{-1}\left(1 - c_{1j}/\lambda_{Rj}\right), \quad (3.4)$$

are respectively, the vector of the sensitivities of asset j's demand to the speculator's private signals, and the sensitivity of the demand for asset j to the equilibrium price of the asset, and c_{1j}, $\mathbf{c}_{2j}$, and $Var[v_j|\boldsymbol{s}_k, p_{Rj}]$ are defined in the appendix.

The next proposition proves existence and uniqueness of the equilibrium in the restricted system.

Proposition 6 *In the restricted system there exists a unique equilibrium in linear strategies. The equilibrium is symmetric and the price vector is given by (3.3), while the demand parameters are implicitly defined by (3.4).*

Existence and uniqueness of equilibrium are not obvious results given that speculators' equilibrium trading intensities come from the solution of a system of two cubic equations. In the appendix, I show how to simplify the system, reducing it to a solvable cubic.

The next proposition characterizes the equilibrium parameters.

Proposition 7 *In the unique linear equilibrium of the restricted system*

1. *$a_{Rji} > 0$ if and only if $\rho_\epsilon\sqrt{\tau_{\epsilon_j}/\tau_{\epsilon_i}} < \rho_v\sqrt{\tau_{v_j}/\tau_{v_i}}$;*
2. *(a) $a_{Rjj} = \gamma\tau_{\epsilon_j}(1 - \gamma^{-1}a_{Rji}Cov[\epsilon_1, \epsilon_2]) > 0$ and (b) $\lambda_{Rj} > 0$;*
3. *if $\rho_\epsilon = 0$, $a_{Rjj} = \gamma\tau_{\epsilon_j}$ and $a_{Rji} \neq 0$;*
4. *if $\rho_\epsilon\sqrt{\tau_{\epsilon_j}/\tau_{\epsilon_i}} = \rho_v\sqrt{\tau_{v_j}/\tau_{v_i}}$, $a_{Rjj} = \gamma\tau_{\epsilon_j}$, $a_{Rji} = 0$, and $b_{Rj} = -a_{Rjj}$.*

The interpretation of these results is as follows. For part 1, suppose an informed speculator trading asset 1 receives two "high" signals s_{k1}, s_{k2}. This may be the effect of either fundamental information, or of errors in the signals. The first possibility is more likely the stronger is the correlation of asset payoffs compared to error terms' correlation and the higher is the relative dispersion of asset payoffs compared to error terms' relative dispersion. In this case, indeed, the effect of fundamental information *dominates* the effect of errors in the signal vector.

For part 2 (a) suppose that $a_{R12} > 0$. This means that an informed agent increases her speculative position in asset 1 upon receiving "good news" about asset 2. However, if $\rho_\epsilon > 0$, good news about asset 1 may come from the joint effect of signal error terms. Therefore, the trader scales down the weight she puts on s_{k1} the more, the higher is the trading intensity she puts on s_{k2}. For 2 (b), the impossibility of observing more than one order flow when pricing an asset eliminates the multicollinearity effects that occur in the unrestricted system. Therefore, the matrix $\mathbf{\Lambda}_R$ is positive definite. [12]

For part 3, the intuition is that a given signal s_{ki} is useful in trading an asset $j \neq i$ insofar as it carries information about v_j or about the error term ϵ_{kj}. As the correlation across error terms vanishes, s_{ki} is still useful for the information it contains about v_j. Therefore, speculators use it in trading asset j.

Result 4 is not surprising given what we said above. If $\rho_\epsilon \sqrt{\tau_{\epsilon_j}/\tau_{\epsilon_i}} = \rho_v \sqrt{\tau_{v_j}/\tau_{v_i}}$, there is no way for a speculator to disentangle error terms from information by pooling the two signals she receives. As a consequence $a_{Rji} = 0$.[13]

Remark 4 Notice that $\mathbf{A} = \mathbf{A}_R \Leftrightarrow a_{ji} = a_{Rji}$. Thus, a trader's sensitivity to private information differs across the two systems as long as her responsiveness to *additional* private information (the signal s_{ki} used when trading asset j) do not coincide.

Example 2 As done for the unrestricted system, let us consider an example of a trader's strategy in the restricted system:

$$X_{Rkj}(\boldsymbol{s}_k, p_{Rj}) = a_{Rjj}(s_{kj} - \bar{v}_j) + a_{Rji}(s_{ki} - \bar{v}_i) + b_{Rj}(\bar{v}_j - p_{Rj}).$$

[12] The reason for this fact here is different from the one in Caballé and Krishnan (1992). In their case the positive definiteness of the matrix mapping order flows into prices is a shortcoming of the hypothesis of imperfect competition across insiders that prevents the existence of unexploited arbitrage opportunities.

[13] Notice that the previous proposition *does not* imply that if $\mathbf{\Pi}_{\boldsymbol{v}}$ and $\mathbf{\Pi}_{\boldsymbol{\epsilon}}$ are diagonal, then $p_{Rj} = p_j$. Indeed, as long as $\rho_u \neq 0$, even though speculators in the unrestricted system do not combine the information contained in their signals, market makers can still learn from the correlation across noise terms and exploit this information when pricing assets.

Again, k's trading intensity in asset j is the composition of 2 effects: a *direct* one stemming from the informational advantage the speculator has over the rest of the market in asset j, and an *indirect* one coming from the informational advantage she has on the remaining asset, to the extent that she received correlated signals. Suppose $\rho_\epsilon\sqrt{\tau_{\epsilon_j}/\tau_{\epsilon_i}} < \rho_v\sqrt{\tau_{v_j}/\tau_{v_i}}$, and that $s_{kj} > \bar{v}_j$, $s_{ki} > \bar{v}_i$. As the effect of fundamental information *dominates* the effect of errors in the signal vector, the speculator reinforces her belief that the asset value is high and increases her long position. If $\bar{v}_j > p_{Rj}$, such a long position is further increased because of the low price the market gives to the asset. [14]

I conclude the section by considering the *symmetric* case (i.e. the case where the precision matrices are doubly symmetric). This simplifies the model and gives the following corollary of propositions 6 and 7.

Corollary 2 *In the symmetric case there exists a unique linear symmetric equilibrium of the restricted system, where informed speculators' trading intensities are implicitly defined by the following system of equations:*

$$\begin{aligned} a_{R1} &= \gamma\left(\tau_\epsilon - \frac{\rho_\epsilon\tau_\epsilon((1-\rho_v^2)(\rho_\epsilon\tau_\epsilon-(1-\rho_\epsilon^2)a_{R1}a_{R2}\tau_u)+(1-\rho_\epsilon^2)\rho_v\tau_v)}{\tau_\epsilon(1-\rho_v^2)+(1-\rho_\epsilon^2)((a_{R2})^2(1-\rho_v^2)\tau_u+\tau_v)}\right), \\ a_{R2} &= \gamma\left(-\rho_\epsilon\tau_\epsilon + \frac{\tau_\epsilon((1-\rho_v^2)(\rho_\epsilon\tau_\epsilon-(1-\rho_\epsilon^2)a_{R1}a_{R2}\tau_u)+(1-\rho_\epsilon^2)\rho_v\tau_v)}{\tau_\epsilon(1-\rho_v^2)+(1-\rho_\epsilon^2)((a_{R2})^2(1-\rho_v^2)\tau_u+\tau_v)}\right), \end{aligned} \tag{3.5}$$

where $a_{R1} = (\mathbf{A}_R)_{11} = (\mathbf{A}_R)_{22}$ *and* $a_{R2} = (\mathbf{A}_R)_{12} = (\mathbf{A}_R)_{21}$. *In this equilibrium, (1)* $a_{R2} \geq 0$, *if and only if* $\rho_\epsilon - \rho_v \leq 0$; *(2)* $a_{R1} = \gamma\tau_\epsilon - \rho_\epsilon a_{R2} > 0$, $\lambda_R > 0$.

Clearly, the intuitions given for proposition 7 carry over to the above corollary.

4 The unrestricted system vs. the restricted system

In this section, I study equilibrium behavior in the two systems and compare its effects on price informativeness and traders' welfare. I first show that different trading mechanisms generate different incentives to collect *and* exploit private information. In particular, the unrestricted system tilts traders' incentives towards collecting *non* payoff-relevant multidimensional information. Conversely, the restricted system enhances incentives to gather payoff-relevant multidimensional information. I then study how these incentives translate into trading aggressiveness and assess the impact this has on price efficiency and traders' welfare.

[14] Numerical simulations show that $b_{Rj} > 0$.

4.1 Value of private information and trading aggressiveness

In section 2 I have shown that in a one-asset world a trader's aggressiveness is related to the *type* of order she submits. In this section I show that in a multi-asset world not only the *type* of the order, but also the way prices are *formed* contributes to shape a trader's aggressiveness. I start by determining the value of an *additional* signal in both the unrestricted and in the restricted system. Intuitively, the extent to which a trader makes use of her signals should depend on the *value* she attributes to them. Such a *value* in turn should depend on the informational content of prices *and* on the number of prices the trader's strategy depends on, to the extent that private and public information (equilibrium prices) are partial substitutes. Based on this intuition, I show that in general traders' valuation for additional private information differs across the two systems. I then relate a trader's value for private information to her aggressiveness, showing that the *more* a trader values additional information the *more* aggressively she trades. Finally, I conclude the section showing that besides affecting speculative aggressiveness, trading mechanisms also influence the way traders interpret private information.

Let $\phi(s_{ki}||s_{kj},\boldsymbol{p})$ and $\phi(s_{ki}||s_{kj},p_{Rj})$ be the *maximum* prices a trader k is willing to pay in order to observe s_{ki} when she already possesses s_{kj} - i.e. the value of *additional* private information - respectively in the unrestricted and in the restricted system. [15]

Proposition 8 *In the unrestricted system, the value of additional private information is given by*

$$\begin{aligned}
&\phi(s_{ki}||s_{kj},\boldsymbol{p}) = \\
&\quad \frac{\gamma}{2}\ln\left(1+(\rho_\epsilon{}^2/D)\left((1-\rho_v^2)\gamma^2\tau_{\epsilon_j}\tau_{\epsilon_i}(\tau_{\epsilon_j}\tau_{u_j}+\tau_{\epsilon_i}\tau_{u_i}-2\rho_\epsilon\rho_u\sqrt{\tau_{\epsilon_j}\tau_{u_j}\tau_{\epsilon_i}\tau_{u_i}})\right.\right.\\
&\quad\left.\left. +(1-\rho_v^2)(1-\rho_u^2)(2-\rho_\epsilon^2)\tau_{\epsilon_j}\tau_{\epsilon_i}+(1-\rho_u^2)(\tau_{\epsilon_i}\tau_{v_j}+\tau_{\epsilon_i}\tau_{v_j}-2\rho_\epsilon\rho_v\sqrt{\tau_{\epsilon_j}\tau_{v_j}\tau_{\epsilon_i}\tau_{v_i}})\right)\right),
\end{aligned}$$

where $D>0, \forall\rho_\epsilon\in(-1,1)$ and is defined in the appendix.

Proposition 8 confirms the intuition of corollary 1. Indeed, $\phi(s_{ki}||s_{kj},\boldsymbol{p})\geq 0$ if and only if $\rho_\epsilon{}^2\geq 0$: informed speculators have an incentive to collect additional

[15] Technically speaking, $\phi(s_{ki}||s_{kj},\boldsymbol{p})$ and $\phi(s_{ki}||s_{kj},p_{Rj})$ are trader k's certainty equivalents for the information contained in signal s_{ki} when she starts off having information s_{kj} and uses the prices $\boldsymbol{p}$ in the unrestricted mechanism (or the price p_{Rj} in the restricted one), while all other traders *do not* observe both private signals. Concentrating on the case where traders' private information is unidimensional simplifies calculations and allows to obtain closed form solutions for the value of information. Numerical simulations for the case where *all* traders have multidimensional private information confirm all the results obtained in this section.

private information *if and only if* error terms are correlated. In a system where market makers exploit cross-asset information by looking at different order flows (*price formation mechanism*) and traders submit multi price contingent orders (*order type*), any *added value* that multidimensional private information may have rests upon error terms correlation.

Proposition 9 *In the restricted system, the value of additional private information is given by*

$$\phi(s_{ki}||s_{kj}, p_{Rj}) = \frac{\gamma}{2}\ln\left(1 + \frac{\tau_{v_j}\tau_{\epsilon_j}(\rho_\epsilon\sqrt{\tau_{\epsilon_i}/\tau_{\epsilon_j}} - \rho_v\sqrt{\tau_{v_i}/\tau_{v_j}})^2}{(\tau_i + \tau_{\epsilon_i})((1-\rho_v^2)\tau_{\epsilon_j} + (1-\rho_\epsilon^2)\tau_{v_j})}\right),$$

where $\tau_i = \tau_{v_i} + (\gamma\tau_{\epsilon_i})^2\tau_{u_i}$.

According to proposition 9, in the restricted system *additional* private information has value *even* when private signals are not correlated through error terms, i.e. $\phi(s_{ki}||s_{kj}, p_{Rj}) \geq 0$ even if ${\rho_\epsilon}^2 = 0$. Indeed, the impossibility for market makers to observe more than one order flow when pricing an asset (*price formation mechanism*) and the inability for traders to draw inferences from different sources of public information (*order type*), maximize traders' incentives to gather additional private information.

The intuition for these results reminds one of the Grossman and Stiglitz (1980) impossibility results. In their model, when there is no noise, the price conveys all the information about the fundamentals and traders have no incentive to buy a private signal. On the other hand, in my model, when market makers observe *all* order flows and traders condition their orders on *all* prices, *(i)* prices exploit all the correlated information about fundamentals and *(ii)* (when $\rho_\epsilon = 0$) traders have no incentive to collect *both* signals when placing their orders. Therefore, in this case, restricting the amount of public information market makers (and informed traders) observe enhances traders' incentives at gathering private information.

The next proposition relates speculators' incentives to gather private information to the aggressiveness with which they speculate on each signal.

Proposition 10

1. *When* $\rho_v = \rho_u = 0$ *and* $\rho_\epsilon \neq 0$, $\phi(s_{ki}||s_{kj}, \boldsymbol{p}) > \phi(s_{ki}||s_{kj}, p_{Rj})$ *and* $a_{jj} > a_{Rjj}$, $|a_{ji}| > |a_{Rji}|$*;*

2. *when* $\rho_\epsilon = \rho_u = 0$ *and* $\rho_v \neq 0$, $\phi(s_{ki}||s_{kj}, \boldsymbol{p}) < \phi(s_{ki}||s_{kj}, p_{Rj})$ *and* $a_{jj} = a_{Rjj}$, $|a_{ji}| < |a_{Rji}|$;

3. *when* $\rho_\epsilon = \rho_v = 0$ *and* $\rho_u \neq 0$, $\phi(s_{ki}||s_{kj}, \boldsymbol{p}) = \phi(s_{ki}||s_{kj}, p_{Rj}) = 0$ *and* $a_{jj} = a_{Rjj} = \tau_{\epsilon_j}$, $|a_{ji}| = |a_{Rji}| = 0$.

Proposition 10 shows that, controlling for correlation coefficients, there is a *monotone* relationship between the value of *additional* private information and traders' signal aggressiveness: incentives to *collect* private information go hand-in-hand with incentives to *exploit* private information.

In particular, when $\rho_v = \rho_u = 0$ and $\rho_\epsilon \neq 0$, *additional* private information is more valuable in the unrestricted system than in the restricted system. Indeed, in this case *additional* information is useful to disentangle the effect of error terms from the private signals. In the unrestricted system this can be done comparing signals with prices whereas in the restricted system traders compare signals with prior means. As prices represent a *better* estimate of the fundamentals, traders in the unrestricted system are better able to assess the extent of their signal bias. Therefore, they value *more* private information and speculate *more* aggressively on it.

Conversely, when $\rho_\epsilon = \rho_u = 0$ and $\rho_v \neq 0$, *additional* private information is more valuable in the restricted system than in the unrestricted system. In this case *additional* information is useful to improve the estimation of each asset payoff. In the unrestricted system, market makers take advantage of multi-order flows observation and exploit this possibility. This destroys the incentives informed speculators have to gather extra information, minimizing its value as well as speculators' trading aggressiveness. In the restricted system, market makers only observe the order flow of the asset they price. This, in turn, enhances the incentives of informed speculators to collect additional information and maximizes their trading aggressiveness.

Finally, when $\rho_\epsilon = \rho_v = 0$ and $\rho_u \neq 0$ additional private information cannot be used neither to disentangle error terms nor to improve fundamentals' estimation. Thus, speculators' value and trading aggressiveness in the two systems coincide.

Remark 5 It is worth clarifying here the relationship between the result on aggressiveness of section 2 (see proposition 3) and proposition 10. The fact that speculators' aggressiveness in the restricted system is *higher* than in the unrestricted system when private signals are correlated only through payoffs, may seem to contrast with the intuition formed in the one-asset benchmark of section 2. Indeed, insofar as traders in the restricted system fail to condition their demand on all the sources of information related to the asset payoff - as they do in the market order market - one may think that as a result they should also trade *less* aggressively. However, notice that prices in the restricted system *do not* depend on cross order-flow information. Therefore, speculators' lack of cross-conditioning ability does not expose them to the risk of price movements spurred by events affecting other assets' order-flows (as it is the case in the market order market where a trader placing an order is not shielded against unanticipated price movements).

Remark 6 Enriching the information structure by introducing a common error term affecting all traders' private signals would allow to control for the presence of "industry" effects. Insofar as such an additional source of uncertainty would not disappear in the aggregate order book, traders would thus bear a higher risk for any given order they place (the assumed strong law of large number would not apply in this case). This, in turn, should lead them to scale back their aggressiveness both in the unrestricted and in the restricted mechanism. However, as long as the assumptions over market makers' information sets and speculators' order types do not change, this should not modify proposition 10's conclusions.

To conclude the section, I give a numerical example of the matrices of trading intensities in the two systems. This allows to show how different trading mechanisms affect the way traders interpret private signals.

Example 3 Suppose that

$$\mathbf{\Pi}_{\boldsymbol{v}} = \begin{pmatrix} 1.56 & -2.2 \\ -2.2 & 3.12 \end{pmatrix}, \quad \mathbf{\Pi}_{\boldsymbol{u}} = \begin{pmatrix} 15.62 & -3.49 \\ -3.49 & 0.78 \end{pmatrix}, \quad \mathbf{\Pi}_{\boldsymbol{\epsilon}} = \begin{pmatrix} 1.09 & -0.49 \\ -0.49 & 0.21 \end{pmatrix},$$

and $\gamma = 1$. Then, computing trading intensities in the two systems, gives

$$\mathbf{A} = \gamma \mathbf{\Pi}_{\boldsymbol{\epsilon}}, \quad \mathbf{A}_R = \begin{pmatrix} 1.02 & -0.3 \\ 0.416 & 0.144 \end{pmatrix}.$$

Consider the off-diagonal terms. In the restricted system receiving good news about asset 2 leads a trader to speculate less aggressively on asset 1 (i.e. $a_{R12} < 0$). Given proposition 7, it is easy to see that for this parameter configuration, $\rho_\epsilon \sqrt{\tau_{\epsilon_1}/\tau_{\epsilon_2}} \equiv .3\sqrt{5} > .6\sqrt{.2} \equiv \rho_v \sqrt{\tau_{v_1}/\tau_{v_2}}$, implying that $a_{R12} < 0$. Thus, if for example $s_{k1} = 1$ and $s_{k2} = 4$ for given prices, speculators in the unrestricted system interpret this signal vector as *good news* about asset 1, while in the restricted system they give to the *same* signal vector the *opposite* interpretation.

This example suggests that the mechanism regulating trading activities crucially impacts traders' interpretation of their private signals.

Figure 1 summarizes the differences in trading behavior across the restricted and the unrestricted system for the symmetric case. If correlation across signal error terms and fundamentals is as in regions II, III, V and VI, then $\text{sign}(a_{R2}) = \text{sign}(a_2)$. However, if ρ_ϵ and ρ_v lie in regions I and IV, then speculators behave differently in the two systems e.g. in region I, they put a positive weight on s_{k2} when trading asset 1 in the restricted system, while they do the opposite in the unrestricted system.

Please insert figure 1 here.

4.2 Price informativeness

In a financial market price informativeness is the result of informed speculators' trading activity. This, allowing information to be embedded into the order flows, enables market participants to form an estimation of the fundamentals which, through market clearing, is embedded into the equilibrium price. In section 2 I have shown that in a one-asset economy price informativeness is an increasing function of speculators' trading aggressiveness. Contrary to this result, I will show here that in a multi-asset world a *stronger* aggressiveness does not necessarily translate into an *increased* price informativeness. The reason is as follows. In a one-asset world the more aggressively traders speculate on their signal, the more important is informed trading vis-à-vis noise trading in the aggregate order flow. This, in turn, increases the component of the order flow carrying information about the fundamentals. Therefore, higher aggressiveness directly implies higher efficiency. In a multi-asset world a stronger aggressiveness also increases the importance of informed trading vis-à-vis noise trading in the aggregate order flow; however, to the extent that private signals do not *necessarily* embed payoff-relevant information, this *does not* imply that the component of the order flow conveying information about the fundamentals gets any *larger*. Therefore, it is not always possible to conclude that higher aggressiveness implies higher efficiency.

Price informativeness is defined as the reduction in the unconditional variance of an asset j's payoff due to the observation of the vector of order flows. Thus, in the unrestricted system $I_{p_j} = (\mathrm{Var}[\boldsymbol{v}])_{jj} - (\mathrm{Var}[\boldsymbol{v}|\boldsymbol{z}])_{jj} = (\mathbf{\Pi}_{\boldsymbol{v}}^{-1})_{jj} - (\mathbf{\Pi}^{-1})_{jj}$, while in the restricted system $I_{p_{Rj}} = (\mathrm{Var}[\boldsymbol{v}])_{jj} - (\mathrm{Var}[\boldsymbol{v}|\boldsymbol{z}_R])_{jj} = (\mathbf{\Pi}_{\boldsymbol{v}}^{-1})_{jj} - (\mathbf{\Pi}_R^{-1})_{jj}$. [16] This definition is natural in the unrestricted system, while in the restricted system it corresponds to the point of view of an econometrician interested in estimating the *deep* parameters of the market. Alternatively, it captures the point of view of a trader who, before submitting an order observes the past asset price as well as the one formed in a related market. I will thus say that the unrestricted system prices are *more* informative than those of the restricted system if and only if $I_{p_j} \geq I_{p_{Rj}}$, $j = 1, 2$. [17]

Proposition 11 *When $\rho_v = \rho_\epsilon = 0$, the prices in the unrestricted system and those in the restricted system are equally informative.*

Proof. If $\rho_v = \rho_\epsilon = 0$, $\mathbf{A} = \mathbf{A}_R = \gamma \mathbf{\Pi}_{\boldsymbol{\epsilon}}$ and the result follows. QED

The intuition for proposition 11 is straightforward: when $\rho_v = \rho_\epsilon = 0$ traders' behavior coincides in the two systems. Thus, the information that market makers retrieve from the order flows in the unrestricted system coincides with the one an econometrician would gather from the restricted system.

Proposition 12 *When the information structure is symmetric and $\rho_\epsilon = \rho_u = 0$, for ρ_v small the prices of the restricted system are more informative than those of the unrestricted system.*

According to proposition 10, if order flows are correlated only through payoffs, traders in the restricted system speculate more aggressively than in the unrestricted system. This increases the importance of informed trading vis-à-vis noise trading in the aggregate order flow. Furthermore, to the extent that both private signals contain correlated information about the asset payoff, it also magnifies the component of the order flow carrying information about the fundamentals. As a result, order flows in

[16] I thus measure the informativeness of an equilibrium price with its *ex ante* volatility. See remark 2, page 9.

[17] Such an efficiency comparison is justified whenever we assume that traders estimate each payoff *separately* (i.e. they are *not* interested in forecasting a linear combination of v_1 and v_2). This is actually the case in this model where agents do not trade an index. To simplify the analysis and concentrate on correlation effects, part of the results are obtained restricting attention to the symmetric case. In this way I also abstract from the effects that differences in signals' precisions, payoffs dispersions and noise trader demands' volatilities have on the use of private information. For an efficiency comparison in a one-asset, strategic set up where traders have information both on the fundamental value *and* on the source of noise see Palomino (2001).

the restricted system end up being *more* informative than in the unrestricted system and prices are better estimators of the fundamentals. [18]

This finding is particularly important given the common wisdom that a major benefit of a multi-price contingent system is that of rendering the market more efficient. Indeed, Amihud and Mendelson (1991b) argue that "a mechanism which enables *simultaneous conditioning* of orders for different assets (...) would increase the information available to traders, improve value discovery and reduce volatility." This assertion points at the positive effect that observing multiple sources of correlated information has. By contrast, proposition 12 unveils the negative side of a multi-price contingent system, by uncovering its *feedback* effect on price informativeness.

Numerical simulations support the result also for larger values of $|\rho_v|$. In particular, letting $\rho_v \in \{-0.9, -0.8, \dots, 0.8, 0.9\}$, $\rho_u = \rho_\epsilon = 0$ and $\gamma, \tau_u, \tau_v, \tau_\epsilon \in \{0.2, 0.4, 0.5, 0.6, 0.8, 1, 3, 4\}$ price informativeness is always higher in the restricted system (see figures 2 and 3, panel (a) for an example).

Please insert figure 2 here.

The next proposition shows that traders' stronger aggressiveness is not sufficient for higher price informativeness.

Proposition 13 *When the information structure is symmetric and $\rho_v = \rho_u = 0$, for ρ_ϵ small, there exists an open set of parameters $\gamma, \tau_v, \tau_u, \tau_\epsilon$ such that the prices of the restricted system are more informative than those of the unrestricted system.*

Consider again proposition 10: when order flows are correlated only through private signal error terms, traders in the unrestricted system speculate more aggressively than in the restricted system. Notice, however, that stronger aggressiveness in this case has a positive and a negative effect on price efficiency. Indeed, insofar as the signal about asset i speculators use when trading asset j is *not* correlated with asset j's payoff, order flows end up containing a *larger* amount of both payoff-relevant and *non* payoff-relevant information. As a consequence when order flows are very noisy, because of a very poor prior and/or because of a very dispersed noise traders' demand,

[18] Notice also that the condition given in proposition 12 is sufficient but it is by no means a *necessary* one. It is easy to show that for example when the information structure is symmetric, $\rho_v = \rho_\epsilon \neq 0$ and $\rho_u = 0$, $I_p \leq I_{p_R}$.

the *positive* effect of a stronger aggressiveness offsets its negative effect easing market makers' estimation of the fundamentals in the unrestricted system. Conversely, when order flows are less noisy, owing either to a good prior and/or to a concentrated noise traders' demand, the reverse happens and the *negative* effect of stronger aggressiveness hinders market makers' estimation rendering prices less efficient. [19]

Numerical simulations confirm this intuition. In particular, using the same parameter values defined above for precisions and setting $\rho_\epsilon \in \{-0.9, -0.8, \ldots, 0.8, 0.9\}$, $\rho_u = \rho_v = 0$ price informativeness is higher in the restricted system for *high* values of $\gamma, \tau_\epsilon, \tau_v$ and τ_u and lower for *low* values of $\gamma, \tau_\epsilon, \tau_v$ and τ_u. In figures 4 and 5 (panel (a)), I plot price informativeness of the restricted system (continuous line) and the one of the unrestricted system (dotted line) as a function of ρ_ϵ, when $\gamma = \tau_u = \tau_v = \tau_\epsilon = 0.2$ and when $\gamma = \tau_u = \tau_v = \tau_\epsilon = 3$.

Please insert figures 4 and 5 here.

4.3 Informed expected utility and noise traders' losses

In this section I study traders' welfare in the two systems. Central to this analysis is the role played by the *price impact of trades*, i.e. the extent to which prices move as a result of market makers' order flow observation. While a more reactive market unambiguously increases noise traders' expected losses, the same has a double-edged effect on speculators' expected utility. Indeed, the stronger is the price response to order flows, the larger are speculators' expected gains. However, the more reactive is the price, the more dispersed is the signal the market conveys about the fundamentals and, as a consequence, the larger is speculators' uncertainty about the payoff. For a given trading aggressiveness, the unrestricted system, allowing market makers to observe both order flows, mitigates the price impact of trades, thus lowering noise traders' expected losses. However, as shown in proposition 10, speculators' aggressiveness varies across the two systems and insofar as it impacts market reaction to order flows, welfare comparisons ultimately depend on parameters' configurations.

To fix notation, indicate with $\pi_k = \boldsymbol{x}_k'(\boldsymbol{v} - \boldsymbol{p})$ and with $\pi_{Rk} = \boldsymbol{x}_{Rk}'(\boldsymbol{v} - \boldsymbol{p}_R)$ respectively an informed trader k's profit in the unrestricted and in the restricted

[19] It is "as if" the fact that traders speculate using a signal that is not relevant from the market maker's point of view *increased* the noise present in the order flow.

system. Also let $-E[\boldsymbol{u}'(\boldsymbol{v}-\boldsymbol{p})] = \text{tr}(\boldsymbol{\Lambda}\boldsymbol{\Pi}_{\boldsymbol{u}}^{-1})$ and $-E[\boldsymbol{u}'(\boldsymbol{v}-\boldsymbol{p}_R)] = \text{tr}(\boldsymbol{\Lambda}_R\boldsymbol{\Pi}_{\boldsymbol{u}}^{-1})$ indicate respectively noise traders' expected losses in the unrestricted and in the restricted system.

When private signals are independent, it is possible to obtain an explicit expression for an informed trader's expected utility in the restricted system:

Proposition 14 *When $\rho_v = \rho_\epsilon = 0$, an informed speculator k's expected utility in the restricted system is given by $E[U(\pi_{Rk})] = -|\text{Var}[\boldsymbol{v}-\boldsymbol{p}_R]|^{-1/2}|\boldsymbol{\Pi_\epsilon} + (\text{Var}[\boldsymbol{v}-\boldsymbol{p}_R])^{-1}|^{-1/2}$.*

Proof. Notice that $E[U(\pi_{Rk})] = E[-\exp(-\gamma^{-1}\pi_{Rk})] = -E[E[\exp(-\gamma^{-1}\pi_{Rk})|\boldsymbol{v},\boldsymbol{p}_R]]$ $= -E[\exp(-\gamma^{-1}(E[\pi_{Rk}|\boldsymbol{v},\boldsymbol{p}_R] - (1/2\gamma)\text{Var}[\pi_{Rk}|\boldsymbol{v},\boldsymbol{p}_R]))]$, and applying lemma 3 in the appendix with $\boldsymbol{w} = \boldsymbol{v}-\boldsymbol{p}_R \sim N(\boldsymbol{0}, \text{Var}[\boldsymbol{v}-\boldsymbol{p}_R])$, the result follows. QED

For the unrestricted system, it is well known that $E\left[U(\pi_k)\right] = -|\boldsymbol{\Pi}|^{1/2}\ |\boldsymbol{\Pi} + \boldsymbol{\Pi_\epsilon}|^{-1/2}$. [20] The next result provides ranking across the two systems for the assumed information structure.

Proposition 15 *When $\rho_v = \rho_\epsilon = 0$, informed speculators always prefer to trade in the restricted system and noise traders' expected losses are always higher in the restricted system.*

Proposition 15 captures the effect of market makers' multi-order flow observation ability on traders' welfare. Indeed, when $\rho_v = \rho_\epsilon = 0$, $\mathbf{A} = \mathbf{A}_R = \gamma\boldsymbol{\Pi_\epsilon}$ and informed traders' behavior coincides in the two systems. However, in the unrestricted system market makers' ability to observe both order flows allows them to better disentangle the noise in the signals they observe (i.e. they can better distinguish order flows' realizations due to noise traders' demand pressure from those due to informed traders' activity). This, in turn, mitigates the price impact of trade borne by noise traders and reduces informed speculators' expected payoff, benefitting the former and damaging the latter.

Proposition 16 *When the information structure is symmetric and $\rho_\epsilon = \rho_u = 0$, for ρ_v small, noise traders' expected losses are always higher in the restricted system.*

With the above parameters' configuration, speculators trade more aggressively in the restricted system embedding more payoff-relevant information in the order

[20] See e.g. Admati and Pfleiderer (1987).

flows. This worsens market makers' adverse selection problem in the restricted system making the price impact of trade stronger and noise traders' expected losses higher.

Numerical simulations support the result also for higher values of $|\rho_v|$. In particular, letting τ_ϵ, τ_v, $\tau_u, \gamma \in \{0.2,\ 0.4,\ 0.5,\ 0.6,\ 0.8,\ 1,\ 3, 4\}$ and $\rho_v \in \{-0.9,\ -0.8,\ \ldots,\ 0.8, 0.9\}$ noise traders' expected losses are *always* higher in the restricted system (see figures 2 and 3, panel (b) for an example). As for informed speculators, the same numerical simulations show that when $\tau_\epsilon, \tau_v, \tau_u, \gamma$ are *small*, they are better off in the unrestricted system but when either $\tau_\epsilon, \tau_v, \tau_u$, or γ increase, they are better off in the restricted system (see figures 2 and 3, panel (c) for an example). Indeed, when $\tau_\epsilon, \tau_v, \tau_u, \gamma$ are *small* differences in aggressiveness across the two systems are large and the negative effect of the higher price impact of trades *more* than compensates the positive effect this has on speculators' expected utility in the restricted system. Conversely, as either $\tau_\epsilon, \tau_v, \tau_u$, or γ increase, differences in aggressiveness shrink and the reverse occurs.

Proposition 17 *When the information structure is symmetric and $\rho_v = \rho_u = 0$, for ρ_ϵ small there exists an open set of parameters $\gamma, \tau_v, \tau_u, \tau_\epsilon$ such that noise traders' expected losses are higher in the restricted system.*

Proposition 13 has shown that speculators' stronger aggressiveness has a non-monotone effect on the informativeness of the order flows in the unrestricted system. The above result captures the implications of this effect on market depth. Indeed, numerical simulations show that when $\tau_\epsilon, \tau_v, \tau_u, \gamma$ are *small*, noise traders are worse-off in the unrestricted system. Conversely, when either $\tau_\epsilon, \tau_v, \tau_u$, or γ increase, the reverse happens (see figures 4 and 5, panel (b) for an example).

As for informed speculators, welfare comparisons also depend on parameters values. When $\tau_\epsilon, \tau_v, \tau_u, \gamma$ are *small*, speculators are better off in the unrestricted system; when either $\tau_\epsilon, \tau_v, \tau_u$, or γ increase, the opposite occurs. The intuition is as follows. When $\rho_v = \rho_u = 0$ and $\rho_\epsilon \neq 0$ speculators in the unrestricted system trade more aggressively on their signals. This, in turn, harshens the price impact of trade borne by noise traders increasing their expected losses and speculators' expected utility. As $\tau_\epsilon, \tau_v, \tau_u$ or γ increase, the positive effect of speculators' increased aggressiveness on trade's price impact is more than compensated by market makers' ability to observe both order flows. This benefits noise traders at the expense of informed speculators (see figures 4 and 5, panel (c) for an example).

4.4 Summary of Results

Based on the results obtained in sections 4.1–4.3 we can thus conclude that in a multi-asset market the two analyzed mechanisms have a different impact on speculators' incentives to collect *and* exploit private information. In particular, the unrestricted system tilts traders towards exploiting *non* payoff-relevant information. Conversely, the restricted system enhances incentives to exploit payoff-relevant information. As a consequence speculators in the restricted system tend to embed more *payoff*-related information in the order flows, rendering prices more informative than in the unrestricted system. The flip-side of the coin is, however, that the price impact of trades tends to be higher in the restricted system, increasing noise traders' expected losses.

Table 1 summarizes the comparison results.

Please insert table 1 here.

5 An "Intermediate" System

The results obtained in the previous sections, have shown that a system disseminating a *large* amount of public information may reduce traders' incentives to exploit payoff related, private information. As a consequence, when only correlation across fundamentals affects order flows, the restricted system equilibrium price vector is more informative than the unrestricted system one (in the symmetric model). If this is the case, a system that *partially* allows cross asset information extraction should deliver prices that on the one hand are less informative than those of the restricted system, and on the other hand are more informative than those of the unrestricted system. A similar *intermediate* mechanism is represented by one where market makers can observe both order flows when pricing an asset, while informed speculators bear single price restrictions. The opening call auction of the NYSE gives an example of such a system. There, each *specialist* handles more than one asset and, as a consequence, is able to make cross asset inference at the moment of setting the opening price. [21] In this *intermediate* system, privately informed speculators *do not* hold an informational advantage over market makers. Indeed, while market makers

[21] See O'Hara (1995). Lindsay and Schaede (1990) report that in 1987 "(. . .) the average number [of stocks handled by a specialist] was 3.7 (. . .)"

can only observe a (vector of signals made by a) linear combination of fundamentals and noisy supply, speculators observe private signals and a linear combination of order flows. Therefore, it is never the case that their information set *dominates* the one of market makers.

Formally, in the intermediate mechanism, an informed speculator k conditions her demand for a given asset j on the whole vector of private signals $\boldsymbol{s}_k$ and on the price of asset j only. In any linear equilibrium, private and public information are conditionally independent, thus the speculator's strategy depends on both her signal and the price. In particular, assume that a generic speculator k submits a demand schedule $X_{Ikj}(\boldsymbol{s}_k, p_{Ij})$, indicating the desired position in asset j at every price p_{Ij}, contingent on the available information, and restrict attention to linear equilibria.

The market makers of asset j, observe the order flows of assets $j = 1, 2$: $\mathbf{L}_I(\cdot) = \int_0^1 \boldsymbol{x}_{Ik} dk + \boldsymbol{u}$. Consider a candidate symmetric equilibrium $X_{Ikj}(\boldsymbol{s}_k, p_{Ij}) = \boldsymbol{a}'_{Ij}\boldsymbol{s}_k + \phi_{Ij}(p_{Ij})$, where $\boldsymbol{a}_{Ij}$ is the 2×1 vector of trading intensities and $\phi_{Ij}(\cdot)$ is a linear function of the j-th price. The vector of aggregate order flows is then $\mathbf{L}_I(\cdot) = \boldsymbol{z}_I + \phi_I(\boldsymbol{p}_I)$, where $\boldsymbol{z}_I = \mathbf{A}_I\boldsymbol{v} + \boldsymbol{u}$, is the vector of order flows' informational content and $\mathbf{A}_I = \begin{pmatrix} \boldsymbol{a}'_{Ij} & \boldsymbol{a}'_{Ii} \end{pmatrix}$. Given competition and market makers' risk neutrality, the equilibrium price vector is given by

$$\boldsymbol{p}_I = \boldsymbol{\Lambda}_I \boldsymbol{z}_I + (I - \boldsymbol{\Lambda}_I \mathbf{A}_I)\bar{\boldsymbol{v}}, \tag{5.6}$$

where $\boldsymbol{\Lambda}_I = (\boldsymbol{\Pi}_I)^{-1}(\mathbf{A}_I)'\boldsymbol{\Pi}_{\boldsymbol{u}}$, is the matrix that maps order flows into prices and $\boldsymbol{\Pi}_I = \boldsymbol{\Pi}_{\boldsymbol{v}} + (\mathbf{A}_I)'\boldsymbol{\Pi}_{\boldsymbol{u}}\mathbf{A}_I$ is the precision matrix of $\boldsymbol{v}|\boldsymbol{z}_I$.

Notice that distinct from the restricted system, in this case market makers learn cross asset information independently from informed traders' equilibrium behavior. As a consequence, the equilibrium price of asset j is informationally equivalent to the linear combination of the informational contents of both the order flows.

Proposition 18 *In every linear equilibrium of the intermediate system prices are given by (5.6) and an informed speculator k's demand for asset $j = 1, 2$ is given by $X_{Ikj} = \boldsymbol{j}'\mathbf{A}_I(\boldsymbol{s}_k - \bar{\boldsymbol{v}}) - \boldsymbol{j}'\mathbf{B}_I(\bar{\boldsymbol{v}} - \boldsymbol{p}_I)$, where,*

$$\boldsymbol{j}'\mathbf{A}_I = \gamma \left(Var[v_j|\boldsymbol{s}_k, p_{Ij}] \right)^{-1} \mathbf{c}_{2j}, \text{ and } \boldsymbol{j}'\mathbf{B}_I = -\gamma \left(Var[v_j|\boldsymbol{s}_k, p_{Ij}] \right)^{-1} (1 - c_{1j}), \tag{5.7}$$

indicate respectively the vector of asset j's demand sensitivities to the speculator's private signals and to the equilibrium price of asset j, and c_{1j}, $\mathbf{c}_{2j}$, and $Var[v_j|\boldsymbol{s}_k, p_{Ij}]$ are defined in the appendix. In the symmetric model of the intermediate system a linear equilibrium always exists.

Uniqueness of the equilibrium is an issue. Numerical simulations have been carried out and for different initial conditions the solution of the fixed point problem did not change.

To compare price efficiencies, I run simulations on the three models, using the same parameterization of sections 4.2–4.3. The results broadly accord to intuition: for most parameter values, when only correlation across fundamentals affects order flows, speculators in the restricted system trade more aggressively than in the intermediate system; in turn speculators in the intermediate system trade more aggressively than in the unrestricted system. This induces a price vector in the intermediate system that on the one hand is *less* informative than in the restricted system and on the other hand is more informative than in the unrestricted system. Thus, restricting the amount of public information that informed speculators observe when placing their orders forces them to exploit more aggressively their private information, enhancing the informativeness of order flows (figure 6, panel (c)).

There are however exceptions: when noise traders' demand is very dispersed ($\tau_u \leq 0.2$) and correlation across payoffs is strong ($|\rho_v| \geq 0.8$) the aggressiveness-informativeness ranking between the restricted and the intermediate system is reversed. Owing to high noise traders' demand dispersion, risk-averse speculators in the restricted system suffer from a large conditional volatility of the payoff and scale back their aggressiveness. Conversely, in the intermediate system, market makers' multiple order flows observation dampens the price impact of trades reducing speculators' payoff conditional volatility. As a result, speculators trade more aggressively and embed more information in the order flows rendering prices more informative (figure 6, panel (a)). [22]

Results for noise traders' expected losses are inconclusive: for some parameterizations noise traders are better off in the intermediate system than in the unrestricted one (figure 6, panel (b)) while for other parameterizations the reverse occurs (figure 6, panel (d)). [23]

[22] To analyze this case simulations have been extended letting γ, τ_v, τ_u and $\tau_\epsilon \in \{0.01,\ 0.1,\ 0.2,\ 0.4,\ 0.5, 0.6,\ 0.8, 1, 3, 4\}$. It is interesting to remark that the dampening effect on the price impact of trades only emerges in the intermediate system but *not* in the unrestricted system.

[23] In some simulations noise traders' losses can even be higher in the intermediate system than in the restricted one.

6 Conclusions

Advances in information technology are deeply modifying the way stock market procedures are handled. ITG, a technology company, through its trading platform QuantEX permits a submission strategy ("Pairs") that automatically executes orders "when the spread differential between two stocks reaches a specified level." The Optimark platform provides a system allowing traders to specify different parameters upon which to condition execution and Bondconnect implements a mechanism allowing the exchange of portfolios of assets. These examples testify the need to improve trade execution, allowing more flexibility both in the determination of the *number* of assets to exchange and in the *amount* of trade relevant information to exploit when submitting an order. Motivated by this evidence, I have analyzed two trading systems where competitive speculators exploit multi-dimensional sources of private information, and contrasted their properties on the basis of two different pricing schemes. In the *unrestricted* mechanism, traders submit multi-price contingent demand functions and market makers set prices observing all order flows; in the *restricted* mechanism, speculators submit standard limit orders and market makers bear a single order flow restriction.

The results show that *incentives* to collect private information crucially depend both on the *type* of order traders submit and on the specific *price formation mechanism* one considers. Indeed, to the extent that private and public information are substitutable, a system allowing traders to observe *more* public signals, under some conditions, reduces their incentives to collect multiple private signals, lowering their trading aggressiveness. This, in turn, *reduces* the amount of information embedded in the order flows and ultimately makes a multi-price contingent mechanism *less* efficient than a single price contingent one, in stark contrast with the view that a mechanism of the first type should render prices more informative. The paper thus uncovers the existence of a possible trade-off between the *quantity* of multi dimensional public information that traders can access, and its resulting *quality*.

Many issues are left for future research. In particular, a dynamic extension of the models presented here would allow one to study how information updating through the observation of past prices influences traders' behavior and market properties. [24]

[24] Chan (1992) studies price determination in a multi-asset Kyle (1985) market where in each period n, market makers observe the order flow of the asset they price and the period $n-1$ prices of all the other assets. However, in his case informed speculators' behavior is not modeled, thus the *feedback* effects of prices on private information usage cannot be analyzed.

Also, introducing production in the restricted model would allow to study the interactions among firms' competition, traders' behavior and stock price determination. This last issue seems particularly relevant given that there is virtually no analysis of the links between firms' conduct in the product market and investors' reactions to the resulting stock price effects. [25]

Please insert figure 6 here.

[25] Fishman and Hagerty (1989) and Dow and Rahi (2002) analyze how the information gathered in the market place affects a firm's investment decisions; Gertner, Gibbons, and Scharfstein (1988) investigate how product-market considerations influence an *informed* firm's decision to reveal information to the capital market; Poitevin (1989) shows how a financially-constrained entrant, by signaling information about its leverage to the capital market, spurs a "deep-pocket" incumbent to engage in predatory practices.

7 Appendix

First of all, I state a well known result on multivariate normal random variables (see e.g. Brown and Jennings 1989).

Lemma 3 *Let $Q(\boldsymbol{w})$ be a quadratic function of the vector $\boldsymbol{w}$: $Q(\boldsymbol{w}) = D + \boldsymbol{b}'\boldsymbol{w} - \boldsymbol{w}'\mathbf{A}\boldsymbol{w}$, where $\boldsymbol{w} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $\boldsymbol{\Sigma}$ is non singular. We then have*

$$E[\exp(Q(\boldsymbol{w}))] = |\boldsymbol{\Sigma}|^{-1/2}|2\mathbf{A} + \boldsymbol{\Sigma}^{-1}|^{-1/2}\times$$
$$\exp\left\{D + \boldsymbol{b}'\boldsymbol{\mu} + \boldsymbol{\mu}'\mathbf{A}\boldsymbol{\mu} + \frac{1}{2}(\boldsymbol{b} - \mathbf{A}\boldsymbol{\mu})'(2\mathbf{A} + \boldsymbol{\Sigma}^{-1})^{-1}(\boldsymbol{b} - \mathbf{A}\boldsymbol{\mu})\right\}.$$

Proof of proposition 5.

Given the equilibrium price vector each agent k conditions her estimation of $\boldsymbol{v}$ on $(\mathbf{A}'\boldsymbol{\Pi_u}\mathbf{A})^{-1}(\boldsymbol{\Pi p} - \boldsymbol{\Pi_v}\bar{\boldsymbol{v}})|\boldsymbol{v} \sim N(\boldsymbol{v}, \mathbf{A}^{-1}\boldsymbol{\Pi_u}^{-1}(\mathbf{A}^{-1})')$, and on the received signal $\boldsymbol{s}_k|\boldsymbol{v} \sim N\left(\boldsymbol{v}, \boldsymbol{\Pi_\epsilon}^{-1}\right)$. Owing to CARA preferences, her demand is given by $\mathbf{X}_k(\boldsymbol{s}_k, \boldsymbol{p}) = (\mathrm{Var}[\boldsymbol{v}|\boldsymbol{s}_k, \boldsymbol{p}])^{-1}(E[\boldsymbol{v}|\boldsymbol{s}_k, \boldsymbol{p}] - \boldsymbol{p})$. As $(\mathrm{Var}[\boldsymbol{v}|\boldsymbol{s}_k, \boldsymbol{p}])^{-1} = \boldsymbol{\Pi_v} + \mathbf{A}'\boldsymbol{\Pi_u}\mathbf{A} + \boldsymbol{\Pi_\epsilon} = \boldsymbol{\Pi} + \boldsymbol{\Pi_\epsilon}$ and $E[\boldsymbol{v}|\boldsymbol{s}_k, \boldsymbol{p}] = (\boldsymbol{\Pi} + \boldsymbol{\Pi_\epsilon})^{-1}(\boldsymbol{\Pi}E[\boldsymbol{v}|\boldsymbol{z}] + \boldsymbol{\Pi_\epsilon s}_k)$, it follows that $\mathbf{X}_k(\boldsymbol{s}_k, \boldsymbol{p}) = \gamma\boldsymbol{\Pi_\epsilon}(\boldsymbol{s}_k - \boldsymbol{p})$ (see e.g. DeGroot 1969). QED

Proof of lemma 2.

CARA and normality of the random variables give $X_{Rkj} = \gamma(\mathrm{Var}[v_j|p_{Rj}, \boldsymbol{s}_k])^{-1}$ $(E[v_j|p_{Rj}, \boldsymbol{s}_k] - p_{Rj})$. Then, $E[v_j|p_{Rj}, \boldsymbol{s}_k] = \bar{v}_j + \begin{pmatrix} c_{1j} & \mathbf{c}'_{2j} \end{pmatrix} \begin{pmatrix} (\boldsymbol{j}'\lambda_j^{-1}(\boldsymbol{p} - \bar{\boldsymbol{v}}))' & (\boldsymbol{s}_k - \bar{\boldsymbol{v}})' \end{pmatrix}'$, where the parameter c_{1j} and the vector $\mathbf{c}'_{2j}$ are defined as follows: $\begin{pmatrix} c_{1j} & \mathbf{c}'_{2j} \end{pmatrix} \mathrm{Var}[p_{Rj}, \boldsymbol{s}_k]$ $= \mathrm{Cov}[v_j, \{\boldsymbol{s}_k, p_{Rj}\}]$. Standard normal computations give

$$\mathrm{Var}[p_{Rj}, \boldsymbol{s}_k] = \begin{pmatrix} \boldsymbol{j}'\left(\mathbf{A}_R\boldsymbol{\Pi_v}^{-1}\mathbf{A}'_R + \boldsymbol{\Pi_u}^{-1}\right)\boldsymbol{j} & \boldsymbol{j}'\mathbf{A}_R\boldsymbol{\Pi_v}^{-1} \\ \\ \left(\boldsymbol{j}'\mathbf{A}_R\boldsymbol{\Pi_v}^{-1}\right)' & \boldsymbol{\Pi_v}^{-1} + \boldsymbol{\Pi_\epsilon}^{-1} \end{pmatrix}, \tag{7.8}$$

and $\mathrm{Cov}[v_j, \{p_{Rj}, \boldsymbol{s}_k\}] = \begin{pmatrix} (\boldsymbol{j}'\mathbf{A}_R\boldsymbol{\Pi_v}^{-1}\boldsymbol{j})' & (\boldsymbol{j}'\boldsymbol{\Pi_v}^{-1})' \end{pmatrix}'$.

Inverting (7.8) I obtain

$$(\mathrm{Var}[p_{Rj}, \boldsymbol{s}_k])^{-1} = \begin{pmatrix} D_1^{-1} & -D_1^{-1}\boldsymbol{j}'\mathbf{A}_R(\boldsymbol{\Pi_v} + \boldsymbol{\Pi_\epsilon})^{-1}\boldsymbol{\Pi_\epsilon} \\ \\ -D_1^{-1}\left(\boldsymbol{j}'\mathbf{A}_R(\boldsymbol{\Pi_v} + \boldsymbol{\Pi_\epsilon})^{-1}\boldsymbol{\Pi_\epsilon}\right)' & D_2 \end{pmatrix},$$

where $D_1 = \boldsymbol{j}'(\mathbf{A}_R(\boldsymbol{\Pi_v} + \boldsymbol{\Pi_\epsilon})^{-1}\mathbf{A}'_R + \boldsymbol{\Pi_u}^{-1})\boldsymbol{j}$, and $D_2 = \boldsymbol{\Pi_v}(\boldsymbol{\Pi_v} + \boldsymbol{\Pi_\epsilon})^{-1}\boldsymbol{\Pi_\epsilon} + D_1^{-1}(\boldsymbol{\Pi_v}^{-1} + \boldsymbol{\Pi_\epsilon}^{-1})^{-1}\boldsymbol{\Pi_v}^{-1}\mathbf{A}'_R\boldsymbol{j}\boldsymbol{j}'\mathbf{A}_R\boldsymbol{\Pi_v}^{-1}(\boldsymbol{\Pi_v}^{-1} + \boldsymbol{\Pi_\epsilon}^{-1})^{-1}$. Using the previous covariance matrix and since $\text{Var}[v_j|p_{Rj}, \boldsymbol{s}_k] = \boldsymbol{j}'\boldsymbol{\Pi}_{\boldsymbol{v}}^{-1}\boldsymbol{j} - (\text{Cov}[v_j, \{p_{Rj}, \boldsymbol{s}_k\}])'(\text{Var}[p_{Rj}, \boldsymbol{s}_k])^{-1}(\text{Cov}[v_j, \{p_{Rj}, \boldsymbol{s}_k\}])$, after standard normal calculations one obtains $c_{1j} = (\boldsymbol{j}'\mathbf{A}_R(\boldsymbol{\Pi}_{\boldsymbol{v}} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}})^{-1}\boldsymbol{j})/D_1$,

$$\mathbf{c}_{2j} = \boldsymbol{j}'\left(I - c_{1j}\mathbf{A}_R\right)\left(\boldsymbol{\Pi}_{\boldsymbol{v}} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}}\right)^{-1}\boldsymbol{\Pi}_{\boldsymbol{\epsilon}}, \tag{7.9}$$

and,

$$\text{Var}\left[v_j|p_{Rj}, \boldsymbol{s}_k\right] = \boldsymbol{j}'\left(I - \mathbf{A}_R c_{1j}\right)\left(\boldsymbol{\Pi}_{\boldsymbol{v}} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}}\right)^{-1}\boldsymbol{j}. \tag{7.10}$$

QED

Proof of proposition 6

Equilibrium existence depends on the existence of a solution to the fixed point problem (7.10). To compute the equilibrium, set $j = 1$ (the case $j = 2$ is symmetric) and notice that we can rewrite the system (3.4) as follows:

$$\begin{aligned}
a_{R11} &= \gamma\tau_{\epsilon_1} - \rho_\epsilon a_{R12}\sqrt{\tau_{\epsilon_1}/\tau_{\epsilon_2}}, \\
a_{R22} &= \gamma\tau_{\epsilon_2} - \rho_\epsilon a_{R21}\sqrt{\tau_{\epsilon_2}/\tau_{\epsilon_1}}, \\
\\
a_{R21} &= \frac{\gamma}{1-\rho_\epsilon^2}\left(\tau_{\epsilon_2}\left(\frac{h_{21}}{h_{22}}\right) - \rho_\epsilon\sqrt{\tau_{\epsilon_1}\tau_{\epsilon_2}}\right), \\
a_{R12} &= \frac{\gamma}{1-\rho_\epsilon^2}\left(\tau_{\epsilon_1}\left(\frac{h_{12}}{h_{11}}\right) - \rho_\epsilon\sqrt{\tau_{\epsilon_1}\tau_{\epsilon_2}}\right).
\end{aligned} \tag{7.11}$$

To see this set $h_{11} = ((I - \mathbf{A}_R c_{11})(\boldsymbol{\Pi}_{\boldsymbol{v}} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}})^{-1})_{1,1}$, $h_{12} = ((I - \mathbf{A}_R c_{11})(\boldsymbol{\Pi}_{\boldsymbol{v}} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}})^{-1})_{1,2}$, $h_{21} = ((I - \mathbf{A}_R c_{12})(\boldsymbol{\Pi}_{\boldsymbol{v}} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}})^{-1})_{2,1}$, and $h_{22} = ((I - \mathbf{A}_R c_{12})(\boldsymbol{\Pi}_{\boldsymbol{v}} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}})^{-1})_{2,2}$. Then, $(\text{Var}[v_1|\boldsymbol{s}_k, p_{R1}])^{-1}\mathbf{c}_{21} = \begin{pmatrix} 1 & (h_{12}/h_{11}) \end{pmatrix}\boldsymbol{\Pi}_{\boldsymbol{\epsilon}}$, and $(\text{Var}[v_2|\boldsymbol{s}_k, p_{R2}])^{-1}$ $\mathbf{c}_{22} = \begin{pmatrix} 1 & (h_{21}/h_{22}) \end{pmatrix}\boldsymbol{\Pi}_{\boldsymbol{\epsilon}}$. Finally, explicitly expressing the equilibrium conditions, one obtains (7.11). There are now two cases to consider: the case in which $\rho_\epsilon = 0$, that gives $a_{R11} = \gamma\tau_{\epsilon_1}$ and a cubic equation in a_{R12} and the case in which $\rho_\epsilon \neq 0$. Start by considering the second (the first is just a simplification of it). Substituting the first equation in (7.11) into the last one, gives the following cubic equation in a_{R12}

$$(a_{R12})^3(1-\rho_\epsilon^2)(1-\rho_v^2)\phi_1 + a_{R12}\phi_1\phi_2 + \phi_3 = 0, \tag{7.12}$$

where $\phi_1 = \tau_{\epsilon_1}\tau_{\epsilon_2}(1-\rho_v^2) + \tau_{v_1}\tau_{\epsilon_2} + \tau_{v_2}\tau_{\epsilon_1} + \tau_{v_1}\tau_{v_2}(1-\rho_\epsilon^2) - 2\rho_\epsilon\rho_v\sqrt{\tau_{\epsilon_1}\tau_{\epsilon_2}\tau_{v_1}\tau_{v_2}}$, $\phi_2 = ((1-\rho_v^2)\tau_{\epsilon_2} + (1-\rho_\epsilon^2)\tau_{v_2} + \gamma^2(1-\rho_v^2)\tau_{\epsilon_1}\tau_{\epsilon_2}\tau_{u_1})$, and $\phi_3 = \gamma(\rho_\epsilon\sqrt{\tau_{\epsilon_1}\tau_{v_2}} - \rho_v\sqrt{\tau_{\epsilon_2}\tau_{v_1}})$ $\sqrt{\tau_{\epsilon_2}\tau_{v_2}}\{(\tau_{v_1}(\tau_{v_2}(1-\rho_\epsilon^2) + \tau_{\epsilon_2}) + \tau_{\epsilon_1}(\tau_{v_2} + \tau_{\epsilon_2}(1-\rho_v^2))) - 2\rho_v\rho_\epsilon\sqrt{\tau_{v_2}\tau_{\epsilon_2}\tau_{\epsilon_1}\tau_{v_1}}\}$. The discriminant associated to this equation is $\Delta = 4(\phi_2/(1-\rho_\epsilon^2)(1-\rho_v^2))^3 + 27(\gamma\phi_3/(1-$

$\rho_\epsilon^2)(1-\rho_v^2)\phi_1)^2$, which can be easily proved to be positive. Therefore, the result follows. QED

Proof of proposition 7.

For part 1, rearranging the cubic equation defining a_{R12} gives

$$a_{R12}\underbrace{\phi_1\left((1-\rho_\epsilon^2)(1-\rho_v^2)(a_{R12})^2+\phi_2\right)}_{(1)}+\phi_3=0.$$

It is easy to check that (1) is positive. Therefore for a solution to exist, it must be the case that a_{R12} has a sign opposite to ϕ_3. Since $\phi_3>0 \Leftrightarrow \tau_{\epsilon_1}\mathrm{Cov}[\epsilon_1,\epsilon_2]>\tau_{v_1}\mathrm{Cov}[v_1,v_2]$, the result follows. For part 2, if $\rho_\epsilon=0$ the proof is straightforward. Otherwise, assume that $a_{R11}<0$, then we have $a_{R12}=\rho_\epsilon^{-1}\sqrt{\tau_{\epsilon_2}/\tau_{\epsilon_1}}(\gamma\tau_{\epsilon_1}-a_{R11})$. If $\rho_\epsilon>(<)0$, $a_{R12}>(<)0$ always, a contradiction. Next, given the properties of trading intensities, it is easy to see that (3.2) is always positive. The result follows. Part 3 and 4 follow by manipulating (7.11). QED

Proof of corollary 2.

In a similar way as done before, I obtain the system of equations (3.5). To solve the system, there are two cases to consider: $\rho_\epsilon\neq 0$ and $\rho_\epsilon=0$. If $\rho_\epsilon\neq 0$, multiply the second equation in the system (3.5) by ρ_ϵ and add it to the first. Then

$$a_{R1}=\gamma\tau_\epsilon(1-\rho_\epsilon^2)-\rho_\epsilon a_{R2}. \tag{7.13}$$

Substituting a_{R1} into the second equation in the system (3.5), and rearranging,

$$\begin{aligned}&\gamma\tau_\epsilon\tau_v(1-\rho_\epsilon^2)(\rho_\epsilon-\rho_v)+(a_{R2})^3\tau_u(1-\rho_v^2)(1-\rho_\epsilon^2)\\ +\ & a_{R2}\left(\tau_\epsilon(1-\rho_v^2)+(1-\rho_\epsilon^2)(\tau_v+\gamma^2\tau_\epsilon^2\tau_u(1-\rho_v^2)(1-\rho_\epsilon^2))\right)=0,\end{aligned} \tag{7.14}$$

this is a cubic equation in a_{R2}. Computing the associated discriminant, $\Delta=4((\tau_\epsilon(1-\rho_v^2)+(1-\rho_\epsilon^2)(\tau_v+\gamma^2\tau_\epsilon^2\tau_u(1-\rho_v^2)(1-\rho_\epsilon^2)))/(\tau_u(1-\rho_v^2)(1-\rho_\epsilon^2)))^3+27((\gamma\tau_\epsilon\tau_v(1-\rho_\epsilon^2)(\rho_\epsilon-\rho_v))/(\tau_u(1-\rho_v^2)(1-\rho_\epsilon^2)))^2$, which is always positive. Therefore, there exists a unique real root. When $\rho_\epsilon=0$ the first of (3.5) gives $a_{R1}=\gamma\tau_\epsilon$. Substituting this solution into the second equation, after rearranging, $(a_{R2})^3\tau_u(1-\rho_v^2)+a_{R2}(\tau_v+(1-\rho_v^2)(\gamma^2\tau_\epsilon^2\tau_u+\tau_\epsilon))+\gamma(\rho_v^2-1)\rho_v\tau_v\tau_\epsilon=0$. Computing the discriminant of this cubic, $\Delta=4((\tau_v+(1-\rho_v^2)(\gamma^2\tau_\epsilon^2\tau_u+\tau_\epsilon))/\tau_u(1-\rho_v^2))^3+27(\gamma(\rho_v^2-1)\rho_v\tau_v\tau_\epsilon/\tau_u(1-\rho_v^2))^2$, which is always positive. Therefore also in this case there is a unique real root.

Next, for part *(1)* the argument is as follows. Rearrange the cubic equation (7.14), to get $a_{R2}((a_{R2})^2\tau_u(1-\rho_\epsilon^2)(1-\rho_v^2)+((1-\rho_\epsilon^2)(\tau_v+\gamma^2\tau_\epsilon^2\tau_u(1-\rho_v^2))+\tau_\epsilon(1-\rho_v^2)))$

$+\gamma\tau_\epsilon\tau_v(1-\rho_\epsilon^2)(\rho_\epsilon-\rho_v)=0$. Now, if $\rho_\epsilon-\rho_v>0$, then for a real solution to exist, it must be the case that $a_{R2}<0$ (and vice-versa). For part *(2)*, assume that $a_{R1}<0$, then rearranging (7.13), $a_{R2}=\rho_\epsilon^{-1}\gamma\tau_\epsilon(1-\rho_\epsilon^2)-\rho_\epsilon^{-1}a_{R1}$. According to this condition, if $\rho_\epsilon<0$, then $a_{R2}<0$ for any ρ_v, which is impossible according to the proof of the previous part. If $\rho_\epsilon>0$, then $a_{R2}>0$, no matter ρ_v, again impossible. Hence the result. Computing λ_R, I get $\lambda_R=(((a_{R1})^2+(a_{R2})^2+2a_{R1}a_{R2}\rho_v)\tau_u+\tau_v)^{-1}$ $((a_{R1}+a_{R2}(\rho_v-\rho_\epsilon))\tau_u)$. If $\rho_\epsilon=0$ then it is easy to see that the numerator of the fraction is positive; if $\rho_\epsilon\neq 0$, substituting the expression for a_{R1} and rearranging, the numerator of λ_R becomes $(\gamma\tau_\epsilon(1-\rho_\epsilon^2)+a_{R2}(\rho_v-\rho_\epsilon))\tau_u$, which is positive. The denominator of the depth is also positive since if $\rho_v<0$ and $a_{R2}<0$ then $2\rho_v a_{R1}a_{R2}>0$, while if $\rho_v<0$ and $a_{R2}>0$ then $(a_{R1})^2+(a_{R2})^2+2\rho_v a_{R1}a_{R2}>(a_{R1}-a_{R2})^2>0$. A similar argument can be used in the case $\rho_v>0$. QED

Proof of proposition 8.

The proof is made in three steps: first I determine the ex-ante expected utility of a trader k that only observes one private signal (and conditions on both equilibrium prices). Next, I find her expected utility when she observes both private signals (and conditions on both equilibrium prices). Finally, I determine her certainty equivalent for signal s_i when she already possesses s_j and conditions on both equilibrium prices.

1. Ex-ante expected utility from unidimensional private information.

To determine the value of multidimensional private information in the unrestricted system, I first need to determine how a trader possessing private information about say asset j would trade if she were able to condition her demand on both prices and market makers were able to observe both order flows. Given normality and CARA, a trader k strategy in asset j is given by $X_{kj}(s_{kj},\boldsymbol{p})=\gamma(\text{Var}[v_j|s_{kj},\boldsymbol{p}])^{-1}(E[v_j|s_{kj},\boldsymbol{p}]-p_j)$ $=a_j^*s_{kj}-b_{jj}^*p_j-b_{ji}^*p_i+c_{jj}^*+c_{ji}^*$, where a_j^*, $b_{j\cdot}^*$ and $c_{j\cdot}^*$ are constants to be determined in equilibrium. The vector of strategies that a trader k submits to market makers is thus given by $\mathbf{X}_k(\boldsymbol{s}_k,\boldsymbol{p})=\mathbf{A}^*\boldsymbol{s}_k+\mathbf{B}^*\boldsymbol{p}+\mathbf{c}^*$, where

$$\mathbf{A}^*=\begin{pmatrix} a_1^* & 0 \\ 0 & a_2^* \end{pmatrix},\qquad \mathbf{B}^*=\begin{pmatrix} b_{11}^* & b_{12}^* \\ b_{21}^* & b_{22}^* \end{pmatrix},\qquad \mathbf{c}^*=\begin{pmatrix} c_{11}^* & c_{12}^* \\ c_{21}^* & c_{22}^* \end{pmatrix}.$$

Thus market makers observe the aggregate vector of order flows $\mathbf{A}^*\boldsymbol{v}+\boldsymbol{u}+\mathbf{B}^*\boldsymbol{p}+\mathbf{c}^*$ whose informational content is given by $\boldsymbol{z}^*=\mathbf{A}^*\boldsymbol{v}+\boldsymbol{u}$. Hence, because of competition and risk neutrality $\boldsymbol{p}=E[\boldsymbol{v}|\boldsymbol{z}^*]=\boldsymbol{\Lambda}^*\boldsymbol{z}^*+(I-\boldsymbol{\Lambda}^*\mathbf{A}^*)\bar{\boldsymbol{v}}$, where $\boldsymbol{\Pi}=\boldsymbol{\Pi}_{\boldsymbol{v}}+\mathbf{A}^*\boldsymbol{\Pi}_{\boldsymbol{u}}\mathbf{A}^*$ and $\boldsymbol{\Lambda}^*=\boldsymbol{\Pi}^{-1}\mathbf{A}^*\boldsymbol{\Pi}_{\boldsymbol{u}}$. Because of normality $E[v_j|s_{kj},\boldsymbol{p}]=\bar{v}_i+\begin{pmatrix} e_{11} & \mathbf{e}_{21}' \end{pmatrix}$

$\left(\begin{array}{cc} (s_{kj}-\bar{v}_j) & (\mathbf{A}^*(\boldsymbol{v}-\bar{\boldsymbol{v}})+\boldsymbol{u})' \end{array} \right)'$, where $\left(\begin{array}{cc} e_{11} & \mathbf{e}'_{21} \end{array} \right)\mathrm{Var}[s_{kj},\boldsymbol{p}] = \mathrm{Cov}[v_j,\{s_{kj},\boldsymbol{p}\}]$. Then, $\mathrm{Var}[v_j|s_{kj},\boldsymbol{p}] = \tau_{v_j}^{-1} - \mathrm{Cov}[v_j,\{s_{kj},\boldsymbol{p}\}]'\mathrm{Var}[s_{kj},\boldsymbol{p}]^{-1}\mathrm{Cov}[v_j,\{s_{kj},\boldsymbol{p}\}]$,

$$\mathrm{Var}[s_{kj},\boldsymbol{p}] = \left(\begin{array}{cc} \tau_{v_j}^{-1}+\tau_{\epsilon_j}^{-1} & (\mathbf{A}^*\mathbf{\Pi}_{\boldsymbol{v}}^{-1}\boldsymbol{j})' \\ \mathbf{A}^*\mathbf{\Pi}_{\boldsymbol{v}}^{-1}\boldsymbol{j} & \mathbf{A}^*\mathbf{\Pi}_{\boldsymbol{v}}^{-1}\mathbf{A}^*+\mathbf{\Pi}_{\boldsymbol{u}}^{-1} \end{array} \right),$$

and $\mathrm{Cov}[v_j,\{s_{kj},\boldsymbol{p}\}] = \left(\begin{array}{cc} \tau_{v_j}^{-1} & \mathbf{A}^*\Pi_{\boldsymbol{v}}^{-1}\boldsymbol{j} \end{array} \right)$. Substituting the expressions for $E[v_j|s_{kj},\boldsymbol{p}]$ and $\mathrm{Var}[v_j|s_{kj},\boldsymbol{p}]$, $X_{kj} = \gamma(\mathrm{Var}[v_i|s_{kj},\boldsymbol{p}])^{-1}$ $(\bar{v}_j + e_{11}(s_{kj}-\bar{v}_j) + \mathbf{e}'_{21}(\mathbf{A}^*(\boldsymbol{v}-\bar{\boldsymbol{v}})+\boldsymbol{u})$ $-p_j)$, and identifying parameters $a_1^* = \gamma\tau_{\epsilon_j}$. To find b_{jj} and b_{ji}, substitute $(\mathbf{\Lambda}^*)^{-1}(\boldsymbol{p}-\boldsymbol{v})$ to $\mathbf{A}^*(\boldsymbol{v}-\bar{\boldsymbol{v}})+\boldsymbol{u}$ in the above expression (the two vectors are informationally equivalent given the equilibrium price vector) and rearrange to find $X_{kj}(s_{kj},\boldsymbol{p}) = a_1(s_{kj}-\bar{v}_j)$ $+\gamma(\mathrm{Var}[v_i|s_{kj},\boldsymbol{p}])^{-1}(1-(\mathbf{e}'_{21}(\mathbf{\Lambda}^*)^{-1})_{11})(\bar{v}_j-p_j)$ $-\gamma(\mathrm{Var}[v_j|s_{kj},\boldsymbol{p}])^{-1}(1-(\mathbf{e}'_{21}(\mathbf{\Lambda}^*)^{-1})_{12})(\bar{v}_i-p_i)$. Simplifying the above expression, one finds $\gamma(\mathrm{Var}[v_j|s_{kj},\boldsymbol{p}])^{-1}$ $(1-(\mathbf{e}'_{21}(\mathbf{\Lambda}^*)^{-1})_{11}) = \gamma\tau_{\epsilon_j}$, and $\gamma(\mathrm{Var}[v_j|s_{kj},\boldsymbol{p}])^{-1}(1-(\mathbf{e}'_{21}(\mathbf{\Lambda}^*)^{-1})_{12}) = 0$. Therefore, $X_{kj}(s_{kj},\boldsymbol{p}) = \gamma\tau_{\epsilon_j}(s_{kj}-p_j)$, and

$$\mathbf{A}^* = \mathbf{B}^* = \gamma\left(\begin{array}{cc} \tau_{\epsilon_1} & 0 \\ 0 & \tau_{\epsilon_2} \end{array} \right),$$

while $\mathbf{c}^* = \mathbf{0}$ and $\mathbf{X}_k(\boldsymbol{s}_k,\boldsymbol{p}) = \mathbf{A}^*(\boldsymbol{s}_k-\boldsymbol{p})$. Once found traders' strategies, we can compute their ex-ante utility. Given normality and CARA preferences, $E[U(\pi_k);A^*] = E[-\exp\{-(1/\gamma)(E[\boldsymbol{x}'_k(\boldsymbol{v}-\boldsymbol{p})|\boldsymbol{v},\boldsymbol{p}]$ $-(1/(2\gamma))\mathrm{Var}[\boldsymbol{x}'_k(\boldsymbol{v}-\boldsymbol{p})|\boldsymbol{v},\boldsymbol{p}])\}]$, where $E[\boldsymbol{x}'_k(\boldsymbol{v}-\boldsymbol{p})|\boldsymbol{v},\boldsymbol{p}] = (\boldsymbol{v}-\boldsymbol{p})'\mathbf{A}^*(\boldsymbol{v}-\boldsymbol{p})$ and $\mathrm{Var}[\boldsymbol{x}'_k(\boldsymbol{v}-\boldsymbol{p})|\boldsymbol{v},\boldsymbol{p}] = (\boldsymbol{v}-\boldsymbol{p})'\mathbf{A}^*\mathbf{\Pi}_{\boldsymbol{\epsilon}}^{-1}\mathbf{A}^*(\boldsymbol{v}-\boldsymbol{p})$. Hence, applying lemma 3, $E[U(\pi_k);\mathbf{A}^*] = -E[\exp\{-(1/\gamma)((\boldsymbol{v}-\boldsymbol{p})'(\mathbf{A}^*-(1/(2\gamma))\mathbf{A}^*\mathbf{\Pi}^{-1}\mathbf{A}^*)$ $(\boldsymbol{v}-\boldsymbol{p}))\}] = -|\mathbf{\Pi}|^{1/2}|(2/\gamma)(\mathbf{A}^*-(1/(2\gamma))\mathbf{A}^*\mathbf{\Pi}^{-1}\mathbf{A}^*)+\mathbf{\Pi}|^{-1/2}$.

2. Ex-ante expected utility from multidimensional private information.

The next step is to determine a trader k's strategy in the unrestricted system whenever she is able to observe both private signals contemporaneously. Since traders take market makers prices as given, proposition 1 holds and equilibrium strategies are given by $\mathbf{X}_k(\boldsymbol{s}_k,\boldsymbol{p}) = \mathbf{A}(\boldsymbol{s}_k-\boldsymbol{p})$, where $\mathbf{A} = \gamma\mathbf{\Pi}_{\boldsymbol{\epsilon}}$. By a similar argument the trader ex-ante utility is given by $E[U(\pi_k);\mathbf{A}] = -|\mathbf{\Pi}|^{1/2}|\mathbf{\Pi}+\mathbf{\Pi}_{\boldsymbol{\epsilon}}|^{-1/2}$.

3. The value of multidimensional private information.

Let $\phi(s_{ki}||s_{kj},\boldsymbol{p})$ be the maximum price a trader in the unrestricted system is willing to pay to observe both private signals contemporaneously, $E[U(\pi_k-\phi(s_{ki}||s_{kj},\boldsymbol{p}));\mathbf{A}] = E[U(\pi_k);\mathbf{A}^*]$, solving for $\phi(s_{ki}||s_{kj},\boldsymbol{p})$, $\phi(s_{ki}||s_{kj},\boldsymbol{p}) = (\gamma/2)\ln((|(2/\gamma)$

$(\mathbf{A}^* - (1/(2\gamma))\mathbf{A}^*\mathbf{\Pi}^{-1}\mathbf{A}^*) + \mathbf{\Pi}|)^{-1}|\mathbf{\Pi} + \mathbf{\Pi}_{\boldsymbol{\epsilon}}|)$. Simplifying this expression, one obtains the expression in proposition 8, where

$$D = (1-\rho_\epsilon{}^2)((1-\rho_u^2)(\tau_{\epsilon_2}\tau_{v_1} + \tau_{\epsilon_1}\tau_{v_2} + \tau_{v_1}\tau_{v_2} - 2\rho_\epsilon\rho_v\sqrt{\tau_{\epsilon_1}\tau_{v_1}\tau_{\epsilon_2}\tau_{v_2}}) +$$
$$(1-\rho_v^2)\gamma^2\tau_{\epsilon_1}\tau_{\epsilon_2}(\tau_{\epsilon_1}\tau_{u_1} + \tau_{\epsilon_2}\tau_{u_2} + \gamma^2\tau_{\epsilon_1}\tau_{\epsilon_2}\tau_{u_1}\tau_{u_2} - 2\rho_\epsilon\rho_u\sqrt{\tau_{\epsilon_1}\tau_{u_1}\tau_{\epsilon_2}\tau_{u_2}}) +$$
$$(\gamma^2\tau_{\epsilon_2}^2\tau_{u_2}\tau_{v_1} + \gamma^2\tau_{\epsilon_1}^2\tau_{u_1}\tau_{v_2} + \tau_{\epsilon_1}\tau_{\epsilon_2}(1-\rho_u^2)(1-\rho_v^2)(1-\rho_\epsilon{}^2) - 2\gamma^2\tau_{\epsilon_1}\tau_{\epsilon_2}\rho_u\rho_v\sqrt{\tau_{u_1}\tau_{v_1}\tau_{u_2}\tau_{v_2}})).$$

QED

Proof of proposition 9.

As done for the unrestricted system, the proof is made in three steps: first we determine the ex-ante expected utility of a trader k that only observes one private signal (and conditions on one equilibrium price). Next, we find her expected utility when she observes both private signals (and conditions on one equilibrium prices). Finally, we determine her certainty equivalent for signal s_j when she already possesses s_i and conditions on on equilibrium prices.

1. Ex-ante expected utility from unidimensional private information.

First, I determine the equilibrium in the restricted system where traders only have one private signal about each asset j when trading that asset. Given that market makers cannot observe more than one order flow, the unique equilibrium in this case coincides with proposition 1. In this equilibrium $X_{Rkj}(s_{kj}, p_{Rj}) = \gamma\tau_{\epsilon_j}(s_{kj} - p_{Rj})$, $p_{Rj} = \lambda_{Rj}z_{Rj} + (1 - \lambda_{Rj}\gamma\tau_{\epsilon_j})\bar{v}_j$, $\lambda_{Rj} = (\gamma\tau_{\epsilon_j})\tau_{u_j}/\tau_j$ and $E[U(x_{Rkj}(v_j - p_{Rj}))] = -(\tau_j/(\tau_j + \tau_{\epsilon_j}))^{1/2}$, where $\tau_j = \tau_{v_j} + (\gamma\tau_{\epsilon_j})^2\tau_{u_j}$.

2. Ex-ante expected utility from multidimensional private information.

Next, I determine a trader's k equilibrium strategy when (only) she can observe two private signals in the restricted system. Because of CARA and normality $X_{Rkj}(s_{kj}, p_{Rj}) = \gamma(\mathrm{Var}[v_i|\boldsymbol{s}_k, p_{Rj}])^{-1}(E[v_j|\boldsymbol{s}_{kj}, p_{Rj}] - p_{Rj}) = a^*_{Rjj}s_{kj} + a^*_{Rji}s_{ki} - b^*_{Rj}p_{Rj} + c^*_{Rj}$, where $E[v_j|\boldsymbol{s}_k, p_{Rj}] = \bar{v}_j + \begin{pmatrix} \mathbf{e}'_{11} & e_{21} \end{pmatrix} \begin{pmatrix} (\boldsymbol{s}_k - \bar{\boldsymbol{v}})' & (v_j + (\gamma\tau_{\epsilon_j})^{-1}u_j \end{pmatrix}$ and, $\begin{pmatrix} \mathbf{e}'_{11} & e_{21} \end{pmatrix} \times \mathrm{Var}[\boldsymbol{s}_k, p_{Rj}] = \mathrm{Cov}[v_j, \{\boldsymbol{s}_k, p_{Rj}\}]$. Then, $\mathrm{Var}[v_j|\boldsymbol{s}_k, p_{Rj}] = \tau_{v_j}^{-1}$ $-\mathrm{Cov}[v_j, \{\boldsymbol{s}_k, p_{Rj}\}]'\,\mathrm{Var}[\boldsymbol{s}_k, p_{Rj}]^{-1}\,\mathrm{Cov}[v_j, \{\boldsymbol{s}_k, p_{Rj}\}]$,

$$\mathrm{Var}[\boldsymbol{s}_k, p_{Rj}] = \begin{pmatrix} \mathbf{\Pi}_{\boldsymbol{v}}^{-1} + \mathbf{\Pi}_{\boldsymbol{\epsilon}}^{-1} & \begin{pmatrix} \tau_{v_j}^{-1} & \rho_v/\sqrt{\tau_{v_i}\tau_{v_j}} \end{pmatrix}' \\ \begin{pmatrix} \tau_{v_j}^{-1} & \rho_v/\sqrt{\tau_{v_i}\tau_{v_j}} \end{pmatrix} & \tau_j/((\gamma\tau_{\epsilon_j})^2\tau_{u_j}\tau_{v_j}) \end{pmatrix},$$

and $\mathrm{Cov}[v_j, \{\boldsymbol{s}_k, p_{Rj}\}] = \begin{pmatrix} \tau_{v_j}^{-1} & \rho_v/\sqrt{\tau_{v_j}\tau_{v_i}} & \tau_{v_j}^{-1} \end{pmatrix}'$. Identifying equilibrium parameters: $\begin{pmatrix} a^*_{Rjj} & a^*_{Rji} \end{pmatrix} = (1/\mathrm{Var}[v_j|\boldsymbol{s}_k, p_{Rj}])\gamma\mathbf{e}'_{11}$, $b^*_{Rj} = -\gamma(\mathrm{Var}[v_j|\boldsymbol{s}_k, p_{Rj}])^{-1}$ $(1 -$

$e_{21}/(\lambda_{Rj}(\gamma\tau_{\epsilon_j})))$, and $c^*_{Rj} = -b^*_{Rj}$. Simplifying these expressions $a^*_{Rjj} = \gamma\tau_{\epsilon_j} - \rho_\epsilon a^*_{Rji}\sqrt{\tau_{\epsilon_j}/\tau_{\epsilon_i}}$, $a^*_{Rji} = \gamma((1-\rho_v^2)\tau_{\epsilon_i} + (1-\rho_\epsilon{}^2)\tau_{v_i})^{-1}(\rho_v\tau_{\epsilon_i}\sqrt{\tau_{v_i}\tau_{v_j}} - \rho_\epsilon\tau_{v_i}\sqrt{\tau_{\epsilon_i}\tau_{\epsilon_j}})$, $b^*_{Rj} = (\gamma\tau_{\epsilon_j} + a^*_{Rji}(\rho_v\sqrt{\tau_{v_j}/\tau_{v_i}} - \rho_\epsilon\sqrt{\tau_{\epsilon_j}/\tau_{\epsilon_i}}))$. A trader k that observes two signals (while the others only observe one) has an ex-ante utility $E[U(x_{Rkj}(v_j - p_{Rj})); a^*_{Rjj}, a^*_{Rji}, b^*_{Rj}] = E[-\exp\{-(1/\gamma)(E[x_{Rkj}(v_j - p_{Rj})|v_j, p_{Rj}] - (1/(2\gamma))\mathrm{Var}[x_{Rkj}(v_j - p_{Rj})|v_j, p_{Rj}])\}$, where $E[x_{Rkj}(v_j - p_{Rj})|v_j, p_{Rj}] = (a^*_{Rjj}(v_j - \bar{v}_j) + a^*_{Rji}(v_i - \bar{v}_i) + b^*_{Rj}(\bar{v}_j - p_{Rj}))(v_j - p_{Rj})$ and $\mathrm{Var}[x_{Rkj}(v_j - p_{Rj})|v_j, p_{Rj}] = \tau_{\epsilon_j}^{-1}(v_j - p_{Rj})^2((a^*_{Rjj})^2 + (a^*_{Rji})^2 + 2\rho_\epsilon a^*_{Rjj}a^*_{Rji})$. Then $E[x_{Rkj}(v_j - p_{Rj})|v_j, p_{Rj}] - (1/(2\gamma))\mathrm{Var}[x_{Rkj}(v_j - p_{Rj})|v_j, p_{Rj}] = (v_j - p_{Rj})^2 k_1 + (v_j - p_{Rj})k_2(\bar{v}_j - p_{Rj}) + (v_j - p_{Rj})k_3(v_i - \bar{v}_i)$ where $k_1 = (a^*_{Rjj} - (1/(2\gamma\tau_{\epsilon_j}))((a^*_{Rjj})^2 + (a^*_{Rji})^2 + 2\rho_\epsilon a^*_{Rjj}a^*_{Rji}))$, $k_2 = (b^*_{Rj} - a^*_{Rjj})$ and $k_3 = a^*_{Rji}$. Applying lemma 3, $E[U(x_{Rkj}(v_j - p_{Rj})); a^*_{Rjj}, a^*_{Rji}, b^*_{Rj}] = -|\mathbf{\Sigma}|^{-1/2}|(2/\gamma)\mathbf{\Omega} + \mathbf{\Sigma}^{-1}|^{-1/2}$, where

$$\mathbf{\Sigma} = \begin{pmatrix} \tau_j^{-1} & 0 & (\rho_v/\tau_j^{-1})\sqrt{\tau_{v_j}/\tau_{v_i}} \\ 0 & (\gamma\tau_{\epsilon_j}^2\tau_{u_j})/(\tau_j\tau_{v_j}) & -(\rho_v\tau_{u_j}(\gamma\tau_{\epsilon_j})^2)/(\tau_j\sqrt{\tau_{v_j}\tau_{v_i}}) \\ (\rho_v/\tau_j^{-1})\sqrt{\tau_{v_j}/\tau_{v_i}} & -(\rho_v\tau_{u_j}(\gamma\tau_{\epsilon_j})^2)/(\tau_j\sqrt{\tau_{v_j}\tau_{v_i}}) & \tau_{v_i}^{-1} \end{pmatrix},$$

and

$$\mathbf{\Omega} = \begin{pmatrix} k_1 & k_2/2 & k_3/2 \\ k_2/2 & 0 & 0 \\ k_3/2 & 0 & 0 \end{pmatrix}.$$

3. The value of multidimensional private information.

Let $\phi(s_{ki}||s_{kj}, p_{Rj})$ be the maximum price a trader in the restricted system is willing to pay to observe both private signals contemporaneously, $E[U(x_{Rkj}(v_j - p_{Rj}) - \phi(s_{ki}||s_{kj}, p_{Rj}))] = E[U(x_{Rkj}(v_j - p_{Rj})); a^*_{Rjj}, a^*_{Rji}, b^*_{Rj}]$. Solving for $\phi(s_{kj}||s_{ki}, p_{Rj})$ one obtains the expression in proposition 9. QED

Proof of proposition 10.

For part (1), given the definition of $\phi(s_{ki}||s_{kj}, \boldsymbol{p})$ and $\phi(s_{ki}||s_{kj}, p_{Rj})$, when $\rho_v = \rho_u = 0$, $\phi(s_{ki}||s_{kj}, \boldsymbol{p}) - \phi(s_{ki}||s_{kj}, p_{Rj}) > 0$ if and only if

$$\begin{aligned} &\rho_\epsilon{}^2\tau_{\epsilon_j}\big((2-\rho_\epsilon{}^2)\tau_{\epsilon_i}^2(\tau_{v_j} + \tau_{\epsilon_j}) + (3-\rho_\epsilon{}^2)\tau_{\epsilon_i}\tau_{\epsilon_j}\tau_i + \gamma^2\tau_{\epsilon_i}\tau_{\epsilon_j}(\tau_{\epsilon_i}\tau_{u_i}(\tau_i + \tau_{v_i}) + \\ &\quad \tau_{\epsilon_j}\tau_{u_j}(\tau_i + \tau_{\epsilon_i})) + \rho_\epsilon{}^4\tau_{\epsilon_i}\tau_{v_j}\tau_i + \tau_{\epsilon_j}\tau_{v_i}^2 + (1-\rho_\epsilon{}^2)\tau_{v_j}\tau_i(3\tau_{\epsilon_i} + \tau_i)\big) \times \\ &\quad \big((1-\rho_\epsilon{}^2)(\tau_i + \tau_{\epsilon_i})(\tau_{\epsilon_j} + \tau_{v_j}(1-\rho_\epsilon{}^2))(\tau_{\epsilon_i}\tau_{\epsilon_j}(1-\rho_\epsilon{}^2) + \\ &\quad \gamma^2\tau_{\epsilon_i}\tau_{\epsilon_j}(\tau_{\epsilon_i}\tau_{u_i} + \tau_{\epsilon_j}\tau_{u_j} + \gamma^2\tau_{\epsilon_i}\tau_{\epsilon_j}\tau_{u_i}\tau_{u_j}) + \tau_{v_i}(\tau_j + \tau_{\epsilon_j}) + \tau_{v_j}(\tau_{\epsilon_i} + \gamma^2\tau_{\epsilon_i}\tau_{u_i}))\big)^{-1} > 0. \end{aligned} \tag{7.15}$$

By inspection, the last inequality is always satisfied, and the result follows. Turning to trading aggressiveness, I will prove the result by contradiction. Suppose that for

$\rho_v = \rho_u = 0$, $|a_{ji}| \leq |a_{Rji}|$. First, let's show that it cannot be that $|a_{ji}| = |a_{Rji}|$, for if this was the case then, rewriting (7.12)

$$-\gamma\rho_\epsilon(\tau_{\epsilon_j}(\tau_{\epsilon_i}+\tau_{v_i})+\tau_{v_j}(\tau_{\epsilon_i}+\tau_{v_i}(1-{\rho_\epsilon}^2)))\frac{\tau_{\epsilon_j}\sqrt{\tau_{\epsilon_i}\tau_{\epsilon_j}}}{(1-{\rho_\epsilon}^2)^2}((1+\gamma^2\tau_{\epsilon_j}\tau_{u_j})(1-{\rho_\epsilon}^2)+\gamma^2{\rho_\epsilon}^2\tau_{\epsilon_j}). \tag{7.16}$$

The last equation is null if and only if $\rho_\epsilon = 0$, hence $|a_{ji}| \neq |a_{Rji}|$. Next, suppose $|a_{ji}| < |a_{Rji}|$ and choose w.l.o.g. $\rho_\epsilon > 0$ (i.e. $a_{Rji} < 0$). Hence, assume $a_{Rji} < -\gamma\rho_\epsilon\sqrt{\tau_{\epsilon_j}\tau_{\epsilon_i}}/(1-{\rho_\epsilon}^2)$. Substituting $-\gamma\rho_\epsilon\sqrt{\tau_{\epsilon_j}\tau_{\epsilon_i}}/(1-{\rho_\epsilon}^2)$ into (7.12) its sign should thus be positive. However, as shown by (7.16), when $\rho_\epsilon > 0$, this equation is always negative. A similar argument can be given in the case $\rho_\epsilon < 0$. Thus, $|a_{Rji}| > |a_{Rji}|$. Finally, let us show that $a_{jj} > a_{Rjj}$. Consider again $\rho_\epsilon > 0$. I have just shown that in this case $a_{Rji} > -\gamma\rho_\epsilon\sqrt{\tau_{\epsilon_j}\tau_{\epsilon_i}}/(1-{\rho_\epsilon}^2)$ or that $-a_{Rji} < \gamma\rho_\epsilon\sqrt{\tau_{\epsilon_j}\tau_{\epsilon_i}}/(1-{\rho_\epsilon}^2)$. Multiply both sides of the last inequality by $\rho_\epsilon\sqrt{\tau_{\epsilon_j}/\tau_{\epsilon_i}}$ and add $\gamma\tau_{\epsilon_j}$. Rearranging this gives $a_{Rjj} \equiv \gamma\tau_{\epsilon_j} - \rho_\epsilon a_{Rji}\sqrt{\tau_{\epsilon_j}/\tau_{\epsilon_i}} < \gamma\tau_{\epsilon_j}/(1-{\rho_\epsilon}^2)$. A similar argument can be given for $\rho_\epsilon < 0$. Hence, $a_{jj} > a_{Rjj}$ and the result follows.

For part (2), since for $\rho_\epsilon = \rho_u = 0$ and $\rho_v \neq 0$, $\phi(s_{ki}||s_{kj}, \boldsymbol{p}) = 0$ and $\phi(s_{ki}||s_{kj}, p_{Rj}) > 0$, the result follows immediately. Moreover, if $\rho_\epsilon = 0$ then $a_{ji} = 0$, while $|a_{Rji}| > 0$, hence $|a_{Rji}| > |a_{ji}|$, whereas $a_{Rjj} = a_{jj} = \gamma\tau_{\epsilon_j}$.

For part (3), if $\rho_\epsilon = \rho_v = 0$, then $\phi(s_{ki}||s_{kj}, \boldsymbol{p}) = \phi(s_{ki}||s_{kj}, p_{Rj}) = 0$ and trading intensities coincide across the two systems. QED

Before proving propositions 12 and 13, I need the following lemma

Lemma 4 *When the information structure is symmetric, price informativeness in the two systems is given by*

(a) *If* $\rho_v = \rho_\epsilon = 0$ $I_p = \tau_v^{-1} - (\tau_v^2(1-\rho_u^2) + \tau_u^2 a_1^4 + 2\tau_u\tau_v a_1^2)^{-1}(\tau_v(1-\rho_u^2) + a_1^2\tau_u)$, *and* $I_{p_R} = I_p$.

(b) *If* $\rho_u = \rho_\epsilon = 0$ $I_p = \tau_v^{-1} - (\tau_v^2 + a_1^4\tau_u^2(1-\rho_v^2) + 2a_1^2\tau_u\tau_v)^{-1}(\tau_v + a_1^2\tau_u(1-\rho_v^2))$, *and* $I_{p_R} = \tau_v^{-1} - (\tau_v^2 + \tau_u^2(1-\rho_v^2)(a_{R1}^2 - a_{R2}^2)^2 + 2\tau_u\tau_v(a_{R1}^2 + a_{R2}^2 + 2\rho_v a_{R1}a_{R2}))^{-1}$ $(\tau_v + (a_{R1}^2 + a_{R2}^2)\tau_u(1-\rho_v^2))$.

(c) *If* $\rho_u = \rho_v = 0$, $I_p = \tau_v^{-1} - (\tau_v^2 + \tau_u^2(a_1^2 - a_2^2)^2 + 2\tau_u\tau_v(a_1^2 + a_2^2))^{-1}$ $(\tau_v + (a_1^2 + a_2^2)\tau_u)$ *and* $I_{p_R} = \tau_v^{-1} - (\tau_v^2 + \tau_u^2(a_{R1}^2 - a_{R2}^2)^2 + 2\tau_u\tau_v(a_{R1}^2 + a_{R2}^2))^{-1}(\tau_v + (a_{R1}^2 + a_{R2}^2)\tau_u)$.

Proof. From the properties of the multivariate normal random variable, $\mathbf{\Pi} = \mathbf{\Pi}_{\boldsymbol{v}} + \mathbf{A}'\mathbf{\Pi}_{\boldsymbol{u}}\mathbf{A}$, and an analogous formula holds for the restricted system. Given this, the above expressions follow from matrix algebra. QED

Proof of proposition 12.

Suppose $\rho_\epsilon = \rho_u = 0$. Implicitly differentiating (7.14), one can see that I_{p_R} is convex in ρ_v and has a local minimum in $\rho_v = 0$. The same result can be obtained for I_p. Perform a second order expansion of I_{p_R} and I_p around $\rho_v = 0$ to get $I_{p_R}(\rho_v) = I_{p_R}(0)+(\rho_v^2/2)(\partial^2 I_{p_R}/\partial\rho_v^2)|_{\rho_v=0}+R_1(0)$ and $I_p = I_p(0)+(\rho_v^2/2)(\partial^2 I_p/\partial\rho_v^2)|_{\rho_v=0}+R_2(0)$, where $(\partial^2 I_{p_R}/\partial\rho_v^2)|_{\rho_v=0} = (2a_{R1}^2\tau_u\tau_v((\tau_\epsilon + a_{R1}^2\tau_u)^2 + 3\tau_v(\tau + 2\tau_\epsilon + \tau_v)))/(\tau^3(\tau+\tau_\epsilon)^2)$, $(\partial^2 I_p/\partial\rho_v^2)|_{\rho_v=0} = (2a_1^2\tau_u\tau_v)/\tau^3$ and $\tau = \tau_v + a_1^2\tau_u$, $a_1 = \gamma\tau_\epsilon$. As for $\rho_v = 0$, $\mathbf{A} = \mathbf{A}_R$ then $I_{p_R}(0) = I_p(0)$ and $I_{p_R} - I_p = (\rho_v^2/2)(\tau^3(\tau+\tau_\epsilon)^2)^{-1}(2a_1^2\tau_u\tau_v^2(4\tau_\epsilon + a_1^2\tau_u + 5\tau_v))$, which is always positive. QED

Proof of proposition 13

Suppose $\rho_v = \rho_u = 0$. Implicitly differentiating (7.14), one can see that I_{p_R} is convex in ρ_ϵ and has a local minimum in $\rho_\epsilon = 0$. As for I_p, it has a stationary point in $\rho_\epsilon = 0$. Perform a second order expansion of I_{p_R} and I_p around $\rho_\epsilon = 0$ to get $I_{p_R}(\rho_v) = I_{p_R}(0)+(\rho_\epsilon^2/2)(\partial^2 I_{p_R}/\partial\rho_\epsilon^2)|_{\rho_\epsilon=0}+R_1(0)$ and $I_p = I_p(0)+(\rho_\epsilon^2/2)(\partial^2 I_p/\partial\rho_\epsilon^2)|_{\rho_\epsilon=0}+R_2(0)$, where $(\partial^2 I_{p_R}/\partial\rho_\epsilon^2)|_{\rho_\epsilon=0} = (\tau^3+(\tau+\tau_\epsilon)^2)^{-1}\ (2\gamma^2\tau_\epsilon^2\tau_u\tau_v\ (\tau_\epsilon\tau(2+\gamma^2\tau_\epsilon\tau_u)+\gamma^4\tau_\epsilon^4\tau_u^2+3\tau_v^2))$, $(\partial^2 I_p/\partial\rho_\epsilon^2)|_{\rho_\epsilon=0} = -(2\gamma^2\tau_\epsilon^2\tau_u(\gamma^2\tau_\epsilon^2\tau_u - 3\tau_v))/\tau^3$ and $\tau = \tau_v + a_1^2\tau_u$, $a_1 = \gamma\tau_\epsilon$. As for $\rho_\epsilon = 0$, $\mathbf{A} = \mathbf{A}_R$ then $I_{p_R}(0) = I_p(0)$ and $I_{p_R} - I_p \geq 0$ if and only if $F(\tau_\epsilon) \equiv \gamma^4\tau_\epsilon^4\tau_u^2 + \gamma^2\tau_\epsilon^3\tau_u + \gamma^2\tau_\epsilon^2\tau_u\tau_v - 3\tau_\epsilon\tau_v - 4\tau_v^2 \geq 0$. As $F(\tau_\epsilon) > \gamma^2\tau_\epsilon^2\tau_u\tau_v - 3\tau_\epsilon\tau_v - 4\tau_v^2$ and $\gamma^2\tau_\epsilon^2\tau_u\tau_v - 3\tau_\epsilon\tau_v - 4\tau_v^2 \geq 0$ if and only if $\tau_\epsilon \geq (3+\sqrt{9+16\gamma^2\tau_u\tau_v})/2\gamma^2\tau_u$, the result follows. QED

Proof of proposition 15.

Simplifying the formulas in the text, $E[U(\pi_k)] = -((\tau_1\tau_2 - \rho_u^2\tau_{v_1}\tau_{v_2})/((\tau_1+\tau_{\epsilon_1})(\tau_2+\tau_{\epsilon_2}) - \rho_u^2(\tau_{v_1}+\tau_{\epsilon_1})(\tau_{v_2}+\tau_{\epsilon_2})))^{1/2}$, and $E[U(\pi_{Rk})] = -((\tau_1^2\tau_2^2)/(\tau_1\tau_2(\tau_1+\tau_{\epsilon_1})(\tau_2+\tau_{\epsilon_2}) - \gamma^4\rho_u^2\tau_{\epsilon_1}^3\tau_{\epsilon_2}^3\tau_{u_1}\tau_{u_2})^{1/2}$, where $\tau_j = \tau_{v_j} + a_{jj}^2\tau_{u_j}$. Therefore, $E[U(\pi_{Rk})] > E[U(\pi_k)]$ if and only if $\tau_1\tau_2(\tau_{v_2}\tau_{u_1}\tau_{\epsilon_1}^3(\tau_2+\tau_{\epsilon_2}) + \tau_{v_1}\tau_{u_2}\tau_{\epsilon_2}^3(\tau_1+\tau_{\epsilon_1})) + \rho_u^2\tau_{v_1}\tau_{v_2}\tau_{\epsilon_1}^3\tau_{\epsilon_2}^3\tau_{u_1}\tau_{u_2} > 0$. The last condition is always satisfied and the result holds. As for noise traders, a standard argument of multi variate statistics shows that their expected losses are higher in the unrestricted system if and only if $\mathrm{tr}(\mathbf{\Lambda}^R - \mathbf{\Lambda})\mathbf{\Pi}_{\boldsymbol{u}}^{-1} \leq 0$. As $E[\boldsymbol{u}'(\boldsymbol{v} - \boldsymbol{p})] = \gamma((\tau_{\epsilon_2}\tau_1 + \tau_{\epsilon_1}\tau_2 - \rho_u^2(\tau_{\epsilon_2}\tau_{v_1} + \tau_{\epsilon_1}\tau_{v_2}))/(\tau_1\tau_2 - \rho_u^2\tau_{v_1}\tau_{v_2}))$, while $E[\boldsymbol{u}'(\boldsymbol{v} - \boldsymbol{p}_R)] = \gamma((\tau_{\epsilon_1}\tau_2 + \tau_{\epsilon_2}\tau_1)/(\tau_1\tau_2))$, the result follows. QED

Proof of proposition 16

Suppose $\rho_\epsilon = \rho_u = 0$. Perform a second order Taylor expansion of $\lambda_R(\rho_v)$ around $\rho_v = 0$, then $\lambda_R(\rho_v) = \lambda_R(0)+(\rho_v^2/2)(\partial^2\lambda_R/\partial\rho_v^2)|_{\rho_v=0}+R_1(0)$, where $(\partial^2\lambda_R/\partial\rho_v^2)|_{\rho_v=0} =$

$-(\tau^2(\tau+\tau_\epsilon)^2)^{-1}(2\gamma\tau_\epsilon\tau_u\tau_v\ (a_1^4\tau_u^2+a_1^2\tau_u(\tau_\epsilon+\tau_v)-\tau_v(\tau_\epsilon+\tau_v))$. In the same way, for the unrestricted system $\lambda(\rho_v)=\lambda(0)+(\rho_v^2/2)(\partial^2\lambda/\partial\rho_v^2)|_{\rho_v=0}+R_2(0)$, where $(\partial^2\lambda/\partial\rho_v^2)|_{\rho_v=0}=-\tau^{-3}2a_1^3\tau_u^2\tau_v$. As $\lambda_R(0)=\lambda(0)$, $\lambda_R(\rho_v)-\lambda(\rho_v)=(\rho_v/2)^2(\tau^3(\tau+\tau_\epsilon)^2)^{-1}\ (2\gamma\tau_\epsilon\tau_u\tau_v\ (a_1^4\tau_u^2\tau_\epsilon+\tau_v^2(\tau_\epsilon+\tau_v)\ +a_1^2\tau_u(\tau_\epsilon+\tau_v)^2))>0$. QED

Proof of proposition 17

Suppose $\rho_v=\rho_u=0$. Perform a second order Taylor expansion of $\lambda_R(\rho_\epsilon)$ around $\rho_\epsilon=0$, then $\lambda_R(\rho_\epsilon)=\lambda_R(0)+(\rho_\epsilon^2/2)(\partial^2\lambda_R/\partial\rho_\epsilon^2)|_{\rho_\epsilon=0}+R_1(0)$, where $(\partial^2\lambda_R/\partial\rho_\epsilon^2)|_{\rho_\epsilon=0}=-(\tau^2(\tau+\tau_\epsilon)^2)^{-1}\ (2a_1\tau_u\tau_v(a_1^2\tau_u(\tau+\tau_\epsilon)\ -\tau_v(\tau_\epsilon+\tau_v)))$. In the same way, for the unrestricted system $\lambda(\rho_\epsilon)=\lambda(0)+(\rho_\epsilon^2/2)(\partial^2\lambda/\partial\rho_\epsilon^2)|_{\rho_\epsilon=0}+R_2(0)$, where $(\partial^2\lambda/\partial\rho_\epsilon^2)|_{\rho_\epsilon=0}=-2a_1\tau_u\tau_v(\tau_v-3a_1^2\tau_u)/\tau^3$. As $\lambda_R(0)=\lambda(0)$, $\lambda_R(\rho_\epsilon)-\lambda(\rho_\epsilon)=(\tau^3(\tau+\tau_\epsilon)^2)^{-1}(2\gamma^6\tau_\epsilon^5\tau_u^3+5\gamma^4\tau_\epsilon^4\tau_u^2\ +4\gamma^2\tau_\epsilon^2\tau_u\tau_v-\tau_v^2\ +3\gamma^2\tau_\epsilon^3(\tau_u+\gamma^2\tau_u^2\tau_v)+\tau_\epsilon(\gamma^2\tau_u\tau_v^2-\tau_v))$. The last equation is a quintic in τ_ϵ, therefore it always has at least one real solution. The result follows. QED

Proof of proposition 18.

The proof follows from 2 claims.

Claim 1 *In every linear equilibrium of the intermediate system, informed strategies are as in proposition 18.*

Proof. In this model market makers observe each order flow, therefore, $\boldsymbol{p}_I=E[\boldsymbol{v}|\boldsymbol{z}_I]$ $=\boldsymbol{\Lambda}_I\boldsymbol{z}_I+(I-\boldsymbol{\Lambda}_I\mathbf{A}_I)\bar{\boldsymbol{v}}$, $\boldsymbol{z}^I=\mathbf{A}_I\boldsymbol{v}+\boldsymbol{u}$. Notice that this implies that each price p_{Ij} is informationally equivalent to the linear combination $\boldsymbol{j}'\boldsymbol{\Lambda}_I\boldsymbol{z}_I$, where $\boldsymbol{\Lambda}_I=(\boldsymbol{\Pi}_I)^{-1}(\mathbf{A}_I)'\boldsymbol{\Pi}_{\boldsymbol{u}}$ is the matrix of market depth in the intermediate model. CARA preferences and normality of the distributions imply $X_{Ikj}=\gamma(\mathrm{Var}[v_j|p_{Ij},\boldsymbol{s}_k])^{-1}\times(E[v_j|p_{Ij},\boldsymbol{s}_k]\ -p_{Ij})$. Standard normal calculations give

$$\mathrm{Var}[p_{Ij},\boldsymbol{s}_k]=\begin{pmatrix}\boldsymbol{j}'\boldsymbol{\Lambda}_I\left(\mathbf{A}_I\boldsymbol{\Pi}_{\boldsymbol{v}}^{-1}(\mathbf{A}_I)'+\boldsymbol{\Pi}_{\boldsymbol{u}}^{-1}\right)(\boldsymbol{\Lambda}_I)'\boldsymbol{j} & \boldsymbol{j}'\boldsymbol{\Lambda}_I\mathbf{A}_I\boldsymbol{\Pi}_{\boldsymbol{v}}^{-1}\\ \\ \left(\boldsymbol{j}'\boldsymbol{\Lambda}_I\mathbf{A}_I\boldsymbol{\Pi}_{\boldsymbol{v}}^{-1}\right)' & \boldsymbol{\Pi}_{\boldsymbol{v}}^{-1}+\boldsymbol{\Pi}_{\boldsymbol{\epsilon}}^{-1}\end{pmatrix}, \tag{7.17}$$

and $\mathrm{Cov}[v_j,\{p_{Ij},\boldsymbol{s}_k\}]=(\ (\boldsymbol{j}'\boldsymbol{\Lambda}_I\mathbf{A}_I\boldsymbol{\Pi}_{\boldsymbol{v}}^{-1}\boldsymbol{j})'\quad(\boldsymbol{j}'\boldsymbol{\Pi}_{\boldsymbol{v}}^{-1})'\ )'$. Step-wise inversion of (7.17) gives

$$(\mathrm{Var}[p_{Ij},\boldsymbol{s}_k])^{-1}=\begin{pmatrix}D_1^{-1} & -D_1^{-1}\boldsymbol{j}'\boldsymbol{\Lambda}_I\mathbf{A}_I(\boldsymbol{\Pi}_{\boldsymbol{v}}+\boldsymbol{\Pi}_{\boldsymbol{\epsilon}})^{-1}\boldsymbol{\Pi}_{\boldsymbol{\epsilon}}\\ \\ -D_1^{-1}\left(\boldsymbol{j}'\boldsymbol{\Lambda}_I\mathbf{A}_I(\boldsymbol{\Pi}_{\boldsymbol{v}}+\boldsymbol{\Pi}_{\boldsymbol{\epsilon}})^{-1}\boldsymbol{\Pi}_{\boldsymbol{\epsilon}}\right)' & D_2\end{pmatrix},$$

where $D_1 = \boldsymbol{j}'\mathbf{\Lambda}_I(\mathbf{A}_I(\mathbf{\Pi}_{\boldsymbol{v}} + \mathbf{\Pi}_{\boldsymbol{\epsilon}})^{-1}(\mathbf{A}_I)' + \mathbf{\Pi}_{\boldsymbol{u}}^{-1})(\mathbf{\Lambda}_I)'\boldsymbol{j}$, and $D_2 = \mathbf{\Pi}_{\boldsymbol{v}}(\mathbf{\Pi}_{\boldsymbol{v}} + \mathbf{\Pi}_{\boldsymbol{\epsilon}})^{-1}\mathbf{\Pi}_{\boldsymbol{\epsilon}} + D_1^{-1}(\mathbf{\Pi}_{\boldsymbol{v}}^{-1} + \mathbf{\Pi}_{\boldsymbol{\epsilon}}^{-1})^{-1}\mathbf{\Pi}_{\boldsymbol{v}}^{-1}(\mathbf{A}_I)'(\mathbf{\Lambda}_I)'\boldsymbol{j}\boldsymbol{j}'\mathbf{\Lambda}_I\mathbf{A}_I\mathbf{\Pi}_{\boldsymbol{v}}^{-1}(\mathbf{\Pi}_{\boldsymbol{v}}^{-1} + \mathbf{\Pi}_{\boldsymbol{\epsilon}}^{-1})^{-1}$. Using the previous matrix and the formula $\mathrm{Var}[v_j|p_{Ij}, \boldsymbol{s}_k] = \boldsymbol{j}'\mathbf{\Pi}_{\boldsymbol{v}}^{-1}\boldsymbol{j} - (\mathrm{Cov}[v_j, \{\boldsymbol{s}_k, p_{Ij}\}])'(\mathrm{Var}[p_{Ij}, \boldsymbol{s}_k])^{-1} \times \mathrm{Cov}[v_j, \{\boldsymbol{s}_k, p_{Ij}\}]$, I obtain $\mathrm{Var}[v_j|p_{Ij}, \boldsymbol{s}_k] = \boldsymbol{j}'(I - \mathbf{\Lambda}_I\mathbf{A}_I c_{1j})(\mathbf{\Pi}_{\boldsymbol{v}} + \mathbf{\Pi}_{\boldsymbol{\epsilon}})^{-1}\boldsymbol{j}$. Finally, indicating with c_{1j} and $\mathbf{c}_{2j}$ the *weights* that a trader k puts on p_{Ij} and $\boldsymbol{s}_k$ in her estimation of v_j, standard normal computations give: $c_{1j} = (D_1)^{-1}(\boldsymbol{j}'\mathbf{\Lambda}_I\mathbf{A}_I(\mathbf{\Pi}_{\boldsymbol{v}} + \mathbf{\Pi}_{\boldsymbol{\epsilon}})^{-1}\boldsymbol{j})$ and $\mathbf{c}_{2j} = \boldsymbol{j}'(I - c_{1j}\mathbf{\Lambda}_I\mathbf{A}_I)\ (\mathbf{\Pi}_{\boldsymbol{v}} + \mathbf{\Pi}_{\boldsymbol{\epsilon}})^{-1}\mathbf{\Pi}_{\boldsymbol{\epsilon}}$.

QED

Claim 2 *A linear equilibrium of the symmetric model exists.*

Proof. As done in the restricted system, to compute the equilibrium in the symmetric case of the intermediate system, rewrite (5.7) as follows:

$$a_{I1} = \gamma\tau_\epsilon - \rho_\epsilon a_{I2}, \quad a_{I2} = (\gamma\tau_\epsilon/(1-\rho_\epsilon^2))((h_{21}/h_{22}) - \rho_\epsilon), \tag{7.18}$$

where $h_{21} = ((I - \mathbf{\Lambda}_I\mathbf{A}_I c_{12})\ (\mathbf{\Pi}_{\boldsymbol{v}} + \mathbf{\Pi}_{\boldsymbol{\epsilon}})^{-1})_{2,1}$ and $h_{22} = ((I - \mathbf{\Lambda}_I\mathbf{A}_I c_{12})(\mathbf{\Pi}_{\boldsymbol{v}} + \mathbf{\Pi}_{\boldsymbol{\epsilon}})^{-1})_{2,2}$. The second equation in (7.18) is a polynomial $\varphi(a_{I1}, a_{I2}; \gamma, \tau_\epsilon, \tau_v, \tau_u, \rho_\epsilon, \rho_v, \rho_u)$. Taking limits, $\lim_{a_{I2}\to\infty} \varphi(a_{I1}, a_{I2}; \cdot) = -\infty$ and $\lim_{a_{I2}\to-\infty} \varphi(a_{I1}, a_{I2}; \cdot) = \infty$. Therefore, for an initial guess of the trading intensities, I can numerically compute the fixed point for each set of parameters.

QED

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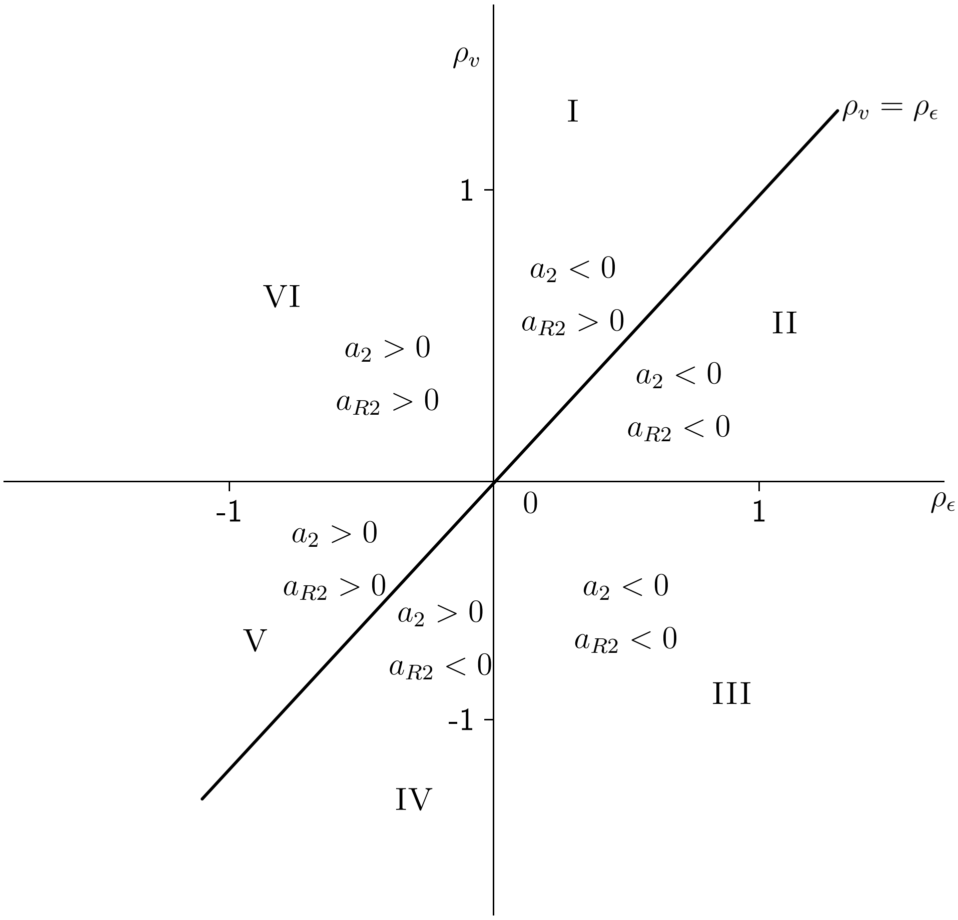
Figure 1: Comparing trading intensities across the restricted and the unrestricted system.



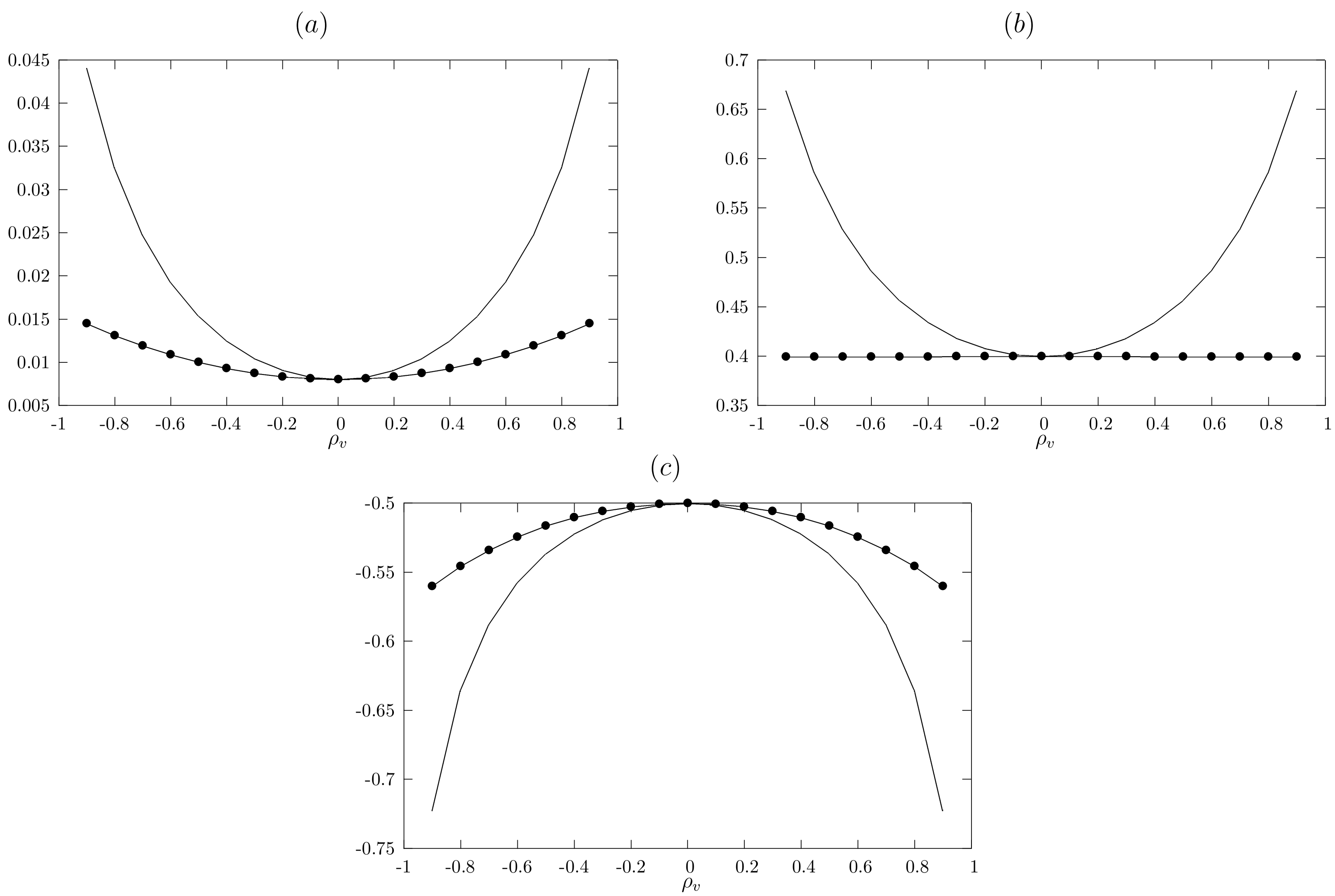


Figure 2: Price informativeness (panel (a)), noise traders' expected losses (panel (b)) and speculators' expected utility (panel (c)) in the restricted and the unrestricted symmetric systems (respectively, continuous and bulleted line) as a function of ρ_v when $\gamma = \tau_u = \tau_v = \tau_\epsilon = 0.2$ and $\rho_\epsilon = \rho_u = 0$.

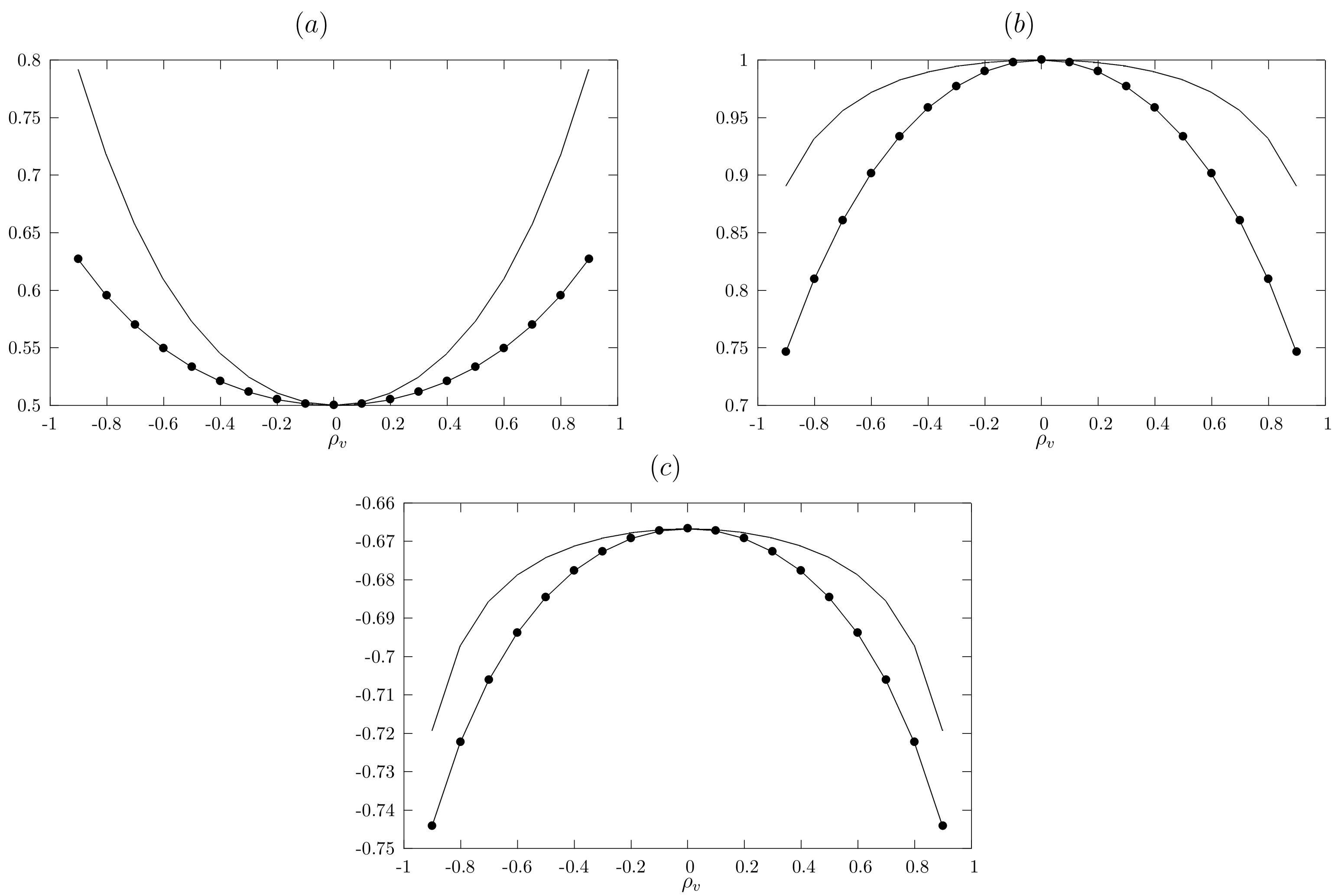


Figure 3: Price informativeness (panel (a)), noise traders' expected losses (panel (b)) and speculators' expected utility (panel (c)) in the restricted and the unrestricted symmetric systems (respectively, continuous and bulleted line) as a function of ρ_v when $\gamma = \tau_u = \tau_v = \tau_\epsilon = 1$ and $\rho_\epsilon = \rho_u = 0$.

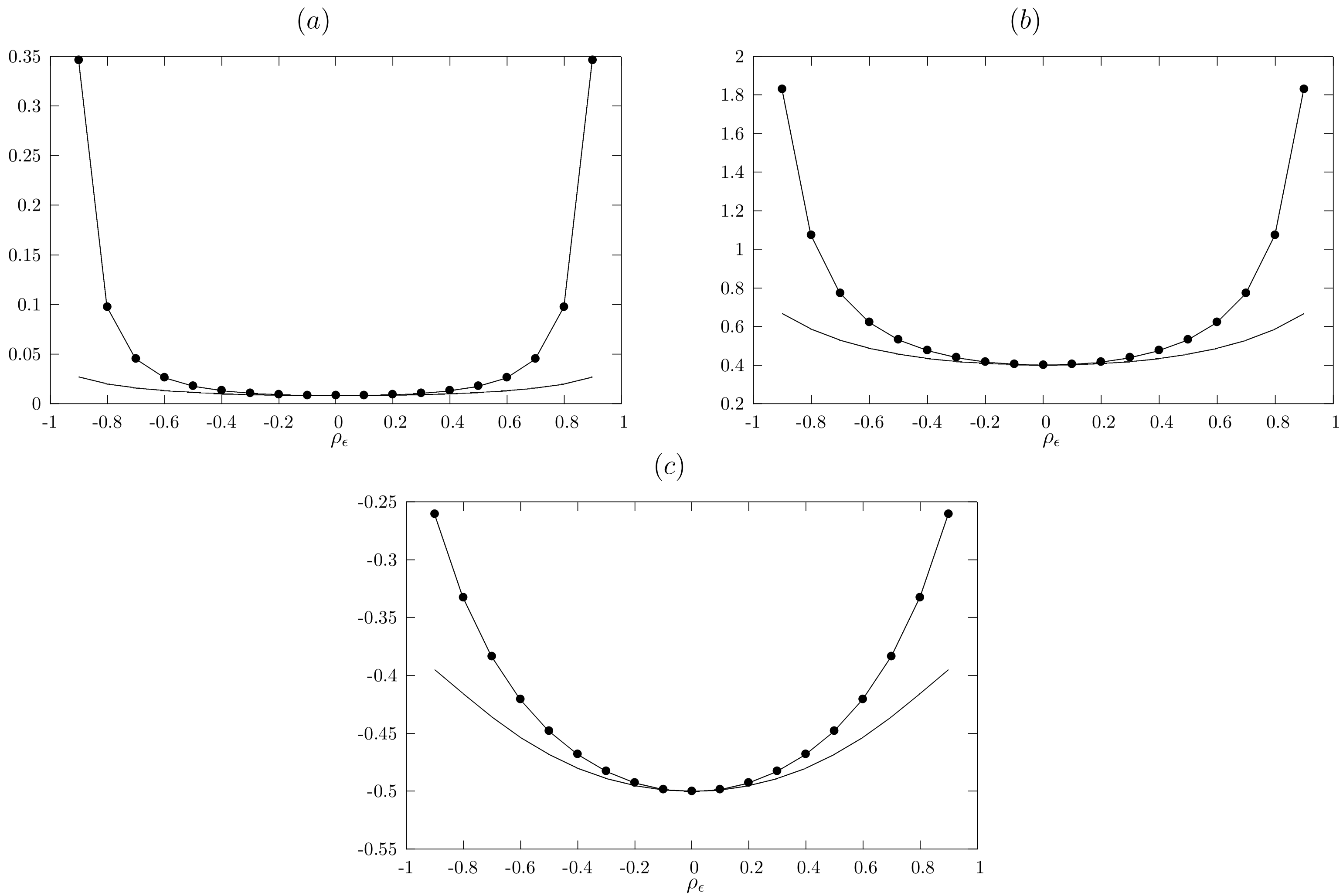


Figure 4: Price informativeness (panel (a)), noise traders' expected losses (panel (b)) and speculators' expected utility (panel (c)) in the restricted and the unrestricted symmetric systems (respectively, continuous and bulleted line) as a function of ρ_ϵ when $\gamma = \tau_u = \tau_v = \tau_\epsilon = 0.2$ and $\rho_v = \rho_u = 0$.

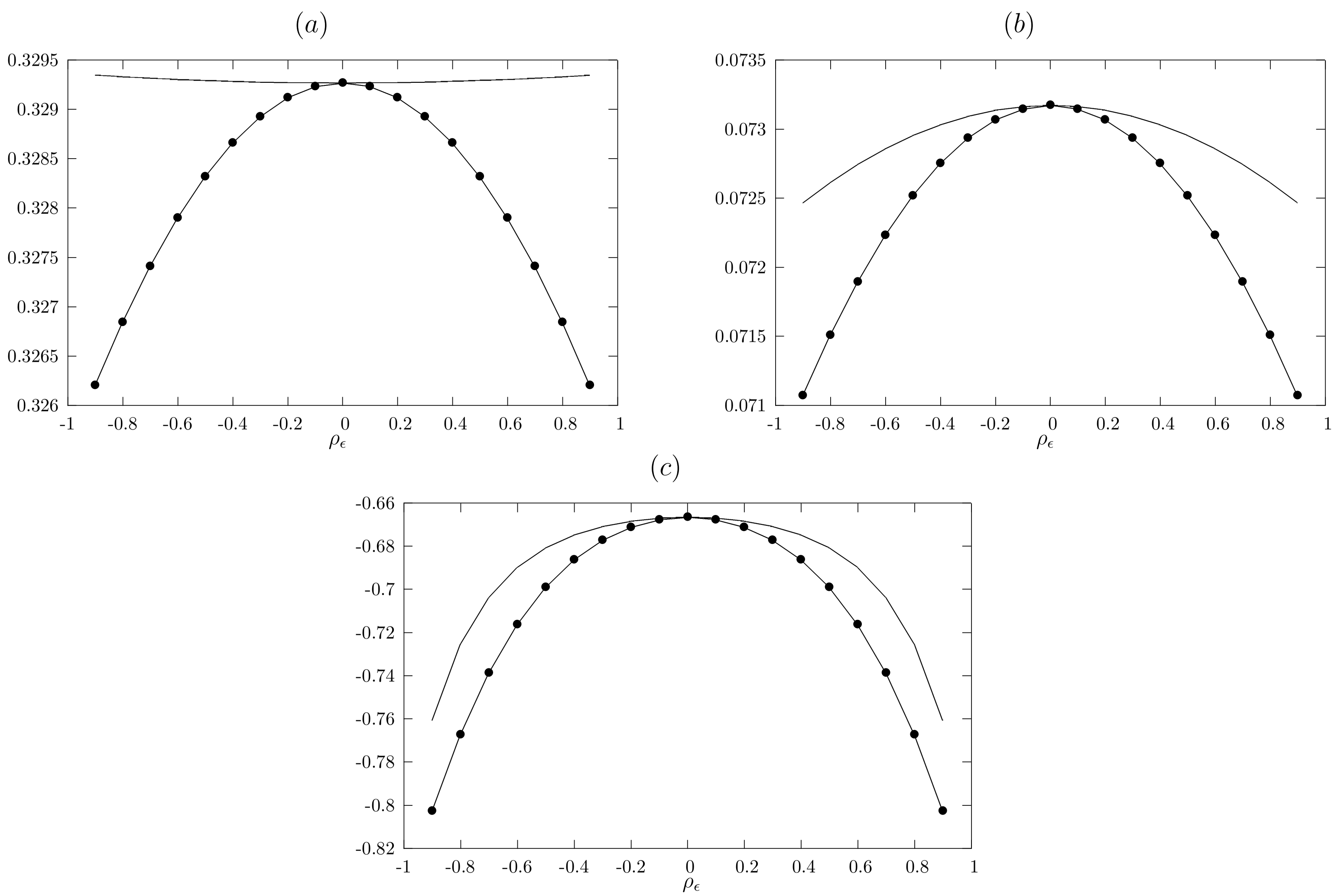


Figure 5: Price informativeness (panel (a)), noise traders' expected losses (panel (b)) and speculators' expected utility (panel (c)) in the restricted and the unrestricted symmetric systems (respectively, continuous and bulleted line) as a function of ρ_ϵ when $\gamma = \tau_u = \tau_v = \tau_\epsilon = 3$ and $\rho_v = \rho_u = 0$.

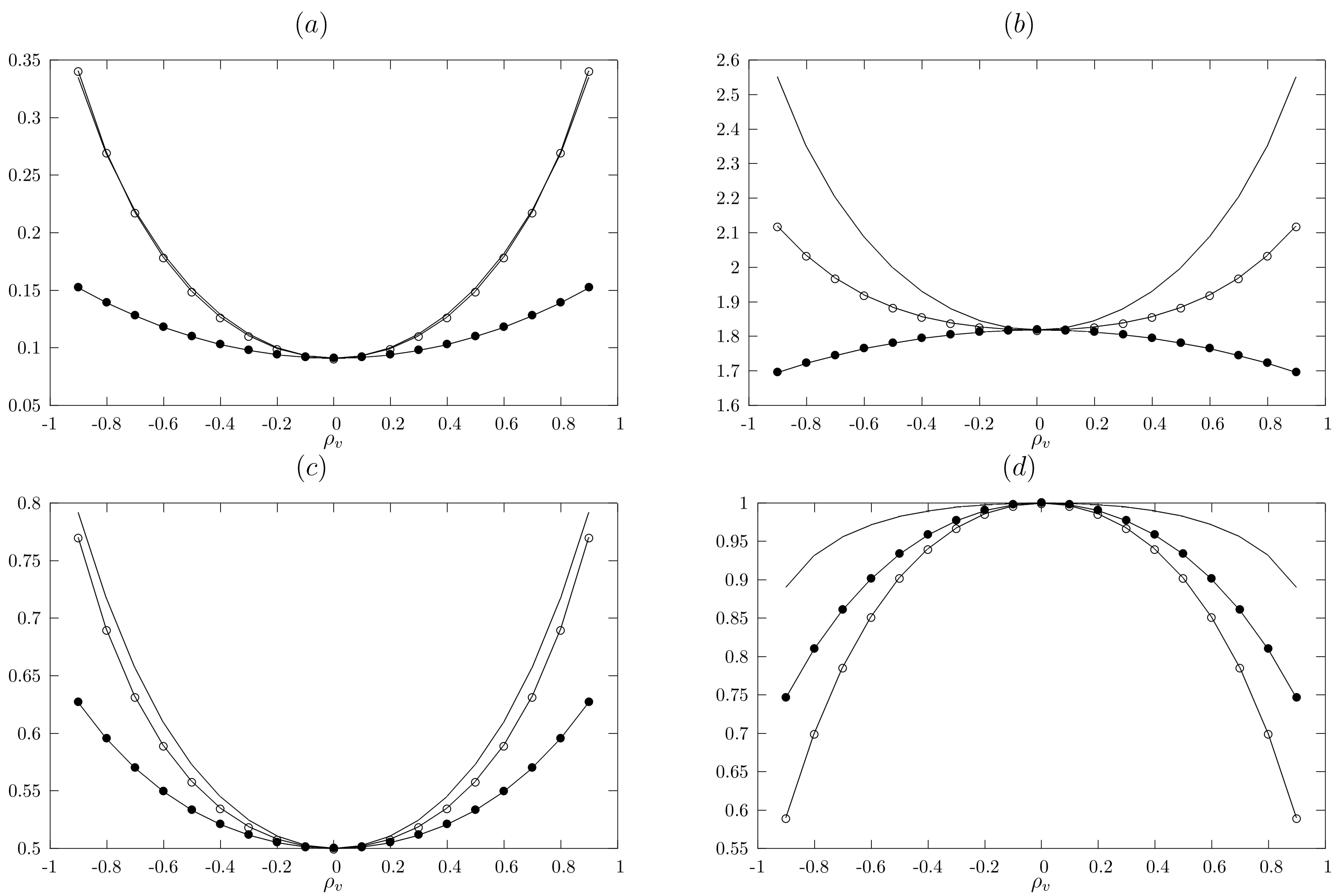


Figure 6: Price informativeness and noise traders' expected losses in the restricted, intermediate and unrestricted symmetric systems (respectively, continuous, circled, and bulleted line) as a function of ρ_v when $\gamma = \tau_v = \tau_\epsilon = 1$ and $\tau_u = 0.1$ (panel (a) and (b)) and when $\gamma = \tau_v = \tau_\epsilon = \tau_u = 1$ (panel (c) and (d)), for $\rho_\epsilon = \rho_u = 0$.

Table 1: Summary of the comparison results.

	$\rho_v = \rho_\epsilon = 0,\ \rho_u \neq 0$	$\rho_u = \rho_\epsilon = 0,\ \rho_v \neq 0$	$\rho_u = \rho_v = 0,\ \rho_\epsilon \neq 0$
$a_{jj} - a_{Rjj}$	$= 0^{a}$	$= 0^{a}$	$> 0^{a}$
$\lvert a_{ji}\rvert - \lvert a_{Rji}\rvert$	$= 0^{a}$	$< 0^{a}$	$> 0^{a}$
$\phi(s_{ki}\|\|s_{kj}, \boldsymbol{p}) - \phi(s_{ki}\|\|s_{kj}, p_{Rj})$	$= 0^{a}$	$< 0^{a}$	$> 0^{a}$
$I_{p_j} - I_{p_{Rj}}$	$= 0^{a}$	$< 0^{b}$	*depends*[c]
$-E[\boldsymbol{u}'(\boldsymbol{v} - \boldsymbol{p})] - (-E[\boldsymbol{u}'(\boldsymbol{v} - \boldsymbol{p}_R)])$	$> 0^{a}$	$> 0^{b}$	*depends*[c]
$E[U(\pi_k)] - E[U(\pi_{Rk})]$	$< 0^{a}$	*depends*[d]	*depends*[d]

[a] Result obtained in the general model.
[b] Result obtained for ρ_v *small*, in the symmetric model and with numerical simulations.
[c] Result obtained for ρ_ϵ *small*, in the symmetric model and with numerical simulations.
[d] Result obtained in the symmetric model with numerical simulations.